Probabilistic tools

Central Limit Theorem
Let $(X_n)_{n\geq 1}$ be a real and independent sequence with same law such that $\mu = \mathbb{E}[X_1]$ and $\mathbb{V}[X_1] = \sigma^2$ are defined ($\mathbb{V}[X_1] \leq +\infty$). Noting $\bar{X}_n = \frac{1}{n}(X_1 + ... + X_n)$, we have:

$$\sqrt{n}\frac{(\bar{X}_n - \mu)}{\sigma} \sim_{n\to\infty} \mathcal{N}(0,1)$$

Spectral Theorem
Let M be a symmetric matrix with real coefficients. Then it exists U orthogonal and D diagonal with real coefficients such that $M = UDU^T$.

Inferential statistics

Likelihood method

This method consists on finding the parameter that maximizes the likelihood:
$L(x_1, ..., x_n; \theta) = f(X|\theta) = \prod_{i=1}^{n} f_\theta(x_i; \theta)$ which is the product of densities across all samples.

Intuitively, we want to find the θ that maximizes a certain event, that is, obtaining some data X (which is why we have $X|\theta$).
We often use the log in order to get rid of power coefficients appearing with the product.
likelihood equation: $\frac{d}{d\theta} ln(L(x_1, ..., x_n; \theta)) = 0$

Exploratory statistics