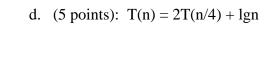
Question 1 (20 points): Solve the recurrence relations below.

a. (5 points):
$$T(n) = T(n-1) + n$$

b. (5 points):
$$T(n) = T(\alpha n) + T((1-\alpha)n) + cn$$
 (0 < \alpha < 1)

c. (5 points):
$$T(n) = 4T(n/2) + n^3$$



Question 2 (10 points) Describe the Mergesort and Quicksort (not the randomized version) algorithms, and explain the similarity and difference between both algorithms. You should give the algorithm details like how the merging function or the partition function works.

Question 3 (10 points) What are the minimum and maximum numbers of elements in a heap of height *h*? Explain your answer.



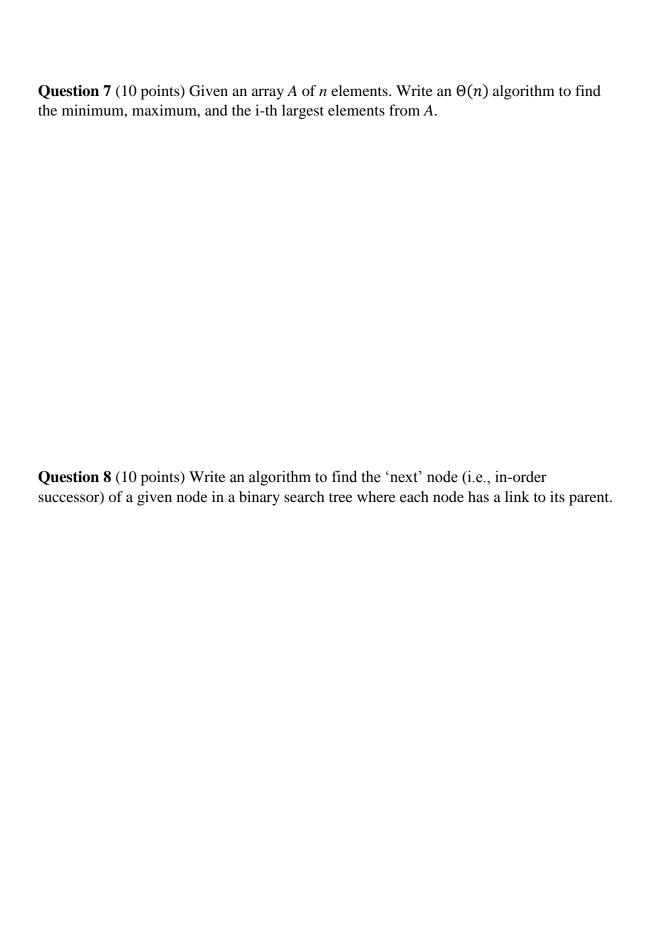
Question 6 (10 points) Read the algorithm of counting sort below, and answer the following questions

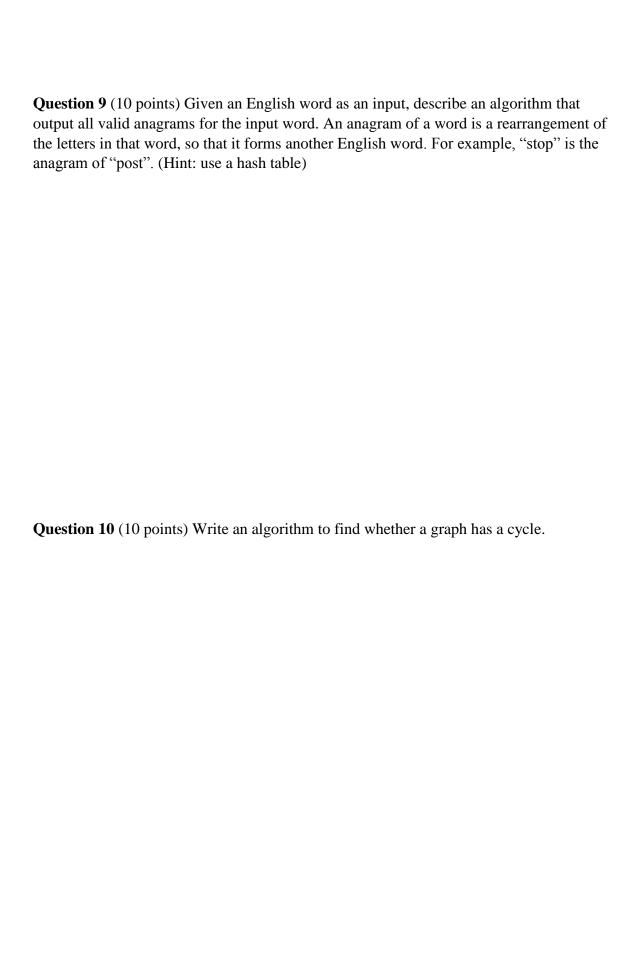
COUNTING-SORT(A, B, k)

1 let C[0 ... k] be a new array **for** i = 0 **to** kC[i] = 0**for** j = 1 **to** A.length C[A[j]] = C[A[j]] + 1**for** i = 1 **to** kC[i] = C[i] + C[i-1]**for** j = A.length **down to** 1 B[C[A[j]]] = A[j]C[A[j]] = C[A[j]] - 1

a. (7 points) Suppose that we were to rewrite the for loop header in line 8 of the COUNTING-SORT as **for** j = A.length **to** 1. Show that the algorithm still works properly. Is the modified algorithm stable?

b. (3 points) Is it a good idea to use counting sort to sort a set of n integers in the range of $[1, n^3]$? Please explain your answer.





Question 11 (10 points) Show that a graph has a unique minimum spanning tree if, for
every cut of the graph, there is a unique light edge crossing the cut. Show that the
converse is not true by giving a counterexample.

Question 12 (10 points): Tell whether each of (a) - (e) is TRUE, FALSE, or OPEN.

- (i) $\Pi_1 \in P$,
- (ii) $\Pi_2 \in NP$ -complete,
- (iii) $\Pi_3 \in NP$.

- a. $\Pi_1 \in NP$
- b. $\Pi_2 \in P$
- c. Π_2 is polynomially reducible to Π_1
- d. If P = NP, then $\Pi_2 \notin P$
- e. Π_3 is polynomially reducible to Π_2