INDIAN INSTITUTE OF INFORMATION TECHNOLOGY KOTTAYAM

IMA 211: Probability, Statistics, and Random Processes

Assignment

1. A 60 percent of new drivers have had driver education. During their first year, new drivers without driver education have probability 0.08 of having an accident, but new drivers with driver education have only a 0.05 probability of an accident. What is the probability a new driver has had driver education, given that driver has had no accident in the first year?

2. As a part of a business strategy, randomly selected 20% of new internet service subscribers receive a special promotion from the provider. A group of 10 neighbours signs for the service. What is the probability that at least 4 of them get a special promotion?

3. Customers of an internet service provider initiate new accounts at the average rate of 10 accounts per day.
   (a) What is the probability that more than 8 new accounts will be initiated today?
   (b) What is the probability that more than 16 accounts will be initiated within 2 days?

4. A software package consists of 12 programs, five of which must be upgraded. If four programs are randomly chosen for testing.
   (a) What is the probability that at least two of them must be upgraded?
   (b) What is the expected number of programs, out of the chosen four, that must be upgraded?

5. A computer user tries to recall her password. She knows it can be one of four possible passwords. She tries her passwords until she finds the right one. Let X be the number of wrong passwords she uses before she finds the right one. Find $E(X)$ and $V(X)$.

6. Suppose that the average household income in some country is 900 coins, and the standard deviation is 200 coins. Assuming the normal distribution of income,
   (a) Compute the proportion of the "middle class" whose income is between 600 and 1200 coins.
   (b) The government of the country decided to issue food stamps to the poorest 3% of households. Below what income will families receive food stamps?

7. Upgrading a certain software package requires the installation of 82 new files. Files are installed consecutively. The installation time is random, but on average, it takes 15 sec to install one file, with a variance of 16 sec^2. What is the probability that the whole package will be upgraded in less than 20 minutes?

8. Determine the value of c that makes the function $f(x,y) = cxy$ is a joint PDF over the range $0 < x < 3$ & $0 < y < x$. Determine the following.
   (a) $P(X < 1, Y < 2)$ (b) $E(Y|X = 1)$ (c) $P(Y > 2|X = 1)$

9. A sample of 3 observations $(x_1 = 0.4,\ x_2 = 0.7,\ x_3 = 0.9)$ is collected from a continuous distribution with density

$$f_X(x) = \begin{cases} \theta x^{\theta-1}, & \text{if } 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

Estimate θ by (a) Method of moments (b) Maximum Likelihood Method

10. Installation of a certain hardware takes random time with a standard deviation of 5 minutes, A computer technician installs this hardware on 64 different computers, with the average installation time of 42 minutes. Give a 95% confidence interval for the true mean installation time.

11. Salaries of entry-level computer engineers have normal distribution with unknown mean and variance. Three randomly selected computer engineers have salaries (in $ 1000s) 30, 50, and 70. Construct a 90% confidence interval for the average salary of an entry-level computer engineer.

12. A random sample of 20 students yielded a mean of 72 and a variance of 16 for scores on a college placement test in Mathematics. Assuming the scores to be normally distributed, construct a 98% confidence interval for σ^2.

13. The federal government awarded grants to the agricultural departments of 9 universities to test the yield capabilities of two new varieties of wheat. Each variety was planted on a plot of equal area at each university, and the yields, in kilograms per plot, were recorded as follows: Construct a 90% confidence interval for σ_1^2/σ_2^2.

University	1	2	3	4	5	6	7	8	9
Variety 1	38	23	35	41	44	29	37	31	38
Variety 2	45	25	31	38	50	33	36	40	43