1. An Unstable Calculation

n	Closed-Form Value	Recursive Value	Absolute Error	Relative Error
0	1.0	1.0	0.0	0.0
1	0.333333333333	0.333333	9.93410748107e-09	2.98023224432e-08
2	0.111111111111	0.111111	5.29819064732e-08	4.76837158259e-07
3	0.037037037037	0.0370373	2.16342784749e-07	5.84125518823e-06
4	0.0123456790123	0.0123466	8.71809912319e-07	7.06166028978e-05
5	0.00411522633745	0.00411871	3.48814414362e-06	0.0008476190269
6	0.00137174211248	0.00138569	1.39522571909e-05	0.0101711954921
7	0.000457247370828	0.000513056	5.58089223023e-05	0.122054113075
8	0.000152415790276	0.000375651	0.000223235614917	1.46464886947
9	5.08052634253e-05	0.000943748	0.000892942551319	17.5757882376
10	1.69350878084e-05	0.00358871	0.00357177023583	210.909460655
11	5.64502926948e-06	0.0142927	0.0142870812639	2530.91358466
12	1.88167642316e-06	0.0571502	0.0571483257835	30370.9634027
13	6.27225474386e-07	0.228594	0.228593318278	364451.584977
14	2.09075158129e-07	0.914374	0.914373307961	4373419.18641
15	**6.96917193763e-08**	**3.65749**	**3.6574932832**	**52481030.9737**

Table 1: Closed-form values and recursive values for the expression $\left(\frac{1}{3}\right)^n$ as well as errors of the recursive value compared to the closed-form value.

2. Finite Differnce Approximation and Convergence

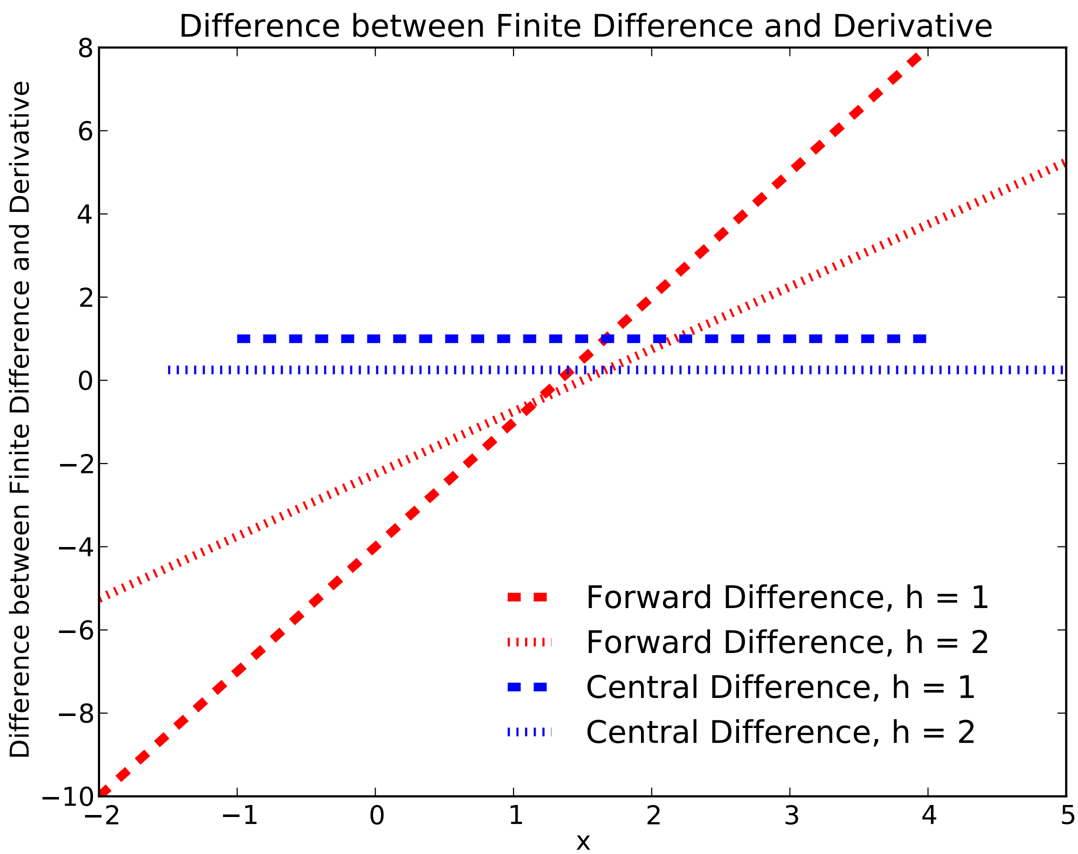

Figure 1: A plot of $f'(x;h) - f'(x)$, or the difference between the finite difference of $f(x) = x^3 - 5x^2 + x$ and the derivative of that function, on the interval $[-2, 6]$ for both forward differencing and central differencing, each at two values of the step-size h.

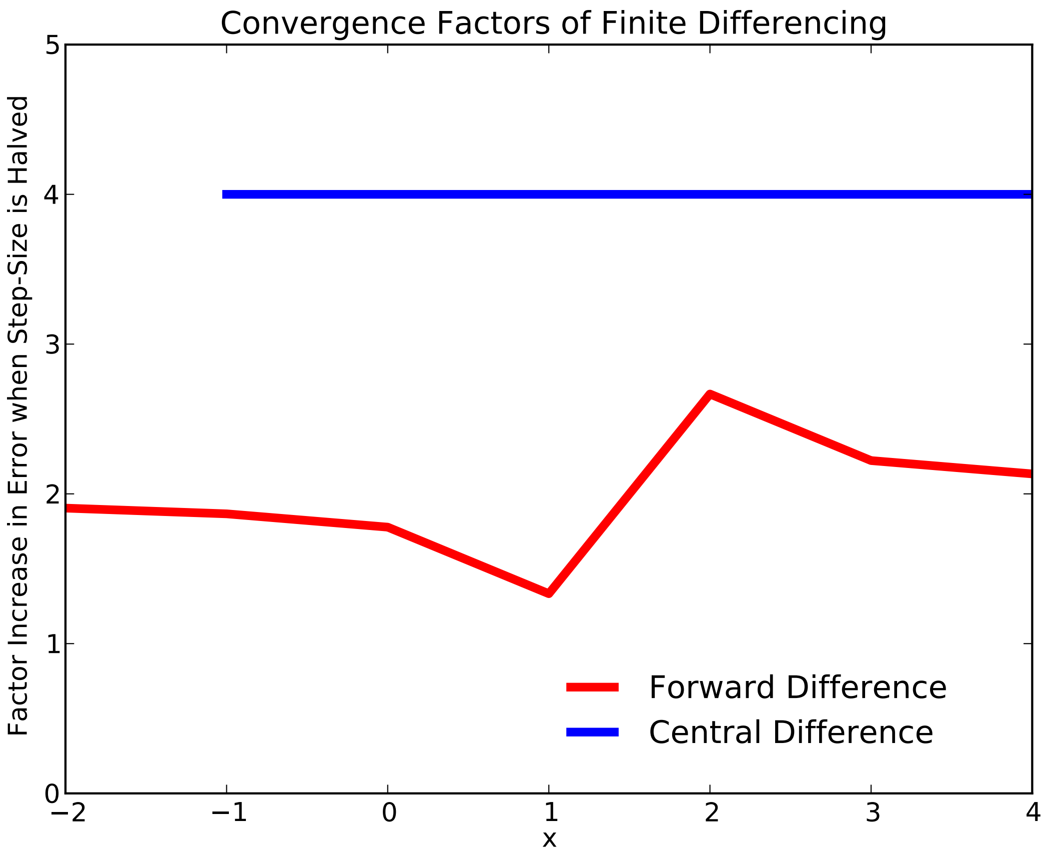

Figure 2: A plot of the ratios of the differencing errors between $h_1 = 1$ and $h_2 = 2$, for both forward and central differencing. We expect these ratios to be equal to the convergence factor $\left(\frac{h_2}{h_1}\right)^n = 2^n$, where n is the convergence order of the finite differencing: $n = 1$ for forward differencing and $n = 2$ for central differencing. Indeed, we see that the ratio is approximately 2 for forward differencing and 4 for central differencing as expected.

3. Second Derivative

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(x) + \frac{h^3}{6}f'''(x) + \mathcal{O}(h^4)$$
$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2}f''(x) - \frac{h^3}{6}f'''(x) + \mathcal{O}(h^4)$$
$$\Rightarrow f(x+h) + f(x-h) = 2f(x) + h^2 f''(x) + \mathcal{O}(h^4)$$
$$\boxed{f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} + \mathcal{O}(h^2)} \quad (1)$$

4. Interpolation: Cepheid Lightcurve

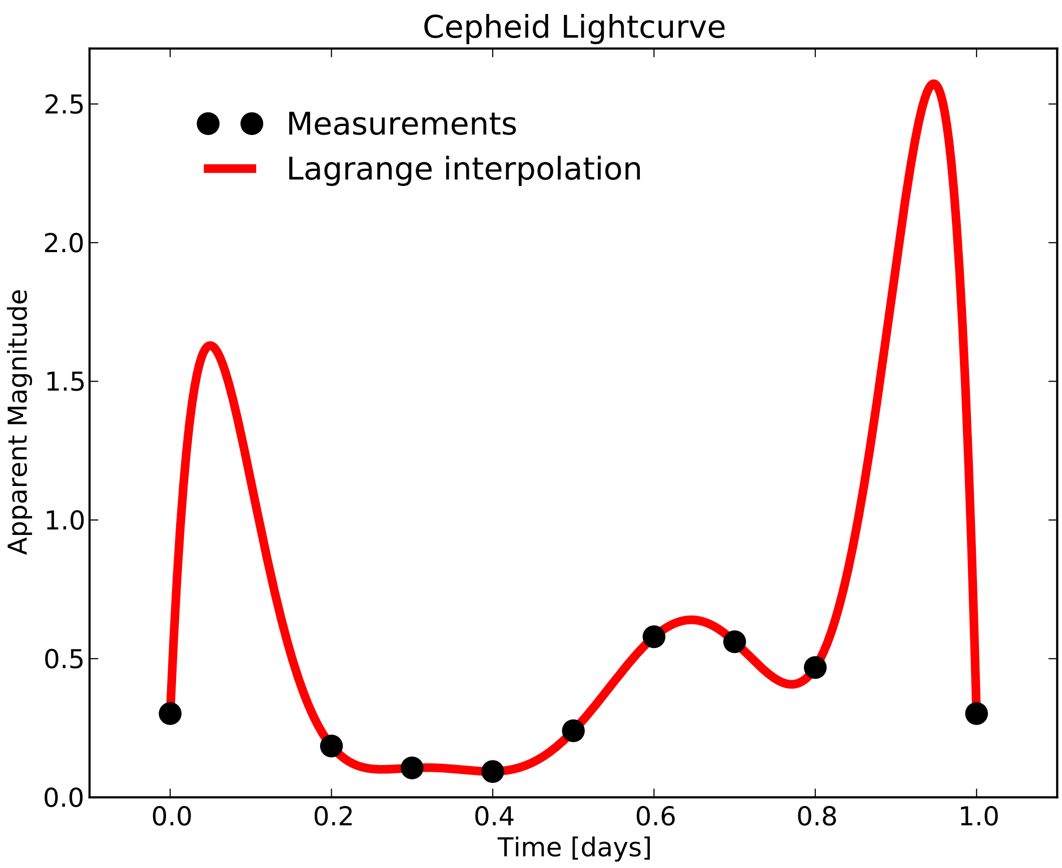

Figure 3: Apparent magnitude versus time for a Cepheid star. Actual measurements are represented as black points. Drawn in red is a Lagrange interpolation through the data.

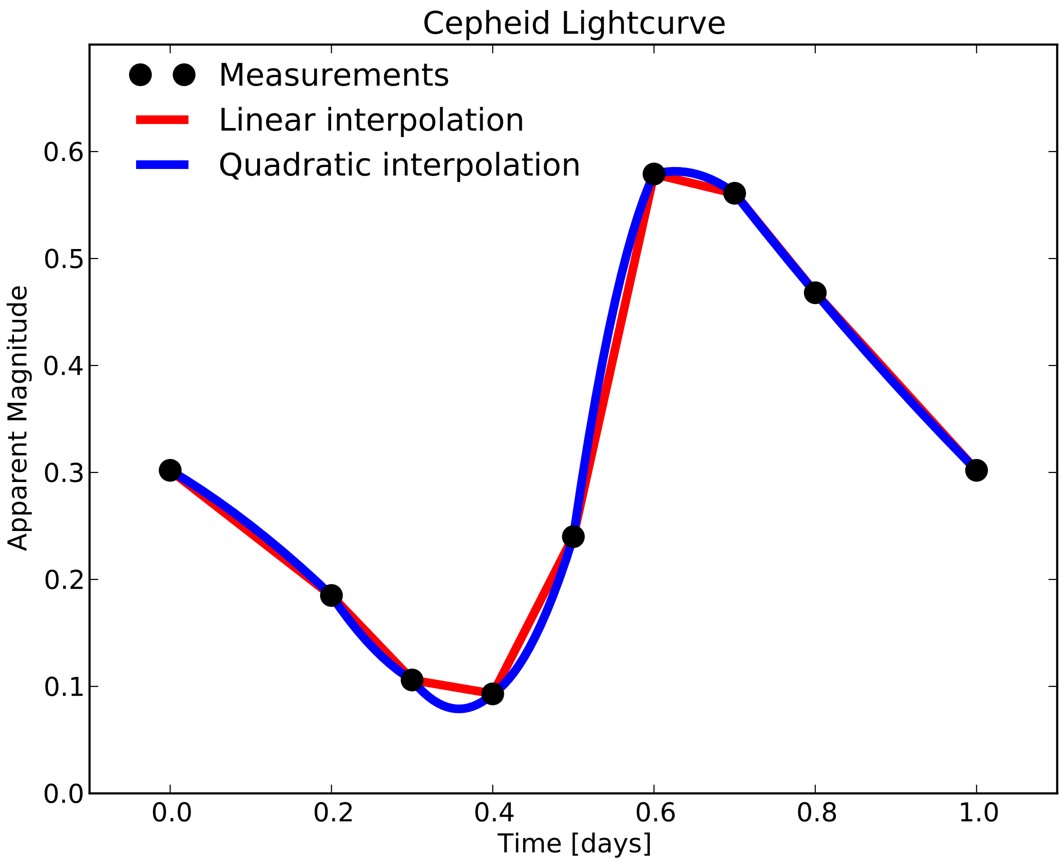

Figure 4: Apparent magnitude versus time for a Cepheid star. Actual measurements are represented as black points. Drawn in red is a piecewise-linear interpolation through the data, and drawn in blue is a piecewise-quadratic interpolation through the data.

5. More Cepheid Lightcurve Interpolation

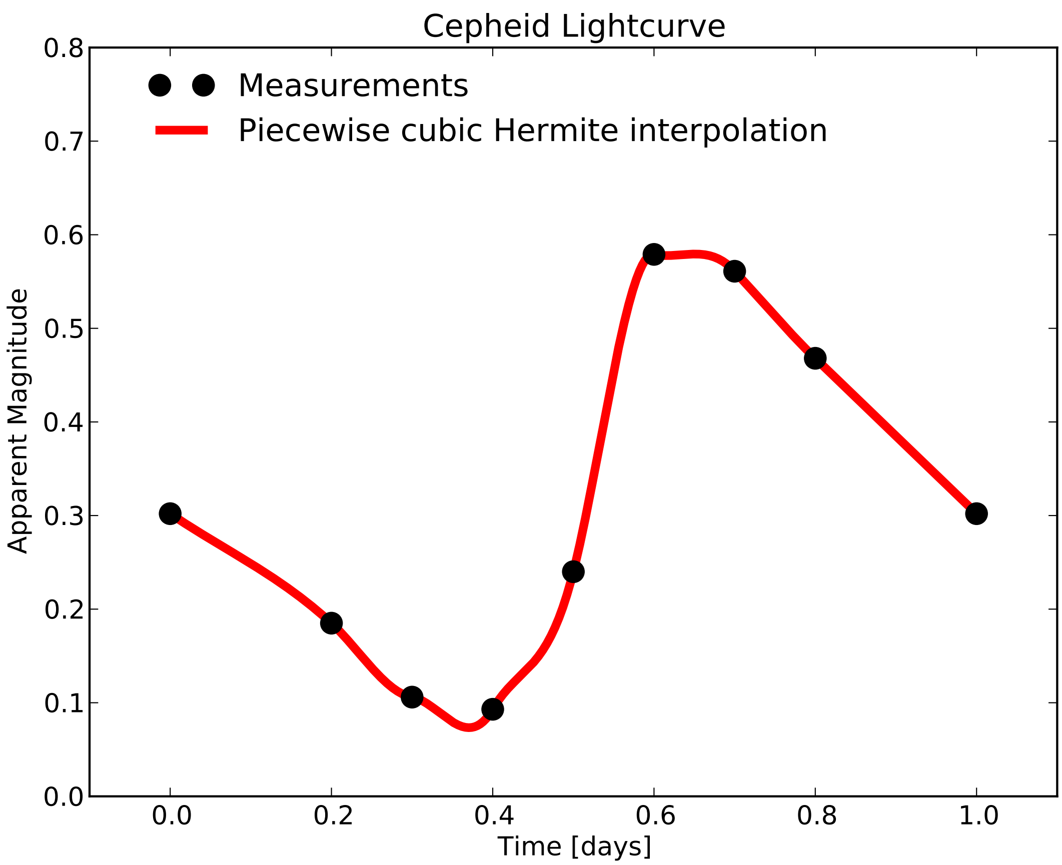

Figure 5: Apparent magnitude versus time for a Cepheid star. Actual measurements are represented as black points. Drawn in red is a piecewise cubic Hermite interpolation through the data.

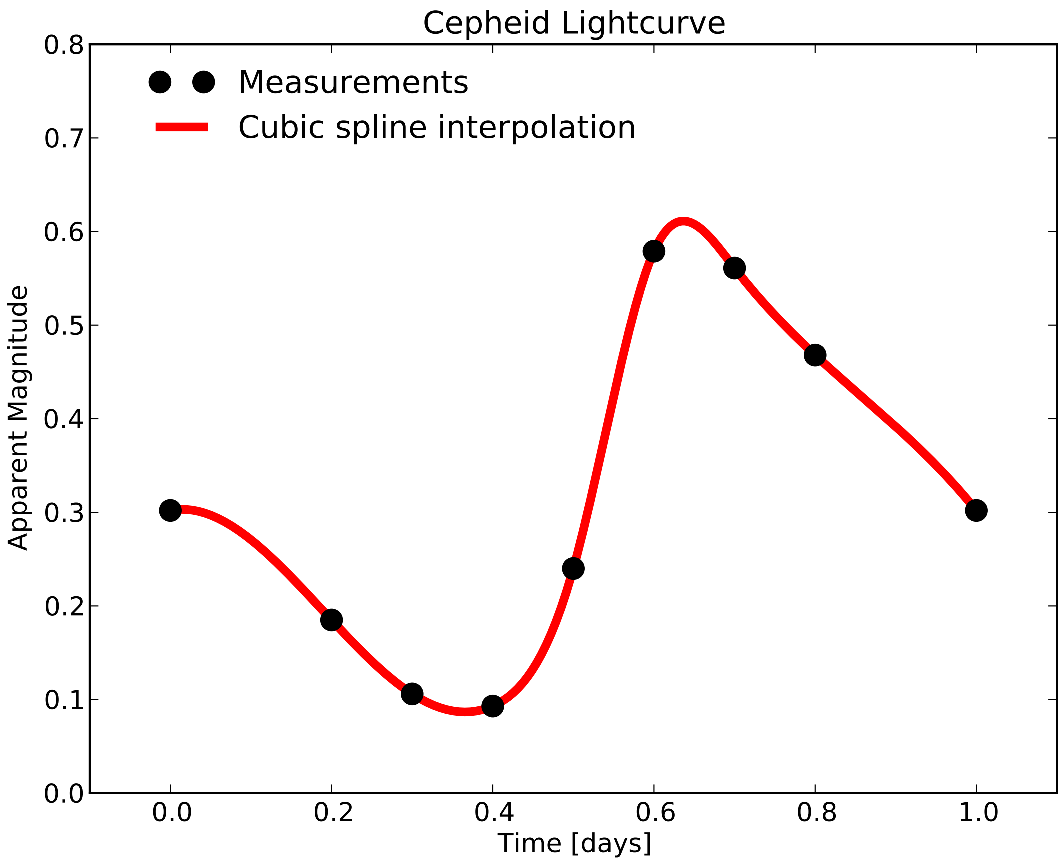

Figure 6: Apparent magnitude versus time for a Cepheid star. Actual measurements are represented as black points. Drawn in red is a cubic spline interpolation through the data.