EXAM INSTRUCTIONS

The time limit for the final exam is 120 minutes. The exam is closed book; thus, no notes are allowed. Using your browser to access other websites, while the exam is in progress, is also prohibited.

As was previously stated (in the first lesson and in the course syllabus), to ensure academic integrity *Proctorio* will be used as a proctoring tool in this exam. Please note that Proctorio will require access to your device's web camera and the audio input, it'll also monitor your internet traffic and provide a detailed report marked with any suspicious activity. All monitored data feeds--audio, video, traffic, and so on--will be reviewed, and those found to be in violation of any of the college's rules on academic integrity will risk getting a failing grade for the course (as outlined in the course syllabus).

In this vein, you're advised to **abstain from obstructing or otherwise tampering with your video or audio** hardware in an attempt to "fool/game" the system. All offenses will be thoroughly investigated and the offending individual will risk failing this class.

All questions are compulsory.

Vectors

1. Definitions:
   a. Vectors
   b. Subspace of vector space
   c. Span
   d. Orthogonality
   e. Linear independence
   f. Hilbert space
   g. Inner product space
2. Computations with vectors
   a. Add and subtract (n-dimensional) vectors
   b. Multiply (n-dimensional) vectors by a constant
   c. Compute the inner product between two vectors (finite and infinite dimensional vectors)
   d. Compute p-norm of vectors (finite & infinite dimensional)
   e. Inner product between (finite & infinite dimensional) vectors

Bases

1. Definitions
   a. Operator norm (formula)
   b. A self-adjoint operator
   c. Define a Reisz Basis
   d. The difference between orthogonal, orthonormal, and biorthogonal bases
2. Computations
   a. Computing the transpose of a real-valued matrix
   b. Computing the Hermitian (or conjugate-transpose) of complex-valued matrices
   c. Computing the condition number of a matrix (using eigenvalues or singular values)
   d. Computing inner product of vectors using Perseval's equality
   e. Computing the synthesis operator for a given set of vectors
   f. Computing the dual basis

Fourier Transform

1. Definitions
   a. 1D and 2D Fourier Transform integral
   b. Inverse FT for 1D and 2D
   c. FT of a triangle function
   d. FT of a square function

2. Computations
    a. Computing Fourier transform for simple functions (Boxcar functions, sinusoidal functions, Dirac deltas, and a sum of Dirac deltas)
    b. Identify effects of filtering, from images out inputs, outputs and the TFs of imaging systems.

Sampling and Interpolation

1. Definitions
    a. Define sampling
    b. Define interpolation
    c. Nyquist frequency
    d. Sampling frequency
    e. Sampling interval
2. Computations
    a. Computing sampling interval/frequency given a set of samples in space/time
    b. Computing sampling interval from the sampling frequency
    c. Computing sampling frequency from the sampling interval
    d. Compute the bandwidth of a function from the plot of its frequency spectrum
    e. Calculate minimum sampling frequency to prevent aliasing from frequency spectrum of continuous time signal

Linear Space Invariant Imaging Systems

1. Definitions
    a. Properties of an LSI system
    b. Define convolution and its formula (for 2D and 1D)
    c. Convolution property of the FT
    d. Input-Output relationship for an LSI system
    e. Definition of the acronyms PSF, OTF, MTF, PTF
    f. Convolution properties (Linearity, Boundedness, and so on)
2. Computations
    a. Computing convolution graphically

Linear Space Invariant Imaging Systems (Discrete or Matrix-Vector form)

1. Definitions
    a. Transfer function for discrete LSI system
    b. Properties of a discrete LSI
    c. Able to describe the two key steps for obtaining a fully discrete forward model: (sample/discretize image space, and discretize object space) – *<u>see lecture 16-17, slide 6 for summary</u>*
    d. Range space
    e. Null space
2. Computations
    a. Computing convolution graphically
3. Hardware
    a. Key function of the **microlens** in a detector element
    b. Function of the **color filter** in a detector element

Discrete Convolution

1. Definitions
    a. Discrete convolution formula
2. Computations
    a. Computing the discrete convolution between two sequences, each of length at most five terms.

Inverse problems (Intro Lectures 17-19)

1. Definitions
    a. Forward model and forward problem

    b. List the two classes of forward models
    c. Inverse problem
    d. Well-posedness and ill-posedness in the Hadamard sense
    e. Condition number of matrix (forward model) as test for the conditioning of inverse problem
    f. Moore-Penrose inverse formula
    g. Tikhonov-Regularized inverse (Optimization problem and the Tikhonov regularized inverse)
2. Computations
    a. *None*

Interative methods

1. Definitions
    a. Role of number of iterations as the regularization parameter
    b. Key advantages of iterative methods
    c. Key idea of gradient method
2. Computations
    a. *None*

Statistical methods

1. Definitions
    a. Meaning of MLE
    b. Meaning of MAP
    c. Which of MAP or MLE is a Bayesian method
2. Computations
    a. Description of the steps taken to compute the MLE

Nonlinear Regularization

1. Definitions
    a. Identify TV regularized problem
    b. Identify L1 regularized problem
    c. Meaning of TV and what prior it imposes
    d. What prior L1 regularization imposes
2. Computations
    a. *None*

Intro to Optics

1. Definitions
    a. Geometric vs Wave optics
    b. How pinhole camera works
    c. Lens types (positive and negative)
    d. Field of view
    e. The waves of the EM spectrum
2. Computations
    a. *None*