# Multi-layer SEIR model

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Equations for the unvaccinated population $(S, E_u, I_u, R_u, D_u, E_r, I_r, R_r, D_r)$:

$$\dot{S} = -\beta(t)\frac{I_u}{N}S - v(t) \tag{1}$$
$$\dot{E}_u = \beta(t)\frac{I_u}{N}S - \theta E_u - r_E(t) \tag{2}$$
$$\dot{I}_u = \theta E_u - (\gamma + \mu)I_u - r_I(t) \tag{3}$$
$$\dot{R}_u = \gamma I_u \tag{4}$$
$$\dot{D}_u = \mu I_u \tag{5}$$
$$\dot{E}_r = -\theta E_r + r_E(t) \tag{6}$$
$$\dot{I}_r = \theta E_r - (\gamma + \mu)I_r + r_I(t) \tag{7}$$
$$\dot{R}_r = \gamma I_r \tag{8}$$
$$\dot{D}_r = \mu I_r \tag{9}$$

Equations for the vaccinated population $(S^*, E_u^*, I_u^*, R_u^*, D_u^*, E_r^*, I_r^*, R_r^*, D_r^*, V^*)$:

$$\dot{S}^* = -\beta^*(t)\frac{I_u^*}{N}S^* + \alpha v(t) \tag{10}$$
$$\dot{E}_u^* = \beta^*(t)\frac{I_u^*}{N}S^* - \theta E_u^* - r_E^*(t) \tag{11}$$
$$\dot{I}_u^* = \theta E_u^* - (\gamma^* + \mu^*)I_u^* - r_I^*(t) \tag{12}$$
$$\dot{R}_u^* = \gamma^* I_u^* \tag{13}$$
$$\dot{D}_u^* = \mu^* I_u^* \tag{14}$$
$$\dot{E}_r^* = -\theta E_r^* + r_E^*(t) \tag{15}$$
$$\dot{I}_r^* = \theta E_r^* - (\gamma^* + \mu^*)I_r^* + r_I^*(t) \tag{16}$$
$$\dot{R}_r^* = \gamma^* I_r^* \tag{17}$$
$$\dot{D}_r^* = \mu^* I_r^* \tag{18}$$
$$\dot{V}^* = v(t) - \alpha v(t) \tag{19}$$

Merging unvaccinated and vaccinated equations:

$$\dot{\tilde{S}} = \dot{S} + \dot{S}^* = -\beta(t)\frac{I_u}{N}S - \beta^*(t)\frac{I_u^*}{N}S^* - (1-\alpha)v(t) \tag{20}$$
$$\dot{\tilde{E}}_u = \dot{E}_u + \dot{E}_u^* = \beta(t)\frac{I_u}{N}S + \beta^*(t)\frac{I_u^*}{N}S^* - \theta(E_u + E_u^*) - r_E(t) - r_E^*(t) \tag{21}$$
$$\dot{\tilde{I}}_u = \dot{I}_u + \dot{I}_u^* = \theta(E_u + E_u^*) - (\gamma + \mu)I_u - (\gamma^* + \mu^*)I_u^* - r_I(t) - r_I^*(t) \tag{22}$$
$$\dot{\tilde{R}}_u = \dot{R}_u + \dot{R}_u^* = \gamma I_u + \gamma^* I_u^* \tag{23}$$
$$\dot{\tilde{D}}_u = \dot{D}_u + \dot{D}_u^* = \mu I_u + \mu^* I_u^* \tag{24}$$
$$\dot{\tilde{E}}_r = \dot{E}_r + \dot{E}_r^* = -\theta(E_r + E_r^*) + r_E(t) + r_E^*(t) \tag{25}$$

$$\dot{\tilde{I}}_r = \dot{I}_r + \dot{I}_r^* = \theta(E_r + E_r^*) - (\gamma + \mu)I_r - (\gamma^* + \mu^*)I_r^* + r_I(t) + r_I^*(t) \tag{26}$$

$$\dot{\tilde{R}}_r = \dot{R}_r + \dot{R}_r^* = \gamma I_r + \gamma^* I_r^* \tag{27}$$

$$\dot{\tilde{D}}_r = \dot{D}_r + \dot{D}_r^* = \mu I_r + \mu^* I_r^* \tag{28}$$

$$\dot{\tilde{V}} = \dot{V}^* = (1 - \alpha)v(t) \tag{29}$$

Let $\chi_S = \frac{S^*}{S}$