HW2

1. a.) $T(n) = 3T(n/27) + 1$

$a = 3$, $b = 27$, $f(n) = 1$, $\log_{27}(3) = 1/3$

$f(n) \le O(n^{1/3 - \varepsilon})$, $\varepsilon = 1/3$

$= O(n^0)$

$= O(1)$

$\boxed{\therefore T(n) = \Theta(n^{1/3})}$

b.) $T(n) = 7T(n/8) + \lg n$

$a = 7$, $b = 8$, $f(n) = \lg(n)$, $\lg_8(7) \approx 0.94$

$f(n) \le O(n^{.94 - \varepsilon})$, $\varepsilon = 0.1$

$\le O(n^{.84})$

$\boxed{\therefore T(n) = \Theta(n^{.94})}$

c.) $T(n) = 2T(n/4) + n$

$a = 2$, $b = 4$, $f(n) = n$, $\lg_4(2) = 1/2$

$f(n) \ge \Omega(n^{1/2 + \varepsilon})$, $\varepsilon = 1/2$

$= \Omega(n)$

$2\left(\frac{n}{4}\right) \le cn \Rightarrow \frac{n}{2} \le cn$, $c = 1/2$

$= \frac{n}{2}$

$\boxed{\therefore T(n) = \Theta(n)}$

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Hw 2

1. d.) $T(n) = 2T(n/4) + n^2$

$a = 2,\ b = 4,\ f(n) = n^2,\ \lg_4(2) = 1/2$

$f(n) \geq \Omega(n^{1/2+\varepsilon})$, $\varepsilon = 3/2$

$= \Omega(n^2)$

$2\left(\frac{n}{4}\right)^2 \leq cn^2 \Rightarrow \frac{n^2}{8} \leq cn^2$, $c = 1/8$

$= \frac{n^2}{8}$

$\boxed{\therefore T(n) = \Theta(n^2)}$

e.) $T(n) = 2T(n/4) + n^{1/2}\lg n$

$a = 2,\ b = 4,\ f(n) = \sqrt{n}\lg n,\ \lg_4(2) = 1/2$

$f(n) = \Theta(\sqrt{n})$

$\boxed{\therefore T(n) = \Theta(\sqrt{n}\lg^2 n)}$

2. There is no change. 27 > 17 ∧ 3 , no swaps

HW2

3. Build Maxheap(A) = [25, 13, 20, 8, 7, 17, 2, 5, 4]

Then we ~~see~~ swap last and first
& run max heapify

$A = [2, 4, 5, 7, 8, 13, 17, 20, 25]$

which is sorted array

4. $T(n) = 2T(n/2 - 3) + n$

$a = 2$, $b = 2$, $f(n) = n$, $\lg_2(2) = 1$, $k = 0$

$f(n) = \Theta(n^1)$

$\boxed{\therefore T(n) = \Theta(n\lg(n))}$ which is in $\Omega(n \lg n)$

5. a.) There are $\boxed{23}$ swaps, 5 in build max heap and 18 are from heapsort re-orders. I get these numbers from programing it.

b.) There will be more comparisons but same number of swaps.

c.) It would increase to $O(n^2)$ because of the extra comparisons.

HW2

6. a) $T(n) = 2T(n/2) + \sin(n)$? $\boxed{\text{No}}$

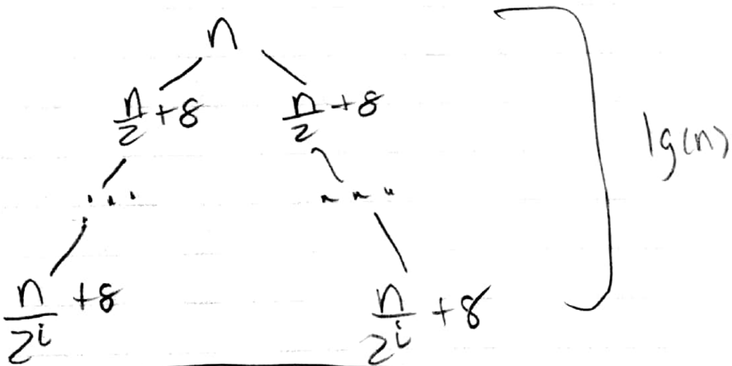
↳ Cant use, $\sin(n)$ range is $[-1, 1]$, and running time cant be negative.

b) $T(n) = T(n/2) + n\sin(n) + 2n$? $\boxed{\text{Yes}}$

$$= T(n/2) + n(2 + \sin(n))$$

$\boxed{\text{Yes}}$ because $n(2+\sin(n))$ will always be a positive number, unlike a)

7. $T(n) = 2T(n/2 + 8) + n$



$\boxed{\therefore T(n) = \Theta(n \lg n)}$, upper and lower band

HW2

8. Loop invariant: The start of every iter of the for loop inside the Build Max Heap is a root of Max Heap

Any thing below $\lfloor \frac{n}{2} \rfloor$ has no leaf and doesn't need max heapify

When i is decremented after each iter down to 0 establishes loop invariant.

∴ When $i = 0$ we know the loop terminates and know there is a max heap