Simulation of Single Server Single Queue system

1. Develop the solution using C/C++/java/scilab/R/python
2. Calculate the six parameters given on slide-2
3. Compare the calculated values from table with the values obtained through direct formulae.

- Suppose inter-arrival times are determined by rolling a die. If the numbers 6,1,4,3,6,5 are rolled this means that the customers arrive at times: 0,6,7,11,14,20,25
- Suppose also that the service time for each customer is : 2, 3, 1, 1, 1, 1, 2
- The following table represents the time spent by customers in the system

Customer	Arrival time	Service begin	Service time	Service ends	Wait	Idle_server
1	0	0	2	2	0	6
2	6	6	3	9	0	0
3	7	9	1	10	2	0
4	11	11	1	12	0	1
5	14	14	1	15	0	2
6	20	20	1	21	0	5
7	25	25	2	27	0	4

- The Customer 3 has to wait for 2 time units before being served since the server is still busy serving customer 2.
- After serving customer 3(at time10) the server is idle for 1 time unit customer 4 arrives, which is then served immediately.

Exercise: Single queue, single server

In the example given on last slide, calculate

1. The average waiting time per customer :
    1. Total waiting time/number of customers
2. The probability of a customer to wait in the queue.:
    1. Total number of waiting customers/Total number of customers
3. The proportion of idle time for servers:
    1. idle time of server/total run time
4. The average service time:
    1. Total service time/number of customers
5. The average arrival time:
    1. Sum of interarrival times/ Total customers arrived
6. The average time that a customer has to spend in the system :
    1. Total time spent / number of customers

Equations of a single server Queue model

- If the items/customers arrive for service according to a Poisson distribution with mean λ.
- Assume also FIFO, no balking and no limit on queue size.
- The service time is taken as following the negative exponential distribution, with mean service time μ.
- The system as per Kendal notional can be written as
  - M/M/1/∞, where
  - Two M are two mean values of lamda and mu, 1 is one server and ∞ is the length of the waiting queue/buffer length.

Average number in system,

$$\overline{S} = \frac{\lambda}{\mu - \lambda}$$

Average number in queue,

$$\overline{q} = \overline{S} - \frac{\lambda}{\mu} = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

Average queueing,

$$\overline{W} = \overline{S} \times \frac{1}{\mu} = \frac{\lambda}{\mu(\mu - \lambda)}$$

Average time in system,

$$\overline{t} = \overline{W} + \frac{1}{\mu} = \frac{1}{\mu - \lambda}$$

Probability of no customer in system,

$$P(0) = 1 - \frac{\lambda}{\mu}$$

(it is required that $\lambda/\mu < 1$)

Average number in the queue when it is not empty,

$$\overline{q} = \frac{\mu}{\mu - \lambda}$$