Bean Machine Physics Modeling

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***Abstract*—The bean machine is used to demonstrate the central limit theorem. This research examines the effect shifting, adding and removing pegs from the bean machine has on the resulting distribution along the base of the board.**

1. Introduction

The bean machine, or Galton Box, is a vertical board with pegs placed in interleaved rows. A ball is dropped into the top of the "machine" and falls bouncing off the pegs until it lands in a bin at the base. Given a large enough board this results in an approximately normal distribution along the base, centered at the middle of the board.

In the research that follows, we have created a physics simulation of a similar board, and examined the changes in distribution that result from moving the pegs. The simulation was created using NVidia PhysX, and differs from the original game in two ways. First, because the simulation is deterministic the ball is dropped at evenly spaced locations along the top of the board in small increments. Second, in order to prevent the ball from falling forward off the board, the board has been tilted at a 45 degree angle.

The data gathered through the research we performed for this report will be used to suggest ideal board configurations and layouts, as well as fair scoring for the bins at the base of the machine. The report is split into five major sections: this introduction, details of the simulation plans, raw results of the simulation execution, interpreted results of the simulation execution, and finally the suggested board layout and scoring systems. After the conclusion of this paper follows an appendix containing the exact board specifications used for the simulations.

2. Simulation Details

For the purpose of this simulation we have created 5 different peg configurations. Each configuration describes a board which is 20" by 20", tilted at 45 degrees. The board has walls on each side. The configuration specifies the locations of a number of pins, each of which is exactly 1" in diameter. A ball (2" in diameter) is dropped at one-thousand evenly-spaced intervals along the top of the board.

2.1 DEVS Formalism

- ***X*** is the set of input events: {ball falls, hits peg}
- ***Y*** is the set of output events - {land in bucket}
- ***S*** is the set of sequential states (or also called the set of partial states) - {the xy position of the ball as it falls through the game, the x position of the ball drop}
- $ta : S \rightarrow T^{\infty}$ is the time advance function which is used to determine the lifespan of a state - {advance the physics engine, so the time advance of the physics engine)
- $S_{ext} : Q \times X \rightarrow S$ is the external transition function which defines how an input event changes a state of the system, where $Q = \{(s, t_e) | s \in S, t_e \in (\mathbb{T} \cap [0, ta(s)])\}$ is the set of total states, and t_e is the elapsed time since the last event:
- The set of partial states intersected with the state of the peg configuration, the state transition is the physx engine telling the ball to report where it is in the grid after doing the physics work on it.

2.2 First Simulation

For the first simulation the pegs were arranged as closely to the original bean machine layout as possible. Pegs were placed along the base of the board in such a way as to divide it into seven two-inch bins at the base, just wide enough for the ball.

2.3 Second Simulation

For the second simulation the pegs were placed starting half-way into the wall on either side, then moving three inches each peg for every other row. For the in-between rows a peg was placed in the middle of each set of two adjacent pegs on the row above it. There are 5 rows, starting at the bottom and moving up three inches per row. This was done to increase the spacing from that found in the first simulation, thus giving the ball more room to move.

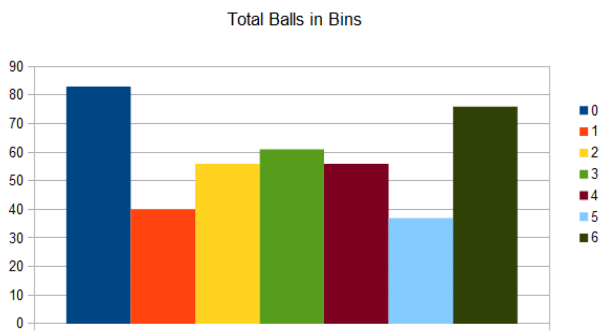
2.4 Third Simulation

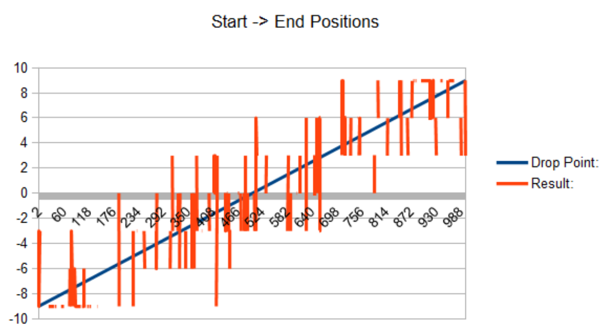
For the third and final simulation we arranged the pegs in the shape of the letter "Z." In order to prevent the ball falling straight down the board without any collisions we placed additional pegs around the center of the board, in the most open spaces to complicate the path of the ball.

3. Simulation Results

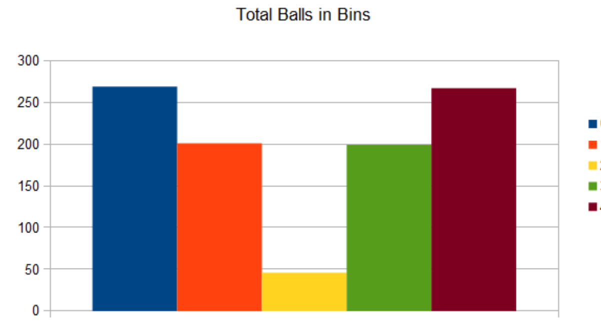
For each simulation we recorded the landing point of the ball after each drop. Here we have detailed the results of those records both in the form of a bar graph depicting the number times the ball fell into each bin, as well as a line-graph depicting the drop and landing locations for each simulation.

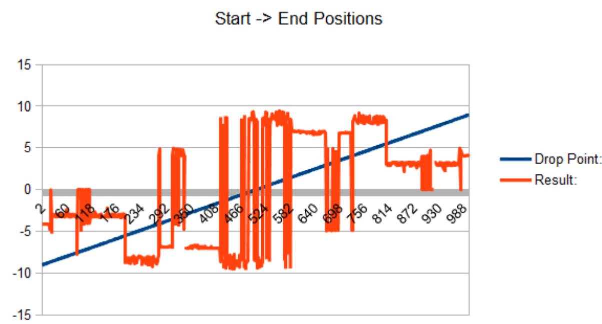
3.1 First Simulation



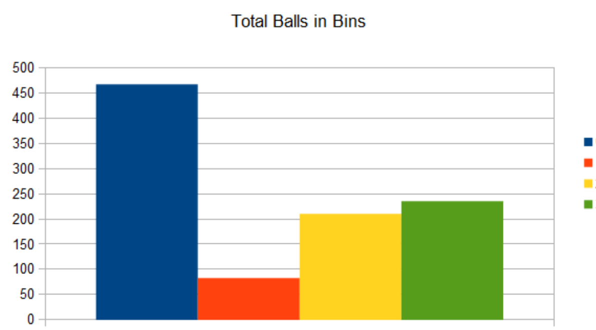


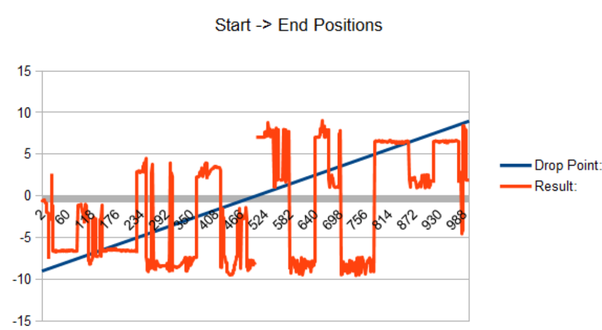
3.2 Second Simulation





3.3 Third Simulation





4. Result Interpretation

4.1 First Simulation

Due to the constricted nature of the board, the ball fell on numerous occasions directly onto the top of a pin with zero horizontal velocity. This combined with the deterministic nature of the simulation meant that the ball would balance perfectly on the pin. This is denoted in the line graph by the blank space between many of the results. The distribution, however, remains interesting. Due again to the deterministic nature of the simulation, we couldn't drop the ball in the exact center each time as in a traditional bean machine. As a result, we retained the normal-curve appearance near the center bins, but had a large jump near the edges from all the drops near one side or the other.

4.2 Second Simulation

Opening up the board a bit solved the problem we ran into with the ball balancing on a peg, but unfortunately resulted in bins of different sizes. As a result, the center bin is only two-inches wide, while the others are either three inches, or three and a half on the outside. While the distribution represents the smaller bin in the center, it seems to contradict the larger bins on the edges. We're attributing this to the half-pegs along the walls, forcing the ball to bound away from the edges more often and towards the center.

4.3 Third Simulation

Perhaps the most interesting result, the ball tended heavily towards the left side of the board, falling into the far left bin nearly 50% of the time. However, only very rarely falling into the bin adjacent to it. This seems to be the result of the placement of the middle pegs, which were used to prevent the ball falling straight down the board.

5. Conclusion

Out first question was to see if there was any best position for the ball to be dropped from. Our results for the third simulation seem to indicate that it makes no difference, strangely the ball is more likely to land in bin 1. This is somewhat mirrored in the second simulation, as shown in the line-graph, there is a large disconnect between the drop location and the end position. However for the first set there is far less variance.

Our second question was one of how to score the bins. For all the above simulations, scores should be assigned to the bins based inversely on the number of runs which resulted in the ball falling into that bin. For a system intended to compete with humans, one should also examine the difference between the drop location and the landing location. Even if the ball often lands on the left side, if it only did so when dropped on the right half of the board a human player would be unlikely to ever drop a ball into it.

6. APPENDIX

6.1 PEGS1.TXT

45
-10 6.5
10 6.5
-9 5.5
-6 5.5
-3 5.5
0 5.5
3 5.5
6 5.5
9 5.5
-7.5 2.5
-4.5 2.5
-1.5 2.5
1.5 2.5
4.5 2.5
7.5 2.5
-10 0.5
10 0.5
-9 -0.5
-6 -0.5
-3 -0.5
0 -0.5
3 -0.5
6 -0.5
9 -0.5
-7.5 -3.5
-4.5 -3.5
-1.5 -3.5
1.5 -3.5
4.5 -3.5
7.5 -3.5
-10 -5.5
10 -5.5
-9 -6.5
-6 -6.5
-3 -6.5
0 -6.5
3 -6.5
6 -6.5
9 -6.5
-7.5 -9.5
-4.5 -9.5
-1.5 -9.5
1.5 -9.5
4.5 -9.5
7.5 -9.5

6.2 PEGS2.TXT

28
-10 6.5
-5.5 6.5
-1.5 6.5
1.5 6.5
5.5 6.5
10 6.5
-7.5 2.5
-3.5 2.5
0 2.5
3.5 2.5
7.5 2.5
-10 -1.5
-5.5 -1.5
-1.5 -1.5
1.5 -1.5
5.5 -1.5
10 -1.5
-7.5 -5.5
-3.5 -5.5
0 -5.5
3.5 -5.5
7.5 -5.5
-10 -9.5
-5.5 -9.5
-1.5 -9.5
1.5 -9.5
5.5 -9.5
10 -9.5

6.3 PEGS3.TXT

17
-10 6.5
-5 6.5
0 6.5
5 6.5
10 6.5
-5 2.5
2.5 2.5
-7.5 -1.5
0 -1.5
7.5 -1.5
-2.5 -5.5
5 -5.5
-10 -9.5
-5 -9.5
0 -9.5
5 -9.5
10 -9.5