ARTIFICIAL INTELLIGENCE II

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Unit-3

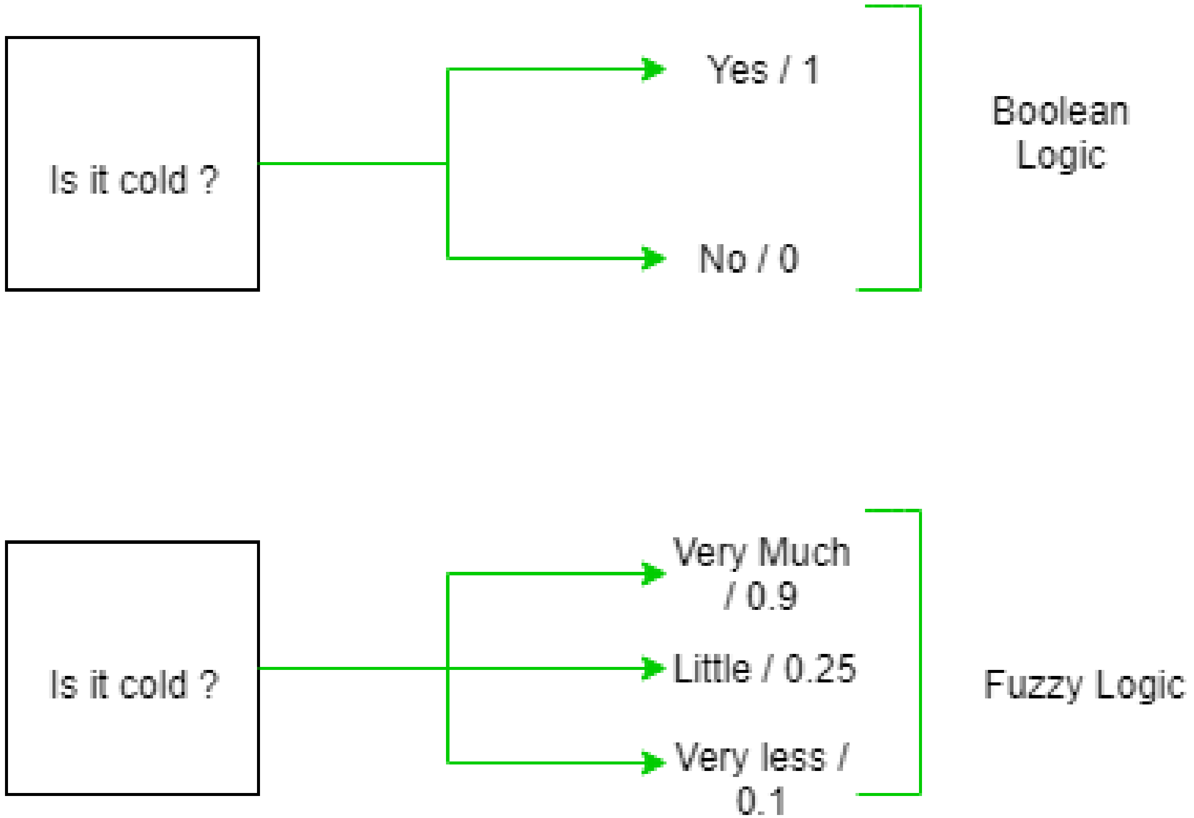
Uncertain Reasoning

Introduction to Fuzzy Logic and Fuzzy sets

The term **fuzzy** refers to things that are not clear or are vague. In the real world many times we encounter a situation when we can't determine whether the state is true or false, their fuzzy logic provides very valuable flexibility for reasoning. In this way, we can consider the inaccuracies and uncertainties of any situation.

Fuzzy Logic is a mathematical method for representing vagueness and uncertainty in decision-making, it allows for partial truths, and it is used in a wide range of applications. It is based on the concept of membership function and the implementation is done using Fuzzy rules.

Introduction to Fuzzy Logic and Fuzzy sets



Introduction to Fuzzy Logic and Fuzzy sets

Fuzzy Logic is a form of many-valued logic in which the truth values of variables may be any real number between 0 and 1, instead of just the traditional values of true or false. It is used to deal with imprecise or uncertain information and is a mathematical method for representing vagueness and uncertainty in decision-making.

Fuzzy Logic is based on the idea that in many cases, the concept of true or false is too restrictive, and that there are many shades of gray in between. It allows for partial truths, where a statement can be partially true or false, rather than fully true or false.

Fuzzy Logic is used in a wide range of applications, such as control systems, image processing, natural language processing, medical diagnosis, and artificial intelligence.

ARCHITECTURE

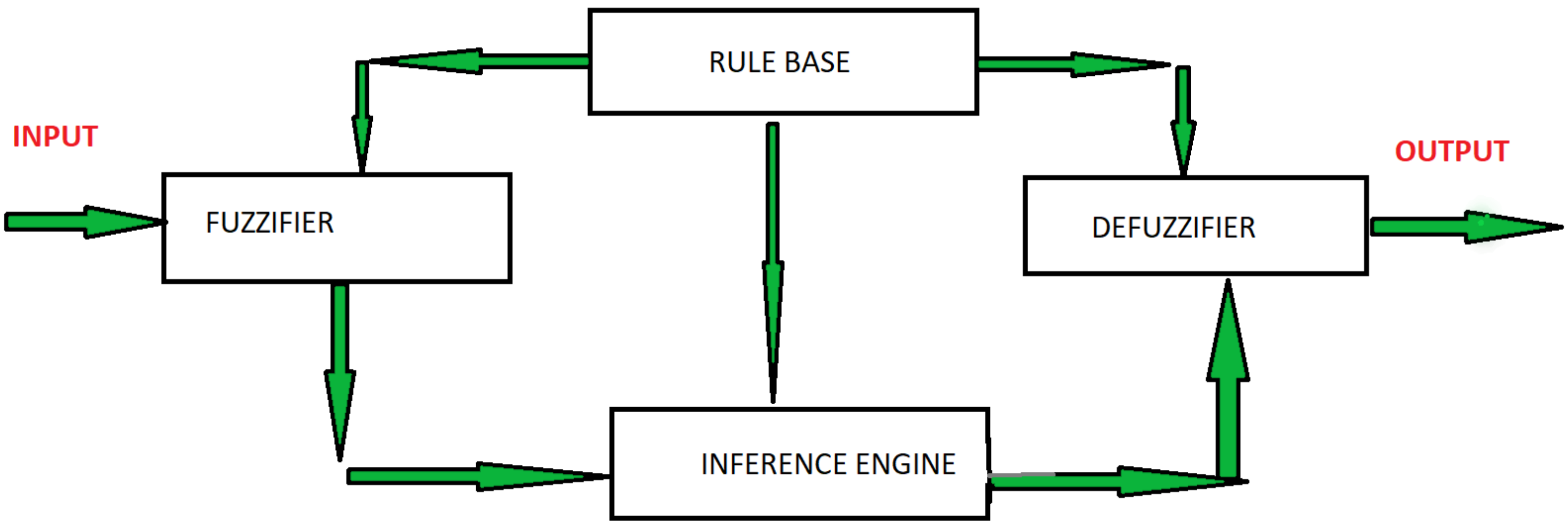
RULE BASE: It contains the set of rules and the IF-THEN conditions provided by the experts to govern the decision-making system, on the basis of linguistic information. Recent developments in fuzzy theory offer several effective methods for the design and tuning of fuzzy controllers. Most of these developments reduce the number of fuzzy rules.

FUZZIFICATION: It is used to convert inputs i.e. crisp numbers into fuzzy sets. Crisp inputs are basically the exact inputs measured by sensors and passed into the control system for processing, such as temperature, pressure, rpm's, etc.

INFERENCE ENGINE: It determines the matching degree of the current fuzzy input with respect to each rule and decides which rules are to be fired according to the input field. Next, the fired rules are combined to form the control actions.

DEFUZZIFICATION: It is used to convert the fuzzy sets obtained by the inference engine into a crisp value. There are several defuzzification methods available and the best-suited one is used with a specific expert system to reduce the error.

ARCHITECTURE



FUZZY LOGIC ARCHITECTURE

Membership function

Definition: A graph that defines how each point in the input space is mapped to membership value between 0 and 1. Input space is often referred to as the universe of discourse or universal set (u), which contains all the possible elements of concern in each particular application.

There are largely three types of fuzzifiers:

Singleton fuzzifier
Gaussian fuzzifier
Trapezoidal or triangular fuzzifier

What is Fuzzy Control?

- It is a technique to embody human-like thinkings into a control system.
- It may not be designed to give accurate reasoning but it is designed to give acceptable reasoning.
- It can emulate human deductive thinking, that is, the process people use to infer conclusions from what they know.
- Any uncertainties can be easily dealt with the help of fuzzy logic.

Advantages of Fuzzy Logic System

- This system can work with any type of inputs whether it is imprecise, distorted or noisy input information.
- The construction of Fuzzy Logic Systems is easy and understandable.
- Fuzzy logic comes with mathematical concepts of set theory and the reasoning of that is quite simple.
- It provides a very efficient solution to complex problems in all fields of life as it resembles human reasoning and decision-making.
- The algorithms can be described with little data, so little memory is required.

Disadvantages of Fuzzy Logic Systems

- Many researchers proposed different ways to solve a given problem through fuzzy logic which leads to ambiguity. There is no systematic approach to solve a given problem through fuzzy logic.
- Proof of its characteristics is difficult or impossible in most cases because every time we do not get a mathematical description of our approach.
- As fuzzy logic works on precise as well as imprecise data so most of the time accuracy is compromised.

Application

It is used in the aerospace field for altitude control of spacecraft and satellites.
It has been used in the automotive system for speed control, traffic control.
It is used for decision-making support systems and personal evaluation in the large company business.
It has application in the chemical industry for controlling the pH, drying, chemical distillation process.
Fuzzy logic is used in Natural language processing and various intensive applications in Artificial Intelligence.
Fuzzy logic is extensively used in modern control systems such as expert systems.
Fuzzy Logic is used with Neural Networks as it mimics how a person would make decisions, only much faster. It is done by Aggregation of data and changing it into more meaningful data by forming partial truths as Fuzzy sets.

Introduction to Fuzzy Logic and Fuzzy sets

• The word “fuzzy” means “vaguness (ambiguity)”.

• Fuzziness occurs when the boundary of a piece of information is not clear-cut.

•Fuzzy sets - 1965 Lotfi Zadeh as an extension of classical notation set.

•Classical set theory allows the membership of the elements in the set in **binary terms.**

• Fuzzy set theory permits membership function valued in the interval [0,1].

Introduction to Fuzzy Logic and Fuzzy sets

Example:

Words like young, tall, good or high are fuzzy.

• There is no single quantitative value which defines the term young.

•For some people, age 25 is young, and for others, age 35 is young.

• The concept young has no clean boundary.

•Age 35 has some possibility of being young and usually depends on the context in which it is being considered.

Fuzzy set theory is an extension of classical set theory where elements have degree of membership.

Introduction to Fuzzy Logic and Fuzzy sets

- In real world, there exist much fuzzy knowledge (i.e. vague, uncertain inexact etc).
- Human **thinking and reasoning (analysis, logic,** interpretation) frequently involved **fuzzy information.**
- Human can give satisfactory answers, which are probably true.
- Our systems are unable to answer many question because the systems are designed based upon classical set theory (Unreliable and incomplete).
- We want, our system should be able to cope with unreliable and incomplete information.
- Fuzzy system have been provide solution.

Introduction to Fuzzy Logic and Fuzzy sets

Classical set theory

- Classes of objects with sharp boundaries.
- A classical set is defined by Crisp(exact) boundaries, i.e., there is no uncertainty about the location of the set boundaries.
- Widely used in digital system design

Let's consider a classical set example involving shapes. Suppose we have a set called "Geometric Shapes," and we want to define the elements (objects) that belong to this set.

The set "Geometric Shapes" can be defined as follows:

Geometric Shapes = {circle, square, triangle, rectangle}

Introduction to Fuzzy Logic and Fuzzy sets

Fuzzy set theory

- Classes of objects with unsharp boundaries.
- A fuzzy set is defined by its ambiguous boundaries, i.e., there exists uncertainty about the location of the set boundaries.
- Used in fuzzy controllers.

Let's suppose A is a set which contains following elements:

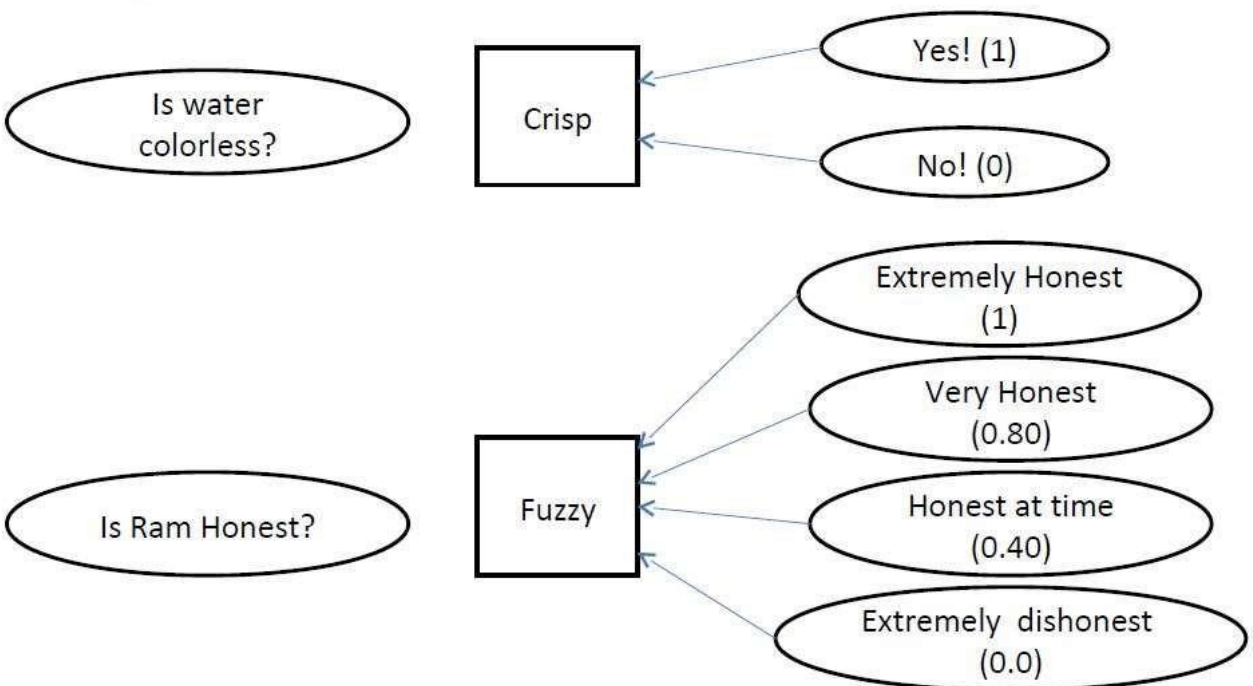
$A = \{(X_1, 0.6), (X_2, 0.2), (X_3, 1), (X_4, 0.4)\}$

And, B is a set which contains following elements:

$B = \{(X_1, 0.1), (X_2, 0.8), (X_3, 0), (X_4, 0.9)\}$

Crisp Set Vs Fuzzy Set

Example



Fuzzy vs crips

Fuzzy sets

Fuzzy set theory

• Fuzzy sets theory is an extension of classical set theory.

• Elements have varying degree of membership. A logic based on two truth values,

• ***True and False is sometimes insufficient when describing*** human reasoning.

• Fuzzy Logic uses the whole interval between 0 (false) and 1 (true) to describe human reasoning.

•A Fuzzy Set is any set that allows its members to have different degree of membership, called **membership function, having interval [0,1].**

Fuzzy sets

- **Fuzzy Logic is derived from fuzzy set theory**
- Many degree of membership (between 0 to 1) are allowed.
- Thus a membership function **µA**

(x) is associated with a fuzzy
sets A such that the function maps every element of universe of discourse X to the interval [0,1].

- The mapping is written as: µA(x): X ? [0,1].
- Fuzzy Logic is capable of handing inherently imprecise

(vague or inexact or rough or inaccurate) concepts

Fuzzy sets

- **Fuzzy set is defined as follows:**
- If X is an universe of discourse and x is a particular element of X, then a fuzzy set A defined on X and can be written as a collection of ordered pairs

$A = \{(x, \mu A(x)), x \in X \}$

Example

- Let X = {g1, g2, g3, g4, g5} be the reference set of students.
- Let A be the fuzzy set of “smart” students, where “smart” is fuzzy term.

A = {(g1,0.4)(g2,0.5)(g3,1)(g4,0.9)(g5,0.8)}

Here A indicates that the smartness of g1 is 0.4 and so on

Membership Function

Membership Function

- The membership function fully defines the fuzzy set
- A membership function provides a measure of *the degree of similarity of an element to a fuzzy set*

Membership functions can

–either be chosen by the user arbitrarily, based on the user’s experience (MF chosen by two users could be different depending upon their experiences, perspectives, etc.)

– Or be designed using machine learning methods (e.g., artificial neural networks, genetic algorithms, etc.)

Fuzzification

Fuzzification

Fuzzification is the process of transforming a crisp set to a fuzzy set or a fuzzy set to a fuzzier set, i.e.,crisp quantities are converted to fuzzy quantities. This operation translates accurate crisp input values into linguistic variables.

A Fuzzy set $\tilde{A} = \{\mu_i/x_i | x_i \in X\}$, a common fuzzification algorithm is performed by keeping μ_i constant and x_i being transformed to a fuzzy set $Q(x_i)$ depicting the expression about x_i. The fuzzy set $Q(x_i)$ is referred to as the kernel of fuzzification. The fuzzified set A can be expressed as

$$\tilde{A} = \mu_1 Q(x_1) + \mu_2 Q(x_2) + \cdots + \mu_n Q(x_n)$$

where the symbol ~ means fuzzified.

This process of fuzzification is called support fuzzification (s-fuzzification). There is another method of fuzzification called *grade fuzzification* (g-fuzzification) where X_i is kept constant and μ_i is expressed as a fuzzy set. Thus, using these methods, fuzzification is carried out.

Defuzzication

Defuzzification is the process of representing a fuzzy set with a crisp number. Internal representations of data in a fuzzy system are usually fuzzy sets. But the output frequently needs to be a crisp number that can be used to perform a function such as commanding a valve to a desired position in a control application or indicate a problem risk index.

the complete structure of a Fuzzy Logic System. Once all input variable values are translated into respective linguistic variable values, the fuzzy inference step evaluates the set of fuzzy rules that define the evaluation. The result of this is again a linguistic value. The defuzzification step translates this linguistic result into a numerical value.

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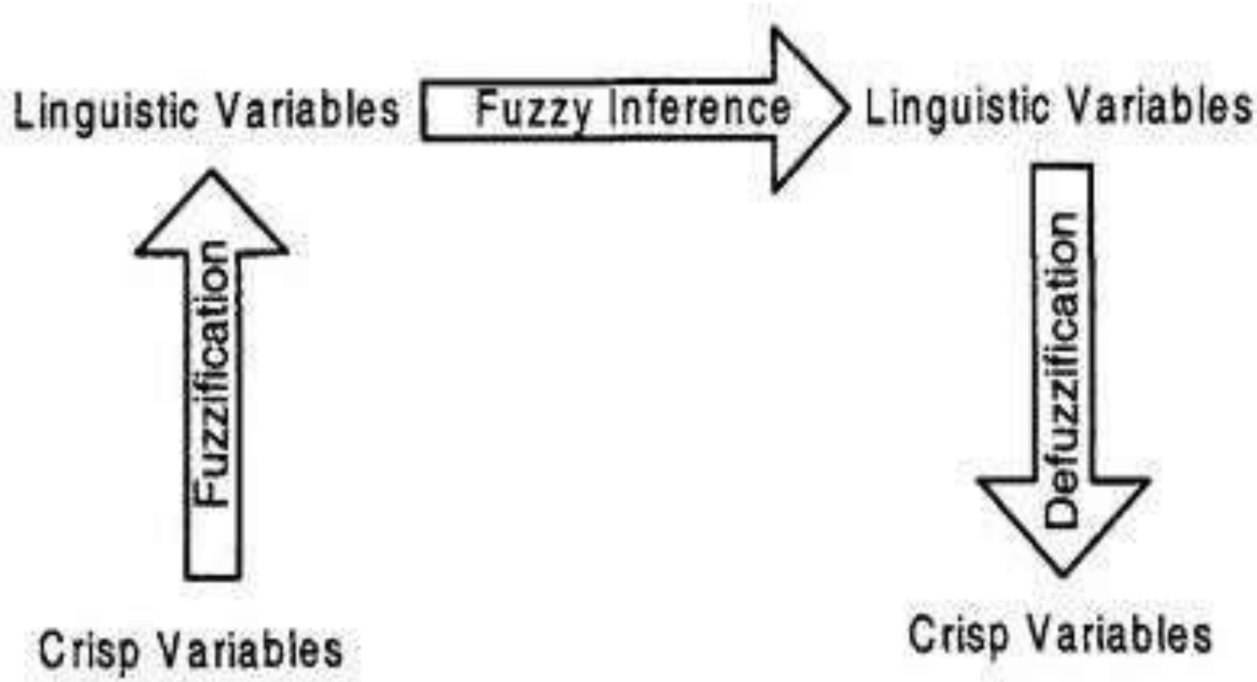
Defuzzication



Fuzzy Set Operation

Given X to be the universe of discourse and Ã and Ḃ to be fuzzy sets with µA(x) and µB(x) are their respective membership function, the fuzzy set operations are as follows:

Union:

µA U B(x) = max (µA(x), µB(x))

Intersection:

µA Ç B(x) = min (µA(x), µB(x))

Complement:

µA (x) =1- µA(x)

Fuzzy Set Operation

Example:

$A = \{(x1,0.5),(x2,0.7),(x3,0)\}$ $B = \{(x1,0.8),(x2,0.2),(x3,1)\}$

Union:

$A \cup B = \{(x1,0.8),(x2,0.7),(x3,1)\}$

Because

$\mu A \cup B(x1) = \max(\mu A(x1), \mu B(x1))$

$= \max(0.5,0.8)$

$= 0.8$

$\mu A \cup B(x2) = 0.7$ and $\mu A \cup B(x3) = 1$

Fuzzy Set Operation

Example:

A = {(x1,0.5),(x2,0.7),(x3,0)} B = {(x1,0.8),(x2,0.2),(x3,1)}

Intersection:

A Ç B = {(x1,0.5),(x2,0.2),(x3,0)}

Because

µA Ç B(x1) = min (µA(x1), µB(x1))

= max(0.5,0.8)

= 0.5

µA Ç B(x2) = 0.2 and µA Ç B(x3) = 0

Fuzzy Set Operation

Example:

A = {(x1,0.5),(x2,0.7),(x3,0)}

Complement:

Ac = {(x1,0.5),(x2,0.3),(x3,1)}

Because

µA (x1) =1- µA(x1)

= 1 – 0.5

= 0.5

µA (x2) = 0.3 and µA (x3) = 1

Linguistic variable

- **Linguistic variable: A *linguistic variable is a*** variable whose values are sentences in a natural or artificial language.
- **For example, the values of the fuzzy variable h***eight could be tall, very tall, very very tall, somewhat tall, not very tall, tall but not very tall, quite tall, more or less tall.*
- *Tall is a linguistic value or primary term.*
- If ***age is a linguistic variable then its term set is:***
- *T(age) = { young, not young, very young, not very young,…… middle aged, not middle aged, … old, not old, very old, more or less old, not very old,…not very young and not very old,…}.*

▪ A linguistic variable carries with it the concept of fuzzy set qualifiers, called hedges.

▪ Hedges are terms that modify the shape of fuzzy sets. They include adverbs such as very, somewhat, quite, more or less and slightly.

Propositions and Connectives

Propositional Logic

A proposition is a collection of declarative statements that have either a truth value "true” or a truth value "false". A propositional consists of propositional variables and connectives. The propositional variables are dented by capital letters (A, B, etc). The connectives connect the propositional variables.

A few examples of Propositions are given below −

"Man is Mortal", it returns truth value “TRUE”

"12 + 9 = 3 – 2", it returns truth value “FALSE”

The following is not a Proposition −

"A is less than 2" − It is because unless we give a specific value of A, we cannot say whether the statement is true or false.

Propositions and Connectives

Connectives

In propositional logic, we use the following five connectives –

OR (VV)

AND (AA)

Negation/ NOT (¬¬)

Propositions and Connectives

OR (VV)

The OR operation of two propositions A and B (written as AVBAVB) is true if at least any of the propositional variable A or B is true.

The truth table is as follows –

A	B	A v B
True	True	True
True	False	True
False	True	True
False	False	False

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Propositions and Connectives

AND (AA)

The AND operation of two propositions A and B (written as AABAAB) is true if both the propositional variable A and B is true.

The truth table is as follows –

A	B	A ∧ B
True	True	True
True	False	False
False	True	False
False	False	False

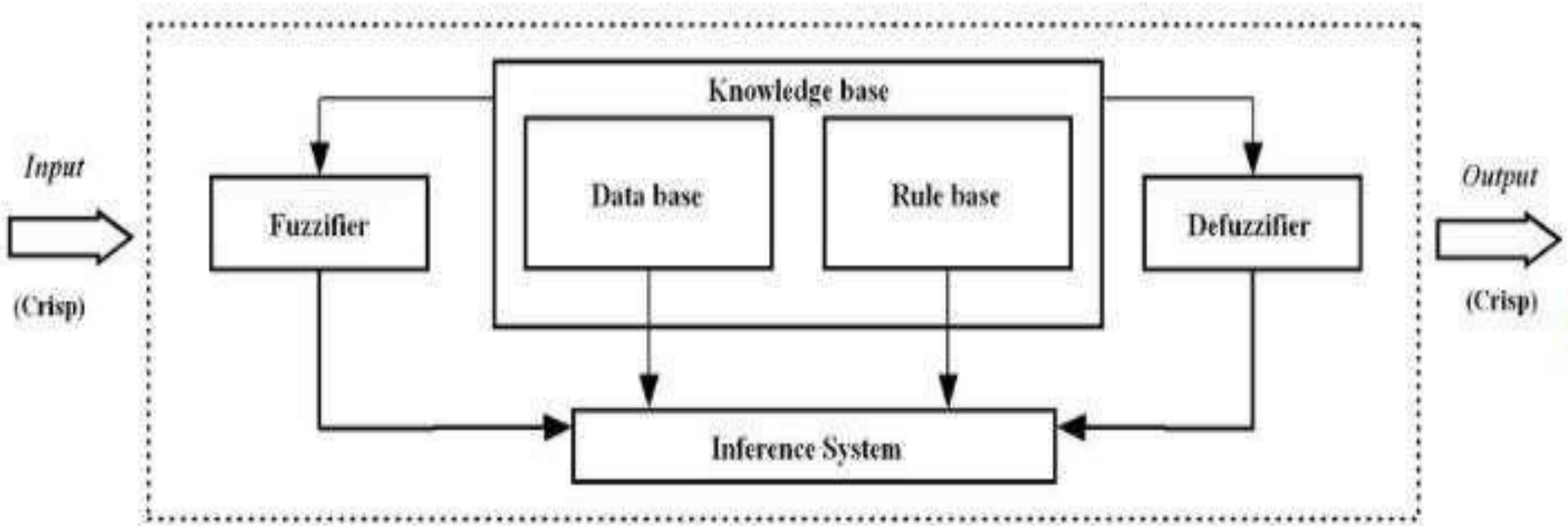
Inference Fuzzy Rules

- Fuzzy rules are useful for modeling human thinking, perception (Opinion,view) and judgement.

- A fuzzy if-then rule is of the form “If x is A then y is B” where A and B are linguistic values defined by fuzzy sets on universes of discourse *X and Y,* respectively.

- “x is A” is called *antecedent and “y is B” is called consequent.*

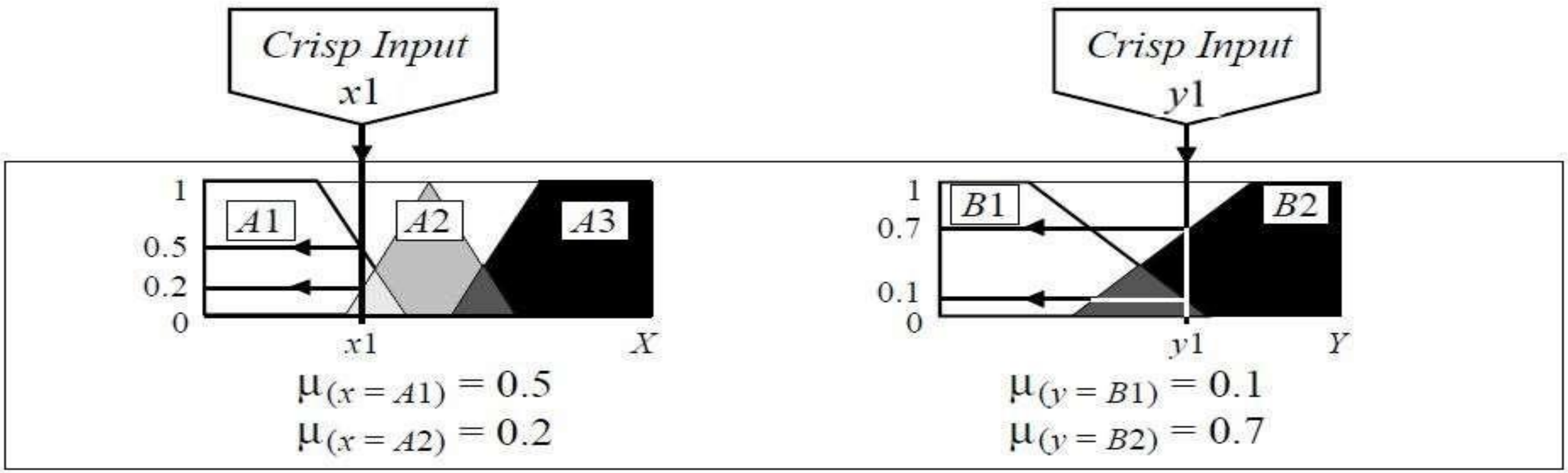
Fuzzy Rule Based Systems



Fuzzy Rule Based Systems

Step 1: Fuzzification

The first step is to take the crisp inputs, x1 and y1 (project funding and project staffing), and determine the degree to which these inputs belong to each of the appropriate fuzzy sets.



Fuzzy Rule Based Systems

Step 2: Rule Evaluation

The second step is to take the fuzzified inputs,

m(x=A1) = 0.5,

m(x=A2) = 0.2,

m(y=B1) = 0.1 and m(y=B 2) = 0.7,

and apply them to the antecedents of the fuzzy rules.

If a given fuzzy rule has multiple antecedents, the fuzzy operator (AND or OR) is used to obtain a single number that represents the result of the antecedent evaluation.

This number (the truth value) is then applied to the consequent membership function.

Fuzzy Rule Based Systems

To evaluate the disjunction of the rule antecedents, we use the **OR fuzzy operation. Typically, fuzzy e**xpert systems make use of the classical fuzzy operation **union:**

µA U B(x) = max (µA(x), µB(x))

Similarly, in order to evaluate the conjunction of the rule antecedents, we apply the **AND fuzzy operation intersection:**

µA Ç B(x) = min (µA(x), µB(x))

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