COMPUTER SCIENCE 3P03 (FALL 2020)

Algorithms

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Assignment #4

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1. We can use this black box to solve the subset whose sum is b in the following way:

Run the input $\{x_1, x_2, \dots, x_n\}$ and k through the box. If it returns YES we know a subset exists. If NO, stop now, there isn't one.

If there is a subset we can find it by removing the i^{th} element and running the input and k through the box again. If it returns YES there is a subset that exists without that i^{th} element, so we can discard it. If it returns NO that element is necessary for the subset so we have to put it back. Increase i and repeat this process. When we get to the end of the input array we will have the subset left over.

ex/ $[2,3,4]$ $k=6$

$[2,3,4] \rightarrow \boxed{Y}$, there is a subset. Begin search.

$[3,4] \rightarrow \boxed{N}$, put 2 back it is necessary

$[2,4] \rightarrow \boxed{Y}$, 3 not necessary

$[2] \rightarrow \boxed{N}$, 4 is necessary, this is the end of input so subset is $[2,4]$

2. a) We can show the greedy solution does not always work with a counter-example:

$$S = \{1003, 885, 854, 771, 734, 486, 281, 121, 83, 62\}$$

$S_1 = \{1003, 771, 486, 281, 83\}$ $S_1 = 2624$

$S_2 = \{885, 854, 734, 121, 62\}$ $S_2 = 2656$

$$|S_1 - S_2| = |2624 - 2656| = 32$$

Counter:

$S_1 = \{1003, 885, 486, 281\}$ $S_1 = 2655$

$S_2 = \{854, 771, 734, 121, 83, 62\}$ $S_2 = 2625$

$$|S_1 - S_2| = |2655 - 2625| = 30$$

$30 < 32$, 32 not optimal, greedy not optimal.

2. b) To Show this problem is NP-Complete we need to:

1. Show it is in NP
2. Give a reduction from a known NP-Complete problem

1. This partition problem is in NP:

We can verify in polynomial time whether S_1 and S_2 have sums less than or equal to K. If they do then we accept this certifizate.

2. The Subset Sum Problem (which is NP-Complete) reduces to the Partition Problem:

Given that PARTITION(S,K) needs us to find S_1 and S_2 such that the $|sum(S_1) - sum(S_2)| \leq K$, then PARTITION(S,0) would mean that $|sum(S_1) - sum(S_2)| \leq 0$

This is only true if $sum(S_1) = sum(S_2)$. So we can reduce SUBSET(S,K) which wants to find 2 subsets with sum equal to K, to PARTITION(S,0).

Since this known NP-Complete problem reduces to the PARTITION problem, and the PARTITION is in NP, it is in NP-Complete.

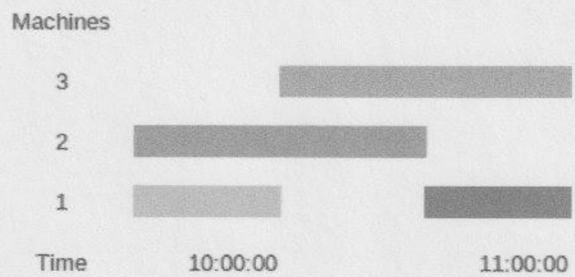
3. Since he has a map and knows he can travel K miles on 1 bottle, he should check his map at the start and choose to fill it at the watering hole that is farthest from his current position but is within K miles.

Since we are choosing the farthest within K miles there is no watering hole farther we could have selected without running out of water. Then any other optimal solution would have selected the same watering hole or one closer to the starting point, so this algorithm is doing no worse than any other optimal solution. This argument holds for all other stops between the start of journey and end.

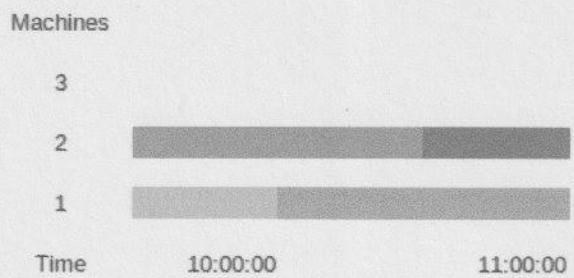
Therefore this algorithm cannot do worse than any other optimal solution, making it optimal.

4. In the first image we can see that the Jobs have been scheduled using the greedy solution of "shortest job first". However this is not optimal since the jobs could be better scheduled in terms of "best fit". This counter example shows that the greedy solution does not always provide an optimal solution.

1



2



5. We can treat the longest increasing subsequence like a DAG in the following way:

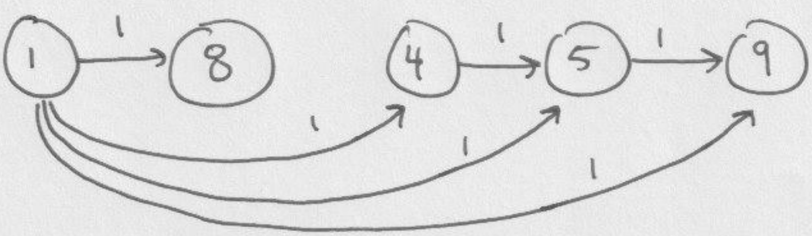
Let S be a sequence of n distinct integers stored in an array: S[1], S[2]... S[n]. We can take each element and make it a vertex in a graph G.

We can connect these vertices with an edge of weight 1 directed $V_i \rightarrow V_j$, so long as:

1. $\text{Vertex}_i < \text{Vertex}_j$
2. Vertex_i comes before Vertex_j in the array.

Condition 1 means we will have an ascending sequence and condition 2 means we will have no cycles.

ex/ S[1, 8, 4, 5, 9], find LIS for 1



$1 \rightarrow 8 = 1$

$1 \rightarrow 4 \rightarrow 5 \rightarrow 9 = 3$ ← here is LIS

$1 \rightarrow 5 \rightarrow 9 = 2$

$1 \rightarrow 9 = 1$