Computer Vision Exercise 6

1 a) $E = \sum_{i=1}^{n} \| x_i' - Mx_i + t \|^2 \quad M? \; t?$

since $\|v\| = \sqrt{v^T v} \quad \|v\|^2 = v^T v$

$$E = \sum_{i=1}^{n} (x_i' - Mx_i + t)^T (x_i - Mx_i + t)$$

gradients:

(1) $\frac{\partial E}{\partial M} = \frac{\partial}{\partial M}((x_i' - Mx_i + t)^T (x_i' - Mx_i + t)) = -2 \sum_{i=1}^{n} (x_i' - Mx_i + t) x_i^T$

(2) $\frac{\partial E}{\partial t} = \frac{\partial}{\partial M}((x_i - Mx_i + t)^T (x_i - Mx_i + t)) = -2 \sum_{i=1}^{n} (x_i' - Mx_i + t)$

b) (1) & (2) $= 0 \qquad Sh = u$

$\sum_{i=1}^{n} (x_i' - Mx_i + t) x_i^T = 0$

$\sum_{i=1}^{n} x_i' x_i^T - \sum (Mx_i + t) x_i^T = 0$

$\sum (Mx_i + t) x_i^T = \sum_{i=1}^{n} x_i' x_i^T$

$Sh = u$

$\left(M \sum_{i=1}^{n} x_i + nt\right) x_i^T = \left(\sum_{i=1}^{n} x_i'\right) \cdot x_i^T \quad / \div x_i^T$

$M \sum_{i=1} x_i + n \cdot t = \sum_{i=1}^{n} x_i' \rightarrow$ for every x_i: $x_i' = M_1 x_i + M_2 y_i + t_i$ (same for y_i) $\qquad M = \begin{pmatrix} M_1 & M_2 \\ M_3 & M_4 \end{pmatrix}$

c) $h = S^{-1} u \qquad x_i' = M_1 x_i + M_2 y_i + t_1$

$(0,0) \rightarrow (1,2)$

$1 = M_1 0 + M_2 0 + t_1 \qquad t_1 = 1$

$2 = M_3 0 + M_4 0 + t_2 \qquad t_2 = 2$

$(1,0) \rightarrow (3,2)$

$3 = M_1 1 + M_2 0 + t_1$

$3 = M_1 + 1$

$M_1 = 2$

$2 = M_3 1 + M_4 0 + t_2$

$2 = M_3 + 2$

$0 = M_3$

$(0,1) \rightarrow (1,4)$

$1 = M_1 0 + M_2 1 + t_1$

$1 = M_2 + 1$

$0 = M_2$

$4 = M_3 0 + M_4 1 + t_2$

$4 = M_4 + 2$

$2 = M_4$

2

(1) $x' = sRx + t \Leftrightarrow \begin{pmatrix} x' \\ y' \end{pmatrix} = s \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix}$

a) $v' = x_2' - x_1' \qquad v = x_2 - x_1$

since $v \cdot v' = \|v\| \|v'\| \cos\theta$

$v \times v' = \|v\| \|v'\| \sin\theta$

$\Rightarrow \cos\theta = \frac{v \cdot v'}{\|v\| \|v'\|} \qquad \sin\theta = \frac{v \times v'}{\|v\| \|v'\|}$

b) $s = \frac{\|v'\|}{\|v\|} = \frac{\sqrt{(x_2' - x_1')^2 + (y_2' - y_1')^2}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$

c) (1) $\Rightarrow x' = s\cos\theta x - s\sin\theta y + t_x \qquad t_x = x' - s\cos\theta x + s\sin\theta y$

$y' = s\sin\theta x + s\cos\theta y + t_y \qquad t_y = y' - s\sin\theta x - s\cos\theta y$

d) $\overset{x_1}{\left(\tfrac{1}{2}, 0\right)} \rightarrow \overset{x_1'}{(0,0)} \qquad \overset{x_2}{\left(0, \tfrac{1}{2}\right)} \rightarrow \overset{x_2'}{(-1,-1)}$

$v' = \begin{pmatrix} -1 - 0 \\ -1 - 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \qquad v = \begin{pmatrix} 0 - \frac{1}{2} \\ \frac{1}{2} - 0 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$

$\|v\| = \sqrt{\left(-\tfrac{1}{2}\right)^2 + \left(\tfrac{1}{2}\right)^2} = \sqrt{\tfrac{1}{4} + \tfrac{1}{4}} = \sqrt{\tfrac{1}{2}} = \frac{1}{\sqrt{2}}$

$\|v'\| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$

$\cos\theta = \frac{+\frac{1}{2} + -\frac{1}{2}}{\frac{1}{\sqrt{2}} \cdot \sqrt{2}} = 0 \qquad \hookrightarrow \theta = 90$

$\sin\theta = \sin(90) = 1$

$s = \frac{\sqrt{2}}{\sqrt{\frac{1}{2}}} = 2$

$t_x = 0 - 2 \cdot 0 \cdot \frac{1}{2} + 2 \cdot 1 \cdot 0 = 0$

$t_y = 0 - 2 \cdot 1 \cdot \frac{1}{2} - 2 \cdot 0 \cdot 0 = -1$