

12.33 A : adjacency matrix of G .

$\lambda_1(A) \geq$ avg degree G .

$$\lambda_1(A) = \max_1 \frac{x^T A x}{x^T x} \geq \frac{1^T A 1}{1^T 1} = \frac{\sum_{(u,v) \in E} A(u,v)}{n} = \frac{\sum_u d(u)}{n} = \bar{d}$$

where \bar{d} is avg degree.

$$\therefore \lambda_1(A) \geq \bar{d}.$$