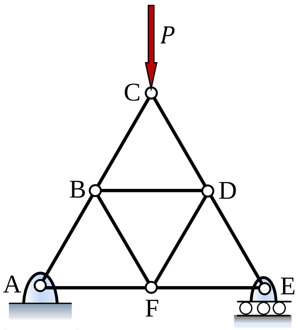
PART-A

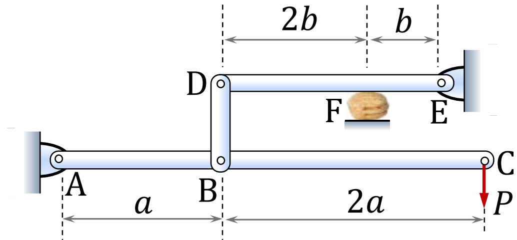
Answer all 10 questions. Each question carries 2.5 Marks. There is no step marking. Final answer should be filled in at the designated space.

Q1. In the truss shown in the figure, the force $P = 10$ kN and all nine members of the truss have equal length. The magnitude of the force carried by member BD is ___**0**___ kN.



$A_x = 0$. From Symmetry, $F_{AF} = F_{EF}$. Then from pin F, $F_{BF} = F_{DF} = 0$

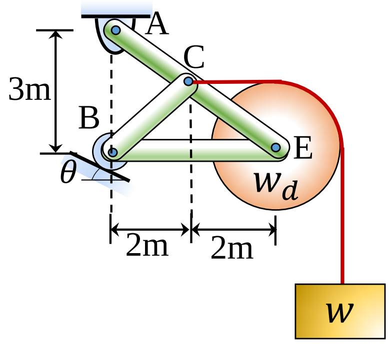
So, $\boldsymbol{F_{BD} = 0}$.



Q2. In the simple machine shown in the figure, if the force $P = 20$ kN, then the crushing force applied on the nut shell at point F is ___**180**___ kN.

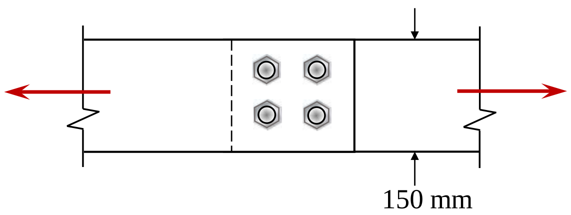
Double levering concept in machines: Taking moment at A in FBD of ABC, $F_{BD} = 3P$. Likewise, taking moment at E in FBD of DE, $N_F = 3F_{BD}$, i.e. $\boldsymbol{N_F = 9P}$.

Q3. In the frame shown in the figure, ACE is a rigid member, $W = 2$ kN and the pulley self-weight $W_d = 1$ kN.



The force in member BE becomes zero if angle $\theta =$ ___**53.13**___ °.

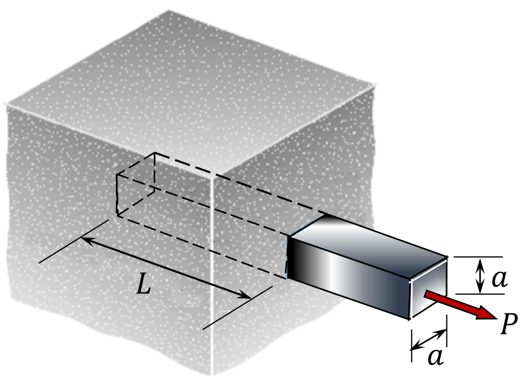
When N_B aligns with BC, force in BE=0, $\theta = 90 - \tan^{-1} 1.5/2$.



Q4. Two 10 mm thick steel plates are fastened together by means of four 20 mm bolts that fit tightly into the holes. If the joint transmits a tensile force of 88 kN, the maximum average normal stress in the plates is ___**80**___ MPa.

Critical section has two holes. Thus, $\sigma = F/(w - 2d)t = 88000/((150 - 2 * 20) * 10) =$ **80 MPa**.

Q5. A force P is applied as shown to a steel reinforcing bar that has been embedded in a block of concrete. The allowable average normal stress in the steel is 200 MPa, and the average allowable bond stress between the concrete and the bar is 10 MPa. The steel bar has 20 mm × 20 mm square cross-section. Neglecting the normal stresses between the concrete and the end of the bar, the smallest length L for which the full allowable normal stress in the bar can be developed is ___**100**___ mm.



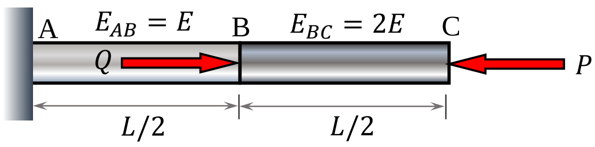
$P = \sigma_{\text{allow}} A = 200 * 20 * 20 = 80000$ N

$\tau_{\text{bond}} = P/4aL$ or $L = P/4a\tau_{\text{bond}} = 80000/(4 * 20 * 10) =$ **100 mm**.

Q6. A cylindrical pressure vessel, with radius r in its cylindrical portion, is constructed of a metal sheet of thickness t, where $t \ll r$. With a factor of safety of 2 for normal stress, the cylindrical pressure vessel can be pressurized to maximum internal pressure p. If a spherical pressure vessel of radius r is fabricated with the same metal sheet of thickness t, and the factor of safety used for the design is 4 for normal stress then the maximum allowable internal pressure is __**1**__ $\times p$.

For cylindrical vessel, $\sigma_\theta = p_{\text{cyl}} r/t = \sigma_{\text{u}}/2$. $\sigma_{\text{u}} = 2p_{\text{cyl}} r/t$

For spherical vessel, $p_{\text{sph}} r/2t = \sigma_{\text{u}}/4 = p_{\text{cyl}} r/2t$, so $p_{\text{sph}} = p_{\text{cyl}} = \boldsymbol{p}$



Q7. Two cylindrical bars AB and BC of equal dimension are joined at B and the assembly is fixed to a wall at A. The modulus of elasticity of bar BC is twice that of bar AB. Under loading by two forces P and Q, as shown, there will be no deflection at point C if Q/P = __**1.5**__ .

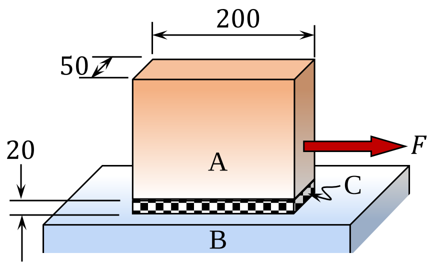
For load P alone, $\delta_C = -P\left(\frac{L}{2}\right)/(2AE) - P\left(\frac{L}{2}\right)/(AE) = -3PL/(4AE)$

For load Q alone, $\delta_C = \delta_B = QL/(2AE)$

Superposition gives $QL/(2AE) - 3PL/(4AE) = 0$. Hence, $\boldsymbol{Q = 3P/2}$

Q8. Rigid block A weighing 500 N is kept over fixed surface B with a 20 mm thick rubber block C placed in between. The bottom surface of C is rigidly bonded to B whereas block A can slide over C with coefficient of static friction being 0.8. The modulus of rigidity/shear modulus of the rubber is 10 MPa.

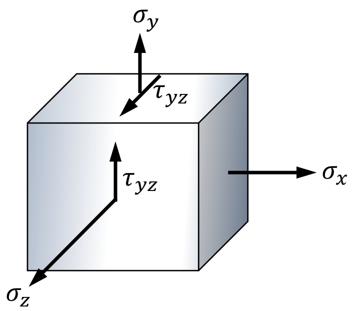
If force F is gradually increased from zero, then just before block A starts sliding, it would be displaced in the horizontal direction by __**80**__ µm.



All dimensions in mm

$F_{max} = 0.8 * 500 = 400$ N and $\tau_{max} = 400/(200 * 50) = 0.04$ MPa.

$\gamma = 0.04/10$=0.004 rad. $\delta = \gamma * 20 = 0.08$ mm = **80 µm**.

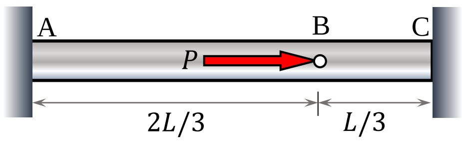


Q9. A small cubic element of each side measuring 10 mm in unstressed condition is subjected to stresses as shown where $\sigma_x = 150$ MPa, $\sigma_y = 100$ MPa, $\sigma_z = 50$ MPa and $\tau_{yz} = 60$ MPa. If the Poisson's ratio and bulk modulus of the material are 0.3 and 200 GPa, respectively, then the change in volume of the element due to application of the given stresses is __**0.5**__ mm^3.

$\sigma_h = (\sigma_x + \sigma_y + \sigma_z)/3 = 100$ MPa. $\epsilon_v = \sigma_h/\beta$ and $\Delta V = \epsilon_v V$

$\Delta V = (100 \times 10^6)/(200 \times 10^9) \times 10^3 = \mathbf{0.5}$ **mm^3**.

Q10. Bar AC is fixed to two walls at A and C. Initially, the bar is unstressed. When a force $P = 10$ kN is applied at point B, as shown, the reaction force at the wall at A will be __**3.333 or –3.333**__ kN.



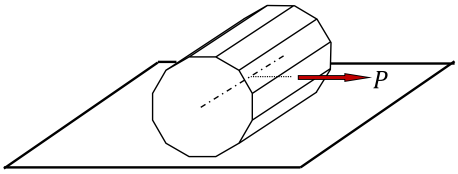
For load P alone, free expansion $\delta_C = P(2L/3)/AE$ and for reaction at C alone, $\delta_C = -R_C L/AE$

$P(2L/3)/AE - R_C L/AE = 0$ gives $R_C = 2P/3$. $R_A = P - R_C = P/3 = \mathbf{3.333}$ **kN or –3.333 kN**.

PART-B

Answer all five (5) questions. Each question carries 4 Marks. There is partial step marking. Final answer should be filled in at the designated space.

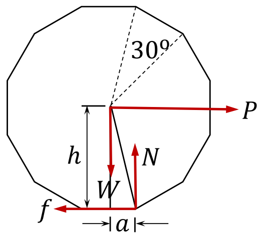
Q1. A 2 m long prismatic cylinder whose cross-section is a dodecagon (12 sided regular polygon) with each side measuring 10 cm is resting on a flat surface. The coefficient of static friction at the ground contact is 0.3. The minimum force P (applied at the center of mass) required to initiate motion of the prismatic cylinder is 5359 N.



When at rest, the bearing stress caused by the cylinder at the ground contact is **100** kPa.

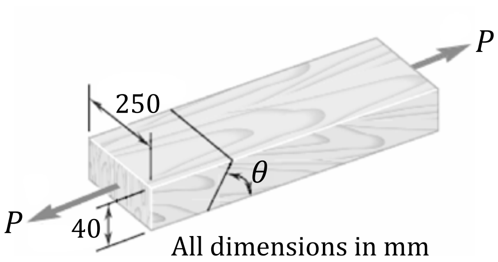
To initiate sliding motion, $P = 0.3W$ gives $W = 17863$ N.

To initiate tipping motion, $P = Wa/h$ where $a/h = \tan 15^{\circ}$, so $W = 20000$ N. For minimum P, maximum W is considered.



Contact area is $A = 0.1 \times 2 = 0.2$ m^2. So, $\sigma_b = W/A = \mathbf{100\ kPa}$.

Q2. Two wooden members of uniform rectangular cross section are joined by the simple glued scarf splice shown. The maximum allowable normal and shearing stresses in the glued splice are 750 kPa and 500 kPa, respectively.



(a) If $\theta = 30^{\circ}$, the largest allowable load P is **11547** N.

(b) If $\theta = 60^{\circ}$, the largest allowable load P is **10000** N.

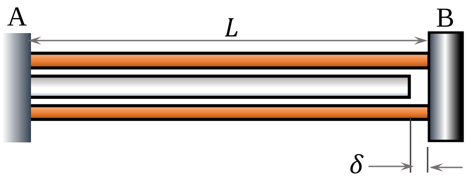
(a) $\sigma_0 = P/A = P/10000$ MPa, $\sigma = |\sigma_0 \cos^2(-60)| < 0.75$ and $\tau = |(\sigma_0/2)\sin(-120)| < 0.5$

This gives $P = 7500/\cos^2 60 = 30000$ N and $P = 5000 \times 2/\sin 120 = \mathbf{11547\ N}$.

(b) $\sigma_0 = P/A = P/10000$ MPa, $\sigma = |\sigma_0 \cos^2(-30)| < 0.75$ and $\tau = |(\sigma_0/2)\sin(-60)| < 0.5$

This gives $P = 7500/\cos^2 30 = \mathbf{10000\ N}$ and $P = 5000 \times 2/\sin 60 = 11547$ N.

Q3. Two 1cm $\times$ 1cm square steel bars with $L = 2$ m length, elastic modulus $E_s = 200$ GPa and coefficient of thermal expansion $\alpha_s = 10 \times 10^{-6}$ K^{-1} are fixed at end A and joined to a rigid block at end B. A 2cm $\times$ 2cm square brass bar in the middle with elastic modulus $E_B = 100$ GPa and coefficient of thermal expansion $\alpha_B = 20 \times 10^{-6}$ K^{-1} is fixed at end A and free at the other end, with a clearance $\delta = 1$ mm from rigid block B, as shown in the figure. If the temperature of the bars is uniformly raised by 100 °C, then the horizontal deflection of block B will be **2.5** mm.

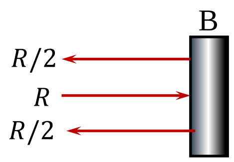


Under free expansion, $\delta_S = \alpha_S L\Delta T = 10 \times 10^{-6} \times 2000 \times 100 = 2$ mm, $\delta_B = \alpha_B L\Delta T = 20 \times 10^{-6} \times 2000 \times 100 = 4$ mm > 1 mm gap. $\delta_B - \delta > \delta_S$. So, steel with have tension and brass will have compression. If deflection of B is Δ and the cross-section of brass bar is A, then

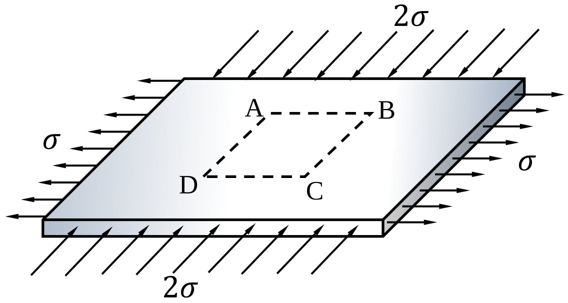
$$\Delta = L\alpha_s\Delta T + \frac{(R/2)L}{E_s(A/4)} = L\alpha_B\Delta T - \delta - \frac{RL}{E_B A}$$

$$\frac{2RL}{E_s A} + \frac{RL}{E_B A} = L(\alpha_B - \alpha_s)\Delta T - \delta = 1 \text{ mm or } \frac{2RL}{E_s A} + \frac{2RL}{E_s A} = 1 \text{ mm}$$

$$\frac{RL}{E_s A} = \frac{1}{4}\text{mm. So, } \Delta = L\alpha_s\Delta T + \frac{2RL}{E_s A} = 2 + \frac{2}{4} = \mathbf{2.5\ mm}.$$



Q4. A square ABCD of 200 mm side is scribed on an unstressed metal plate of thickness 15 mm. When subjected to normal stresses as shown in the figure, the length of side AB increased by 0.14 mm and that of side BC reduced by 0.22 mm. The modulus of rigidity of the metal is 25 GPa,



(a) The Poisson's ratio of the metal is __**0.2**__.

(b) The value of the stress σ is __**30**__ MPa.

$\epsilon_x = 0.14/200 = 7 \times 10^{-4}$ and $\epsilon_y = -0.22/200 = -11 \times 10^{-4}$

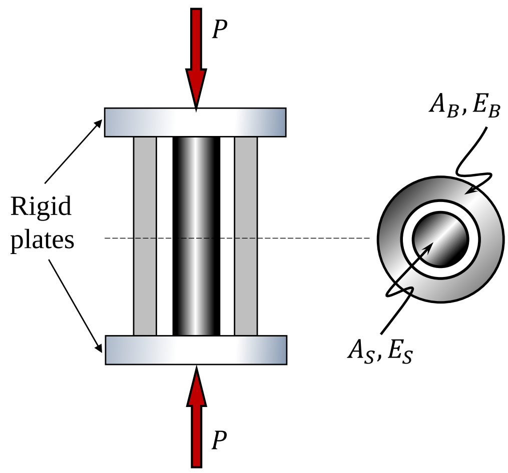
$\epsilon_x = \sigma/E + \vartheta \times 2\sigma/E = (\sigma/E)(1+2\vartheta)$ and $\epsilon_y = -\vartheta \times \sigma/E - 2\sigma/E = -(\sigma/E)(2+\vartheta)$

So, $-\epsilon_y/\epsilon_x = (2+\vartheta)/(1+2\vartheta) = 11/7$ or $14 + 7\vartheta = 11 + 22\vartheta$ or $15\vartheta = 3$, $\boldsymbol{\vartheta = 0.2}$.

Given $G = E/(2 \times (1+\vartheta)) = 80$ GPa, $E = 2G(1+\vartheta) = 60$ GPa.

From $\epsilon_x = (\sigma/E)(1+2\vartheta)$, $\sigma = E\epsilon_x/(1+2\vartheta) = 60 \times 10^3 \times 7 \times 10^{-4}/1.4 =$ **30 MPa**.

From $\epsilon_y = -(\sigma/E)(2+\vartheta)$, $\sigma = -E\epsilon_y/(2+\vartheta) = 60 \times 10^3 \times 11 \times 10^{-4}/2.2 = 30$ MPa.



Q5. A steel cylinder is held inside a brass tube/sleeve of same length by means of two rigid plates at two ends. A compressive force P is applied as shown. The cross-sectional area and elastic modulus of steel cylinder are A_S and E_S, respectively, and those for the brass tube are A_B and E_B, respectively. If $E_S = 1.5E_B$ and $A_S = 0.5A_B$ then the force carried by the steel member is __**42.85**__ % of the total compressive force.

There are two parallel springs of stiffness

$$K_s = \frac{A_s E_s}{L} \text{ and } K_B = \frac{A_B E_B}{L}$$

Let the common deflection be δ.

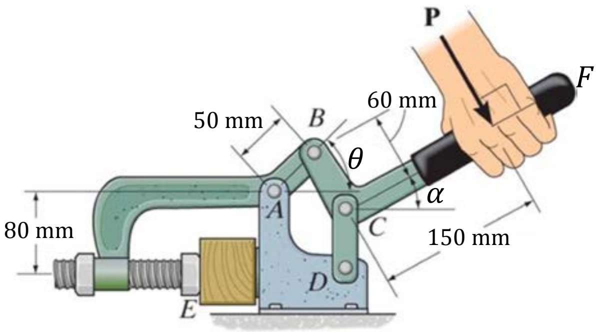
Then the force carried by the steel and brass bars are $F_S = K_s\delta$ and $F_B = K_B\delta$, respectively.

$$F_S/P = \frac{K_s\delta}{(K_s\delta + K_B\delta)} = \frac{A_s E_s}{A_s E_s + A_B E_B} = \frac{A_s E_s}{A_s E_s + 2A_s E_s/1.5} = \frac{1}{1+4/3} = \frac{3}{7} = \mathbf{0.4285}$$

PART-C

Answer any four (4) questions. Each question carries 12.5 Marks. There is step marking. Detailed solution (with FBDs) is necessary.

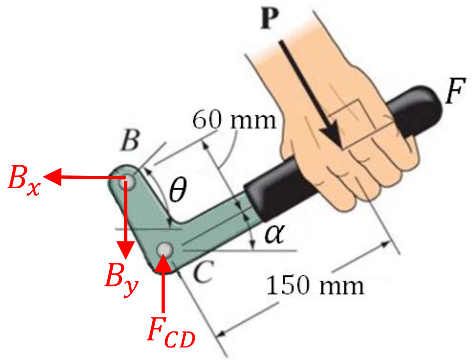
Q1. A force $P = 150$ N is applied to the handle of the toggle clamp to hold a wooden block at E in place. The geometric dimensions are shown in the figure with $\angle BCF = 90^{o}$. The angles $\alpha = 30^{o}$ and $\theta = 45^{o}$. Neglecting the self weight of the wooden block, determine the horizontal clamping force applied at E.

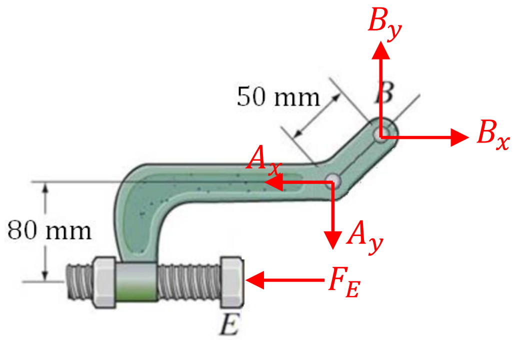


Space for Solution

CD is a two-force member.

Then the FBDS of the handle and the clamp are given below.





From FBD of lever,

$\sum F_x = 0$ gives

$$B_x = P \sin \alpha = \frac{P}{2} = 75 \text{ N}.$$

$\sum M_z = 0$ at pint C gives

$$B_y \times 60 \sin \alpha = 150 \times P - B_x \times 60 \cos \alpha$$

$$B_y = \frac{150 \times 150 - 75 \times 60 \cos 30}{60 \sin 30} = 620.096 \text{ N}.$$

From FBD of clamp,

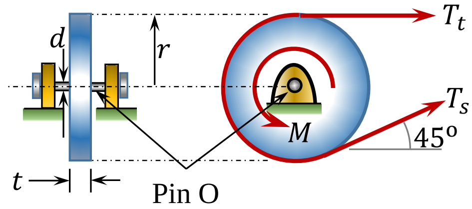
$\sum M_z = 0$ at pint A gives

$$F_E \times 80 = B_y \times 50 \cos \theta - B_x \times 50 \sin \theta$$

$$F_E = \frac{620.096 \times 50 \cos 45 - 75 \times 50 \sin 45}{80} = \mathbf{240.9\ N}.$$

The horizontal clamping force at E is **240.9** N.

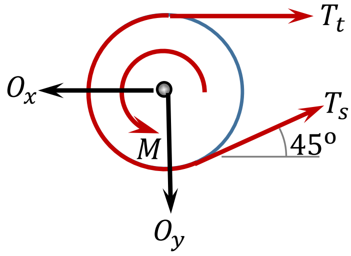
Q2. The belt on the pulley of radius $r = 0.25$m and thickness $t = 0.1$m is on the verge of slipping when transmitting the maximum allowable torque $M = 50$ Nm. The coefficient of friction between the pulley and the belt is $\mu = 0.28$ and the angle of wrap is 225^o (see Figure). The pin at O is frictionless and has 5mm diameter.



(a) Determine the maximum average shear stress developed in the pin.
(b) Find the maximum average bearing stress at the pin and the pulley contact.

Space for Solution

Since the pulley is on the verge of slipping,



$$T_t/T_s = e^{\mu\beta} = e^{0.28\times225\times\frac{\pi}{180}} = 3.003$$

$$M = r \times (T_t - T_s) \text{ or } 50 = 0.25 \times 2.003 \times T_s$$

So, $T_s = 99.85$ N and $T_t = 299.55$ N.

Reaction forces at pin O are

$O_x = T_s \cos 45 + T_t = 370.2$ N and $O_y = T_s \sin 45 = 70.6$ N.

Resultant reaction force is $R = \sqrt{O_x^2 + O_y^2} = 376.87$ N.

(a) Since there is double shear, average shear stress

$$\tau = R/(2A_b) = 376.87/(2 \times \pi \times 5^2/4) = \mathbf{9.597\ MPa}$$

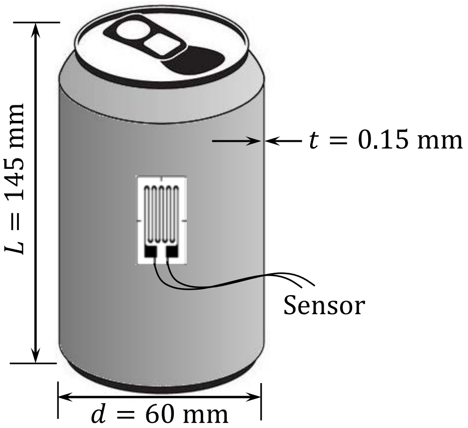
(b) Average bearing stress on the pin is

$$\sigma_b = R/(dt) = 376.87/(5 \times 100) = \mathbf{0.7537\ MPa}$$

The maximum average shear stress in the pin is <u>**9.597**</u> MPa.

The maximum average bearing stress at the pin and the pulley contact is <u>**0.7537**</u> MPa.

Q3. A strain gage is attached to a softdrinks canister in the longitudinal direction, as shown in the figure. The cannister with aluminum body can be approximated as a cylindrical pressure vessel with 60 mm diameter, 145 mm length, and 0.15 mm wall thickness. When the canister is opened, the strain gauge reading shows tthat the logitudinal strain reduced by 170×10^{-6} in comparison to the original state. Using this information:



(a) Calculate the original internal gage pressure within the canister.

(b) Determine the change in the canister's diameter in the cylindrical portion after depressurization.

For aluminum, consider Young's modulus $E = 70$ GPa and Poisson's ratio $\nu = 0.33$.

<u>Space for Solution</u>

According to Hooke's law:

$$\epsilon_{xx} = \frac{1}{E}(\sigma_{xx} - \nu\sigma_{\theta\theta})$$

Also for a cylindrical pressure vessel:

$$\sigma_{xx} = \frac{pr}{2t};\ \sigma_{\theta\theta} = \frac{pr}{t}$$

Hence,

$$\epsilon_{xx} = \frac{1}{E}(\sigma_{xx} - \nu\sigma_{\theta\theta}) = \frac{1}{E}\left(\frac{pr}{2t} = \frac{\nu pr}{t}\right) = \frac{pr}{2tE}(1 - 2\nu)$$

From the above equation

$$p = \frac{2E\epsilon_{xx}t}{r(1-2\nu)} = 2 \times (70 \text{ GPa}) \times \frac{(170 \times 10^{-6})}{(1 - 2 \times 0.33)} \times \frac{0.15 \text{ mm}}{30 \text{ mm}} = \mathbf{350000\ Pa}$$

Again, from Hooke's Law:

$$\epsilon_{\theta\theta} = \frac{1}{E}(\sigma_{\theta\theta} - \nu\sigma_{xx}) = \frac{1}{E}\left(\frac{pr}{t} - \frac{\nu pr}{2t}\right) = \frac{(2-\nu)pr}{2Et}$$

$$= \frac{(2 - 0.33) \times 350000 \text{ Pa} \times 30\text{mm}}{2 \times 70 \text{ GPa} \times 0.15\text{mm}} = 0.000835$$

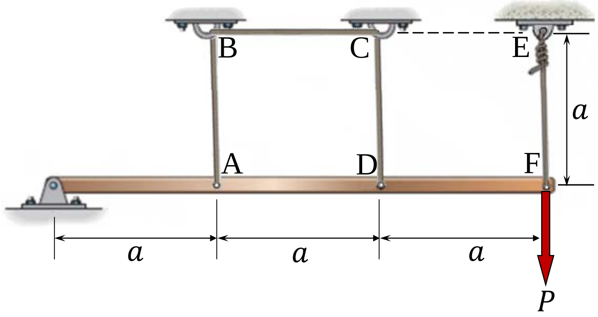
Now change in diameter can be calculated as:

$$\Delta d = d \times \epsilon_{\theta\theta} = 60 \times 0.000835 = \mathbf{0.05\ mm}$$

The original internal pressure within the canister is <u>**0.35**</u> MPa.

The change in the canister's diameter after depressurization is <u>**0.05**</u> mm.

Q4. A rigid beam of negligible weight is supported by two strings, AD and EF, of equal cross-section and same material. The string between points A and D passes through friction-less eyelets at B and C. Initially, all the strings are just taut. When a force $P = 12$ kN is applied at point F, find the tension developed in the string EF.

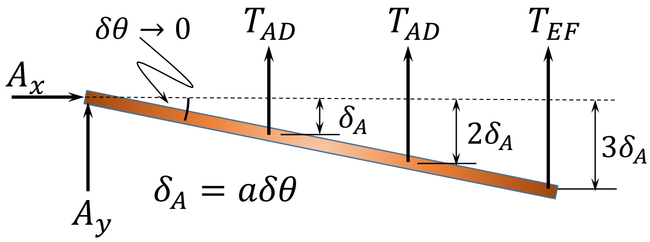


Space for Solution

Let the beam tilt by incremental angle $\delta\theta$ due to load. Then $\delta_F \cong 3a\delta\theta$, $\delta_D \cong 2a\delta\theta$ and $\delta_A \cong a\delta\theta$

So, string AD extends by $\delta_A + \delta_D = 3a\delta\theta$ and string EF extends by $3a\delta\theta$.

Length of string AD is $3a$ and that of string EF is a.



Tensions

$$T_{AD} = AE \times 3a\delta\theta/3a$$

$$T_{EF} = AE \times 3a\delta\theta/a$$

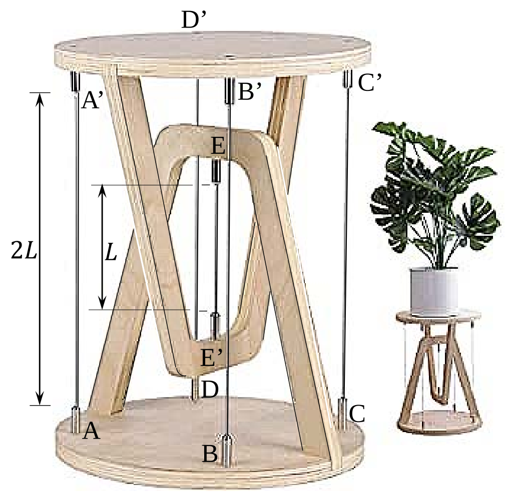
Thus, $T_{EF} = 3T_{AD}$.

In FBD of bar, taking $\sum M_z = 0$ at point A, $aT_{AD} + 2aT_{AD} + 3aT_{EF} = 3aP$

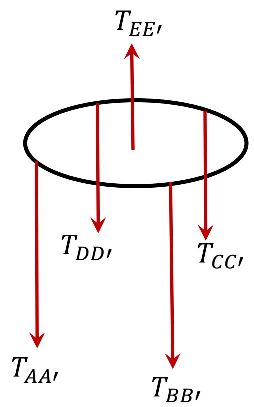
Or $T_{AD} + 2T_{AD} + 9T_{AD} = 3P$, i.e. $T_{AD} = P/4$ and $\boldsymbol{T_{EF} = 3P/4 = 9}$ **kN**.

The tension developed in the string EF is **9** kN.

Q5. A modern minimalist furniture designed based on the principle of tensegrity structure is shown in the figure. The structure has negligible self-weight. Here, Points A to E are on one rigid body and points A' to E' are on another rigid body. The two rigid bodies are connected through strong metal wires (of same material and cross-section) and the platform ABCD is kept on flat ground. Initially, the tension in wire EE' is adjusted so that equal 75 N tension results in each of the remaining four wires. The length of the wire EE' is L and that of each of the remaining four wires is $2L$. When a weight of 300 N, e.g. a plant pot, is kept centrally on the top surface A'B'C'D', find the tension in the wire EE'.



Space for Solution



$$T_{AA'} = T_{BB'} = T_{CC'} = T_{DD'} = 75 \text{ N}.$$

$$T_{EE'} = T_{AA'} + T_{BB'} + T_{CC'} + T_{DD'} = 300 \text{ N}.$$

Let the platform deflect by δ downwards. So, EE' extends by δ and length of others reduced by δ. Then

$$\Delta T_{EE'} = (AE/L)\delta$$

$$\Delta T_{AA'} = \Delta T_{BB'} = \Delta T_{CC'} = \Delta T_{DD'} = -(AE/2L)\delta = -\Delta T_{EE'}/2$$

$$W + (T_{AA'} + \Delta T_{AA'}) + (T_{BB'} + \Delta T_{BB'}) + (T_{CC'} + \Delta T_{CC'}) + (T_{DD'} + \Delta T_{DD'}) = T_{EE'} + \Delta T_{EE'}$$

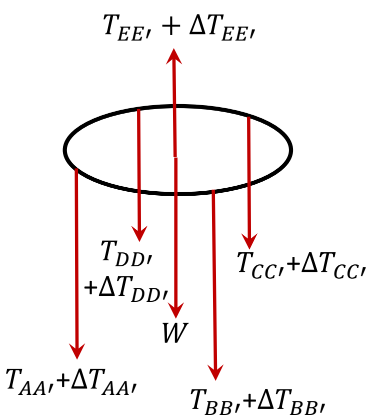
So,

$$W + \Delta T_{AA'} + \Delta T_{BB'} + \Delta T_{CC'} + \Delta T_{DD'} = \Delta T_{EE'}$$

$$W - 4 \times \Delta T_{EE'}/2 = \Delta T_{EE'}$$

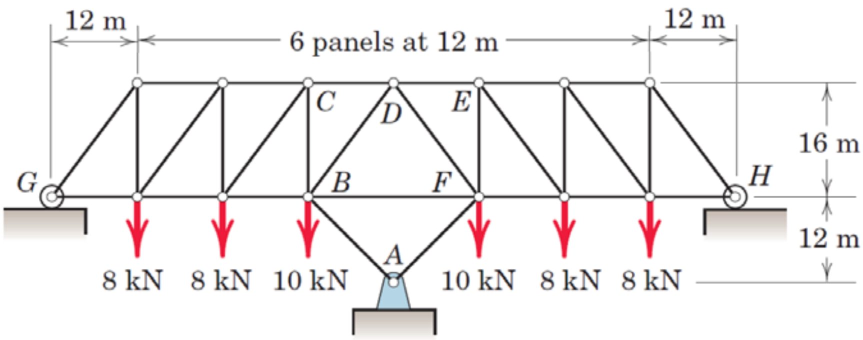
$$\Delta T_{EE'} = W/3$$

New tension in EE' is $T_{EE'} + \Delta T_{EE'} = 300 + 300/3 = \mathbf{400}$ **N.**



The tension in the wire EE' (after application of 300 N load) is **400** N.

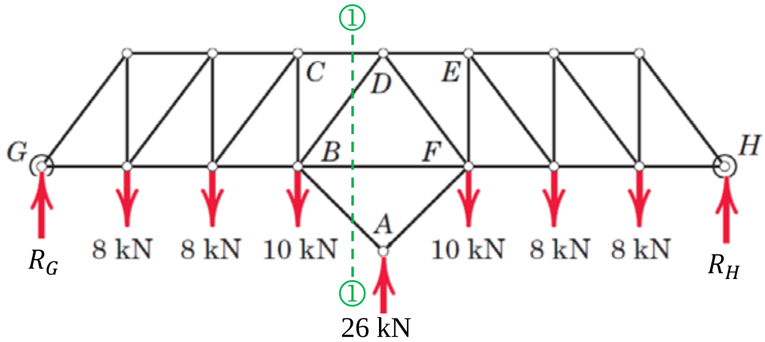
Q6. If it is known that the center pin A supports one half of the vertical loading (external forces) shown, determine the force in member BF and its nature (tensile/compressive).

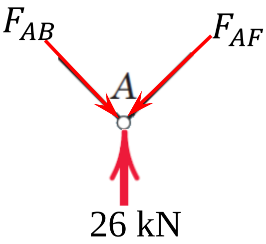


Space for Solution

Consider FBD of the entire truss.
$R_A = 26$ kN,

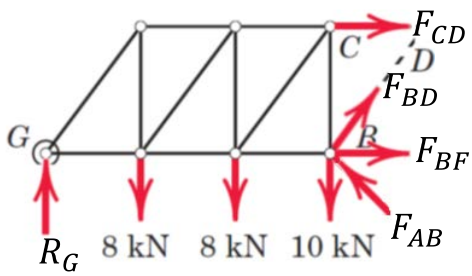
$\sum M_z = 0$ at pint H gives $R_G = 13$ kN and $\sum F_y = 0$ gives $R_H = 13$ kN.





From FBD of Pin A,
$\sum F_x = 0$ gives $F_{AB} = F_{AF}$ and $\sum F_y = 0$ gives $F_{AB}/\sqrt{2} + F_{AF}/\sqrt{2} = R_A$
So, $F_{AB} = 26/\sqrt{2} = 18.385$ kN (C).

Consider FBD of the left part of section ①-①.



$\sum M_z = 0$ at pint D gives

$$F_{BF} \times 16 - \left(\frac{F_{AB}}{\sqrt{2}}\right) \times 16 - \left(\frac{F_{AB}}{\sqrt{2}}\right) \times 12 + 10 \times 12 + 8 \times 24 + 8 \times 36 - R_G \times 48 = 0$$

$$F_{BF} \times 16 - 13 \times 16 - 13 \times 12 + 10 \times 12 + 8 \times 24 + 8 \times 36 - 13 \times 48 = 0$$

Or

$$F_{BF} = \frac{-10 \times 12 - 8 \times 24 - 8 \times 36 + 13 \times 48 + 13 \times 16 + 13 \times 12}{16} \text{ kN}$$

$F_{BF} = 24.25$ kN or $\boldsymbol{F_{BF} = 24.25}$ **kN (T)**

The force in member BF is **24.25** kN and its nature is **Tensile**.