## Exercise 1. Homogeneous coordinates.

a) The equation of a line in the plane is

$$ax + by + c = 0.$$

Show that by using homogeneous coordinates this can be written as

$$\mathbf{x}^{\mathsf{T}}\mathbf{l} = 0$$

where 
$$\mathbf{l} = \begin{pmatrix} a & b & c \end{pmatrix}^{\top}$$
.

Let 
$$ax + by + c = z^T l = 0$$
 where  $z = \begin{pmatrix} x \\ y \end{pmatrix}$   $A = \begin{pmatrix} a \\ b \end{pmatrix}$  vector
$$z l = \begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} a \\ b \\ c \end{pmatrix} = ax + by + c \quad (=0)$$

b) Show that the intersection of two lines  $\mathbf{l}$  and  $\mathbf{l}'$  is the point  $\mathbf{x} = \mathbf{l} \times \mathbf{l}'$ .

Given that 
$$l = ax + by + c_1$$
 and  $l' = ax + by + c_2$   
We can get a system of equations by rearrangement
$$ax + b_1 y = -c_1$$

$$a_2 x + b_2 y = -c_2$$

$$\begin{array}{ccc}
\times & = 0 & \times & = 0 \\
(\times) & (A_1) & (A_2) & (A$$

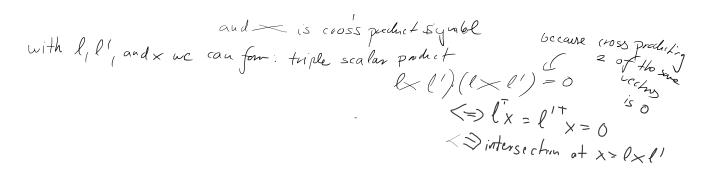
we need to solve for x, y, which satisfies both equations.

Let  $x = l \times l' = \begin{bmatrix} i & i & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{bmatrix} = \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$   $\begin{pmatrix} 1 \\ 1 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \\ 4 \end{pmatrix} \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix}$ 

triple scalar product:  $(a \times b)^T c = [a, b, c]^T$   $x = l \times l'$  where x is a point (inhuscetion point)

and  $x = l \times l'$  with l, l', and  $x = l \times l'$  we can form: triple scalar product

because (10ss pradicting



**Intersection of lines.** Given two lines  $\mathbf{l} = (a, b, c)^\mathsf{T}$  and  $\mathbf{l}' = (a', b', c')^\mathsf{T}$ , we wish to find their intersection. Define the vector  $\mathbf{x} = \mathbf{l} \times \mathbf{l}'$ , where  $\times$  represents the vector or cross product. From the triple scalar product identity  $\mathbf{1}.(\mathbf{1}\times\mathbf{1})=\mathbf{1}.(\mathbf{1}\times\mathbf{1})=\mathbf{1}.(\mathbf{1}\times\mathbf{1})$  see that  $\mathbf{1}^T\mathbf{x}=\mathbf{1}^T\mathbf{x}=0$ . Thus, if  $\mathbf{x}$  is thought of as representing a point, then  $\mathbf{x}$  lies on the both lines  $\mathbf{1}$  and  $\mathbf{1}'$ , and hence is the intersection of the two lines. This shows:

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c) Show that the line through two points  $\mathbf{x}$  and  $\mathbf{x}'$  is  $\mathbf{l} = \mathbf{x} \times \mathbf{x}'$ .

Line joining points. An expression for the line passing through two points  ${\bf x}$  and  ${\bf x}'$ may be derived by an entirely analogous argument. Defining a line l by  $l = x \times x'$ , it may be verified that both points x and x' lie on 1. Thus

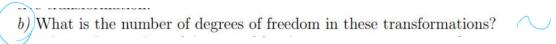
**Result 2.4.** The line through two points x and x' is  $l = x \times x'$ .

d) Show that for all  $\alpha \in \mathbb{R}$  the point  $\mathbf{y} = \alpha \mathbf{x} + (1 - \alpha) \mathbf{x}'$  lies on the line through points  $\mathbf{x}$  and  $\mathbf{x}'$ .

Due to y expressing a ratio between x and x (like with bariscutric coordinates)
y is a combination of x and x y lies on the line those 2 coordnites form

## Exercise 2. Transformations in 2D.

a) Use homogeneous coordinates and give the matrix representations of the following transformation groups: translation, Euclidean transformation (rotation+translation), similarity transformation (scaling+rotation+translation), affine transformation, projective transformation.



towolation:

$$det + (t_{x}, t_{y})$$
 be the vector which we want for towns late with

 $X' = (x+t_{x})$ 
 $X' = (x+t_{y})$ 
 $X' = (x+t_{y})$ 

$$X' = X \cdot P = \begin{pmatrix} P_{11} \times + P_{12} & y + P_{15} \\ P_{21} \times + P_{22} & y + P_{23} \end{pmatrix}$$

$$P_{31} \times + P_{32} y + P_{33}$$

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Exercise 3. Planar projective transformation.

The equation of a line on a plane, ax + by + c = 0, can be written as  $\mathbf{l}^{\top}\mathbf{x} = 0$ , where  $\mathbf{l} = [a\ b\ c]^{\top}$  and  $\mathbf{x}$  are homogeneous coordinates for lines and points, respectively. Under a planar projective transformation, represented with an invertible  $3 \times 3$  matrix  $\mathbf{H}$ , points transform as

$$\mathbf{x}' = \mathbf{H}\mathbf{x}$$
.

a) Given the matrix  $\mathbf{H}$  for transforming points, as defined above, define the line transformation (i.e. transformation that gives  $\mathbf{l}'$  which is a transformed version of  $\mathbf{l}$ ).

## 2.3.1 Transformations of lines and conics

**Transformation of lines.** It was shown in the proof of theorem 2.10 that if points  $\mathbf{x}_i$  lie on a line 1, then the transformed points  $\mathbf{x}_i' = H\mathbf{x}_i$  under a projective transformation lie on the line  $\mathbf{l}' = \mathbf{H}^{-\mathsf{T}}\mathbf{l}$ . In this way, incidence of points on lines is preserved, since  $\mathbf{l}'^{\mathsf{T}}\mathbf{x}_i' = \mathbf{l}^{\mathsf{T}}\mathbf{H}^{-1}\mathbf{H}\mathbf{x}_i = 0$ . This gives the transformation rule for lines:

Under the point transformation x' = Hx, a line transforms as

$$\mathbf{l}' = \mathbf{H}^{-\mathsf{T}} \mathbf{l}. \tag{2.6}$$

One may alternatively write  $1^{\prime T} = 1^{T}H^{-1}$ . Note the fundamentally different way in which lines and points transform. Points transform according to H, whereas lines (as rows) transform according to  $H^{-1}$ . This may be explained in terms of "covariant" or "contravariant" behaviour. One says that points transform *contravariantly* and lines transform *covariantly*. This distinction will be taken up again, when we discuss tensors in chapter 15 and is fully explained in appendix 1(p562).