Practical 4: Predictive model criticism and model comparison using WinBUGS

1. **Bristol Royal Infirmary example**

   The data below are taken from the analysis presented to the Bristol Royal Infirmary Inquiry of data on mortality following paediatric cardiac surgery at Bristol and other specialist centres.

```
list(r=c(25, 24, 23, 25, 42,24,53,26,25,58,41,31),
      n=c(187,323,122,164,405,239,482,195,177,581,143,301))
```

   The data are also available in file `bristol-data.odc`.

   12 hospitals were considered in the analysis. The data refer to the total number (n) of open heart surgery operations carried out on children under 1 year old by each centre between April 1991 and March 1995, and the number of deaths (r) within 30 days of the operation.

   One of the aims of the analysis is to explore whether an assumption of exchangeability is reasonable for the mortality rates in all 12 hospitals, or whether there is any evidence that the mortality rates for one or more of the hospitals do not appear to be drawn from the same (parent) distribution as the rest. Additionally, the literature shows a consistent relationship between volume of surgery (i.e. the number of operations carried out) and the outcome for a range of operations and so it may be appropriate to consider whether volume of surgery (n) should be included as a covariate in this analysis.

   (a) Define a set of plausible candidate models for these data. A binomial likelihood $r_i \sim \text{Binomial}(p_i, n_i)$ seems reasonable for these data (similar to the model used for the Surgical example in practical 1), so focus attention on comparing different models for the hospital-specific mortality rates. Some potential models to consider include:

(1)	$p_i \sim \text{Uniform}(0,1)$	independent effects
(2)	$\text{logit}\ p_i \sim \text{Normal}(\mu, \tau)$	all hospitals exchangeable with Normal prior
(3)	$\text{logit}\ p_i \sim \text{Student-t}(\mu, \tau, 4)$	all hospitals exchangeable with t_4 prior
(4)	$\begin{cases} \text{logit}\ p_i \sim \text{Normal}(\mu, \tau) \\ \qquad\qquad i = 1, .., 10, 12 \\ \text{logit}\ p_{11} \sim \text{Normal}(0, 0.000001) \end{cases}$	hospitals 1–10, 12 exchangeable with Normal prior; independent effect for hospital 11 (outlier model)
$(5) - (7)$	$\text{logit}\ p_i = q_i + \beta * n_i$ $q_i \sim$ priors as for $\text{logit} p_i$ in (2)-(4)	covariate models

   i. Include statements in your model code to calculate posterior and mixed predictive p-values to help identify whether any of the hospitals appear to have unusual mortality rates.

      Hints:
      Calculate posterior and mixed predictive p-values based on comparing observed

and predicted number of deaths in each hospital (see lecture notes).
Remember that you will need to calculate *mid p-values* to summarise conflict between discrete measures. For example, to calculate $\Pr(r_i^{\text{pred}} > r_i)$ where r_i^{pred} and r_i are discrete quantities, use the following WinBUGS code:

```
prob[i] <- step(r.pred[i]-r[i]-0.001) + 0.5 * equals(r.pred[i]+r[i])
```

ii. Calculate the DIC for each model.

(b) Fit each model in turn and examine the predictive p-values (you may find it easier to set up a script to run the models, and just change the name of the model file and initial values file for each new run). Which hospital(s) appear to have an unusual mortality rate? Does changing the model (for example, including volume as a covariate) help to explain the unusual rates or do the p-values remain extreme for all models?

(c) Use the DIC to compare your models. Which is the best supported model?