

$$9. \quad k_\pi = \sum_{\sigma \geq \pi} \text{sign}(\pi, \sigma) \lambda(\pi, \sigma)! f_\sigma \quad f_\pi = \sum_{\sigma \geq \pi} \lambda(\pi, \sigma)! k_\sigma$$

$$10. \quad a_\pi = \sum_{\sigma \leq \pi} (\text{sign } \sigma) \lambda(\sigma, \pi)! h_\sigma \quad h_\pi = \sum_{\sigma \leq \pi} (\text{sign } \sigma) \lambda(\sigma, \pi)! a_\sigma$$

## Appendix 2: The inner product on $\tilde{\mathcal{P}}$

1.  $(\tilde{h}_\pi, \tilde{k}_\sigma) = n! \delta_{\pi\sigma}$  Could there be an extra n! in every single one of these things????
2.  $(\tilde{s}_\pi, \tilde{s}_\sigma) = \delta_{\pi\sigma} \frac{n!}{|\mu(0, \pi)|}$
3.  $(\tilde{h}_\pi, \tilde{s}_\sigma) = n! \zeta(\sigma, \pi)$ , where  $\zeta(\sigma, \pi) = \begin{cases} 1 & \text{if } \sigma \leq \pi \\ 0 & \text{if not} \end{cases}$
4.  $(\tilde{a}_\pi, \tilde{s}_\sigma) = n! (\text{sign } \sigma) \zeta(\sigma, \pi)$
5.  $(\tilde{a}_\pi, \tilde{a}_\sigma) = n! \lambda(\sigma \wedge \pi)!$
6.  $(\tilde{a}_\pi, \tilde{h}_\sigma) = \begin{cases} n! & \text{if } \pi \wedge \sigma = 0 \\ 0 & \text{if } \pi \wedge \sigma \neq 0 \end{cases}$
7.  $(\tilde{k}_\pi, \tilde{k}_\sigma) = n! \sum_{\tau \geq \pi \vee \sigma} \frac{\mu(\pi, \tau) \mu(\sigma, \tau)}{|\mu(0, \tau)|}$
8.  $(\tilde{a}_\pi, \tilde{k}_\sigma) = n! (\text{sign } \sigma) \lambda(\sigma, \pi)! \zeta(\sigma, \pi)$
9.  $(\tilde{h}_\pi, \tilde{h}_\sigma) = n! \lambda(\sigma \wedge \pi)!$
10.  $(\tilde{k}_\pi, \tilde{s}_\sigma) = n! \frac{\mu(\pi, \sigma)}{|\mu(0, \sigma)|} \zeta(\pi, \sigma)$
11.  $(\tilde{f}_\pi, \tilde{k}_\sigma) = n! \sum_{\tau \geq \pi \vee \sigma} \frac{|\mu(\pi, \tau)| \mu(\sigma, \tau)}{|\mu(0, \tau)|}$
12.  $(\tilde{f}_\pi, \tilde{f}_\sigma) = (\text{sign } \pi) (\text{sign } \sigma) n! \sum_{\tau \geq \pi \vee \sigma} \frac{\mu(\pi, \tau) \mu(\sigma, \tau)}{|\mu(0, \tau)|}$
13.  $(\tilde{f}_\pi, \tilde{h}_\sigma) = n! \lambda(\pi, \sigma)! \zeta(\pi, \sigma)$
14.  $(\tilde{f}_\pi, \tilde{s}_\sigma) = (\text{sign } \pi) (\text{sign } \sigma) n! \frac{\mu(\pi, \sigma)}{|\mu(0, \sigma)|} \zeta(\pi, \sigma)$
15.  $(\tilde{f}_\pi, \tilde{a}_\sigma) = n! (\text{sign } \pi) \delta_{\pi\sigma}$

$\pi$  and  $\sigma$  are set partitions of  $[n]$ .  
 $\mu$  is the Mobius function in  $\Pi_n$

Doubilet's notation	Standard notation
k	m
a	e
h	h
s	p
f	f

$\text{sign}(\pi) = (-1)^{(\text{number of even blocks of } \pi)}$   
 $= (-1)^{(\text{rank of } \pi \text{ in partition lattice})}$

## Appendix 3: The Kronecker inner product on $\tilde{\mathcal{P}}$

1.  $[\tilde{s}_\pi, \tilde{s}_\sigma] = \frac{n!}{|\mu(0, \pi)|} \delta_{\pi\sigma} \tilde{s}_\pi$
2.  $[\tilde{h}_\pi, \tilde{s}_\sigma] = n! \zeta(\sigma, \pi) \tilde{s}_\sigma$
3.  $[\tilde{a}_\pi, \tilde{s}_\sigma] = n! (\text{sign } \sigma) \zeta(\sigma, \pi) \tilde{s}_\sigma$