Saurabh Hinduja.

49 +20 = 69

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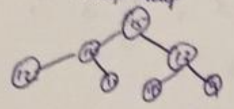
*Introduction to Theory of Algorithms, Midterm I. This is a closed book, closed notes exam and no calculators are allowed. Sep. 30th, 2015*

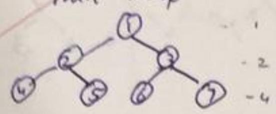
**Question 1** (10 points) What are max-heap and min-heap? Please give examples of max-heap and min-heap.

Max-heap is when the root is maximum and it decreases as we go to leaves. Min-heap is when the root is minimum and it increases as we go to the leaves. Example.

Max heap



min heap



**Question 2** (10 points): Can MAX-Heapify be implemented in a non-recursive way? If your answer is yes, please write the implementation code. If your answer is no, please explain why. Yes, it can be implemented in non-recursive way.

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```
Max-heapify.
For i = lg(A.length) to 0 (Decrease).
    ~~For j = A.length to 0~~   j = 2^(log i)
    if A[j] > A[i]
        key = A[j]
        A[j] = A[i]
        A[i] = key.
    if A[j+1] > A[i]
        key = A[j+1]
        A[j+1] = A[i]
        A[i] = key.
```

Start from the last leaf and. Compare it with each root.

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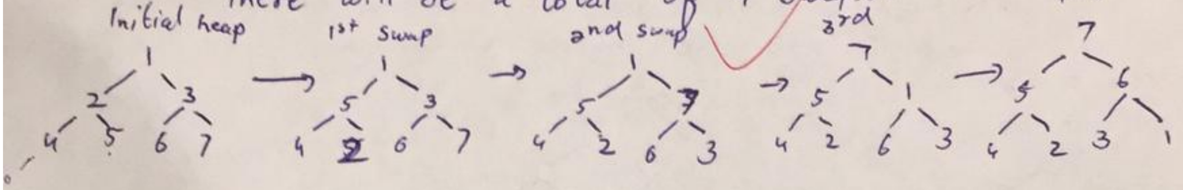
**Questions 3** (10 points): Fill True or False in the table below.

| $f(n)$ | $g(n)$ | $f(n) = O(g(n))$ | $f(n)=\Omega(g(n))$ | $f(n)=\Theta(g(n))$ |
|---|---|---|---|---|
| $2n^3 + 3n$ | $100n^2 + 2n + 100$ | False | True. | False. |
| $50n+\lg n$ | $10n+\lg\lg n$ | ~~False~~ True | True. | True |
| $50n\lg n$ | $10n\lg\lg n$ | ~~False~~ True. | True | True. |
| $\lg n$ | $lg^2n$ | False | True. | False. |
| $n!$ | $5^n$ | True | False | False. |

**Question 4** (30 points): Given an array A = [1, 2, 3, 4, 5, 6, 7]

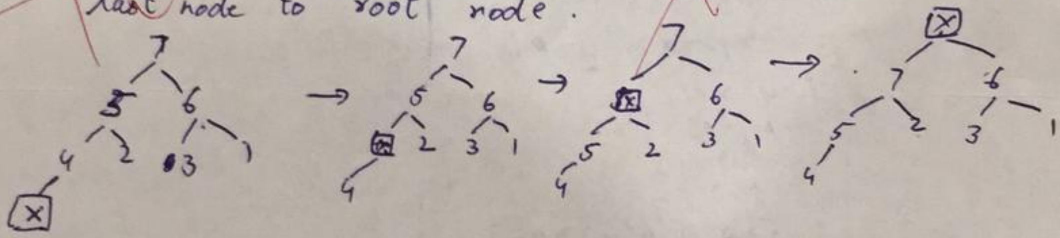
(a) What is the exact number of swap operations for BUILD_MAX_HEAP to build a max-heap on this array? Explain your reason.

There will be a total of 4 swaps.

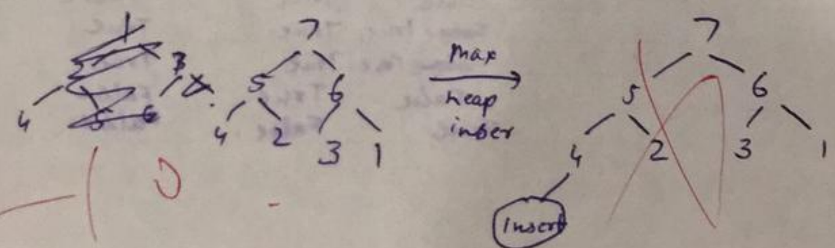



(b) What is the exact number of swap operations for Max-Heap-Insert to build a max-heap on this array? Explain your reason.

Total number of swap operations is 3. To swap from the last node to root node.



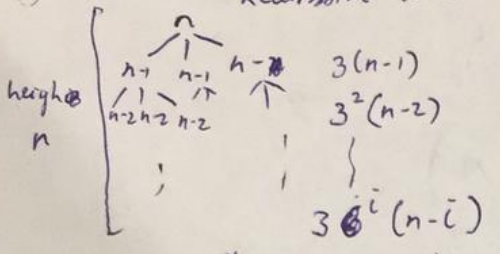
(c) Draw the max-heap that is built by calling Max-Heap-Insert.



**Question 5** (30 points): Solve the recurrence relations below.

a. (5 points): $T(n) = 3T(n-1) + n$

Recursive tree.



$n^{th}$ term $= 3^{n-1}$

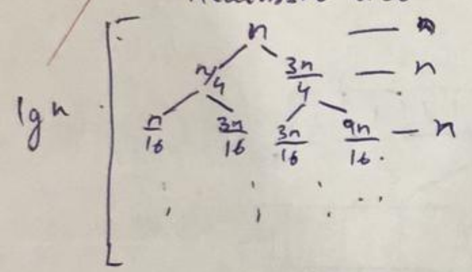
$$\text{Sum} = \sum_{0}^{n-1} 3^i (n-i)$$

$$3^n \frac{n(n+1)}{2}$$

$$\text{Ans} = O\left(3^n n(n+1)\right).$$

b. (10 points): $T(n) = T(n/4) + T(3n/4) + n$

Recursive tree.



$$\text{Ans} = O(n \lg n).$$

c. (5 points): $T(n) = 6T(n/7) + \lg n$

Masters theorem.

$a = 6,\ b = 7,\ f(n) = \lg n.$

$$f(n) = O\left(n^{\log_6 7 + \epsilon}\right)$$

$$\text{Ans}\ O(\lg n)$$

$f(n)$ $n \lg^2 n$

d. (5 points): $T(n) = 2T(n/2) + n\lg^2 n$

Masters theorem.

$a = 2 \quad b = 2 \quad k = 1 \quad p = 2.$

Ans $\cancel{O(\lg \log_2 \lg^2 n)}$,

$$= O(\lg^3 n).$$

e. (5 points): $T(n) = 2T(n/4) + \sqrt{n}/\lg n$

Masters theorem.

$a=2 \quad b=4 \quad k=1/2 \quad p=-1$

Ans $O(n^{\log_2 4} \lg n)$.

$= O(n^2 \lg n)$.

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$\Omega\left(\frac{\sqrt{n}}{\lg n}\right) \quad n^{\log_4 2} \quad \frac{\lg n}{\sqrt{n}}$

$f(n) = \frac{\sqrt{n}}{\lg 2}$

$f(n)$

**Question 6** (10 points) Is merge sort an in-place algorithm? Describe how merge sort works.

In Merge Sort we have 2 sorted arrays and we merge them in ~~in~~ ~~order~~ Order.

Answer Y/N - first

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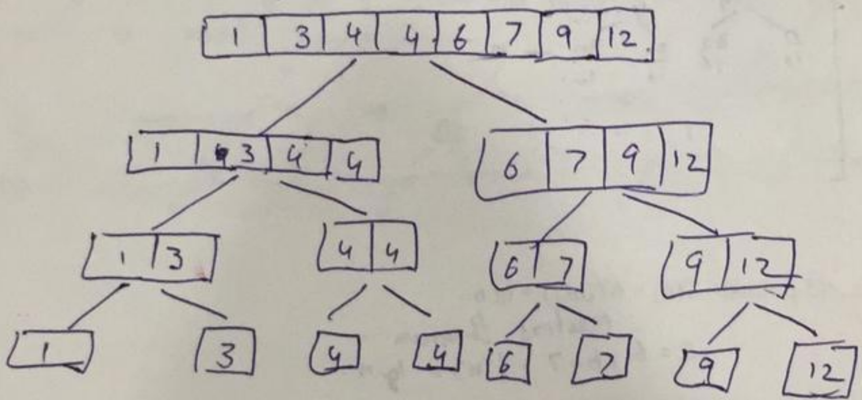


As shown in the diagram above in Merge sort the element of independent arrays are compared and then merged into one array. This procedure is repeated recurssively until we have only one array.