GENERAL FOUNDATION PROGRAMME

Module Notes

SPRING 2021-2022

Mathematics 02

Module Code: GFP-MATH02

Glossary of Mathematical Terms

S.No	Terms in English	Terms in Arabic
1	Cumulative frequency	التكرار التراكمي
2	Decay	تسوس
3	Determine	يحدد
4	Exponential Function	الدالة الأسية
5	Intercept	تقاطع
6	Logarithmic Function	الدالة اللوغارتمية
7	Mean	المتوسط
8	Median	الوسط
9	Mode	المنوال
10	Permutation	التباديل
11	Probability	الإحتمالات
12	Quadratic Equation	معادلة من الدرجة الثانية
13	Standard Deviation	الإنحراف المعياري
14	Symmetry	تناظر / تماثل

Formulae Sheet

Functions

For the graph of a quadratic function $ax^2 + bx + c$,

a) vertex is given by the formula $(x, y) = \left(\frac{-b}{2a},\ f\left(\frac{-b}{2a}\right)\right)$

b) axis of symmetry is the line $x = \frac{-b}{2a}$

c) The maximum or minimum value of a quadratic function $f(x) = ax^2 + bx + c$ occurs at $x = \frac{-b}{2a}$

If $a > 0$ then the minimum value is $f\left(\frac{-b}{2a}\right)$

If $a < 0$ then the maximum value is $f\left(\frac{-b}{2a}\right)$

Quadratic Formula

For any real numbers a, b and c, the solution to $ax^2 + bx + c = 0$ is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Exponents and Logarithms

Laws of Logarithms

- $\log_a A + \log_a B = \log_a (AB)$
- $\log_a A - \log_a B = \log_a \left(\frac{A}{B}\right)$
- $\log_a A^n = n \log_a A$
- $\log_a 1 = 0$
- $\log_a a = 1$

Exponential Growth and Decay

If A – Final Amount, P – Initial Amount, r – the relative Rate of growth, t – time

The exponential growth is given by the formula, $A = Pe^{rt}$

The exponential decay is given by the formula, $P = P_0 e^{-rt}$

Trigonometry

Law of sines

- $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Law of cosines

- $a^2 = b^2 + c^2 - 2bc\cos A$
- $b^2 = a^2 + c^2 - 2ac\cos B$
- $c^2 = a^2 + b^2 - 2ab\cos C$

Arithmetic Mean

Mean ($\bar{x}$) = $\frac{1}{N}\sum_{i=1}^{n} f_i x_i$, where N = $\sum_{i=1}^{n} f_i$

Learning Outcomes

After the completion of this module, student should be able to:

a) Understand the concepts of quadratic, exponential and logarithmic functions; sketch their graphs and apply in real life problems.

b) Solve problems related to quadratic, exponential and logarithmic equations.

c) Solve two variable linear equations/inequalities graphically and algebraically.

d) Use the law of sines and cosines to solve triangle and real-life problems

e) Understand the basic concepts of data handling and measures of central tendency.

f) Understand fundamentals of permutation and combination and basic concept of probability.

Chapter 1- FUNCTIONS

Objectives:

- Understand the definition of function, domain and range
- Understand the concepts of graphs by transformation

1.1 Function – Definition, Domain and Range

A relation from Set X to Set Y is called a function if each element of X is related to exactly one element in Y.

It is often written as "f(x)" where x is the input value.

Example

Determine whether y is a function of x: Give reasons.

$y^2=x+3$

Solution:

$$y^2=x+3$$

$$y= \pm\sqrt{x+3}$$

If x = 0,

$$y= \pm\sqrt{x+3}$$

If you put any value for x, you will get two values for y. So y is not a function of x.

Domain

- The set of inputs for a function is called the *domain* of the function. To define a function fully, the domain must be stated.
- If the domain is not stated it is assumed that the domain is a set of real numbers.

Range

The set of output is called the *range* of the function.

The notation f(x) represents the output values of a function,

So for $f: x \rightarrow x^2 \quad for\ x \in R$ we have $f(x)= x^2$

Example:

If $f(x) = 2x^2 - 5$, $x \in R$, find f(3) and f(-1)

As f(x) is the output of the mapping, f(3) is the output when 3 is the input, i.e. f(3) is the value of $2x^2 - 5$ when x = 3

$f(3) = 2(3)^2 - 5 = 13$

$f(-1) = 2(-1)^2 - 5 = -3$

Exercise

1. Find whether y is a function of x . Give reasons.

   a) $3y^2+5x=6$ b) $8x+2y=6$ c) $y^3-3x=4$
2. If $f(x) = 3x^3 + 4$, find f(0) and f(− 2)
3. If $f(x) = 5x^2 - 3x$, find f(− 1) and f(5)

Chapter-2 QUADRATIC FUNCTION

Objectives:

- Understand the definition of axis of symmetry and vertex
- Determine the maximum or minimum of a quadratic function.
- Determine the zeros of a quadratic function.
- Solve quadratic equations using the quadratic formula.
- Identify the concept: Intercepts of a quadratic function
- Sketch the graph the quadratic functions
- Real life problems involving quadratic functions.

Quadratic functions

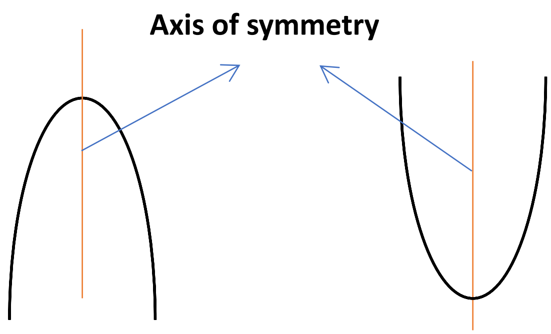
A **quadratic function** is one of the form $\mathbf{f(x) = ax^2 + bx + c}$, where **a**, **b**, and **c** are numbers with **a** not equal to zero.

The graph of a quadratic function is a parabola

2.1 Axis of symmetry, Vertex and maximum or minimum

The axis of symmetry is the line which divides a parabola into two equal halves that are reflections of each other

Axis of symmetry $x = \frac{-b}{2a}$



Example

Find the axis of symmetry of the function

$f(x) = x^2 + 4x - 3$

Solution:

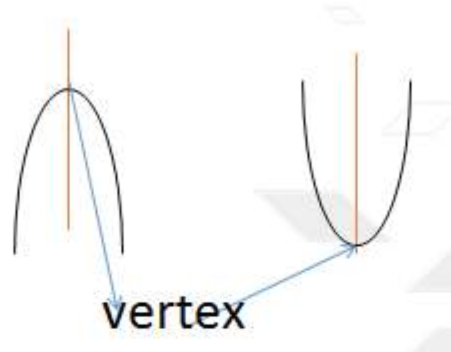
a = 1, b = 4

Axis of symmetry, x = $\frac{-b}{2a} = \frac{-4}{2\times 1} = -2$

Vertex

Vertex is the lowest or highest point on the graph of a quadratic function $f(x) = ax^2 + bx + c$

Vertex (x,y) = $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$



Example

Find the vertex of the function f(x) = $-2x^2 + 4x - 5$

Solution:

$a = -2, \quad b = 4$

$x = \frac{-b}{2a} = \frac{-4}{2\times(-2)} = 1$

$y = f(1) = -2(1)^2 + 4(1) - 5 = -3$

Vertex = $(1, -3)$

Exercise:

1) Find the axis of symmetry for the following:
    a) $y = x^2 + 4x - 2$
    b) 2) $y = x^2 + \frac{x}{2}$
    c) 3) $y = \frac{-1}{2}(x+5)^2$

2) Find the vertex for the following functions:
    a) $f(x) = 3x^2 - 9x - 6$
    b) $f(x) = \frac{5}{6}x^2 + 3x$
    c) $f(x) = (x-7)^2 + 2$
    d) $f(x) = -(x+1)^2 - 2$

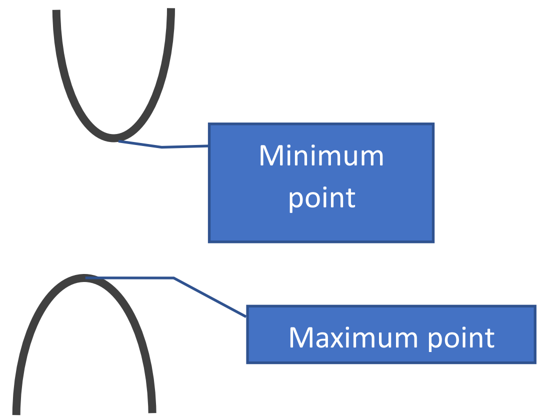
Maximum or minimum value

The **maximum** or **minimum** value of a quadratic function

$f(x) = ax^2 + bx + c$ occurs at $x = \frac{-b}{2a}$

If a > 0 then the minimum value is $f\left(\frac{-b}{2a}\right)$

If a < 0 then the maximum value is $f\left(\frac{-b}{2a}\right)$



Example

Find the maximum or minimum value of each quadratic functions.

a) $f(x) = x^2 + 4x$

Solution: a = 1, b = 4, c = 0

$x = \frac{-b}{2a} = \frac{-4}{2\times1} = -2$

$f(-2) = (-2)^2 + 4(-2) = -4$

Since a = 1 >0 the function has the minimum value, The minimum value is -4 .

b) $f(x) = -2x^2 + 4x - 5$

Solution $a = -2$, $b = 4$, $c = -5$

$x = \frac{-b}{2a} = \frac{-4}{2\times(-2)} = 1$

$f(1) = -2(1)^2 + 4(1) - 5 = -3$

Since a = -2 <0 , the function has the maximum value.

Maximum value is -3

Exercise:

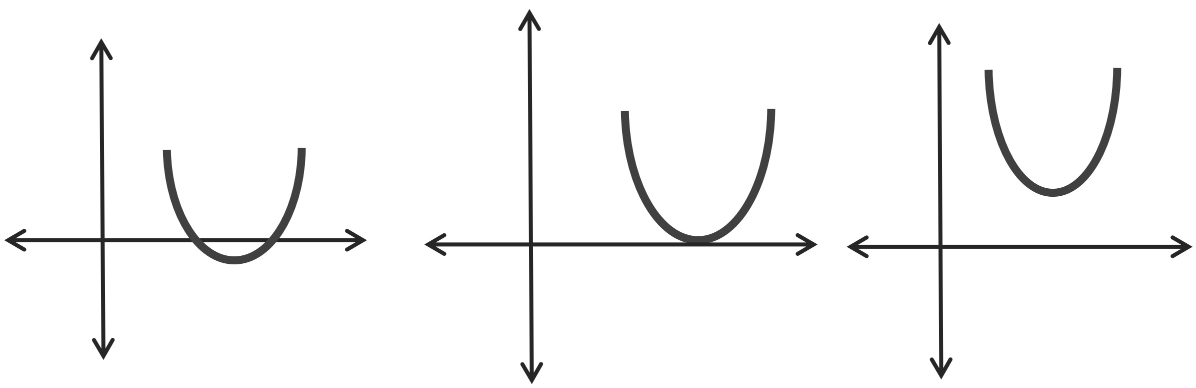
Find the maximum or minimum value of the quadratic equations:

1) $f(x) = x^2 + 14x - 14$
2) $f(x) = x^2 - 6x + 17$
3) $f(x) = -x^2 + 7x + 11$

Zeros of a Quadratic Function

The graph of a quadratic function $f(x)=ax^2 + bx + c$ is a parabola. A parabola can cross the x axis once, twice or never. These points of intersection are called the **x intercepts** or zeros.

To find the zeros of a quadratic function set $f(x)=0$ and solve the equation $ax^2 + bx + c = 0$ by using the quadratic formula.



Quadratic Equations

The Standard Form of a quadratic equation is

$ax^2 + bx + c = 0$, Where a, band c are known numbers a$\neq 0$ and x is a variable

Quadratic Formula

The solution to the equation $ax^2+bx+c=0$ is given by quadratic formula.

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Where d = b^2- 4ac is called the **discriminant**.

When d > 0	When d = 0	When d < 0
There are two distinct real zeros. That is Two x-intercepts	There is only one zero. That is one x-intercept.	There are imaginary or complex (*a + bi* form) zeros. That is no x-intercept. The imaginary unit (number) is *i*. Facts: $i = \sqrt{-1}$ and $i^2 = -1$

Example 01. Solve using the quadratic formula

$x^2 - 4x - 8 = 0$

Solution: a=1, b = – 4 , c = – 8

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1(-8)}}{2 \times 1}$$

$$x = \frac{4 \pm \sqrt{16+32}}{2} \qquad x = \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 4\sqrt{3}}{2}$$

$$x = \frac{2(2 \pm 2\sqrt{3})}{2}$$

$$x = 2 \pm 2\sqrt{3}$$

Example 02. Solve using the quadratic formula

$x^2 - 4x + 5 = 0$

Solution: a=1, b = – 4 , c = 5

$$\boldsymbol{x} = \frac{-\mathbf{b} \pm \sqrt{\mathbf{b}^2 - 4\mathbf{ac}}}{2\mathbf{a}}$$

$$\mathbf{x} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1(5)}}{2 \times 1}$$

$$\mathbf{x} = \frac{4 \pm \sqrt{16-20}}{2}$$

$$\mathbf{x} = \frac{4 \pm \sqrt{-4}}{2} = \frac{4 \pm \mathbf{i}2}{2}$$

$\sqrt{-4} = \sqrt{-1 \times 4}$

$= \sqrt{-1} \times \sqrt{4}$

$= i \times 2 = i2$

$\sqrt{-1} = i$

$$\mathbf{x} = \frac{2(2 \pm \mathbf{i})}{2}$$

$$\mathbf{x} = 2 \pm \mathbf{i}$$

Exercise

1. $x^2 - 5x + 6 = 0$
2. $x^2 - 4x + 5 = 0$
3. $x^2 - 2x + 1 = 0$
4. $3x^2 - 5x + 12 = 0$

5. $x^2 + 9x + 20 = 0$

6. $6x^2 + 9x - 6 = 0$

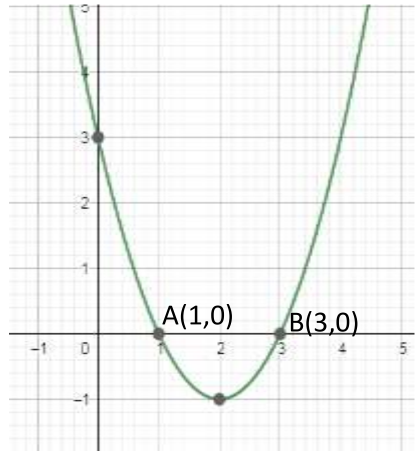
7. $x^2 + 16 = 0$

2.2 X and Y Intercepts

X intercept

The x intercept is a point where the line crosses the x axis. At this point, the y coordinate is zero.

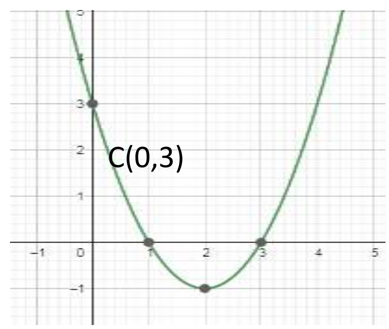
To determine the x intercept, we set y = 0 and solve for x.



Y intercept

The y intercept is the point where the line crosses the y axis. At this point the x coordinate is zero.

To determine the y intercept we set x = 0 and solve for y.



Example:

Find the x and y intercepts of the graph y = $3x^2$ – x.

Solution:

To find the y intercepts, set x = 0

y = $3x^2$ – x = 3(0) – 0 = 0

So the y intercept is 0.

To find the x intercept, set y = 0.

Solution:

y = $3x^2$ – x

0 = $3x^2$-x

Or

Use Quadratic Formula to solve *x*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a = 3, b = -1, c = 0

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(0)}}{2(3)}$$

x = 0 and x = $\frac{1}{3}$

x(3x—1) = 0   x = 0 3x-1 = 0   x = 0 or 3x = 1   x = 0 or x = $\frac{1}{3}$

So x intercepts are x = 0 and x = $\frac{1}{3}$

Exercise:

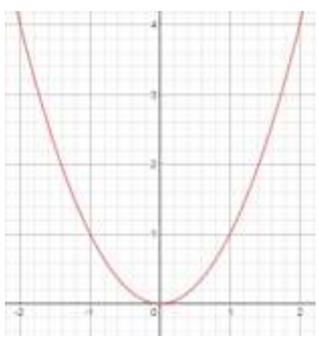
Find the x and y intercepts of the given graphs:

1. $y = x^2 + 5x + 6$
2. $y = x^2 - 9x$
3. $y = x^2 - 5x$

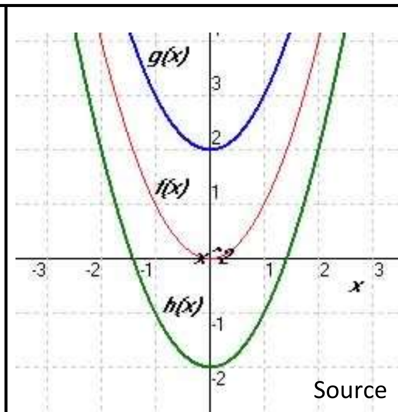
2.3 Transformation of Quadratic Functions

we will discuss how quadratic graphs can be transformed from the original graph.

Basic Graph

$y = x^2$ **Square Function**	

Vertical shifting

Vertical Transformation When k is +, shift Up When k is (-), shift Down K is a unit	f(x) => g(x) = f(x) + k	Shift Up g(x) = x^2 + k Shift Down g(x) = x^2 - k	 Source :Pindling(2019)

Example :

Let f(x)= x^2 be the base function. Describe the graph a) g(x) = x^2+2 and

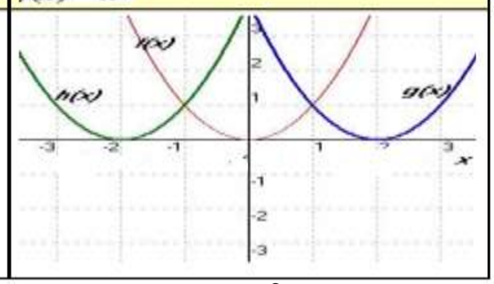
b) h(x) = $x^2 - 2$

Solution:

a) The base graph f(x) shifted up 2 units vertically.

b) The base graph f(x) shifted down 2 units vertically.

Horizontal shifting

Transformation	On $f(x)$	Example: $f(x) = x^2$	Illustration for $f(x) = x^2$
Horizontal Translation When h is , shift Right When h is ( ), shift Left	$f(x)$ => $g(x) = f(x - h)$	Shift Right $g(x) = (x - 2)^2$ Shift Left $h(x) = (x + 2)^2$	

Source :Pindling(2019)

Example.

Let f(x)= x^2 , a base function. Describe the graph a) g(x) = $(x - 2)^2$and

b) h(x) = $(x + 2)^2$

Solution

a) The base graph f(x) shifted 2 units to the right horizontally.

b) The base graph f(x) shifted 2 units to the left horizontally.

Example.

Let f(x)= x^2 , a base function. Describe the graph

a) g(x) = $(x - 2)^2$ and

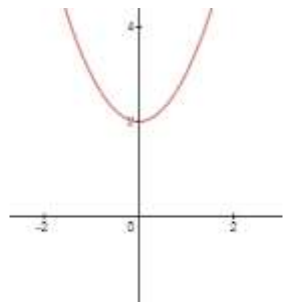
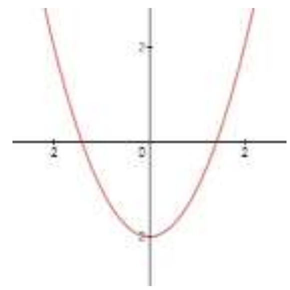
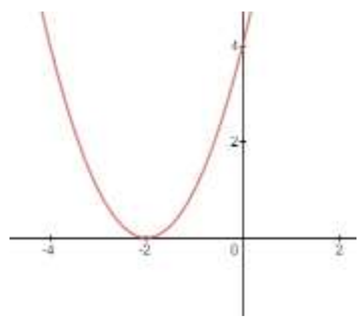
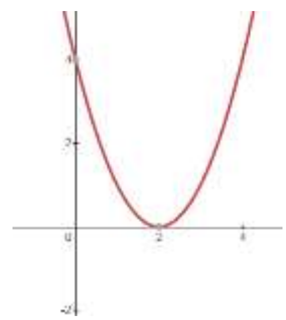
b) h(x) = $(x + 2)^2$

Solution

a) The base graph f(x)=x^2 shifted 2 units to the right horizontally.

b) The base graph f(x)=x^2 shifted 2 units to the left horizontally.

Transformation of Graphs

$y = x^2$			
$y = x^2 + 2$		$y = x^2 - 2$	
$y = (x + 2)^2$		$y = (x - 2)^2$	

Exercise

1. Let f(x)= x^2 , a base function. Describe the graph

   a) g_1(x) = $(x-3)^2$ and b) g_2(x) =$x^2 + 3$

3. How will the function f(x)=x^2 be, when it is

   a) shifted left 1unit b) shifted up 6 units

   c) shifted down 7 units d) shifted right 4 units

4. Draw the base function f(x) = x^2 and its transformed functions

a) shifted left 5 units	b) shifted up 5 units

c) shifted down 3 units	d) shifted right 4 units

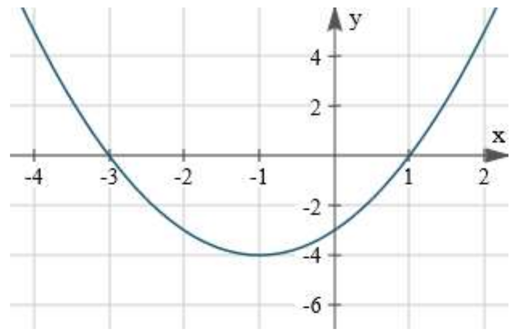
2.4 The Graph of the Quadratic Function

The graph of a quadratic equation y = ax^2 +bx + c is a parabola.

Shape of the parabola

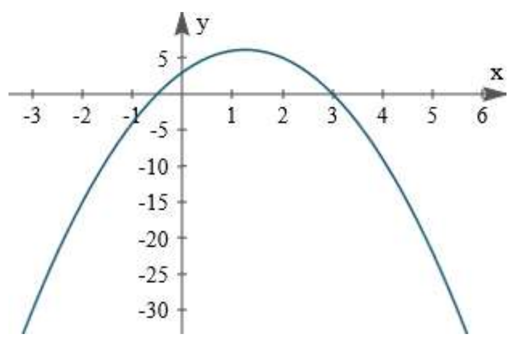
If a>0, then the parabola opens upwards (U-shaped) and has a minimum point

Eg. $Y = x^2+2x-3$



If $a < 0$, then the parabola opens downwards (n-shaped) and has a maximum point

Eg. y = $-2x^2$ + 5x + 3



Steps to sketch the graph

To sketch the graph of the quadratic function:

- Find the vertex of the given function.
- Determine whether the point is maximum or minimum
- Find the x and y intercepts.
- Use these points to plot the graph.

Example

Sketch the graph of $f(x) = 2x^2 - 8x + 6$

Solution:

a= 2, b= -8 , c = 6

Since *a*=2, *a*>0 hence the function has minimum point and it opens upwards

x = $\frac{-b}{2a}$ = $\frac{8}{2\times 2}$ = **2**

The *y* value of the minimum point is

$y=2(2)^2-8(2)+6= -2$

So the vertex is (2,−2)

y- Intercept

The *y*-intercept is found by substituting *x*=0

$y=2(0)^2 -8(0)+6=6$

The y-intercept is (0,6)

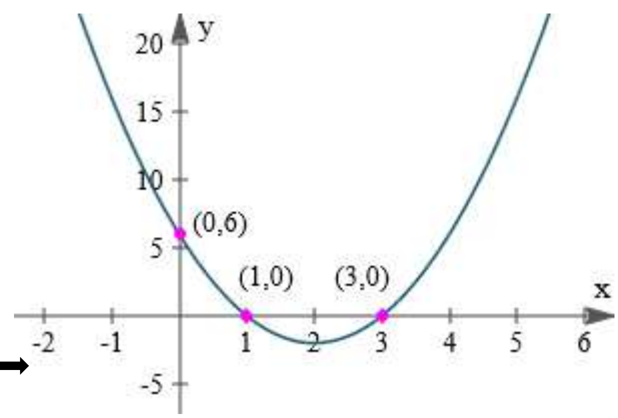
x-intercepts

$2x^2-8x+6=0$

$2(x^2-4x+3)=0$ $2(x-1)(x-3)=0$

X= 1, 3

Using the above points, the sketch of the curve will be



Exercise

Sketch the following quadratic function:

1) $y = -x^2 + x + 6$

2) $y = x^2 - 6x + 9$

3) $y = -4x - 3 - x^2$

4) $y = x^2 - 8x + 7$

2.5 Application of Quadratic Functions

Solution of problems involving quadratic functions

- Translate the word problem into mathematic language and form a quadratic equation.
- Solve the quadratic equation
- Translate the solution into verbal language and avoid the solution which does not related to the problem.

Example: 1

The length of a hall is 5m more than its breadth. Find the function that models its area A in terms of its width x. If the area of the floor of the hall is 84 m^2, what are the length and the breadth of the hall?

Solution:

Let the breadth be x, Length = 5 + x Area = length × breadth

A(x) = x(5 + x) A(x) = $5x + x^2$

A= 84 So, x(5 + x) = 84

$5x + x^2 - 84 = 0$ x= 7 (Using Quadratic formula)

Breadth = 7 m, length = 12 m.

Example: 2

The sum of two numbers is 48 and their product is 432. Find the numbers.

Solution:

Let the first number be x.

Second number is 48 – x.

x(48-x)= 432

$48x - x^2 - 432 = 0$

a=-1, b = 48 , c=-432

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-48 \pm \sqrt{(48)^2(-4)(-1)(-432)}}{2(-1)}$$

$$= \frac{-48 \pm \sqrt{2304-1728}}{-2}$$

$$= \frac{-48 \pm 24}{-2} = 12 , 36$$

First number = 12 Second number = 48-12 = 36

Example:3 (Maximum or Minimum question)

Example

A ball is thrown upward from the top of a 50 m high building at a speed of 85 m per second. The ball's height above ground can be modeled by the equation H(t)=$-16t^2$+85t+50.

a. When does the ball reach its maximum height?

b. What is the maximum height of the ball?

Solution:

The ball reaches its maximum height at the vertex of the parabola.

$$t = \frac{-b}{2a} = \frac{-85}{2\times -16} = 2.7$$

The ball reaches its maximum height after 2.7 seconds

To find the maximum height, find the *y* coordinate of the vertex of the parabola

$$y = H(2.7) = -16(2.7)^2 + 85\times 2.7 + 50 = 163$$

The ball reaches a maximum height of 163 m

Exercise: (Maximum or Minimum questions)

1) The path of a rock is given by the function h(t) = $-3t^2$+30t+73 where h is the height in the metres and t is the time in seconds .
   a) What is the maximum height of the rock?
   b) At what time does the rock reach its maximum height?

2) A rock is thrown upward from the top of a 112m high cliff overlooking the ocean at a speed of 96 m per second. The rock's height above ocean can be modeled by the equation H(t)=$-16t^2$+96t+112.

   a) When does the rock reach its maximum height?

   b) What is the maximum height of the rock?

Exercises:

a) The length of a rectangular parking lot is 2 times as long as it is wide. Find the function that models its area A in terms of its width. If the area is 1800 $m^{2,}$ find the length and width.
b) The sum of two positive numbers is 36 and their product is 323. Find the two numbers.
c) The perimeter of a rectangular field is 82m and its area is 400m^2. Find the breadth of the rectangle.

d) The area of a rectangular plot is $528m^2$. The length of the plot is one metre more than twice its breadth. Find the length and the breadth of the plot.

e) A rectangle has length x meters and width (x-5) meters. Find a function that models the area of the rectangle. The rectangle has an area of 36 m^2, find the value of x.

Reference

- Pindling.org. (2019). *3.4 Transformation of Graphs*. [online] Available at: http://www.pindling.org/Math/CA/By_Examples/3_4_Transformation/3_4_Transformation.html [Accessed 17 Aug. 2019].

- https://vdocuments.site/route404.html. (Full reference unavailable, due to removal of original web source).
- waymaker, l. (2019). *Graphing Linear Equations | College Algebra*. [online] Courses.lumenlearning.com. Available at: https://courses.lumenlearning.com/waymakercollegealgebra/chapter/graph-linear-equations/ [Accessed 2 Jan. 2019].

Chapter 3- LINEAR EQUATIONS AND INEQUALITIES

Objectives:

- Solve linear equations with two variables
- Sketch the graph of linear equations in two variables.
- Solve the graph of two or three linear inequalities of two variables

Linear Equations

An equation of the form ax + by + c = 0, where a, b, c are real numbers, a ≠ 0, b ≠ 0 is called a linear equation in two variables.

Example:

i) $\mathrm{x} + 2\mathrm{y} = 3$ ii) $-2\mathrm{x} + 3\mathrm{y} = 5$

iii) $\frac{3\mathrm{x}}{2} - \frac{5\mathrm{y}}{3} + 12 = 0$

3.1 Solving System of Linear Equations

There are three different ways to solve a system of linear equations in two variables.

- Substitution method
- Elimination method
- Graphing

Substitution Method

Solve : $\mathrm{x} + \mathrm{y} = 3$

$\mathrm{x} - \mathrm{y} = 1$

Step 1

Solve one equation for either variable.

$$x + y = 3$$

$$y = 3 - x$$

Step 2

Substitute the value in the second equation

$x - y = 1$ $\qquad\qquad$ $x - (3 - x) = 1$

$x - 3 + x = 1$ $\qquad\qquad$ $2x = 1+3$

$2x = 4$ $\qquad\qquad$ $x=2$

Step 3

Solve for the second variable.

$x + y = 3$

$2 + y = 3$

$y = 1$

Exercise :

Solve using the substitution method:

1. $x + y = 5$
   $y = x + 3$
2. $x + y = 2$
   $3x + 6y = 16$
3. $3u + z = 15$
   $u + 2z = 10$
4. $u + 6y = 32$
   $u + 3y = 17$
5. $2b + v = 13$
   $b + v = 8$

Elimination Method

Solve: $5x + 3y = 10$

$10x - 6y = 0$

Solution:

Step 1

Write both equations in the form Ax + By = C.

Step 2

Multiply one or both equations by a number that will create an opposite coefficient for either x or y if needed.

$$5x + 3y = 10 \quad \longrightarrow \quad 1$$

$$10x - 6y = 0 \quad \longrightarrow \quad 2$$

1 → Multiply by 2 $\qquad 10x + 6y = 20$

$$\underline{10x - 6y = 0}$$

$$20x \qquad = 20$$

$$x = 1$$

Step 3

Substitute what you get from step 2 in the other equation.

$$10x - 6y = 0$$

$$10 - 6y = 0$$

$$6y = -10$$

$$y = \frac{10}{6}$$

Exercise

Solve using the elimination method:

1. $6u + v = 18$

   $5u + 2v = 22$

2. $3a + 5u = 17$

   $2a + u = 9$

3. $x - 7y = -17$

   $5x + 2y = -11$

4. $6x - 5y = 8$

   $-12x + 2y = 0$

Sketch the following system of linear equations graphically.

$$3x - 2y = 4$$
$$2x + y = 5$$

Solution:

Consider the two linear equations.

$3x - 2y = 4$

$2y = 3x - 4$

$y = \dfrac{3x-4}{2}$

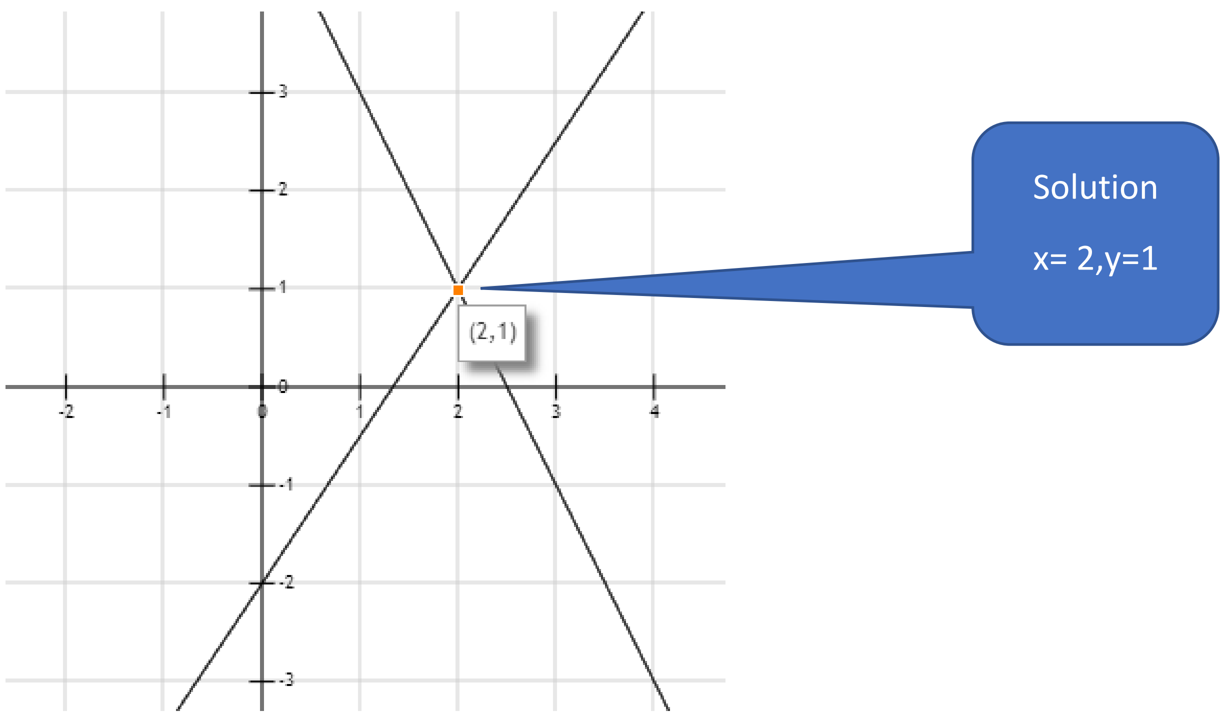
x	0	2
y	-2	1

$2x + y = 5$

$y = 5 - 2x$

x	0	2
y	5	1

Solution graphically



Sketch the graph of the following system of linear equations.

1) $x - 4y + 14 = 0$
   $3x + 2y - 14 = 0$

2) $x + y = 3$
   $2x + 5y = 12$

3) $x + y = 4$

   $2x - 3y = 3$

4) $x - y = 0$
   $2x - y = 2$

3.2 Linear Inequalities

Sketch the graph of linear inequalities in two variables.

Example: Show graphically the solution set of the linear inequalities.

$$3x + 4y \leq 12, \qquad 4x + 3y \leq 12,$$

Step 1

Convert the inequalities into an equation.

$3x + 4y = 12,$ → 1 $\qquad$ $4x + 3y = 12,$ → 2

Step 2

Put y = 0 in both equations to get the point where the line meets the x axis.

Similarly put x = 0 in both equations to get the point where the line meets with y axis.

y =0 $\qquad$ x=0 in 1

x=4 $\qquad$ y=3

3x + 4y =12 meets the axes at A(4,0) B(0,3)

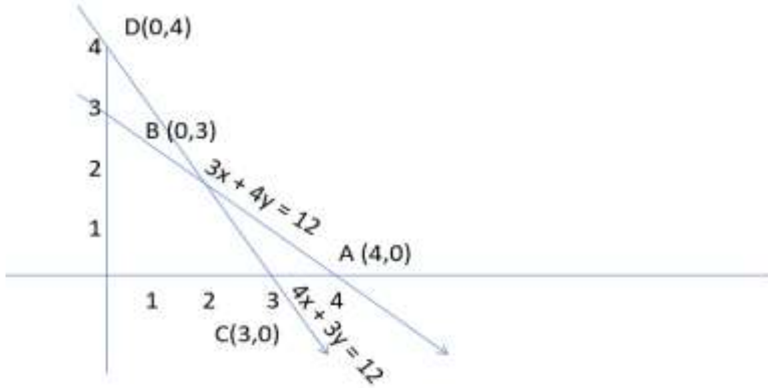
y=0 and $\qquad$ x =0 in 2

x = 3 , $\qquad$ y=4

4x + 3y = 12 meets the axes at C (3,0) , D (0,4).

Step 3

Join the points from in step 2 to get the graph of the line



Step 4

Choose a point (0,0) not lying on the given lines. Substitute its coordinates in the inequalities. If the inequalities are satisfied, shade the position of the plane which contains the chosen point. Otherwise shade the position which does not contain the chosen point.

1 → $3 \text{ x } 0 + 4 \text{ x } 0 \ \leq 12$

$$0 \leq 12$$

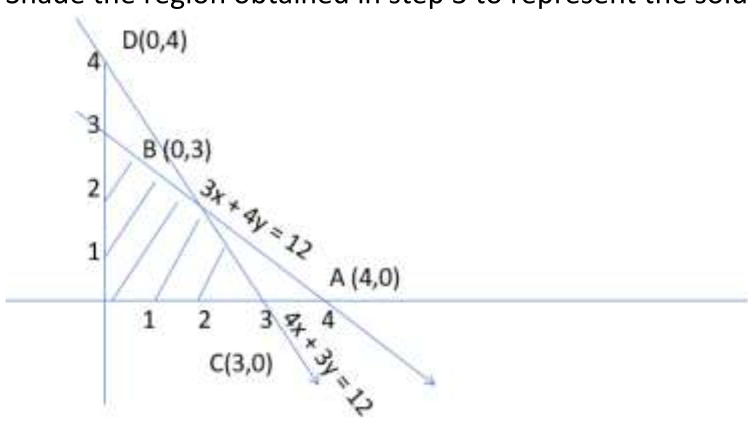
2 → $4 \text{ x } 0 + 3 \text{ x } 0 \leq 12$

$$0 \leq 12$$

(0,0) satisfies inequality 1 and 2

Step 4

Shade the region obtained in step 3 to represent the solution set



Exercise :

1) **Draw the diagram of the solution set of the linear inequalities.**

$a)\ x + y \leq 5$

$4x + y \geq 4$

$b)\ x + y \geq 1$

$7x + 9y \leq 63$

$c)\ x - y \leq 1$

$x + 2y \leq 8$

2) **Draw the diagram of the solution set of the linear inequalities.**

$a) 3x + y \geq 12$

$y \geq 1$

$x \geq 0$

$b)\ x + y < 5$

$2x - y > 0$

$-x + 5y > -20$

$c)\ x \geq 2$

$-3x + y < -1$

$4x + 3y < 12$

$d)\ y \leq \frac{2}{3}x + 3$

$y \geq -x$

$x \leq 2$

Reference: https://english.eagetutor.com/content/graph-of-a-linear-equation-in-two-variables-sp-1660494426

Chapter -4 EXPONENTIAL AND LOGARITHMIC FUNCTION

Objectives:

- Understand the inverse relationship between exponents and logarithms and use this relationship to solve related problems.
- Solve logarithmic equations
- Solve exponential and logarithmic equations
- Sketch the graphs of exponential and logarithmic functions.
- Solve real life problems involving exponential functions.
- Compare simple interest and compound interest.

Logarithm

The logarithm is the inverse operation to exponentiation.

$$\boxed{\log_b N = x}$$

Base

Logarithm to base 10 is called the **common logarithm** and

base 'e' is called the **natural logarithm**.

Common logarithm $\log x = \log_{10} x$

Natural logarithm $\ln x = \log_e x$

4.1 The inverse relationship between exponents and logarithms

For any real number $a > 0$ and $a \neq 1$

If $a^c = b$ **then** $\log_a b = c$

In this definition $\log_a b = c$ is called **logarithmic form** and

$a^c = b$ is called **exponential form**.

For example : $10^2 = 100$ can be written in log form as

$\log_{10} 100 = 2$

Exercise:

Rewrite the following using log:

1) $5^2 = 25$

2) $7^{a-b} = y$

3) $8^{-1} = \frac{1}{8}$

Rewrite the following using power

1) $\log_6 216 = 3$

2) $\log 5(\frac{1}{125}) = -3$

3) $\log_{\frac{1}{3}} 81 = -4$

4) $\log_{\frac{3}{2}}(\frac{27}{8}) = 3$

Solve the following:

$\log_4 16 = x + 3$

Solution:

$4^{x+3} = 16$

$= 4^2$

$x + 3 = 2$

$x = -1$

Exercise:

Solve the following.

1. $\log_3 x = 2$
2. $\log_5 2x - 1 = 2$
3. $\log_3 81 = 3x + 1$
4. $\log_{x+1} 49 = 2$

4.2Laws of Logarithms

Log $_a$ A + log $_a$ B = log $_a$ (AB) for Eg log $_a$ 5 + log $_a$ 3 = log $_a$ (15)

Log $_a$ A - log $_a$ B = log $_a$ $(\frac{A}{B})$ for Eg log $_a$ 20 - log $_a$ 4 = log $_a$ (5)

Log $_a$ A^n = n log A for Eg log $_a$ 3^2 = 2log $_a$ (3)

Log $_a$ a = 1 For Eg. log $_7$ 7 = 1

Simplify using the laws of logarithms

a) log $_3$ 7 + log $_3$ 4

Solution: log $_3$ 7 + log $_3$ 4 = log $_3$ (7 x 4) = log $_3$ 28

b) log $_2$ 25 - log $_2$ 5

Solution: log $_2$ 25 - log $_2$ 5 $= \log_2\left(\frac{25}{5}\right)$ $= \log_2 5$

c) log $_4$ 4

Solution: log $_4$ 4 =1

d) 2log 3 + 2 log 7

Solution:

2 log 3 + 2 log 7 = log 3 ^{2} + log 7 ^{2}

= log 9 + log 49 = log (441)

4.3 Logarithmic and exponential Equations

1) log $_a$ 2x - log $_a$ 2 = log $_a$ (2x -1)

Solution:

$\log_a\left(\frac{2x}{2}\right)$ = log $_a$ (2x-1)

$\frac{2x}{2}$ = 2x-1

x = 2x-1

-x = -1 x = 1

Exercise:

Solve the following.

a) $\log_7 5x + \log_7 2 = \log_7 (2x + 1)$
b) $\log_3 36x - \log_3 6x = \log_3 (4x + 3)$
c) $\log a^4 - \log a^2 = \log 3a + \log 4$

Solving exponential Equations

Exponential Equation with base a

1) Solve $3^{2x+1} = 7$

Solution: $3^{2x+1} = 7$

$\log 3^{2x+1} = \log 7$

$2x+1 \log 3 = \log 7$

$2x + 1 = \frac{\log 7}{\log 3}$ $\quad 2x = 0.77 \quad x = 0.38$

Exponential equation with base e

1) $e^{5x+1} = 300$

Solution: $e^{5x+1} = 300$

take ln on both sides

$5x + 1 \ln e = \ln 300$ $\quad$ note: $\ln e = 1$

$5x+1 = \ln 300$ $\quad 5x+1 = 5.703$ $\quad x = 0.940$

Exercise 1

Solve the following:

a) $10^{2x-1} = 25$

b) $8 e^x = 24$

c) $4^{3x+2} = 9$

d) $\sqrt{e^{3x+1}} = 4$

Graphs of Exponential and Logarithmic Functions

Graphing exponential functions is similar to the graphing you have done before.

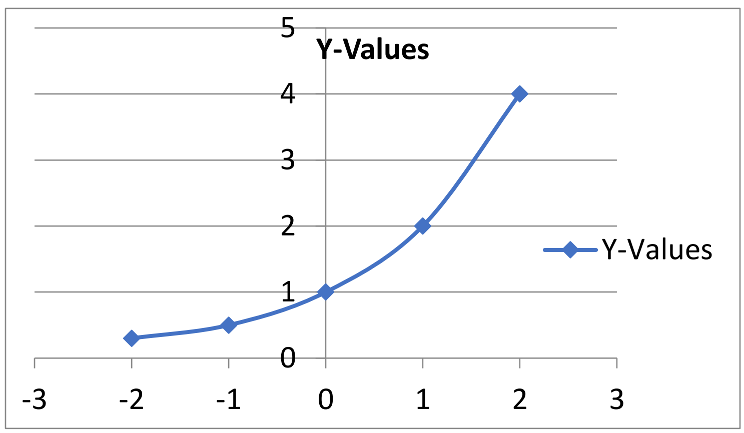
However, by the nature of exponential functions, their points tend either to be very close to one fixed value or else to be too large to be conveniently graphed.

Example:

Draw a graph for the function $f(x) = 2^x$

Solution:

X	-2	-1	0	1	2
f(x)	0.25	0.5	1	2	4



Exercise 2:

Draw the graphs of the following:

1) $f(x) = e^{x}$
2) $f(x) = 5^{x}$
3) $f(x) = e^{2x}$
4) $f(x) = e^{x+1}$
5) $f(x) = e^{-x}$

Graphs of Logarithmic Functions

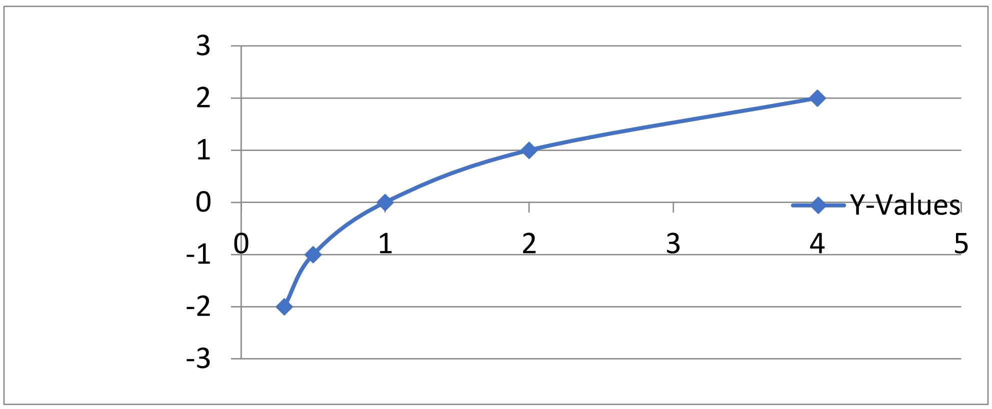
The inverse of exponential function is a logarithmic function. Graphs of both the functions are reflections of each other.

Draw the graph of the function f(x) = $\log_2 x$.

Solution:

Let y= $\log_2 x$ $\Rightarrow$ x = 2^y put different values for y, find the corresponding x values.

X	0.25	0.5	1	2	4
f (x) = y	-2	-1	0	1	2



Exercise 3:

Draw the graphs of the following:

1) f(x) = $\log_3 x$
2) f(x) = $\log_5 x$
3) f(x) = $\log_2 (x + 1)$
4) f(x) = $\log_2 (x - 1)$

4.4 Applications of Exponential Functions

Exponential Growth

The exponential growth formula is $\mathbf{A = p\,e^{rt}}$

P – Initial value

A – Final value

r - Rate of growth

t - Time

Example1

1) The population of a city is p = 250342 $e^{0.012t}$ where t = 0 represents the population in the year 2000.

a) Find the population of the city in the year 2010
b) Find the population of the city in the year 2015
c) Find the number of years required for the population to reach 320000

Solution:

a) P = 250342, t = 10

P = 250342 $e^{0.012 \times 10}$

= 282259.82

=282260 people.

b) P = 250342, t = 15

P = 250342 $e^{0.012 \times 15}$

= 299713.8 = 299714 people.

c) A = P e^{r}t

320000 = 250342 $e^{0.012t}$

$\underline{320000}$ = $e^{0.012t}$

250342

$\ln\left(\frac{320000}{250342}\right) = e^{0.012t}$

ln(0.1278) = 0.012t

0.245 = t

20.46 years = t

Example 2

How long will it take 1000 RO to double if invested at 8% interest compounded continuously.

Solution:

$A = P e^{r t}$

P = 1000 A = 2000, r= 8% or 0.08

$2000 = 1000 e^{0.08 t}$

$2 = e^{0.08 t}$

$\ln 2 = 0.08t$

$8.66 = t$

Exercise:

1) The number of bacteria in a culture is modeled by the function $b(t) = 300 e^{0.45t}$, where t is measured in hours.
   a) What is the initial number of bacteria?
   b) How many bacteria are there in the culture after 3 hours?
   c) After how many hours will the number of bacteria reach 5000?
2) The population (P) of a colony of insects t days after the commencement of an experiment is given by the exponential law $p = 20000 e^{0.03t}$. Find the following:
   a) Population at the end of ten days.
   b) Number of days for the population to be 40000.
3) A scientist starts with 100 bacteria in an experiment. After 5 days, she discovers that the population has grown to 350. Find the rate of growth.
4) The initial population of rabbits in a lab is 30. After 50 days the population is 400. When will the rabbit population reach 500?

Exponential decay

Solving an exponential decay problem is very similar to working with population growth. The value of r will be negative in the case of decrease or decay.

$P = Po e^{-rt}$

Example

The number of milligrams of a drug in a person's body after t hours is given by the formula $P = 20 e^{-0.4t}$

1) Find the amount of the drug after 2 hours.
2) Find the amount of the drug after 5 hours.

Solution:

1) t = 2

$D = 20\ e^{-0.4 \times 2}$

$= 20\ e^{-0.8}$

= 8.987

2) After 2 hours, 8.987 milligrams of the drug are left in the body.

$D = 20^{\ e\ -04 \times 5}$

= 2.707

Exercise:

1) The value of a car can be modeled by the equation $V = 6580\ e^{-xt}$, where t is the age of the car in years and x is a constant.
   a) State the value of the car when it is new.
   b) After two years the value of the car will be R.O 4300. Find the value of x.

2) At the start of an experiment there are 8000 bacteria. After 2 hours, 1000 bacteria are dead. If the death rates are exponential, how long will it take for the population to reach 5000?

4.5 Simple Interests and compound interest

There are two different ways of calculating interest -- simple interest and compound interest

Simple interest

Each year, the interest is calculated as a percentage of the principal,

$$\mathbf{I = P\,r\,t}$$

I = interest

P = principal

r = interest rate

t = time

Example:

Ahmed borrowed R.O 3,000 for 4 years at a 5% interest rate. How much interest did he pay?

I = Prt

I = 3,000 × 5% × 4 years   I = 3000 × 0.05 × 4 = R.O 600

Compound interest

When you deposit money into an interest-bearing account, or take out a line of credit, the interest that accumulates is added to the principal, and the next interest calculation is done on both the principal and the interest.

The formula for annual compound interest, **including the principal sum**, is:

$$\boldsymbol{A = P\left(1 + \frac{r}{n}\right)^{nt}}$$

A = the future value of the investment/loan, including interest
P = the principal investment amount (the initial deposit or loan amount)
r = the annual interest rate (decimal)
n = the number of times that interest is compounded per year
t = the number of years the money is invested or borrowed for

Compound interest

Interest can be compounded at any interval, but the most common compounding intervals are

Annual: once per year.

Quarterly: four times per year

Half yearly : 2 times per year

Example:

If you deposit R.O 1,000 for 5 years at 4% interest that compounds monthly. Calculate the value after 5 years.

Solution:

$A = P\left(1 + \frac{r}{n}\right)^{nt}$

$= 1000\left(1 + \frac{0.04}{12}\right)^{5\times 12}$

= 1220.9

Relation between compound interest and exponential growth

- When we start compounding more and more frequently(continuously), the growth is slowing down and the number of compounding increases, the computed value appears to be approaching some fixed value "e" (exponential).
- The equation for "continual" growth (or decay) is $A = Pe^{rt}$, where "*A*" is the ending amount, "*P*" is the beginning amount (principal, in the case of money), "*r*" is the growth or decay rate (expressed as a decimal), and "*t*" is the time (in whatever unit was used on the growth/decay rate).

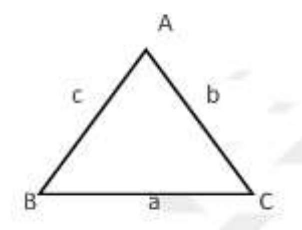
Chapter -5 TRIGONOMETRY

Objective:

- Use the laws sines and cosines to solve a triangle.

5.1 Laws of Sines

Law of Sines



$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

Note:

The sum of the internal angles of a triangle is equal to 180°

A + B + C=180°

Steps to solve the given triangle :

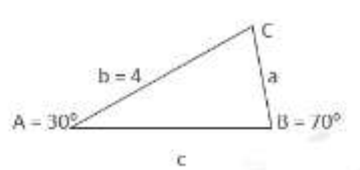
1. Find the unknown angle using the sum of the internal angles equals to 180°

2. Use the sine rule to find one unknown side .

3. Use the sine rule again to find the other unknown side

Exercise

Solve the triangle ABC



Solution:

1. Calculate the unknown angle C :

$A+B+C = 180°$

$C = 180° - (30°+70°) = 80°$

2. Use the sine rule to find the unknown side

$$\frac{c}{sinC} = \frac{b}{sinB}$$

$$c = \frac{bsinC}{sinB} = \frac{4sin80°}{sin70°} = 4.19204$$

3.Use the sine rule again to find the other unknown side

$$\frac{a}{sinA} = \frac{b}{sinB}$$

$$c = \frac{bsinA}{sinB} = \frac{4sin30°}{sin70°} = 2.128$$

The triangle is now solved.

Exercise

Solve the triangle ABC

a) $\angle$ C = 85° $\angle$ A = 65 ° $\angle$ B = 30°, c= 13cm

b) $\angle$ A = 45° $\angle$ B = 122 ° $\angle$ C = 13°, c= 6cm

c) $\angle$ C = 13° $\angle$ A = 22 ° c = 9mm

d) $\angle$ A = 30° $\angle$ B = 65 ° b = 10cm

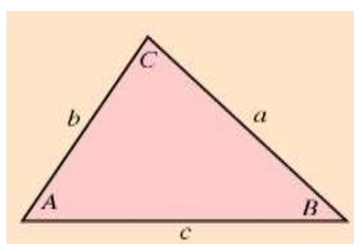
e) $\angle$ A = 70° c = 26cm a = 25cm

5.2 Law of Cosines

$a^2 = b^2 + c^2 - 2bc \cos A$

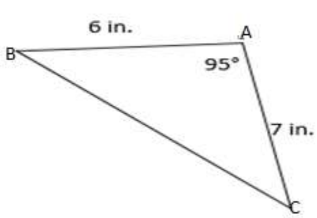
$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$



Example :

Find the length of the third side.



Solution:

$$a^2 = b^2 + c^2 - 2\, b\, c\, Cos\, A$$

$$a^2 = 6^2 + 7^2 - 2 \times 6 \times 7\, Cos\, 95$$

$$a^2 = 92.32$$

$$a = 9.6\, in$$

Practice: Use the Law of Cosines to find the length of the third side for each triangle. Round your answer to the nearest tenth.

Hint: Draw a picture if needed.

1. $m\measuredangle C = 83°$, a = 10 cm. and b = 6 cm.
2. $m\measuredangle C = 22°$, a = 2 in. and b = 9 in.
3. $m\measuredangle D = 74°$, a = 4 m. and b = 2 m.
4. $m\measuredangle X = 59°$, y = 8 mm. and z = 5 mm.
5. $m\measuredangle C = 123°$, d = 9 ft. and e = 12 ft.

Answers: 1) 11.0 cm 2) 7.2 in. 3) 3.9 m. 4) 6.9 mm. 5) 18.5 ft.

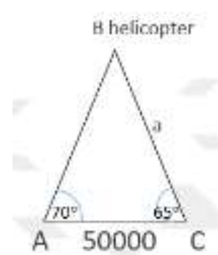
Real Life Problem on Law of Sines and Cosines

Example 1.

A helicopter is hovering between two helicopter pads. The pilot knows that he flew into the air at a 70° angle to get to his current position. He also knows that the two pads are 50,000 feet apart. He wants to practice his descent so that he lands at a 65° angle. If he were to start descending now, how many feet would he have to travel in descent to land at a 65° angle?

Solution:

$$\frac{sinA}{a} = \frac{sinB}{b} = \frac{sinC}{c}$$



Given that ∠ A = 70° , ∠ C= 65° b = 50000

∠B = 180 ° −(70°+65°) = 45°

$$\frac{sinA}{a} = \frac{sinB}{b}$$

$$\frac{sin70}{a} = \frac{sin45}{50000}$$

a = 66446.302 feet

Example 2

Scientists in Houston are trying to figure out the distance from a satellite to Cape Canaveral. They know that the satellite is 530 miles from Houston at an 85º angle of elevation, and Houston and Cape Canaveral, in a straight line from one to the other, are 902 miles apart. How far is the satellite from Cape Canaveral?

Solution

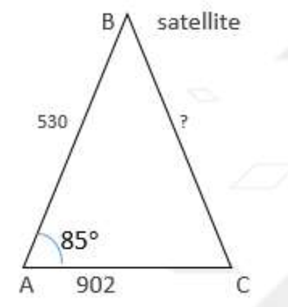
Given that b= 902 , c = 530 ∠A = 85°

$$a^2 = b^2 + c^2 - 2bc\ CosA$$

$=(902)^2 + (530)^2 - 2 \times 902 \times 530\ Cos85\,°$

a = $\sqrt{(902)^2 + (530)^2 - 2 \times 902 \times 530\ Cos85\,°}$

= 1005.57 miles



Chapter -6 BASIC PROBABILITY AND STATISTICS

Objectives:

- Summarise the data into simple graph like bar chart, pie chart
- Summarise the given data into frequency table and histogram
- Understand the concept of permutation and combination
- Understand basic probability concepts. Compute the probability of simple events using tree diagrams.
- Understand basic concepts of mean, mode, median.

6.1 Permutations and Combinations

Factorial

The **Factorial** of a number is the multiplication of that number by every smaller number down to 1.

The **Factorial Notation** is **n!,** where **n** represents the number and the "**!**" indicates the factorial process.

Note the following: By definition 0! = 1

Example: 8! = 8 * 7 *6 * 5 * 4 * 3 * 2 *1 = 40,320

Permutation and combination

Permutation
A Permutation is an **arrangement** of n objects in a **<u>specific order</u>** using r objects at a time.

$$_nP_r = \frac{n!}{(n-r)!}$$

Eg: Out of three students (A, B, C), we need to arrange 2 for the positions President and Vice President. The number of ways is,

President	**Vice President**
A	B
A	C
B	A
B	C
C	A
C	B

There are 6 different ways!!

Example: A news program has time to present 2 of four available news stories. How many ways can the evening news be set up?

Checking the process:

If we let A, B, C, D represent the four shows, then the possible show orders would be:

AB BA CA DA

AC BC CB DB

AD BD CD DC

Twelve (12) possible presentations where order matters.

$$_nP_r = \frac{n!}{(n-r)!}$$

$$_nP_2 = \frac{4!}{(4-2)!}$$

$$_4P_2 = \frac{4*3*2*1}{2*1}$$

$$_4P_2 = \frac{24}{2} = 12$$

Example

Find the number of ways of forming 4 letter word from a 7-letter word.

Solution:

Total number of letters n = 7

Number of letters for selection r = 4

$p_4^7 = \frac{7!}{(7-4)!} = \frac{7!}{3!} = 840$

Combinations

A Combination is an arrangement of items in which order does not matter. It is the selection of some or all of a number of different objects.

ORDER DOES NOT MATTER!

A combination is the **selection** of r objects from n objects **<u>without regard to order</u>**.

$$_nC_r = \frac{n!}{(n-r)!r!}$$

Combinations: Examples

A news program has time to present 2 of four available news stories. How many **different sets of stories** can be presented on the evening news?

Checking the process:

If we let A, B, C, D represent the four shows, then the possible show orders would be:

AB BA CA DA

AC BC CB DB

AD BD CD DC

However, AB and BA represent the presentation of the same two stories. If order does not matter, one of these two may be deleted. Repeating the process results in: AB, AC, AD, BC, BD, CD,

Six (6) different presentations where order does not matter.

$$_nC_r = \frac{n!}{(n-r)!r!}$$

$$_nC_2 = \frac{4!}{(4-2)!2!}$$

$$_nC_2 = \frac{4*3*2*1}{2*1*2*1}$$

$$_nC_2 = \frac{24}{4} = 6$$

Example

1. How many ways are there of choosing 4 pens from 52 pens?

Solution:

n = 52   r = 4

$c_4^{52} = \frac{52!}{4!(52-4)!} = \frac{52!}{4!48!} = 270725$

Exercise

1) Evaluate the following:
   a) $5!$ b) $9! - 4!$ c) $\frac{10!}{8!2!}$
2) In how many ways can you select a committee of 5 people from a group of 14 members?
3) Verify that $2\times\ C_4^7 = C_4^8$

6.2 Probability

Probability is how likely something is to happen.

The probability of an event A happening is given by P(A).

P(A) = number of outcomes favourable to A

total number of possible outcomes

The probability of event A is the number of ways event A can occur divided by the total number of possible outcomes.

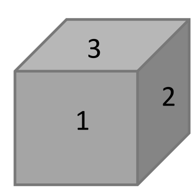
Sample Space

A sample space is the set of all possible outcomes in an experiment.

Example 1 : Tossing a coin. Write the sample space

Possible outcomes are head or tail.

Sample space, *S* = {head, tail}



Example 2: Tossing a dice . Write the sample space

Possible outcomes are the numbers 1, 2, 3, 4, 5, and 6

Sample space, S = {1, 2, 3, 4, 5, 6}.

Example 3: Two coins are tossed. Sample Space = {HH, HT, TH, TT}, H – Head, T - Tail

Example 3: Two coins are tossed. Represent all the sample space

Sample Space = {HH, HT, TH, TT}, H – Head, T - Tail

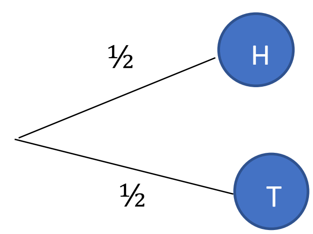
Tree Diagram

Tree diagrams allow us to see all the possible outcomes of an event and calculate their probability.
Each branch in a tree diagram represents a possible outcome.

Examples

A coin is tossed once. Find the probability of getting a i) head ii) tail.

Solution:



When a coin is tossed, the sample space is $\{H, T\}$

P (getting a head) = ½

P (getting a tail) = ½

So the sum of the probabilities is 1

P(A) = P (getting head)

P(B) = P (getting tail)

Complement of A

P(A) = P (getting head)

P(B) = P (getting tail)

Here P(B) is same as probability of 'not A'. We denote the event "not A" by $\bar{A}$ (Complement of A)

$$P(A) + P(\bar{A}) = 1$$

≤ P ≤ 1.

Example

If P (A) = 0.08, what is the probability of "not A".

Solution:

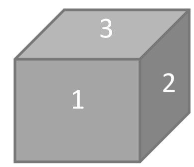
P (A) + P ($\bar{A}$) = 1

0.08 + P ($\bar{A}$) = 1

P ($\bar{A}$) = 1 – 0.08

= 0.92

Example:



A dice is thrown once.

What is the probability of each outcome?

What is the probability of rolling an even number?

What is the probability of rolling a number greater than 5?

Solution:

The sample space is {1,2,3,4,5,6}

p(getting 1) = $\frac{1}{6}$, p(getting 2) =$\frac{1}{6}$, p(getting 3) = $\frac{1}{6}$,

p(getting 4) = $\frac{1}{6}$, p(getting 5) = $\frac{1}{6}$, p(getting 6) = $\frac{1}{6}$,

Number of even numbers is 3 (2,4,6). So

p(getting an even number) = $\frac{3}{6}$ = $\frac{1}{2}$

P(getting a number greater than 5) = $\frac{1}{6}$

Example:

A box contains 2 blue, 3 white, and 5 red balls. If a ball is drawn at random from the box, what is the probability that will be

a) red, b) blue and c) not white

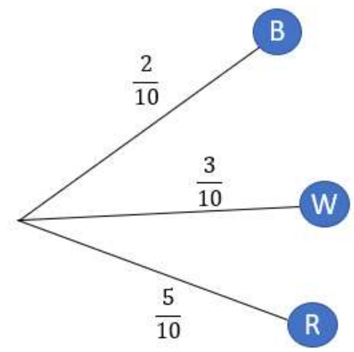
Solution:

Number of possible outcomes = 2+3+5= 10

P(getting a red ball) = $\frac{5}{10} = \frac{1}{2}$

P(getting a blue ball) = $\frac{2}{10} = \frac{1}{5}$

P(not getting white ball) = $\frac{2+5}{10} = \frac{7}{10}$



Exercise:

1) A number from 1 to 15 is chosen at random. What is the probability of choosing i) an odd number, ii) an even number, iii) a number less than 5, iv) a number between 3 and 10.
2) A glass jar contains 6 red, 5 green, 8 blue and 3 yellow marbles. If a single marble is chosen at random from the jar, what is the probability of choosing i) a red marble, ii) a yellow marble, iii)not a blue marble.
3) What is the probability of choosing a vowel from the English alphabet?
4) What is the probability of getting a 7 after rolling a single dice wi th numbers 1 to 6.

6.3 Representation of data

Bar Chart

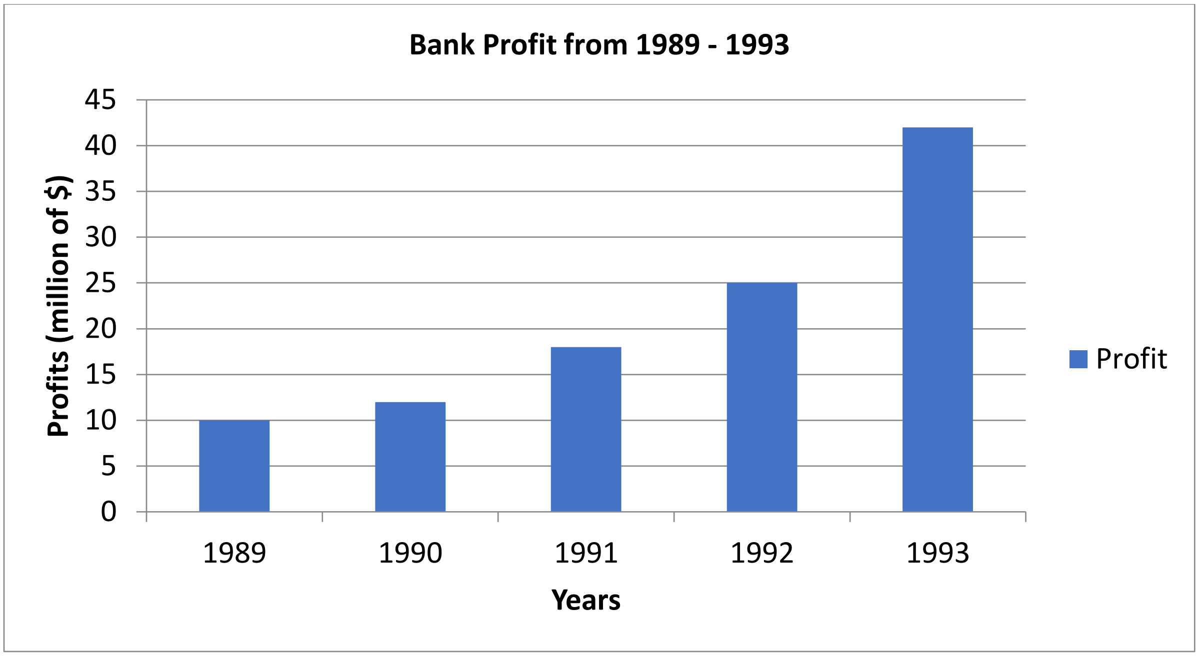
A bar chart uses bars to show comparisons between categories of data. These bars can be displayed horizontally and vertically.

A bar graph will always have two axes. One axis will generally have numerical values and the other will describe the types of categories being compared.

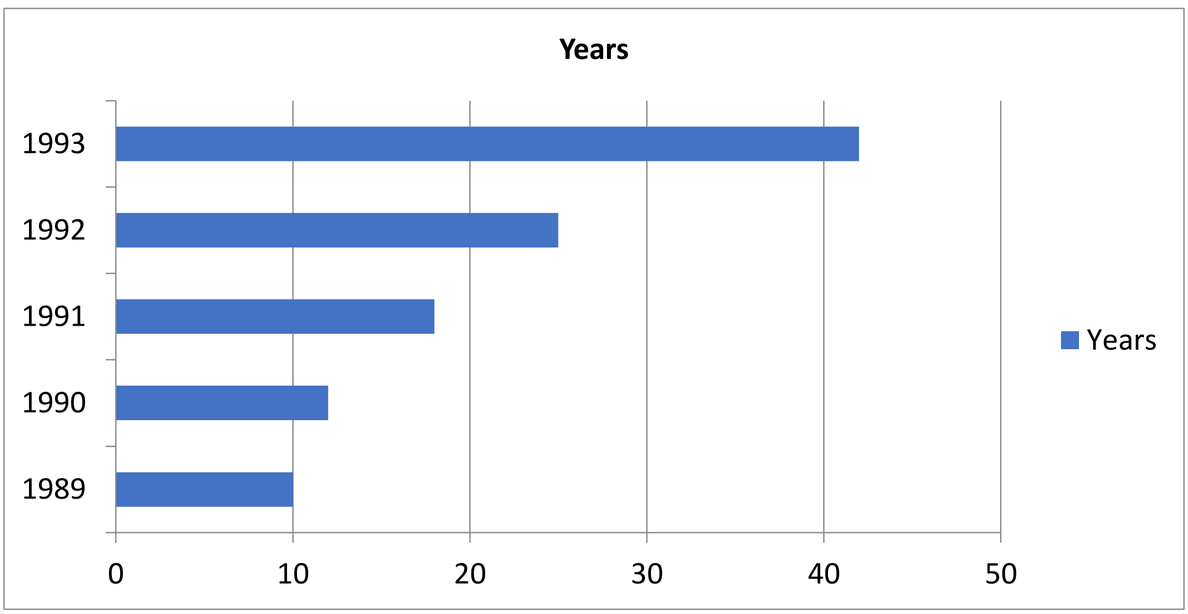
Heights of the bars are proportional to the frequencies.

Draw a bar chart to represent the profits of a bank for 5 years.

Years	1989	1990	1991	1992	1993
Profits (millions of $)	10	12	18	25	42



Bar graphs can also be **horizontal**, like this:



Exercises

1) Draw a bar chart to represent the following data and interpret the chart.

Day	Number of hotdogs sold
Monday	10
Tuesday	12
Wednesday	40
Thursday	36
Friday	15

2) The data below shows the percentage of the total expenditures of a company during 2003.

Heading	Infrastructure	Transport	Advertisement	Taxes	Salary
Expenditure (%)	20	12	15	10	20

3) The following data shows the production of fertilizers by a company (in 1000 tons) over the years

Year	2006	2007	2008	2009	2010
Production (in 1000 tons)	25	40	75	20	80

4) The following bar chart shows the trends of foreign direct investments (FDI) into Oman from Europe.

Year	1999	2000	2001	2002	2003	2004
Euro (in millions	10	15	12	35	30	55

Pie Chart

A special chart that uses "pizza slices" to show relative sizes of data.

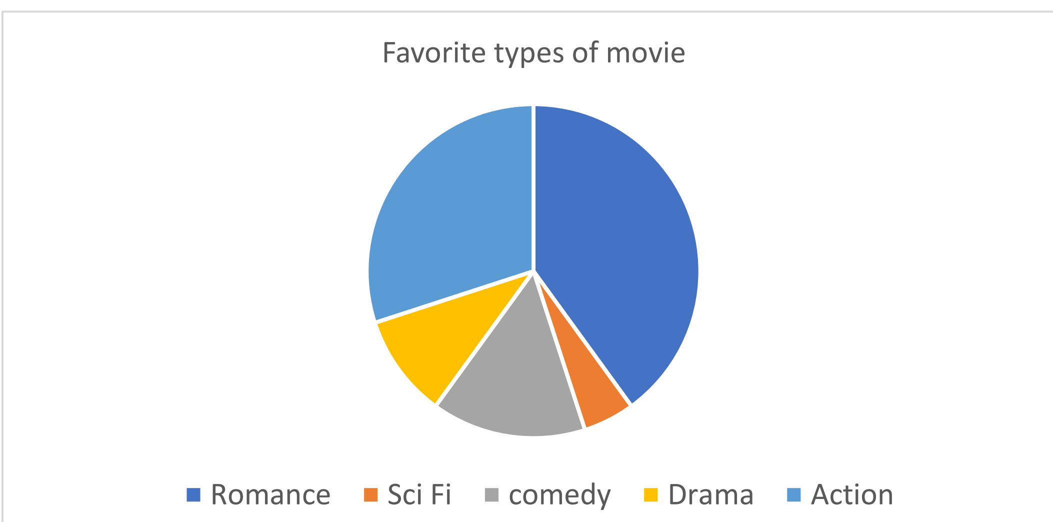
Favorite Type of Movie					
Romance	Sci Fi	Comedy	Drama	Action	TOTAL
8	1	3	2	6	20

Show the data in a pie chart

Solution:

A full circle has **360 degrees**, so we do this calculation:

Romance	**Sci Fi**	**Comedy**	**Drama**	**Action**	**TOTAL**
8	1	3	2	6	20
8/20 × 360° = 144°	1/20 × 360° = 18°	3/20 × 360° = 54°	2/20 × 360° = 36°	6/20 × 360° = 108°	360°



Exercise:

a) Draw a pie chart.

Type of vehicle	Number of vehicles
Car	140
Motorbike	70
Vans	55
Buses	5

b) Ninety people were asked which newspaper they read. Draw a pie chart

Newspaper	Number of people
Times of Oman	45
Muscat Daily	20
Others	15
I don't read the newspaper	10

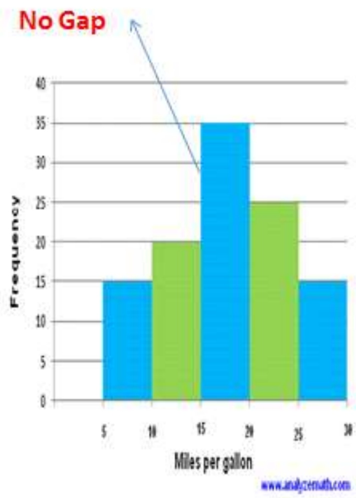
Histogram

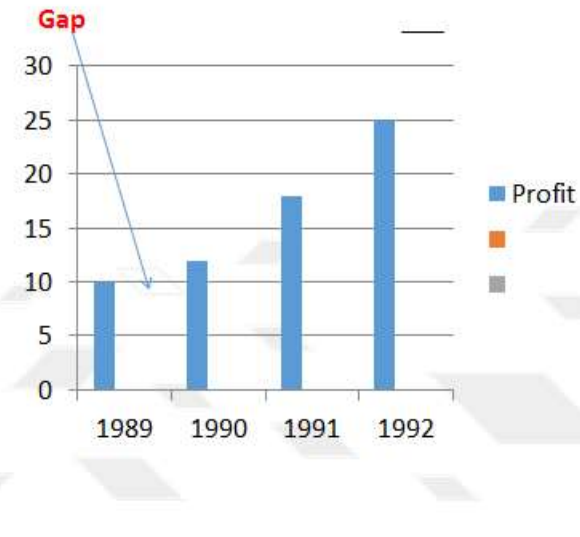
A histogram is a display of statistical information that uses rectangles to show the frequency of data items in successive numerical intervals of equal size.

A histogram has two axes: the x axis and the y axis.

The x axis represents the classes. The y axis contains frequency

Difference between Histogram and Bar chart

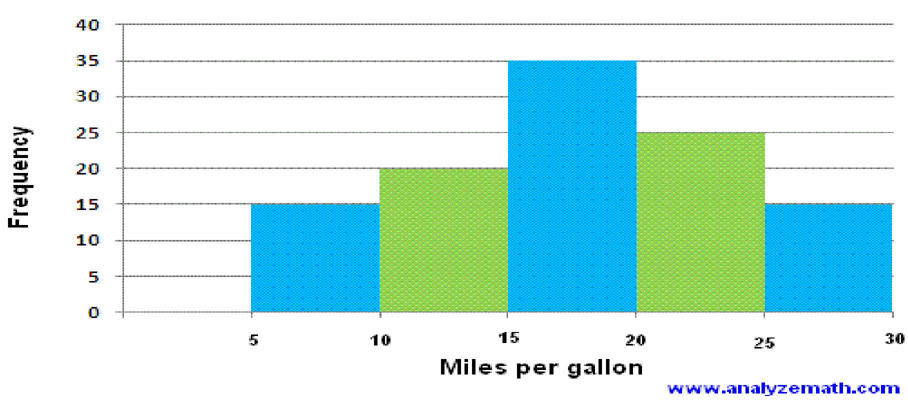




Example:

The fuel efficiency (in miles per gallon) of 110 cars are shown below. Draw the histogram.

Miles per gallons	**5-10**	**10-15**	**15-20**	**20-25**	**25-30**
Age of cars	15	20	35	25	15



Exercise

1) Draw the histogram.

Number of texts	**0-2**	**2-4**	**4-6**	**6-8**
Number of students	1	3	4	4

2) Draw the histogram.

Height of Trees	**100 -150**	**150- 200**	**200-250**	**250-300**
Number of Trees	5	30	25	50

6.4 Measures of Central Tendency

In statistics, a central tendency is a central value for distribution. Measures of central tendency are often called averages.

The most common measures of central tendency are the arithmetic mean, the median and the mode.

Mean

The mean is equal to the sum of all the values in the data set divided by the number of values in the data set.

If $x_1, x_2 \ldots\ldots x_n$ are observations then $(\bar{x}) = \frac{1}{n}\sum_{i=1}^{n} x_i$

Example

1) Find the mean of the data 12,15, 9, 16, 8, 5.

Solution:

$$\overline{x} = \frac{1}{n} \sum xi$$

$$= \frac{1}{6} \; (12+15+9+16+8+5)$$

$$= \frac{65}{6}$$

$$= 10.8$$

Mean of Grouped Data

If $x_1, x_2 \ldots\ldots x_n$ are observations and $f_1, f_2 \ldots f_n$ are the corresponding frequencies, then

mean $\bar{x} = \frac{1}{N}\sum_{i=1}^{n} x_i f_i$, **N** $= \sum_{i=1}^{n} f_i$

Example:

Rashid did a survey of the number of pets owned by his classmates, with the following result. Find the mean.

Number of Pets	Frequency
0	4
1	12
2	8
3	2
4	1
5	2
6	1

Solution

x_i No. of Pets	f_i Frequency	$x_i f_i$
0	4	0
1	12	12
2	8	16
3	2	6
4	1	4
5	2	10
6	1	6
	N = 30	$\Sigma xifi$ = 54

$$x = \frac{1}{N}\sum xifi$$

$$= \frac{1}{30} \times 54 = 1.8$$

Exercise:

1) Noora read a magazine article and recorded the number of letters in each word. Her results are shown in the table. Calculate the mean.

No. of letters	1	2	3	4	5	6
Frequency	2	8	15	30	22	16

2) Ahmed rolled a dice a number of times and recorded his results in a table. Calculate the mean score.

Scores	1	2	3	4	5	6
Frequency	7	8	6	4	7	8

3) Find the average marks received by a group of students from the following:

Marks obtained	10	20	30	40	50
Number of Students	6	8	5	7	6

4) The mean marks scored by 100 students was found to be 40. Later on it was discovered that a score of 53 was misread as 83. Find the correct mean.

Mode

The mode is the value of **observation having the maximum frequency**.

Example:

Find the mode of the data 25, 30, 35, 25, 30, 25, 35, 25, 35

Solution:

Value	Frequency
25	4
30	2
35	3

Maximum frequency is 4, So **Mode = 25**

Exercise:

Find the mode of the following:

1) 26,27, 28,27,28,26,27,28,28

2) 171, 173, 171,172,171, 172,171, 173, 171,173

Median

The median is the middle number in a set of numbers. To calculate the median of any set of numbers, you need to write the numbers in order.

To find the median numbers

1) Put all the numbers in numerical order- either increasing or decreasing order.
2) If there is an odd number of observations, then

   median = $\left(\frac{n+1}{2}\right)^{th}$ observation.
3) If there is an even number of observations, then

   median = mean of $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ observation.

Example:

Find the median of 5, 11, 12, 4, 8, 21

Solution:

Arrange the numbers in order 4, 5, 8, 11, 12, 21 n=6

Median = mean of 8 and 11.

$= \frac{8+11}{2} = 9.5$

Example:

Find the median of 7, 10, 5, 6, 8

Solution:

Arrange the numbers in order

5, 6, 7, 8, 10

N = 5

Median = 3rd observation. = 7

Exercise:

1) Find the median of the following:
   a) 38,42,36, 44, 39, 34, 40, 43
   b) 25, 23, 19, 24, 12, 7, 17
2) Find the median of the data 25, 18, 12, 24, 32, 23, 19, 29, 27. If 12 is replaced by 21, what will the new mean be?

Reference:

CMS, C. (n.d.). *When to use a Bar Chart*. [online] Chartblocks.com. Available at: https://www.chartblocks.com/en/support/faqs/faq/when-to-use-a-bar-chart [Accessed 2 Apr. 2019].