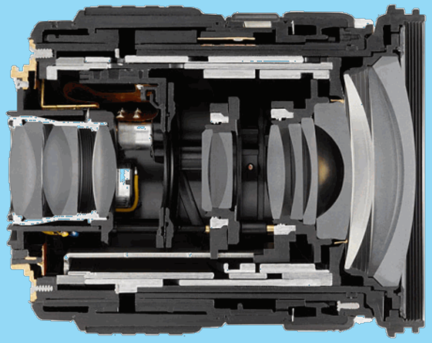
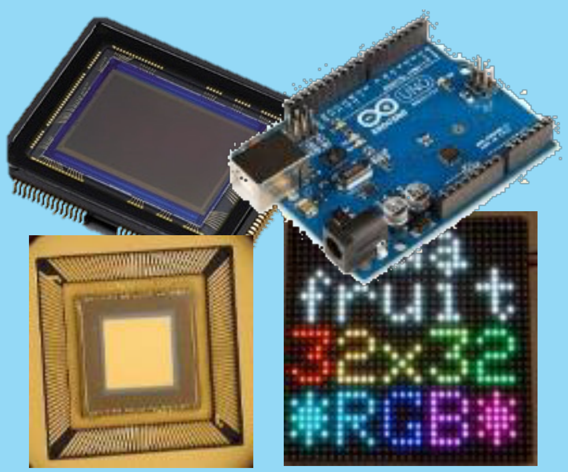


Optics

+



Sensors & devices

+

Signal processing & algorithms

COMPUTATIONAL METHODS FOR IMAGING (AND VISION)

LECTURE 12: LSI IMAGING SYSTEMS

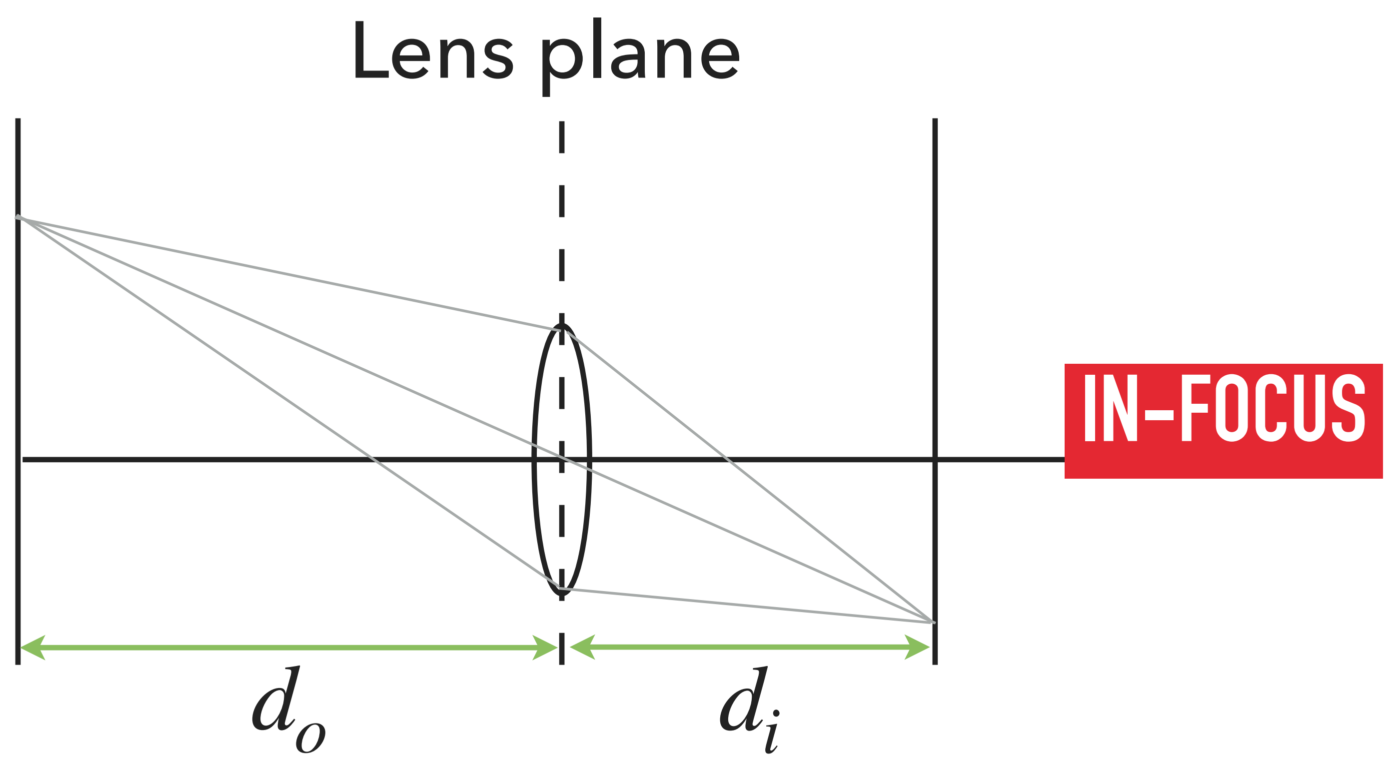
PROF. JOHN MURRAY-BRUCE

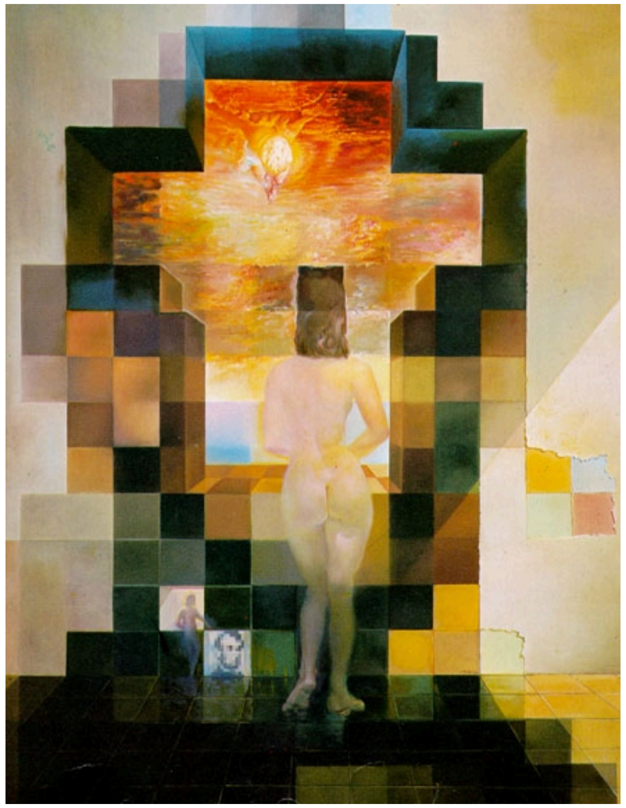
LSI IMAGING SYSTEMS

- CAMERA
- TOMOGRAPHY
- OPTICAL MICROSCOPY
- HOLOGRAPHY

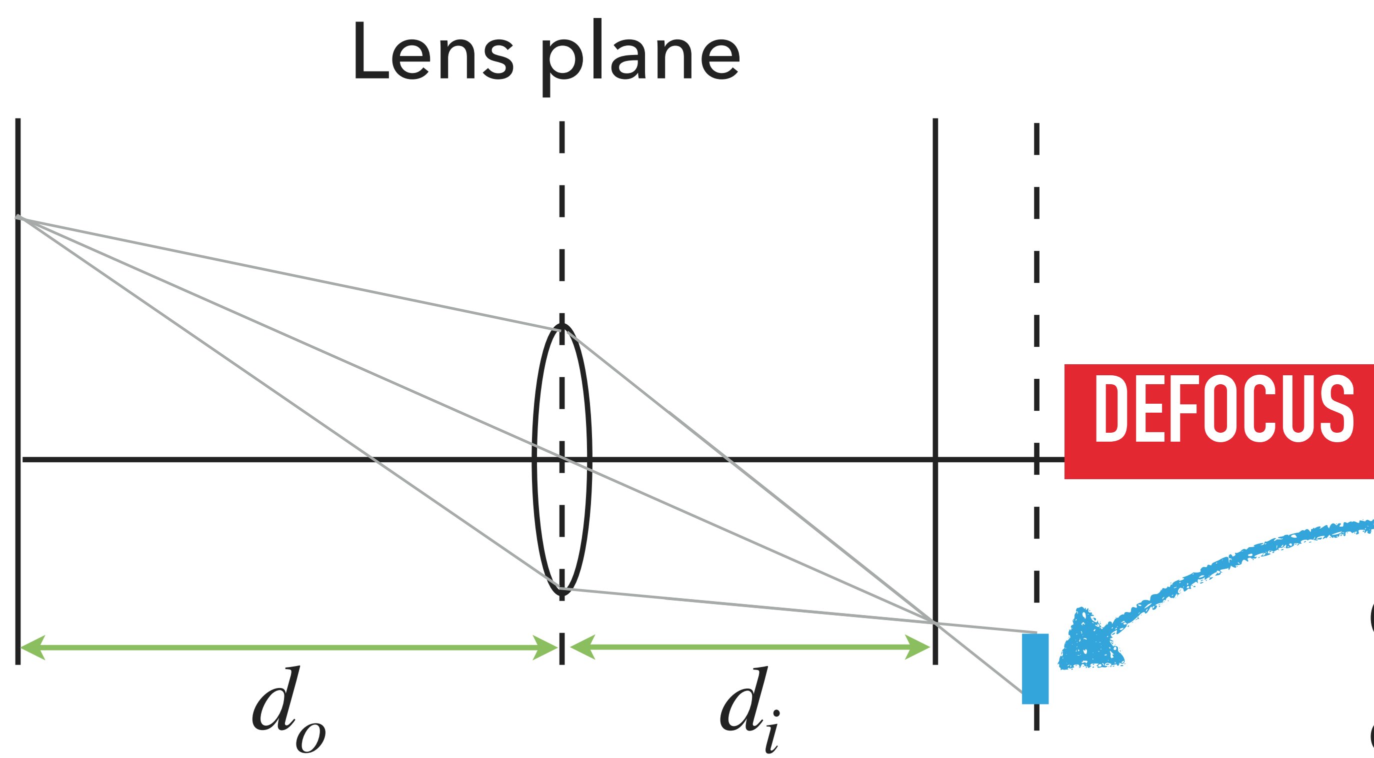
EXAMPLES

CAMERA



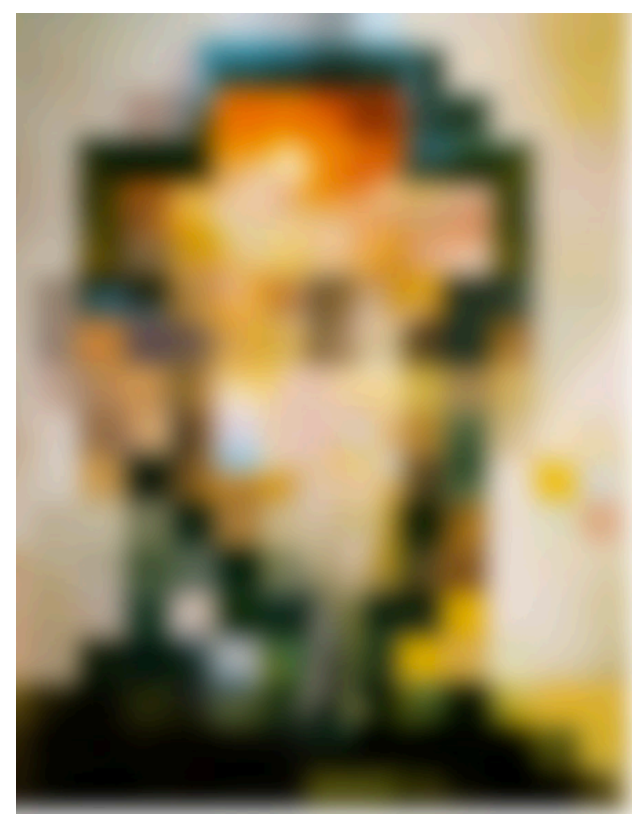


CAMERA



Circle of confusion

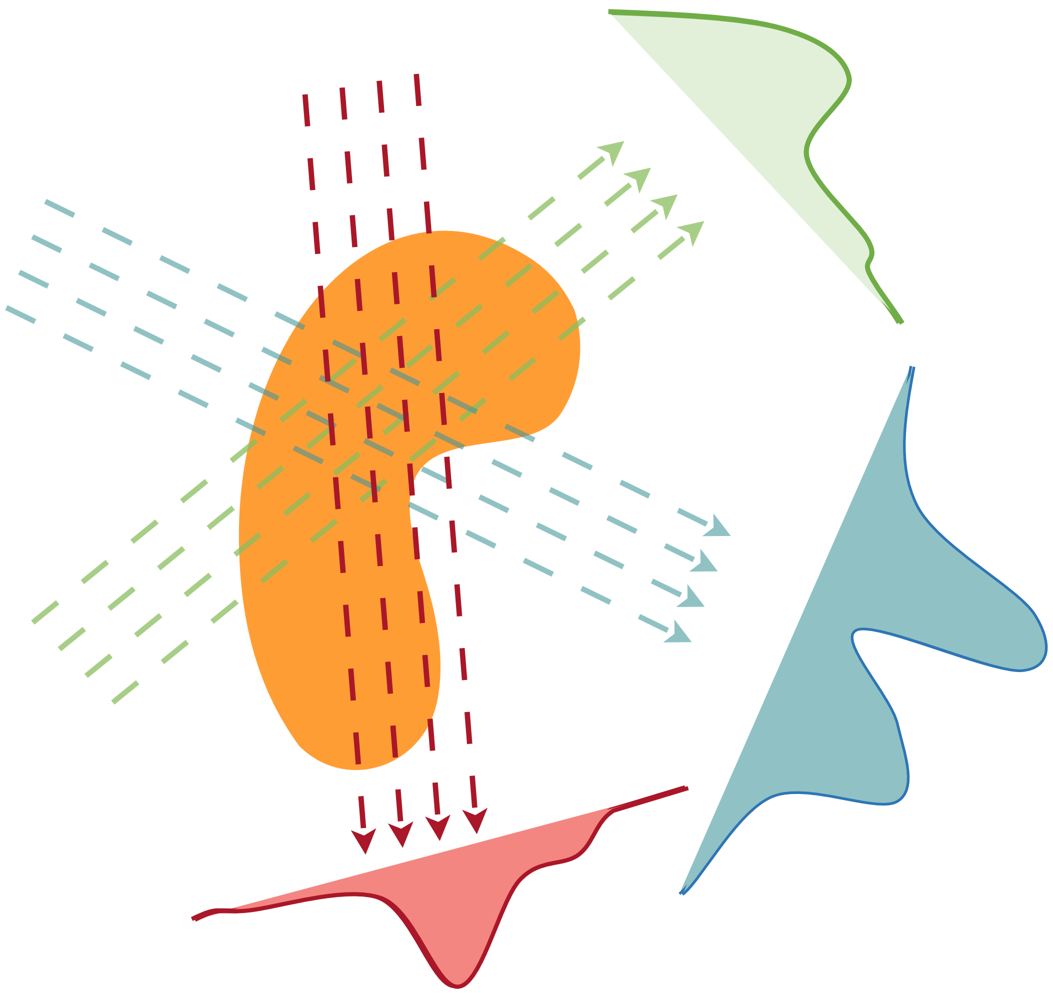
It is so-called because the light is no longer focussed a single point, but is instead spread over some larger circular region.

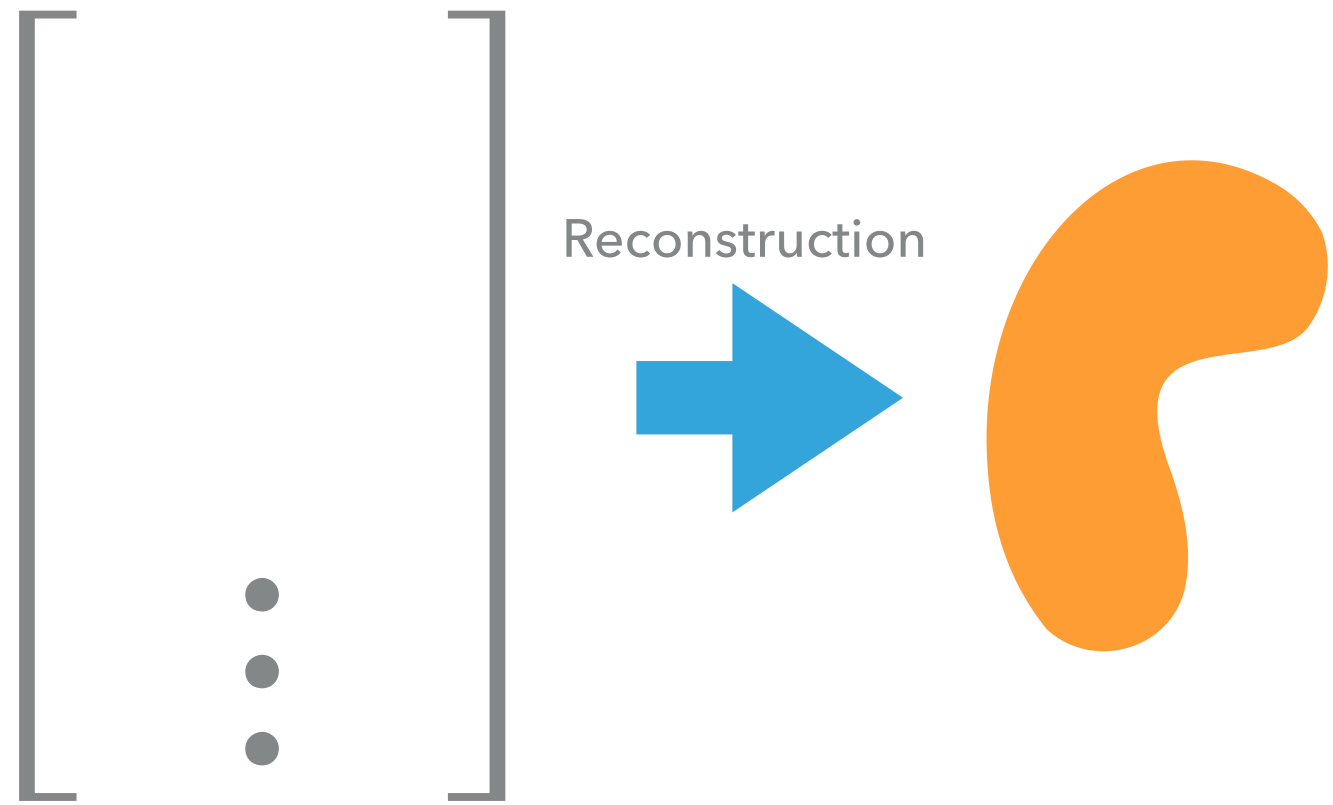


TOMOGRAPHY

Tomo - "to slice"

Graphy - "to write"

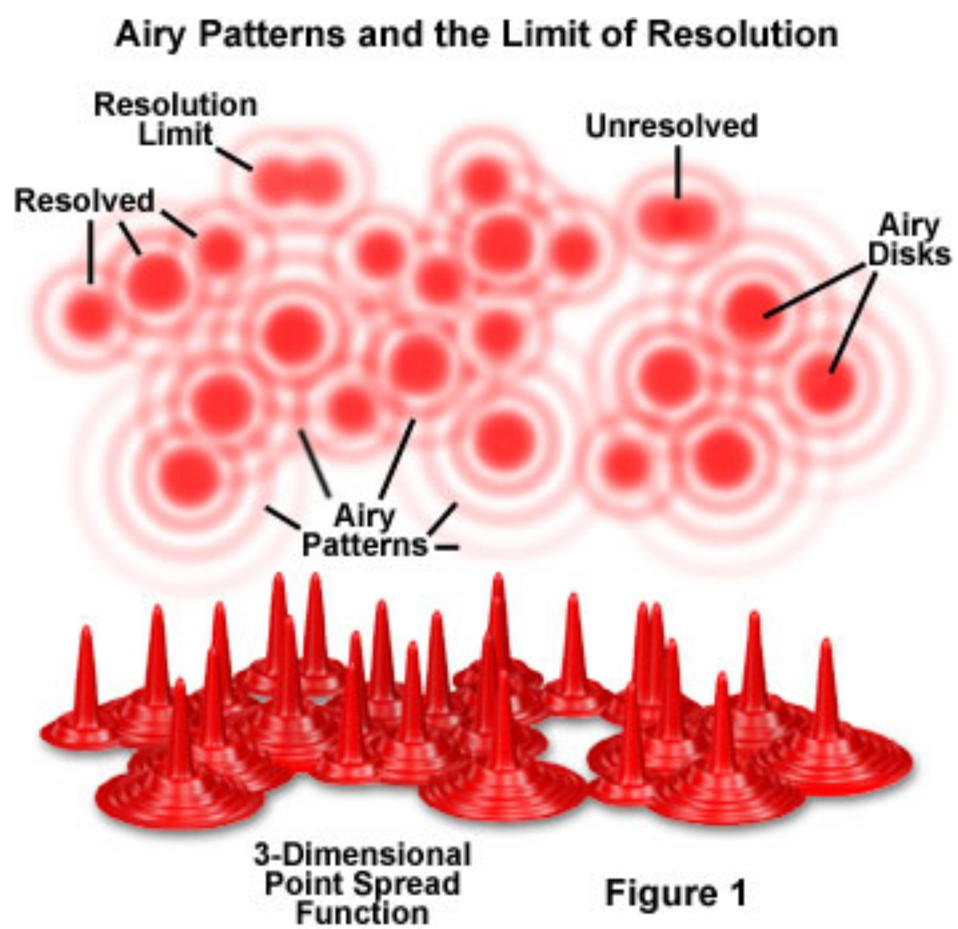




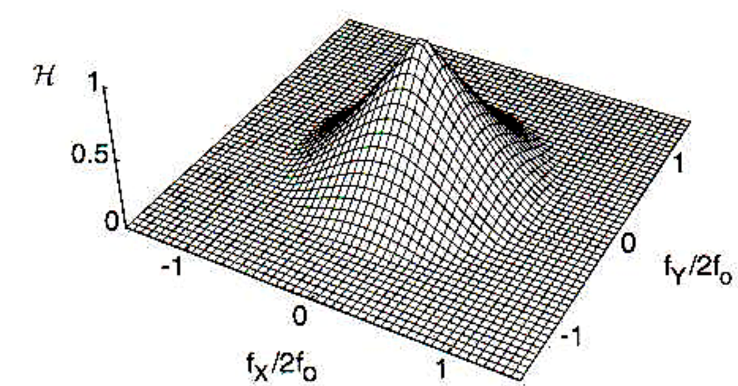
- Take many 1D projections of a 2D object
- Reconstruct original 2D object from measurement of those measured 1D projections

- The operator A maps from object space $\mathscr{X}$ to image space $\mathscr{Y}$
  - What is the object space $\mathscr{X}$ and image space $\mathscr{Y}$?
  - Is the operator A linear?

OPTICAL MICROSCOPY
EXAMPLE OF SHIFT-INVARIANT SYSTEM



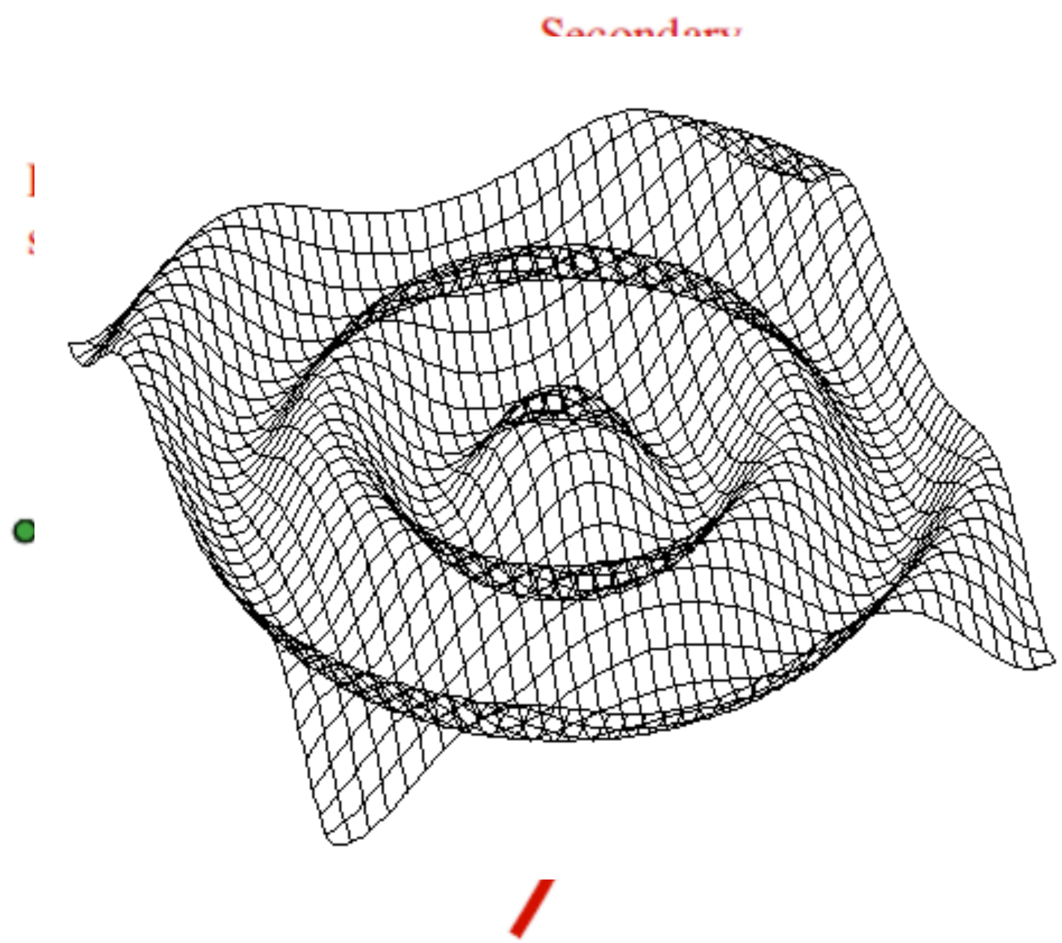
Optical Transfer function



- Operator A maps from object space $\mathscr{X}$ to image space $\mathscr{Y}$
  - What is the object space $\mathscr{X}$ and image space $\mathscr{Y}$?
  - Is the operator A linear?

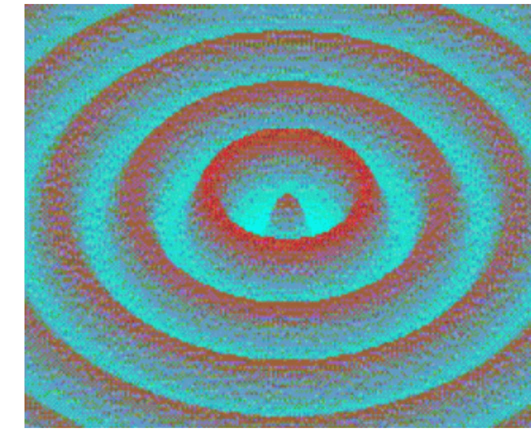
WAVE PROPAGATION

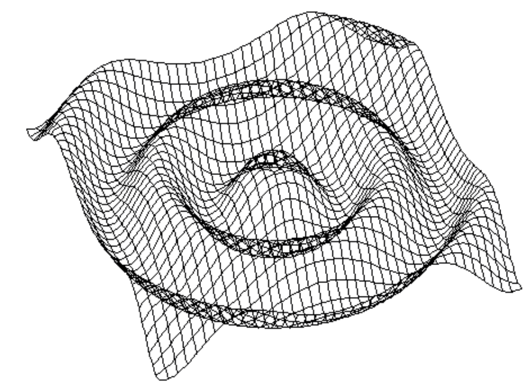
EXAMPLE OF SHIFT INVARIANT SYSTEM

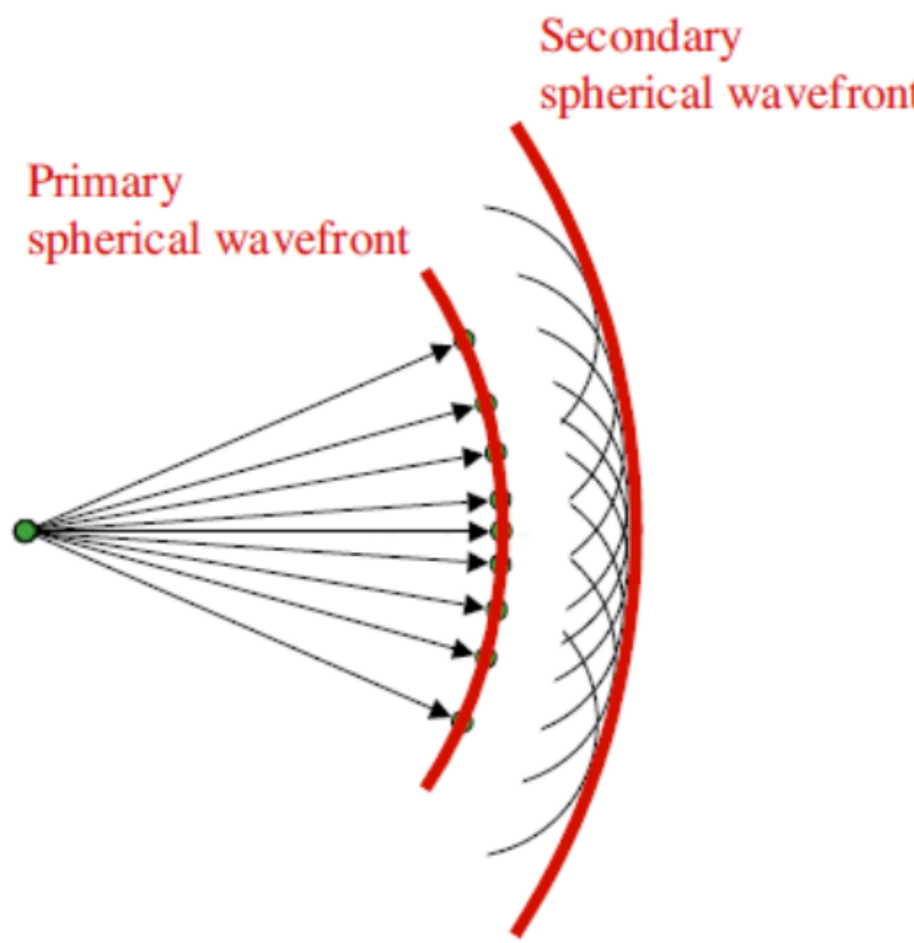


$$g_{\text{out}}(x, y; z) = \frac{e^{\mathrm{j}kz}}{\mathrm{j}\lambda z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_{\text{in}}(x', y') e^{\mathrm{j}k \frac{(x - x')^2 + (y - y')^2}{2z}} \mathrm{d}x' \mathrm{d}y'$$

WAVE PROPAGATION
EXAMPLE OF SHIFT INVARIANT SYSTEM

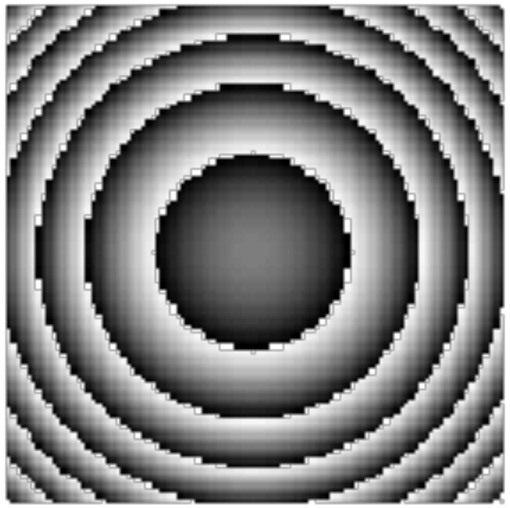
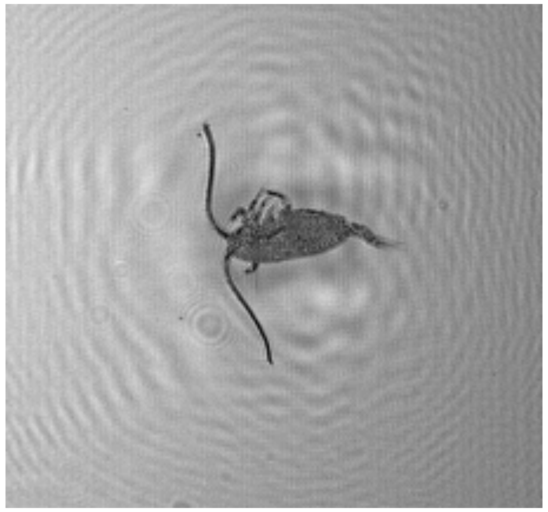
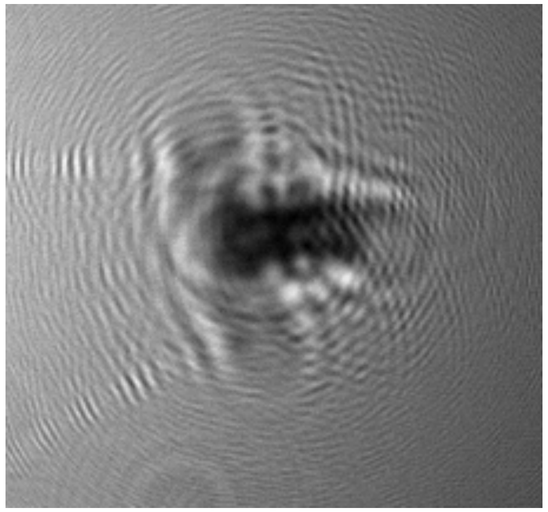






$$g_{\mathrm{out}}(x, y; z) = \frac{e^{\mathrm{j}kz}}{\mathrm{j}\lambda z} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g_{\mathrm{in}}(x', y') e^{\mathrm{j}k\frac{(x - x')^2 + (y - y')^2}{2z}} \, \mathrm{d}x' \mathrm{d}y'$$

HOLOGRAPHY

 $\star$  $=$ 

$$h_z(x,y) = \frac{e^{jkz}}{j\lambda z} e^{jk\frac{(x^2+y^2)}{2z}} \qquad g_{\text{in}}(x,y) \qquad g_{\text{out}}(x,y)$$

$$g_{\text{out}}(x,y;z) = \frac{e^{jkz}}{j\lambda z} \iint g_{\text{in}}(x',y') e^{jk\frac{(x-x')^2+(y-y')^2}{2z}} \, dx'dy'$$

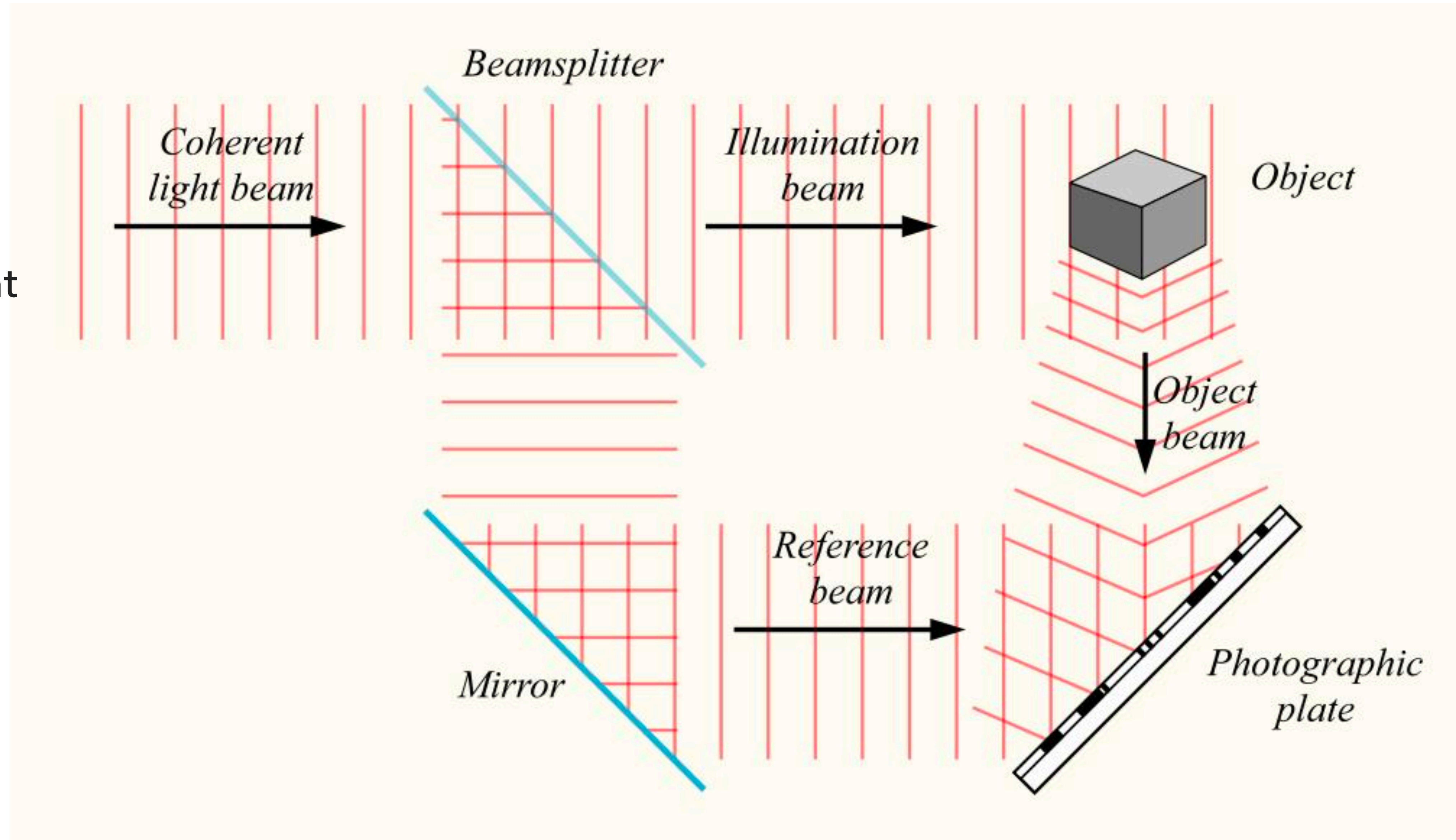
- **Optical Transfer Function:** the FT of $h_z(x,y)$

$$H_z(\omega_x,\omega_y) = e^{j2\pi z/\lambda} e^{-j\lambda z(\omega_x^2+\omega_y^2)}$$

- Range and null spaces?
- Adjoint and inverse operators?

RECORDING A HOLOGRAM

Laser beam is coherent

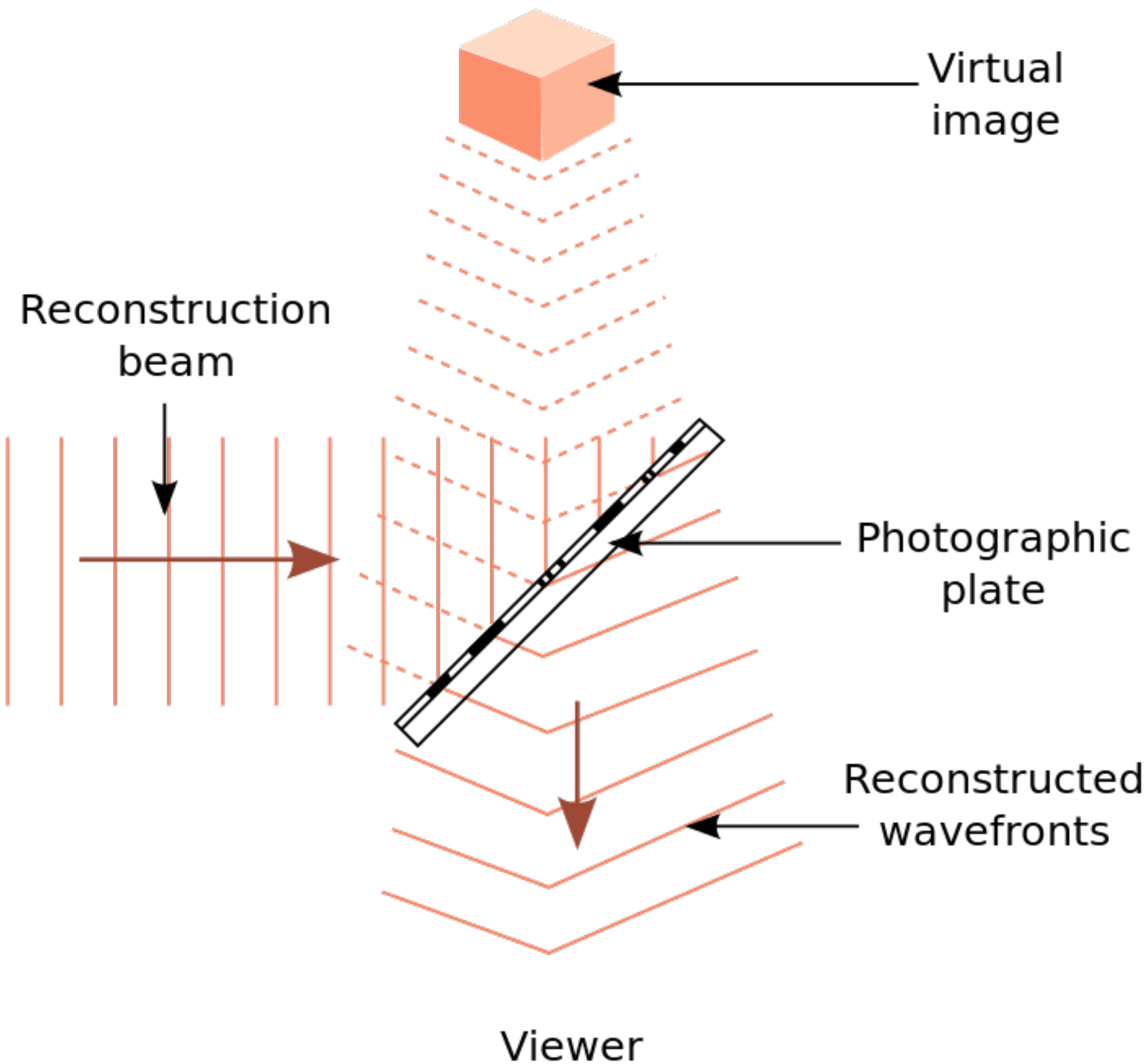


$$H_z(\omega_x, \omega_y) = e^{j2\pi z/\lambda} e^{-j\lambda z(\omega_x^2 + \omega_y^2)}$$

[Image credit: Bob Mellish / Creative Commons]

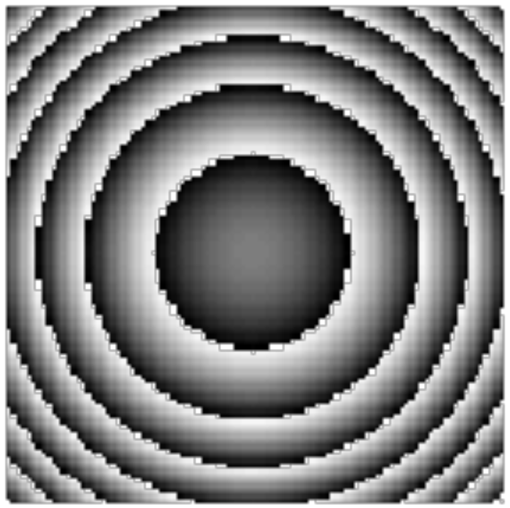
RECORDING A HOLOGRAM

RECONSTRUCTION

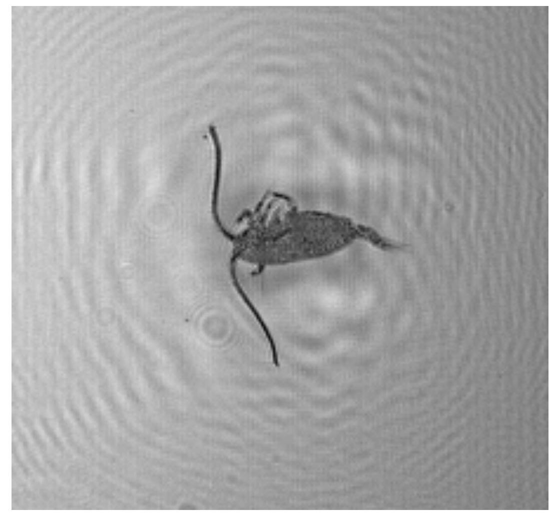


[Image credit: Wikimedia]

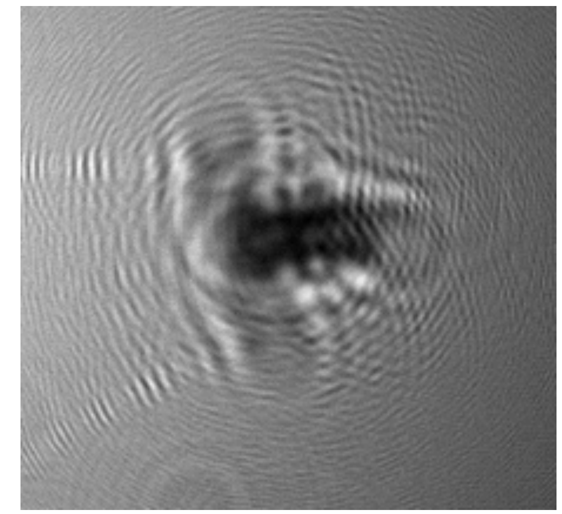
HOLOGRAPHY: DIGITAL BACK-PROJECTION



$\star$



$=$



$$h_z(x, y) = \frac{e^{jkz}}{j\lambda z} e^{jk\frac{(x^2+y^2)}{2z}}$$

$$g_{\text{in}}(x, y)$$

$$g_{\text{out}}(x, y)$$

$$g_{\text{out}}(x, y; z) = \frac{e^{jkz}}{j\lambda z} \int\!\!\int g_{\text{in}}(x', y') e^{jk\frac{(x-x')^2+(y-y')^2}{2z}} \, dx'dy'$$

- **Optical Transfer Function:** the FT of $h_z(x, y)$

$$H_z(\omega_x, \omega_y) = e^{j2\pi z/\lambda} e^{-j\lambda z(\omega_x^2+\omega_y^2)}$$

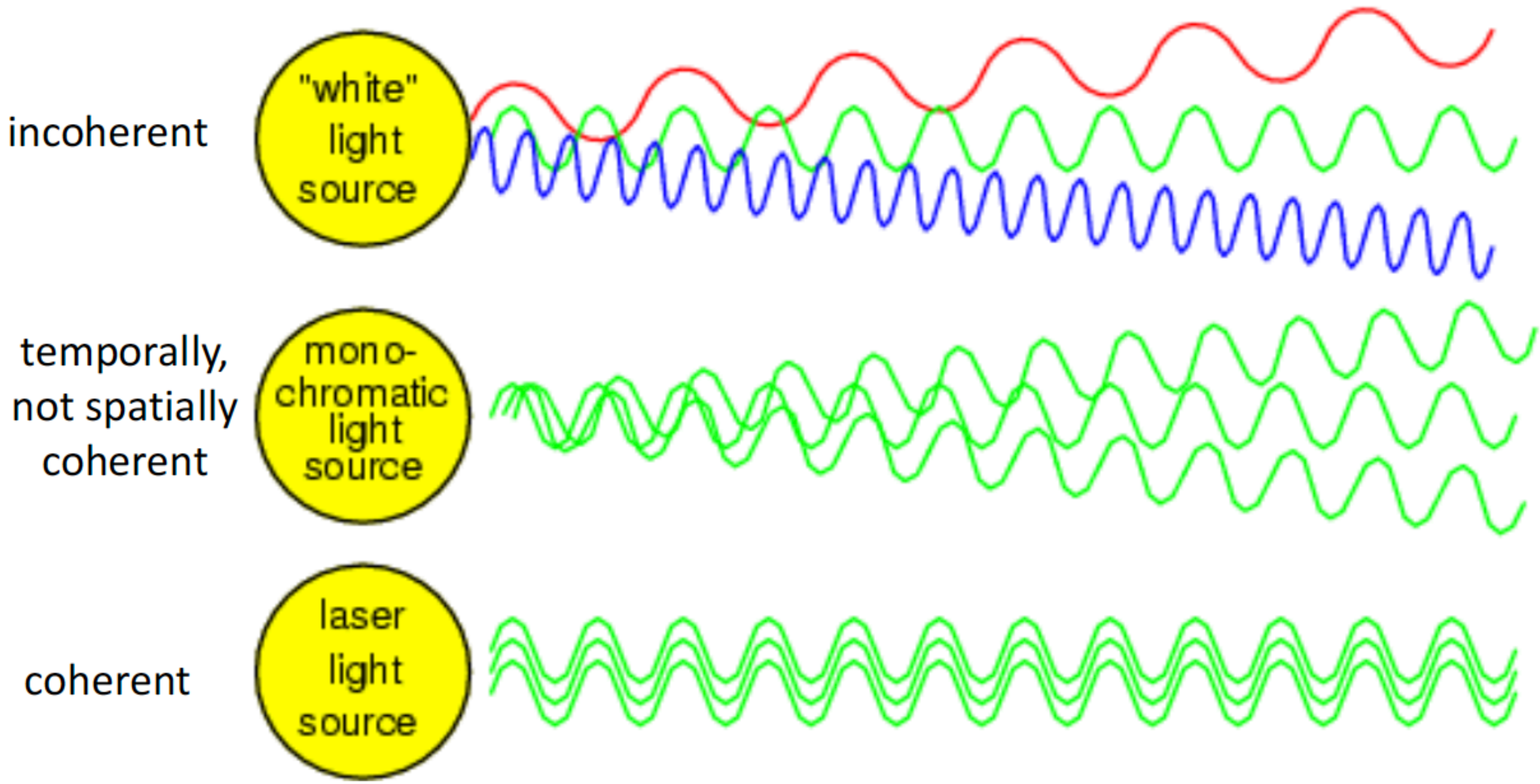
- Range and null spaces?
- Adjoint and inverse operators?

MAKE

$z \longrightarrow (-z)$

RECONSTRUCTION

TYPES OF COHERENCE



[Image credit: Lei Tian (BU)]

LIGHT PROPAGATION IN FREE SPACE

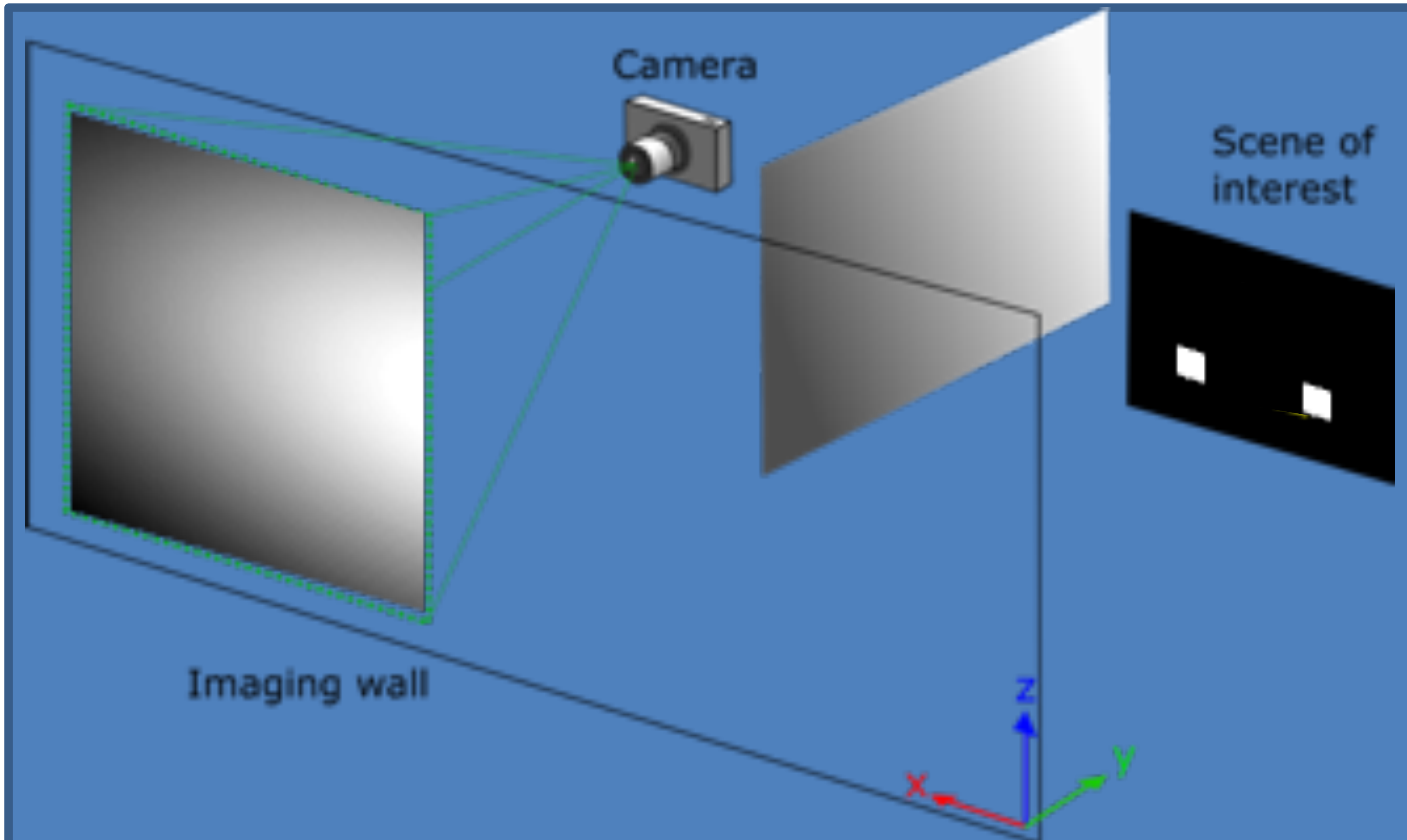
- **Light transport:** the intensity of the light emitted by a point light source has an inversely proportional relationship with the square of the distance from the point source
- Point source is located at $\boldsymbol{p}_{\mathrm{s}} = (p_1, p_2, p_3)$ in 3D space
- The intensity at some position $\boldsymbol{x} = (x_1, x_2, x_3)$ is given by:

$$i(\boldsymbol{x}) = \frac{1}{||\boldsymbol{x} - \boldsymbol{p}_{\mathrm{s}}||^2}$$

LIGHT PROPAGATION IN FREE SPACE

MATLAB

- **Light transport:**



$$y = A\, f$$

IMPULSE RESPONSE FUNCTIONS

TRANSFER FUNCTIONS

MATLAB PRACTICE 7

IMPULSE RESPONSE FUNCTIONS

- **Matlab**
  - Visualizing impulse response functions
  - Visualizing transfer functions

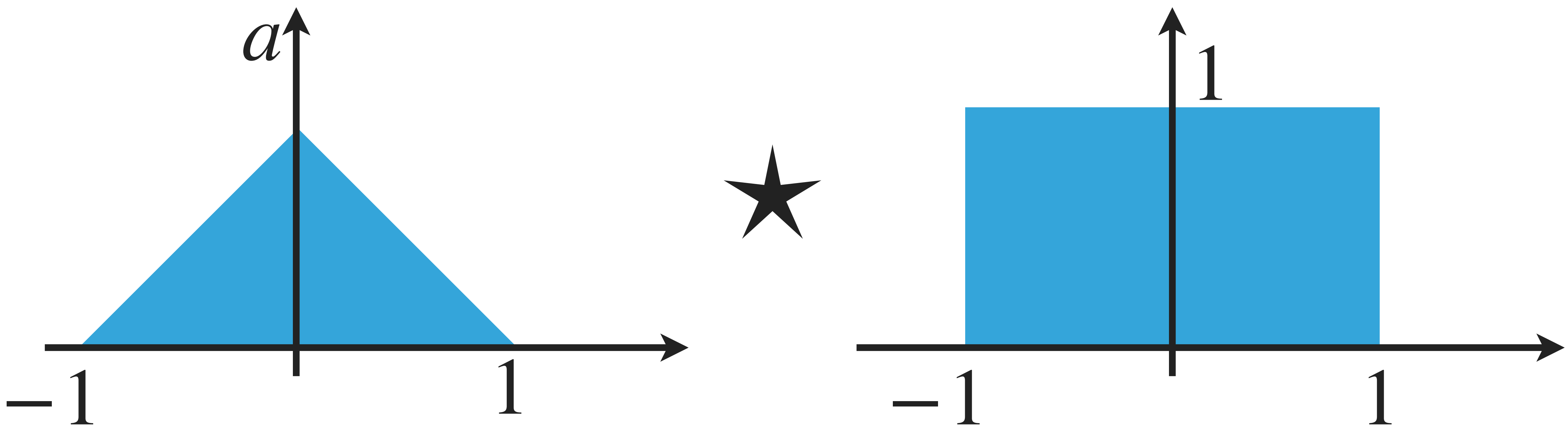
WHAT WE “COVERED” IN THE PREVIOUS LECTURE

- **Convolution operator**
  - Properties - function spaces view
- **LSI imaging systems**
  - Examples: defocus, tomography, digital holography
- **Matlab**
  - Visualizing impulse response functions and transfer functions

BIG QUESTION: WHAT'S IN THE MIDTERM?

PROBLEM 1

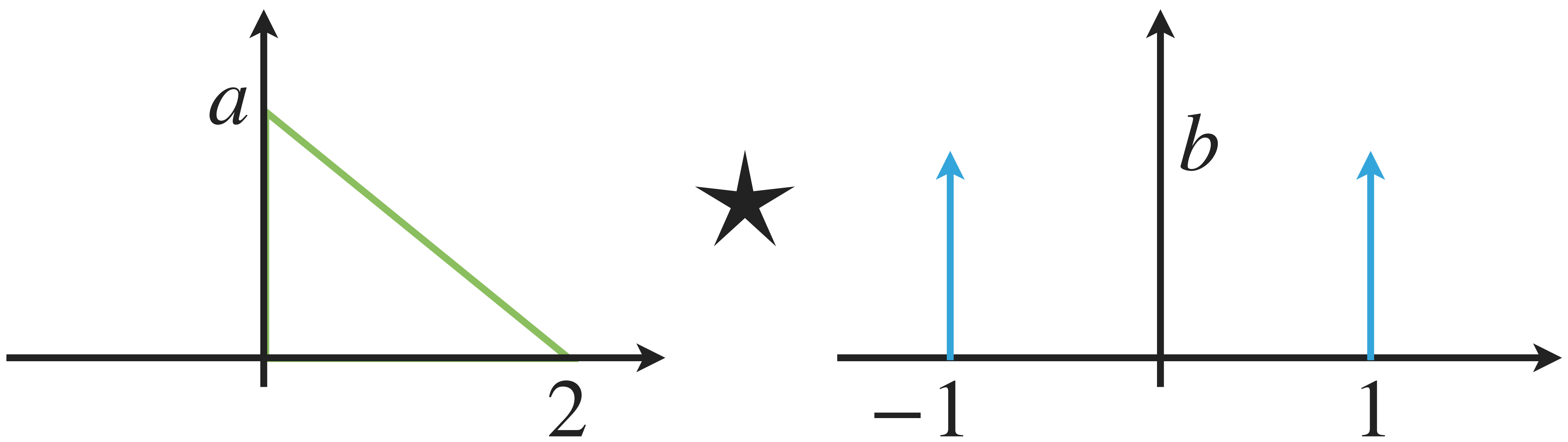
- Convolution example 1:



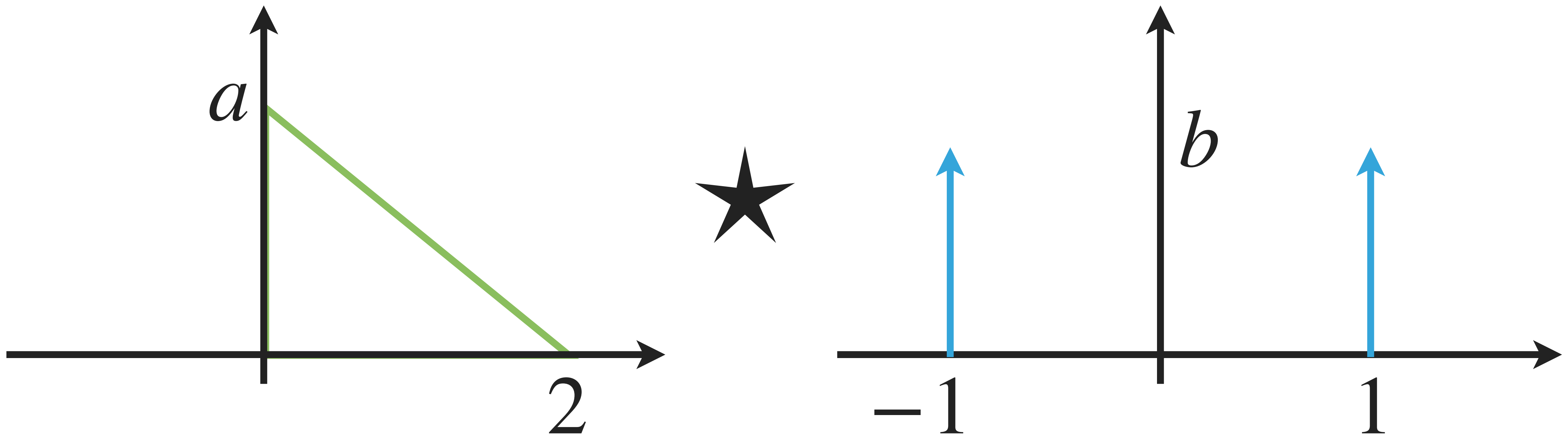
Before we compute the convolution, can anyone figure out the width of the resulting function after convolution?

PROBLEM 1

- Convolution example 2:

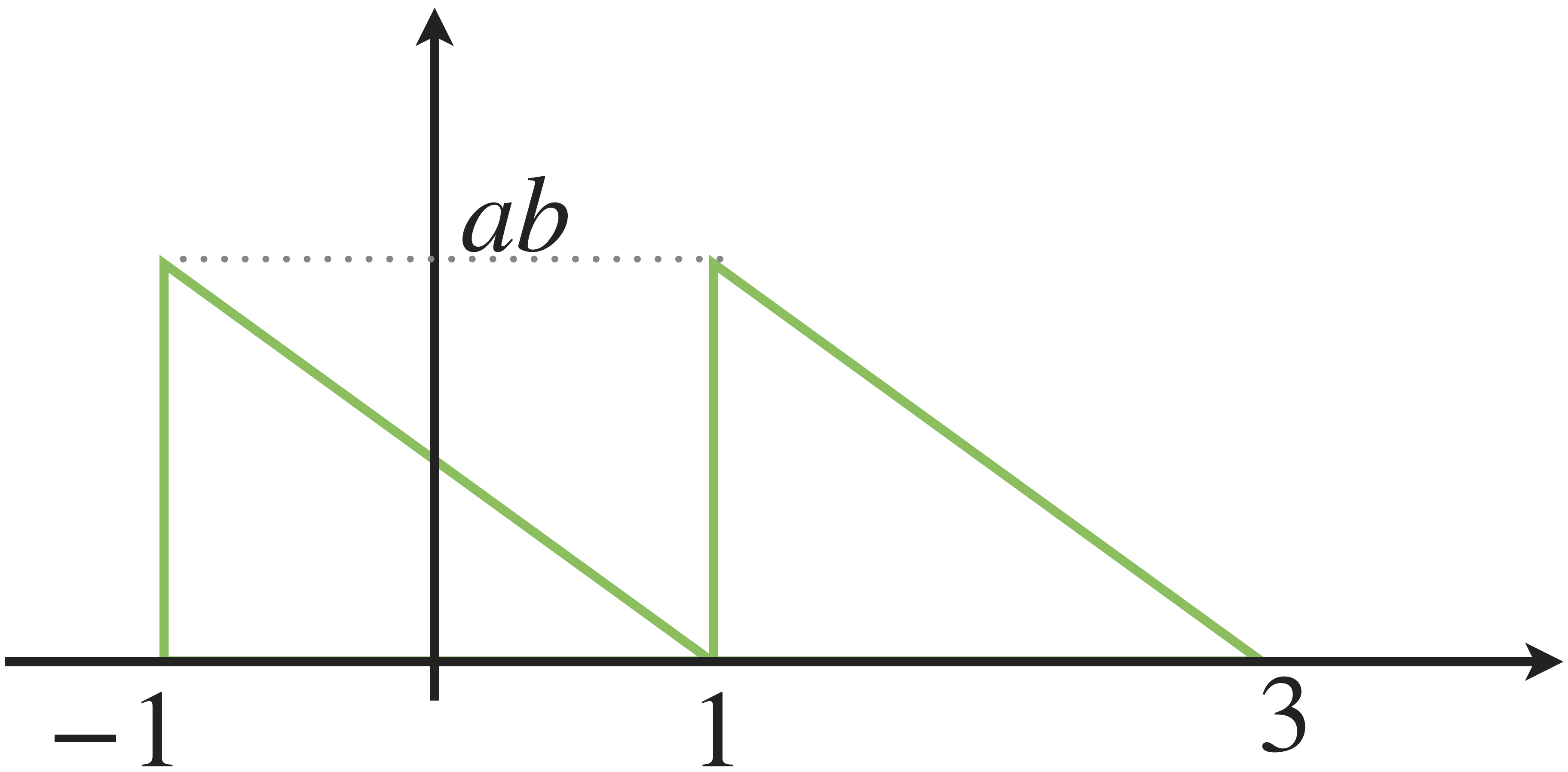


CONVOLUTION QUESTION 1



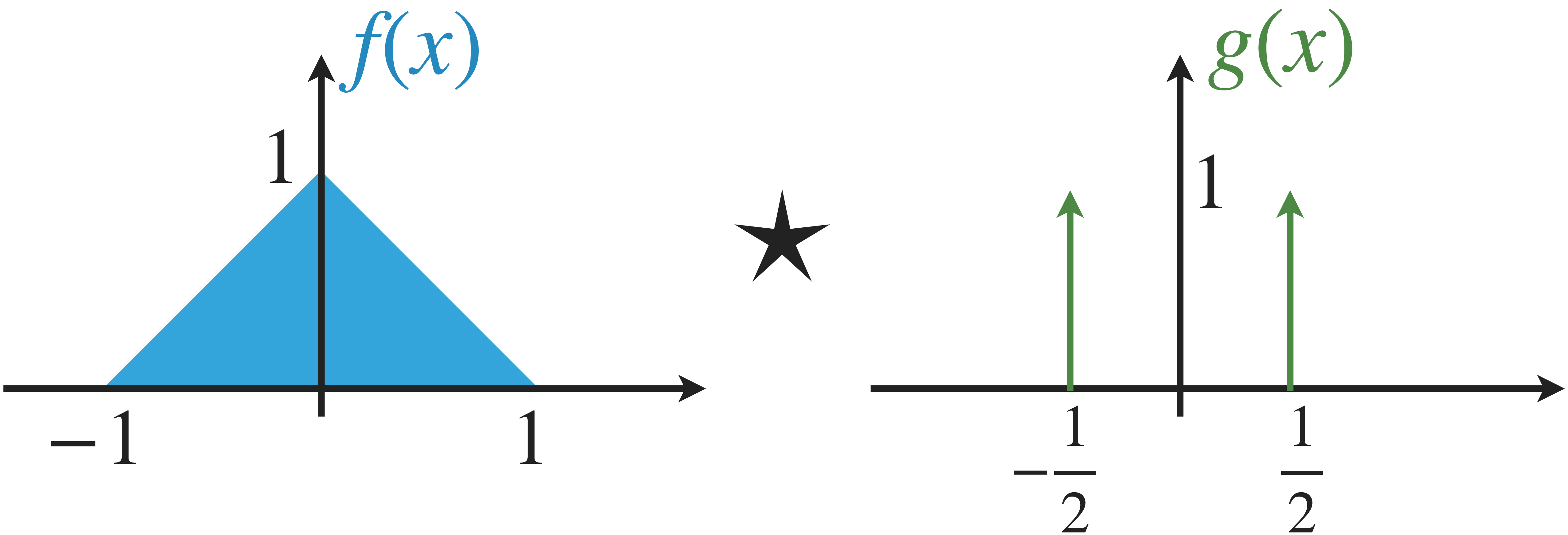
The triangle function is moved to the locations of the Dirac delta functions

CONVOLUTION QUESTION 1
SOLUTION

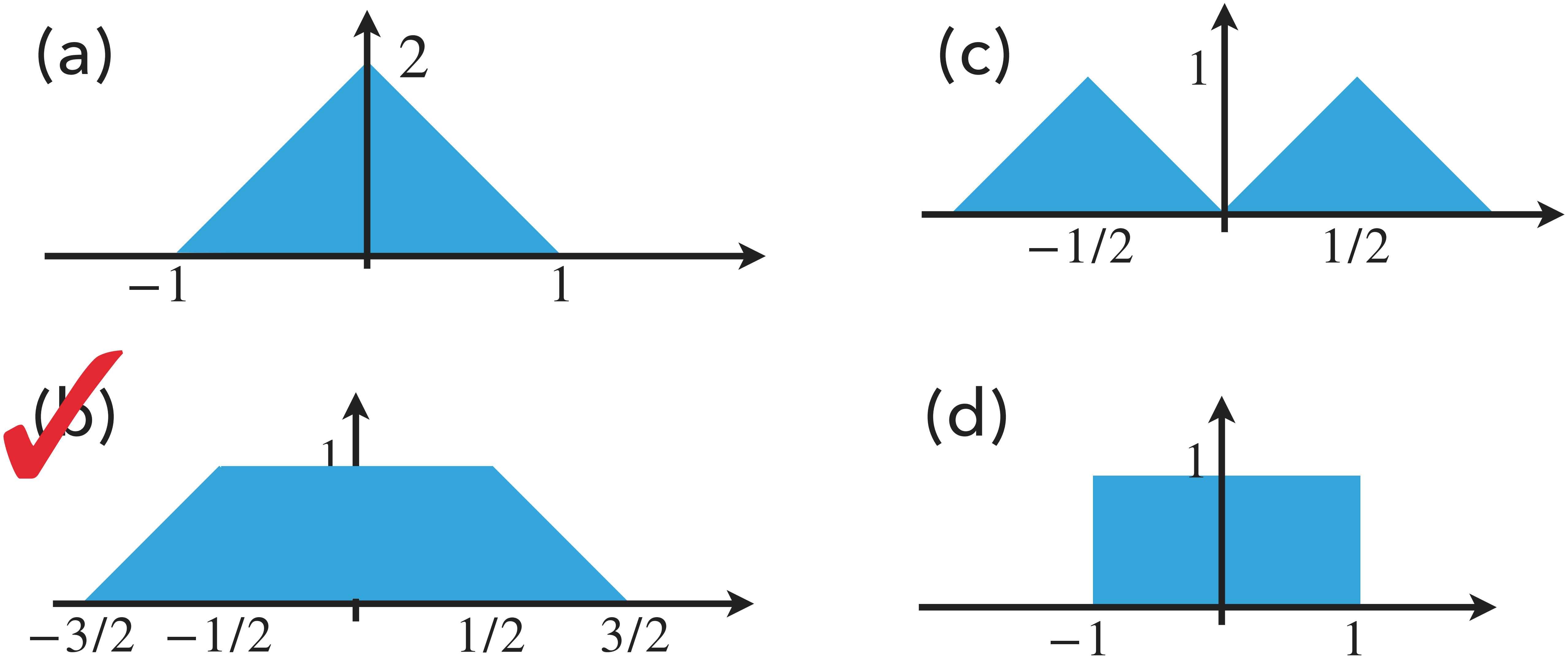


CONVOLUTION QUESTION 2

- Convolution example 2:

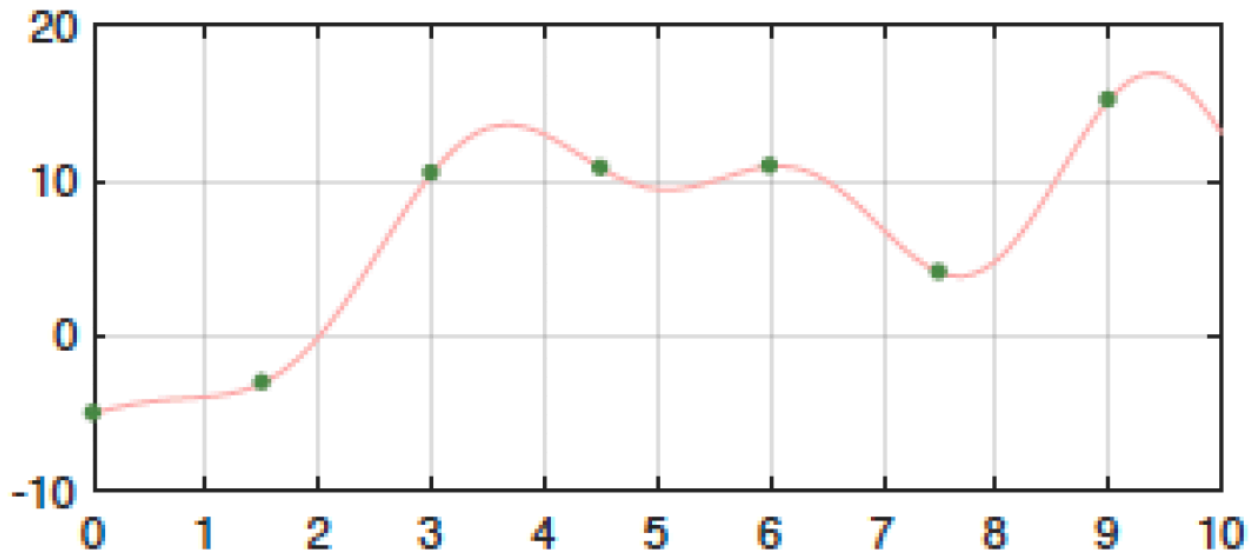


CONVOLUTION QUESTION 2



MOVE TENT TO −1/2 AND +1/2 AND TAKE THE SUM

The figure below shows a continuous function and its sampled version (samples are represented by the dots). What is the sampling frequency?



(a) 1
(b) 1.5
(c) 0.5
(d) None of the above.