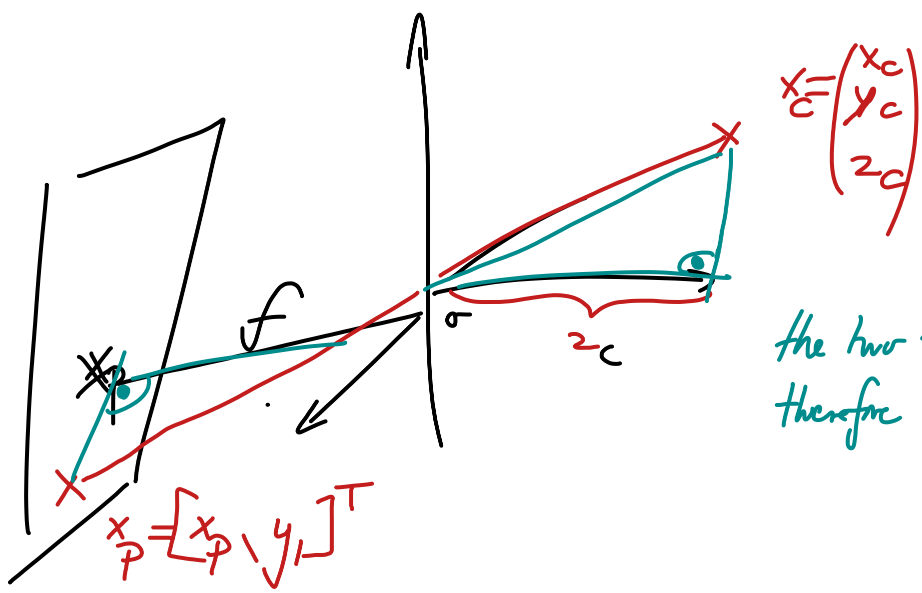
Exercise 1. Pinhole camera.
The perspective projection equations for a pinhole camera are

$$x_p = f\frac{x_c}{z_c}, \qquad y_p = f\frac{y_c}{z_c}, \tag{1}$$

where $\mathbf{x}_p = [x_p, y_p]^\top$ are the projected coordinates on the image plane, $\mathbf{x}_c = [x_c, y_c, z_c]^\top$ is the imaged point in the camera coordinate frame and f is the focal length. Give a geometric justification for the perspective projection equations.
(Hint: Use similar triangles and remember that the image plane is located at a distance f from the projection center and is perpendicular to the optical axis, i.e. the z-axis of the camera coordinate frame.)



the two triangles have the same angles, therefore

$$\frac{x_p}{f} = \frac{x_c}{z_c}$$

$$x_p = f\frac{x_c}{z_c}$$

similarly to y_c thus applies

// cannot copy-paste the exercise screencap due to app change

(2) a)

$$x_p = f\frac{x_c}{z_c} \qquad y_p = f\frac{y_c}{z_c}$$

scalings on x, y; γ is the aspect ratio

origin offset where (u_0, v_0) is the principal point

$$K = \begin{bmatrix} f & 0 & 0 \\ 0 & \gamma f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \frac{u_0}{f} \\ 0 & 1 & \frac{v_0}{f} \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & s & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} f & sf & u_0 \\ 0 & \gamma f & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

shear

m_u, m_v $\quad p = [u_0 \; v_0]$ is the principal point's coordinates

? $u = m_u \cdot x_p + u_0$

$$u = m_u \cdot f \frac{x_c}{z_c} + u_0$$

Similarly:

$$v = m_v \cdot y_p + v_0$$

$$v = m_v \cdot f \frac{y_c}{z_c} + v_0$$

b)

3) Camera's intrinsic calibration matrix:

$$\begin{bmatrix} f & sf & u_0 \\ 0 & \gamma f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \tilde{p} = K x_c$$

$$\Rightarrow \quad K = \begin{bmatrix} m_u f & 0 & u_0 \\ 0 & m_v f & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

4) Camera projection matrix

$$x_c = R x_w + t = \begin{bmatrix} & R & & t_x \\ & & & t_y \\ & & & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{pmatrix} x_w \end{pmatrix} = x_c$$

P matrix $\quad P = K[R|t]$

$$P = \begin{bmatrix} m_u f & 0 & u_0 \\ 0 & m_v f & v_0 \\ 0 & 0 & 1 \end{bmatrix} \cdot x_c$$

a.k.a. extrinsic calibration matrix (?)

$$\cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

vanilla projection matrix

5 Rodrigues formula

$$Rx = \cos\theta\, x + \sin\theta\, u \times x + (1-\cos\theta)(u \cdot x)\, u$$

R is the rotation matrix