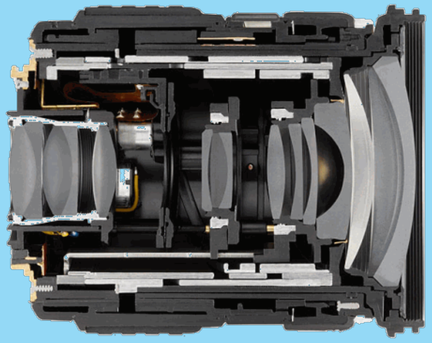
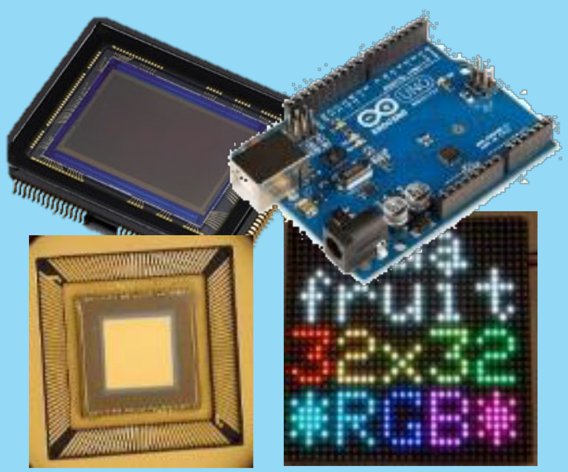


Optics

+



Sensors
&
devices

+

Signal
processing
&
algorithms

COMPUTATIONAL METHODS FOR IMAGING (AND VISION)

LECTURE 25: INTRODUCTORY OPTICS

PROF. JOHN MURRAY-BRUCE

WHERE ARE WE

WE ARE HERE!

10	15-Mar-21	Forward Models and Inverse Problems	Linear Inversion - Inverse problems - Deconvolution and Denoising	IIP 4, Appendix E	**HW 4**	
	17-Mar-21		Intro to Regularized Inversion I - Tikhonov	IIP 5, Appendix E		
11	22-Mar-21	Regularization	Intro to Regularized Inversion II - Iterative methods - Steepest descent	IIP 6		
	24-Mar-21		Statistical methods I - ML estimation - Bayesian estimation	IIP 7.1 - 7.5		
12	29-Mar-21	Forward models and Inverse Problems II	LSV imaging systems: Forward problem - SVD - Inversion	IIP 8.1, 9, 10		
	31-Mar-21	Non-linear Regularization	Beyond L_2-regularization - Sparsity (l_0- and l_1-priors) - TV prior	SMIV 1.1 - 1.5 Papers & Handout		**HW 4**
13	5-Apr-21		Algorithms overview - ISTA/FISTA - ADMM	Papers & Handout		
	7-Apr-21	Introductory Optics	Geometrical/Ray Optics - Rays & pinhole cameras - Lenless imaging and Coded apertures	IIP 8.2, 8.3, 9.5 Papers & Handout		
14	12-Apr-21 14-Apr-21	**Spring Break (no classes)**				
15	19-Apr-21	Applications of Comp. Imaging	INTRO OPTICS	Papers & Handout		
	21-Apr-21		Compressive Imaging and Imaging from few photons	Papers & Handout		
16	26-Apr-21 28-Apr-21	**Group Presentations (Teams)**				
17	3-May-21	***no class**				
	5-May-21	**Final Exam: 12:30 PM - 2:30 PM**				

WAVE–PARTICLE NATURE OF LIGHT

'Light is a particle' – Newton

Short or Long wavelength approximation

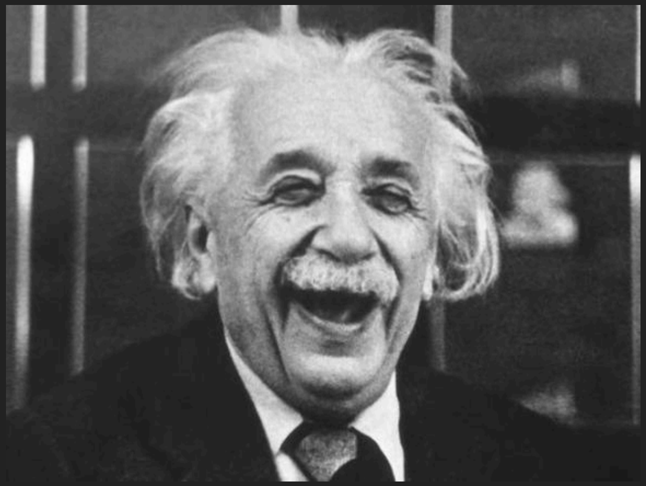
Energy (of particle/photon) described by the Poynting vector

'Light is a wave' – Huygens

Wave Optics description

Light properties described by the wave equations

'Light is both a particle and a wave' – Einstein

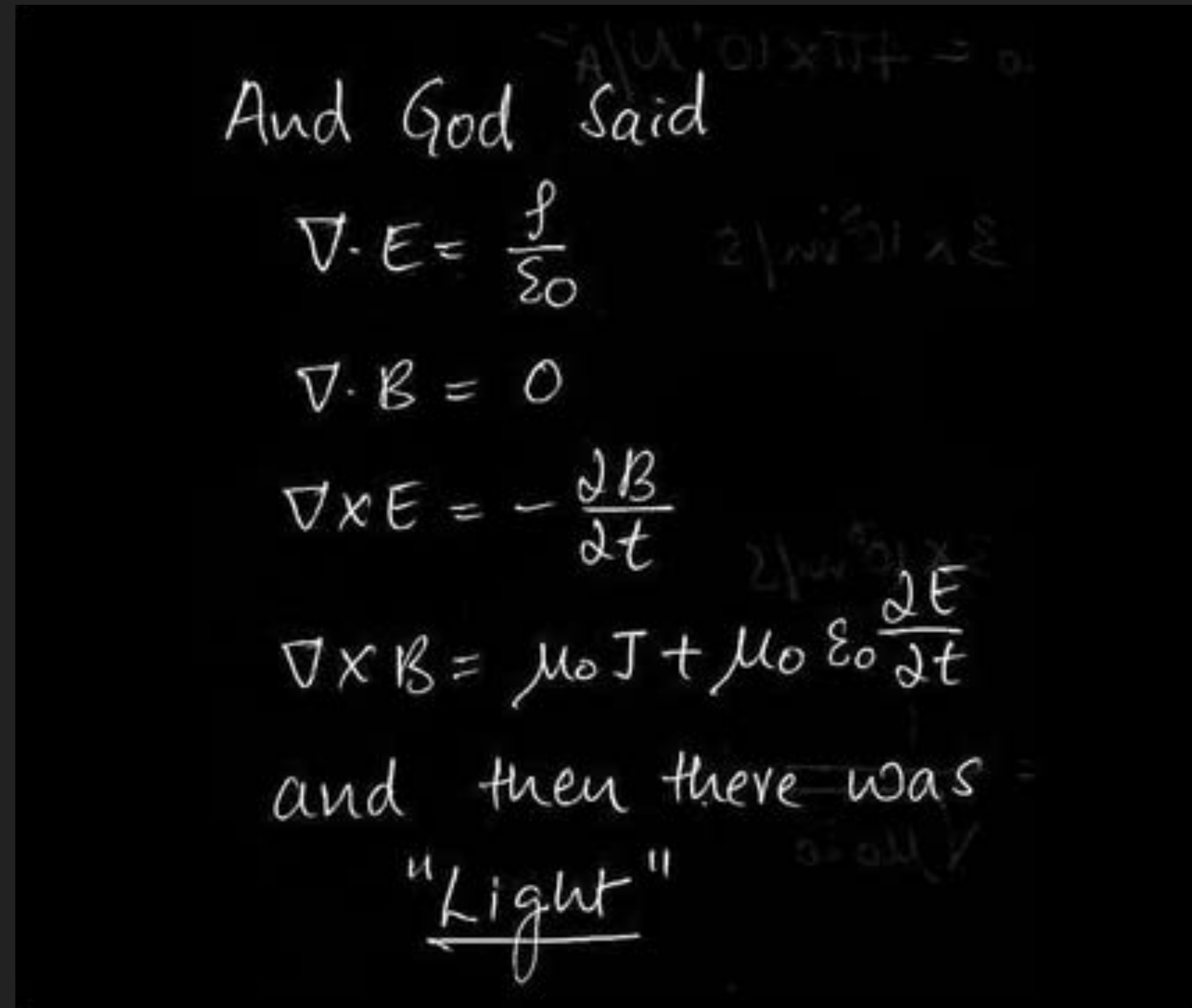


INTRO TO OPTICS
OUTLINE

- **Optics/Light**
  - Wave-Particle Duality
  - Electromagnetic (EM) waves
  - EM spectrum
  - Geometric optics vs Wave optics
- **Pinhole camera**
- **Lens-based camera**
- **Coded-aperture imaging**

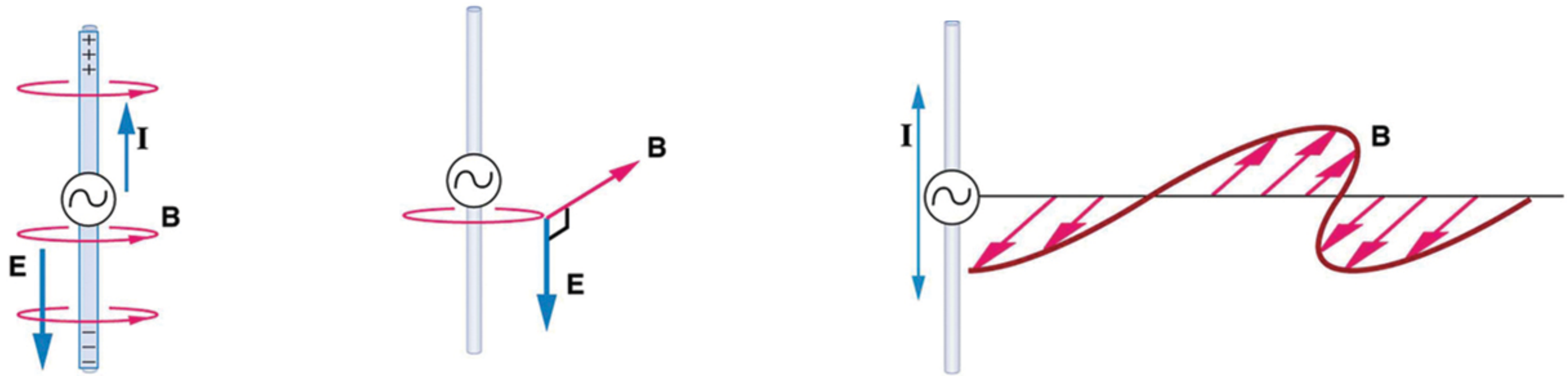
INTRO TO OPTICS
LEARNING OUTCOME

- Be able to describe light as a wave and a particle
- Identify at least waves of the EM spectrum
- Identify fundamental differences between wave and geometric optics
- Describe how a pinhole camera works
- Describe how a concave lens works
- Basic description of aperture-based cameras
- Define numerical aperture, depth-of-field and field-of-view of camera


And God Said
$\nabla \cdot E = \frac{\rho}{\varepsilon_0}$
$\nabla \cdot B = 0$
$\nabla \times E = -\frac{\partial B}{\partial t}$
$\nabla \times B = \mu_0 J + \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$
and then there was
"Light"

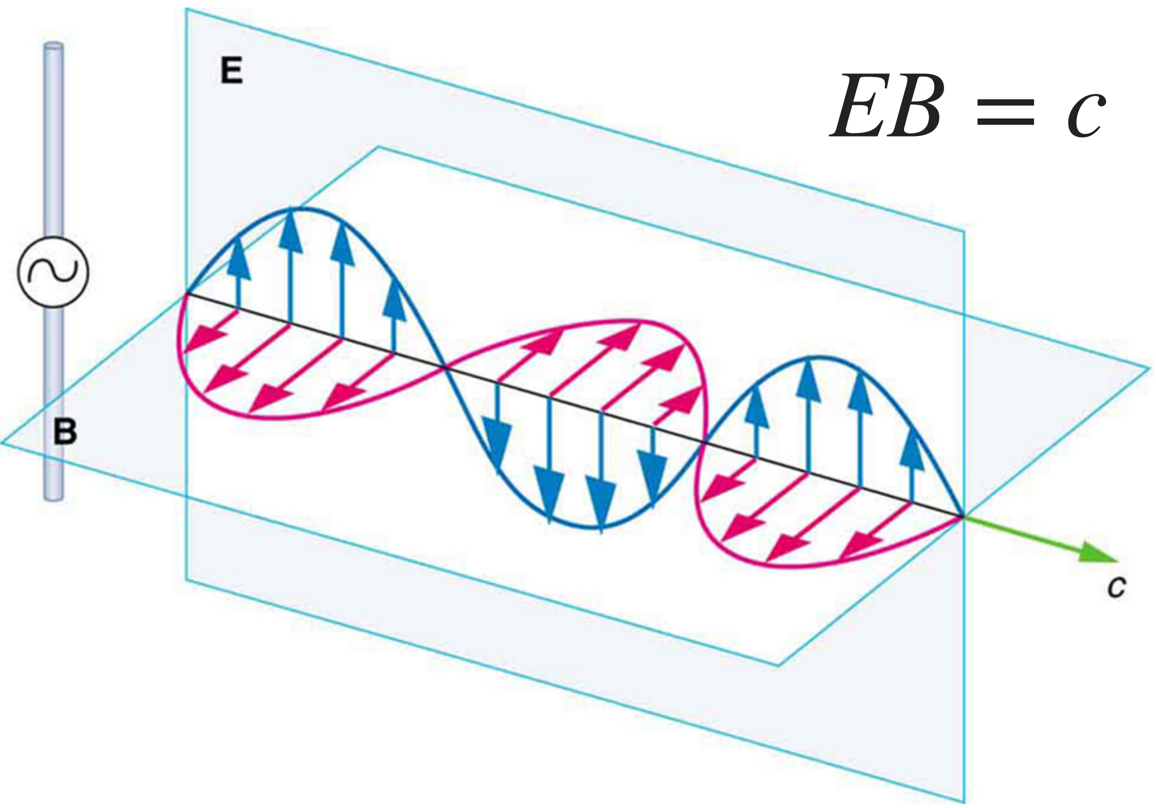
OPTICS

ELECTRIC & MAGNETIC WAVES



- As the current varies, the magnetic field varies in magnitude and direction.
- The current in the antenna produces the circular magnetic field lines.
    - **Current (I)** produces the separation of charge along the wire, which in turn creates the electric field as shown.
    - **Electric and magnetic fields (E and B)** near the wire are **perpendicular**; they are shown here for one point in space.
    - **Magnetic field** varies with current and **propagates away from the antenna** at the speed of light.

ELECTRIC & MAGNETIC WAVES



$$EB = c$$

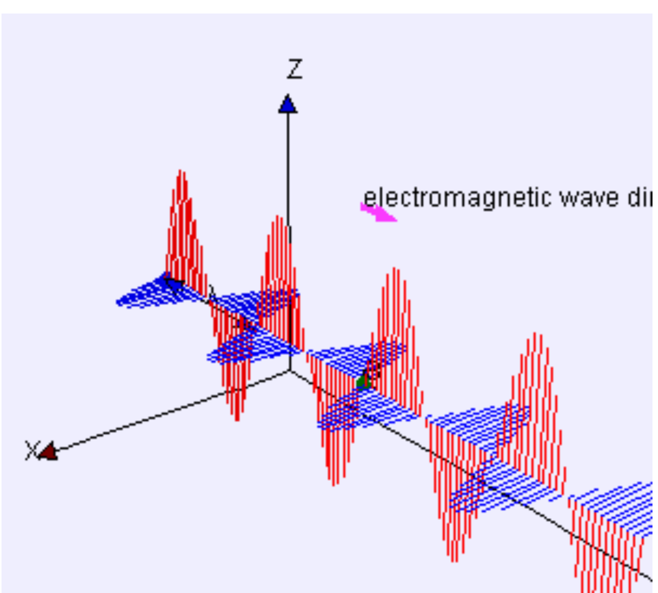
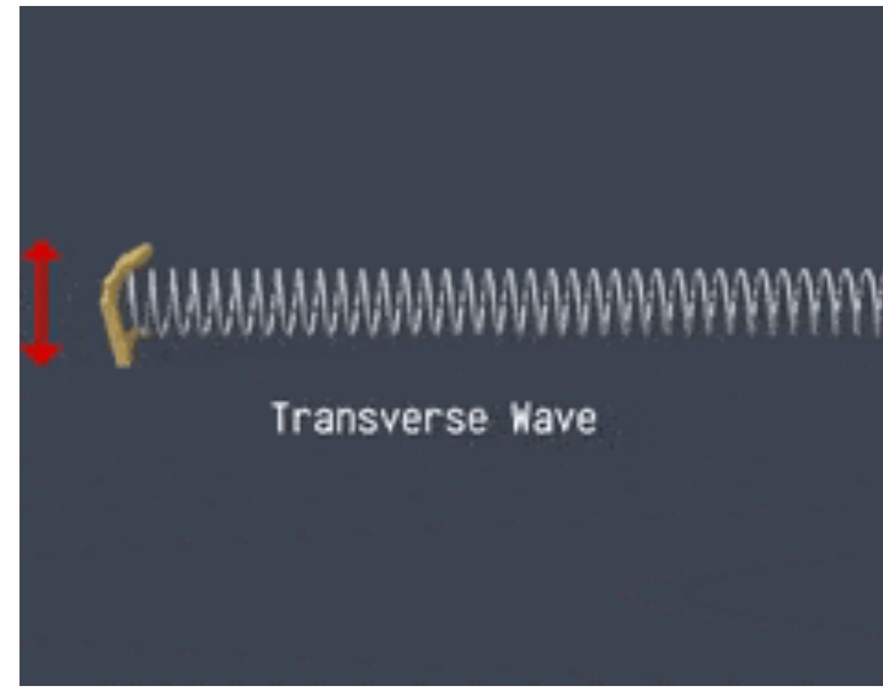


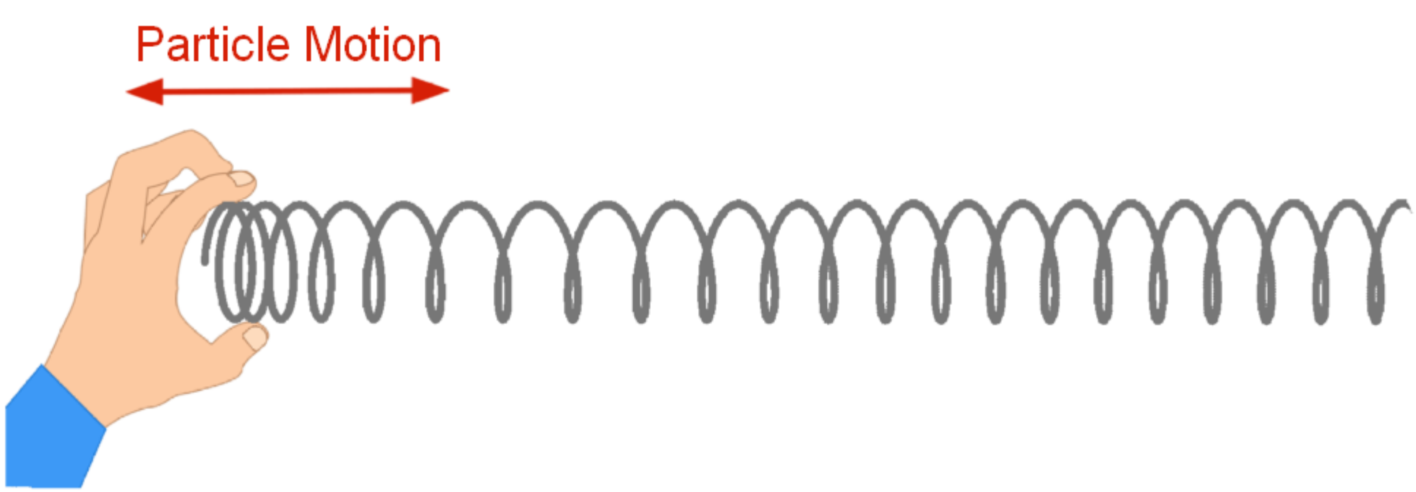
Image credit: Wikipedia

- The **magnetic field lines** also propagate away from the antenna at the speed of light
  - Magnetic part of the wave has the same period and wavelength as the electric part
- E and B field shown together.
  - The electric and magnetic fields produced by a long straight wire antenna are exactly in phase.
  - They are perpendicular to one another and to the direction of propagation, making this a **transverse wave**.
- **E and B fields** are **in-phase**, and they are perpendicular to one another and the direction of propagation.
- The waves are shown only along one direction, but they propagate out in other directions too.

TRANSVERSE VS LONGITUDINAL WAVES

- **Transverse wave:** the direction of oscillation is perpendicular to the direction of propagation (energy transfer).
- Light propagates a transverse wave.





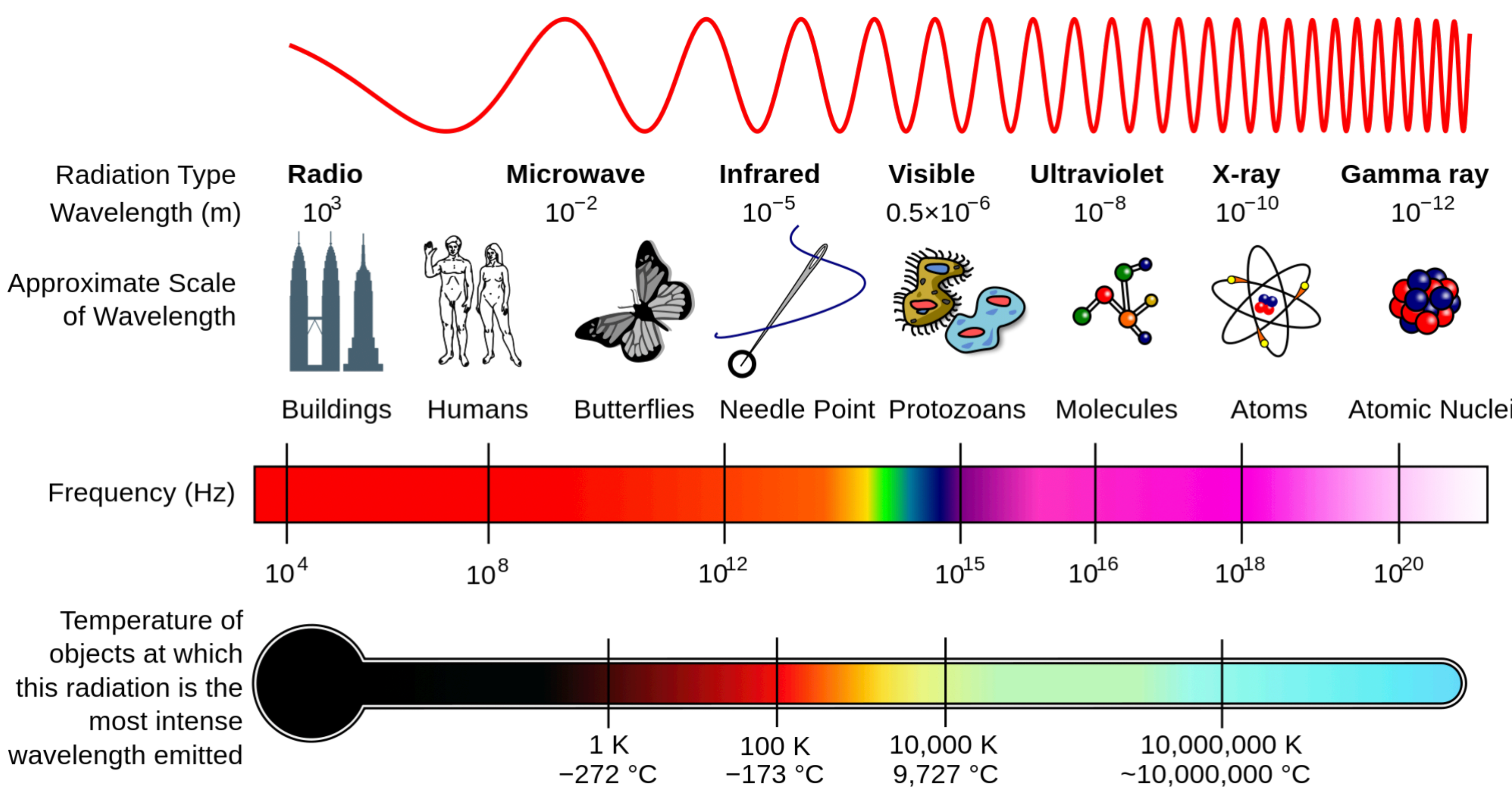
- **Longitudinal wave:** direction of oscillation is parallel to direction of propagation (energy transfer).

OPTICS

- **Optics** is the study of the behavior of visible light and other forms of electromagnetic waves.
- Optics falls into two distinct categories:
  - When electromagnetic radiation, such as visible light, interacts with objects that are **large compared with its wavelength, its motion can be represented by straight lines like rays**. **Ray optics** is the study of such situations and includes lenses and mirrors.
  - When electromagnetic radiation interacts with objects about the **same size as the wavelength or smaller, its wave nature becomes apparent.** For example, observable detail is limited by the wavelength, and so visible light can never detect individual atoms, because they are so much smaller than its wavelength. **Physical or wave optics** is the study of such situations and includes all wave characteristics.

ELECTROMAGNETIC SPECTRUM

- An **electromagnetic wave** has a **frequency** and a **wavelength** associated with it and **travels at the speed of light**
- For EM traveling at the speed of light $c\ \mathrm{ms}^{-1}$:
- $c = f\lambda$



ELECTROMAGNETIC SPECTRUM

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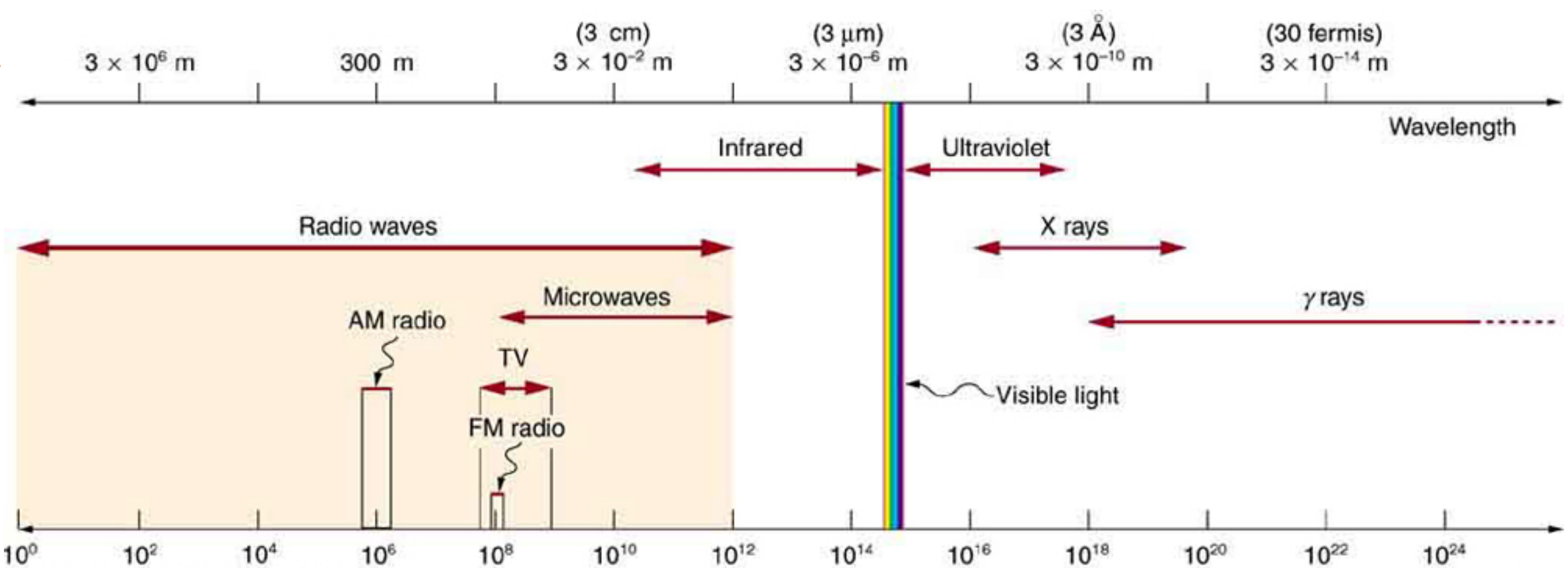


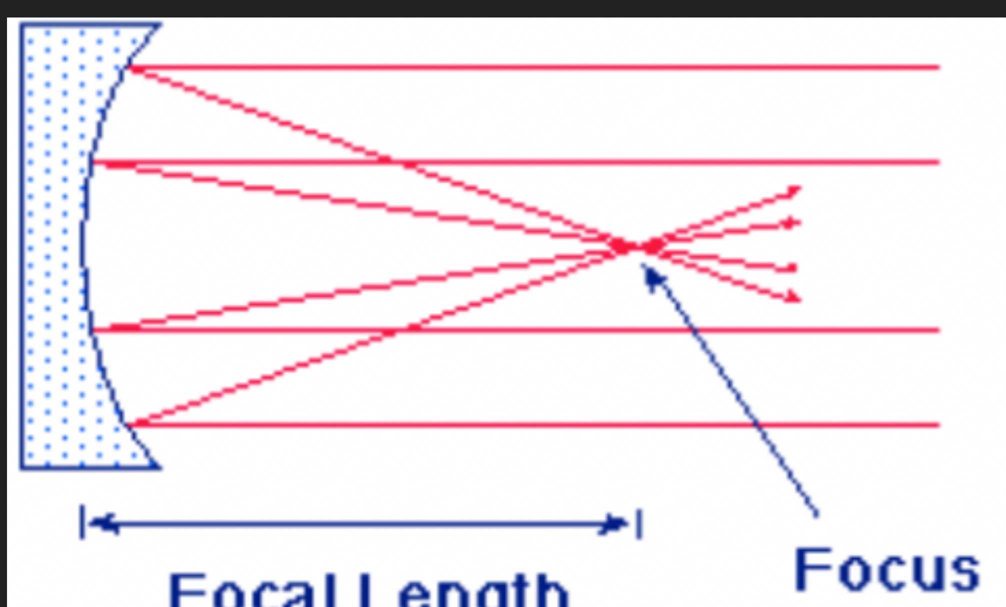
METHOD/MEASUREMENT USED DICTATES HOW TO THINK OF LIGHT

- **Photons:** particles with energy $E = hf$
  - h is Plank's constant ($6.62607015 \times 10^{-34}$Js) and f is the wave frequency
  - Important for calculating detector efficiencies, considering scattering processes, beamsplitting, blackbody radiation
- **Waves:** with interference, vibrate charges in a substance have a polarization.
  - Important for diffraction limit, seeing, gratings, diffraction spikes, polarization effects in scattering
- **Rays**–representing the path of photons (or equivalently wavefront normals)
  - Important for thinking about optics, image quality, aberrations, etc. (not as fundamental as particle and waves, but useful analytically/geometrically)

GEOMETRICAL VS WAVE OPTICS

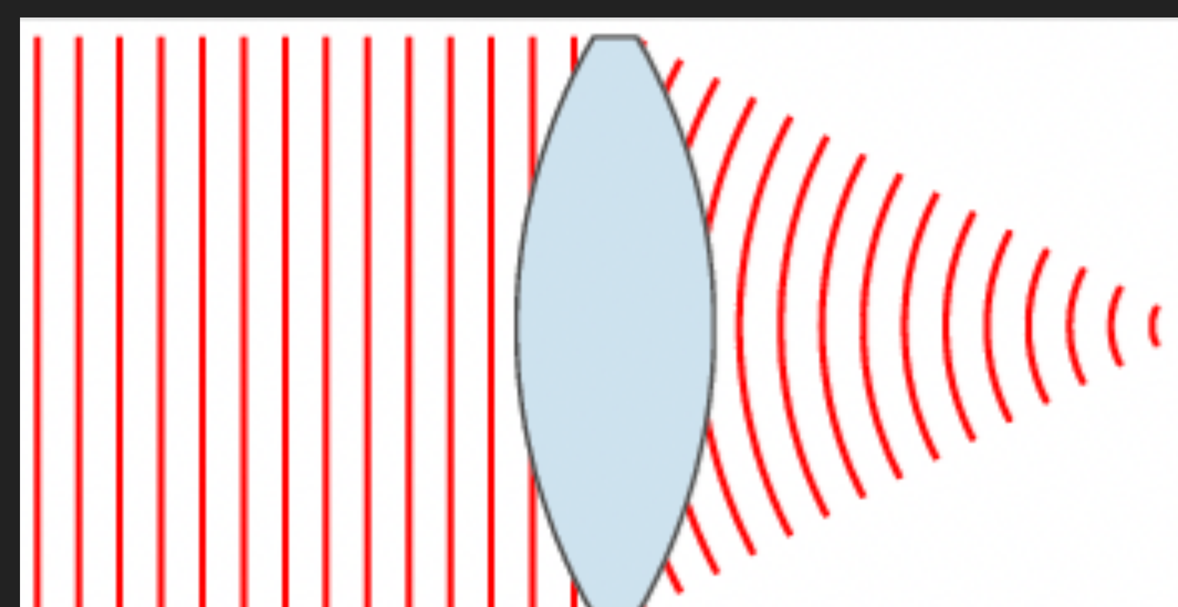
Geometrical/Ray optics

- Traces rays that do not interact with each other
- Geometry-based (law of reflection)
- Intersection of individual rays defines a focal plane and produces an image



Wave optics

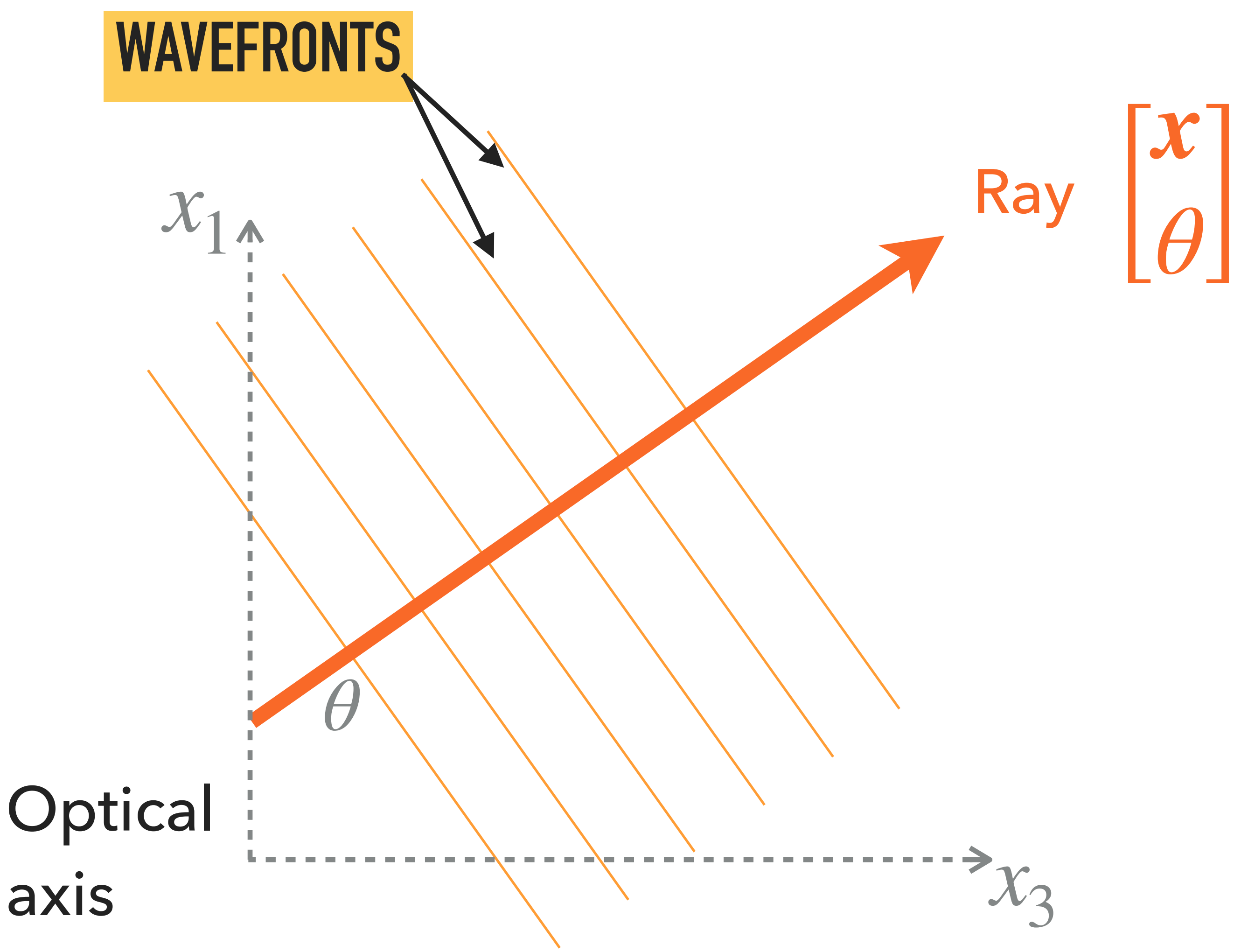
- Wave nature of light
- Uses wavefronts (rather than rays)
- Wave propagation and interference determine illumination
- Interference, diffraction, polarization properties described under this model



WORKING WITH RAYS

GEOMETRIC OPTICS APPROXIMATION

- **Rays move in straight lines through homogenous media**
  - **Can bend in inhomogeneous media**

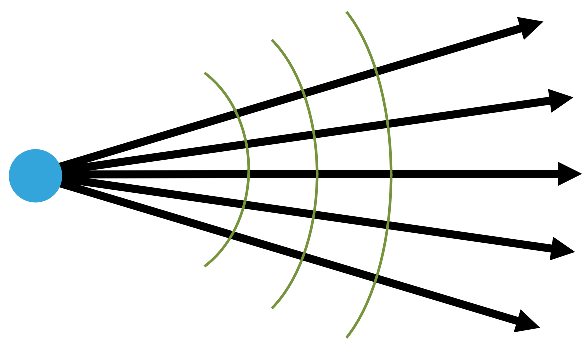


Rays are parameterized by position x and angle of propagation θ

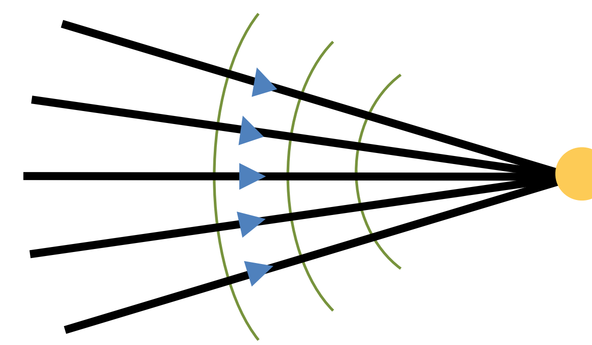
DEFINITIONS

DIVERGING SPHERICAL WAVE

Point object
Point source



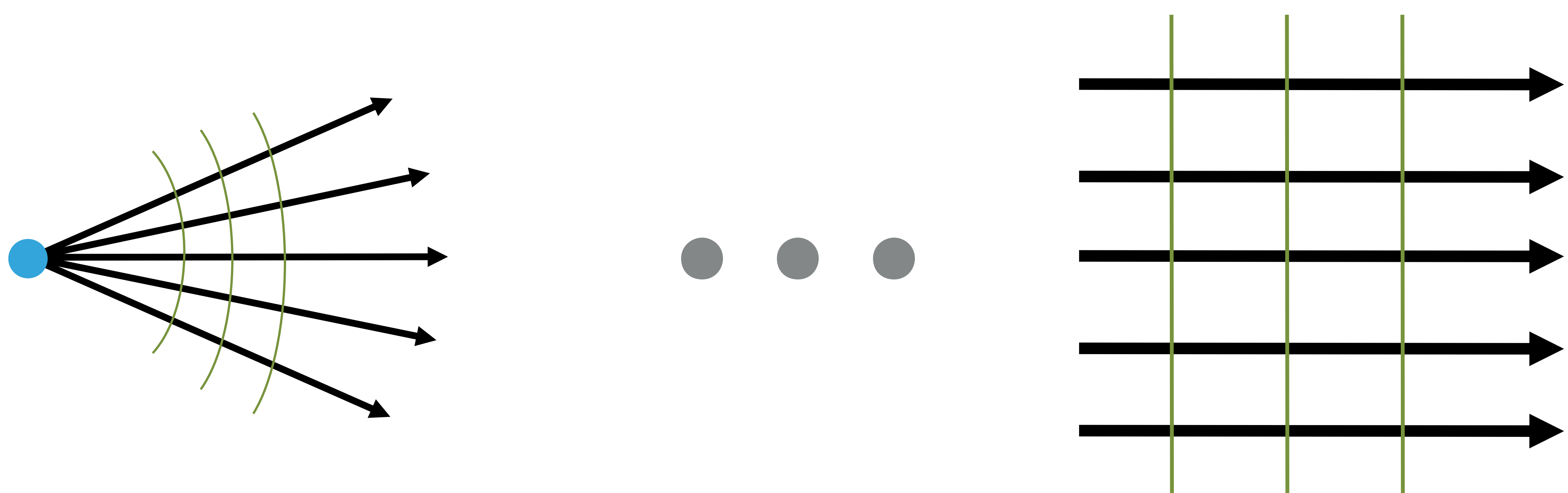
CONVERGING SPHERICAL WAVE



Point image

Wavefronts are perpendicular to rays
Rays travel from left to right

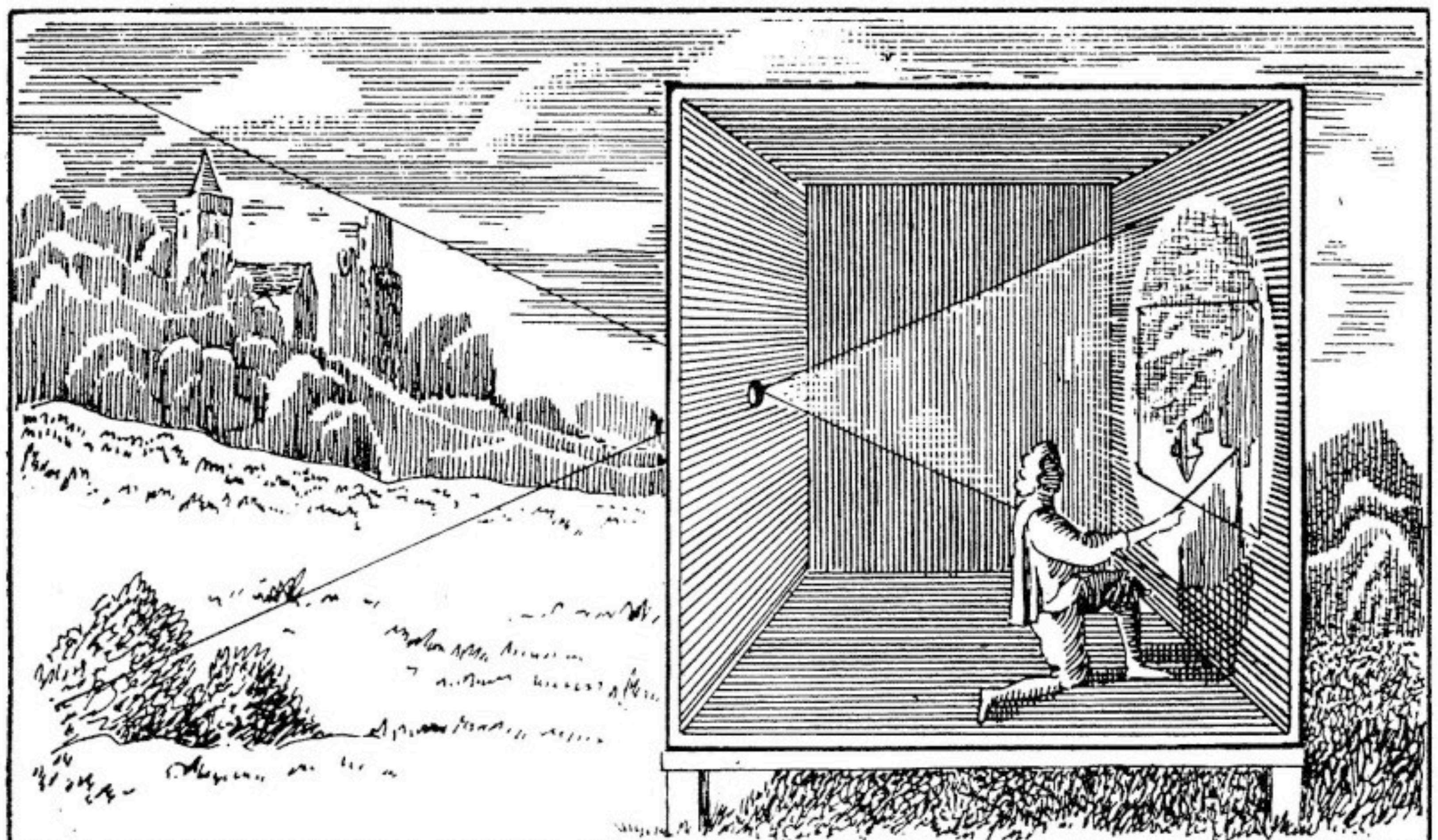
Point object or source at infinity



Spherical wave at infinity becomes a **plane wave**

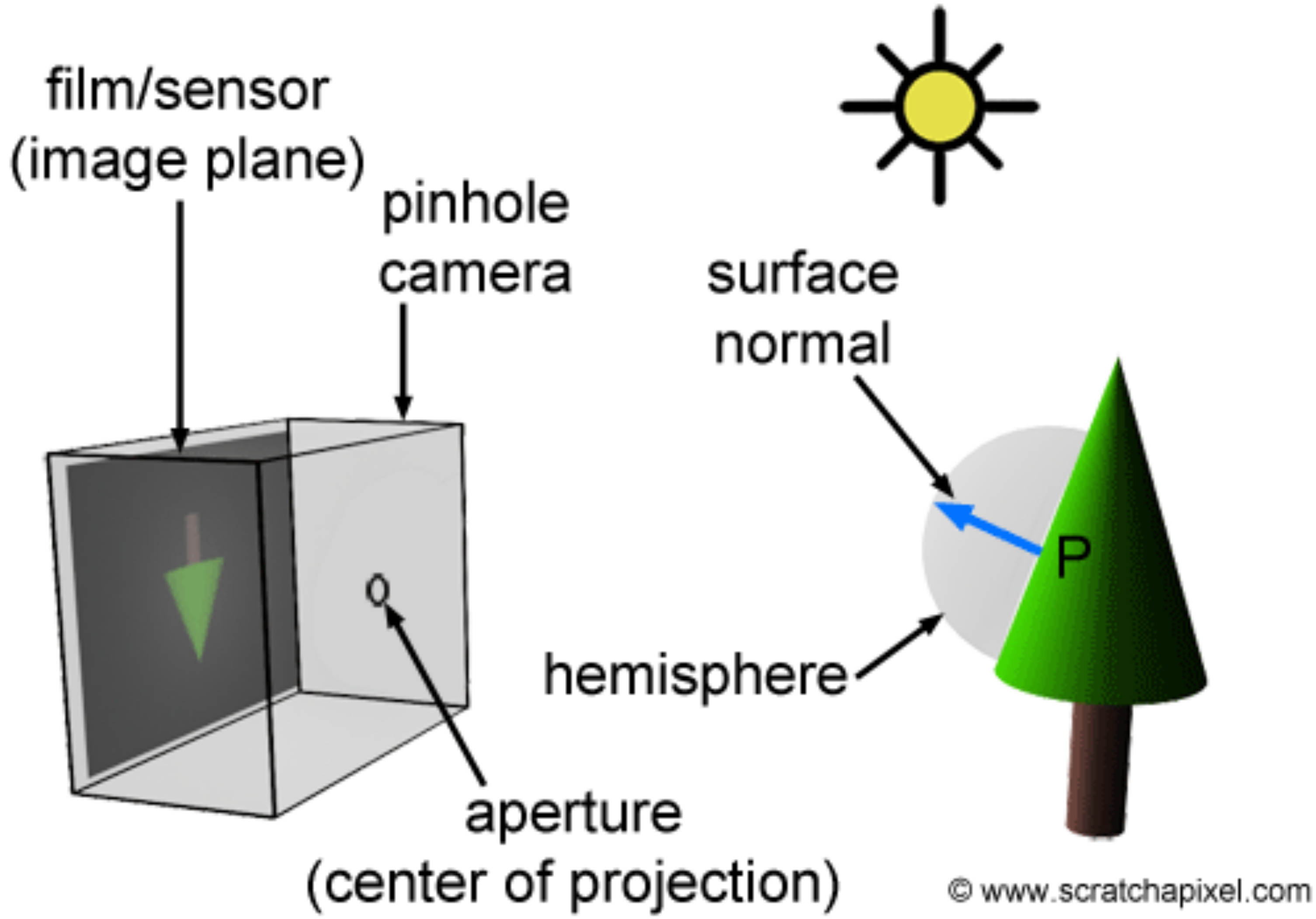
PINHOLE CAMERA
CAMERA OBSCURA

- Imaging explained with rays

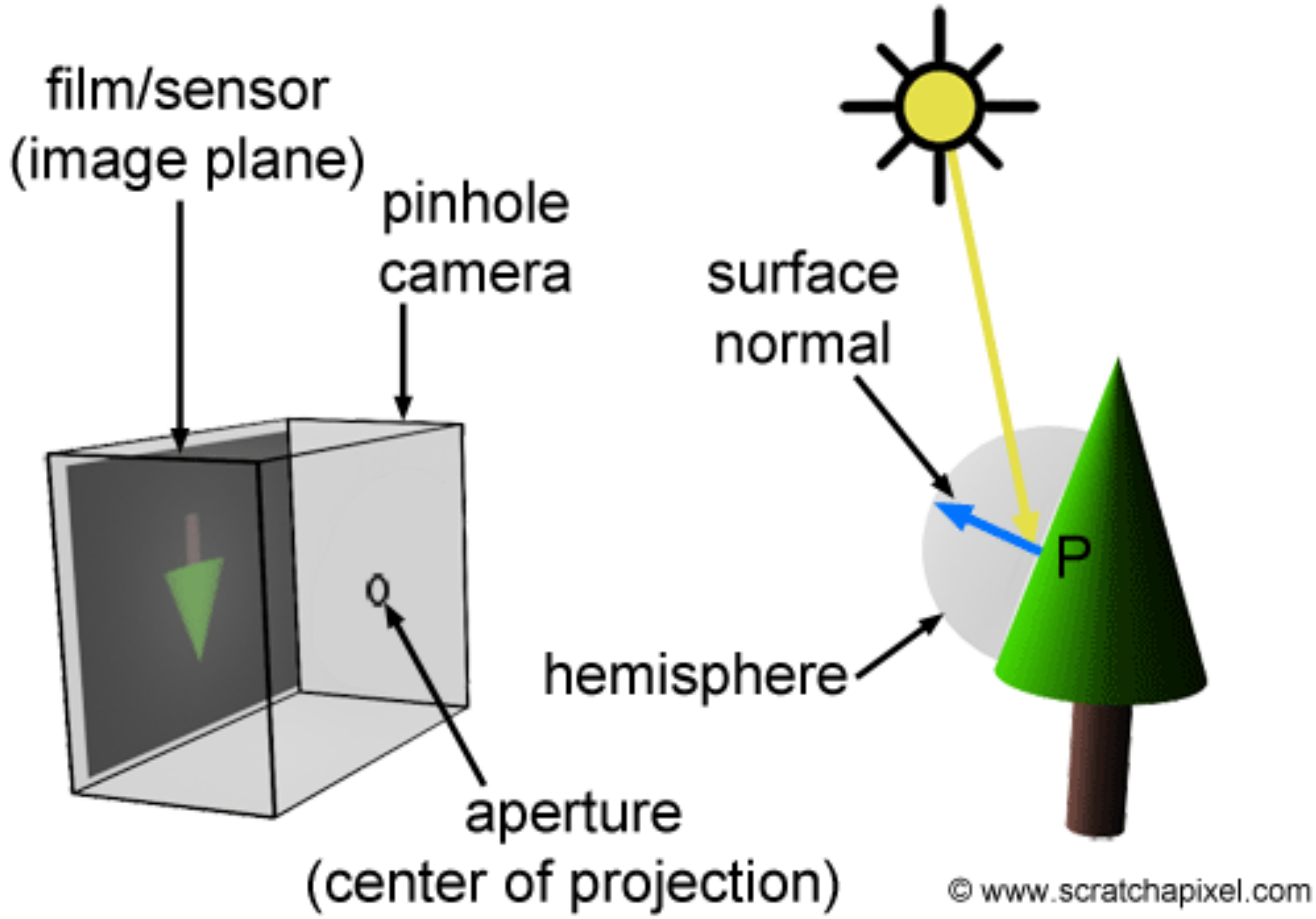


Mo-Ti (Chinese Philosopher, 470 BC – 390 BC)

HOW DOES IT WORK?
IDEAL PINHOLE

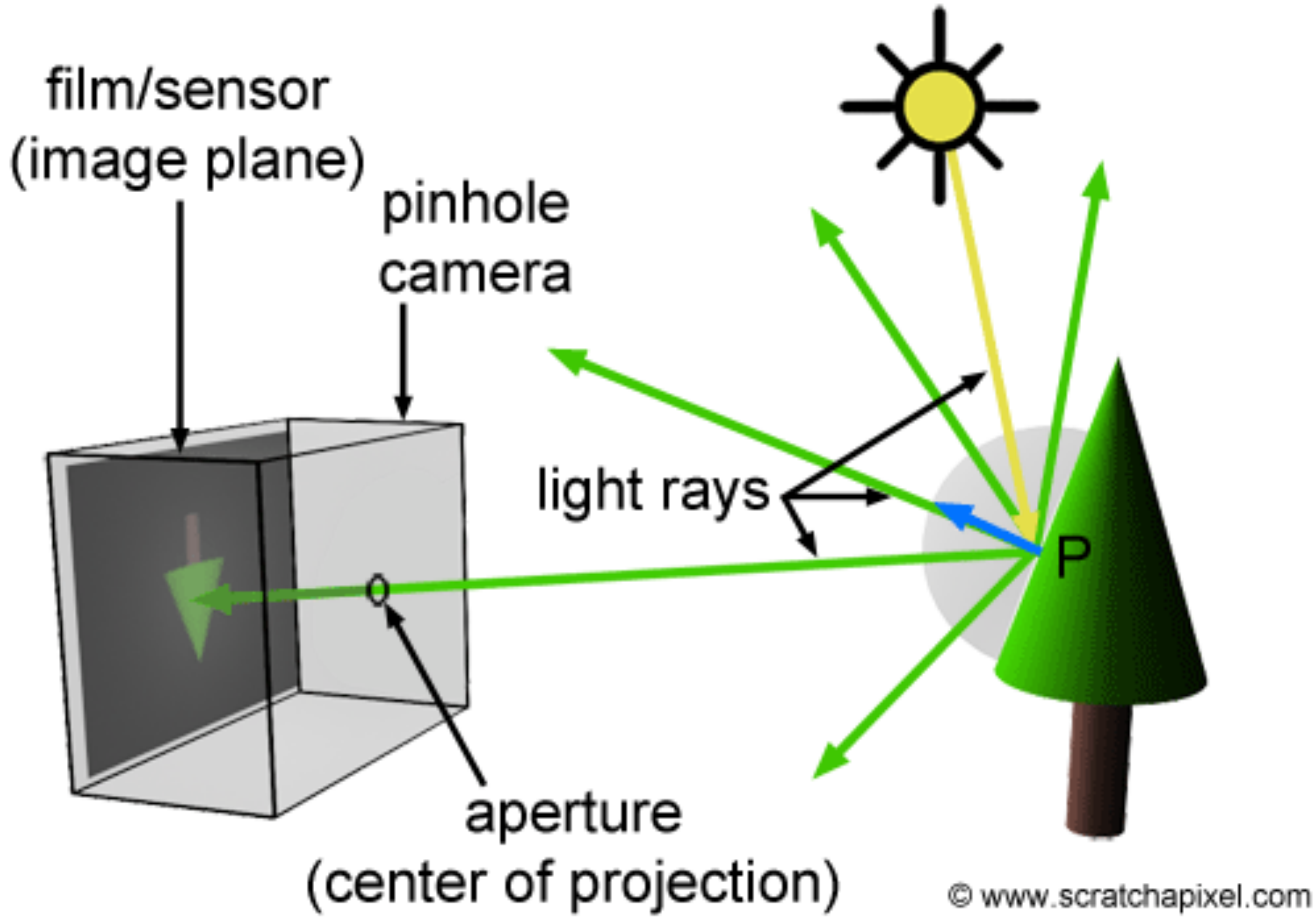


HOW DOES IT WORK?
IDEAL PINHOLE



- Light from source (sunlight) reaches object and reflects off of the object

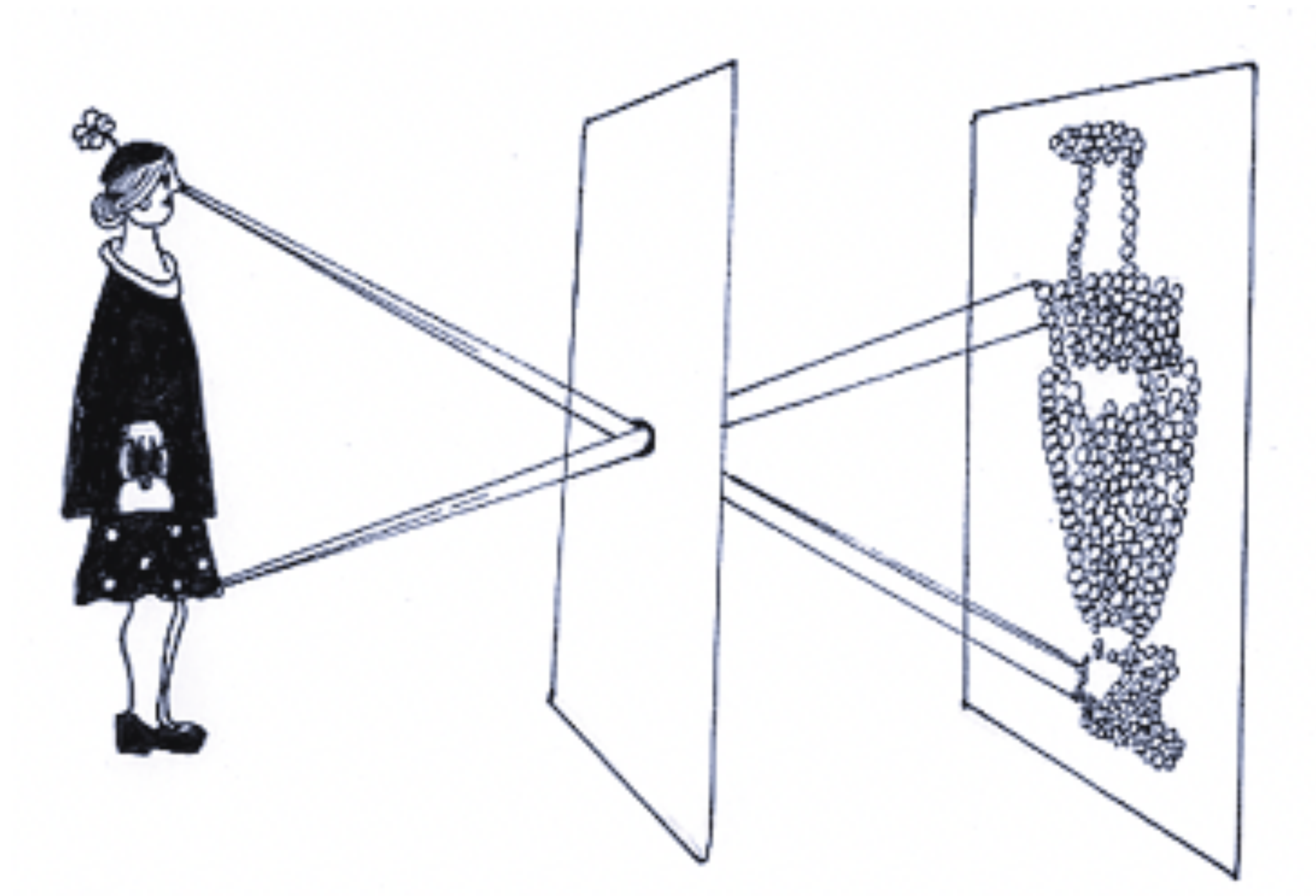
HOW DOES IT WORK?
IDEAL PINHOLE



- The reflected light at each point P, spreads spherically (– a point source at point P)
- Only rays traveling in certain direction can enter the pinhole
- Thus, light from each point P, reaches a unique point in the image plane

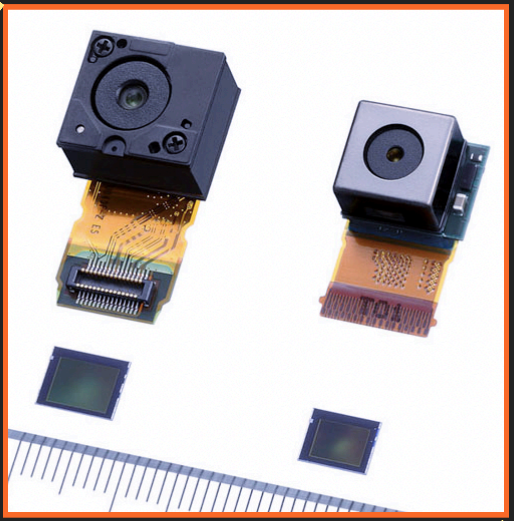
HOW DOES IT WORK?

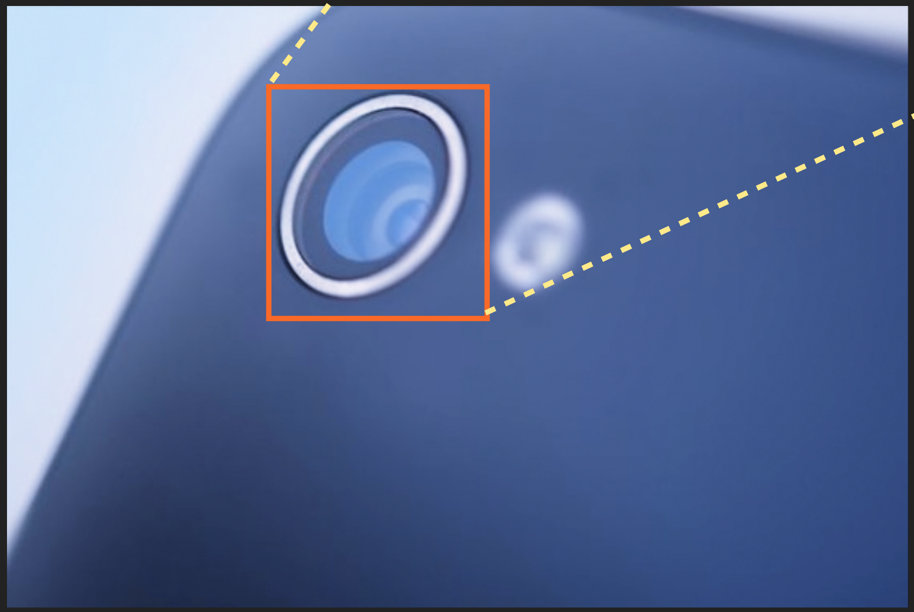
BIGGER PINHOLE?



- More light is collected - **Good** (less noisy)
- However image is blurry - **Bad** (sharp edges become blurred out)

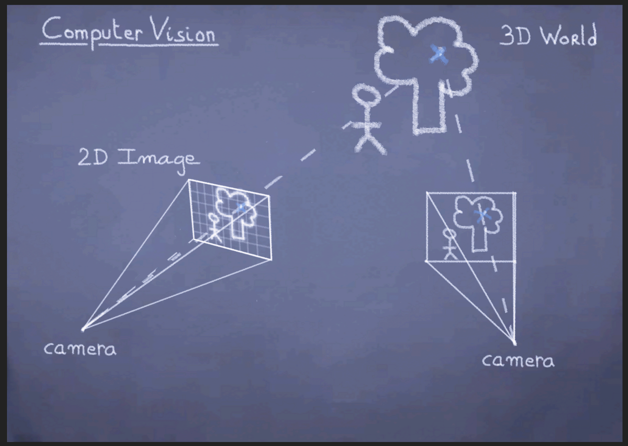
Model for common cameras:
Cellphone, surveillance cameras, etc.

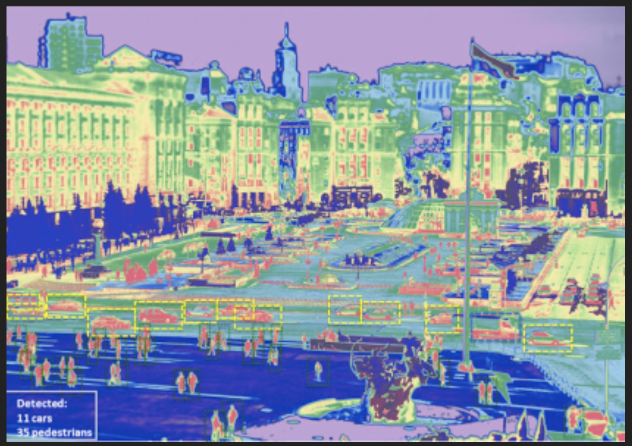




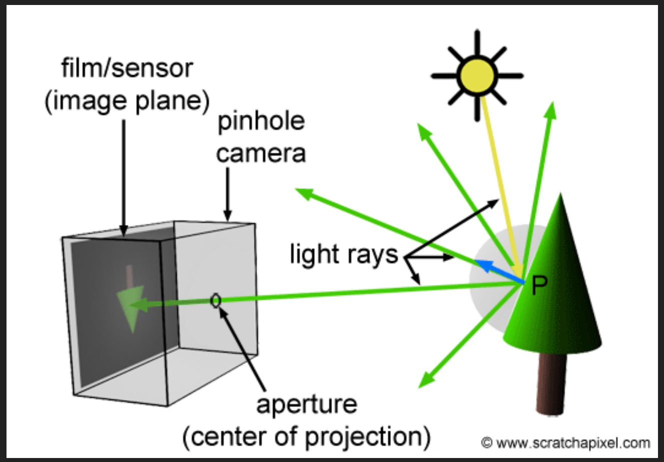
Computer graphics: rendering

Computer vision

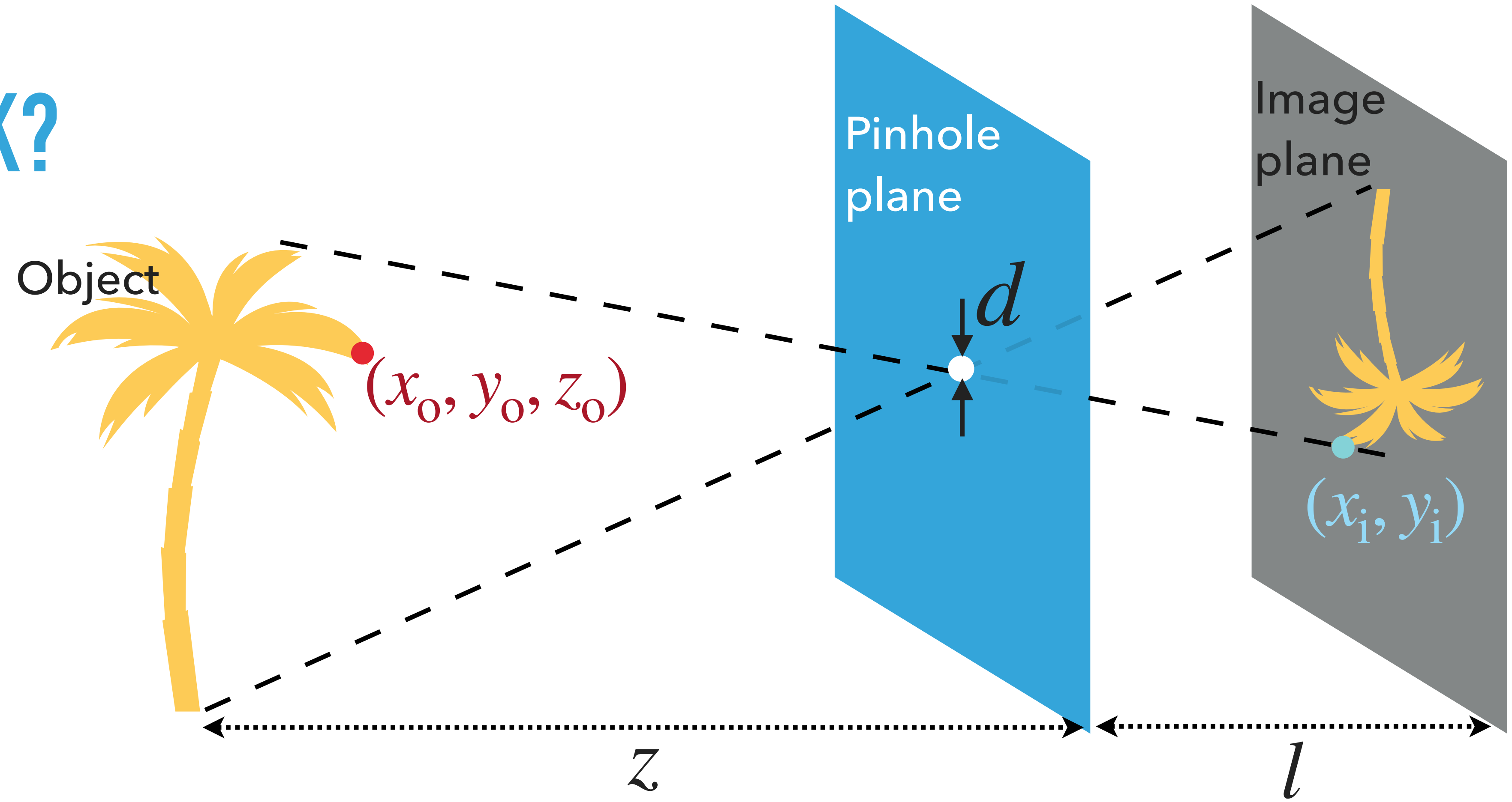




PINHOLE APPLICATIONS



HOW DOES IT WORK?
BIGGER PINHOLE?



$$\boxed{h(x_i, y_i) = \operatorname{circ}\left(\frac{x_i + (l/z_o)x_o}{d + (ld/z_o)}, \frac{y_i + (l/z_o)y_o}{d + (ld/z_o)}\right)}$$

$$\operatorname{circ}(x, y) = \begin{cases} 1 & \sqrt{x^2 + y^2} \leq 0.5 \\ 0 & \text{otherwise} \end{cases}$$

- **Forward model:** a **convolution** in (x, y) and **projection** in z:

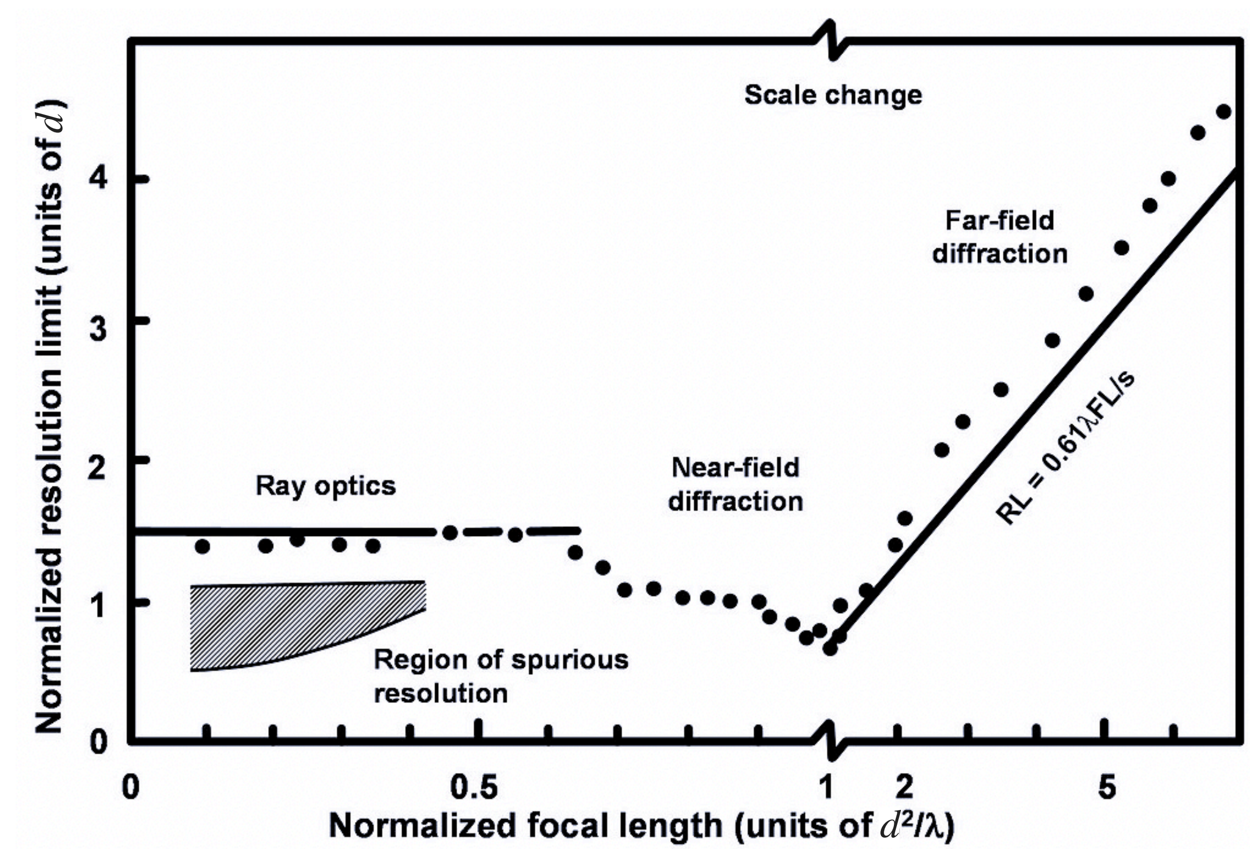
$$\iiint_{(x_o, y_o, z_o)} f(x_o, y_o, z_o) \operatorname{circ}\left(\frac{x_i + (l/z_o)x_o}{d + (ld/z_o)}, \frac{y_i + (l/z_o)y_o}{d + (ld/z_o)}\right) \mathrm{d}x_o \mathrm{d}y_o \mathrm{d}z_o$$

OPTIMAL PINHOLE SIZE

- Smaller pinhole improves resolution, but decreases energy
- Resolution also depends on diffraction:

$$\Delta x = d + \frac{ld}{z_o} + \frac{l\lambda}{d}$$

- Optimal pinhole size
  $d_{opt} = \sqrt{l z_o \lambda / (z_o + l)}$
- **Resolution:**
  $\Delta x_{min} \approx 2\sqrt{l\lambda}$, when $z_o \gg l$



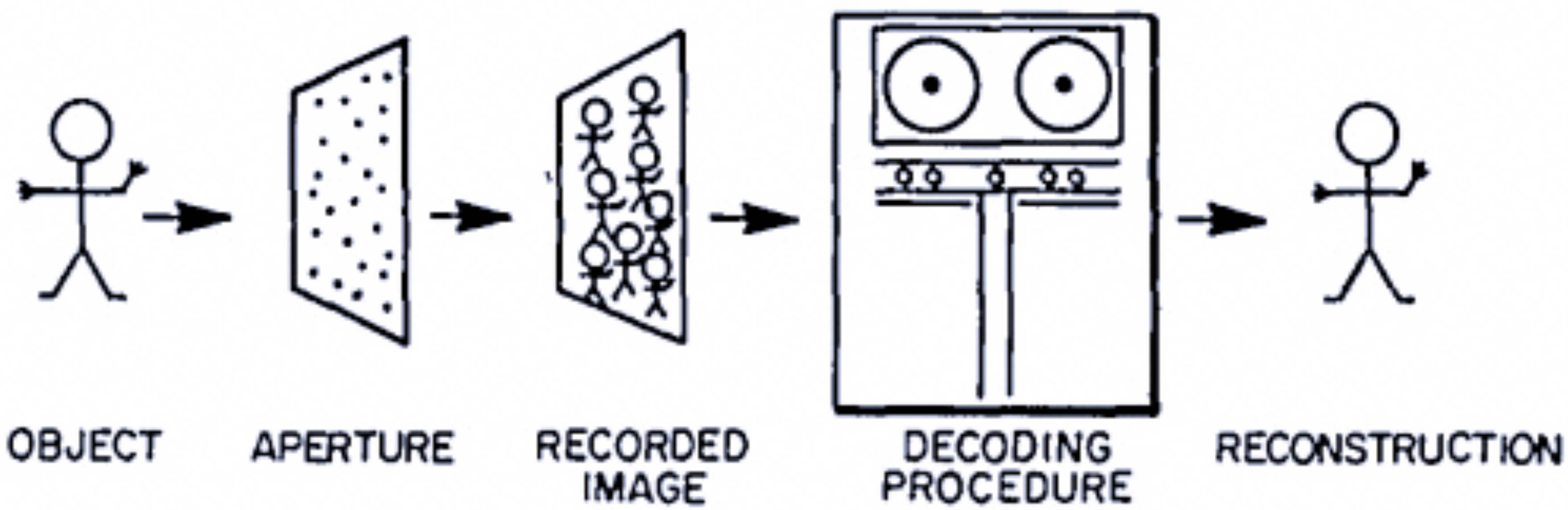
ACCIDENTAL PINHOLE CAMERA
APPLICATION OF PINHOLE

Windows open

Windows almost completely shut (pinhole)

http://people.csail.mit.edu/torralba/research/accidentalcameras/

CODED-APERTURE IMAGING
APPLICATION OF PINHOLE

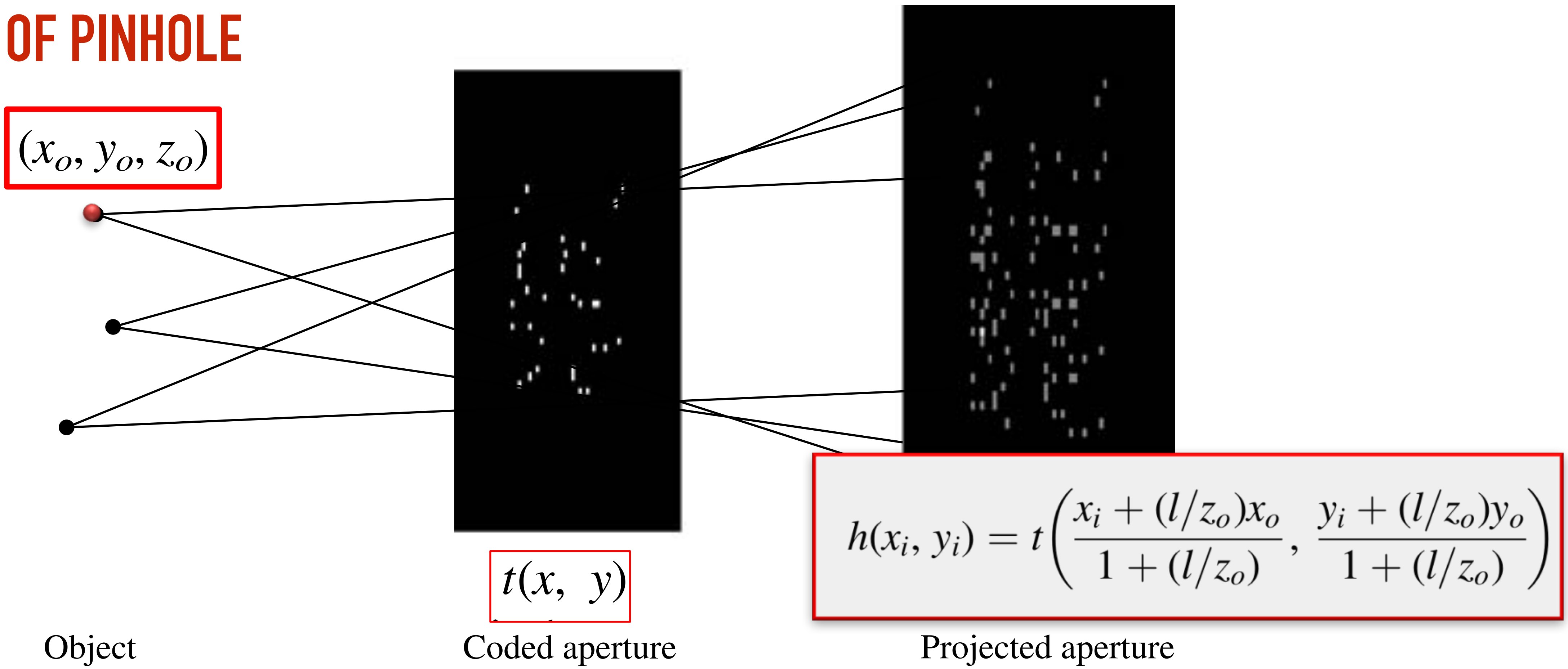


- Widely used in x-rays, gamma rays
  - High-energy rays cannot be focused with lenses or mirrors
  - Blocking light with a known pattern, a coded "shadow" is cast upon a plane
  - Computational reconstruction of original object

E. E. Fenimore and T. M. Cannon, "Coded aperture imaging with uniformly redundant arrays," Appl. Opt. 17, 337-347 (1978)

CODED-APERTURE IMAGING

APPLICATION OF PINHOLE



- **Forward model:** a **convolution** in (x, y) and **projection** in z:

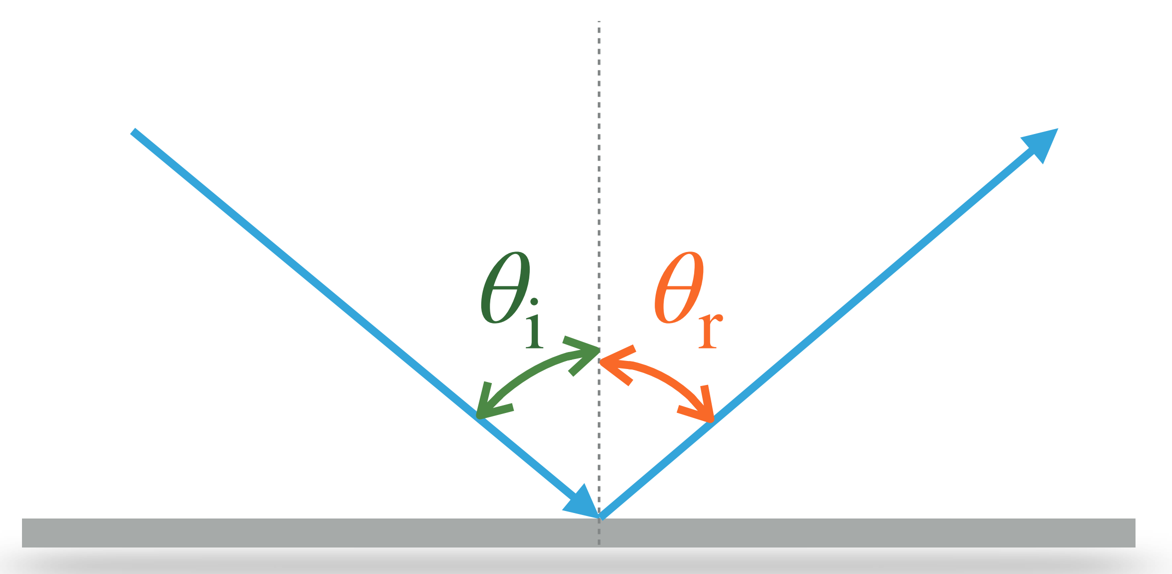
$$\iiint_{(x_o, y_o, z_o)} f(x_o, y_o, z_o)\; t\left(\frac{x_i + (l/z_o)x_o}{d + (ld/z_o)}, \frac{y_i + (l/z_o)y_o}{d + (ld/z_o)}\right) \mathrm{d}x_o \mathrm{d}y_o \mathrm{d}z_o$$

CHANGING DIRECTION OF RAYS

REFLECTION

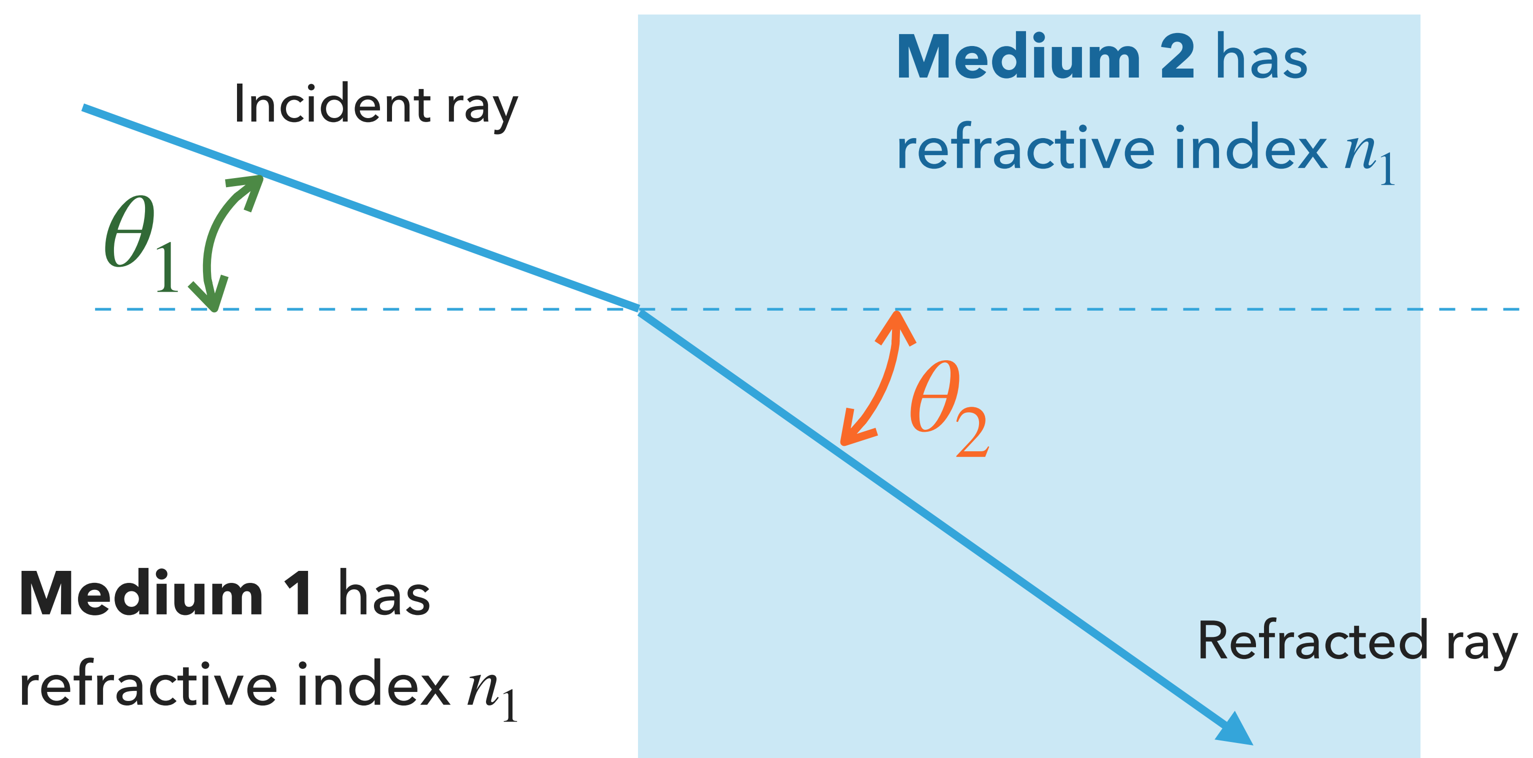
- Reflections off flat (mirror/mirror-like) surface follow the **law of reflection:**

$$\theta_i = \theta_r$$



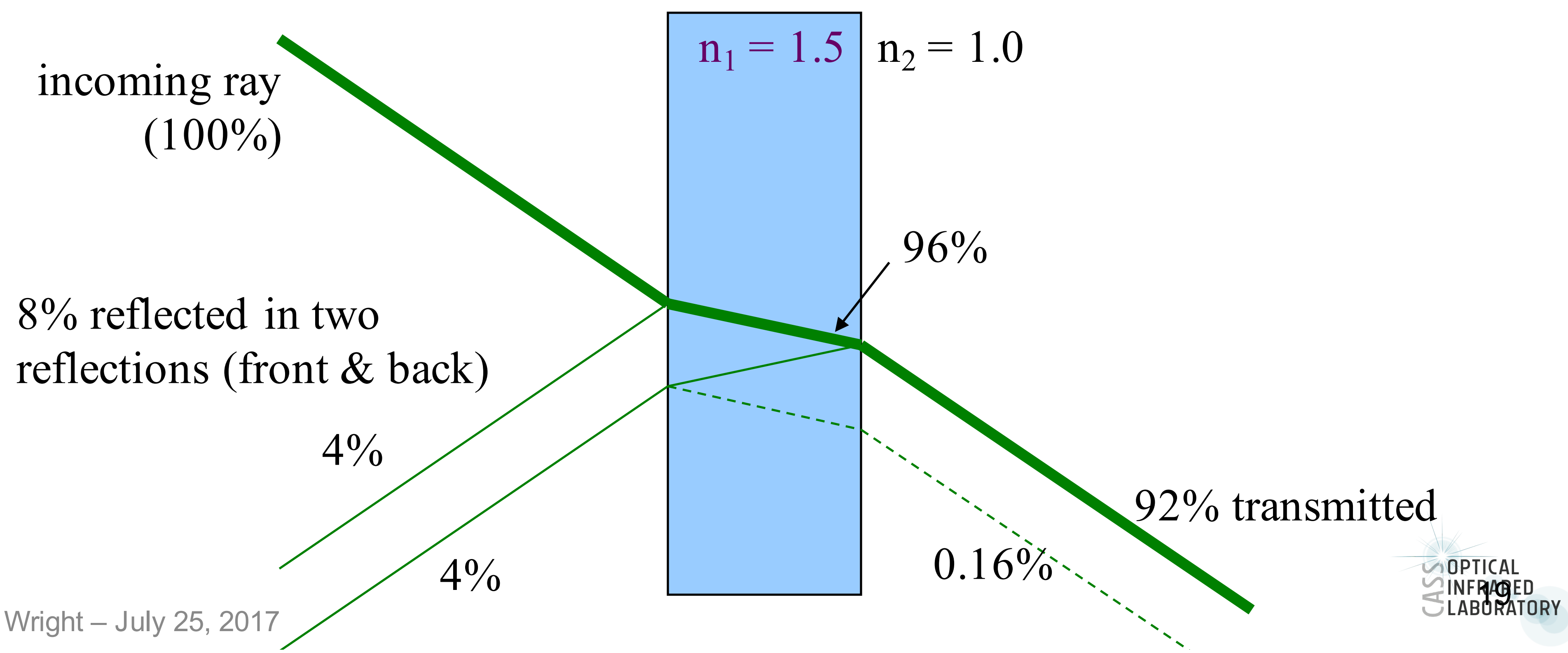
TRANSMISSION REFRACTION, DISPERSION

- Light is **refracted** when it reaches an interface between two media
- **Snell's law:** $n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$
- **Dispersion** is observed if the index of refraction is wavelength dependent $n(\lambda)$
  - Different wavelengths of light are refracted at different amounts



OPTICAL DESIGN

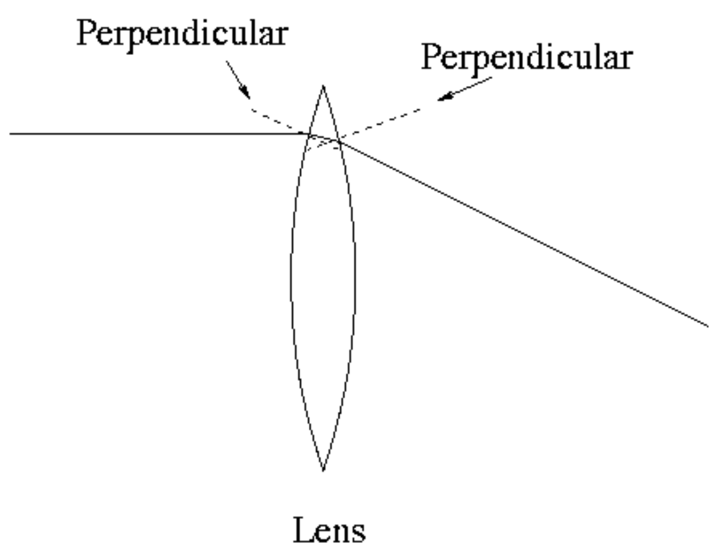
- Let's consider a thick piece of glass ($n = 1.5$), and the associated light paths:
  - Reflection fraction $R = \left(\dfrac{n_1 - n_2}{n_1 + n_2}\right)^2$
  - using $n_1 = 1.5$, $n_2 = 1.0$ (air), $R = (0.5/2.5)^2 = 0.04 = 4\ \%$



S. Wright – July 25, 2017

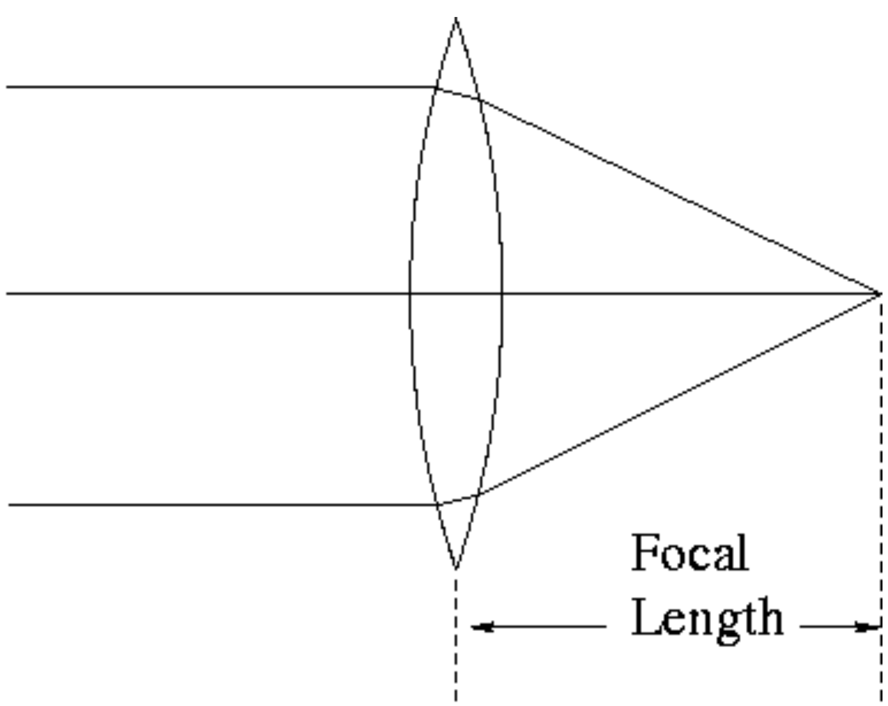
FOCAL IMAGING

GETTING FOCUSSED



- A lens, with front and back curved surfaces, bends light twice, each diverting incoming ray towards centerline.
- Follows laws of refraction at each interface, using *local* surface normal

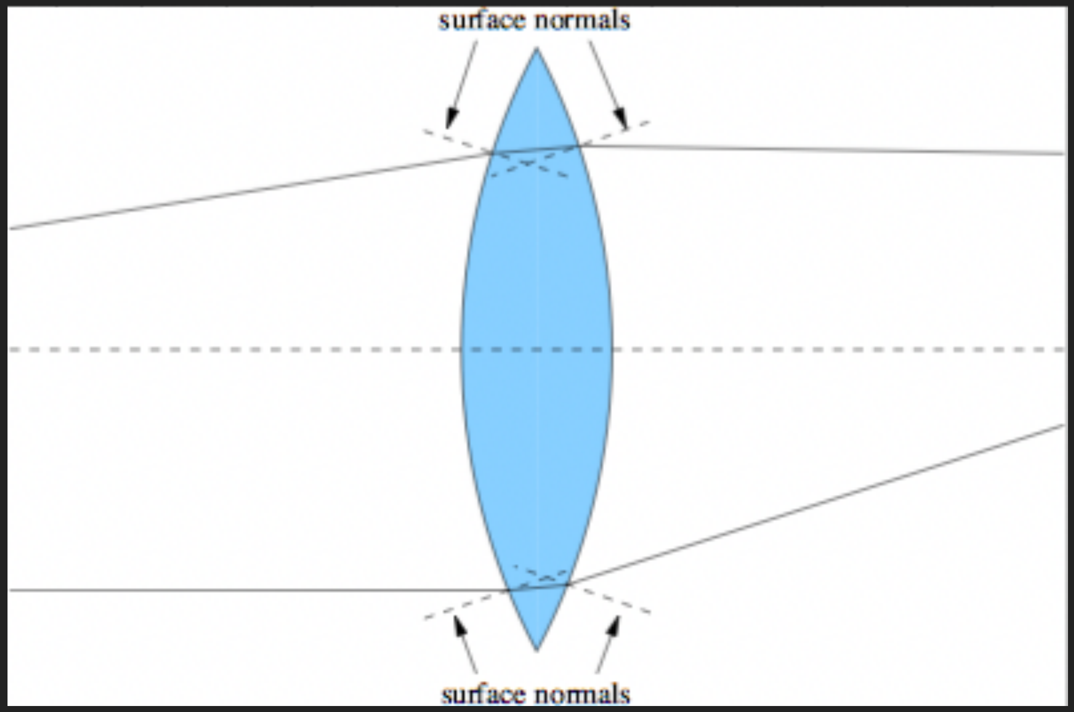
- Parallel rays, coming, for instance from a specific direction (like a distant star) are focused by a convex (positive) lens to a focal point.
  - Placing detector at this point would record an image of the star at a very specific spot on the detector.
  - Lenses map incoming angles into positions in the focal plane.



LENS TYPES

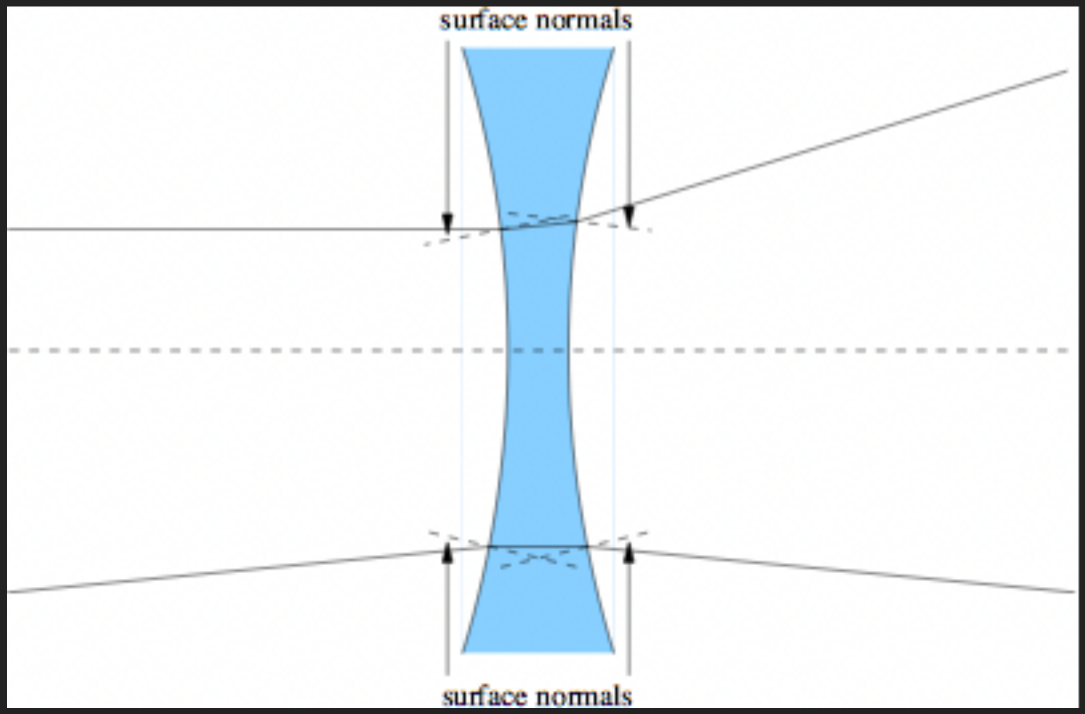
Positive Lenses

- Thicker middle
- Bend rays toward axis
- Forms a real focus



Negative Lenses

- Thinner middle
- Bend rays away from axis
- Form virtual focus



LENSMAKER'S FORMULA

- **Generic lens formula** including radii of curvature (R_1 and R_2) and index of refraction n

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

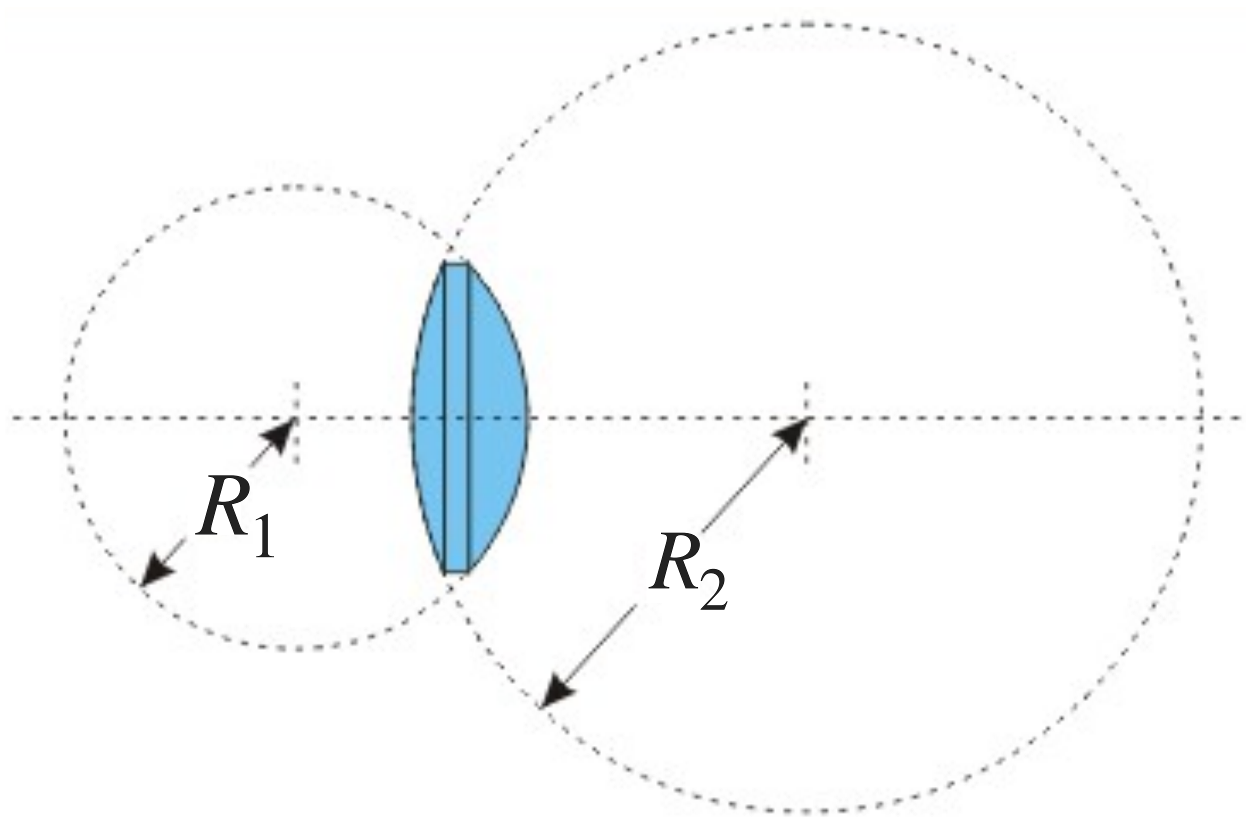
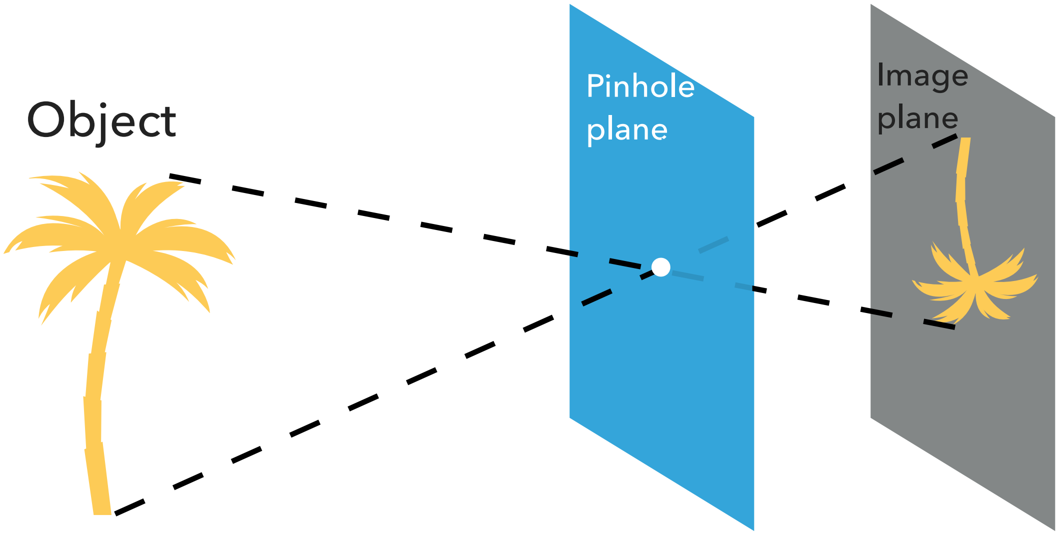


IMAGE GENERATION
PINHOLE VS LENS

- **Pinhole camera**: small hole means that light hitting any particular point on the image plane must have come from a particular direction outside the camera



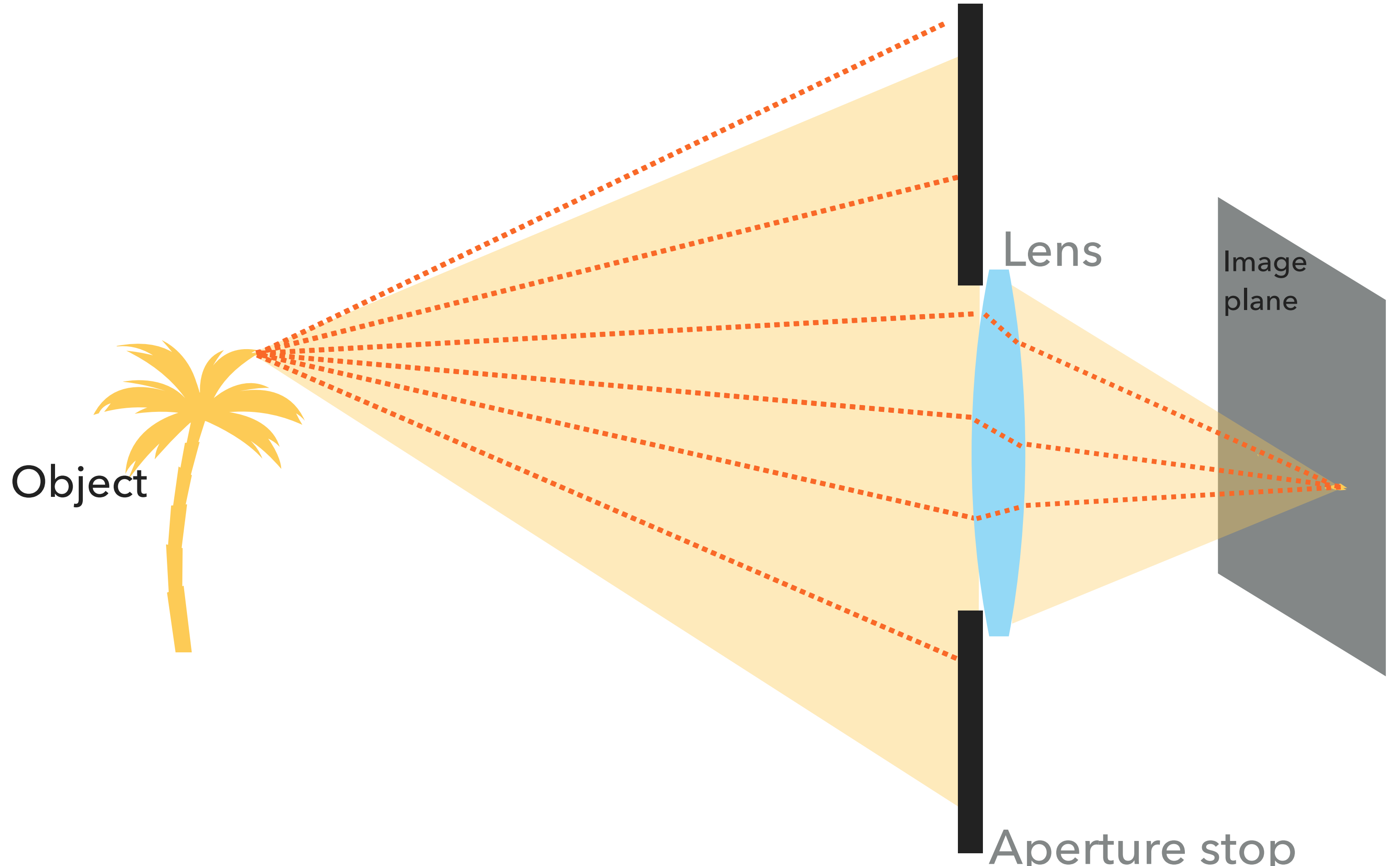


IMAGE GENERATION

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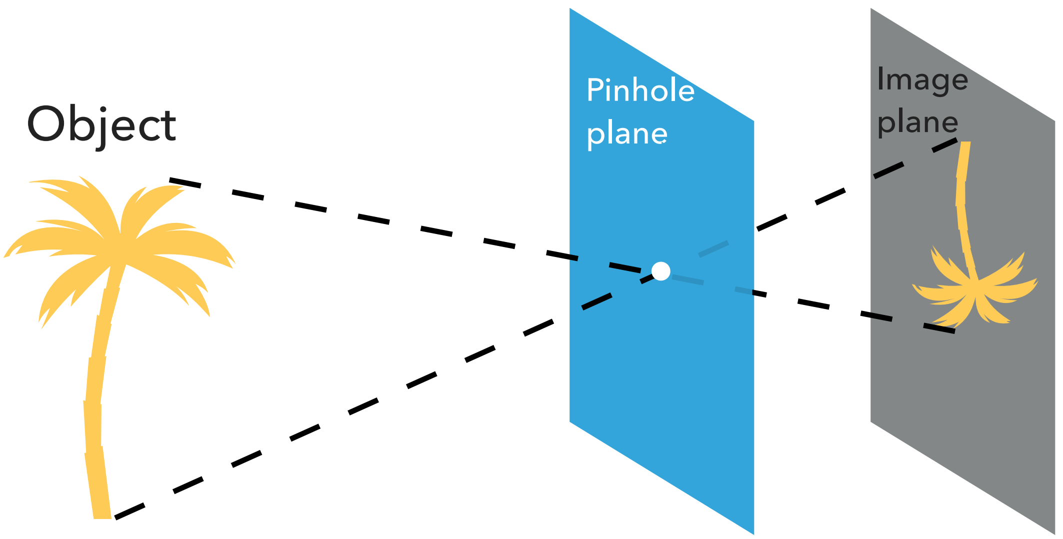


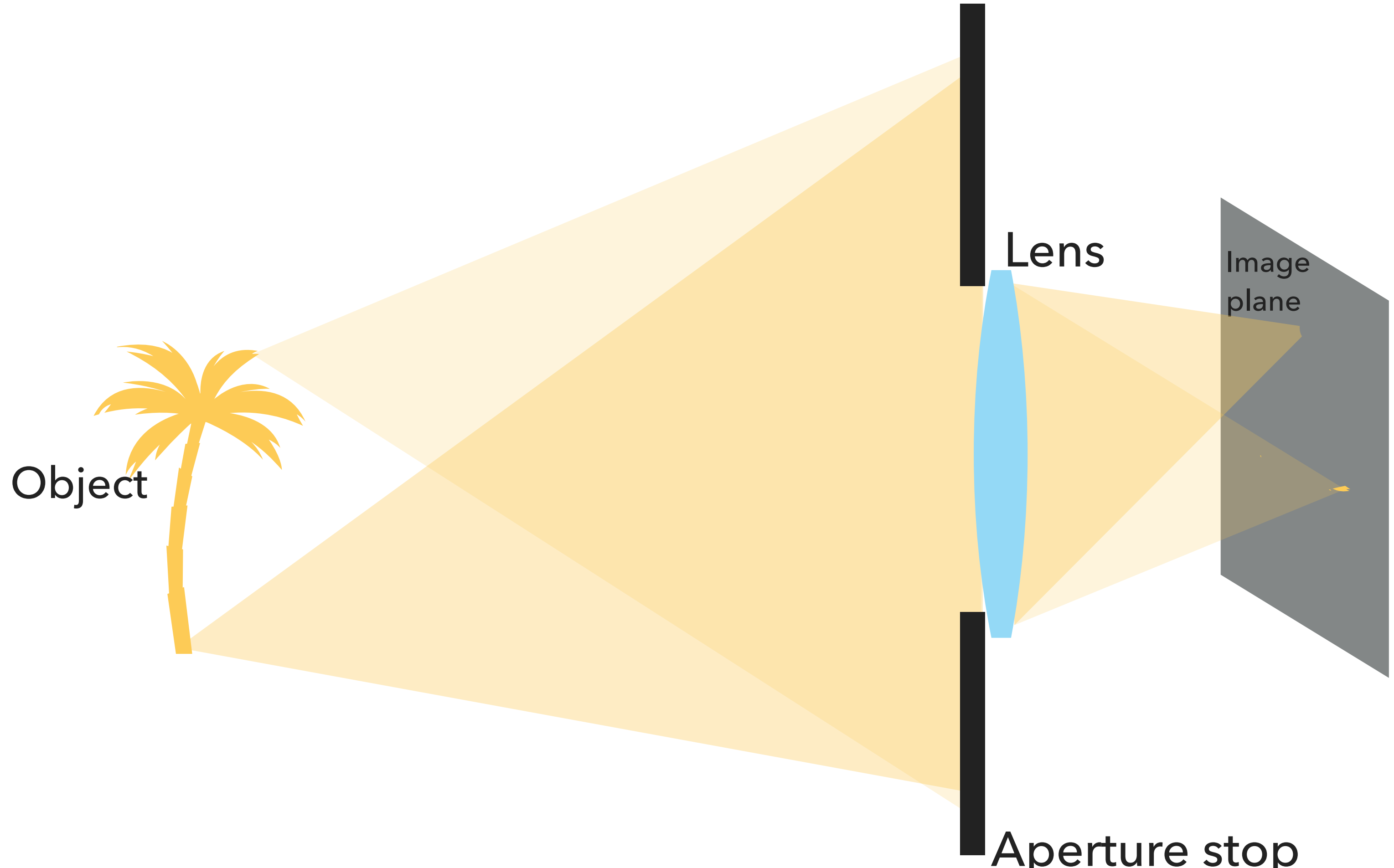
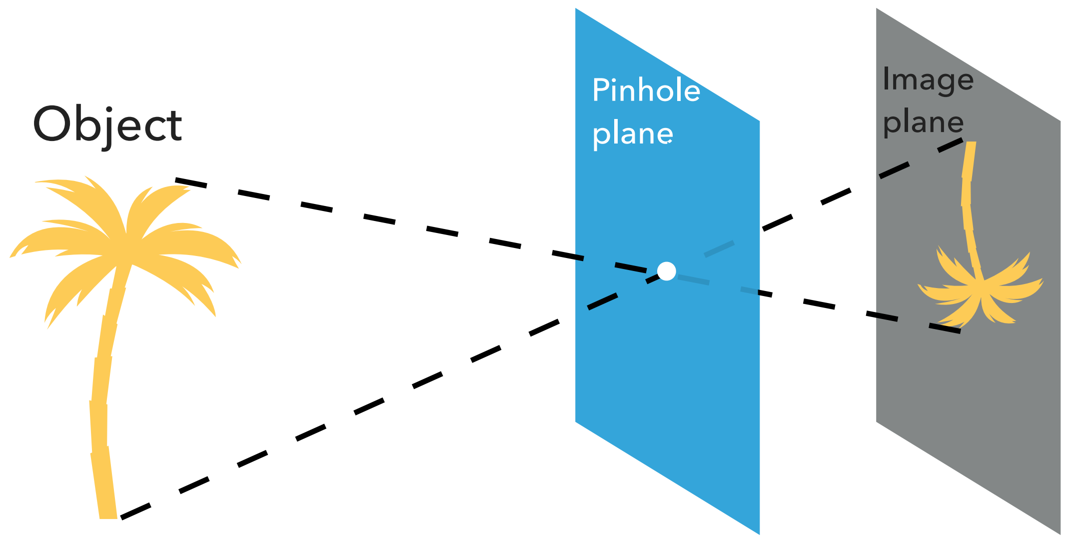


IMAGE GENERATION
PINHOLE VS LENS

- **Pinhole camera**: small hole means that light hitting any particular point on the image plane must have come from a particular direction outside the camera



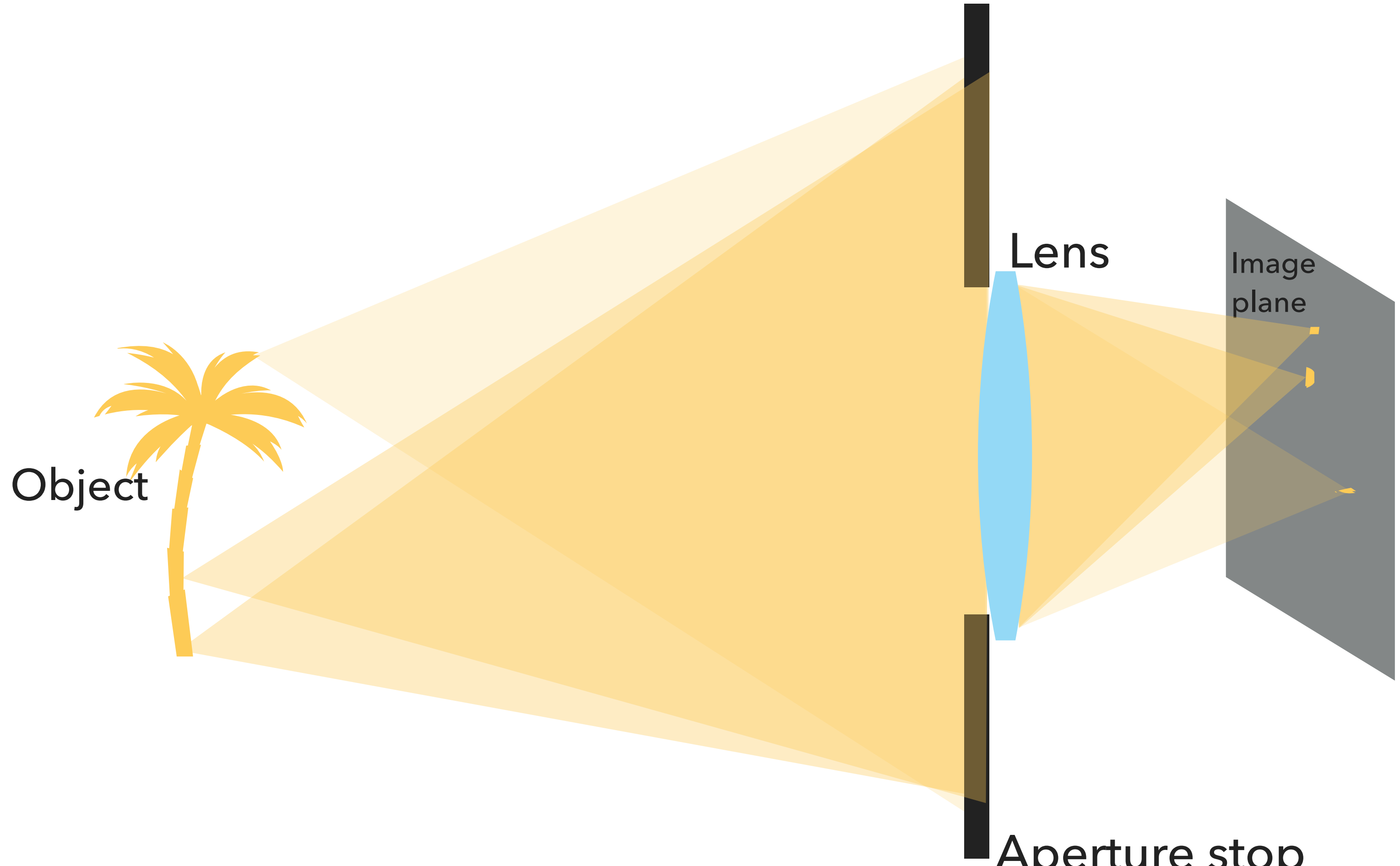


IMAGE GENERATION

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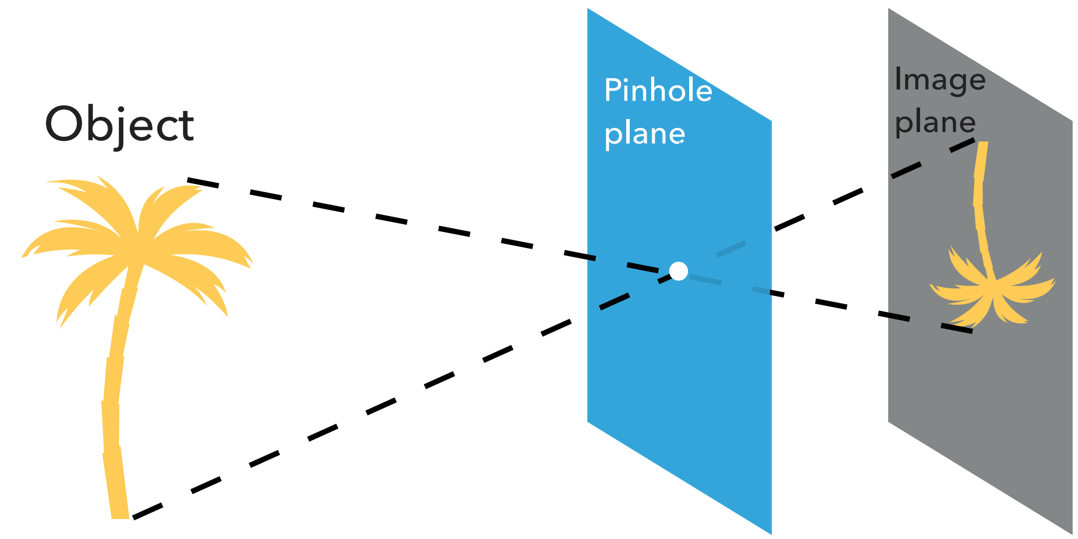


- **Lens-based camera**: any particular point on the image plane corresponds to a particular direction outside the camera.

- **Lenses** have an additional important advantage of collecting more light than a pinhole.

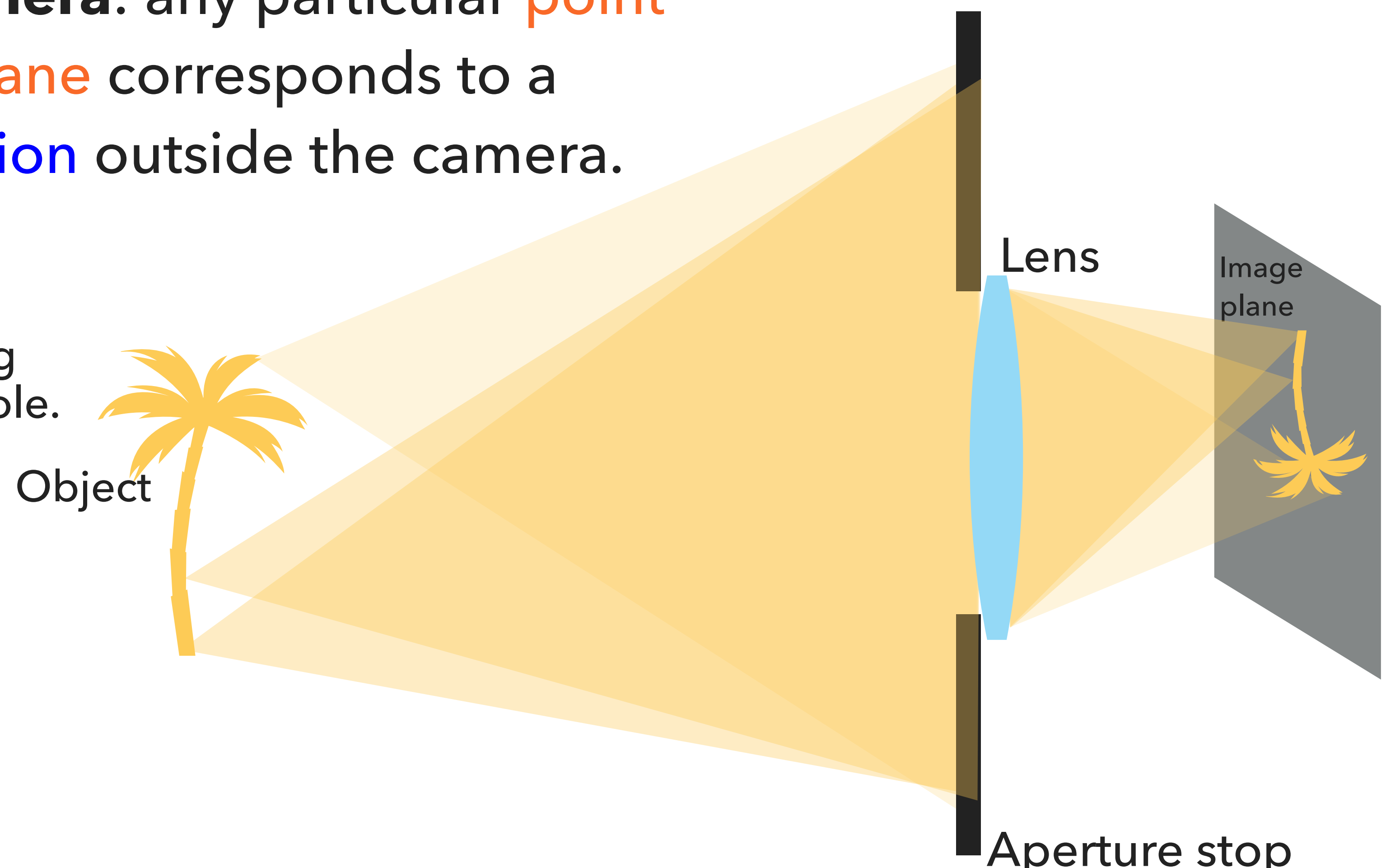
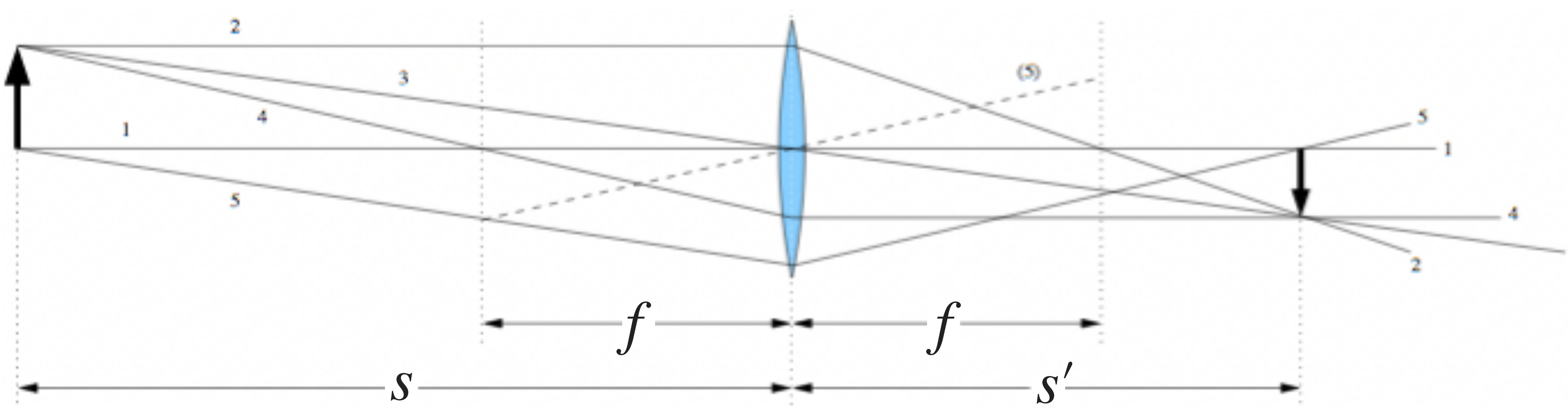


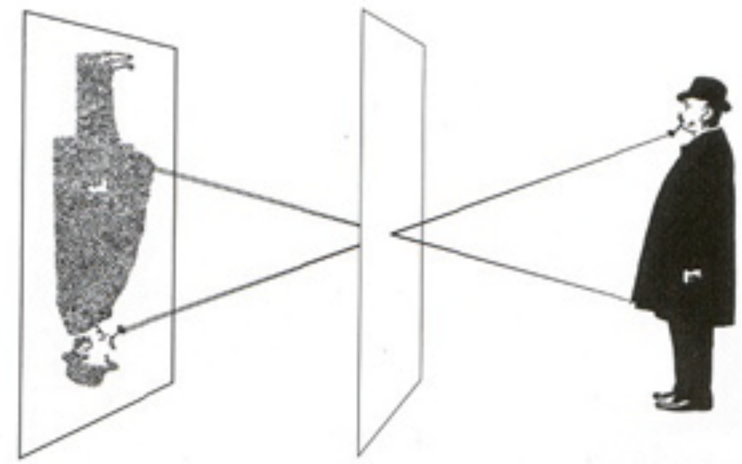
IMAGE FORMATION

- Place arrow (object) on left, trace through image:
  (1) along optical axis (no deflection – optical axis);
  (2) parallel to axis (marginal ray), goes through far focus with optical axis ray;
  (3) through lens center (chief ray);
  (4) through near-side focus, emerges parallel to optical axis;
  (5) arbitrary ray with helper



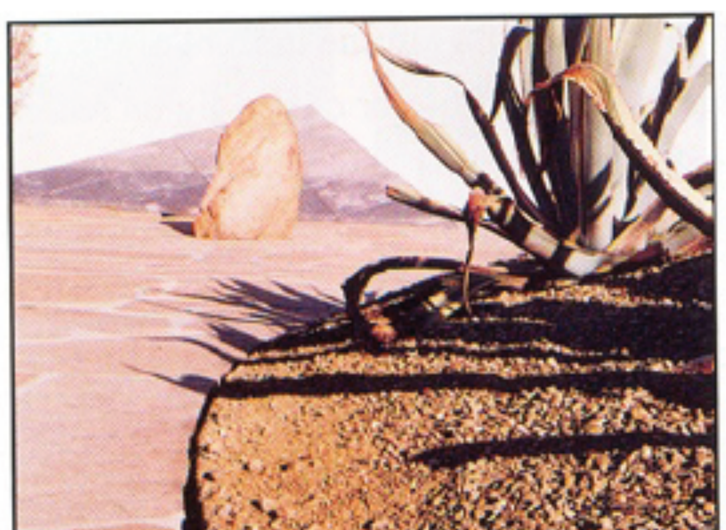
PINHOLE VS LENS IMAGING

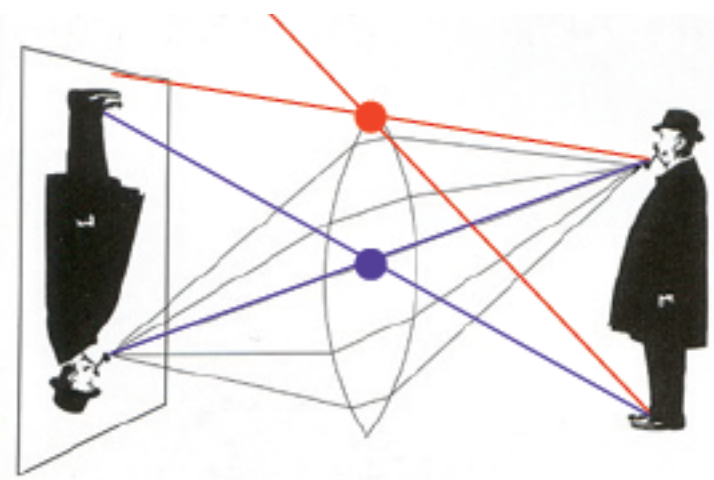
Pinhole camera



To make this picture, the lens of a camera was replaced with a thin metal disk pierced by a tiny pinhole, *equivalent in size to an aperture of f/182. Only a few rays of light from each point on the subject got through the tiny opening, producing a soft but acceptably clear photograph. Because of the small size of the pinhole, the exposure had to be 6 sec long.*

Lens-based camera





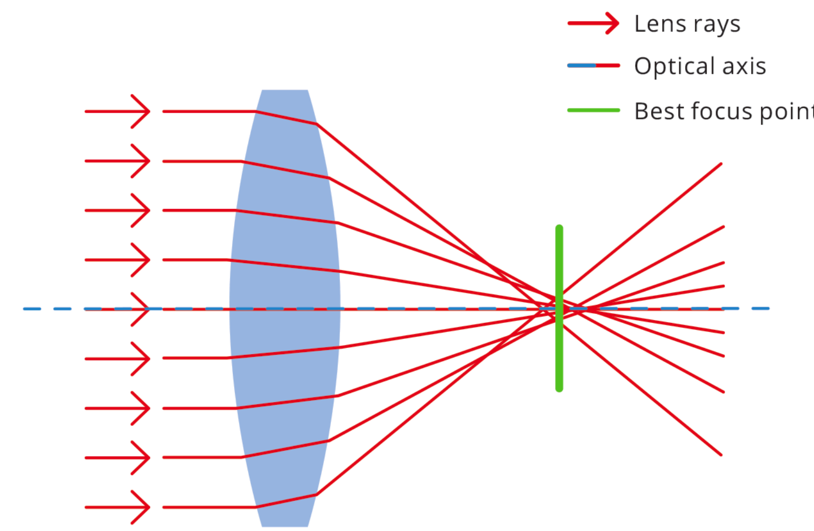
This time, using a simple convex lens with an f/16 aperture, *the scene appeared sharper than the one taken with the smaller pinhole, and the exposure time was much shorter, only 1/100 sec.*

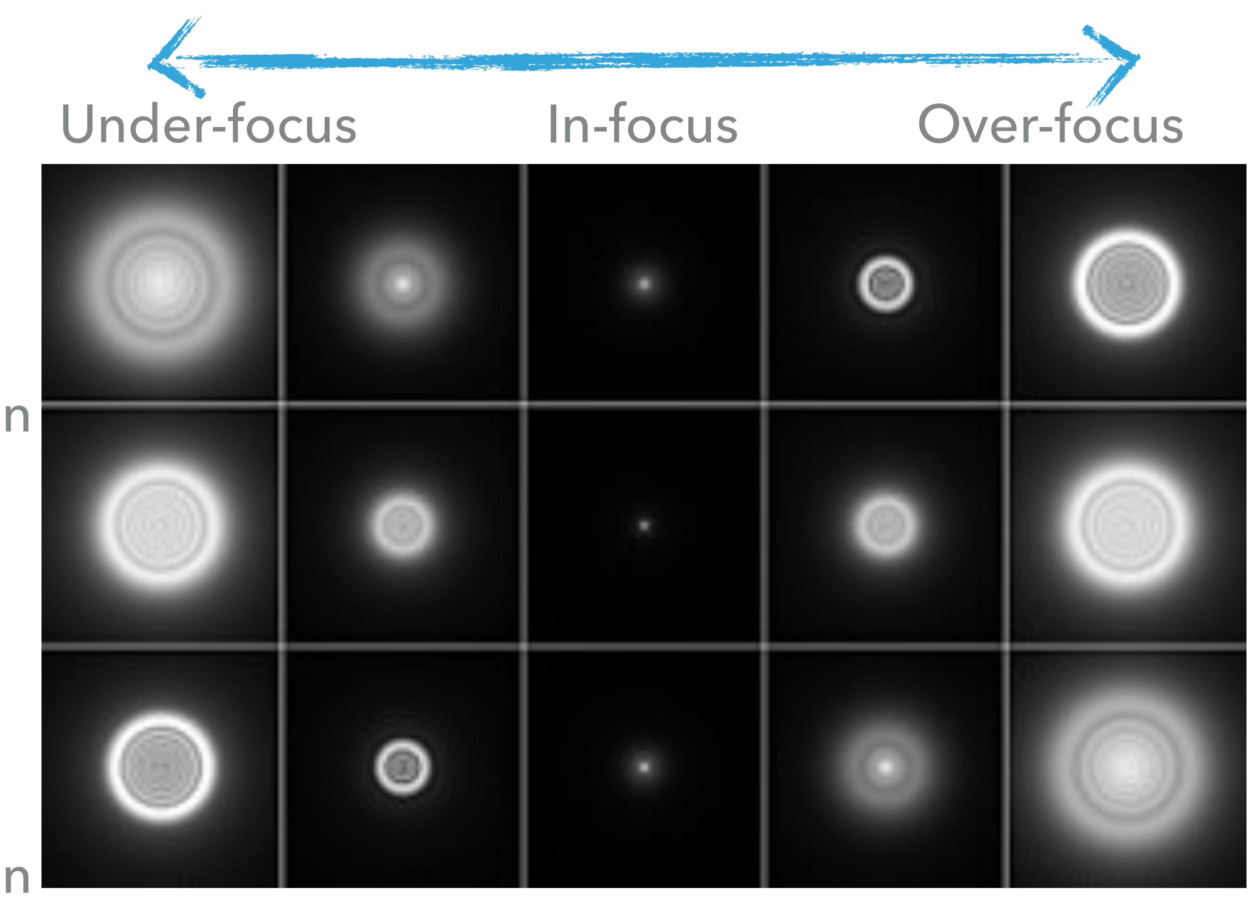
The lens opening was much bigger than the pinhole, letting in far more light, but it focused the rays from each point on the subject precisely so that they were sharp on the film.

From Photography, London *et al.*

ABERRATIONS

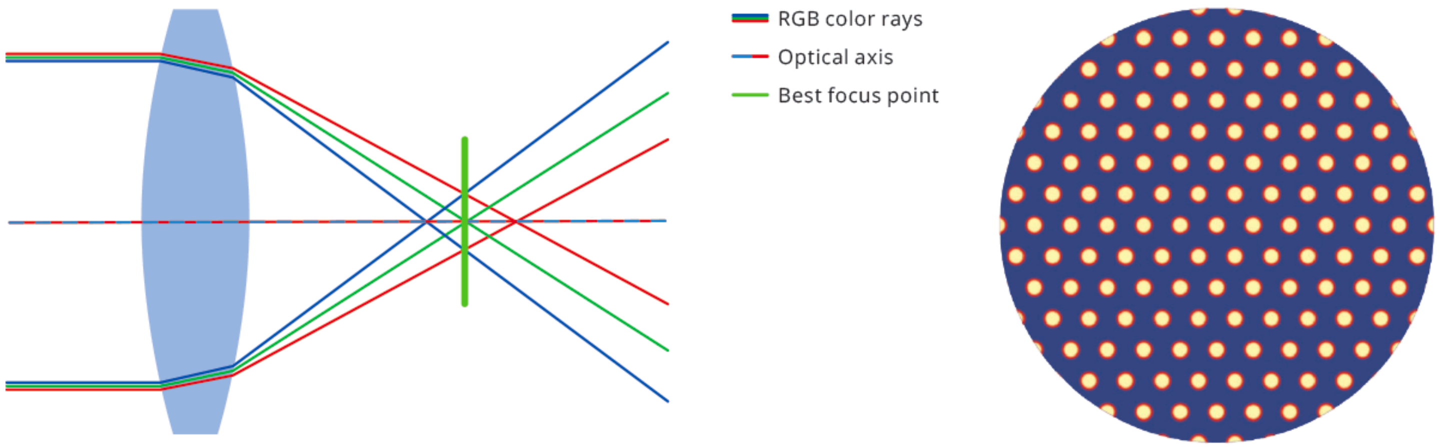
- **Spherical Aberration** -- causes light rays passing through a lens (or reflected from a mirror) at different distances from the optical center to come to focus at different points on the axis. This causes a star to be seen as a blurred disk rather than a sharp point. Most telescopes are designed to eliminate this aberration.



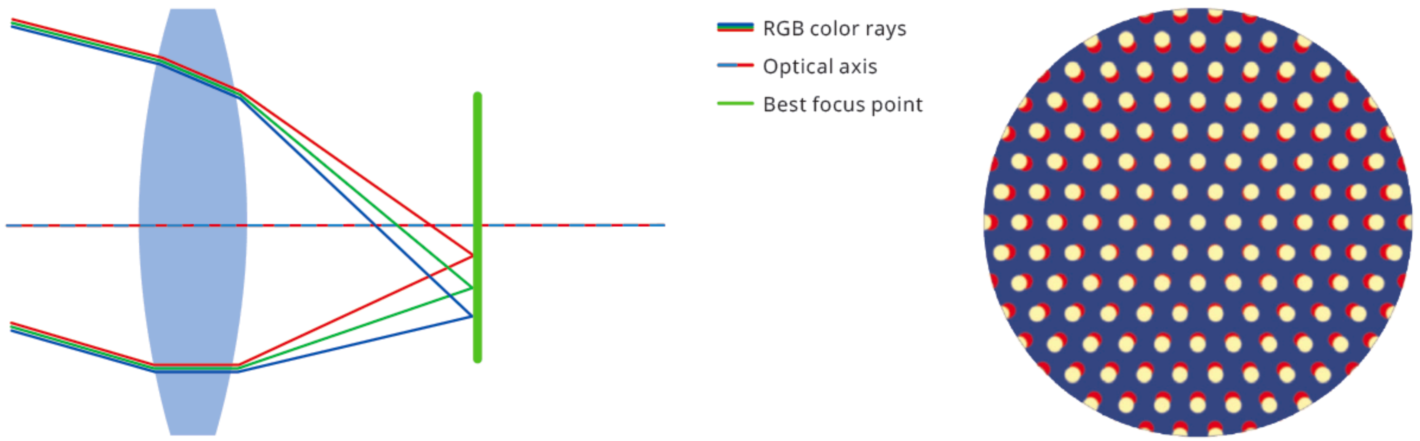


CHROMATIC ABERRATIONS

- **Chromatic Aberration** -- usually associated with objective lenses of refractor telescopes. It is the failure of a lens to bring light of different wavelengths (colors) to a common focus. This results mainly in a faint colored halo (usually violet) around bright stars, the planets and the moon. It also reduces lunar and planetary contrast. It usually shows up more as speed and aperture increase. Achromat doublets in refractors help reduce this aberration and more expensive, sophisticated designs like apochromats and those using fluorite lenses can virtually eliminate it.



Longitudinal/axial chromatic abberation



Lateral/transverse chromatic abberation

FIELD OF VIEW

NUMERICAL APERTURE

DEPTH OF FOCUS

FUNDAMENTAL IMAGING CONSIDERATIONS

FIELD OF VIEW

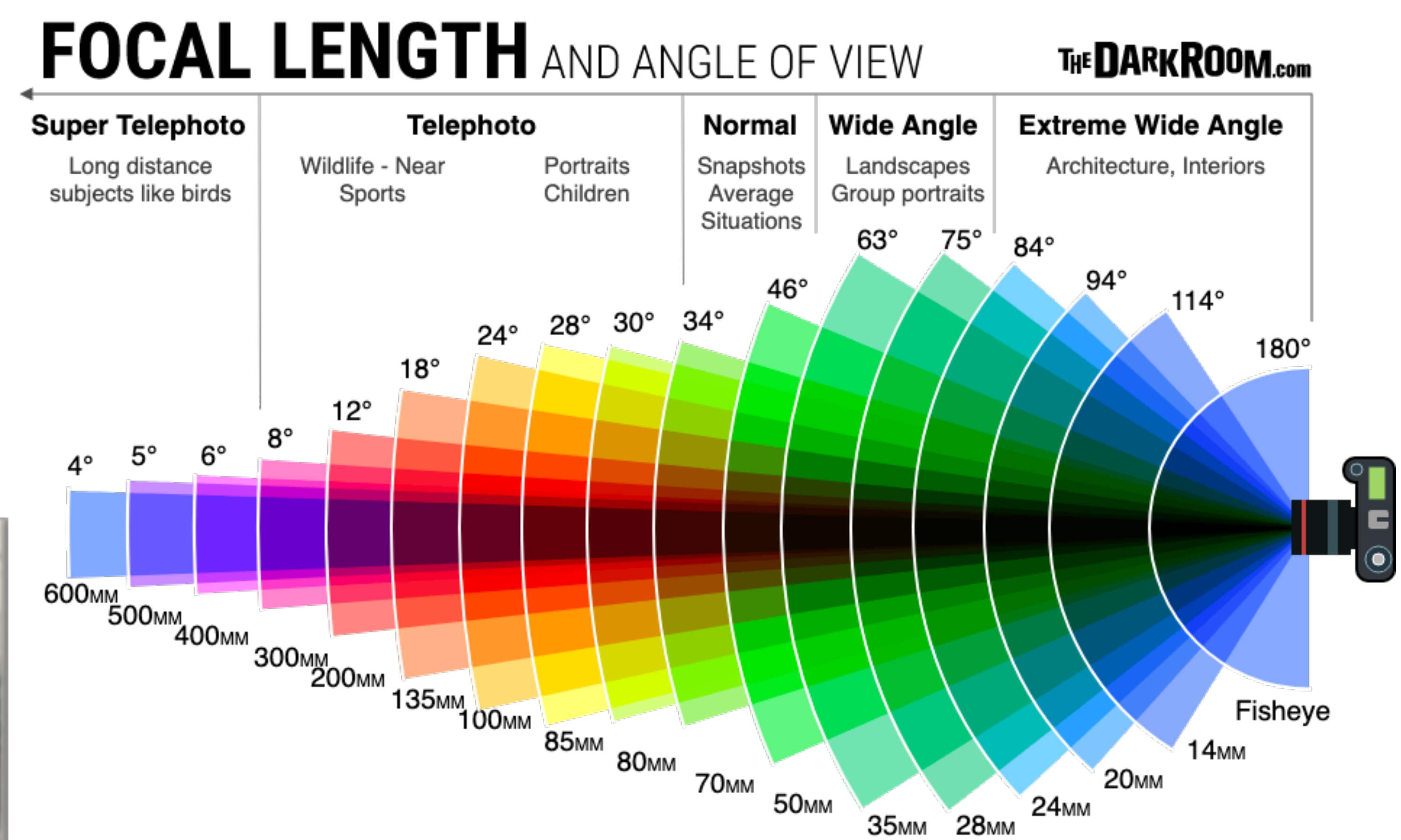
NUMERICAL APERTURE

DEPTH OF FOCUS

FUNDAMENTAL IMAGING CONSIDERATIONS

FIELD OF VIEW (FOV)

- **Field of view (FOV):** the maximum area of a scene that a camera can image.



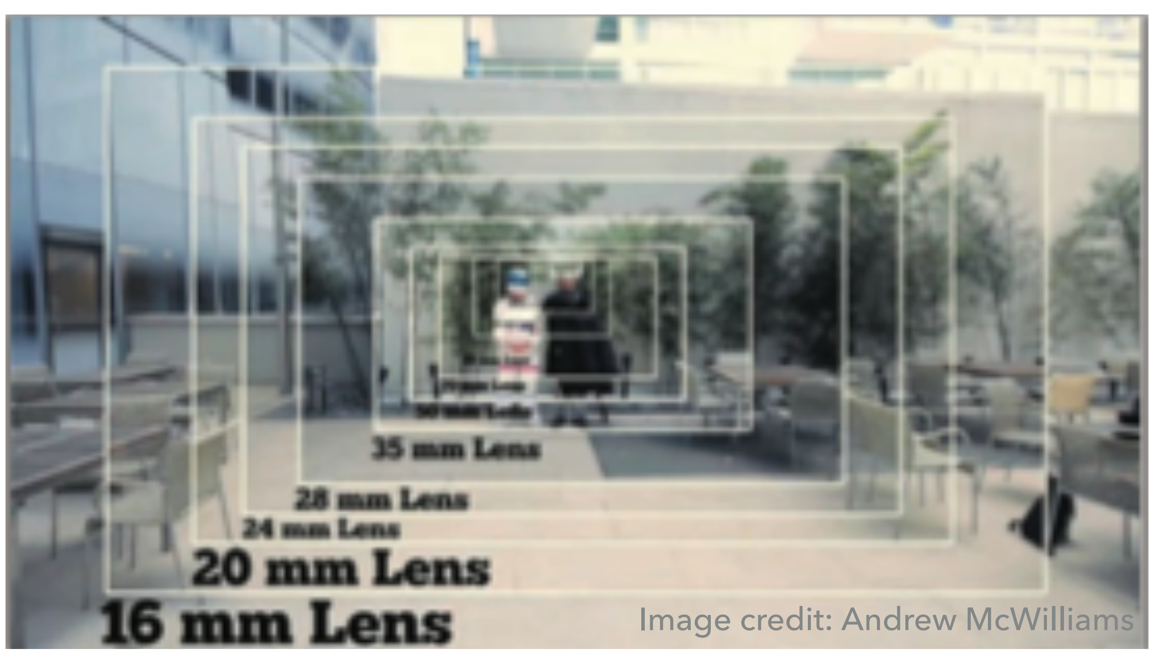


Image credit: Andrew McWilliams

FIELD OF VIEW (FOV)

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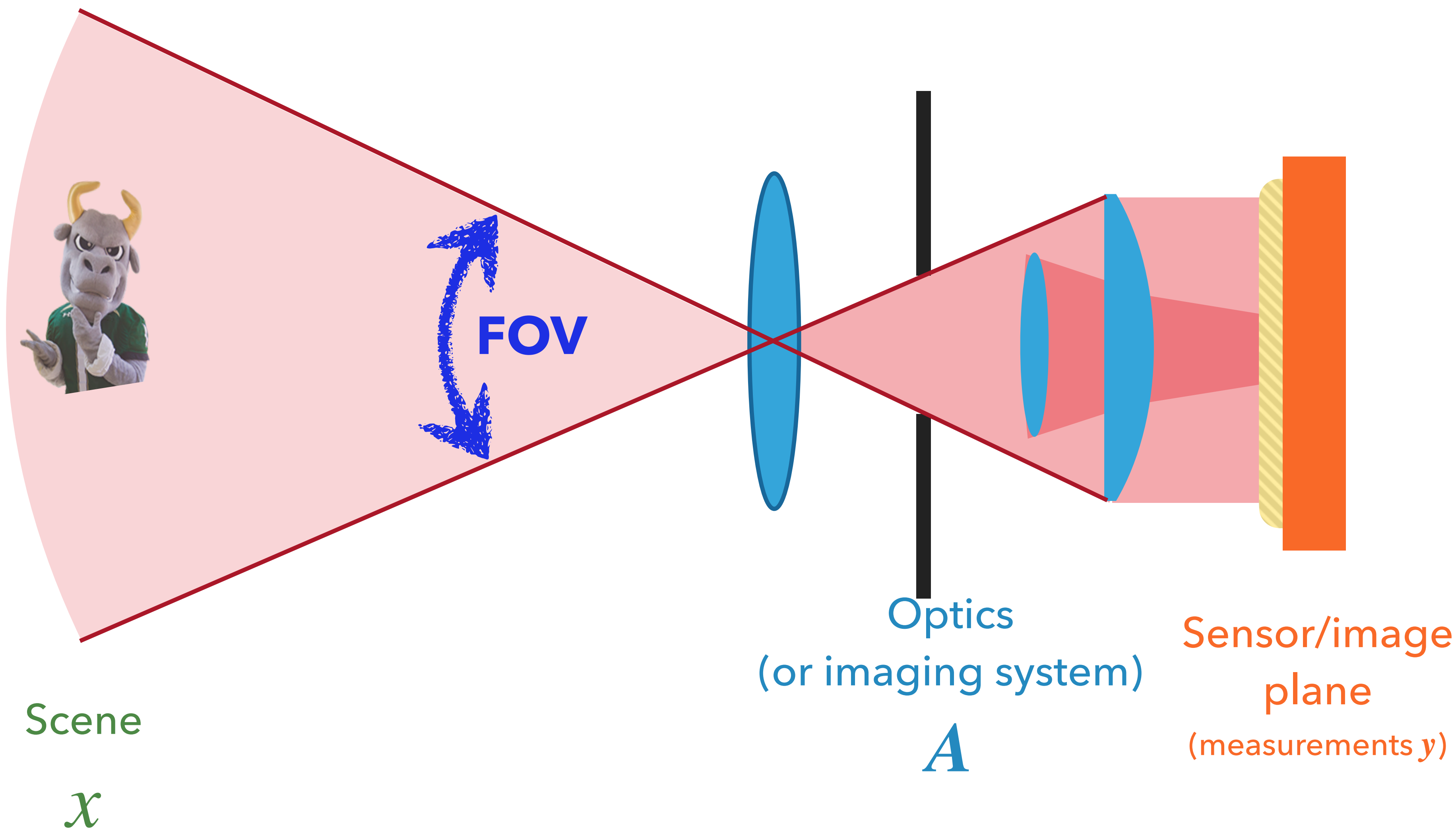
Why is FOV measured in terms of angle?



FIELD OF VIEW

NUMERICAL APERTURE

DEPTH OF FOCUS

FUNDAMENTAL IMAGING CONSIDERATIONS

NUMERICAL APERTURE

- **Numerical Aperture (NA):** describes the range of angles over which the system can accept light.

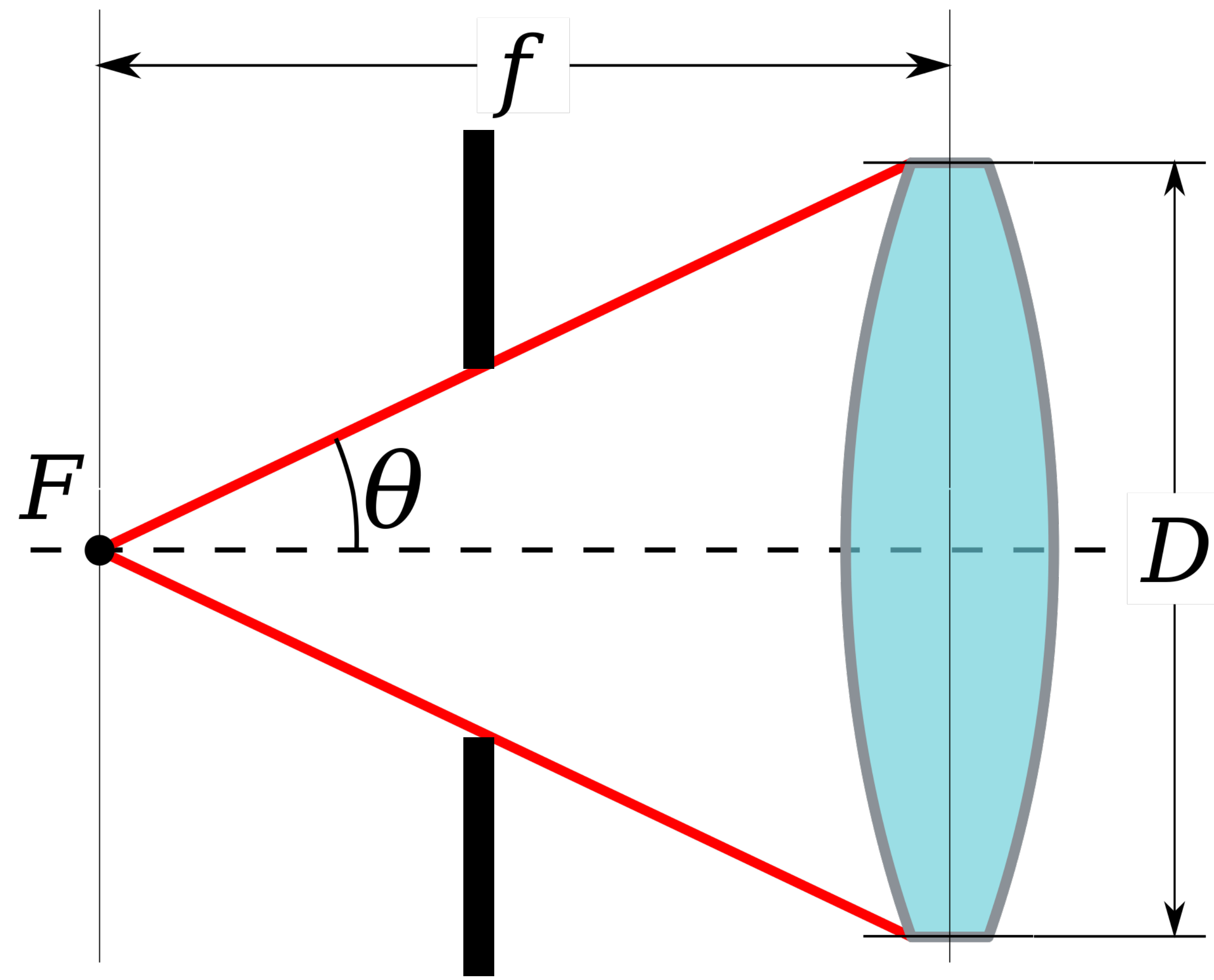
$$\mathrm{NA} = n \sin \theta$$

θ is the **half-angle subtended** by the imaging system from an *axial object*.

n is the refractive index of the medium

In photography, typically use the **f-number** ($f/\#$) instead:

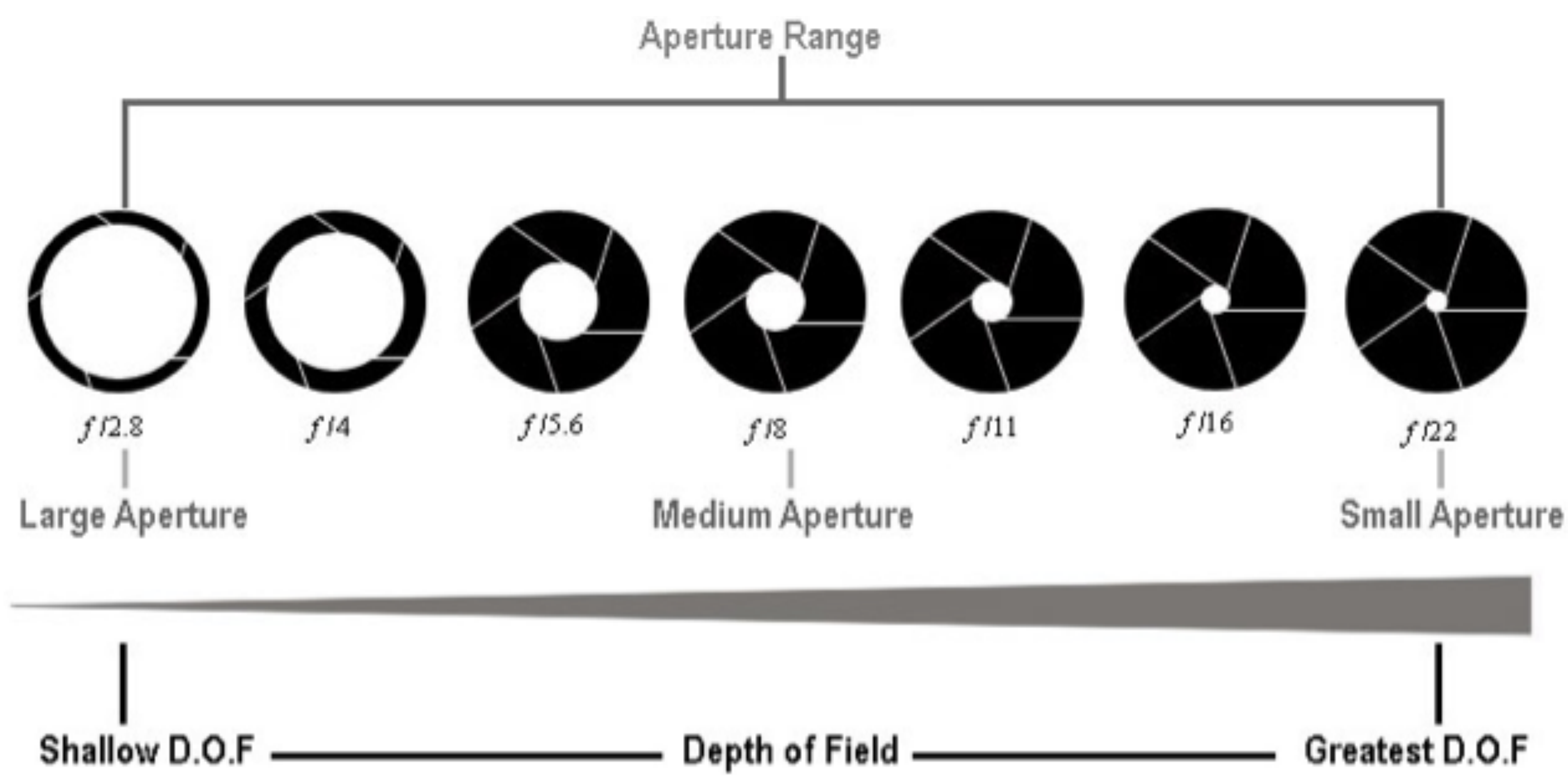
$$f/\# = \frac{1}{2\,(\mathrm{NA})}$$



PHOTOGRAPHY

- The aperture stop controls many things

Lets in more light



Lets in less light

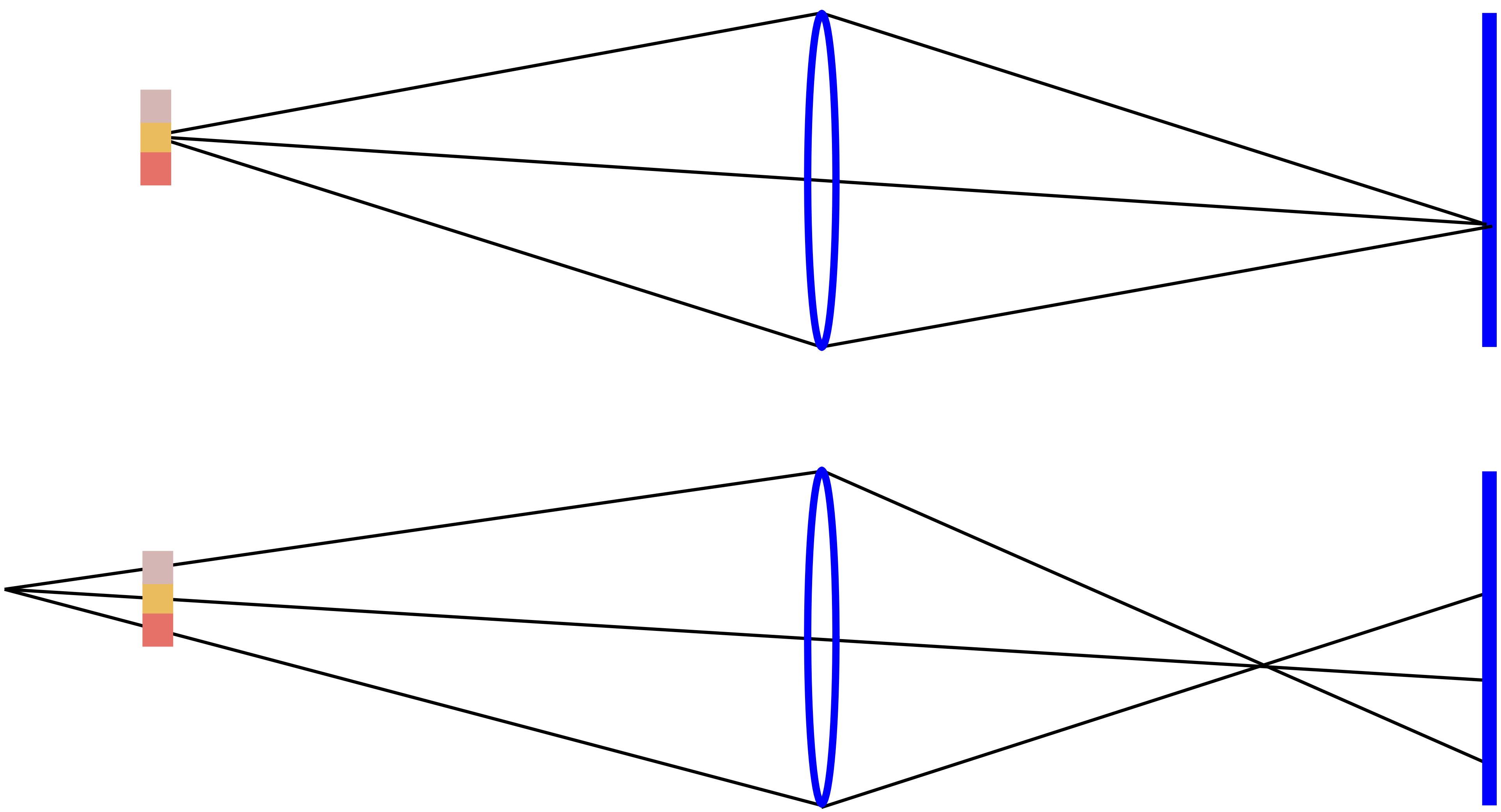
FIELD OF VIEW

NUMERICAL APERTURE

DEPTH OF FOCUS

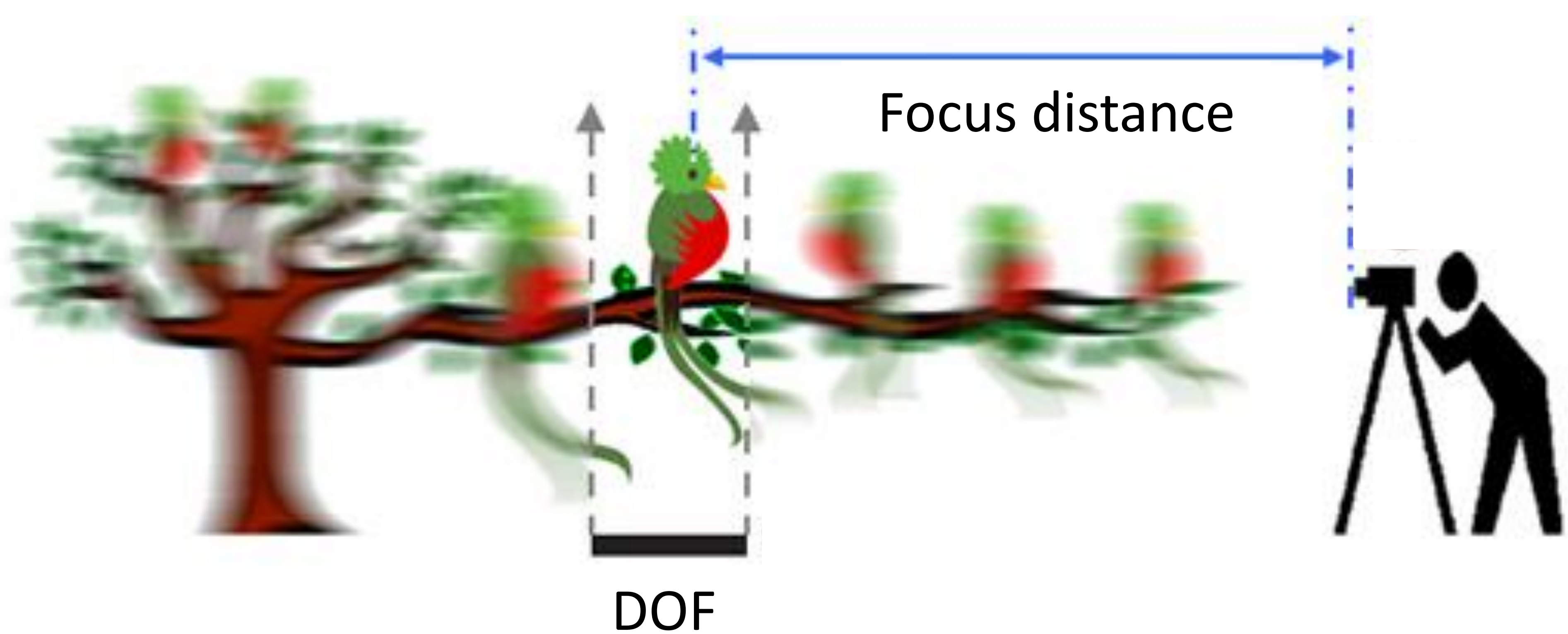
FUNDAMENTAL IMAGING CONSIDERATIONS

IN-FOCUS AND OUT-OF-FOCUS



DEPTH OF FIELD

- For a given image distance, the **depth of field** is the range of object distances for which the object stays in focus



DEPTH OF FIELD

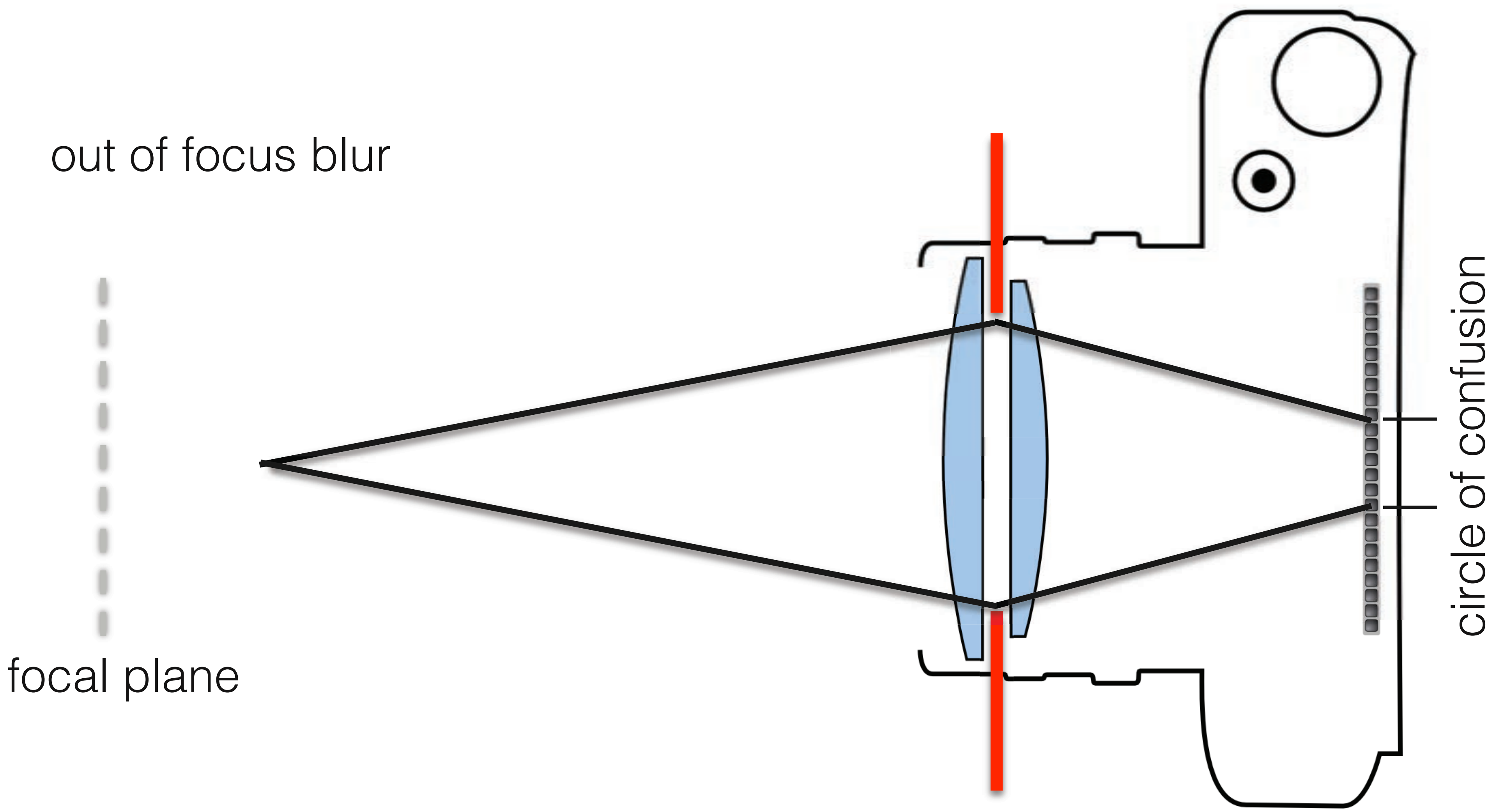
- DOF is proportional to the numerical aperture

f/22

f/8

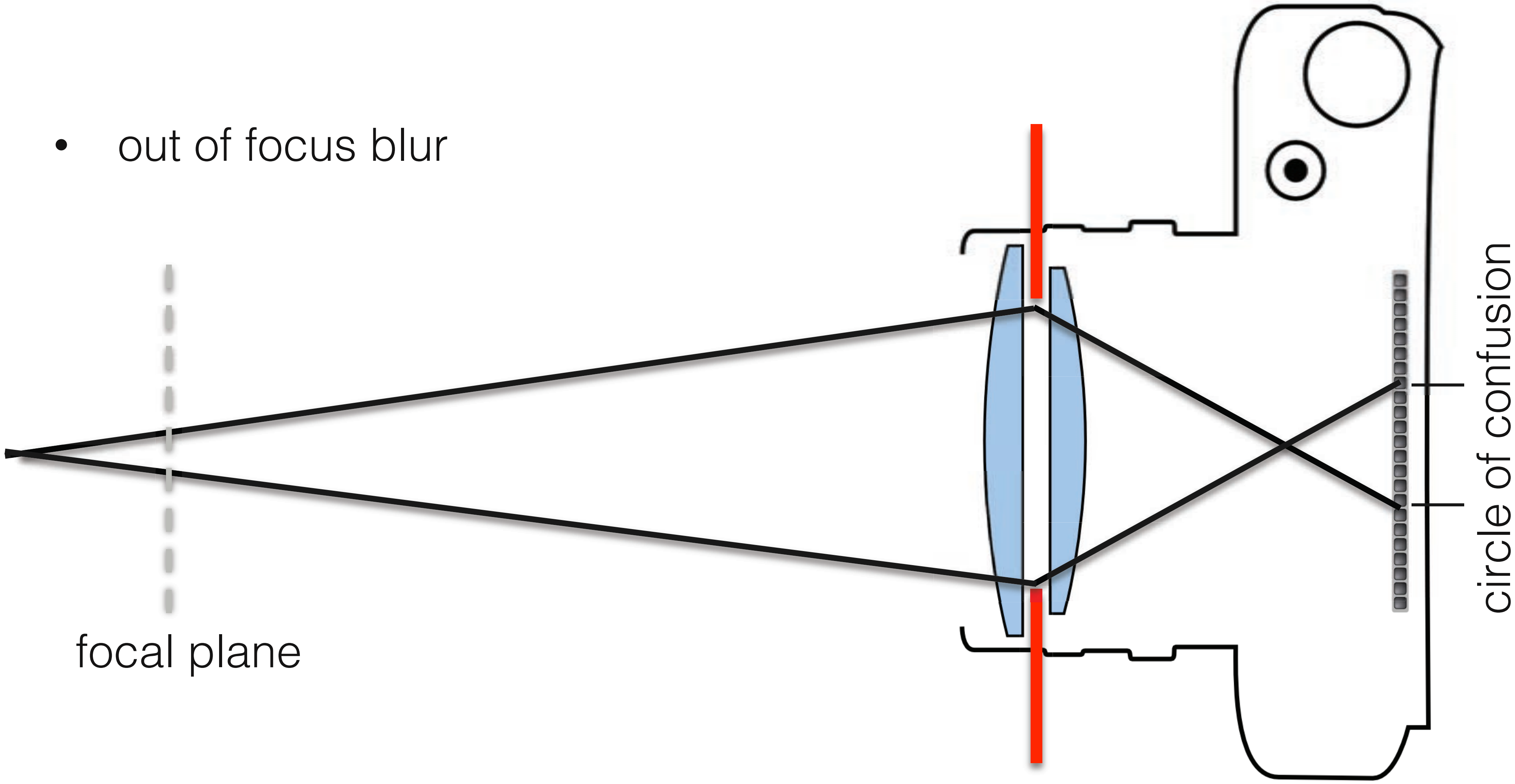
$$f/\# = \frac{1}{2\,(\mathrm{NA})}$$

UNDER FOCUS

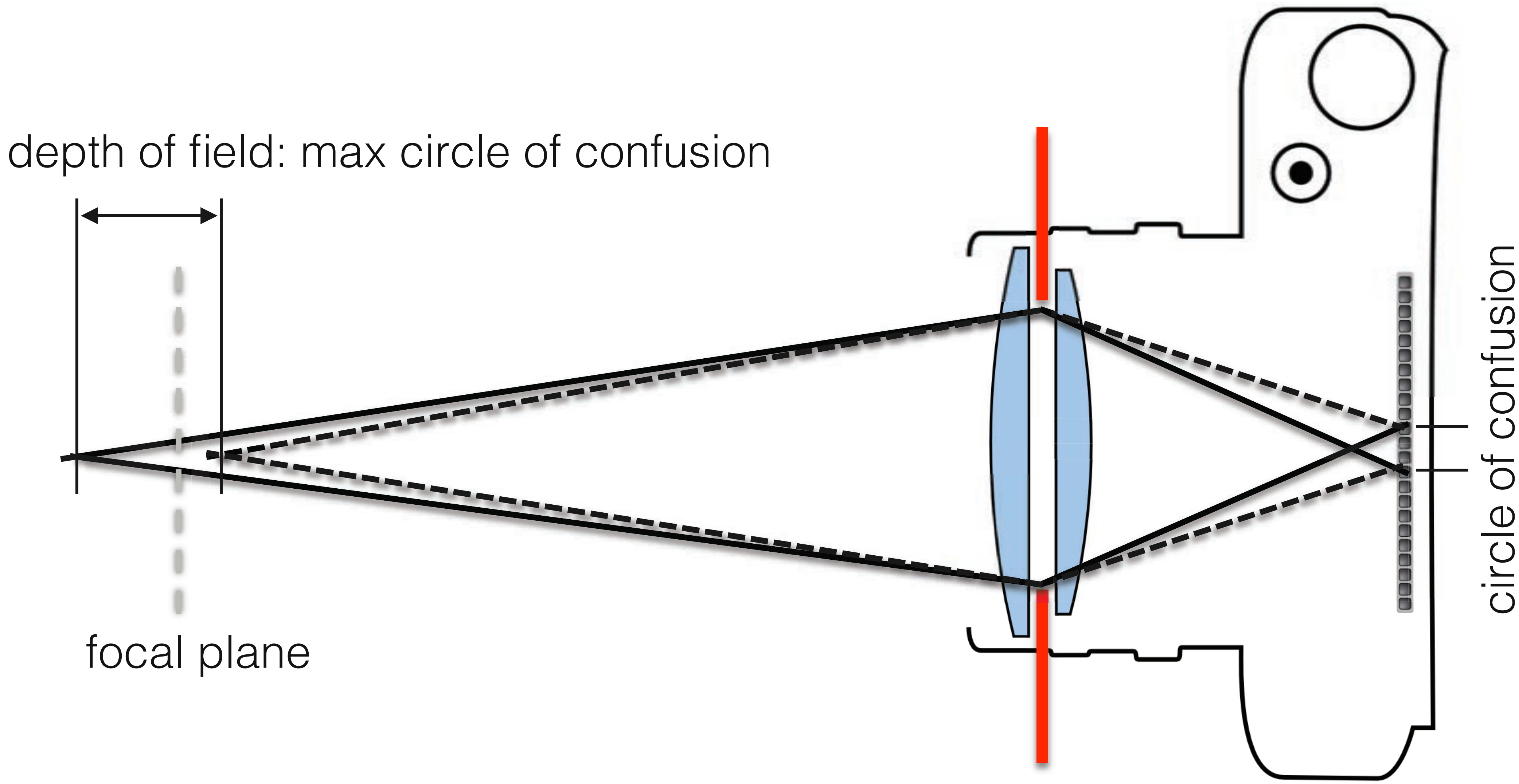


OVER FOCUS

- out of focus blur



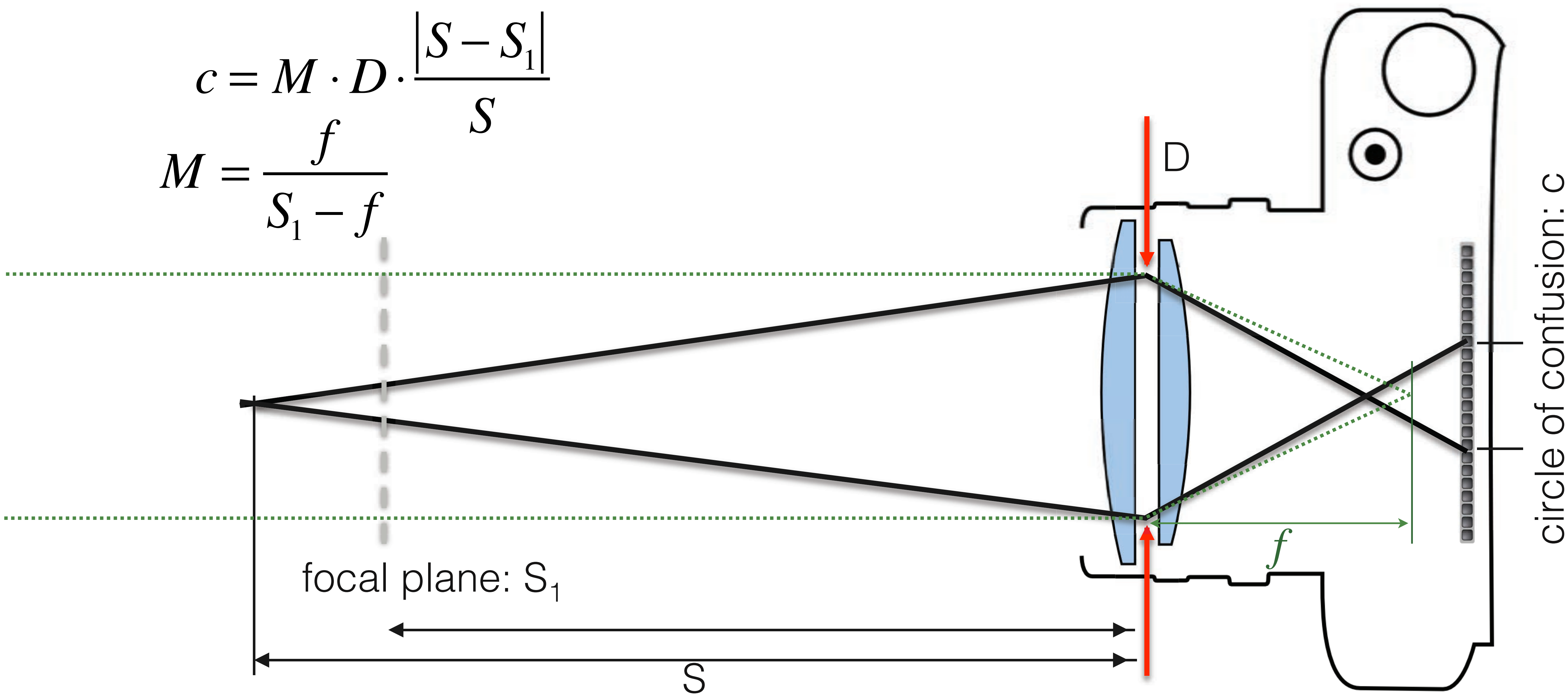
DEPTH OF FIELD



CIRCLE OF CONFUSION

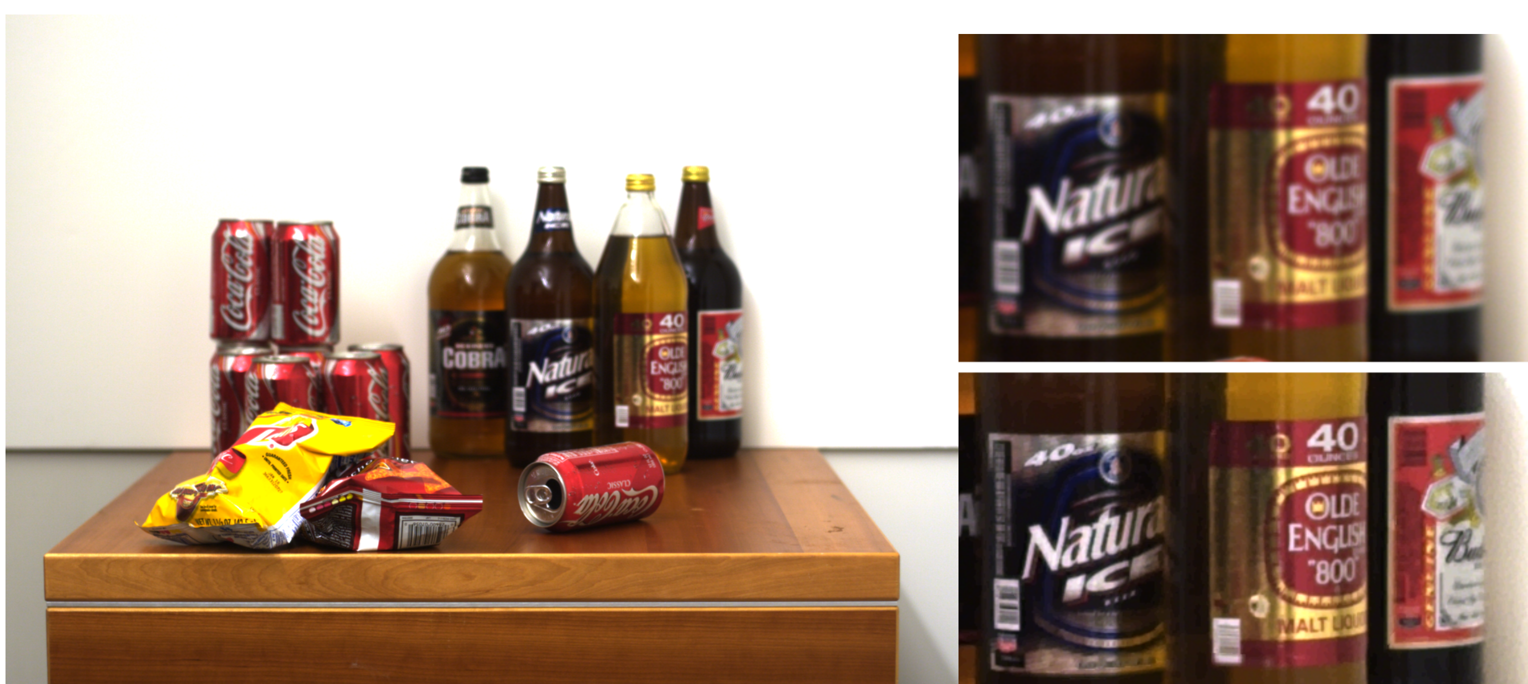
$$c = M \cdot D \cdot \frac{|S - S_1|}{S}$$

$$M = \frac{f}{S_1 - f}$$

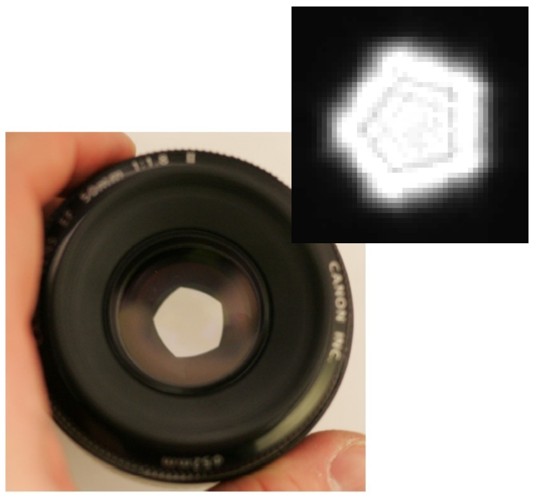


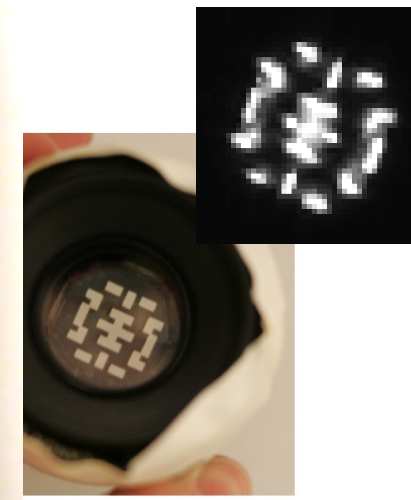
f is the focal distance of the lens

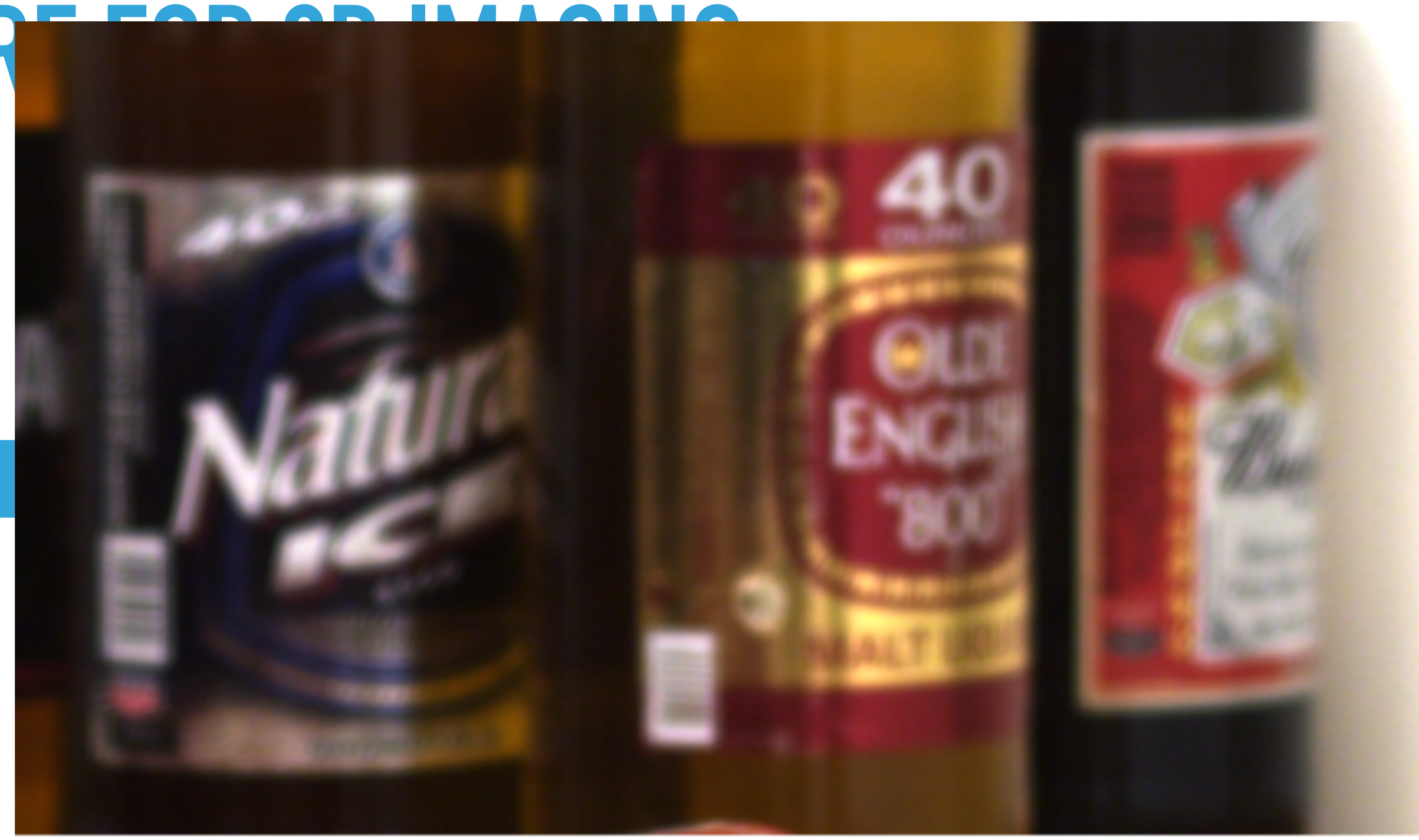
CODED-APERTURE FOR 3D IMAGING

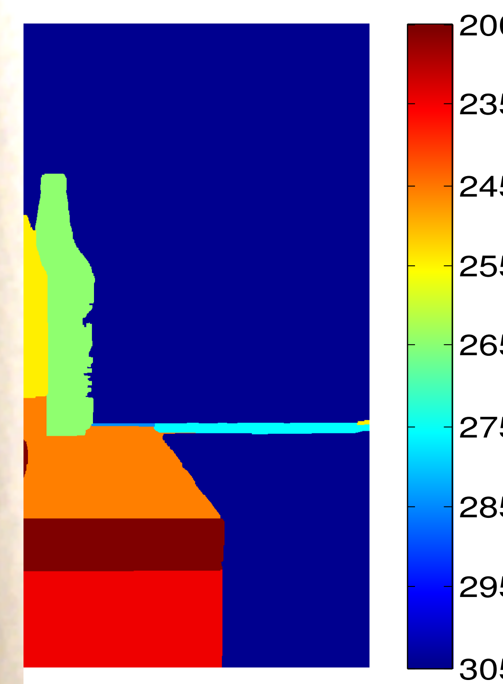


CODED-APERTURE FOR 3D IMAGING









CODED-APERTURE

CODED-APERTURE

NEXT TIME!

RECENT APPLICATIONS OF COMPUTATIONAL IMAGING SYSTEMS