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 $u$  fixed vector

$$\max_{x \in \{0,1\}^n} x^T u u^T (1-x)$$

$$= (x^T u) (u^T (1-x))$$

$$= \left( \sum_i x_i u_i \right) \cdot \left( \sum u_j - \sum_i x_i u_i \right)$$

Since  $u$  is a fixed vector, and  $\sum u_j = M \Rightarrow$  fixed number.

Let  $\sum_i x_i u_i = \sigma \rightarrow$  note that this is equivalent to selecting some set of entries in  $u$  & summing them up.

$$\therefore \max \sigma \cdot (M - \sigma)$$

This is a quadratic in  $\sigma$ , and is maximized at

$$\sigma = \frac{M}{2}.$$

$\therefore$  This optimization problem is partitioning  $u$  into 2 sets such that both ~~area~~ sum of both are as equal as possible. [Formally, the  $\max \sigma (M - \sigma)$  is a concave function, with max at  $\sigma = \frac{M}{2}$ ]