

UNIVERSITY OF EDINBURGH
SCHOOL OF MATHEMATICS
Bayesian Data Analysis

Assignment 3

Students' goals.

229 students aged 7-13 from 9 schools were asked whether popularity or sporting ability was most important to them. The aim is to create an appropriate model that takes into account variability between the schools (if necessary).

Statistical analysis.

Introduce the following random variables. Consider the i th student, $i = 1, 2, \dots, N = 229$, and set $Z_i = 1$ if popularity is more important to the student than sporting ability, and $Z_i = 0$ otherwise.

Possible likelihood:

$Z_i \mid \theta_k \sim \text{Bern}(\theta_k)$, where k is the number of the school the i th student is from, independently given θ_k , $\theta_k \in (0, 1)$, $k = 1, 2, \dots, 9$.

1. Run a Bayesian model for the case $\theta_1 = \theta_2 = \dots = \theta_9 = \theta$ (i.e. $Z_i \mid \theta \sim \text{Bern}(\theta)$ iid) with a non-informative prior for θ (e.g. Jeffreys $B(0.5, 0.5)$ or uniform $B(1, 1)$).
 - (a) Check convergence and autocorrelation. State the number of burn-in iterations. [1 mark]
 - (b) Give the posterior summaries for θ . Record the value of the DIC. [1 mark]
2. Run a Bayesian model with independent non-informative priors for θ_k , $k = 1, \dots, 9$.
 - (a) Check convergence and autocorrelation (if the autocorrelation is high, thin as necessary). State the number of burn-in iterations. [1 mark]
 - (b) Give the posterior summaries for θ_k , $k = 1, \dots, 9$. Record the value of the DIC. [1 mark]
3. Run a Bayesian model with a hierarchical exchangeable prior for θ_k , $k = 1, \dots, 9$:

$$\theta_k \mid a, b \sim \text{Beta}(a, b), \quad k = 1, \dots, 9,$$

with a non-informative prior for $a > 0$ and $b > 0$.

Consider two types of priors:

- 1) $a \sim \Gamma(0.01, 0.01)$ and $b \sim \Gamma(0.01, 0.01)$ independently;
- 2) $a \sim |t_3|$ and $b \sim |t_3|$ independently, where $|t_3|$ the absolute value of Students' t distribution with 3 degrees of freedom with density $p(x) = \frac{4}{\sqrt{3}\pi} \frac{1}{(1+x^2/3)^2}$, $x > 0$.
 - (a) For each of the two types of priors for a and b , check convergence and autocorrelation (if the autocorrelation is high, thin as necessary). State the number of burn-in iterations. Choose the prior such that the posterior distribution converges. [3 marks]
 - (b) Give the posterior summaries for a , b and θ_k , $k = 1, \dots, 9$. Record the value of the DIC. [1 mark]
4. Compare the three models, with identical, independent and the chosen hierarchical prior, by comparing their DIC. Which model is preferable? [2 marks]
(Give any other comments on the comparison of the models.)

Remark. In questions 1(a), 2(a), 3(a), provide history and autocorrelation plots as evidence for your conclusions. WinBUGS code for all 3 models is on Learn.