EECS 499 Algorithmic Robotics

Homework 1 - Basic Concepts

1 Written Assignment

(W1) Suppose we have three boxes: the first one has 4 red balls and 6 green balls, the other has 5 red and 5 green, and the last one has 3 red and 7 green balls. We roll a fair, 6-sided dice, if it falls on 1 or 2, pick a random ball from the first box, if it falls on 3 or 4, we then pick the ball from the second, and if it falls on 5 or 6, we pick from the third box. What is the probability of getting a red ball?

(W2) Suppose that there are 4 red balls, 10 green balls, and 6 yellow balls in one box. Three balls are selected randomly one after another without putting back in. What is the probability that they are all red? What is the probability that two are red and one is green?

(W3) In recent years, it has rained in average 100 days per year. The weather forecast has predicted rain for tomorrow. When it actually rains, the weather forecast correctly forecasts rain 90% of the time. When it doesn't rain, it incorrectly forecasts rain 10% of the time. What is the probability that it will rain tomorrow?

2 Programming Assignment

(P1) Consider a Gaussian distribution $X \sim \mathbf{N}(0,\ 1)$. Generate 3 different set of samples from this distribution for each of the three sample sizes $N = 100$, $N = 1,000$, and $N = 100,000$, and construct the corresponding histograms. Plot the resulting histograms and the underlying probability density function.

(P2) Consider a Uniform distribution $X \sim \mathbf{U}(0, 1.5)$. Generate 3 different set of samples from this distribution for each of the three sample sizes $N = 100$, $N = 1,000$, and $N = 100,000$, and construct the corresponding histograms. Plot the resulting histograms and the underlying probability density function.

(P3) Consider the probability distribution

$$X \sim f(x) = \begin{cases} |x| & x \in [-1, 1] \\ 0 & \text{otherwise} \end{cases},$$

with a domain of $[-2,\ 2]$. Generate a set of samples from this distribution for a sample size of $N = 100,000$ using

rejection sampling, and construct the corresponding histograms. Plot the resulting histogram and the underlying probability density function.

(P4) Consider a mixture of two Gaussians, $f_x = \sum_{i=1}^{2} p_i f_i$, where $p_1 = 0.3$, $p_2 = 0.7$ and $f_1 \sim \mathbf{N}(0,\ 1)$, $f_2 \sim \mathbf{N}(2,\ 0.5)$, with the domain of $[-5,\ 5]$. Generate the samples from this distribution using the sample size of $N = 100,000$, then construct the resulting histogram. Plot the resulting histogram and the underlying probability density function.

(P5) Consider a Gaussian distribution $X \sim \mathbf{N}(1,\ 0.5)$ with the domain of $[-3,\ 3]$, generate its histogram $h(X)$ using sample size of $N = 100,000$. For any $x \in X$, there is a nonlinear transformation $y = x^2$, what is the histogram of Y? Show your results of the histogram of X and the histogram of Y. Set the sample size $N = 1,000$, generate the histogram of X and the histogram of Y, then plot their corresponding density distribution based on the samples of X and Y (you can use "line" function to mark the density distribution, an example is given below).

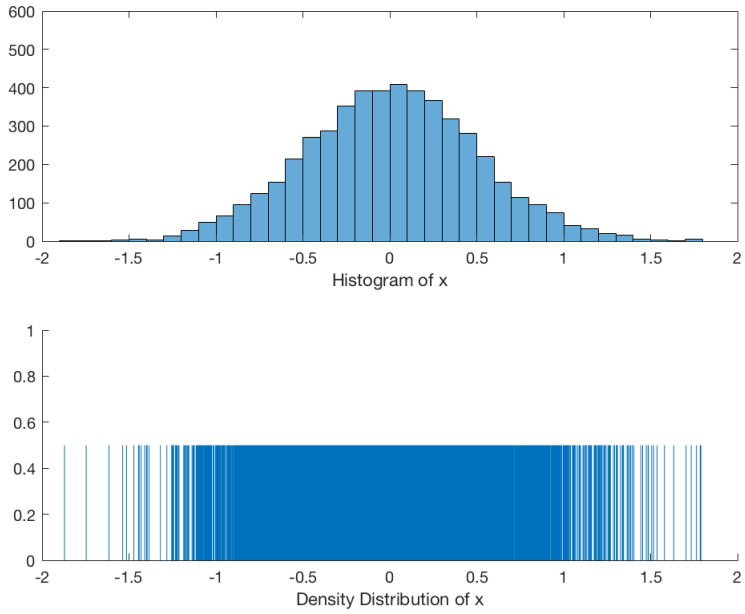

Figure 1: A example of histogram of $X \sim \mathbf{N}(0,\ 0.5)$ and its density distribution.