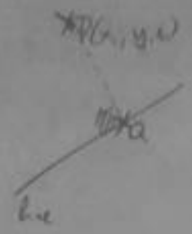
Comp Vision Ex 5

I.

line: $ax + by - d = 0$

$a^2 + b^2 = 1$ coef norm.

Point $P(x_i, y_i)$



since line and P are perpendicular, we can apply this formula (wikipedia).

↳ distance between point and line

$$dist = \frac{|ax + by + c|}{\underbrace{\sqrt{a^2 + b^2}}_{=1}}$$

$$= \frac{|ax + by + c|}{\sqrt{1}} = |ax + by + c|$$

II

$P(x_i, y_i)$

$$E = \sum_{i=1}^{n} (ax_i + by_i - d)^2$$

$$\frac{\partial E}{\partial d} \overset{!}{=} 0$$

$$\left(= \frac{\partial \left(\sum_{i=1}^{n} (ax_i + by_i - d)^2 \right)}{\partial d} = 2(ax_i + by_i - d) \cdot \underbrace{\frac{\partial (ax_i + by_i - d)}{\partial d}}_{-1} = -2(ax_i + by_i - d) \right)$$

$$= \sum_{i=1}^{n} -2(ax_i + by_i - d) = 0 \qquad -2\sum_{i=1}^{n} (ax_i + by_i - d) = 0$$

$$\sum_{i=1}^{n} (ax_i + by_i - d) = 0$$

$$\sum_{i=1}^{n} ax_i + by_i - \sum_{i=1}^{n} d = 0 \qquad \text{/ d is constant}$$

$$\sum_{i=1}^{n} ax_i + by_i - dn = 0$$

$$\sum_{i=1}^{n} ax_i + by_i = dn$$

$$\frac{\sum_{i=1}^{n} ax_i + by_i}{n} = d \iff \frac{a}{n} \cdot \sum_{i=1}^{n} x_i + \frac{b}{n} \sum_{i=1}^{n} y_i = d$$

$$a\bar{x} + b\bar{y} = d$$

III

$$E = \sum_{i=1}^{n} \left(ax_i + by_i - \left(\frac{a}{n} \sum_{i=1}^{n} x_i + \frac{b}{n} \sum_{i=1}^{n} y_i \right) \right)^2$$

$$E = (\dots] - a\bar{x} - b\bar{y})^2 = \sum_{i=1}^{n} (a(x_i - \bar{x}) + b(y_i - \bar{y}))^2 = \left\| \begin{bmatrix} x_1 - \bar{x} & y_1 - \bar{y} \\ \vdots & \\ x_n - \bar{x} & y_n - \bar{y} \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \right\|^2 = (UN)^T (UN) = U^T N^T U N$$

where $N = \begin{pmatrix} a \\ b \end{pmatrix}$

IV

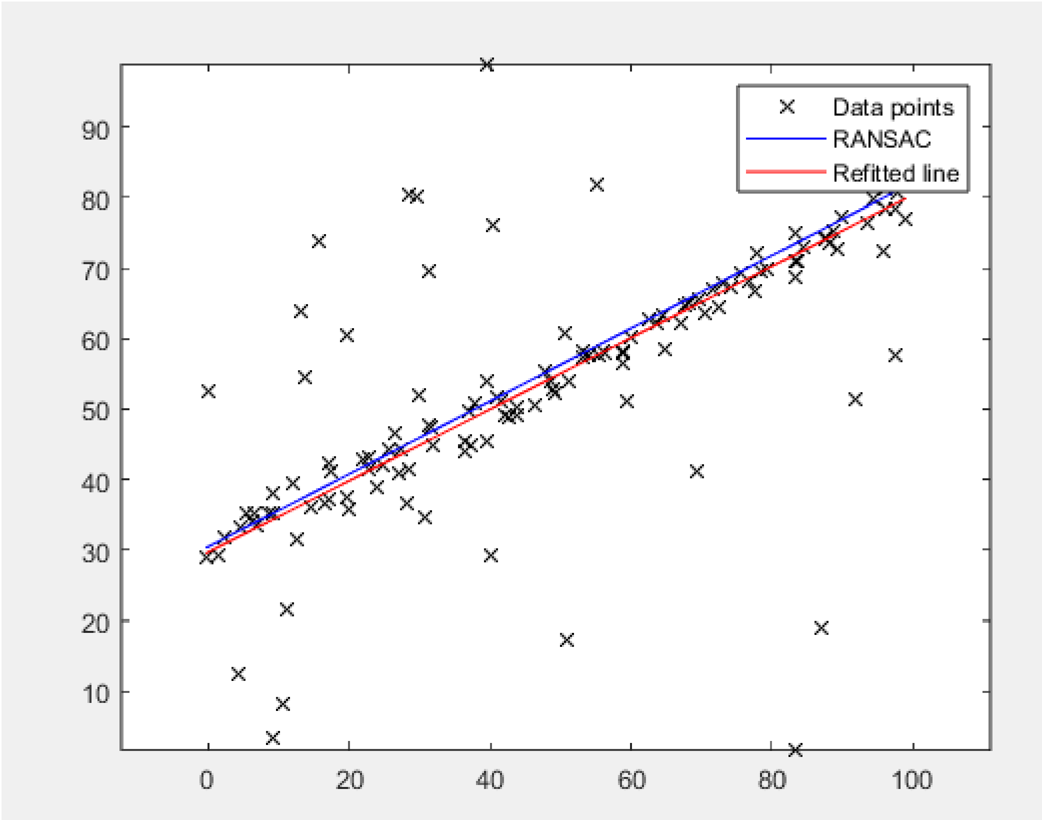
minimize $\| U (a, b)^T \|$ $\quad | a^2 + b^2 = 1 \qquad \|N\|^2 = 1$

$$E = (UN)^T (UN)$$

$$= [a, b] \underbrace{U^T U \begin{bmatrix} a \\ b \end{bmatrix}}_{=0} = 0$$

$$\frac{dE}{dU} = 2(U^T U) N = 0$$

$$(U^T U) N = 0$$




```
Inliers: 101
Best line coeffs: m= 0.516709, b=30.477242
fx >>
```