I worked with my Partner David Vartanyan on this worksheet. It took about 14 astronomical units to complete (2 hours).

The Discrete Fourier Transform

(a)

I implemented the matrix formulation of the discrete Fourier transform in `dft.py`. The relative errors between the output $y(x)$ of my DFT and the output $z(x)$ of `numpy.fft.fft` for a vector x can be considered to be

$$\text{relative error} = \frac{|y - z|}{|z|}.$$

In general y and z are complex vectors. The relative error between my DFT and `numpy`'s FFT for a random vector of ten dimensions was 1.67×10^{-15}.

(b-c)

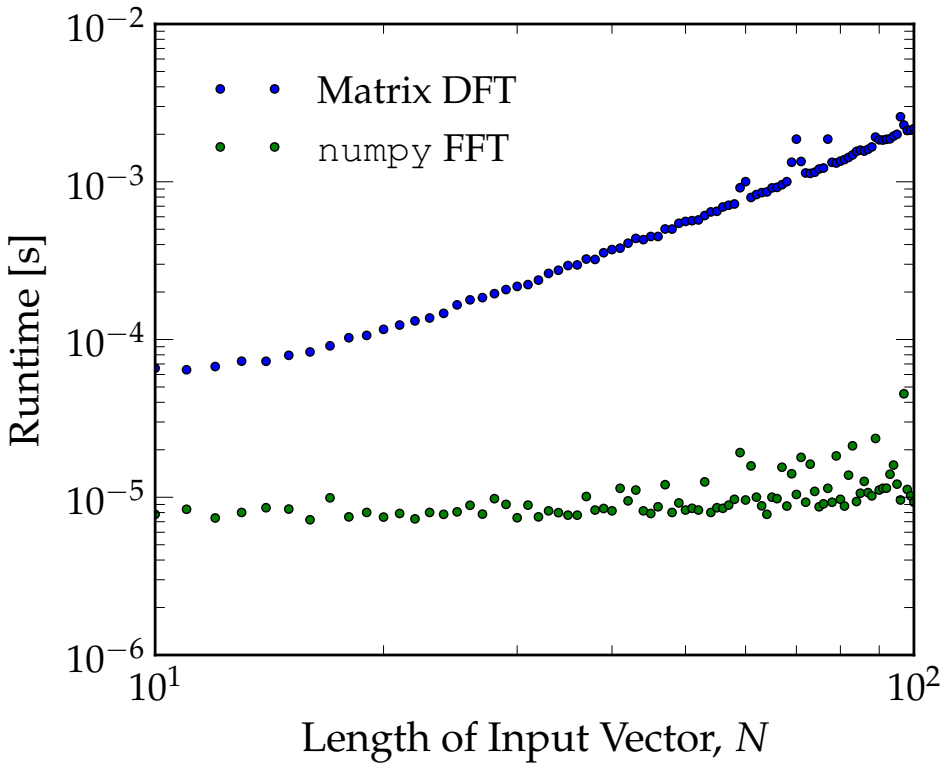

Figure 1: The runtime of two discrete Fourier transform implementations as a function of input vector size. Over the depicted range, the `numpy` FFT appears to execute in constant time, whereas the matrix DFT increases approximately one log unit of runtime for one-half log unit of input vector size, indicating that runtime $\sim N^2$. This plot demonstrates that FFT is *fast*.