I worked with David Vartanyan (my partner) and John Pharo on this worksheet. It took about five hours total time.

1. An Unstable Calculation

n	Closed-Form Value	Recursive Value	Absolute Error	Relative Error
0	1.0	1.0	0.0	0.0
1	0.333333333333	0.333333	9.93410748107e-09	2.98023224432e-08
2	0.111111111111	0.111111	5.29819064732e-08	4.76837158259e-07
3	0.037037037037	0.0370373	2.16342784749e-07	5.84125518823e-06
4	0.0123456790123	0.0123466	8.71809912319e-07	7.06166028978e-05
5	0.00411522633745	0.00411871	3.48814414362e-06	0.0008476190269
6	0.00137174211248	0.00138569	1.39522571909e-05	0.0101711954921
7	0.000457247370828	0.000513056	5.58089223023e-05	0.122054113075
8	0.000152415790276	0.000375651	0.000223235614917	1.46464886947
9	5.08052634253e-05	0.000943748	0.000892942551319	17.5757882376
10	1.69350878084e-05	0.00358871	0.00357177023583	210.909460655
11	5.64502926948e-06	0.0142927	0.0142870812639	2530.91358466
12	1.88167642316e-06	0.0571502	0.0571483257835	30370.9634027
13	6.27225474386e-07	0.228594	0.228593318278	364451.584977
14	2.09075158129e-07	0.914374	0.914373307961	4373419.18641
15	**6.96917193763e-08**	**3.65749**	**3.6574932832**	**52481030.9737**

Table 1: Closed-form values and recursive values for the expression $\left(\frac{1}{3}\right)^n$ as well as errors of the recursive value compared to the closed-form value.

2. Finite Differnce Approximation and Convergence

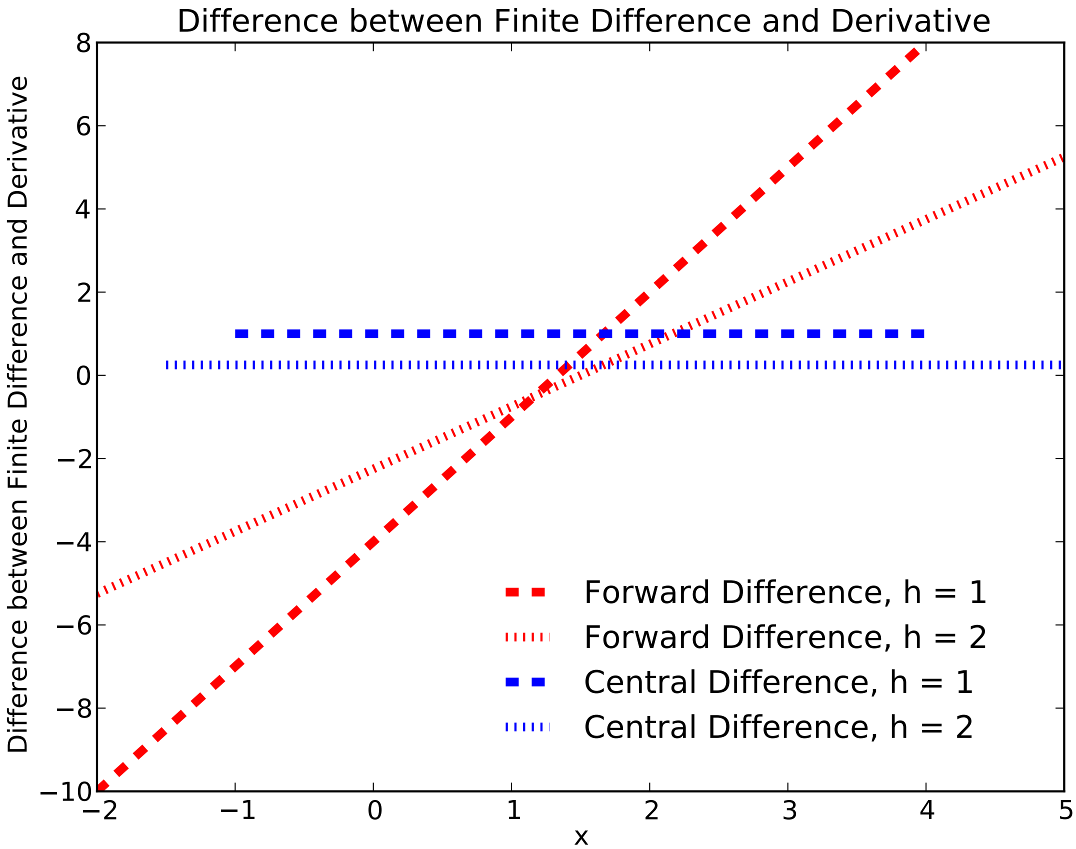

Figure 1: A plot of $f'(x;h) - f'(x)$, or the difference between the finite difference of $f(x) = x^3 - 5x^2 + x$ and the derivative of that function, on the interval $[-2, 6]$ for both forward differencing and central differencing, each at two values of the step-size h.

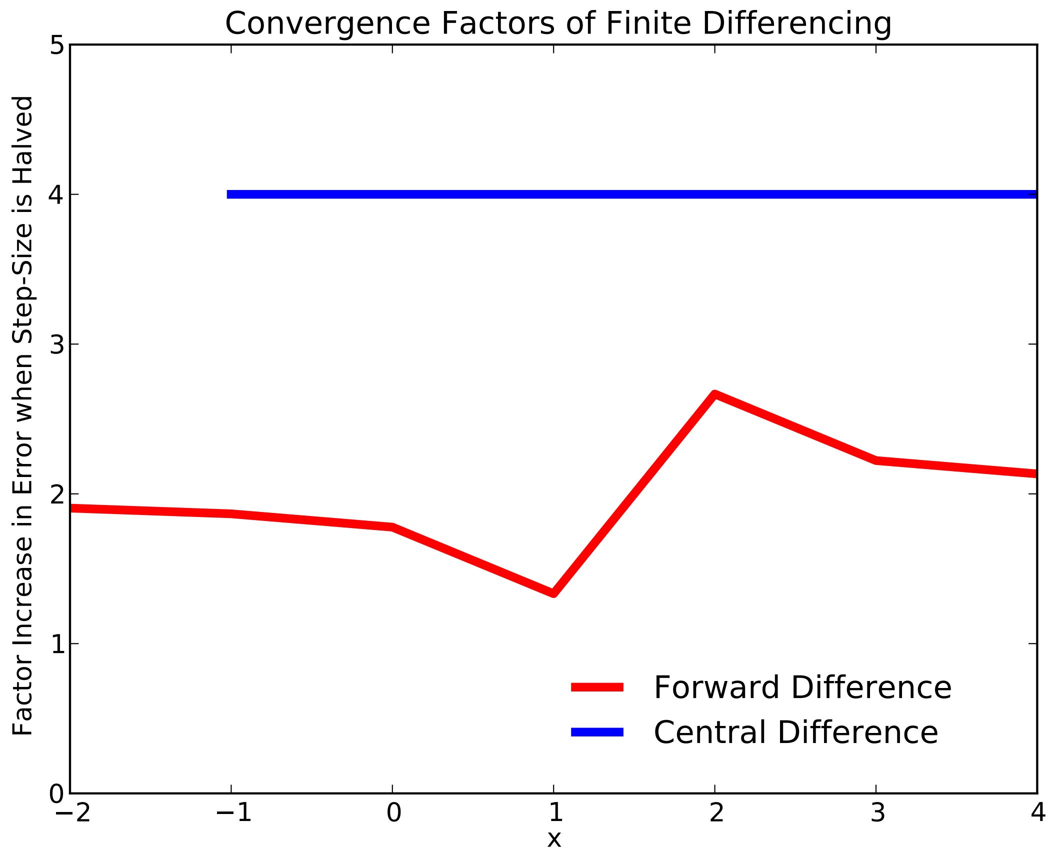

Figure 2: A plot of the ratios of the differencing errors between $h_1 = 1$ and $h_2 = 2$, for both forward and central differencing. We expect these ratios to be equal to the convergence factor $\left(\frac{h_2}{h_1}\right)^n = 2^n$, where n is the convergence order of the finite differencing: $n = 1$ for forward differencing and $n = 2$ for central differencing. Indeed, we see that the ratio is approximately 2 for forward differencing and 4 for central differencing as expected.

3. Second Derivative

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{6}f'''(x) + \mathcal{O}(h^4)$$

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{6}f'''(x) + \mathcal{O}(h^4)$$

$$\Rightarrow f(x+h) + f(x-h) = 2f(x) + h^2 f''(x) + \mathcal{O}(h^4)$$

$$\boxed{f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} + \mathcal{O}(h^2)} \tag{1}$$

4. Interpolation: Cepheid Lightcurve

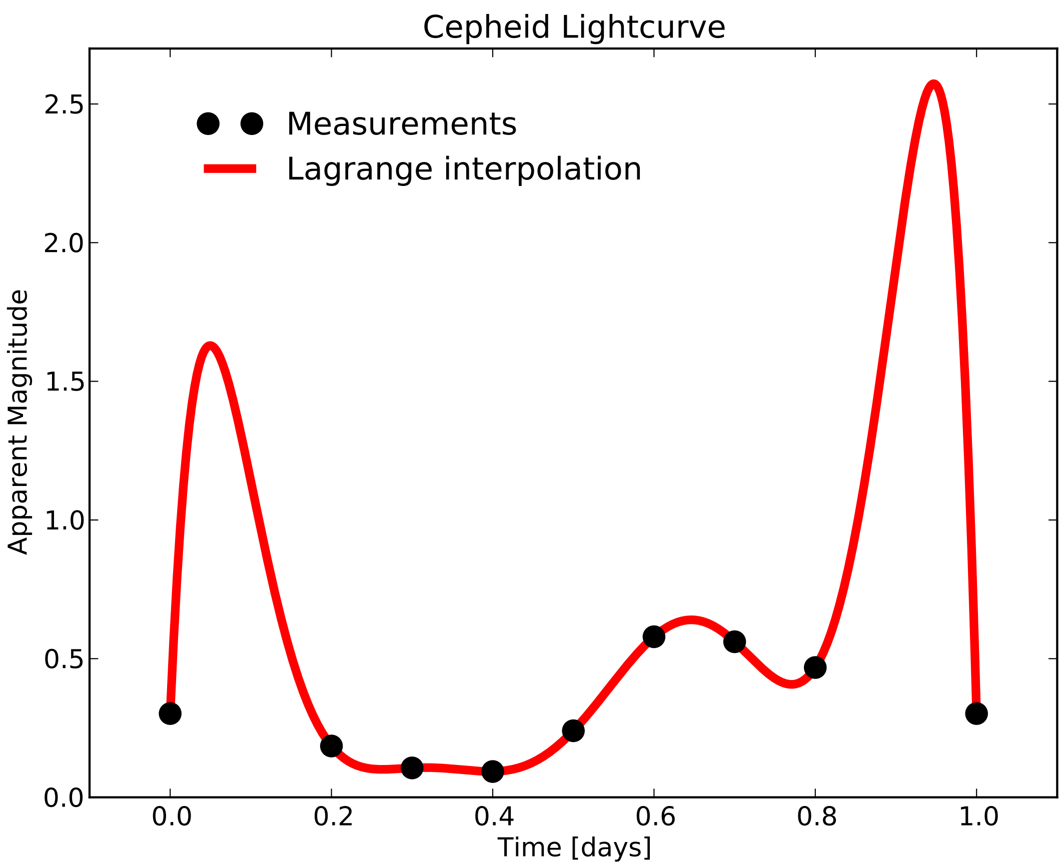

Figure 3: Apparent magnitude versus time for a Cepheid star. Actual measurements are represented as black points. Drawn in red is a Lagrange interpolation through the data.

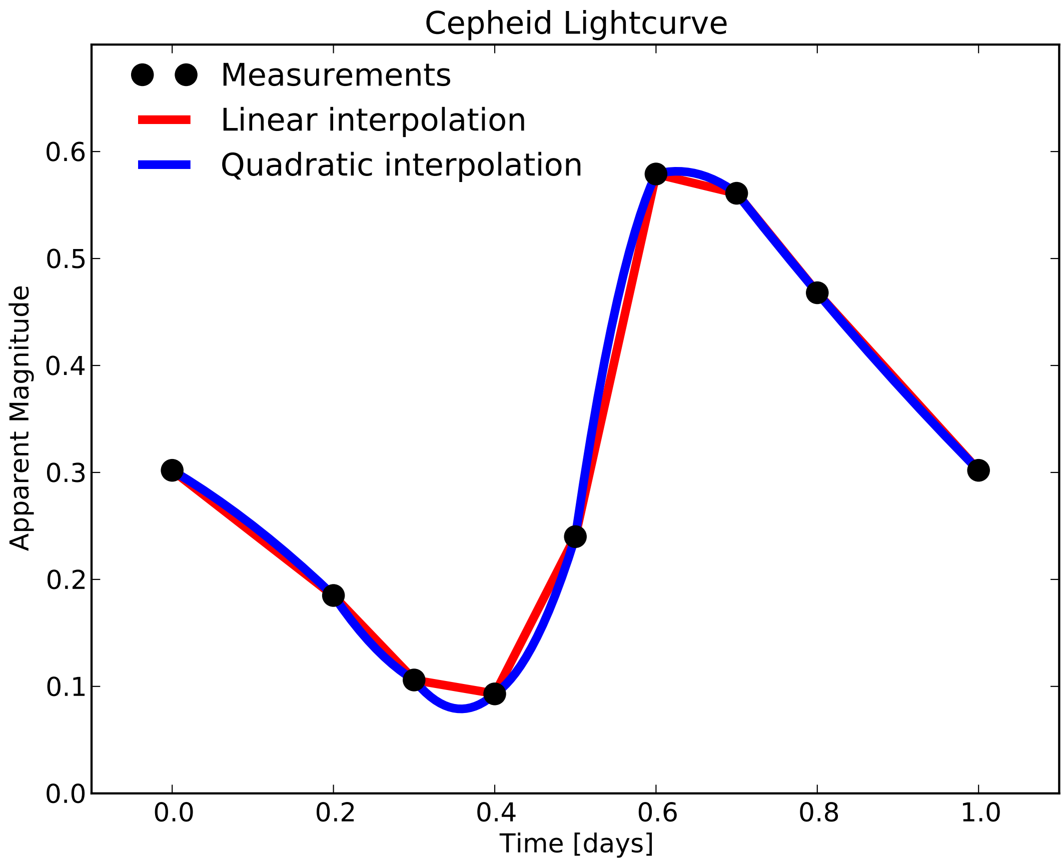

Figure 4: Apparent magnitude versus time for a Cepheid star. Actual measurements are represented as black points. Drawn in red is a piecewise-linear interpolation through the data, and drawn in blue is a piecewise-quadratic interpolation through the data.

5. More Cepheid Lightcurve Interpolation

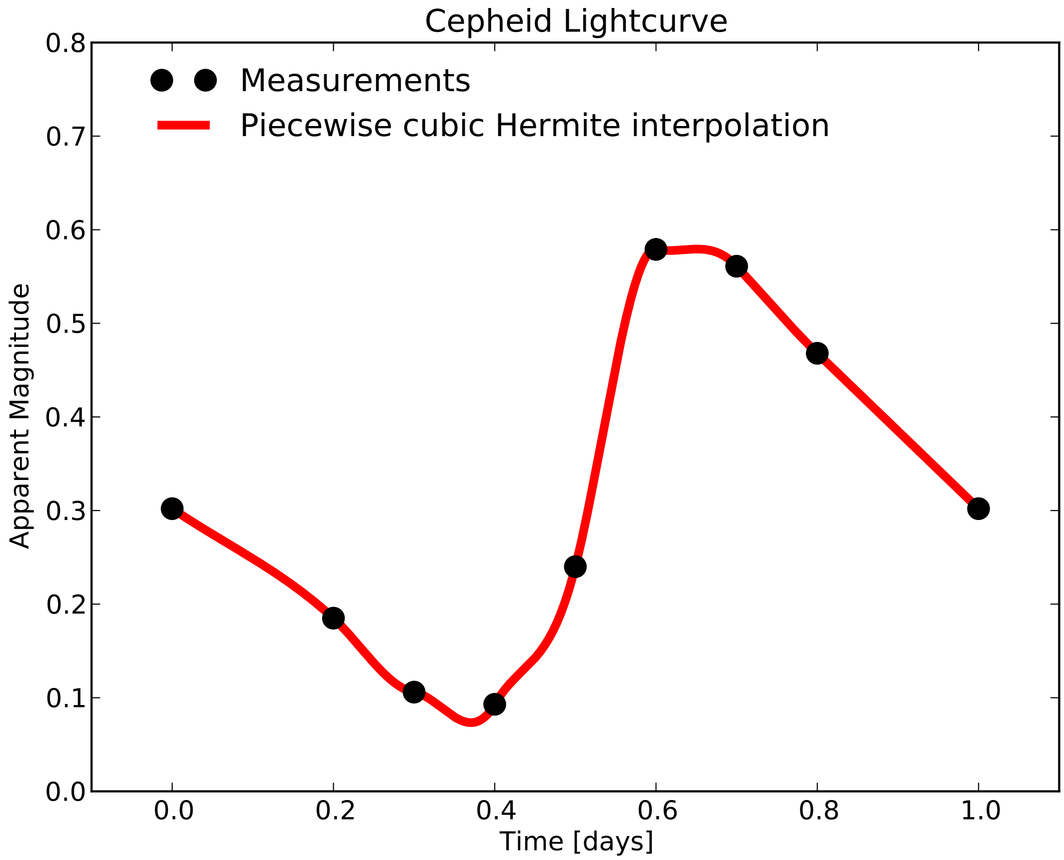

Figure 5: Apparent magnitude versus time for a Cepheid star. Actual measurements are represented as black points. Drawn in red is a piecewise cubic Hermite interpolation through the data.

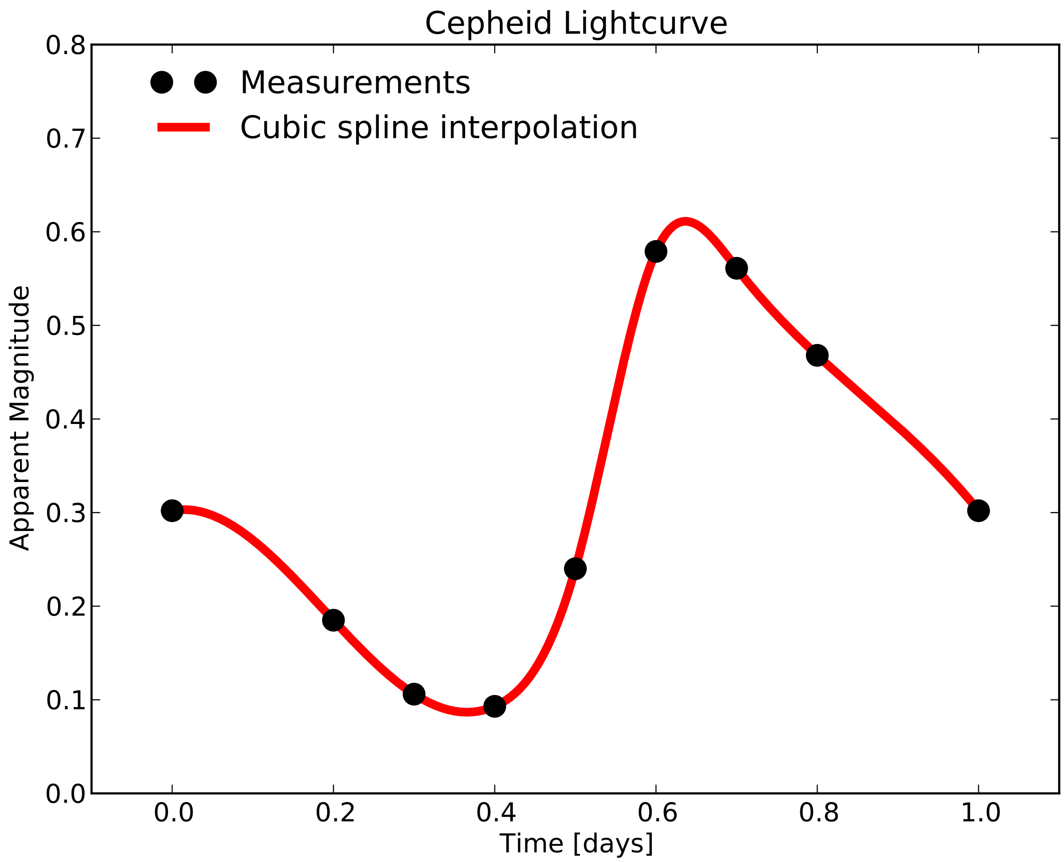

Figure 6: Apparent magnitude versus time for a Cepheid star. Actual measurements are represented as black points. Drawn in red is a cubic spline interpolation through the data.

Comments on the Various Interpolations

The Lagrange interpolation looks nice in the middle of the data, but near the ends it clearly becomes pathologically oscillatory, which is unreasonable. Linear interpolation is more reasonable, but quadratic interpolation is even nicer-looking. Cubic Hermite interpolation looks a little unnatural, with jerky changes within the intervals. The nicest-looking and more reasonable interpolation here is cubic spline.