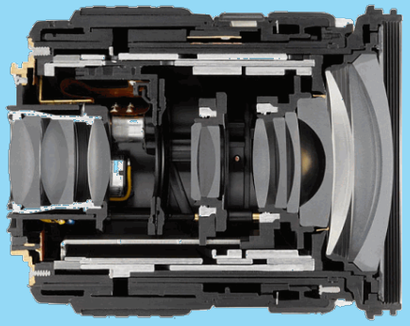
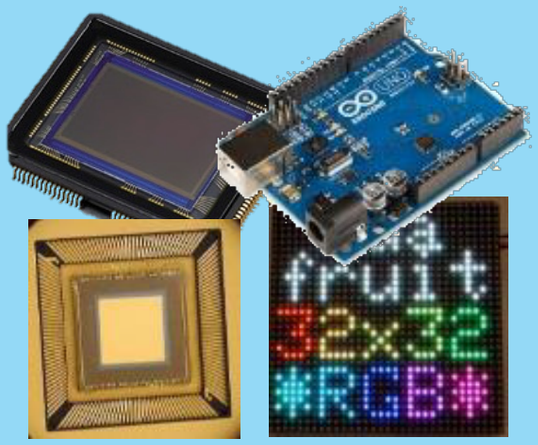


Optics

+



Sensors & devices

+

Signal processing & algorithms

COMPUTATIONAL METHODS FOR IMAGING (AND VISION)

DISCRETE CONVOLUTIONS (EXAMPLES)

PROF. JOHN MURRAY-BRUCE

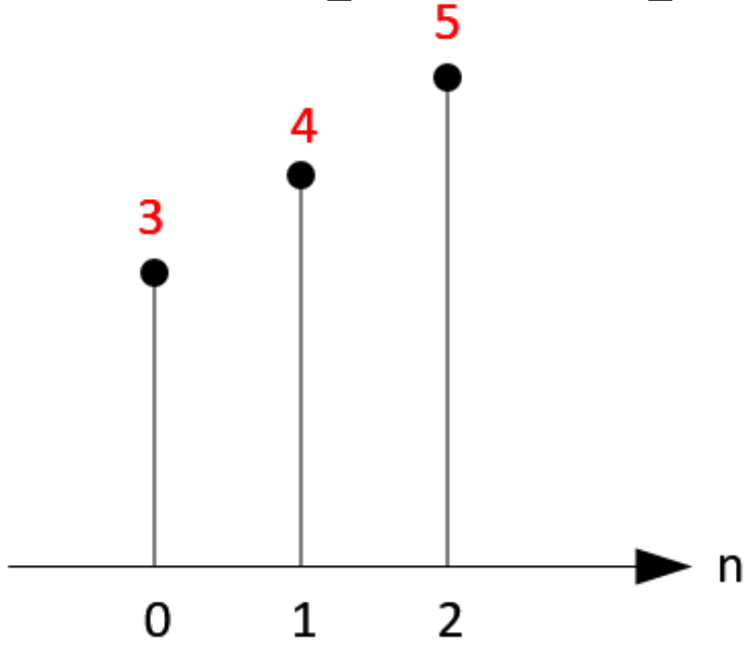
DISCRETE CONVOLUTION

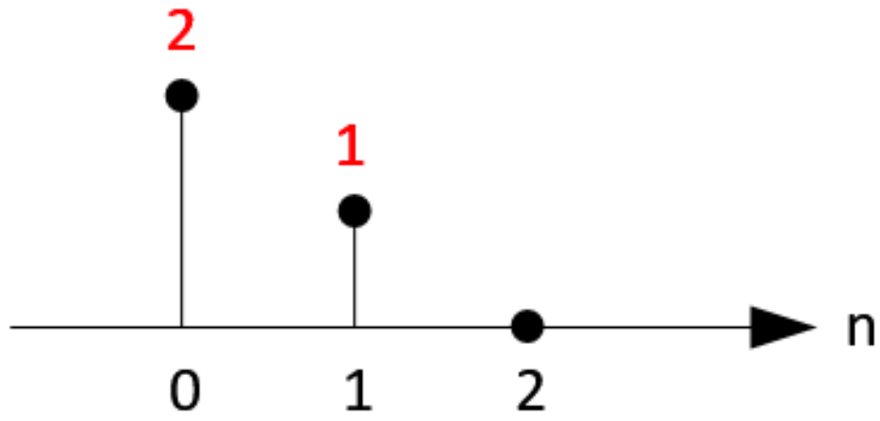
EXAMPLES FOR 1D AND 2D

1D DISCRETE CONVOLUTION

EXAMPLE

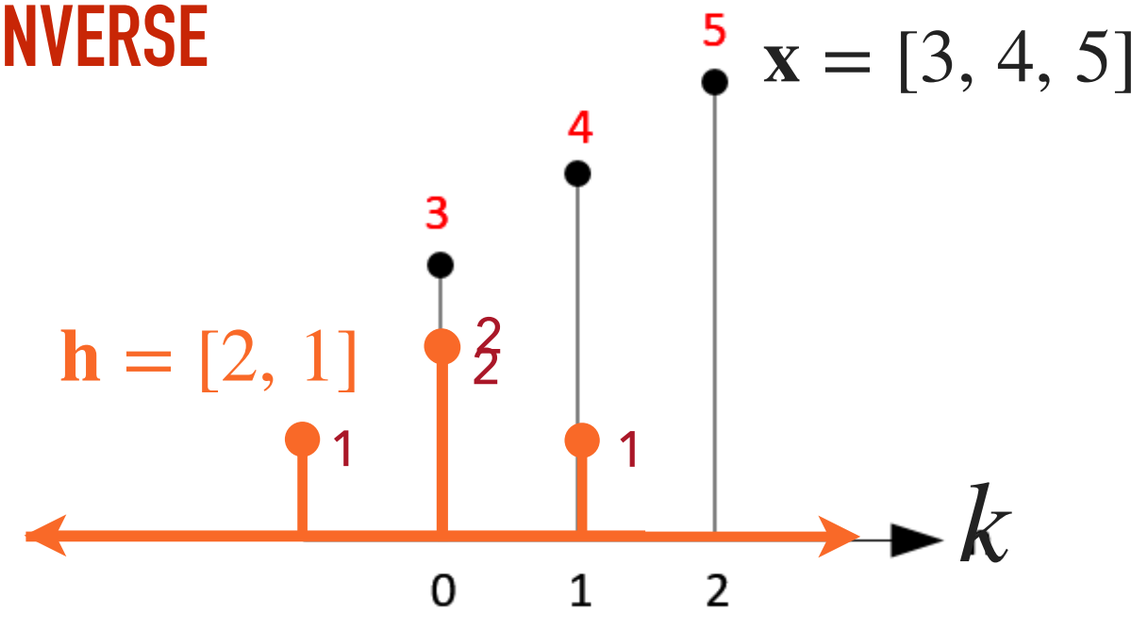
$$x = [3, 4, 5] \qquad h = [2, 1]$$



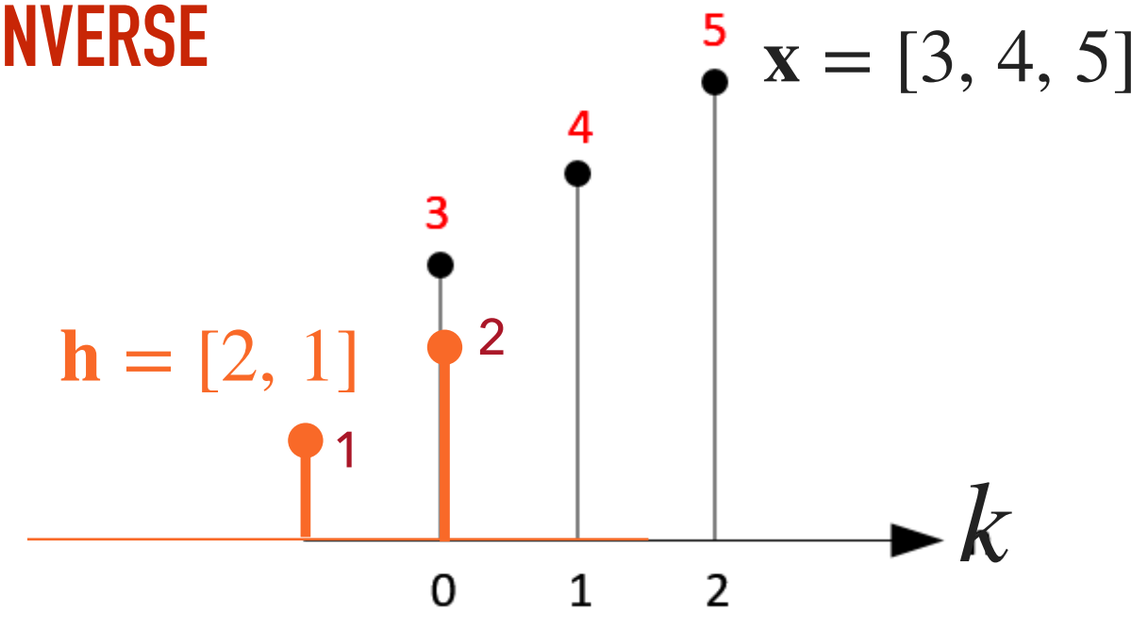


$$g_n = \sum_{k=-\infty}^{\infty} x_k \, h_{n-k}$$

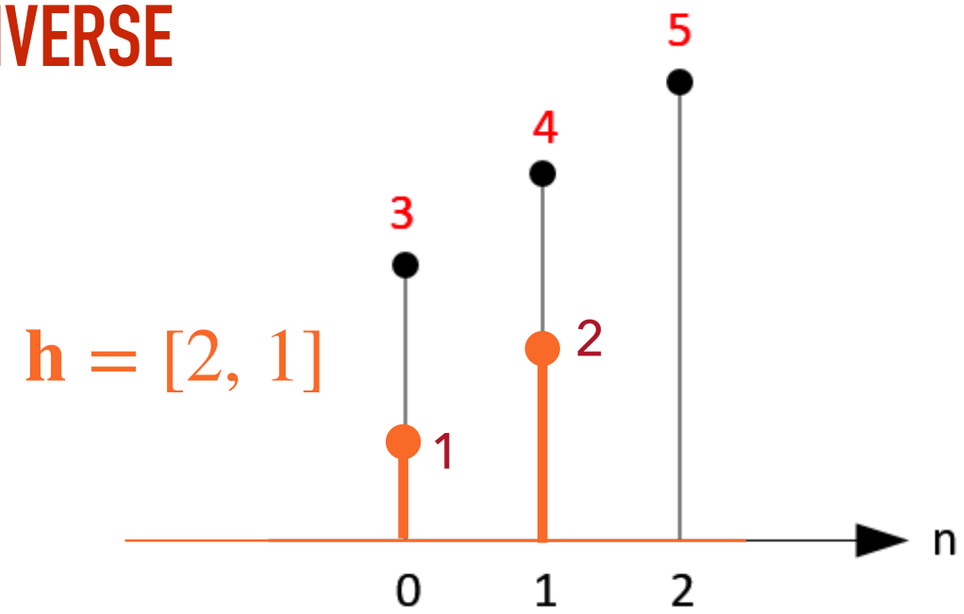
1D DISCRETE CONVOLUTION
MOORE-PENROSE PSEUDO-INVERSE



1D DISCRETE CONVOLUTION
MOORE-PENROSE PSEUDO-INVERSE

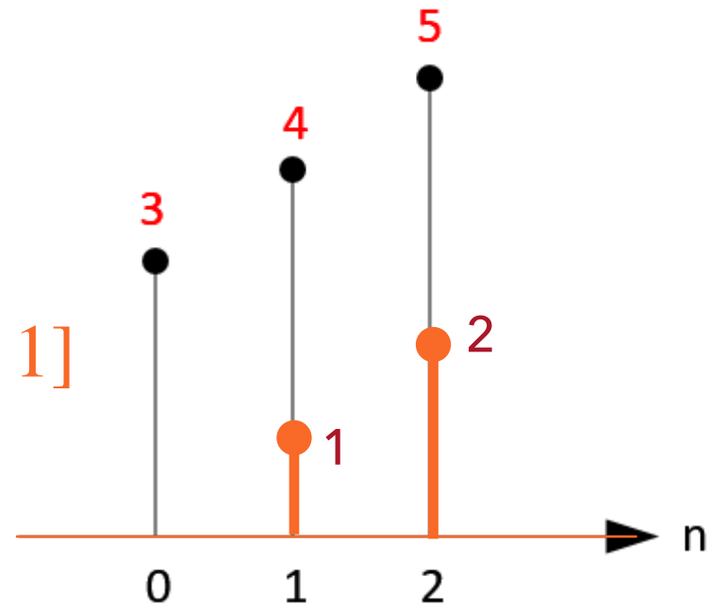


1D DISCRETE CONVOLUTION
MOORE-PENROSE PSEUDO-INVERSE

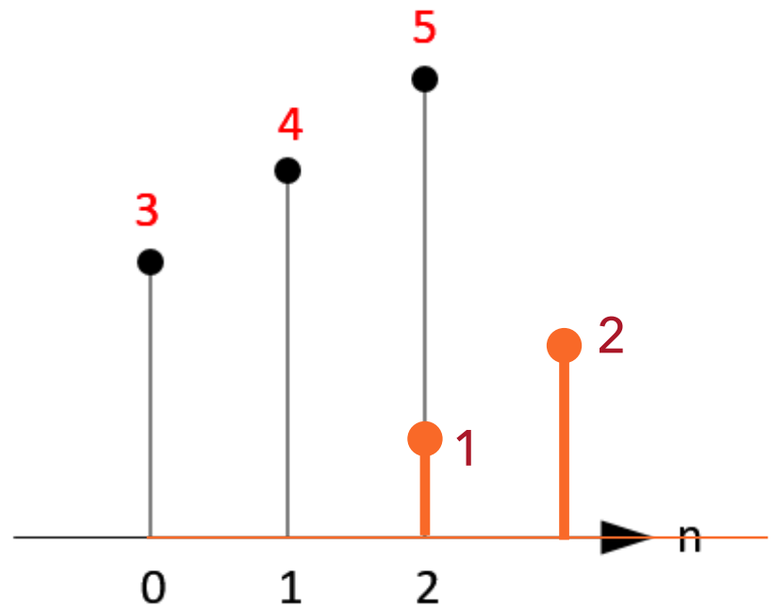


1D DISCRETE CONVOLUTION
MOORE-PENROSE PSEUDO-INVERSE

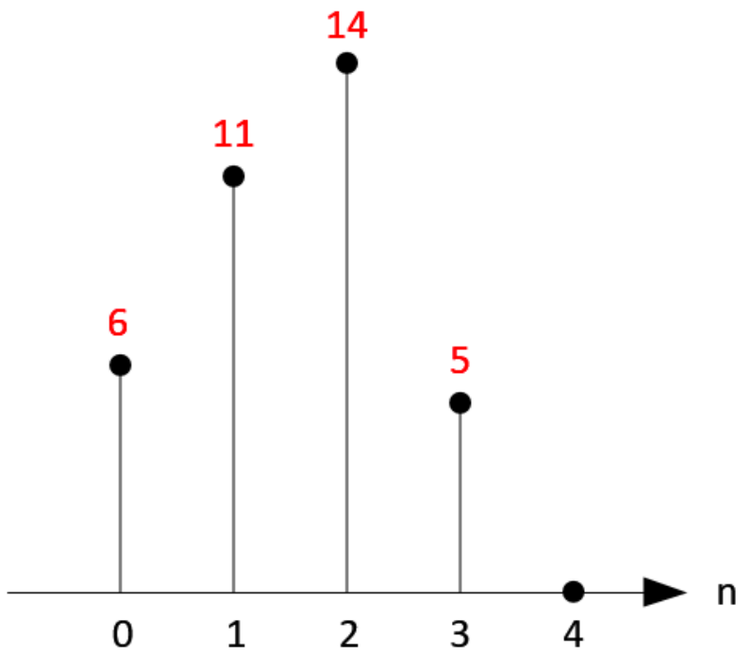
$\mathbf{h} = [2,\ 1]$

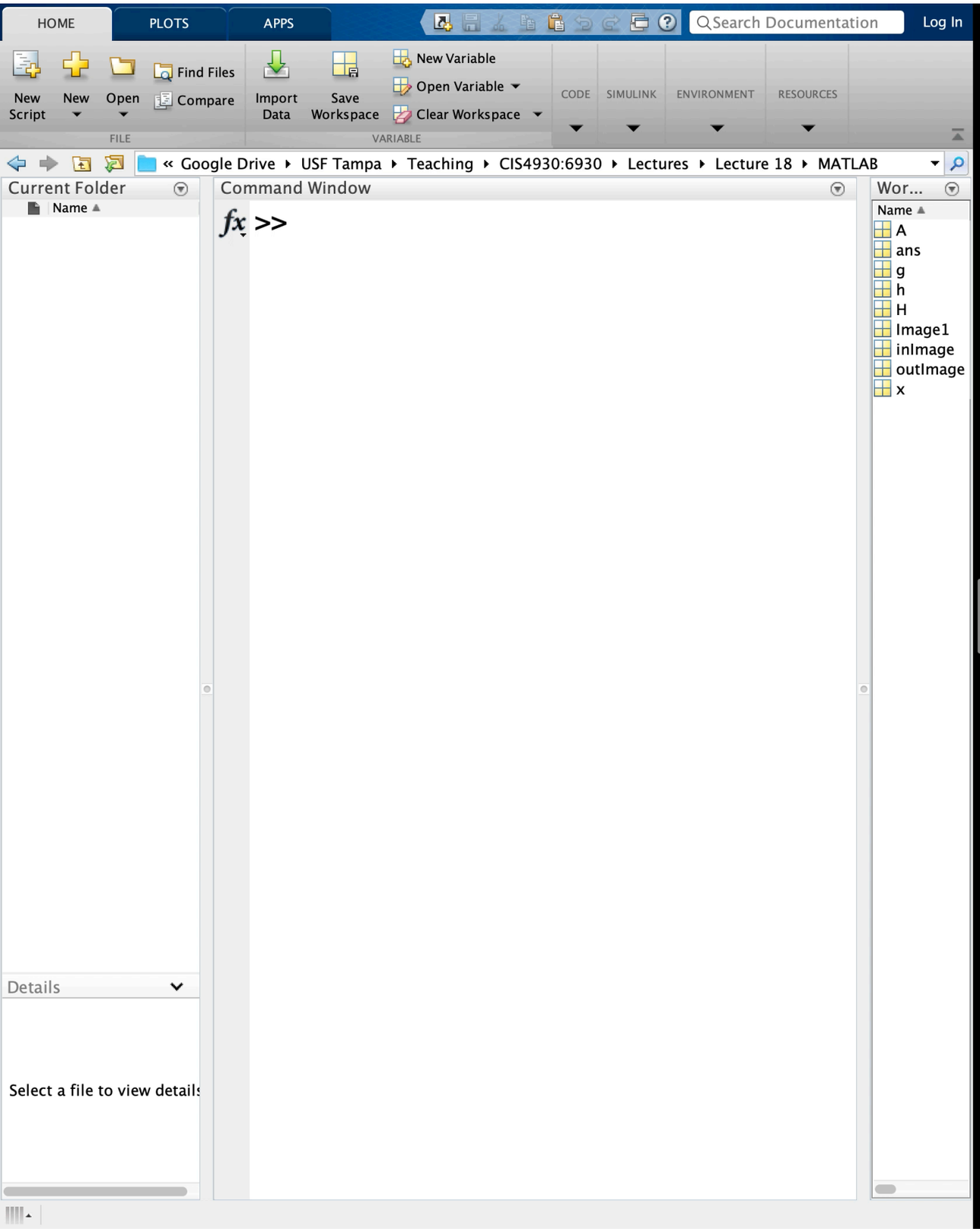


1D DISCRETE CONVOLUTION
MOORE-PENROSE PSEUDO-INVERSE



$$\mathbf{y} = [6, 11, 14, 5]$$

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New Variable
Open Variable
Clear Workspace
CODE
SIMULINK
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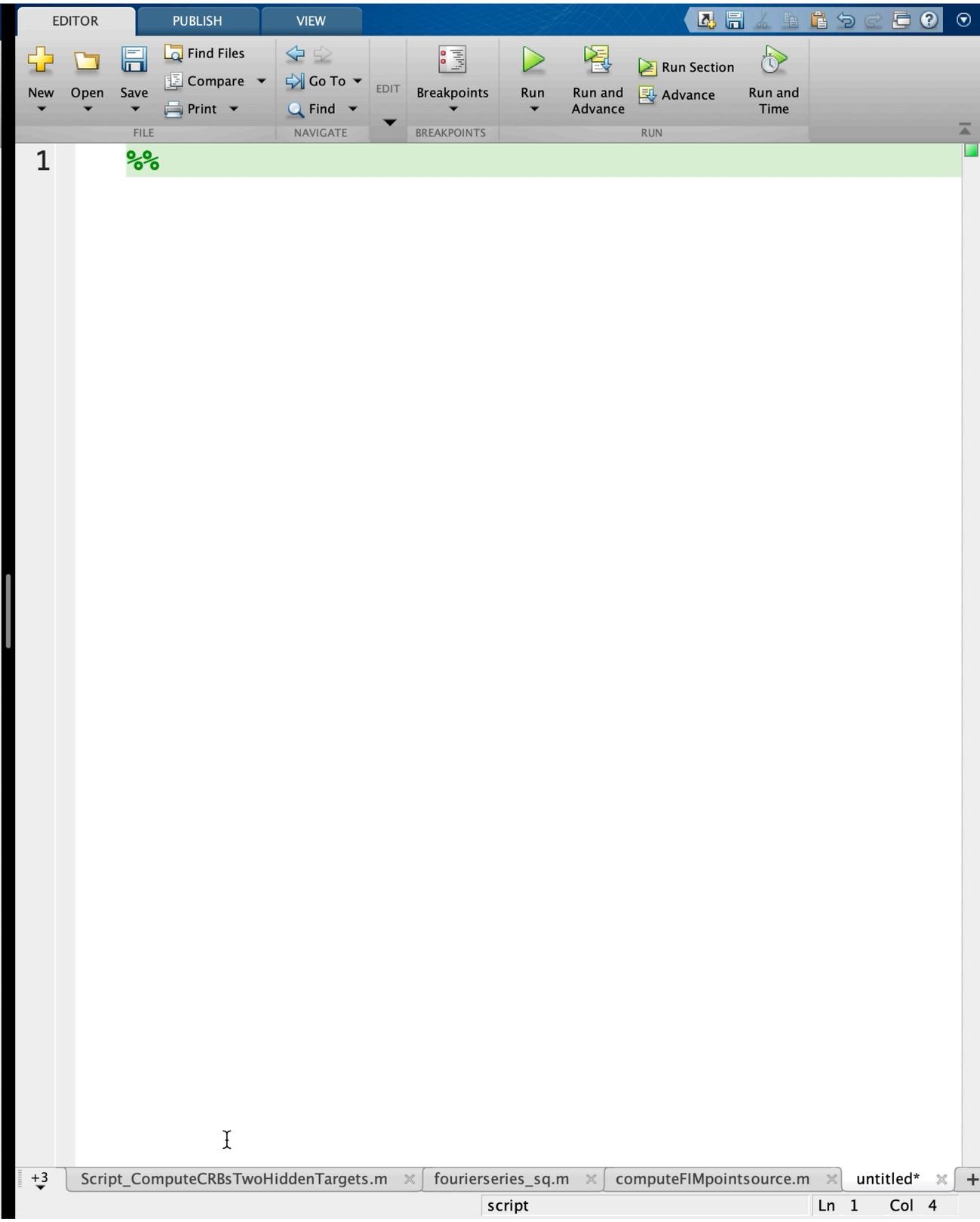
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DISCRETE CONVOLUTION

2D EXAMPLE

2D DISCRETE CONVOLUTION

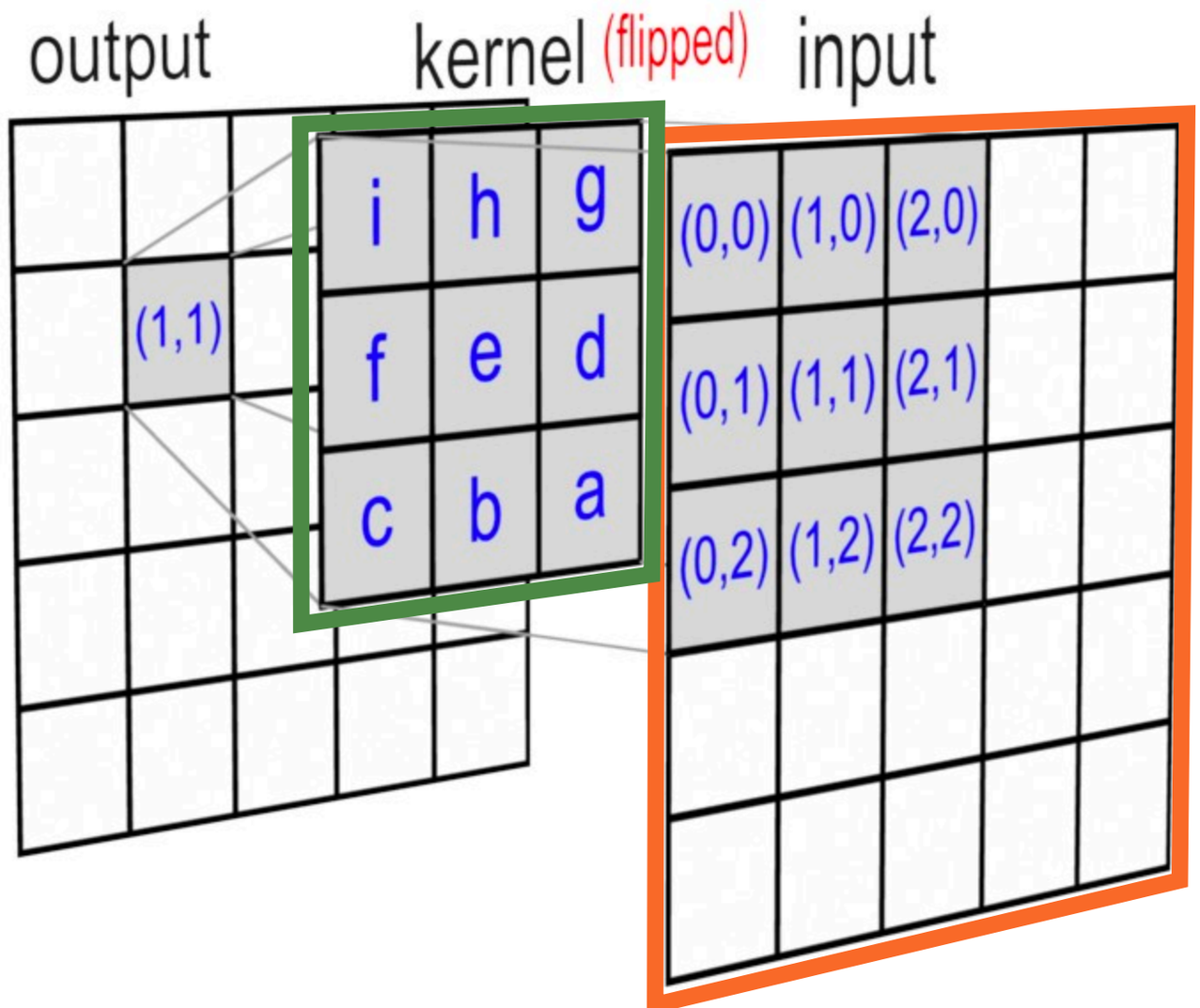
$$\mathbf{A} \star_{2\mathrm{D}} \mathbf{F} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} A_{i,j} F_{i-m,j-n}$$

PSF (also called kernel in CNNs)

$$\mathbf{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Input image

$$\mathbf{F} = \begin{bmatrix} x_{0,0} & x_{1,0} & x_{2,0} & \cdots \\ x_{0,1} & x_{1,1} & x_{2,1} & \cdots \\ x_{0,2} & x_{1,2} & x_{2,2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$



2D DISCRETE CONVOLUTION

1	2	3
4	5	6
7	8	9

X

$\star_{2D}$

-1	-2	-1
0	0	0
1	2	1

H

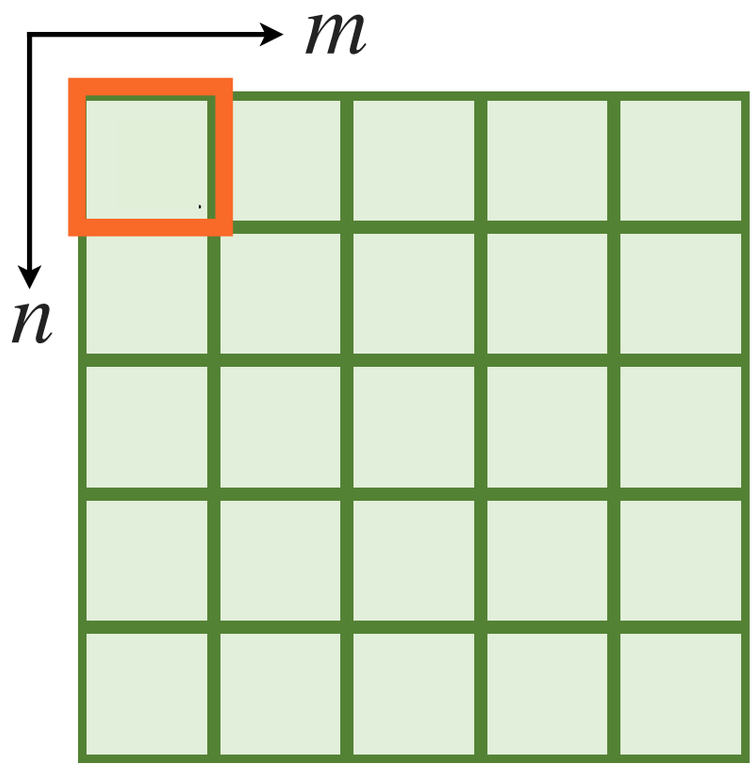
$$\mathbf{X} \star_{2\mathrm{D}} \mathbf{H} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} X_{i,j} H_{i-m,j-n}$$

2D DISCRETE CONVOLUTION

1	2	1
0	0	0
-1	-2	-1

1	2	3
4	5	6
7	8	9

Flip **H** along both dimensions, the center is the (0, 0) element

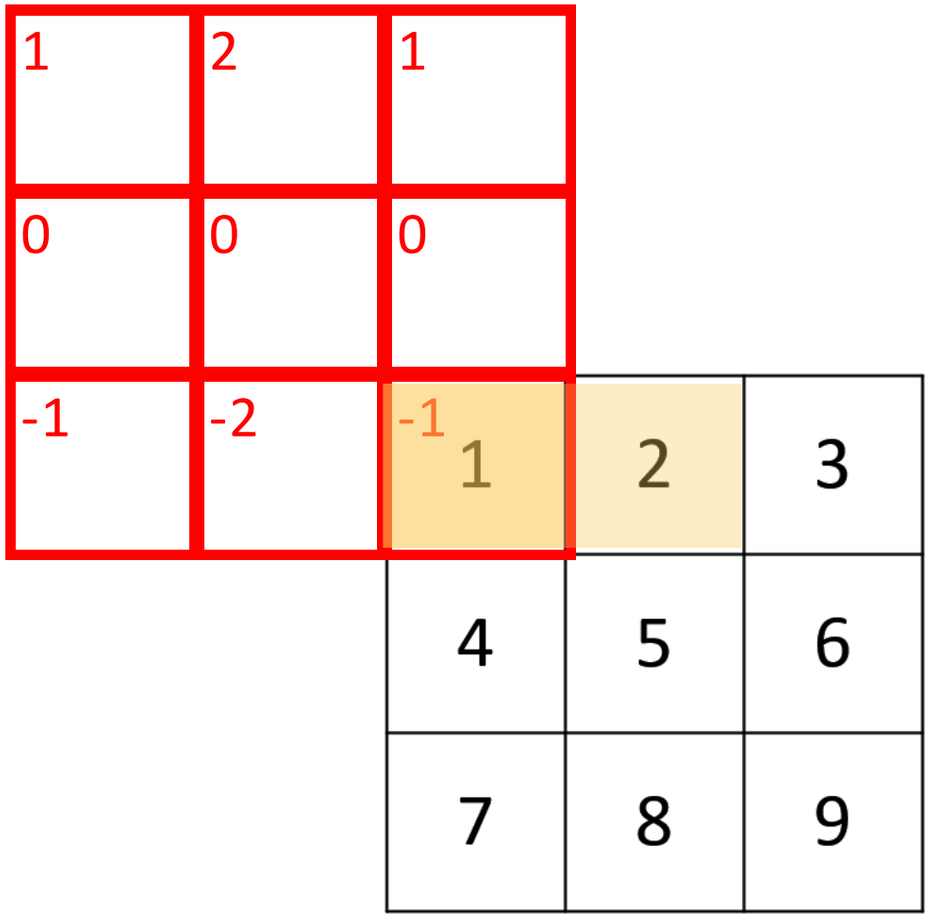


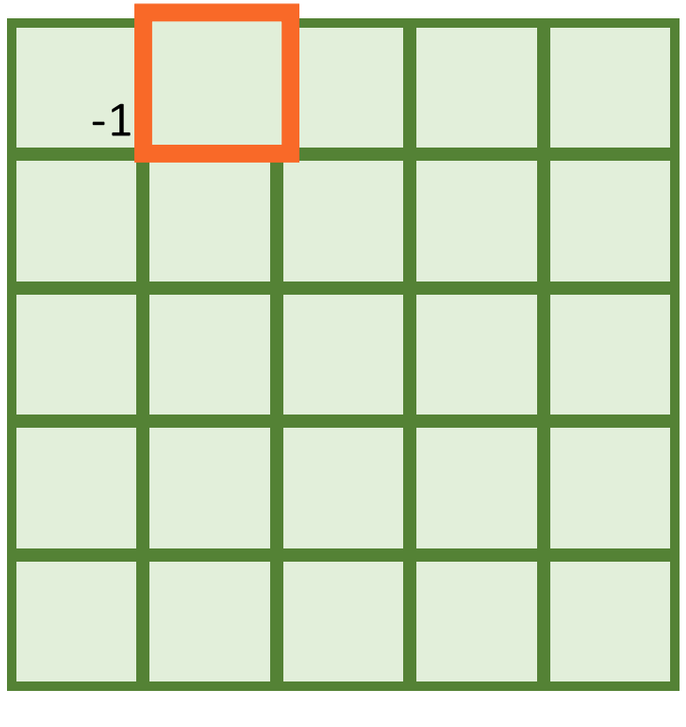
$$g_{0,0} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{0-i,0-j}$$

$$g_{0,0} = x_{0,0}h_{0,0} + x_{1,0}\,h_{-1,0} + x_{2,0}h_{-2,0}$$
$$+x_{0,1}\,h_{1,-1} + x_{1,1}h_{-1,-1} + x_{2,1}h_{-2,-1}$$
$$+x_{0,2}\,h_{0,-2} + x_{1,2}h_{-1,-2} + x_{2,2}h_{-2,-2}$$

$$g_{0,0} = -1 \times 1 = -1$$

2D DISCRETE CONVOLUTION



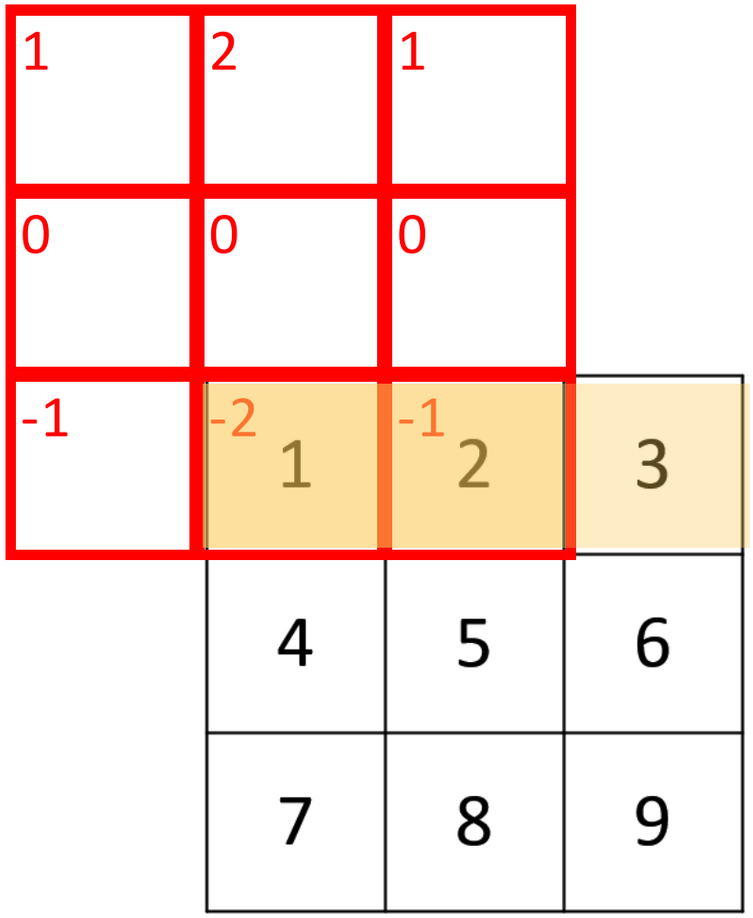


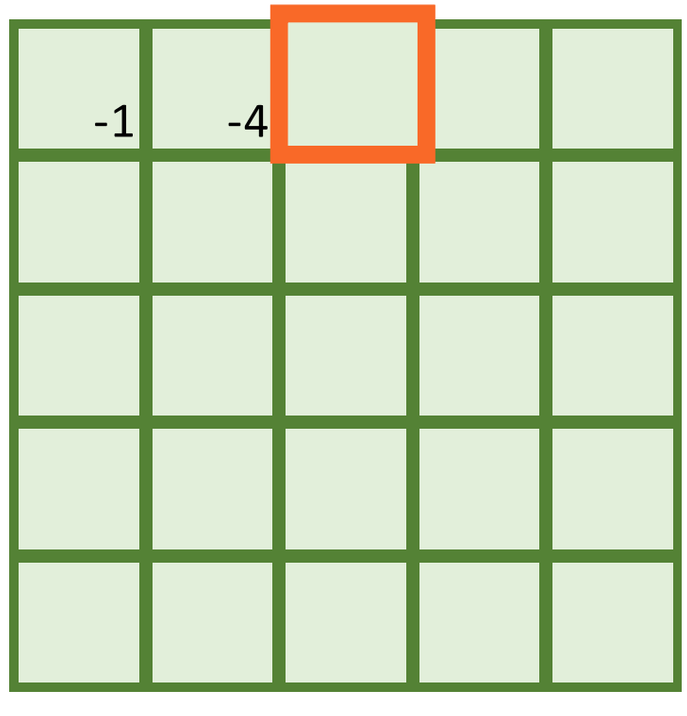
$$g_{1,0} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{1-i,0-j}$$

$$g_{1,0} = x_{0,0}h_{1,0} + x_{1,0}\,h_{0,0} + x_{2,0}h_{-1,0}$$
$$+x_{0,1}\,h_{1,-1} + x_{1,1}h_{0,-1} + x_{2,1}h_{-1,-1}$$
$$+x_{0,2}\,h_{1,-2} + x_{1,2}h_{0,-2} + x_{2,2}h_{-1,-2}$$

$$g_{1,0} = (-2 \times 1) + (-1 \times 2) = -4$$

2D DISCRETE CONVOLUTION



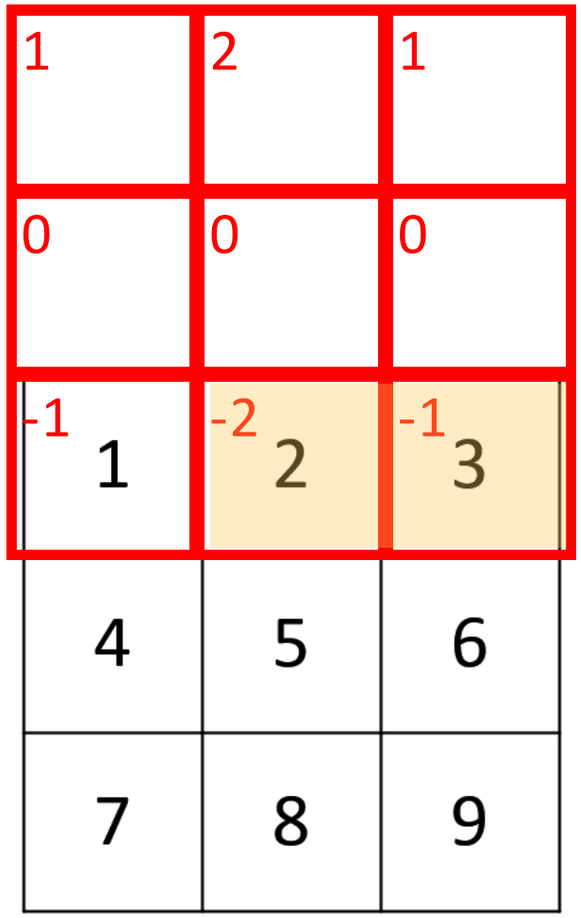


$$g_{2,0} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{2-i,0-j}$$

$$g_{2,0} = x_{0,0}h_{2,0} + x_{1,0}\,h_{1,0} + x_{2,0}h_{0,0}$$
$$+ x_{0,1}\,h_{2,-1} + x_{1,1}h_{1,-1} + x_{2,1}h_{0,-1}$$
$$+ x_{0,2}\,h_{2,-2} + x_{1,2}h_{1,-2} + x_{2,2}h_{0,-2}$$

$$g_{2,0} = (-1 \times 1) + (-2 \times 2) + (-1 \times 3) = -8$$

2D DISCRETE CONVOLUTION

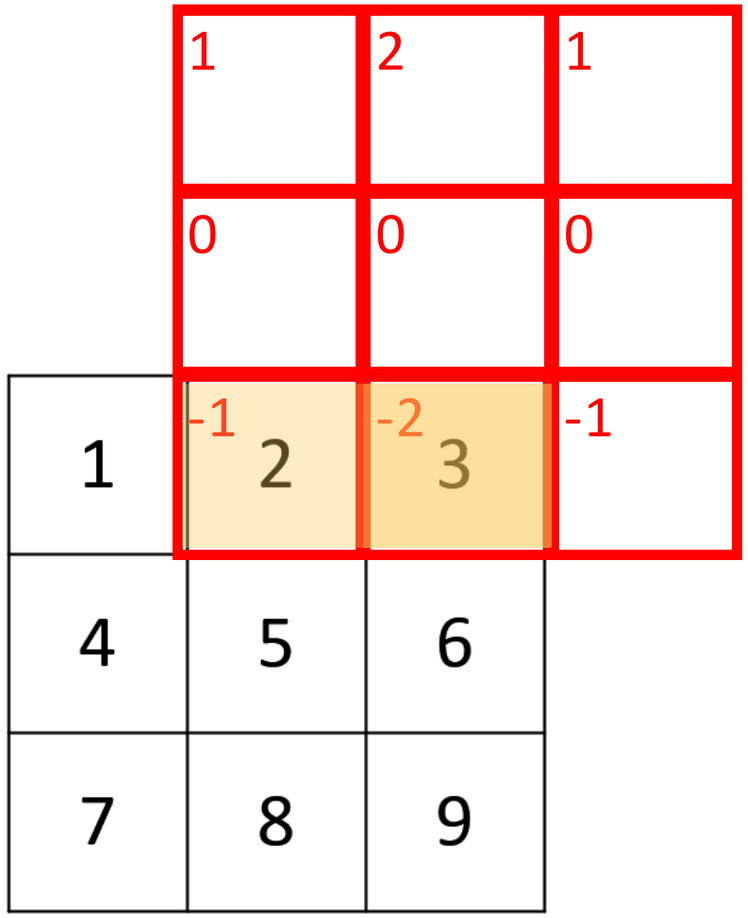


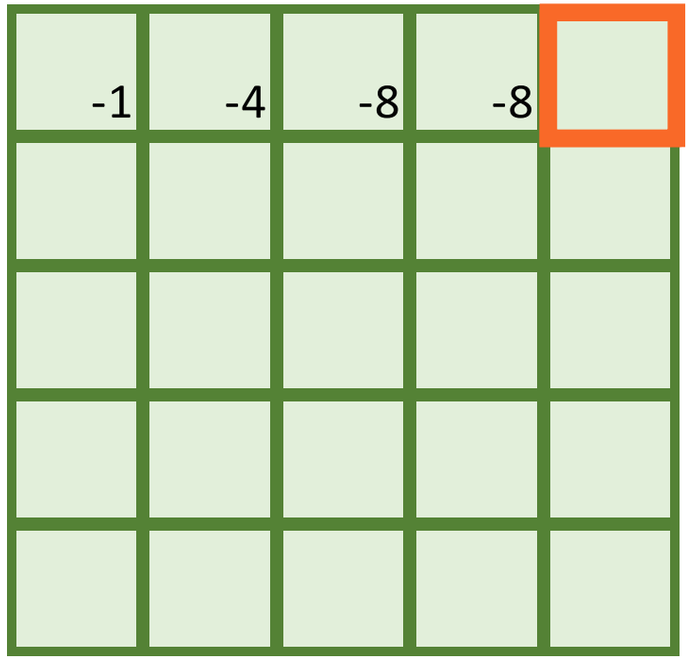
$$g_{3,0} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{3-i,0-j}$$

$$\begin{aligned} g_{3,0} = {} & x_{0,0}h_{3,0} + x_{1,0}\,h_{2,0} + x_{2,0}h_{1,0} \\ & + x_{0,1}\,h_{3,-1} + x_{1,1}h_{2,-1} + x_{2,1}h_{1,-1} \\ & + x_{0,2}\,h_{3,-2} + x_{1,2}h_{2,-2} + x_{2,2}h_{1,-2} \end{aligned}$$

$$g_{2,0} = (-1 \times 2) + (-2 \times 3) = -8$$

2D DISCRETE CONVOLUTION



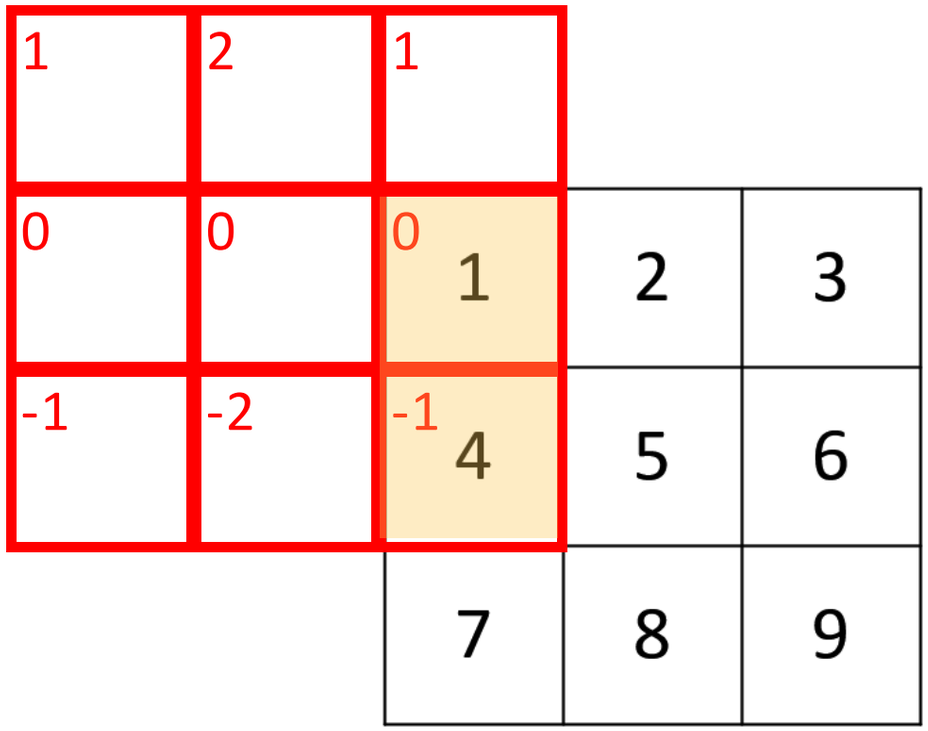


$$g_{4,0} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{4-i,0-j}$$

$$g_{4,0} = x_{0,0}h_{4,0} + x_{1,0}\,h_{3,0} + x_{2,0}h_{2,0}$$
$$+ x_{0,1}\,h_{4,-1} + x_{1,1}h_{3,-1} + x_{2,1}h_{2,-1}$$
$$+ x_{0,2}\,h_{4,-2} + x_{1,2}h_{3,-2} + x_{2,2}h_{2,-2}$$

$$g_{4,0} = (-1 \times 3) = -3$$

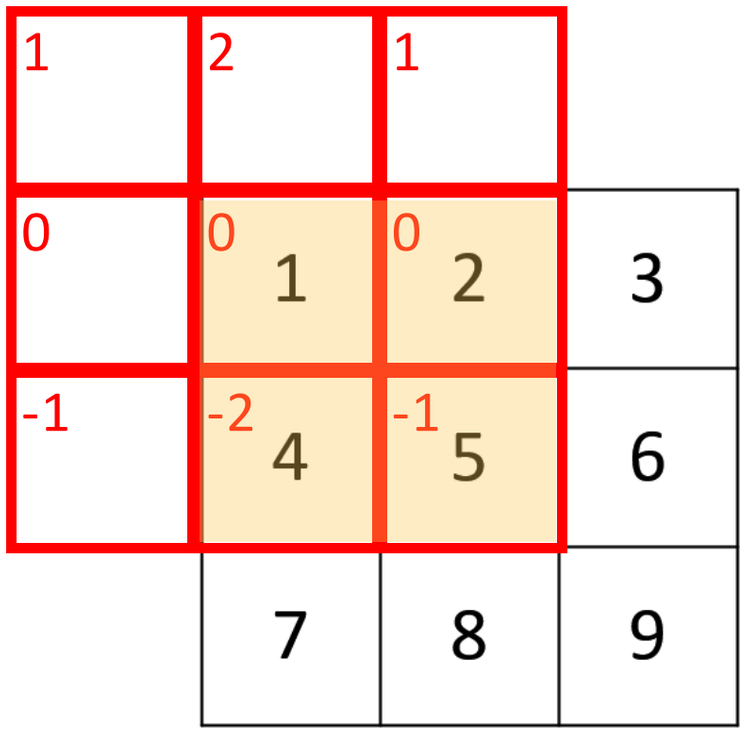
2D DISCRETE CONVOLUTION

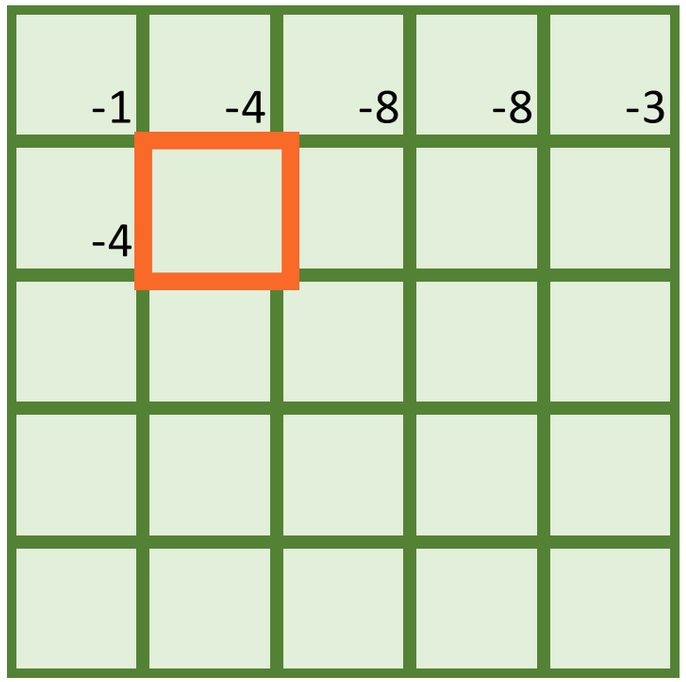


$$g_{0,1} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{0-i,1-j}$$

$$g_{0,1} = (0 \times 1) + (-1 \times 4) = -4$$

2D DISCRETE CONVOLUTION

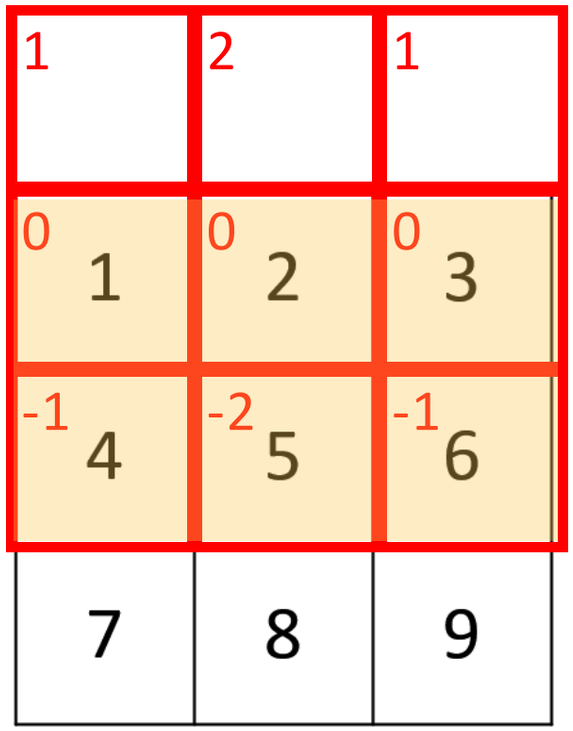


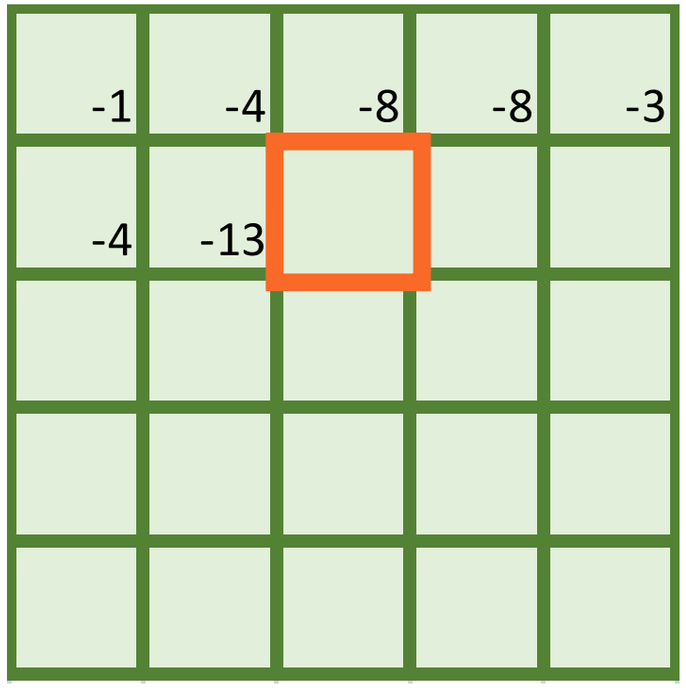


$$g_{1,1} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{1-i,1-j}$$

$$g_{1,1} = (0 \times 1) + (0 \times 2) + (-2 \times 4) + (-1 \times 5) = -13$$

2D DISCRETE CONVOLUTION

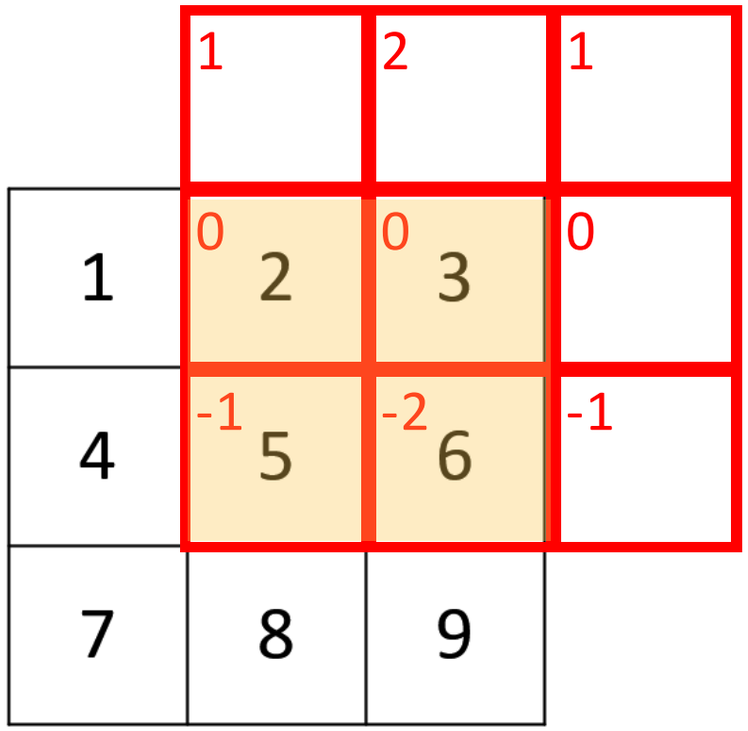


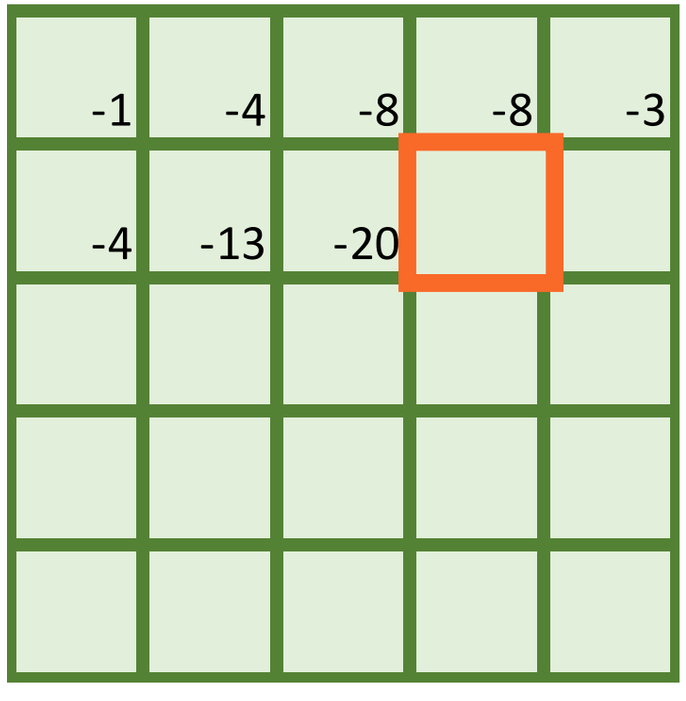


$$g_{2,1} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{2-i,1-j}$$

$$g_{2,1} = (0 \times 1) + (0 \times 2) + (0 \times 3) + (-1 \times 4) + (-2 \times 5) + (-1 \times 6) = -20$$

2D DISCRETE CONVOLUTION

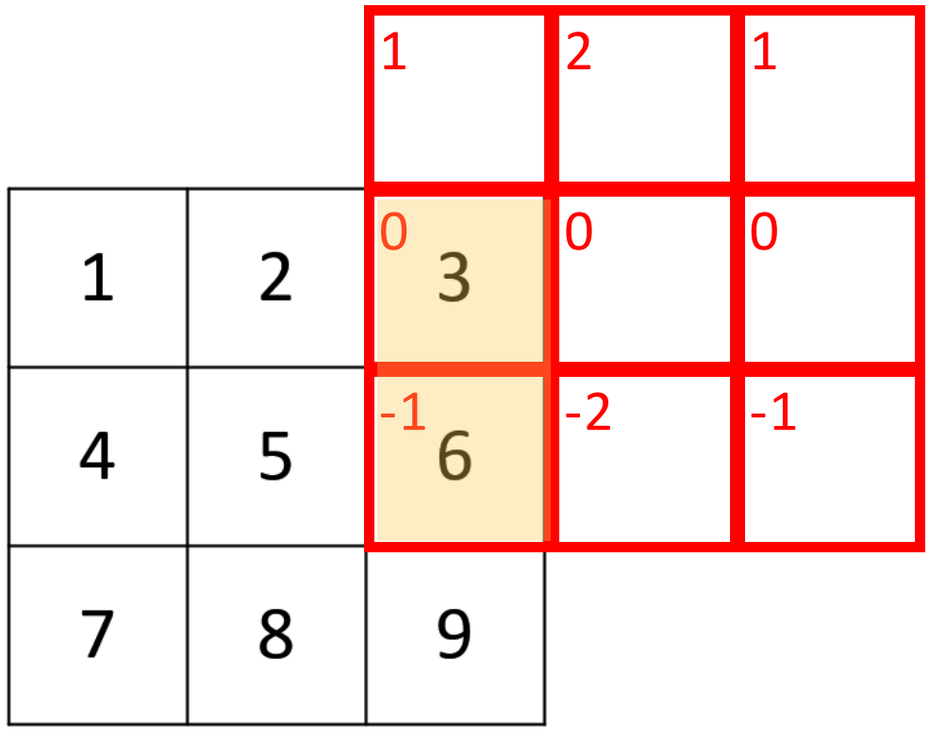


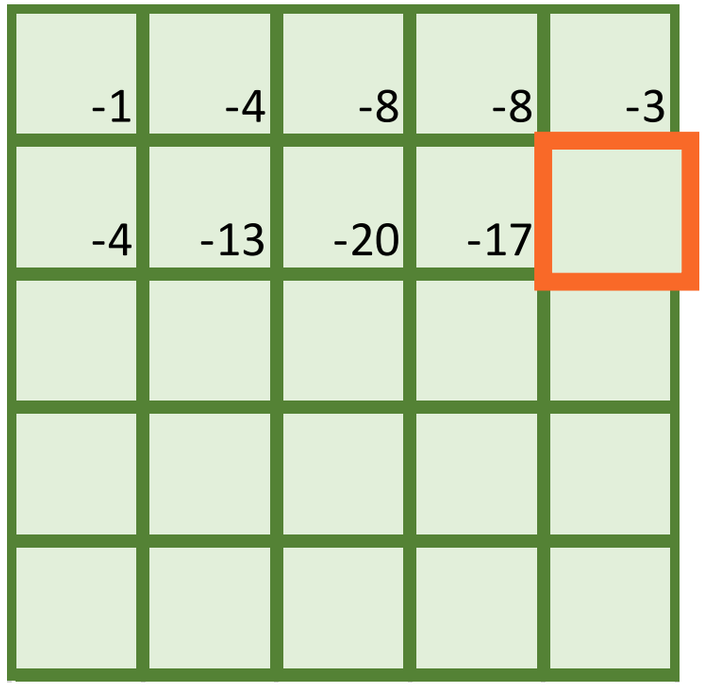


$$g_{3,1} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{3-i,1-j}$$

$$g_{3,1} = (-1 \times 5) + (-2 \times 6) = -17$$

2D DISCRETE CONVOLUTION

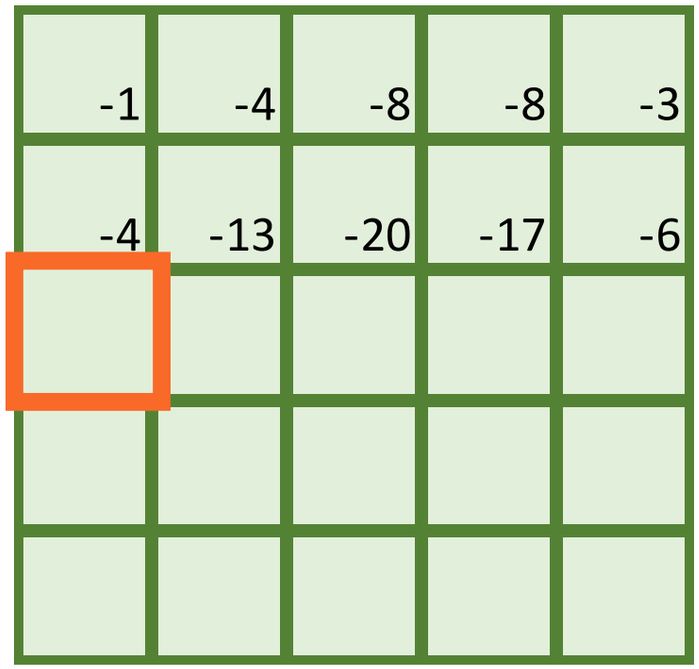




$$g_{4,1} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{4-i,1-j}$$

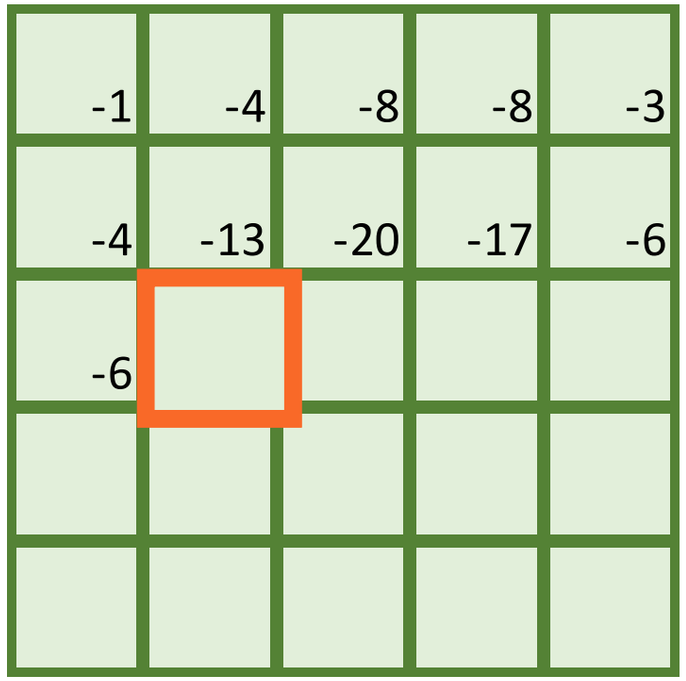
$$g_{4,1} = (-1 \times 6) = -6$$

2D DISCRETE CONVOLUTION

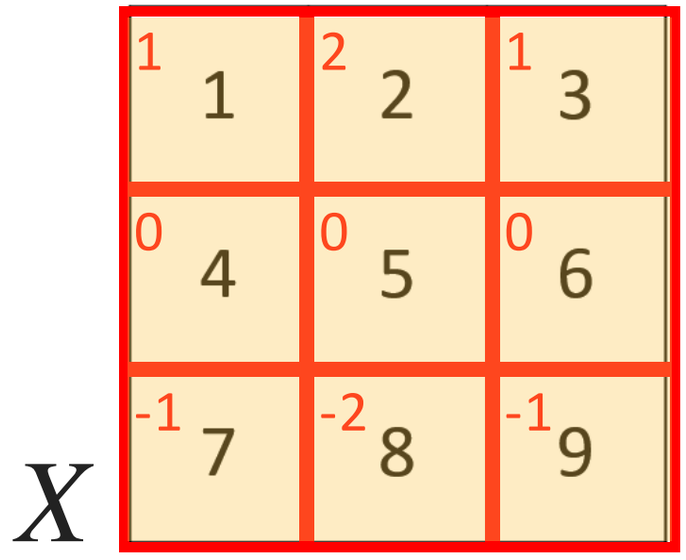


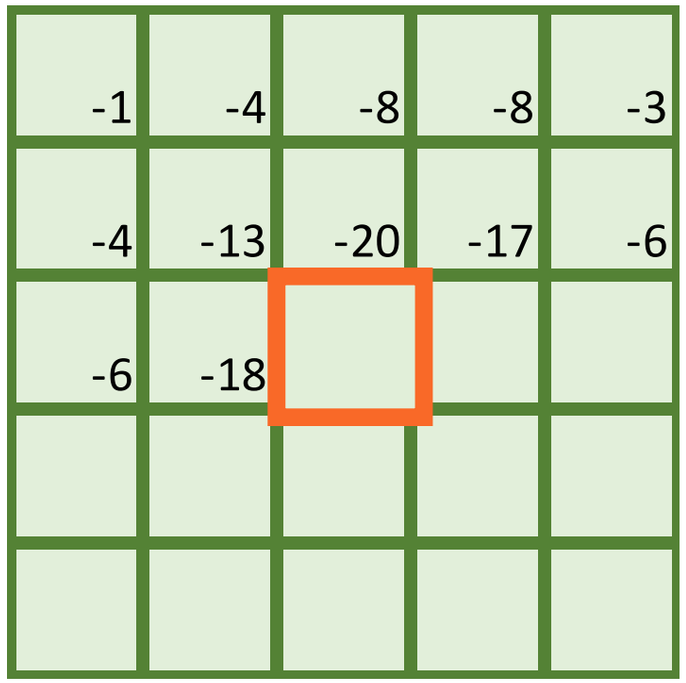
$$g_{0,2} = (-1 \times 6) = -6$$

2D DISCRETE CONVOLUTION



2D DISCRETE CONVOLUTION





$$g_{2,2} = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} x_{i,j} h_{2-i,2-j}$$

$$g_{2,2} = 1 + 4 + 3 - 7 - 16 - 9 = -24$$

2D DISCRETE CONVOLUTION

1	2	3
4	5	6
7	8	9

$\star_{2D}$

-1	-2	-1
0	0	0
1	2	1

=

DISCRETE CONVOLUTION OUTPUT

-1	-4	-8	-8	-3
-4	-13	-20	-17	-6
-6	-18	-24		

Note that we used index i to denote column and j to index row

MATLAB IMPLEMENTATION

- 1D discrete convolution can be implemented in Matlab by using the built-in ‘`conv`’ function
- 2D discrete convolution can be implemented in Matlab by using the built-in ‘`conv2`’ function
- n-Dimension discrete convolution can be implemented in Matlab by using the built-in ‘`convn`’ function

THAT IS ALL!!!