Assignment

Problems

1. Let X be a discrete random variable with probability mass function $p(i) = \frac{1}{5}$, $i = 1, 2, \ldots, 5$, zero elsewhere. Find $M_X(t)$.

2. Let X be a discrete random variable with the probability mass function $p(i) = 2\left(\frac{1}{3}\right)^i$, $i = 1, 2, 3, \ldots$; zero elsewhere. Find $M_X(t)$ and $E(X)$.

3. Let X be a continuous random variable with probability density function
$$f(x) = \begin{cases} 2x & \text{if } 0 \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$$
Find the moment-generating function of X.

4. Let X be a continuous random variable with the probability density function
$$f(x) = \begin{cases} 6x(1-x) & \text{if } 0 \le x \le 1, \\ 0 & \text{otherwise.} \end{cases}$$
   (a) Find $M_X(t)$.
   (b) Using $M_X(t)$, find $E(X)$.

5. Let X be a discrete random variable. Prove that $E(X^n) = M_X^{(n)}(0)$.

6. Suppose that 15 points are selected at random and independently from the interval $(0, 1)$. How many of them can be expected to be greater than $\frac{3}{4}$?

7. A number t is said to be the median of a continuous random variable X if
$$P(X \le t) = P(X \ge t) = \frac{1}{2}.$$
Calculate the median of the normal random variable with parameters μ and σ^2.

8. Show that the gamma density function with parameters (r, λ) has a unique maximum at $\frac{r-1}{\lambda}$.

9. Let X be a gamma random variable with parameters (r, λ). Find the distribution function of cX, where c is a positive constant.

10. Is the following a probability density function? Why or why not?

$$f(x) = \begin{cases} 120x^2(1-x)^4 & 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

11. For what value of c is the following a probability density function of some random variable X? Find $E(X)$ and $\mathrm{Var}(X)$.

$$f(x) = \begin{cases} cx^4(1-x)^5 & 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$