Computer Vision Exercise 8

1. c) When the camera gets rotated, the number of features tracked decreases. Fast movements which smudge the image also seem to untrack features. This happens because KLT tracks contrast within neighbourhoods, therefore it is sensitive to blurring and motion.
   d) Redetecting the trackable features, perhaps after noticing significant loss. Another method could be trying to reduce the motion or at least to restrict it.
2.

$$\Delta \mathbf{p} = H^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^{\mathrm{T}} [T(\mathbf{x}) - I(\mathbf{W}(\mathbf{x}; \mathbf{p}))] \tag{10}$$

where H is the $n \times n$ (Gauss-Newton approximation to the) *Hessian* matrix:

$$H = \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^{\mathrm{T}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right].$$

$$\nabla I \Rightarrow [I_x \; I_y]$$

$$H = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix}$$

$$b = \sum_x \left(\Delta I \frac{\partial W}{\partial p}\right)^T T(x) - I(W(x;p))$$

if we let $T(x) - I(W(x,p)) = -I_t$ and $\nabla I \Rightarrow [I_x I_y]$

$$b = \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix} \leftarrow \sum_x \left[\Delta I \frac{\partial W}{\partial p}\right]^T [-I_t] = \sum_x \begin{bmatrix} I_x \\ I_y \end{bmatrix} (-I_t)$$

Solution given by $(\mathbf{A}^T\mathbf{A})\mathbf{d} = \mathbf{A}^T \mathbf{b}$

$$\begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = - \begin{bmatrix} \sum I_x I_t \\ \sum I_y I_t \end{bmatrix}$$

Let $A = [I_x \; I_y]$

$$\hookrightarrow H = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = A^T A$$

and $\Delta p = \begin{pmatrix} u \\ v \end{pmatrix} \Rightarrow H \Delta p = -b$

this way, we can write $\Delta p = H^{-1}(-b)$

$$\Leftrightarrow \Delta p = H^{-1} \sum_x \left[\nabla I \frac{\partial W}{\partial p}\right]^T [T(x) - I(W(x;p))]$$