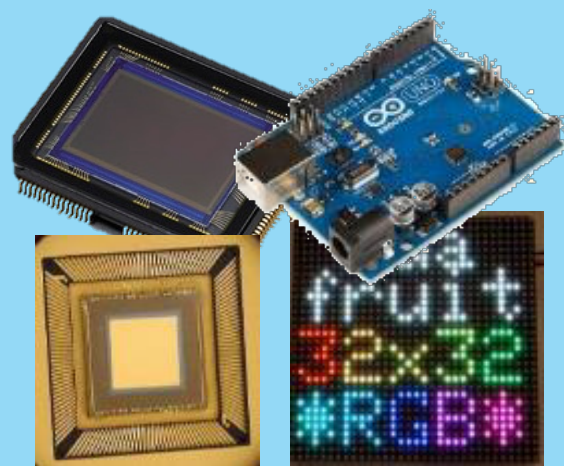


Optics



Sensors
&
devices



Signal
processing
&
algorithms

COMPUTATIONAL METHODS FOR IMAGING (AND VISION)

DISCRETE CONVOLUTIONS (EXAMPLES)

PROF. JOHN MURRAY-BRUCE

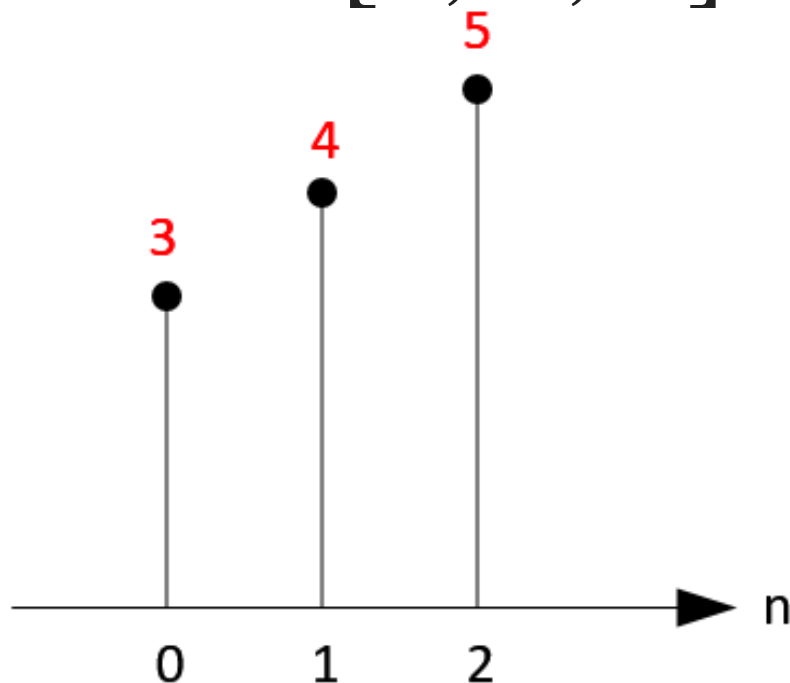
DISCRETE CONVOLUTION

EXAMPLES FOR 1D AND 2D

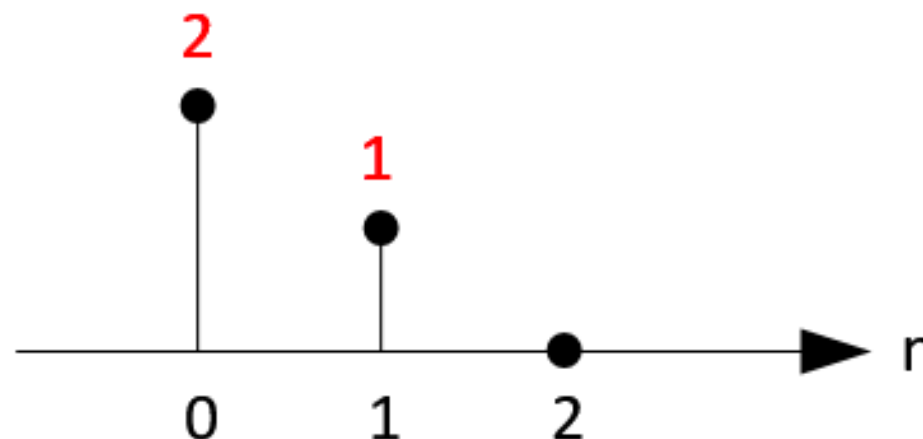
1D DISCRETE CONVOLUTION

EXAMPLE

$$x = [3, 4, 5]$$



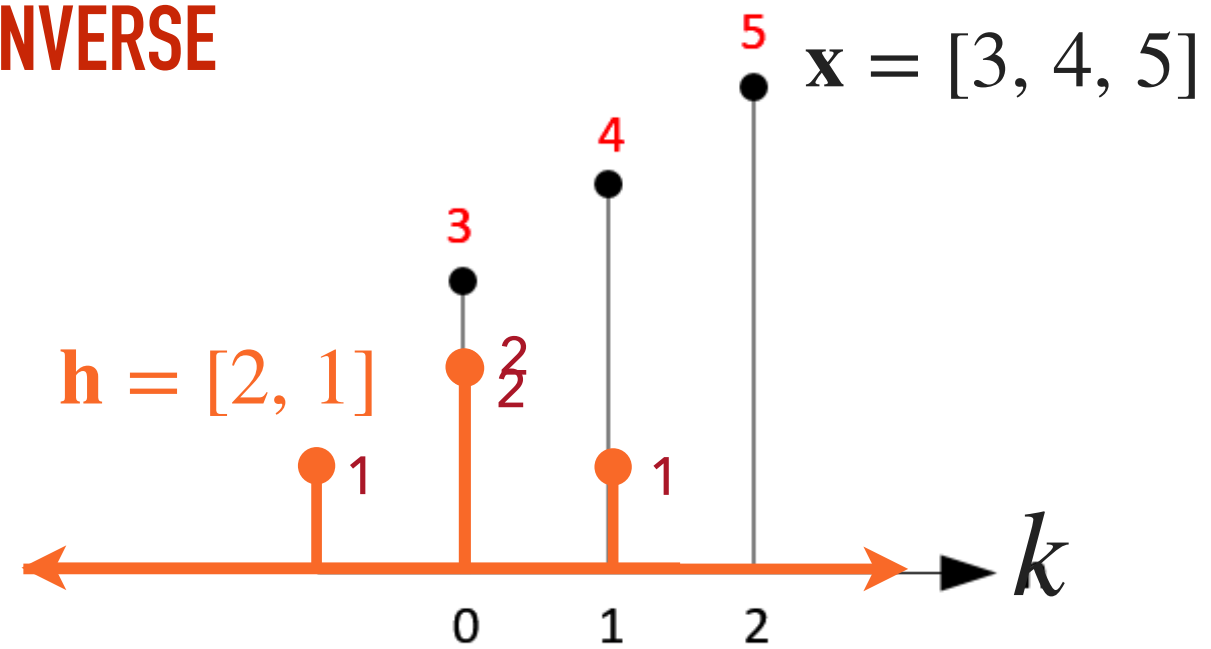
$$h = [2, 1]$$



$$g_n = \sum_{k=-\infty}^{\infty} x_k h_{n-k}$$

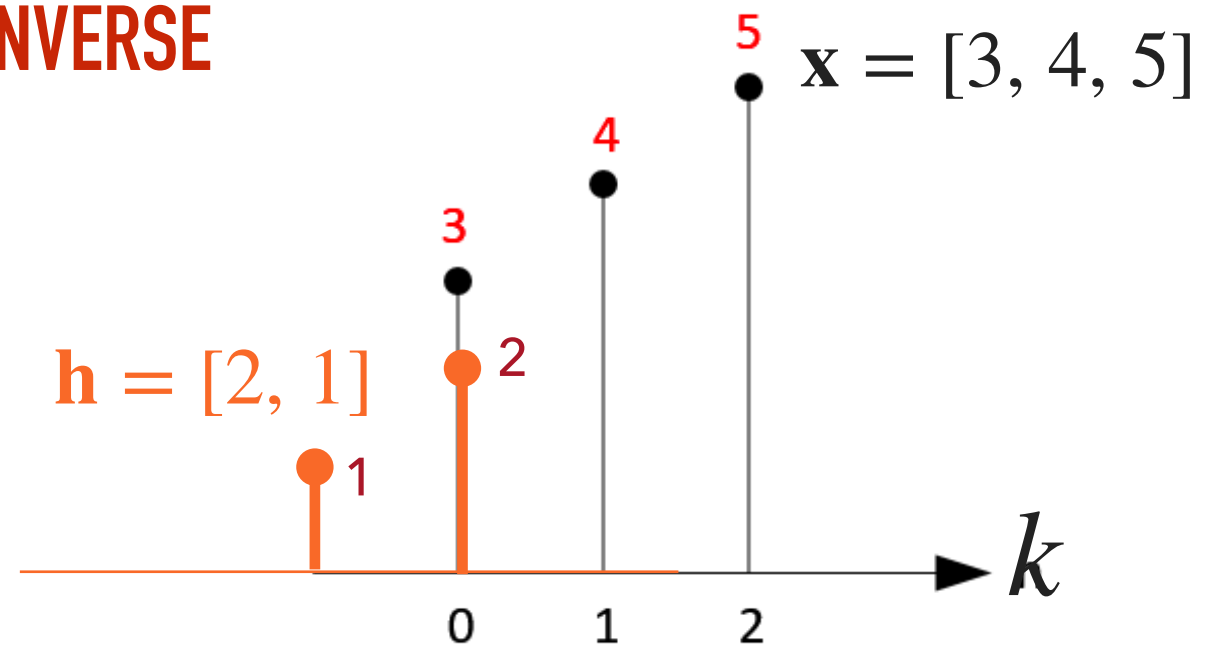
1D DISCRETE CONVOLUTION

MOORE-PENROSE PSEUDO-INVERSE



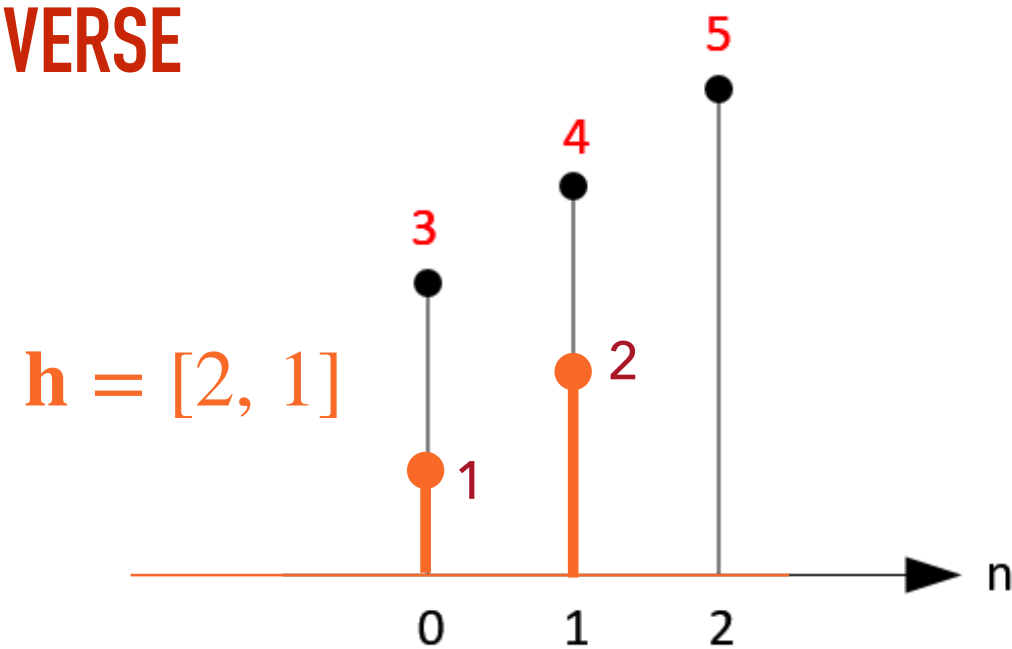
1D DISCRETE CONVOLUTION

MOORE-PENROSE PSEUDO-INVERSE



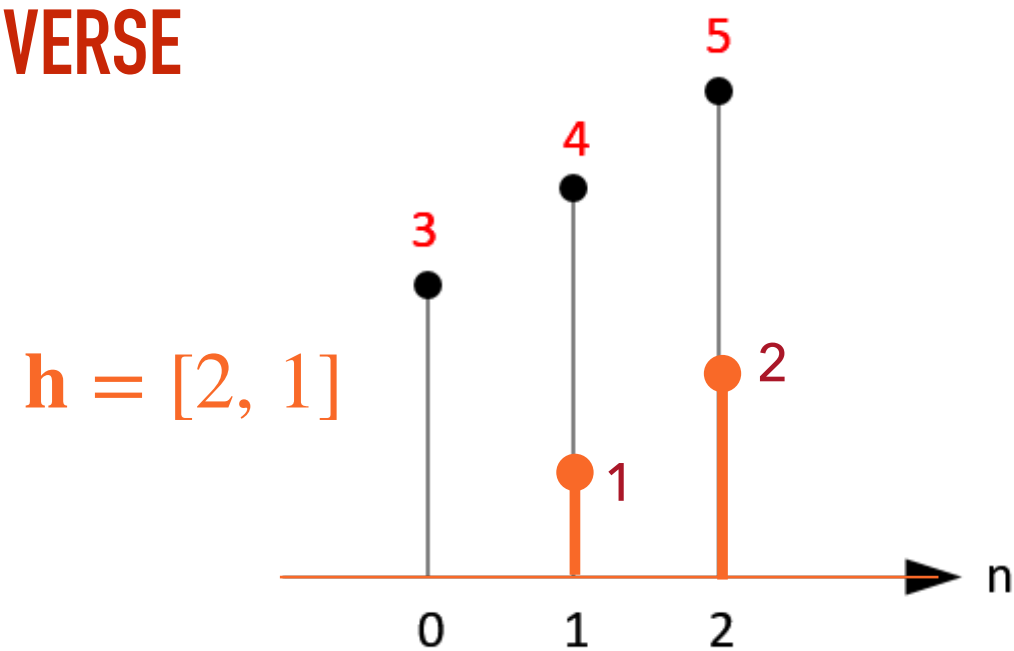
1D DISCRETE CONVOLUTION

MOORE-PENROSE PSEUDO-INVERSE



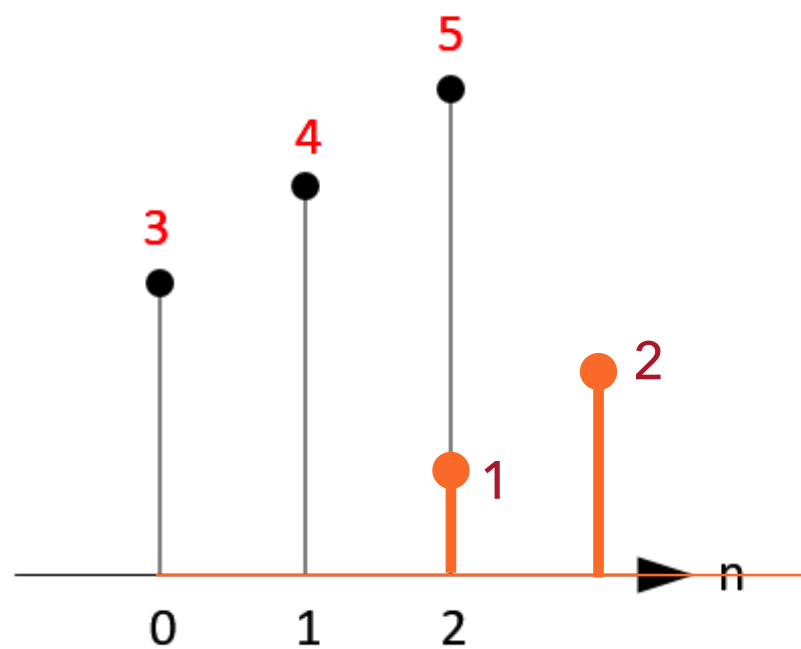
1D DISCRETE CONVOLUTION

MOORE-PENROSE PSEUDO-INVERSE

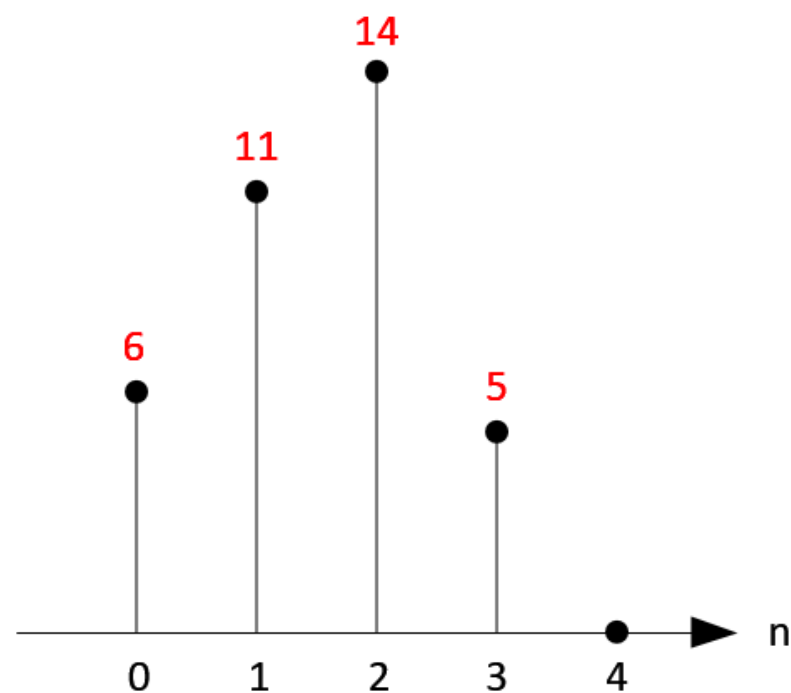


1D DISCRETE CONVOLUTION

MOORE-PENROSE PSEUDO-INVERSE



$$y = [6, 11, 14, 5]$$



The image displays the MATLAB R2019a software interface. The top ribbon includes tabs for HOME, PLOTS, APPS, EDITOR, PUBLISH, and VIEW. The left sidebar contains the Current Folder browser, Command Window, and Workspace browser. The Command Window shows the prompt `fx >>`. The Workspace browser lists variables: A, ans, g, h, H, Image1, inImage, outImage, and x. The Editor window on the right shows a script file with a green status bar at the top. The bottom status bar indicates the current file is `script` at line 1, column 4.

HOME PLOTS APPS Q Search Documentation Log In

New Script New Open Compare Import Data Save Workspace New Variable Open Variable Clear Workspace CODE SIMULINK ENVIRONMENT RESOURCES

« Google Drive » USF Tampa » Teaching » CIS4930:6930 » Lectures » Lecture 18 » MATLAB

Current Folder Name

Command Window `fx >>`

Wor... Name

- A
- ans
- g
- h
- H
- Image1
- inImage
- outImage
- x

EDITOR PUBLISH VIEW

New Open Save Find Files Compare Go To Find EDIT Breakpoints Run Run and Advance Run Section Advance Run and Time

1 %%

Script_ComputeCRBsTwoHiddenTargets.m fourierseries_sq.m computeFIMpointsources.m untitled* script Ln 1 Col 4

DISCRETE CONVOLUTION

2D EXAMPLE

2D DISCRETE CONVOLUTION

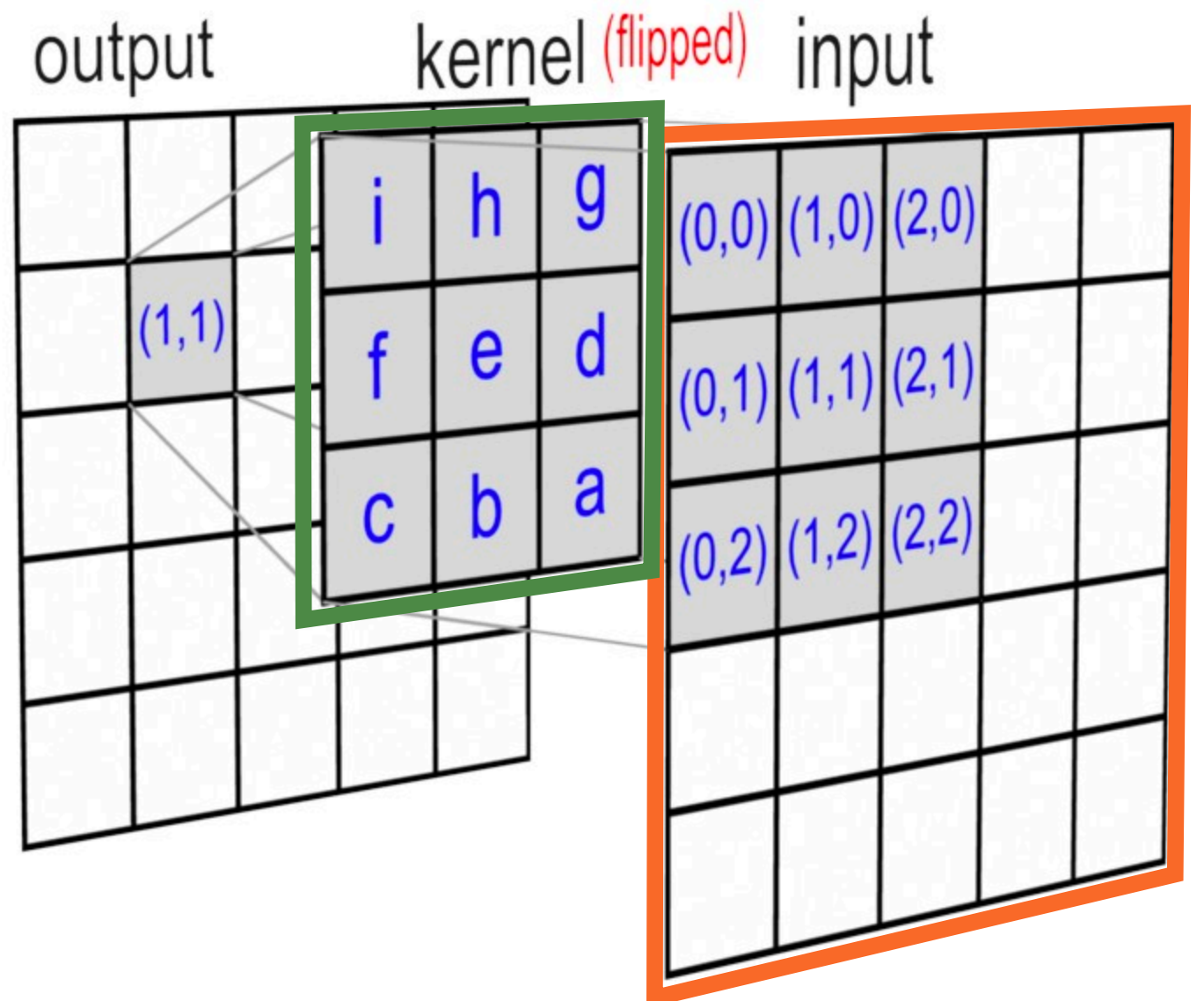
$$\mathbf{A} \star_{2D} \mathbf{F} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} A_{i,j} F_{i-m,j-n}$$

PSF (also called
kernel in CNNs)

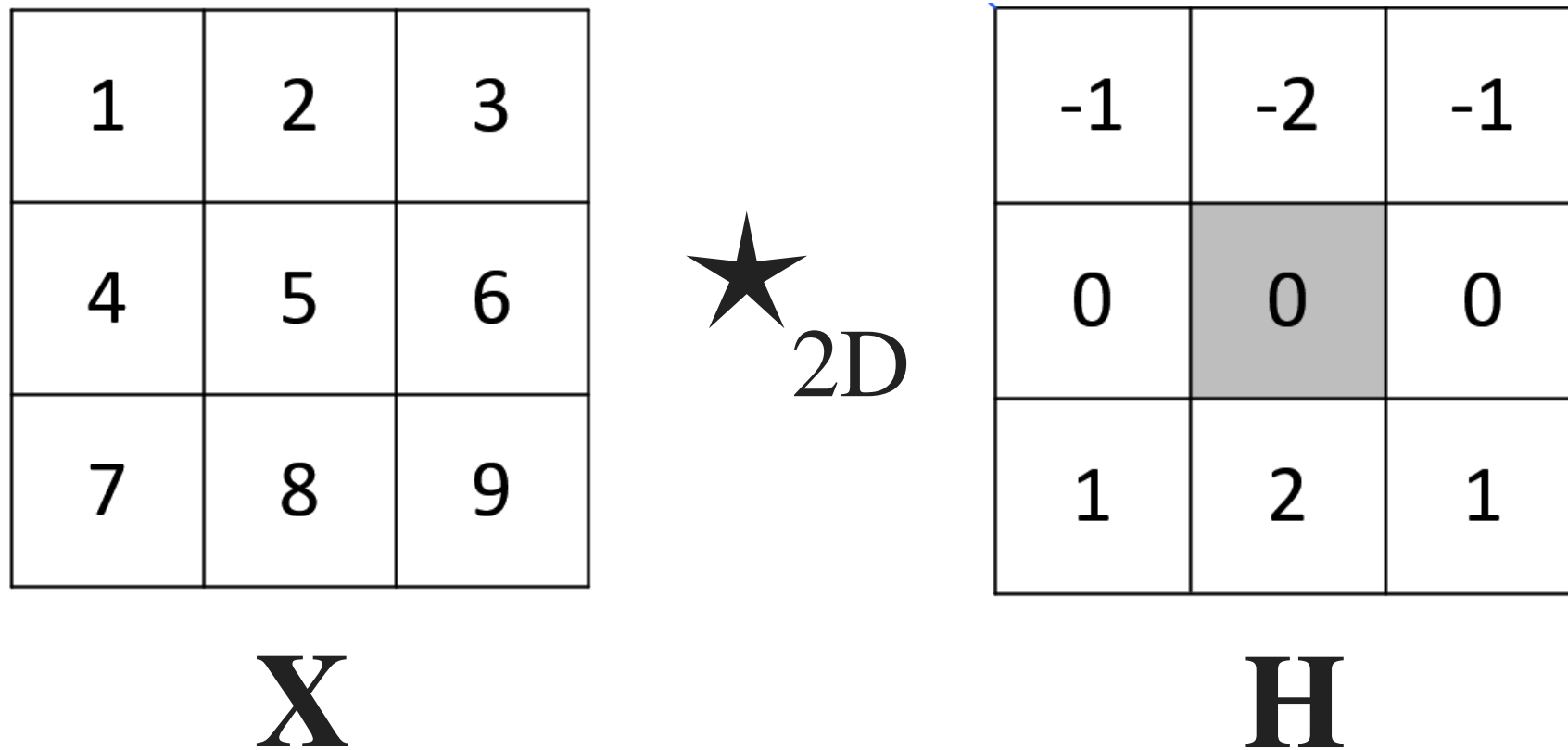
$$\mathbf{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Input image

$$\mathbf{F} = \begin{bmatrix} x_{0,0} & x_{1,0} & x_{2,0} & \cdots \\ x_{0,1} & x_{1,1} & x_{2,1} & \cdots \\ x_{0,2} & x_{1,2} & x_{2,2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$



2D DISCRETE CONVOLUTION

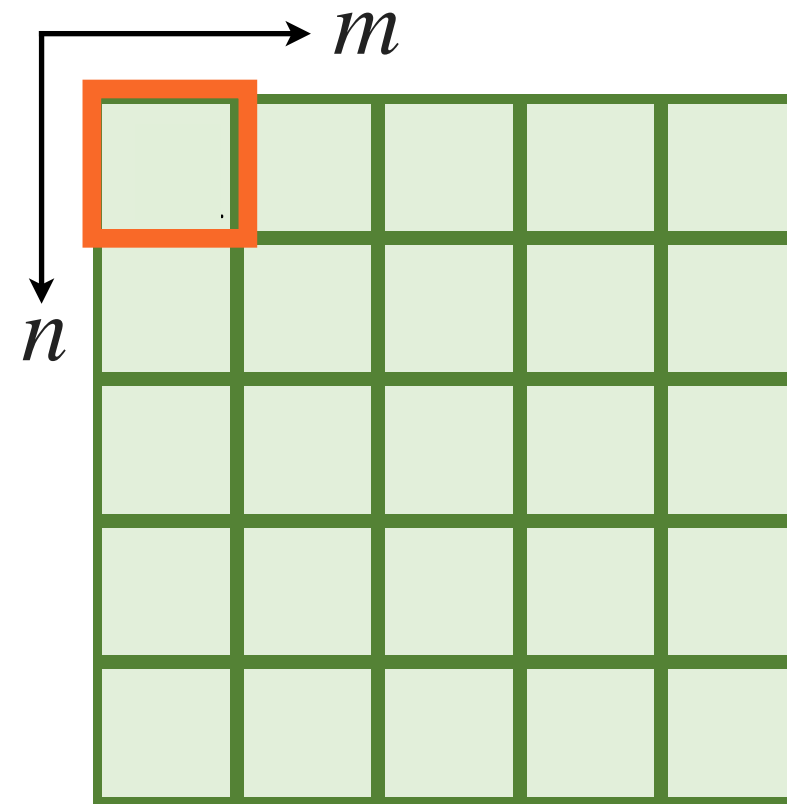


$$\mathbf{X} \star_{2D} \mathbf{H} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} X_{i,j} H_{i-m,j-n}$$

2D DISCRETE CONVOLUTION

1	2	1		
0	0	0		
-1	-2	-1	1	2
			4	5
			7	8
				9

Flip **H** along both dimensions, the center is the (0, 0) element



$$g_{0,0} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{0-i,0-j}$$

$$\begin{aligned} g_{0,0} = & x_{0,0}h_{0,0} + x_{1,0}h_{-1,0} + x_{2,0}h_{-2,0} \\ & + x_{0,1}h_{1,-1} + x_{1,1}h_{-1,-1} + x_{2,1}h_{-2,-1} \\ & + x_{0,2}h_{0,-2} + x_{1,2}h_{-1,-2} + x_{2,2}h_{-2,-2} \end{aligned}$$

$$g_{0,0} = -1 \times 1 = -1$$

2D DISCRETE CONVOLUTION

1	2	1		
0	0	0		
-1	-2	-1	1	2
			4	5
			7	8
				6
				9

	-1			

$$g_{1,0} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{1-i,0-j}$$

$$\begin{aligned} g_{1,0} = & x_{0,0}h_{1,0} + x_{1,0}h_{0,0} + x_{2,0}h_{-1,0} \\ & + x_{0,1}h_{1,-1} + x_{1,1}h_{0,-1} + x_{2,1}h_{-1,-1} \\ & + x_{0,2}h_{1,-2} + x_{1,2}h_{0,-2} + x_{2,2}h_{-1,-2} \end{aligned}$$

$$g_{1,0} = (-2 \times 1) + (-1 \times 2) = -4$$

2D DISCRETE CONVOLUTION

1	2	1	
0	0	0	
-1	-2	-1	3
	1	2	
	4	5	6
	7	8	9

	-1	-4		

$$g_{2,0} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{2-i,0-j}$$

$$\begin{aligned}
 g_{2,0} = & x_{0,0}h_{2,0} + x_{1,0}h_{1,0} + x_{2,0}h_{0,0} \\
 & + x_{0,1}h_{2,-1} + x_{1,1}h_{1,-1} + x_{2,1}h_{0,-1} \\
 & + x_{0,2}h_{2,-2} + x_{1,2}h_{1,-2} + x_{2,2}h_{0,-2}
 \end{aligned}$$

$$g_{2,0} = (-1 \times 1) + (-2 \times 2) + (-1 \times 3) = -8$$

2D DISCRETE CONVOLUTION

1	2	1
0	0	0
-1	1	-2
4	5	6
7	8	9

-1	-4	-8		

$$g_{3,0} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{3-i,0-j}$$

$$\begin{aligned} g_{3,0} = & x_{0,0}h_{3,0} + x_{1,0}h_{2,0} + x_{2,0}h_{1,0} \\ & + x_{0,1}h_{3,-1} + x_{1,1}h_{2,-1} + x_{2,1}h_{1,-1} \\ & + x_{0,2}h_{3,-2} + x_{1,2}h_{2,-2} + x_{2,2}h_{1,-2} \end{aligned}$$

$$g_{2,0} = (-1 \times 2) + (-2 \times 3) = -8$$

2D DISCRETE CONVOLUTION

	1	2	1		
	0	0	0		
1	-1	2	-2	3	-1
4	5	6			
7	8	9			

-1	-4	-8	-8	

$$g_{4,0} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{4-i,0-j}$$

$$\begin{aligned} g_{4,0} = & x_{0,0}h_{4,0} + x_{1,0}h_{3,0} + x_{2,0}h_{2,0} \\ & + x_{0,1}h_{4,-1} + x_{1,1}h_{3,-1} + x_{2,1}h_{2,-1} \\ & + x_{0,2}h_{4,-2} + x_{1,2}h_{3,-2} + x_{2,2}h_{2,-2} \end{aligned}$$

$$g_{4,0} = (-1 \times 3) = -3$$

2D DISCRETE CONVOLUTION

1	2	1		
0	0	0	1	2
-1	-2	-1	4	5
			7	8
				9

	-1	-4	-8	-8	-3

$$g_{0,1} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{0-i,1-j}$$

$$g_{0,1} = (0 \times 1) + (-1 \times 4) = -4$$

2D DISCRETE CONVOLUTION

1	2	1	
0	0	1	2
-1	-2	4	5
	7	8	9

-1	-4	-8	-8	-3
-4				

$$g_{1,1} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{1-i,1-j}$$

$$g_{1,1} = (0 \times 1) + (0 \times 2) + (-2 \times 4) + (-1 \times 5) = -13$$

2D DISCRETE CONVOLUTION

1	2	1
0	0	0
-1	-2	-1
7	8	9

-1	-4	-8	-8	-3
-4	-13			

$$g_{2,1} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{2-i,1-j}$$

$$g_{2,1} = (0 \times 1) + (0 \times 2) + (0 \times 3) + (-1 \times 4) + (-2 \times 5) + (-1 \times 6) = -20$$

2D DISCRETE CONVOLUTION

		1	2	1
1	0	2	0	3
4	-1	5	-2	6
7	8	9		

	-1	-4	-8	-8	-3
	-4	-13	-20		

$$g_{3,1} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{3-i,1-j}$$

$$g_{3,1} = (-1 \times 5) + (-2 \times 6) = -17$$

2D DISCRETE CONVOLUTION

		1	2	1
1	2	0 3	0	0
4	5	-1 6	-2	-1
7	8	9		

-1	-4	-8	-8	-3
-4	-13	-20	-17	

$$g_{4,1} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{4-i,1-j}$$

$$g_{4,1} = (-1 \times 6) = -6$$

2D DISCRETE CONVOLUTION

-1	-4	-8	-8	-3
-4	-13	-20	-17	-6

$$g_{0,2} = (-1 \times 6) = -6$$

2D DISCRETE CONVOLUTION

-1	-4	-8	-8	-3
-4	-13	-20	-17	-6
-6				

2D DISCRETE CONVOLUTION

X

1	1	2	1	3	
0	4	0	5	6	
-1	7	-2	8	-1	9

-1	-4	-8	-8	-3
-4	-13	-20	-17	-6
-6	-18			

$$g_{2,2} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{2-i,2-j}$$

$$g_{2,2} = 1 + 4 + 3 - 7 - 16 - 9 = -24$$

2D DISCRETE CONVOLUTION

1	2	3
4	5	6
7	8	9

★
2D

-1	-2	-1
0	0	0
1	2	1

=

DISCRETE CONVOLUTION
OUTPUT

-1	-4	-8	-8	-3
-4	-13	-20	-17	-6
-6	-18	-24		

Note that we used index i to denote column and j to index row

MATLAB IMPLEMENTATION

- ▶ 1D discrete convolution can be implemented in Matlab by using the built-in '`conv`' function
- ▶ 2D discrete convolution can be implemented in Matlab by using the built-in '`conv2`' function
- ▶ n -Dimension discrete convolution can be implemented in Matlab by using the built-in '`convn`' function

THAT IS ALL!!!
