1.1 Disk rolling on cosine plane

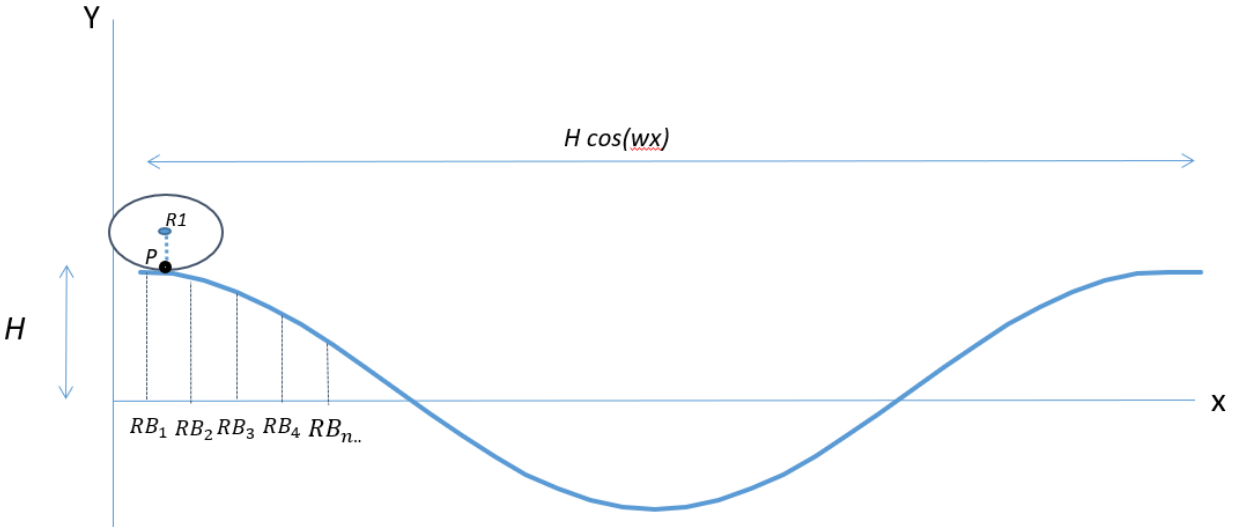

Fig. 3.5 Cosine trajectory

In order to perform numerical simulation of disk rolling down on cosine plane. Three governing equation and plane function need to be considered.

Y function of this plane are

$$Y = H\,cos(\omega x) \tag{1.1.1}$$

The slope can be obtained by differentiate equation 3.3.1

$$Slope = \ -\omega H\,sin(\omega x) \tag{1.1.2}$$

Then, angle of the cosine plane can be calculated

$$\Phi = Arctan(-\omega H\,sin(\omega x)\,) \tag{1.1.3}$$

The radius of the cosine plane are

$$R_B = \cot(\Phi) \tag{1.1.4}$$

Here equation of motion are divided in to three parts with these following conditions

1. Slope < 0 $\equiv$ disk rolling down incline plane
2. Slope = 0 $\equiv$ disk rolling on flat plane
3. Slope > 0 $\equiv$ disk rolling up incline plane

1.2 Disk rolling down incline plane

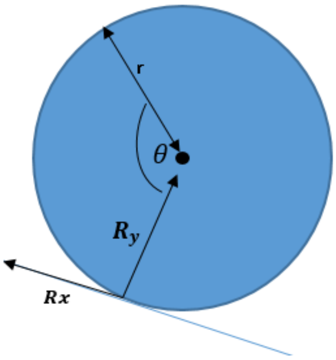

Fig. 3.6 Rolling down incline

Equation of motion when disk rolling down an incline plane

X direction : $m\ddot{x} = m\ g\ sin(\Phi) - R_x($ (1.2.1)

Y direction : $m\ddot{y} = -R_z - m\ g\ cos(\Phi)$ (1.2.2)

Rotational directional $\frac{mr^2}{2}\ddot{\theta} = r\ m\ g\ sin(\Phi) +\ r\ R_x$ (1.2.3)

Contact point X $\ddot{x} = r\ddot{\theta}$ (1.2.4)

Contact point Y $\ddot{y}$ (1.2.5)

Hence, the governing equation when disk rolling down can be shown as

$$\begin{bmatrix} m & 0 & 0 & 1 & 0 \\ 0 & m & 0 & 0 & 1 \\ 0 & 0 & \frac{mr^2}{2} & -r & 0 \\ 1 & 0 & -r & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \\ R_x \\ R_z \end{bmatrix} = \begin{bmatrix} m\ g\ sin(\Phi) \\ -m\ g\ cos(\Phi) \\ r\ m\ g\ sin(\Phi) \\ 0 \\ 0 \end{bmatrix} \tag{1.2.6}$$

1.3 Disk rolling on flat plane

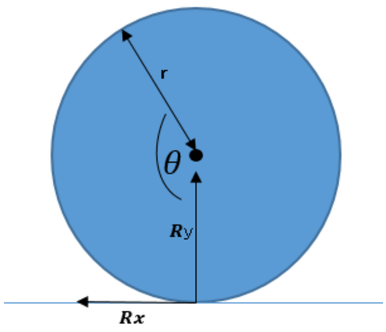

Fig. 3.7 Rolling on flat plane

Equation of motion when disk rolling on a flat plane

X direction : $m\ddot{x} = -R_x$ (1.3.1)

Y direction : $m\ddot{y} = -R_z - mg$ (1.3.2)

Rotational directional $\frac{mr^2}{2}\ddot{\theta} = rR_x$ (1.3.3)

Contact point X $\ddot{x} = r\ddot{\theta}$ (1.3.4)

Contact point Y $\ddot{y}$ (1.3.5)

The governing equation when disk rolling on flat plane

$$\begin{bmatrix} m & 0 & 0 & 1 & 0 \\ 0 & m & 0 & 0 & 1 \\ 0 & 0 & \frac{mr^2}{2} & -r & 0 \\ 1 & 0 & -r & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \\ R_x \\ R_z \end{bmatrix} = \begin{bmatrix} 0 \\ -mg \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{1.3.6}$$

1.4 Disk rolling up incline plane

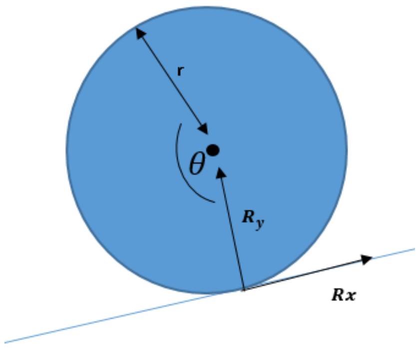

Fig. 3.8 Rolling on incline plane

Equation of motion when disk rolling up an incline plane

X direction : $$m\ddot{x} = -m\ g\ sin(\Phi) - R_x \tag{1.4.1}$$

Y direction : $$m\ddot{y} = -R_z - m\ g\ cos(\Phi) \tag{1.4.2}$$

Rotational directional $$\frac{mr^2}{2}\ddot{\theta} = -r\ m\ g\ sin(\Phi) + r\ R_x \tag{1.4.3}$$

Contact point X $$\ddot{x} = r\ddot{\theta} \tag{1.4.4}$$

Contact point Y $$\ddot{y} \tag{1.4.5}$$

Hence, the governing equation when disk rolling down can be shown as

$$\begin{bmatrix} m & 0 & 0 & 1 & 0 \\ 0 & m & 0 & 0 & 1 \\ 0 & 0 & \frac{mr^2}{2} & -r & 0 \\ 1 & 0 & -r & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\theta} \\ R_x \\ R_z \end{bmatrix} = \begin{bmatrix} -m\ g\ sin(\Phi) \\ -m\ g\ cos(\Phi) \\ -r\ m\ g\ sin(\Phi) \\ 0 \\ 0 \end{bmatrix} \tag{1.4.6}$$