

LESSON 1: THE BUILDING BLOCKS

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Intro to Machine Learning

Princeton Wintersession 2021

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Welcome!

About me:

- Particle physics PhD
- Now working in physics-informed ML
- Excited about scientific ML + AI ethics!

What We'll Cover:

- Conceptual foundation
- Mathematical intuition
- Model building fundamentals
- Application examples

More to Learn:

- Not a deep dive into the mathematics
- Not an exhaustive rundown of models/methods
- Find out what to learn more about

Course Guidelines:

- First time teaching this course
- Please interrupt!
- [Course website](#)
- Setup a local environment or work in Google Collab

Outline for the Course

- **Monday**

- Conceptual overview
- Mathematical foundations

- **Tuesday**

- Neural Networks

- **Wednesday**

- Convolutional Neural Networks
- Recurrent Neural Networks

- **Thursday**

- Generative Models
- Unsupervised Models

- **Friday**

- Designing an ML experiment
- Examples of research applications
- AI Ethics and Algorithmic Fairness
- Other topics

Outline for Today

- **What is ML and What Can It Do?**
- **Supervised Learning: Training**
- **Mathematical Foundations**
- **Introduction to Libraries**
- **Coding Exercises**

Introduction Time!

Why are you interested in ML?

- Name + Affiliation
- What made you want to take this course?
 - What do you most hope to learn?

MACHINE LEARNING

A Conceptual Overview



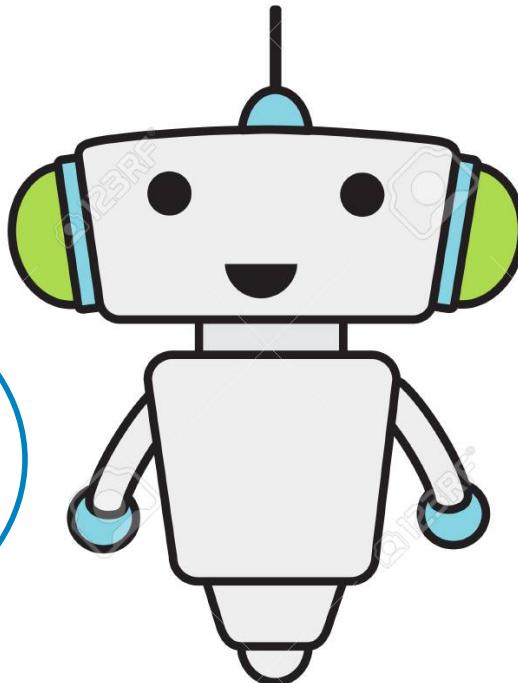
Killer Robots! Or, What is Machine Learning?

A field of study that gives the ability to the computer to learn without being explicitly programed

Algorithms that improve automatically through experience

Computer programs that can access data and use it to learn for themselves

Using statistics to find patterns in large datasets without the patterns being explicitly stated



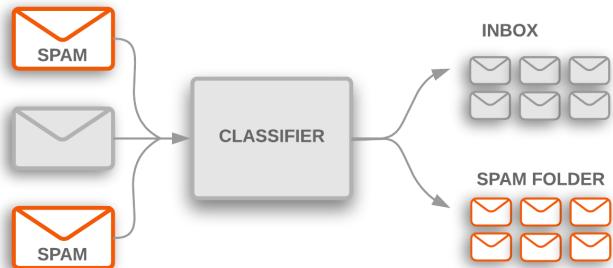
Teaching a computer system how to make accurate predictions when fed data

ML can be a great tool to enable scientific (and other) research, but it doesn't know what it doesn't know

What are Problems for ML?

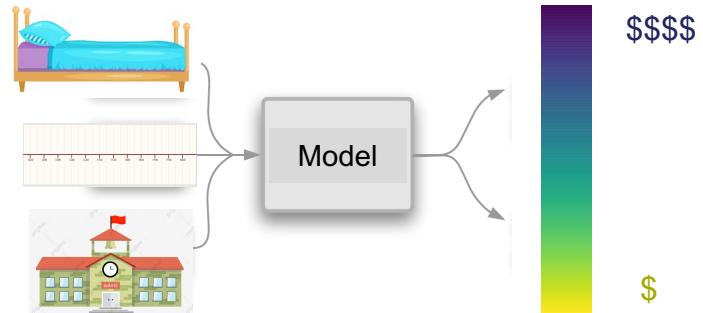
Classification:

Predict a class **label** for an input



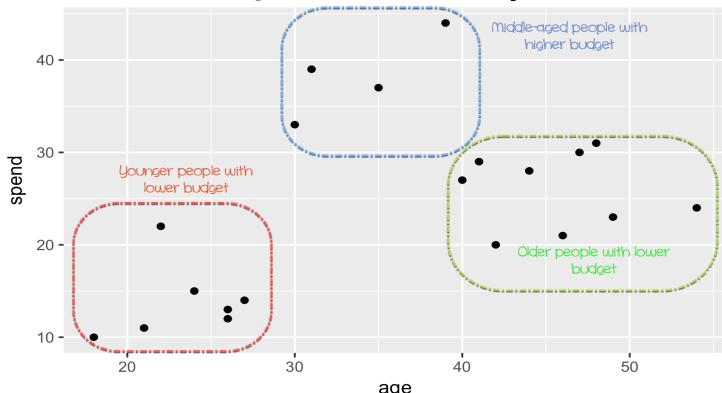
Regression:

Predict a **continuous variable**



Clustering:

Group similar inputs



Generation:

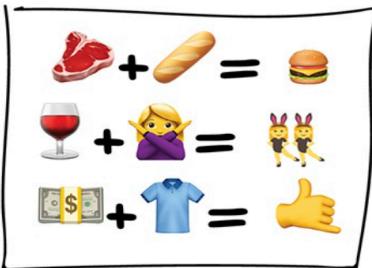
Construct new data within pattern



What are Problems for ML?

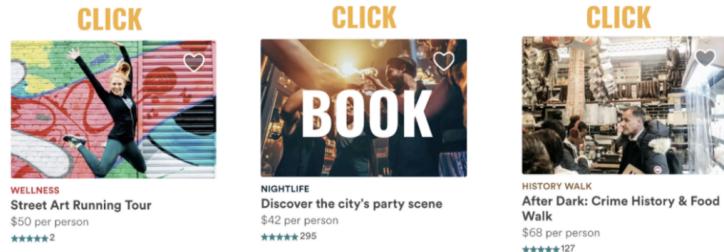
Association Rules:

Identify common patterns in data



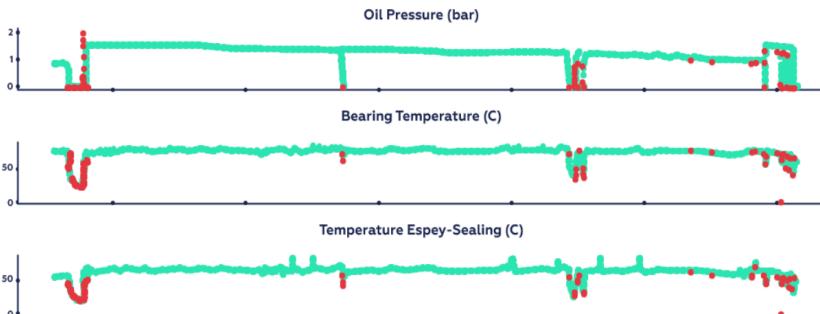
Ranking:

Generate optimal orderings



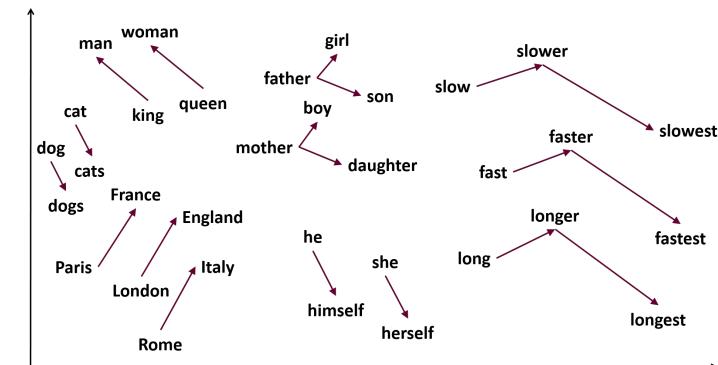
Anomaly Detection:

Identify statistical outliers



Restructuring:

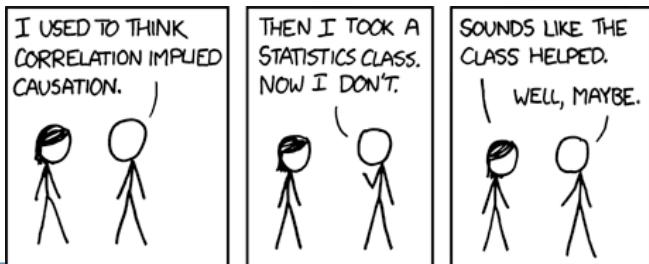
Transform data representations



What are NOT Problems for ML?*

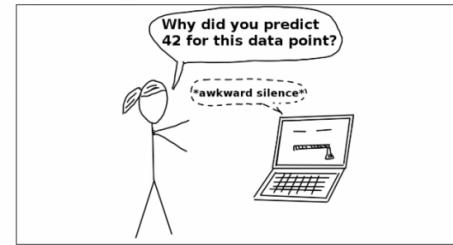
Causation:

Models learn correlations, but can't infer causality or intent



Precise Interpretability:

It's often difficult to understand what a model is learning



Context:

Models are incapable of non-mathematical reasoning

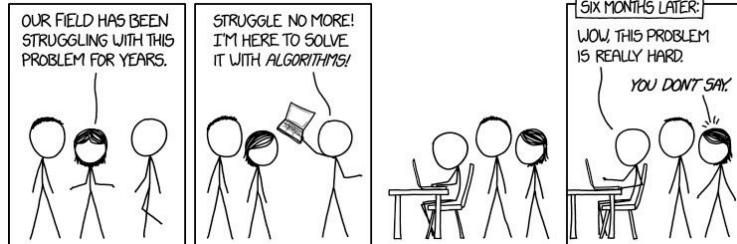


Keaton Patti

I forced a bot to watch over 1,000 episodes of Jerry Springer and then asked it to write an episode of its own. Here is the first page.

Data Limitations:

Models can't fix problems in data or learn without examples

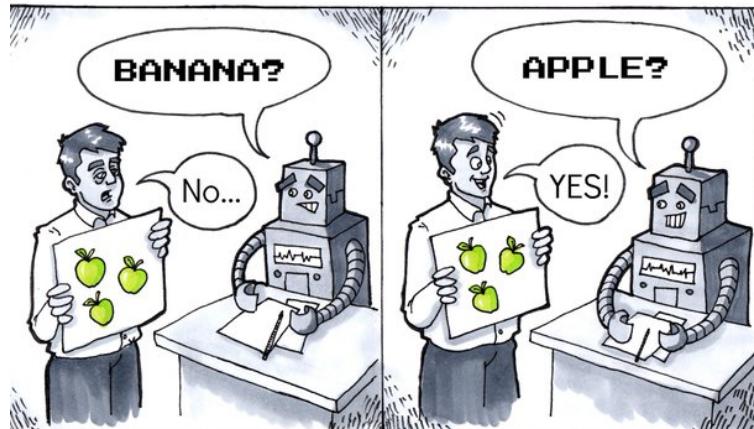


Supervision Required?

Supervised Learning

- Model is provided with **labels**
- Algorithm learns relationship* between features and labels during training

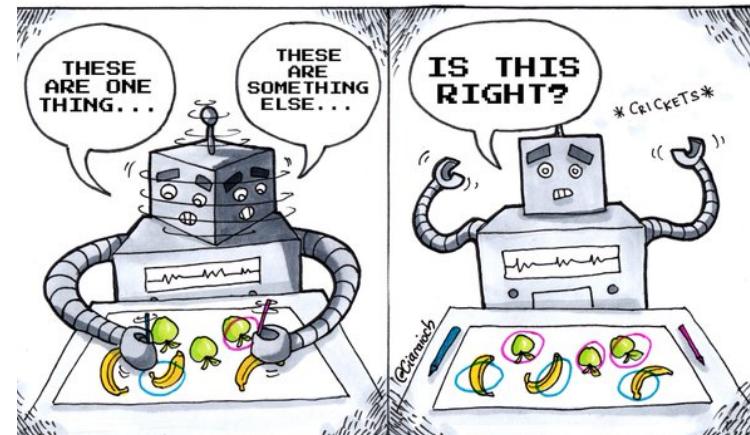
*if that relationship can be expressed as a mathematical function



Un-supervised Learning

- Model takes **unlabeled** or **unclassified** data
- Algorithm learns patterns* or groupings in the data during training

*but no promises that those patterns are useful!

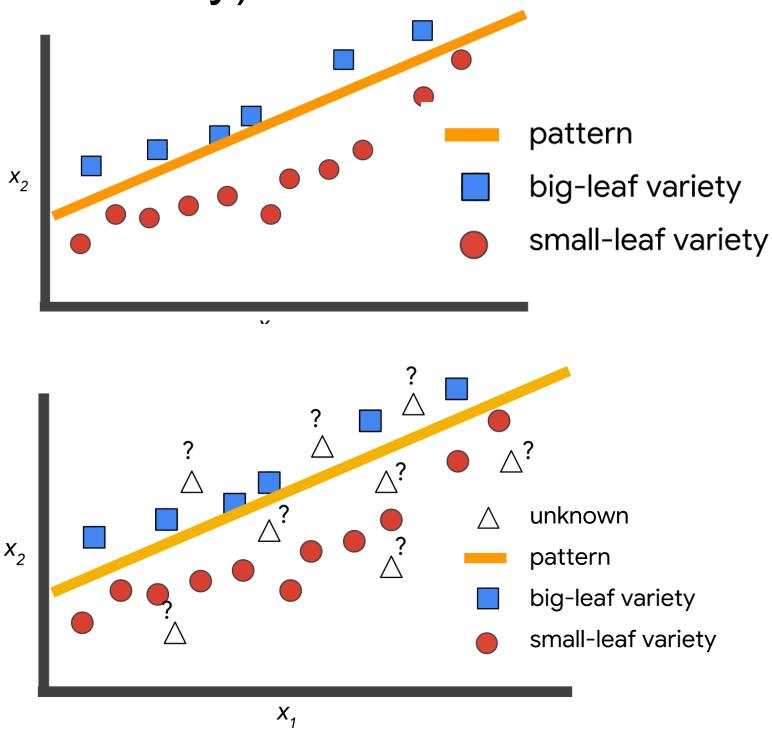


Bonus: Reinforcement Learning – models are rewarded for meeting goals

Supervision Required?

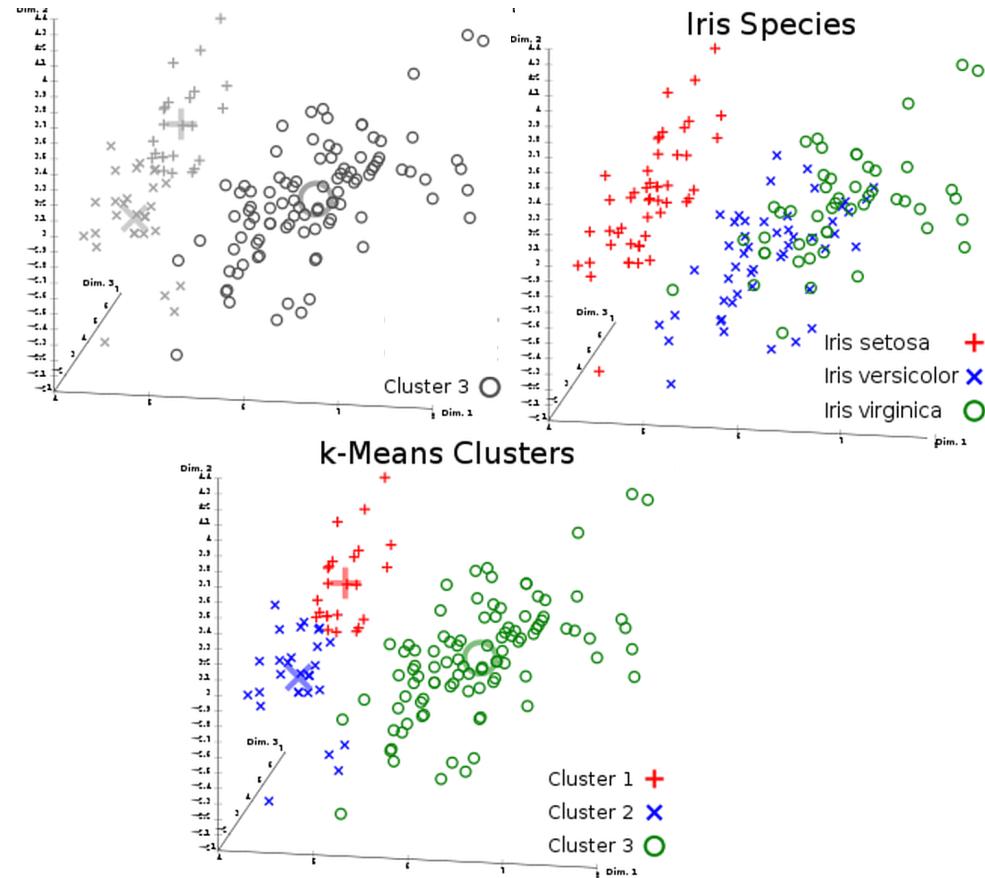
Supervised Learning

Example: distinguishing between two known classes (learn a decision boundary)

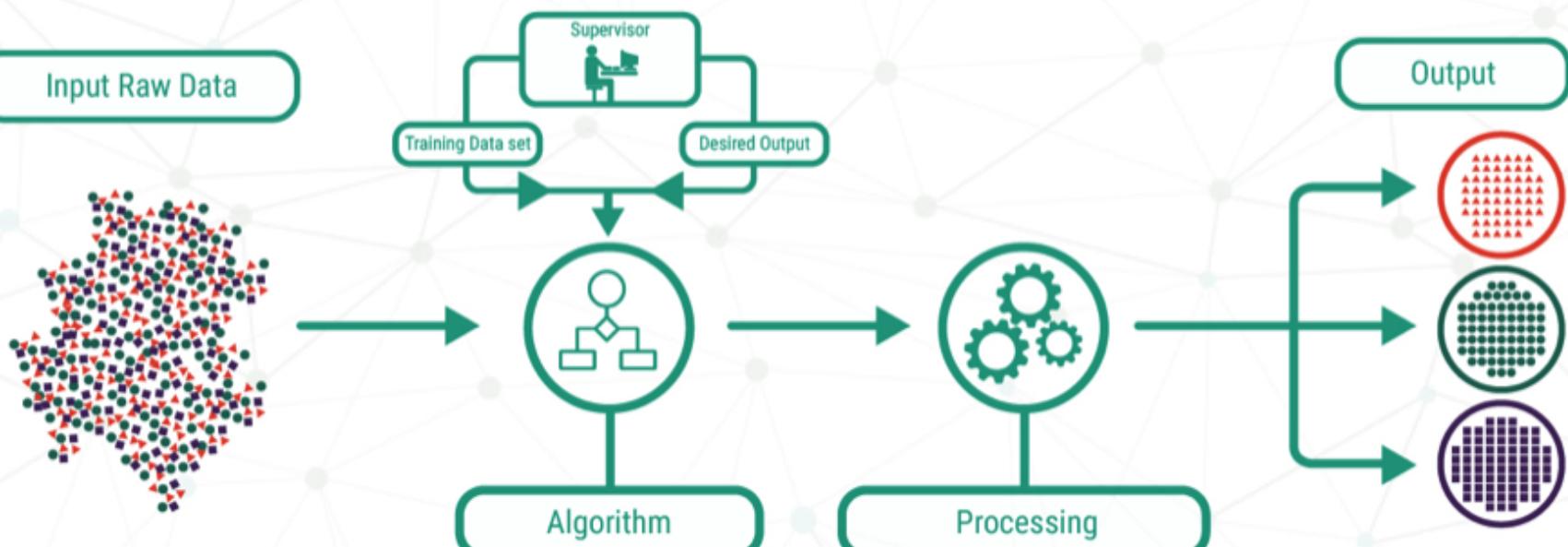


Un-supervised Learning

Example: grouping data into 3 classes (learn relational structure)

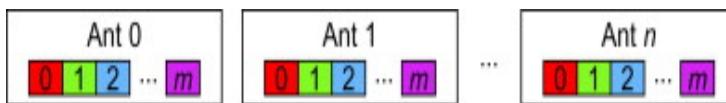


Supervised Algorithms

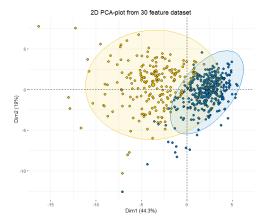
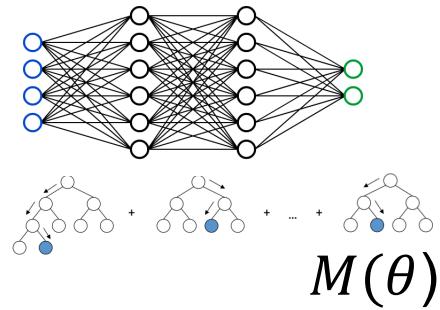


How Do You Actually Do This?

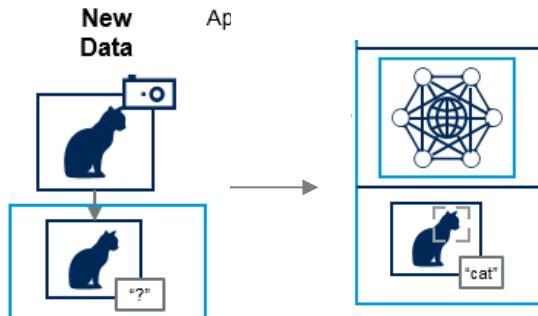
Training data



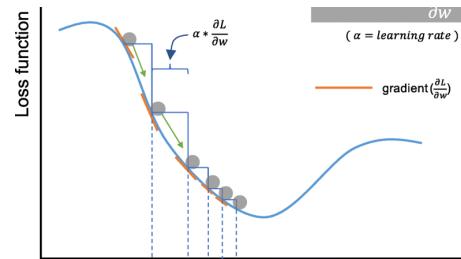
Choose a Model



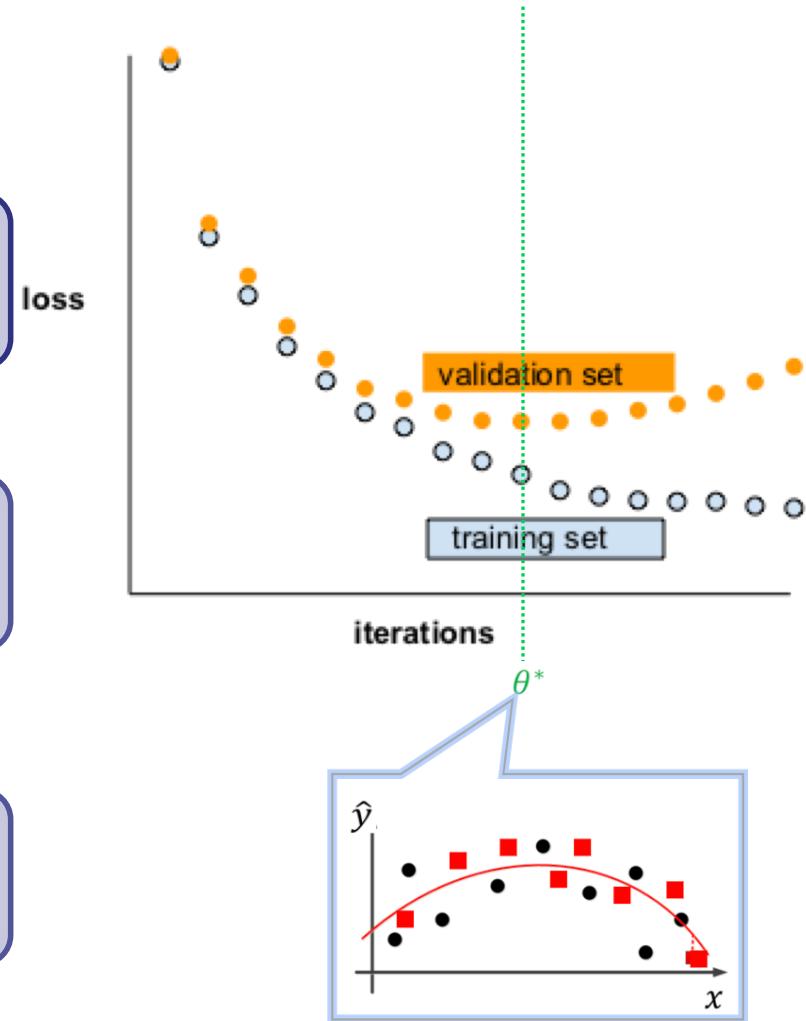
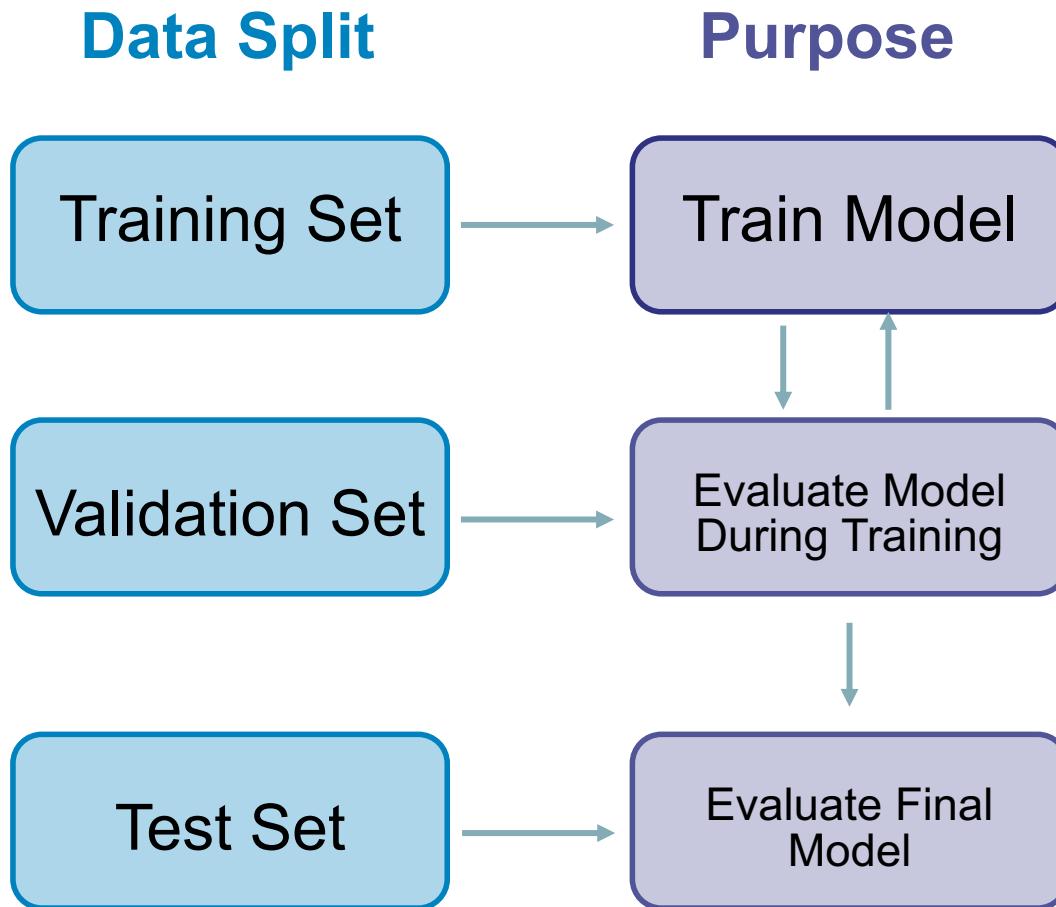
Make an Inference!



Optimize Parameters



Training a Model



Over and Under Fitting

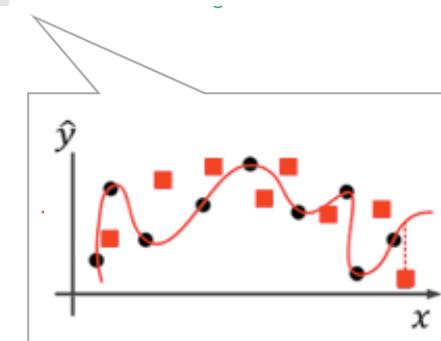
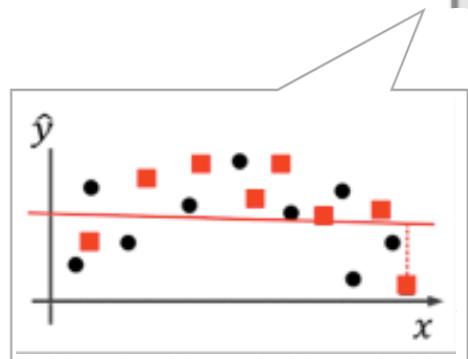
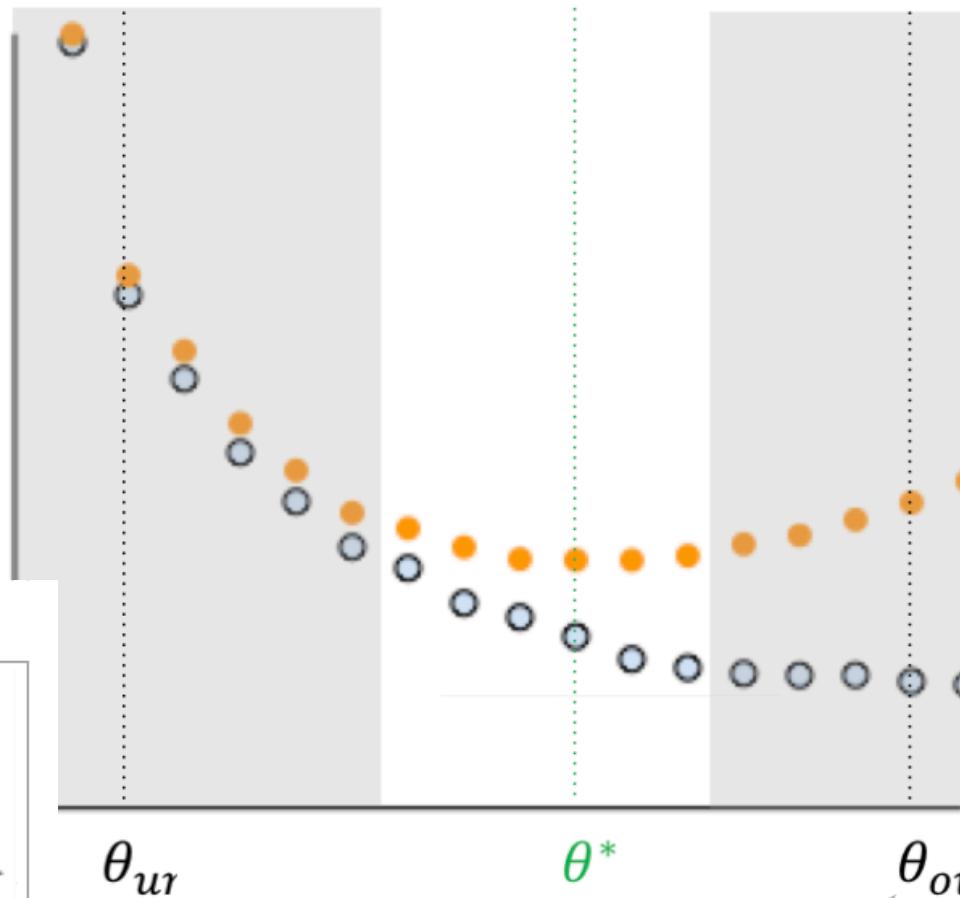


- Didn't learn general patterns in data
- High bias
- Low variance
- **Inaccurate inference**

- Learned specific details of training set
- Low bias
- High variance
- **Inaccurate inference**

Over and Under Fitting

Underfitting Optimum Overfitting



Choosing a Cost Function + Evaluation

Loss Functions

- Common choices:
 - Mean Squared Error

$$MSE = \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n}$$

- Hinge Loss

$$SVM\text{Loss} = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + 1)$$

- Cross Entropy

$$Cross\text{Entropy}\text{Loss} = -(y_i \log(\hat{y}_i) + (1 - y_i) \log(1 - \hat{y}_i))$$

- Considerations:

- Type of model
- Presence of outliers in dataset
- Sensitive outcomes
- Training behavior

[further reading](#)

Evaluation Metrics

- Common choices:

- Accuracy $\frac{Number\ of\ Correct\ predictions}{Total\ number\ of\ predictions\ made}$
- Confusion matrix
- Sensitivity $\frac{TruePositive}{FalseNegative + TruePositive}$
- Specificity $\frac{TrueNegative}{TrueNegative + FalsePositive}$
- False positive rate $\frac{FalsePositive}{TrueNegative + FalsePositive}$
- Precision $\frac{TruePositives}{TruePositives + FalsePositives}$
- Recall $\frac{TruePositives}{TruePositives + FalseNegatives}$
- F1 Score $F1 = 2 * \frac{1}{\frac{1}{precision} + \frac{1}{recall}}$

- Considerations:

- What type of outcomes matter?
- Typically should report multiple

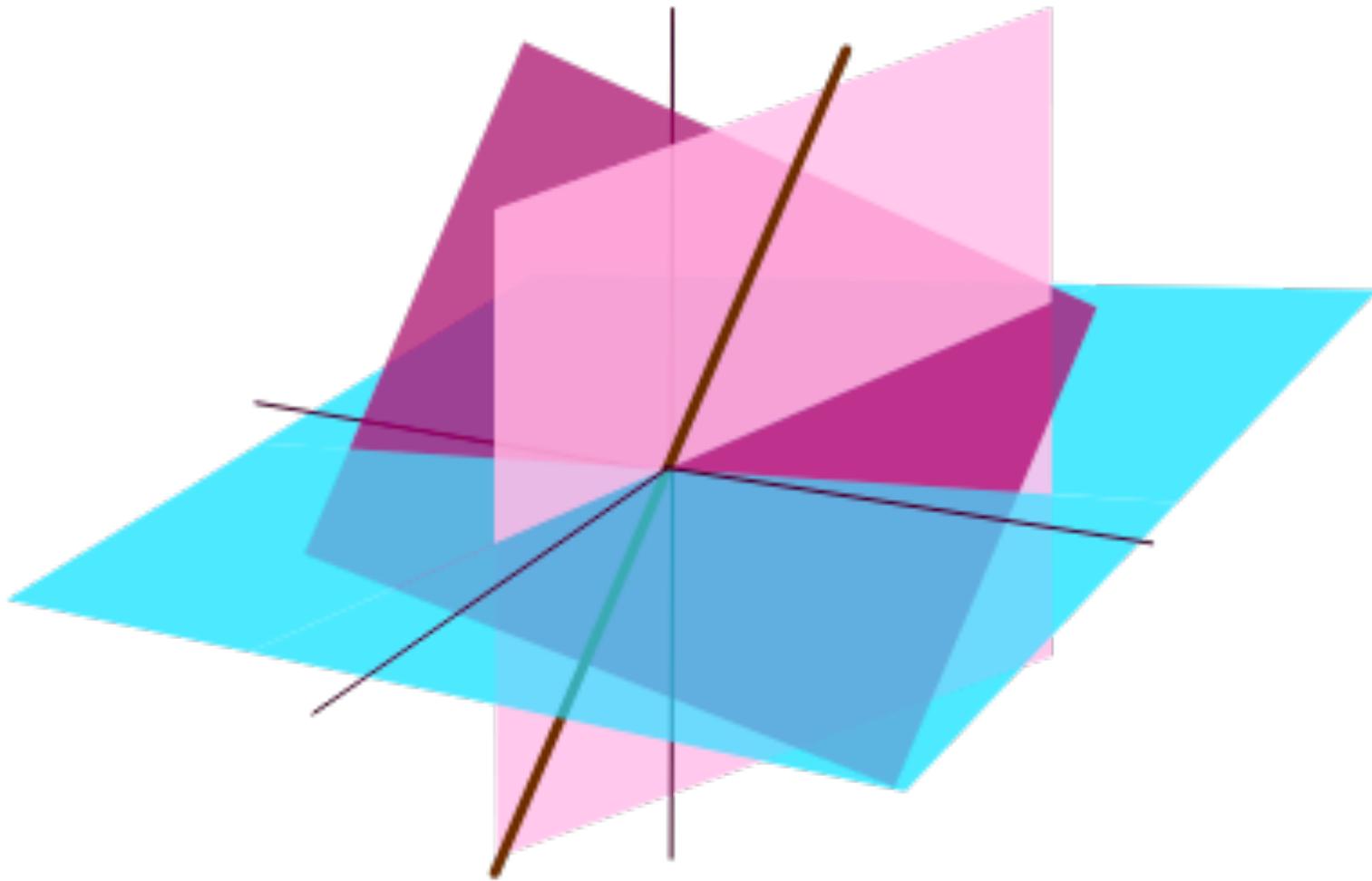
[further reading](#)

Question Time!

Can you think of an example of ML in your daily life?:

- Do you think it's a supervised or un-supervised model?
 - What data could have been used to train it?
 - What might have been the learning objective?
- Can you imagine some important training considerations?

A Quick Linear Algebra Refresher



Matrix Basics

$$\begin{bmatrix} 460 \\ 430 \\ 480 \\ \dots \\ \dots \end{bmatrix}$$

Vectors

- A single column of data
- Can represent a single variable, training labels, model parameters

$$\begin{pmatrix} a_1 & a_2 \\ a_3 & a_4 \end{pmatrix}$$

Where $a_{ij} \in \mathbb{R}$

Matrices

- $m \times n$ matrix: m columns of n elements
- Often represents training data, model parameters, convolutions

Matrix Multiplication

- Dot product of rows and columns
- Rows of first matrix multiplied by columns of second matrix
- Allows dimension transformations

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} * \begin{pmatrix} 7 & 8 \\ 9 & 10 \\ 9 & 12 \end{pmatrix} = \begin{pmatrix} 52 & 64 \\ 127 & 154 \end{pmatrix}$$

Where,

$$[1, 2, 3] * [8, 10, 12] = 1 * 8 + 2 * 10 + 3 * 12 = 64$$

Matrix Types + Operations

$$A = \begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix} \quad A^T = \begin{pmatrix} 1 & -2 \\ 4 & 3 \end{pmatrix}$$

Matrix Transpose

B is transpose of A if $a_{ij} = b_{ji}$

Matrix Inverse

B is inverse of A if $AB=BA$

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 4 & 4 & 5 \\ 6 & 7 & 7 \end{pmatrix} \quad B = \begin{pmatrix} -7 & -7 & 6 \\ 2 & 1 & -1 \\ 4 & 5 & -4 \end{pmatrix}$$

$$AA^T = I = A^T A$$

$$\text{Where, } A^{-1} = A^T$$

Orthogonal Matrix

Square matrix with columns of unit length

Diagonal Matrix

$a_{ij} = 0$ for $i \neq j$, $a_{ii} \neq 0$ for $i=j$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Matrices + Models

Matrix multiplication for model inference

| No. of Rooms (x0) | Size (Square feet) (x1) | Year Built (x2) | No. of Floors (x3) | Price (y) |
|-------------------|-------------------------|-----------------|--------------------|-----------|
| 2 | 1434 | 2010 | 1 | 8500 |
| 3 | 1534 | 2019 | 2 | 9600 |
| 2 | 962 | 1996 | 3 | 25880 |
| ... | ... | ... | ... | ... |

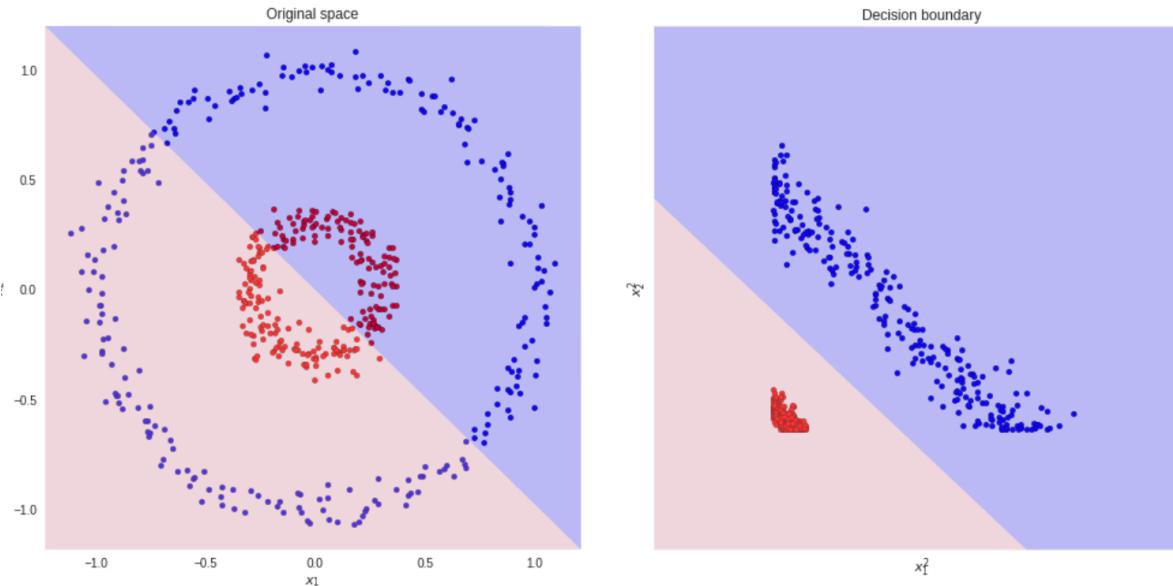
x_i = features of a house

y = target variable

$$x = \begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} \quad \& \quad \theta = \begin{pmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix}$$

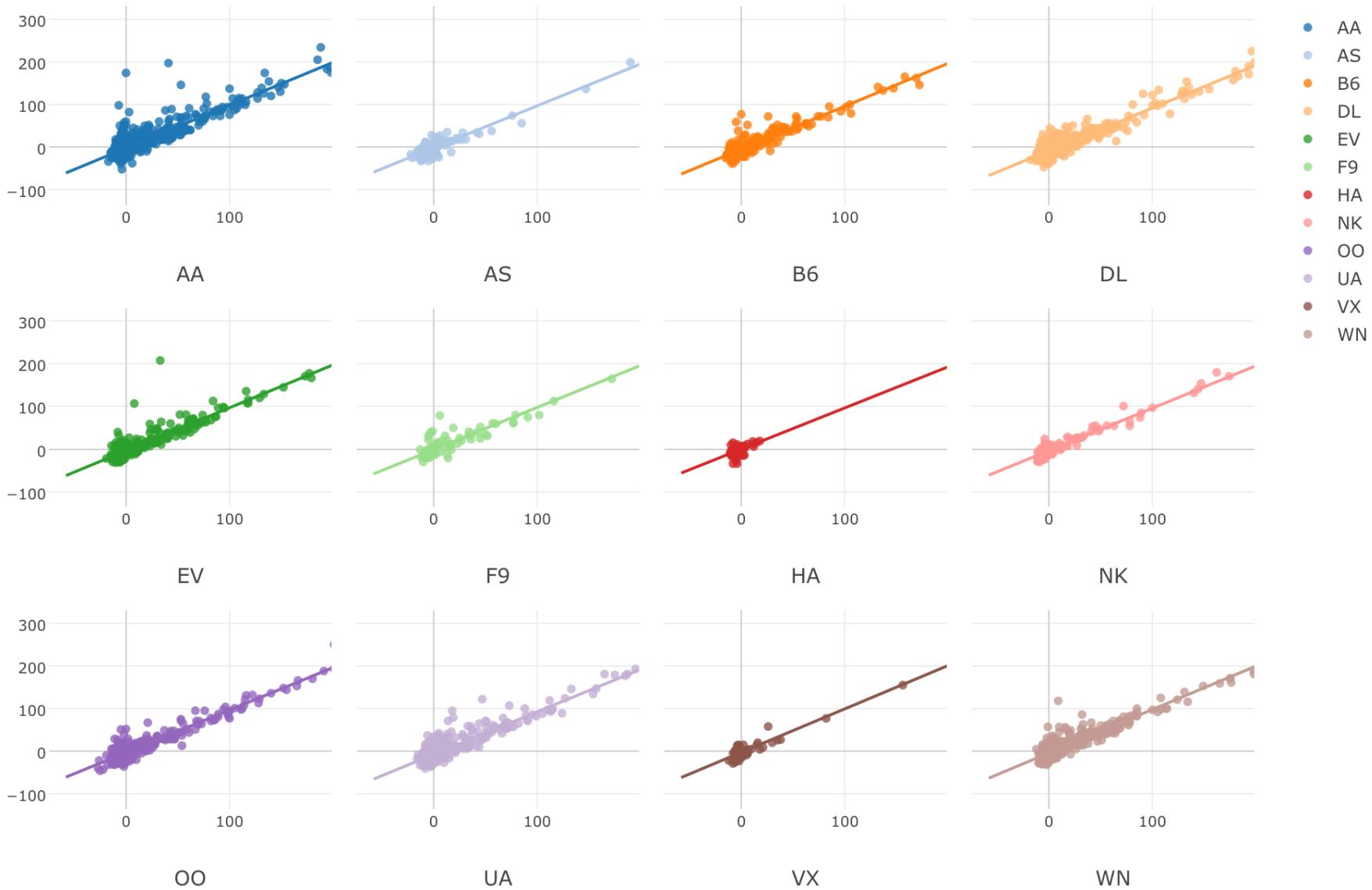
$$h_{\theta}(x) = \theta_0 x_0 + \theta_1 x_1 + \dots + \theta_n x_n = y'$$

Matrix multiplication for data transformation



$$y_n = \mathbf{W}^T \mathbf{x}_n$$

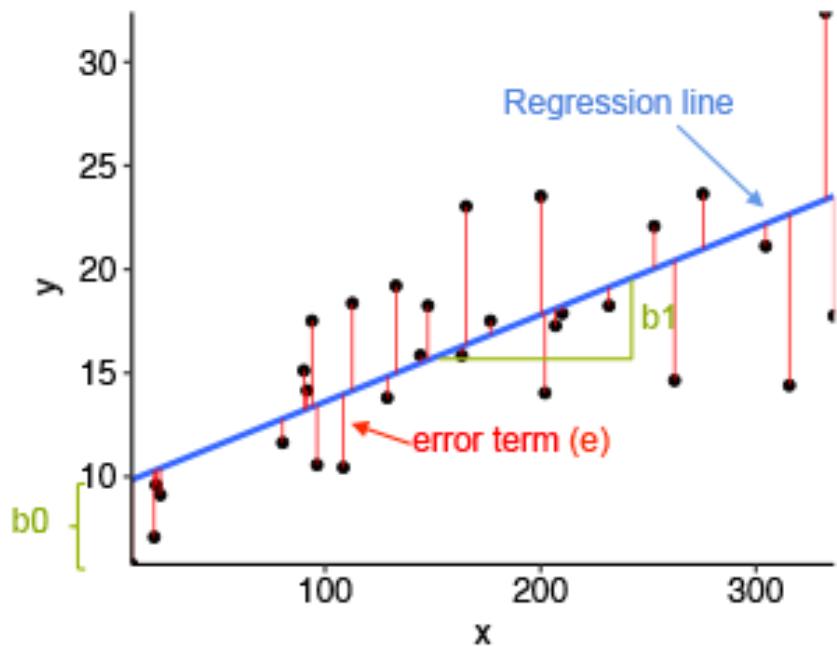
The Simplest ‘ML’ Models



Linear Regression as ML

Fit the set of dependent and independent data of a dataset to a linear line function

Allows prediction of continuous variables



A specific case of linear models

$$y = y(x) \rightarrow y(x_i) = \tilde{y}_i + \epsilon_i = \sum_{j=0}^{n-1} \beta_j x_i^j + \epsilon_i,$$

$$y_0 = \beta_0 + \beta_1 x_0^1 + \beta_2 x_0^2 + \cdots + \beta_{n-1} x_0^{n-1} + \epsilon_0$$

$$y_1 = \beta_0 + \beta_1 x_1^1 + \beta_2 x_1^2 + \cdots + \beta_{n-1} x_1^{n-1} + \epsilon_1$$

$$y_2 = \beta_0 + \beta_1 x_2^1 + \beta_2 x_2^2 + \cdots + \beta_{n-1} x_2^{n-1} + \epsilon_2$$

.....

$$y_{n-1} = \beta_0 + \beta_1 x_{n-1}^1 + \beta_2 x_{n-1}^2 + \cdots + \beta_{n-1} x_{n-1}^{n-1} + \epsilon_{n-1}.$$

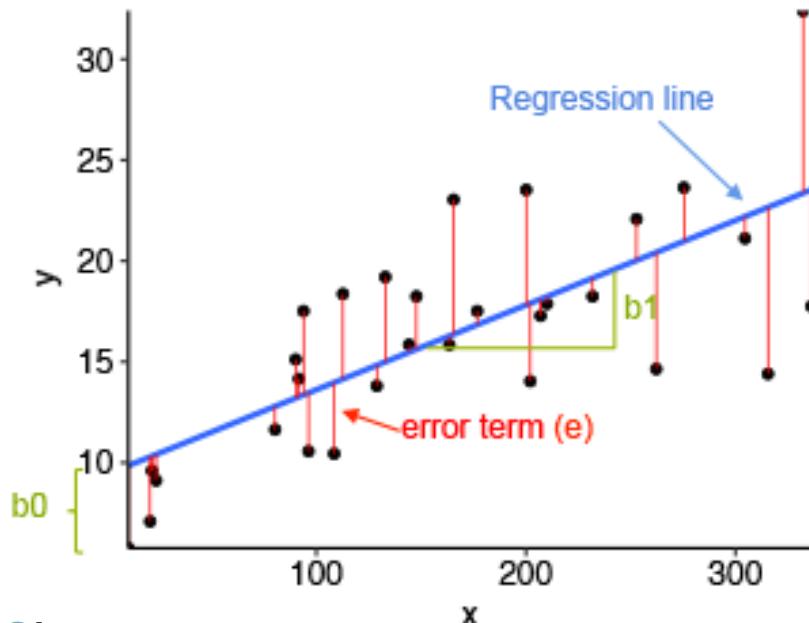
Learn parameters of
 β and ϵ

$$y = X\beta + \epsilon.$$

Linear Regression

Fit the set of dependent and independent data of a dataset to a linear line function

Allows prediction of continuous variables



- Training: Often Ordinary Least Squares
- Inference: use parameters of new data to calculate regression value
- Loss Function: $MSE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2$

[further reading](#)

Pros:

- Easy to implement and train
- Fast inference
- Interpretable

Cons:

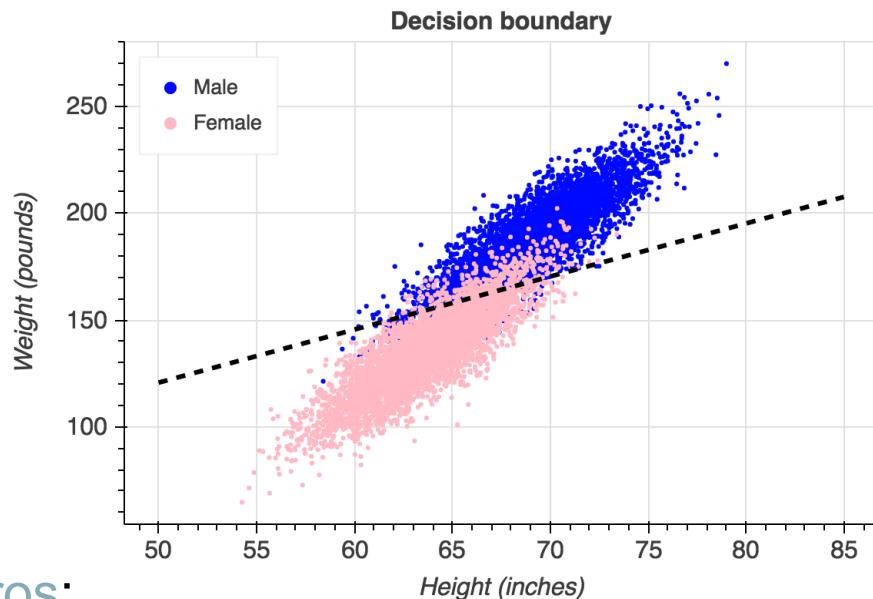
- Assumes linear relationship
- Overfits or doesn't converge with correlated features

Logistic Regression

Estimate the logarithmic probability of a class from linear combination of inputs

Assume probability depends linearly on features:

$$h\theta(X) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 X)}}$$



- **Training:** GD to adjust slope parameters
- **Inference:** select a classification threshold
- **Loss Function:**

$$Cost(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

[further reading](#)

Pros:

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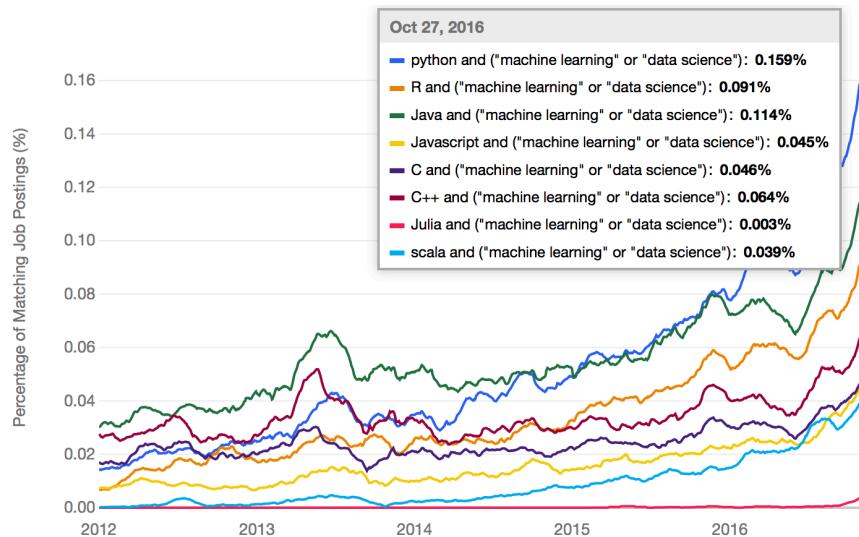
Machine Learning Libraries



Python

Interpreted, high-level, general-purpose programming language

- Most popular language for machine learning
- An extensive ecosystem of open-source libraries
- Easily readable, simple syntax
- Generally no need to compile



```
test_number = 407 # our example is not a prime number

# prime numbers are greater than 1

if test_number > 1:

    # check for factors

    number_list = range(2, test_number)

    for number in number_list:

        number_of_parts = test_number // number

        print(f"{test_number} is not a prime number")

        print(f"{number} times {number_of_parts} is {test_number}")

        break

    else:

        print(f"{test_number} is a prime number")

    else:

        print(f"{test_number} is not a prime number")
```

[further reading](#)

Fundamental Libraries



Provides an easy way to handle arrays (matrices) in python



Provides high-performance data structures and analysis tools (matrix manipulation)



Provides a range of customizable data visualization methods



Provides modules for optimization, linear algebra, and mathematical/scientific data processing

Keras

An open-source library providing intuitive support for neural networks and other ML models

- A user-friendly interface for TensorFlow (Google's ML library)
 - Other common ML library is Pytorch (Facebook)
- Contains building blocks of models, optimization and training methods, and data loading functionality
 - However, limited customization

```
model = Sequential()
model.add(Dense(32, activation='relu', input_dim=100))
model.add(Dense(1, activation='sigmoid'))
model.compile(optimizer='rmsprop',
              loss='binary_crossentropy',
              metrics=['accuracy'])

# Generate dummy data
import numpy as np
data = np.random.random((1000, 100))
labels = np.random.randint(2, size=(1000, 1))

# Train the model, iterating on the data in batches of 32 samples
model.fit(data, labels, epochs=10, batch_size=32)
```

Coding Time!

Happy to answer any questions!

[link](#) to Day 1 notebook

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