I worked with my Partner David Vartanyan on this worksheet. It took about three hours.

(1) Measuring π with an MC Experiment

I implemented an MC experiment that pseudo-randomly and uniformly selects points in the interval $[0,1] \times [0,1]$ and determines whether or not these points are in the unit circle. By symmetry, one fourth of the unit circle is contained within this interval, so the fraction of points within the circle is an estimate of the ratio between one-fourth the circle's area ($\frac{1}{4}\pi$) and the area of the square interval (1). This method is demonstrated pictorially in Figure 1.

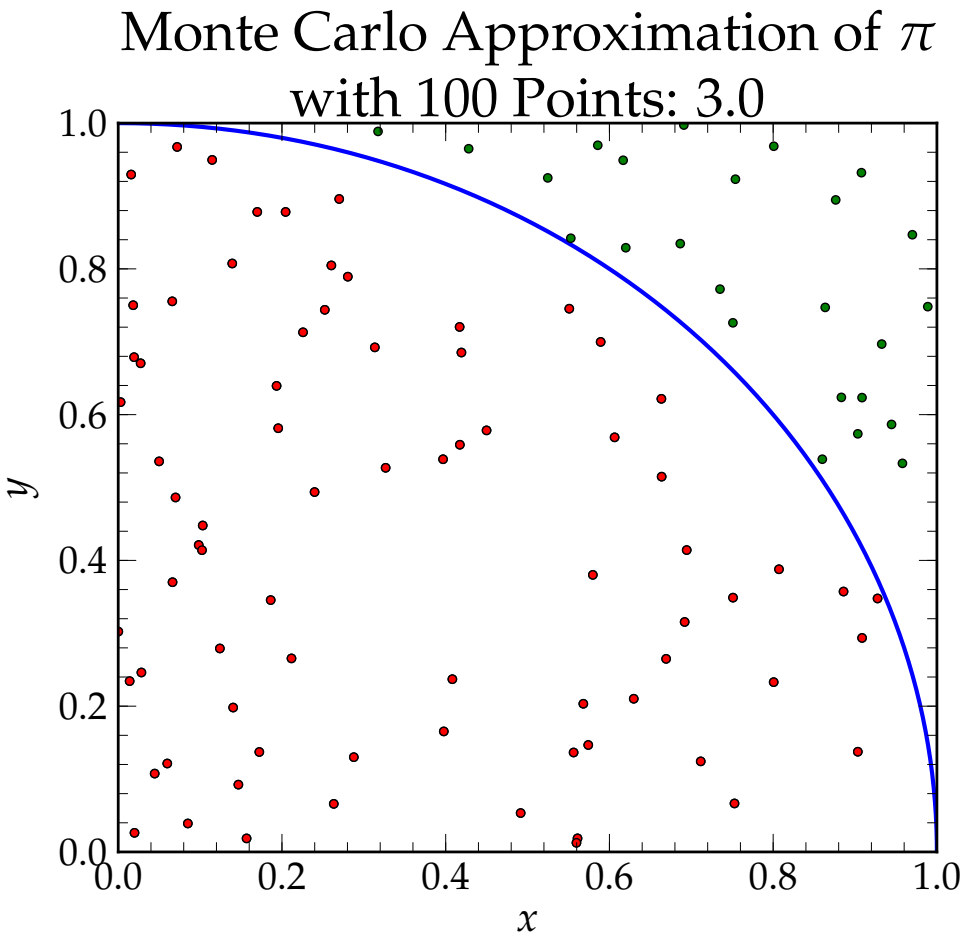

Figure 1: In this experiment, 75 points were internal to the circle (red) and 25 points were external (green), meaning the quarter circle has an approximate area of 0.75; the whole circle thus has approximate area 3.0.

Increasing the total number of points N by a factor of four, we should see a halving in the relative error of our scheme (since MC methods have error $\propto N^{-1/2}$). In the following table, we increase N in factors of four to investigate the resulting change in error.

N	Approximation of π	Relative Error in the Approximation
4	4.0	0.273
16	3.5	0.114
64	3.0	0.045
256	2.984375	0.050
1024	3.08984375	0.016
4096	3.1474609375	0.002
16384	3.13330078125	0.003
65536	3.1416015625	0.000003
262144	3.1450958252	0.001

In general at each new resolution the error goes down by a factor roughly two, but interesting at some resolutions the error seems to regress, and at some resolutions the error jumps down beyond a factor of two. I suppose that's to be expected of a probabilistic method.

(2) The Birthday Paradox

The result of my experiment for various numbers of trials N is given below. The analytical answer is 23, so a relative error is given.

N	Smallest Number of People	Relative Error
4	18	0.227
16	23	0.000
64	23	0.000
256	24	0.045
1024	23	0.000
4096	24	0.045
16384	23	0.000
65536	23	0.000

This really doesn't appear to follow the expected convergence of $N^{-1/2}$, but this is because we are reaching 0 error so quickly (second resolution step). There isn't really a way around this; since our trial numbers are already so small, it's hard for us to ramp up the error any more to really see the convergence behavior.