MAE 263F: Homework 02

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***Abstract*— This document contains the report of the Homework 02 for the course MAE 263F taught at UCLA by Prof. M. Khalid Jawed. The homework has three assignments each of which has multiple sub questions. Detailed explanation and answers to those assignment is provided in the document.**

I. PROBLEM STATEMENT

The file Homework2.m in the GitHub repository includes MATLAB code for a solver that simulates an elastic rod using the discrete Elastic Rods Algorithm. The complete question of the assignment is from the notes, at the end of chapter 7, titled "Discrete Simulation of Slender Structures" by M. Khalid Jawed and Sangmin Lim. A screenshot of the exact the problem set is attached below.

Assignment: An elastic rod with a total length $l = 20$ cm is naturally curved with radius $R_n = 2$ cm. The location of its N nodes at $t = 0$ are

$$\mathbf{x}_k = \left[R_n \cos\left((k-1)\Delta\theta\right), \quad R_n \sin\left((k-1)\Delta\theta\right), \quad 0\right],$$

where $\Delta\theta = \frac{l}{R_n}\frac{1}{N-1}$. The twist angles θ^k $(k = 1, \ldots, N-1)$ at $t = 0$ are 0. The first two nodes and the first twist angle remain fixed throughout the simulation (i.e. one end is clamped). The physical parameters are: density $\rho = 1000$ kg/m^3, cross-sectional radius $r_0 = 1$ mm, Young's modulus $E = 10$ MPa, shear modulus $G = \frac{E}{3}$ (corresponding to an incompressible material), and gravitational acceleration $g = [0, 0, -9.81]^T$. Choose an appropriate time step size Δt and number of nodes N.

- Write a computer program that simulates the deformation of this rod under gravity from $t = 0$ to $t = 5$ s.
- Plot the z-coordinate of the last node ($\mathbf{x}_N$) with time. The solution can be found at the end of this Chapter.

Fig 1. The above figure is a screenshot of the problem statement from the notes chapter 7.

In this report, we detail the methods used to solve the problem, present the results, and draw conclusions based on those findings.

II. METHODOLOGY

This problem set was simulated using the Discrete Elastic Rod method. Additional details about the method can be found in the textbook. To address the problem, the pseudocode provided in the textbook was used as a reference for the simulation. The pseudocode is shown in the image below.

We have used discrete elastic rod method for the simulation. The DER is already given in the textbook. Below is a pseudocode describing the simulation.

Algorithm 1 Simulation Algorithm

```
Require: Input Constants
Require: Mass Matrix
Require: Initial DOF Vector
Require: Reference and Voronoi Length
Require: Reference frame at t = 0
Require: Material frame at t = 0
Require: Reference twist at t = 0
Require: Natural Curvature
Require: Free DOFs

Current time = 0
for timeStep = 1, timeStep++, timeStep < N_Steps do

    Current time ← Current time + Δt

    Guess: q^(1)(current) = q^(1)(previous)

    while Error < Tolerance do
        Compute Reference frame using
        Compute Reference Twist
        Compute Material Frame
        Compute f and J

        f_free ← f(free DOFs)
        J_free ← J(free DOFs)

        Δq_free ← J_free \ f_free
        q^(n+1)(free DOFs) ← q^(n)(free DOFs) − Δq_free

        Error ← sum(abs(f_free))
    end while

    q(current) ← q^(n)(current)
    q̇(current) = (q(current) − q(previous)) / Δt

    Store the reference frame for calculation at the next time step

    EndZ(timeStep) ← q(end)          ▷ This stores the z coordinate of the node at the end

    Plot the current configuration of rod    ▷ This displays the configuration of the rod at each time
end for

Plot EndZ vs time                  ▷ This graph is asked in the question
```

Fig 2. The above figure is the pseudocode for this problem from the notes

The pseudocode describes the simulation process for solving a problem using the Discrete Elastic Rod (DER) method. It begins by defining the required inputs, such as constants, mass matrix, degrees of freedom (DOF), natural curvature, material and reference frames, and initial configurations. The simulation proceeds iteratively for each time step, updating the current time and estimating the state of the system based on the previous time step. Within each iteration, an error tolerance loop is used to refine the reference and material frames and compute the forces and Jacobians for free degrees of freedom. These computations help adjust the system's state until the error falls below a set tolerance. Once the updated state is determined, it is stored for further use. At each step, the z-coordinate of the last node is recorded, and the current configuration of the rod is plotted. The process continues until the specified number of steps is completed, resulting in a graph of the z-coordinate of the last node over time, as required by the problem.

III. ASSIGNMENT RESULTS

We have an elastic rod with a total length of l = 20 cm, which is naturally curved with a radius of Rn = 2, cm. The rod is fixed at one end, with the first two nodes and the initial twist angle remaining constant throughout the simulation. Initially, the rod is shaped like a circular loop. When suspended under gravity, the rod moves for a short period before settling into a resting state. The images below show the initial circular configuration and the almost final shape of the rod after 4.96 seconds of simulation.

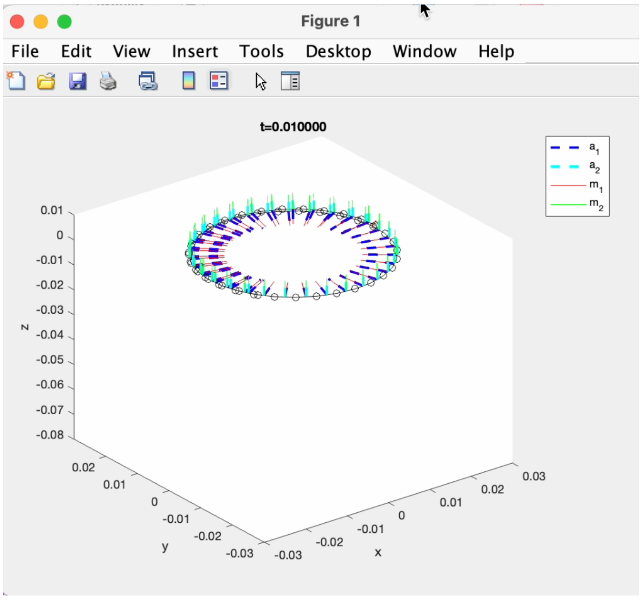

Fig 3. Rod's initial configuration in 3D at time = 0.01 seconds.

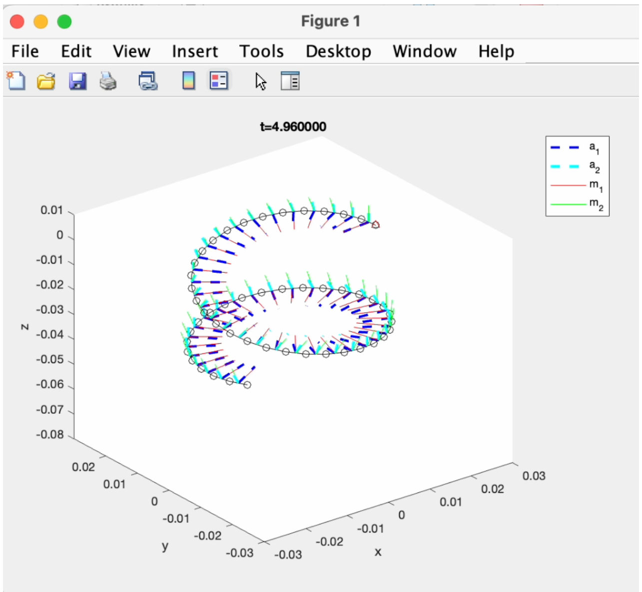

Fig 4. Rod's configuration towards the end of simulation at time = 0.01 seconds.

After completing the simulation, we analyze the required results. The graph below shows how the z-coordinate of the last node changes over time.

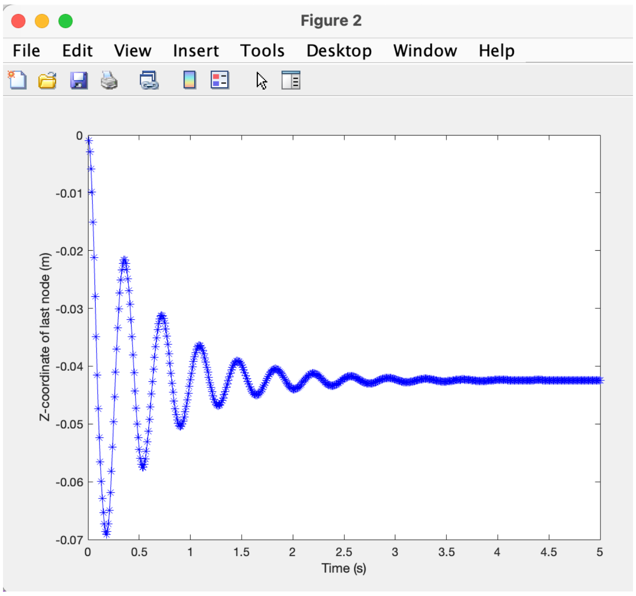

Fig 5. Plot of the Z-coordinate of the last node over time, illustrating the oscillatory behavior and eventual stabilization of the rod during the simulation.

The graph aligns with the solution provided at the end of the chapter in the notes.

IV. Discussion of Results

The simulation successfully modeled the deformation of an elastic rod under the influence of gravity using the Discrete Elastic Rod (DER) method. The initial configuration of the rod, a circular loop, gradually transformed as the rod adjusted to the forces of gravity. The results indicate that the rod reached a stable configuration after a period of oscillation, as observed in the plot of the z-coordinate of the last node over time. The oscillatory motion diminished steadily, showcasing the rod's natural damping behavior before coming to rest.

The pseudocode effectively captured the necessary computational steps, including reference frame updates, force and Jacobian calculations, and error tolerance checks, ensuring accurate results. The results also align with the solution provided in the notes, validating the correctness. This demonstrates the robustness of the DER approach in simulating flexible structures under physical forces. Future improvements could explore finer time steps or additional physical parameters to enhance the simulation's precision further.

References

[1] M. Khalid, Sangmin Lim, "Discrete simulation of slender structures". In Bruin Learn, https://bruinlearn.ucla.edu/courses/193842/files/18268468?module_item_id=6975313.

[2] Professor M. Khalid Jawed, MechAE 263F Course Modules, Link: https://bruinlearn.ucla.edu/courses/193842/modules/items/7007232