Number System

Fraction → Percentage

1 → 100%
1/2 → 50%
1/3 → 33.33%
1/4 → 25%
1/5 → 20%
1/6 → 16.66%
1/7 → 14.28%
1/8 → 12.5%
1/9 → 11.11%
1/10 → 10%
1/11 → 9.09%
1/12 → 8.33%
1/13 → 7.69%
1/14 → 7.14%
1/15 → 6.66%

#

Eg - 37.5% → Fraction

$3 \times 12.5\% = 3 \times \frac{1}{8} = \frac{3}{8}$

Squares, Cubes, factorials

Squares		Cubes	Factorials
1 - 1	16 - 256	1 - 1	
2 - 4	17 - 289	2 - 8	1 - 1
3 - 9	18 - 324	3 - 27	2 - 2
4 - 16	19 - 361	4 - 64	3 - 6
5 - 25	20 - 400	5 - 125	4 - 24
6 - 36	21 - 441	6 - 216	5 - 120
7 - 49	22 - 484	7 - 343	6 - 720
8 - 64	23 - 529	8 - 512	7 - 5040
9 - 81	24 - 576	9 - 729	8 - 40320
10 - 100	25 - 625	10 - 1000	9 - 362880
11 - 121	26 - 676	11 - 1331	10 - 3628800
12 - 144	27 - 729	12 - 1728	
13 - 169	28 - 784	13 - 2197	
14 - 196	29 - 841	14 - 2744	
15 - 225	30 - 900	15 - 3375	

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Non-Terminating & Recurring to Fraction

$2.\overline{53} \rightarrow$ Fraction

$x = 2.\overline{53}$ $100x = 253.\overline{53}$

$99x = 251$

$x = \frac{251}{99}$

$2.5\overline{3} \rightarrow$ Fraction

$10x = 25.\overline{3}$ $100x = 253.\overline{3}$

$90x = 228$

$x = \frac{228}{90}$

Check if a no. is prime or not

$79 \rightarrow$ Take root $\rightarrow \sqrt{79} = 8.xxx$

Divide 79 by prime no.s only till 8

i.e. 2, 3, 5, 7

As none of them divides 79, it is prime

$111 \rightarrow \sqrt{111} = 10.xxx$

(2, 3, 5, 7) 111 is a composite no.

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Rules of Divisibility

2 - Even number
4 - Last 2 digits should be divisible by 4
8 - Last 3 digits should be divisible by 8
5 - Last digits should be 0 or 5
3 - Sum of digits should be divisible by 3
9 - Sum of digits should be divisible by 9
10 - 0 at units place
6 - Should be divisible by 3 & 2
11 - (Sum of digits at even place - Sum of digits at odd place) should be divisible by 11 or should be 0

For 7 or any prime no,

→ Find multiple ending with 9

$7 \times 7 = 49$ $\quad$ $13 \times 3 = 39$ $\quad$ $17 \times 7 = 119$

→ Add 1

50 $\quad$ 40 $\quad$ 120

→ Divide by 10

5 $\quad$ 4 $\quad$ 12

Above is seed no.

Eg) 87415 divisible by 7 or not

→ Seperate units place from rest of the number

8741 $\quad$ 5

→ Multiply seperated units no. with seed no. & add to the remaining number

$8741 + (5 \times 5) = 8741 + 25 = 8766$

→ continue till you get an easy no. to check

$876 + (5 \times 6) = 876 + 30 = 906$

$90 + (5 \times 6) = 90 + 30 = 120$

$12 + (0 \times 6) = 12$

$\therefore$ 87415 is not divisible by 7

⇒ 2^n or 5^n → n digits from units place divisible by 2^n or 5^n

HCF & LCM

No. of factors

$$N = a^x \times b^y \times c^z$$

Factorise N where a, b, c are prime no.s

$$\therefore f = (x+1)(y+1)(z+1)$$

Eg) $60 - 2^2 \times 3^1 \times 5^1$

$$f = (2+1)(1+1)(1+1)$$

$$= 3 \times 2 \times 2$$

$$= 12$$

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Properties of HCF & LCM

LCM × HCF = a × b

a & b are 2 no.s

Co-prime → 2 or more no.s who's HCF is 1

Eg) • $420 - 2^2 \times 3 \times 5 \times 7$

Co-primes → 4, 3, 5, 7

• $12 - 2^2 \times 3$

Co-primes → 4, 3

→ 2 consecutive no.s are always co-prime

HCF & LCM of Fractions

→ HCF = $\frac{\text{HCF of Numerator}}{\text{LCM of Denominator}}$

→ LCM = $\frac{\text{LCM of Numerator}}{\text{HCF of Denominator}}$

Surds & Indices

Indices

→ $a^m \times a^n = a^{m+n}$

→ $\frac{a^m}{a^n} = a^{m-n}$

→ $(a^m)^n = a^{mn}$

→ $\sqrt[n]{a} = a^{1/n}$

→ $\sqrt[n]{a^p} = a^{p/n}$

→ $a^{-n} = \frac{1}{a^n}$

→ $a^0 = 1$ (0^0 is not defined)

→ $(a \times b)^m = a^m \times b^m$

or $a^m \times b^m = (a \times b)^m$

→ $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

or $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

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Properties of Indices

→ If $a^m = a^n$, then $m = n$ if $(a \neq 0, 1, -1)$

→ If $a^m = b^m$, then $a = b$ if m is odd
$a = \pm b$ if m is even

Surds

→ $(\sqrt[n]{a})^n = a$
→ $\sqrt[n]{a} \times \sqrt[n]{b} = \sqrt[n]{ab}$ or $(ab)^{1/n}$
→ $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ or $\left(\frac{a}{b}\right)^{1/n}$ or $\frac{a^{1/n}}{b^{1/n}}$
→ $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ or $(a)^{1/mn}$
→ $(\sqrt[n]{a})^m = a^{m/n}$

Pure ⟷ Mixed Surd

$\sqrt{24} = \sqrt{2\times2\times2\times3} = 2\sqrt{6}$
(Pure) (Mixed)

$3\sqrt{3} = \sqrt{3\times3\times3} = \sqrt{27}$
(Mixed) (Pure)

Positive Square root of a Surd

Eg) $\sqrt{7+4\sqrt{3}} = a+b$

$a^2 + 2ab + b^2 = 7 + 4\sqrt{3}$ $\quad a^2+b^2 = 7$

$a^2+b^2+2ab = 4+3+2\times2\times\sqrt{3}$ $\quad 2ab = \overset{2}{\cancel{4}}\sqrt{3}$

$a^2+b^2+2ab = (2)^2 + (\sqrt{3})^2 + (2)(2)(\sqrt{3})$ $\quad ab = 2\sqrt{3}$

$= (2+\sqrt{3})^2$ $\quad a = 2,\ b = \sqrt{3}$

$a+b = 2+\sqrt{3}$

Eg → $\sqrt{8-2\sqrt{15}} = a-b$

$a^2+b^2-2ab = 8-2\sqrt{15}$

$a^2+b^2 = 8$

$2ab = 2\sqrt{15}$

$ab = \sqrt{15}$

$a = \sqrt{5}$, $b = \sqrt{3}$

$= (\sqrt{5})^2 + (\sqrt{3})^2 - 2(\sqrt{5})(\sqrt{3})$

$(a-b)^2 = (\sqrt{5}-\sqrt{3})^2$

$a-b = \sqrt{5}-\sqrt{3}$

Speed Calculations

→ Square of a no. ending with 5
- Last two digits → 25
- Multiply the no. apart from 5 with the next consecutive no.
- Place the result before 25

Eg) $35^2 = 12\,25$ (3×4) $75^2 = 5625$ (7×8)

→ Multiplying 2 no.s close to base 100

1) $a, b > 100$

102×107

$102 \quad +2$
$107 \quad +7$

10914

2) $a, b < 100$

94×89

$94 \quad -6$
$89 \quad -11$

$83\,66$

3) $a > 100$ & $b < 100$

105×91

$105 \quad +5$
$91 \quad -9$ $\quad \times \; -45$

$9600 - 45$
$= 9555$

→ Split Calculation

Divide no. into simplified forms

Eg) $105 \times 211 = 211 \times 100 + 211 \times 5$
$= 21100 + 1055 = 22155$

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Eg) 23% of 170 $\Rightarrow$ 20% of 170 + 3% of 170
34 + 5.1 = 39.1

Sequence & Series

AP

- Arithmetic Progression or AP - Common difference 'd'

  1st term ← a, a + d, a + 2d . . . . a + (n - 1)d → No. of terms

  $T_n / a_n = a + (n-1)d$

- a, b, c are in AP (Only for 3 terms)

  $c - b = b - a$

  $2b = a + c$

  $b = \frac{a+c}{2}$

  Middle term is AM of other 2 terms

- Sum of terms in AP

  $S_n = \frac{n}{2}(T_1 + T_n)$ → 1st & Last term or equidistant terms from both sides

  $S_n = \frac{n}{2}[2a + (n-1)d]$

- Combination of 2 AP's

  $AP_1 \rightarrow 1, 4, 7, 10 \ldots\ldots \quad d_1 = 3$

  $AP_2 \rightarrow 2, 4, 6, 8 \ldots\ldots \quad d_2 = 2$

  $AP_1 + AP_2 = 3, 8, 13, 18 \ldots \quad d = 5$

  $d_1 + d_2 \Rightarrow d$

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GP or Geometric Progressions

1st term ← a, ar, ar^2 ..... ar^{n-1}

(n = no. of terms; r → common ratio)

$$a_n = ar^{n-1}$$

Sum of terms in GP

$$S_n = a\left[\frac{r^n - 1}{r - 1}\right]$$ when $r > 1$ & $r < -1$

Sum of Infinite terms in GP

$$-1 < r < 1$$

$$S_\infty = \frac{a}{1-r}$$

AGP (Arithmetic Geometric Progression)

Comb. of AP & GP in a single Serie | Sequence

Eg) $2 + \frac{4}{5} + \frac{6}{5^2} + \frac{8}{5^3} \ldots\ldots = N$ — ①

$Nr \rightarrow AP$

$Dr \rightarrow GP$

Multiply LHS & RHS by 5

$$10 + 4 + \frac{6}{5} + \frac{8}{5^2} \ldots = 5N \text{ — ②}$$

② - ①

$$4N = 12 + \frac{2}{5} + \frac{2}{5^2} \ldots\ldots$$

$$4N = 12 + 2\left(\frac{1}{5} + \frac{1}{5^2} \ldots\ldots\right)$$

$$4N = 12 + 2\left(\frac{1/5}{1-1/5}\right)$$

$$4N = 12 + \frac{\cancel{2}}{\cancel{5}} \times \frac{\cancel{5}}{\cancel{4}\,2}$$

$$4N = \frac{25}{2}$$

$$N = \frac{25}{8}$$

Sum of first 'n' natural no.s

→ First 'n' natural no.s $= \frac{n(n+1)}{2}$

→ First 'n' even natural no.s $= n(n+1)$

→ First 'n' odd natural no.s $= n^2$

→ Sum of Squares of first 'n' natural no.s $= \frac{n(n+1)(2n+1)}{6}$

→ Sum of cubes of first 'n' natural no.s $= \left[\frac{n(n+1)}{2}\right]^2$

Combinations

$${}^{a}C_{b} = \frac{a!}{(a-b)!\,b!}$$

Eg) ${}^{5}C_{2} = \frac{5!}{3!\,2!} = \frac{5\times4}{2} = 10$

${}^{6}C_{4} = \frac{6!}{2!\,4!} = \frac{6\times5}{2} = 15$

#. $n \times n! = (n+1)! - n!$

Imp. formula for factorials

#. Sum of n terms whose common difference is in AP

$AP \rightarrow 1, 3, 6, 10 \ldots\ldots$

$d \rightarrow 2, 3, 4 \ldots\ldots$

$T_n = an^2 + bn + c$

$n = 1, \quad T_1 = a + b + c$

$a + b + c = 1$

$n = 2, \quad 4a + 2b + c = 3$

$n = 3, \quad 9a + 3b + c = 6$

$a = \frac{1}{2}, \quad b = \frac{1}{2}, \quad c = 0$

$\cancel{S_n = \frac{\Sigma n^2}{2}} \qquad T_n = \frac{n^2}{2} + \frac{n}{2}$

$S_n = \frac{1}{2}\Sigma n^2 + \frac{1}{2}\Sigma n$

$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$

$= \frac{1}{2} \times \frac{n(n+1)}{2}\left(\frac{2n+1}{3} + 1\right)$

If we want to find Sum of first 13 terms

$S_{13} = \frac{1}{2} \times 13 \times \frac{\cancel{14}^{7}}{\cancel{2}}\left(\frac{\cancel{27}^{9}}{\cancel{3}} + 1\right)$

$= \frac{13 \times 7 \times \cancel{10}^{5}}{\cancel{2}} = \underline{\underline{455}}$

STATS

Standard Deviation

$$\sigma = \sqrt{\frac{a(-m+x)^2 + b(-m+y)^2 \ldots}{n}}$$

σ = Std. deviation
$a, b, c \ldots$ = no. of times values are occuring
$x, y, z \ldots$ = Values
m = mean of values
n = no. of terms

Eg - SD of 5,5,5,8,8,7,9,9

$$m = \frac{56}{8} = 7$$

$$\sigma = \sqrt{\frac{3(-7+5)^2 + 2(-7+8)^2 + 1(-7+7)^2 + 2(-7+9)^2}{8}}$$

$$= \sqrt{\frac{12+2+0+8}{8}} = \sqrt{\frac{\cancel{22}^{11}}{\cancel{8}_4}} = \frac{\sqrt{11}}{2}$$

Relation b/w Mean, Median & Mode

Mode = 3 (Median) − 2 (Mean)

* Range = Highest Value − Lowest Value

* Mean deviation from median/ Mean or mode = 1) Median − each term (|M|)
2) Add the deviations
3) Divide by no. of terms

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(*) Variance $= (\sigma)^2$
↓
Standard deviation

(*) $C.V = \frac{\sigma}{m} \times 100$

C.V → coefficient of variance
m → mean

Prime Numbers

→ 1 - 100 → 25
→ 1 - 200 → 46
→ 1 - 500 → 95
→ 1 - 1000 → 168

Imp (Shortcut)