I worked with my Partner David Vartanyan on this worksheet. It took about four hours.

ODE Integration: Simplified Stellar Structure

(1) The code generates values for the pressure P and the contained mass $M(< r)$ at a given set of radii within a spherical object (a star) based on conditions at the object's center. The starting point for the code is the center of the object; it first assigns the central density that we provide (from which the central pressure can be calculated based on the polytropic equation) and sets the contained mass to zero there. Then, based on the particular integration scheme used, it evaluates the equations for $\frac{\mathrm{d}P}{\mathrm{d}r}$ and $\frac{\mathrm{d}M}{\mathrm{d}r}$ with certain radius, density, and contained mass values to "integrate" these derivatives to the next radius value we are interested in, resulting in approximations for M and P there. The particular method these derivatives are evaluated and used to approximate M and P depends on whether we are using RK2, RK3, or RK4. From the new radius value, $\frac{\mathrm{d}P}{\mathrm{d}r}$ and $\frac{\mathrm{d}M}{\mathrm{d}r}$ are again calculated in the prescribed fashion to approximate M and P at the next interesting radius value, and the process is completed until we reach some maximum cutoff radius. We also track the surface of the star by looking for when the pressure falls below some arbitrary cutoff; the radius of the star becomes the radius of the surface, and $M(< r)$ becomes the total stellar mass for $r > r_{\mathrm{surface}}$.

(2) Running Forward Euler with 1000 grid points, we compute a stellar radius of 1502 km and a stellar mass of 1.45 $M_\odot$.

(3)

To calculate self-convergence factors for the various integration methods, I used

$$Q_{\mathrm{theoretical}} = \frac{h_3^n - h_2^n}{h_2^n - h_1^n}$$

and

$$Q_{\mathrm{actual}} = \frac{|M_{*3} - M_{*2}|}{|M_{*2} - M_{*1}|}$$

where n is the theoretical convergence order. I used $h_1 = 2h_2 = 4h_3$, with $h1 = 2$ km.

Table 1: Self-Convergence of Stellar Mass for Various Integration Methods

Integration Method	n	$Q_{\mathrm{theoretical}}$	Q_{actual}
FE	1	0.500	0.503
RK2	2	0.250	0.248
RK3	3	0.125	0.122
RK4	4	0.062	0.092

Table 1 shows that the actual self-convergence factors are extremely close to their theoretical expected values, except in the case of RK4, where it appears to converge 150% slower than theoretically predicted.

(4)

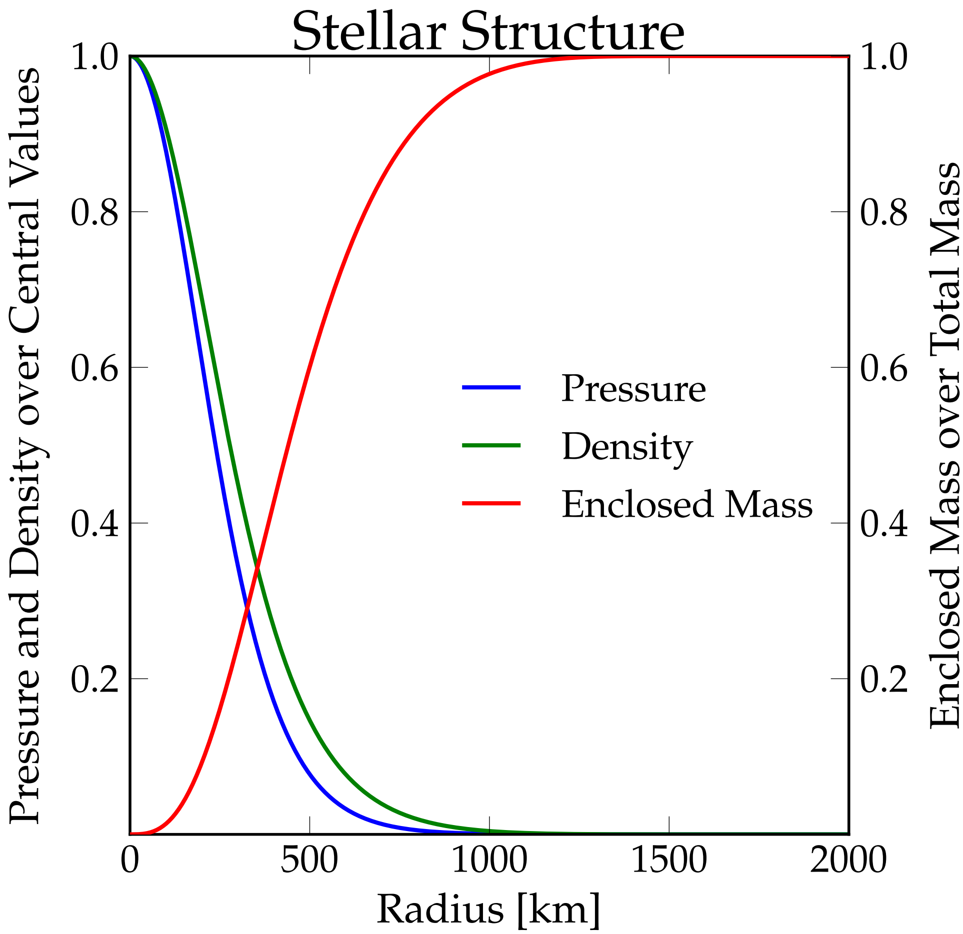

Figure 1: Pressure, density, and enclosed mass as functions of radius in our simulated star. RK4 was used with 10000 grid points.