Daniel Sawyer HW1

1a.
Yes, there is a unique solution. Column vectors: [X, Y, Z] = [1, 2, 3]

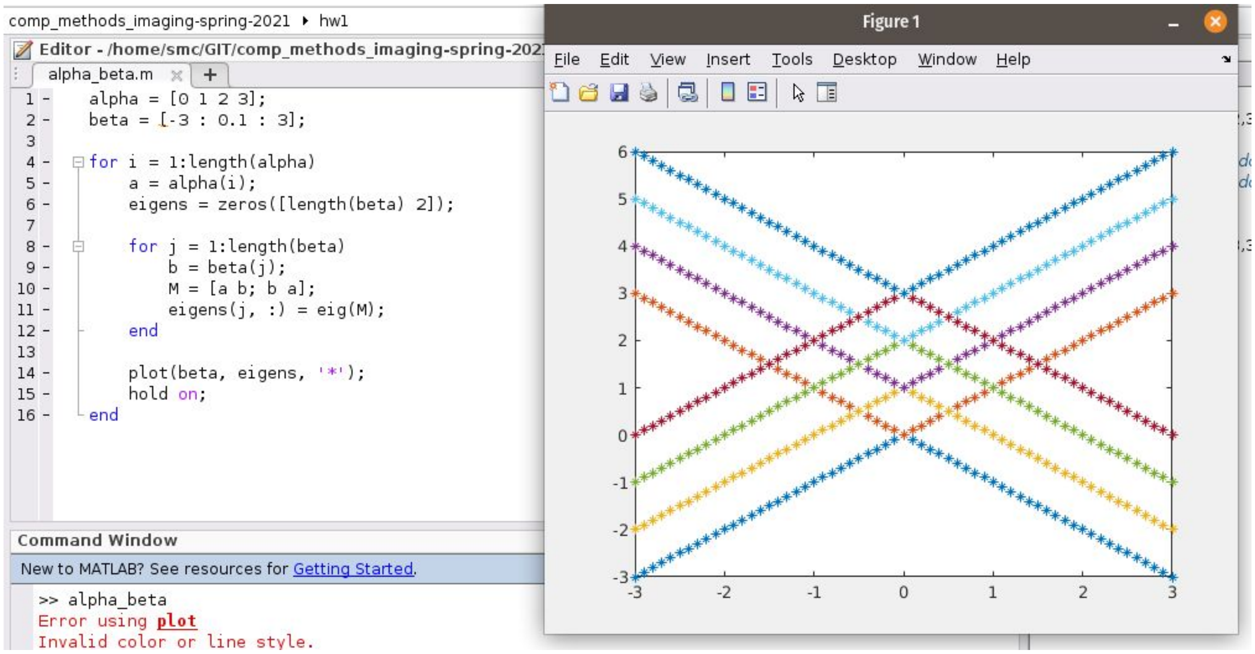
1b.
No unique solution due to A being singular. However, there are infinitely many solutions since X, Y, Z exist though one vector contains both X and Z and Y is constant. $X + 0Y + 2Z = 7$ and $0X + Y + 0Z = 2$

1c.
No solutions exists at all. A is singular and no solutions exist since $0X + 0Y + 0Z = 1$ can never be true.

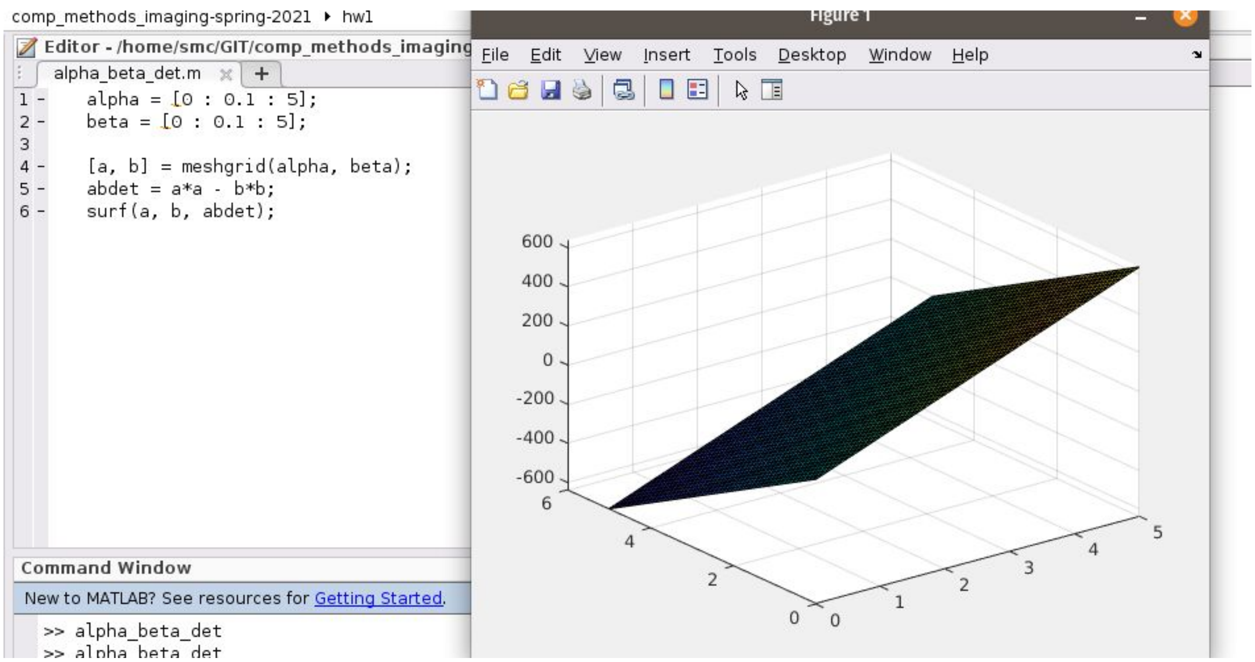
2a.
Eigenvalues of A from matlab, eig(A) = -1, 3
Unit-norm Eigenvectors using matlab is [V,D] = eig(A): V = [-0.7071 0.7071; 0.7071 0.7071]
They are orthogonal since the dot product of the columns of V equal 0. dot(V(:, 1), V(:, 2)) = 0

2b.
det(A) = -3, since det(A) != 0, A is invertible and not singular.
inv(A) = [-0.333333333333333,0.666666666666667;0.666666666666667,-0.333333333333333]
= $[-\frac{1}{3}\ \frac{2}{3};\ \frac{2}{3}\ -\frac{1}{3}]$

2c.
Eigenvalues = a-b, a+b
A simple Eigenvector for a-b would be a column vector [-1; 1] or [1; -1], unit-norm [-0.7071; 0.7071] or [0.7071; -0.7071]
A simple Eigenvector for a+b would be a column vector [1; 1], unit-norm [0.7071; 0.7071]



2d.
B is invertible as long as abs(a) != abs(b).



3a.
||Ux|| = ||x||

Orthogonal matrices have property of M.T = M^-1 and M * M^-1 = M^-1 * M = Identity so M.T * M = M * M.T = Identity if orthogonal. Therefore Ux^-1 * Ux = Ux * Ux^-1 = U.T * U = U * U.T = x^-1 * x = x * x^-1 = x.T * x = x * x.T = Identity. If this holds true, you can say that they are orthogonal and preserves length.

3b.
cos(theta) = (x*y) / (||x||*||y||) = (Ux*Uy) / (||Ux||*||Uy||) and since we proved length is preserved, we can say: (Ux*Uy) = (x*y) and (||Ux||*||Uy||) = (||x||*||y||). Therefore, cos(theta) = (x*y) / (||x||*||y||) = (Ux*Uy) / (||Ux||*||Uy||) is proven true if they are orthogonal.

4a.
Phi1 = [½ 0; sqrt(3)/2 1]
Phi3 = [½ -sqrt(3)/3; sqrt(3)/2 ½]

4b.
Phi1 = [0.5 0; 0.866 1], dual = [2 -1.732; 0 1]
Phi2 = [0.5 -0.866; 0.866 0.5], dual = [0.5 -0.866; 0.866 0.5]

4c.
Phi1 dual inner product = -3.464, not 0 therefore it is not orthonormal
Phi3 dual inner product = 0, since it is 0 it is orthonormal

4d.

5.
True.

6.