Percentages (%)

Absolute % change (Increase | Decrease) $= \dfrac{\text{Final Value} - \text{Initial Value}}{\text{Initial Value}} \times 100$

Multiplying Factor $= \text{Original Value} + \dfrac{\%\ \text{Inc.} \mid \text{Dec.} (OV)}{100\%}$

Eg) y is inc. by 20%.

$$y = y + \frac{20\%(y)}{100\%} = y + \frac{y}{5} = \frac{6}{5}y \text{ or } \underbrace{1.2}_{\text{Multiplying Factor}}y$$

Successive % change

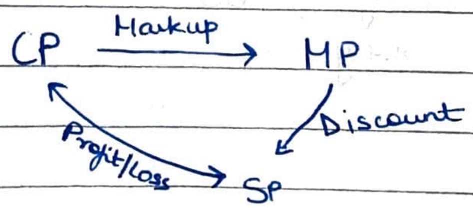
If there is a successive % inc. of a% & b% on Original Value

Net increase $= \left(a + b + \dfrac{ab}{100}\right)$

Net decrease $= \left[(-a) + (-b) + \dfrac{(-a)(-b)}{100}\right] = \left(-a - b + \dfrac{ab}{100}\right)$

Profit & Loss

Selling Price (SP)	Discount
Cost Price (CP)	Markup
Marked ~~Markup~~ Price (MP)	Profit / Loss



DATE
PAGE

→ Markup = HP − CP

→ Discount = HP − SP

→ Profit (SP > CP) = SP − CP

→ Loss (CP > SP) = CP − SP

$$\text{Markup \%} = \frac{HP - CP\ (\text{Markup})}{CP} \times 100$$

$$\text{Discount \%} = \frac{HP - SP\ (\text{Discount})}{HP} \times 100$$

$$\text{Profit \% / Loss \%} = \frac{\text{Profit / Loss}}{CP} \times 100$$

Cases related to Profit & Loss

→ $SP_A = SP_B$

A sold at $x\%$ profit

B sold at $x\%$ loss

Then, $\text{Loss \%} = \frac{x^2}{100}$

→ If $CP_A = CP_B$

A sold at $x\%$ profit

B sold at $x\%$ loss

Then, No Profit No Loss

→ Buy x get y items free

$$\text{Discount \%} = \frac{y}{x+y} \times 100$$

→ Successive Discounts of a% & b% = $\left(a+b-\frac{ab}{100}\right)$

Simple & Compound Interest

→ $SI = \frac{P \times R \times T}{100}$

→ $CI = A - P$

$A = P\left(1+\frac{R}{100}\right)^T$

→ For 1^{st} year, SI = CI

→ For 2 years, ~~SI~~ $CI - SI = \frac{PR^2}{100^2}$

Installments

Time Value of Debt = ~~A+~~ P + SI or CI $= 275 + \frac{275 \times 25 \times 4}{100}$

= 550

SI

	I_1	I_2	I_3	I_4
T_1	x	×	×	×
T_2	$1.25x$	x	×	×
T_3	$1.5x$	$1.25x$	x	×
T_4	$1.75x$	$1.5x$	$1.25x$	x

⇒ $5.5x = 550$

$x = 100$

P = 275, R = 25%, T = 4 yrs

$TV_D = 275 + \frac{25}{16}$

$= 276.5625$

	I_1	I_2	I_3	I_4
T_1	x	×	×	×
T_2	$1.25x$	x	×	×
T_3	$1.5625x$	$1.25x$	x	×
T_4	$1.953125x$	$1.5625x$	$1.25x$	x

⇒ $6.015625x = 276.5625$

$x = 45.97$

Ratios

$$A : B = \frac{A}{B}$$

$$A : B : C \Rightarrow A = \frac{A}{A+B+C} \ldots C = \frac{C}{A+B+C}$$

Proportions & Variations

Direct Proportion $\Rightarrow a \propto b$ or $a = Kb$ → constant

$$K = \frac{a}{b}$$

Inverse Proportion $\Rightarrow a \propto \frac{1}{b}$ or $a = K \cdot \frac{1}{b}$

$$K = ab$$

Mixed Variation $\Rightarrow a \propto b^2$ & $a \propto \frac{1}{\sqrt{c}}$

then $a \propto \frac{b^2}{\sqrt{c}}$ or $a = K \cdot \frac{b^2}{\sqrt{c}}$

$$K = \frac{a\sqrt{c}}{b^2}$$

Partnership

$$\frac{P_1}{P_2} = \frac{C_1 \times T_1}{C_2 \times T_2}$$

P_1 & P_2 → Profits

C_1 & C_2 → Capital Invested

T_1 & T_2 → Time period

Eg)

	A	:	B
C	4	:	3
T	6	:	7
P	?=2	:	7

$$\frac{P_1}{P_2} = \frac{\cancel{4}^{2} \times \cancel{3}}{\cancel{2}\cancel{6} \times 7} = 4\ \frac{2}{7}$$

DATE

PAGE

Means & Weighted Averages

→ Arithmetic Mean or AM = $\frac{a+b}{2}$ or $\frac{a+b+c}{3}$ ..... $\frac{a+b+c.....n}{n}$

→ Geometric Mean or GM = $\sqrt[2]{ab}$ or $\sqrt[3]{abc}$ or $\sqrt[n]{abc....n}$ for 'n' values

→ Harmonic Mean or HM = $\frac{2ab}{a+b}$ or $\frac{2}{1/a + 1/b}$

Note: AM $\geq$ GM $\geq$ HM & AM $\times$ HM = $(GM)^2$

Mean, Median & Mode

For a set of 'n' numbers,

→ Mean = $\frac{n+1}{2}$ (n → odd)

→ Mean = Avg $\left(\frac{n}{2}, \frac{n+1}{2}\right)$ (n → even)

→ Median (odd) = $\left(\frac{n+1}{2}\right)^{th}$ Value when numbers are arranged in ascending / Descending order

→ Median (Even) = ~~Avg $\left(\frac{n+1}{2}\right)$~~ 2 middle values 'a' & 'b' $\left(\frac{a+b}{2}\right)$ [redacted]

→ Mode = Max. occurring Value

Weighted Average

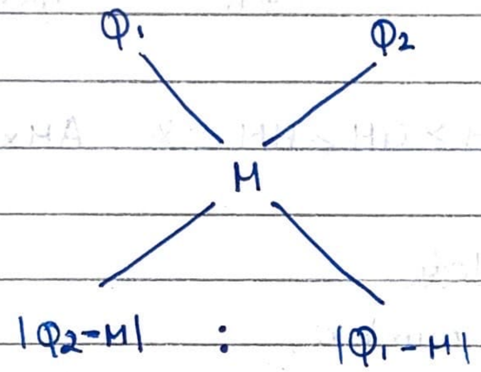
$$W_A = \frac{Q_1W_1 + Q_2W_2 + \dots Q_nW_n}{\cancel{W}\ Q_1 + Q_2 \dots + Q_n}$$

$Q \rightarrow$ Quantities

$W \rightarrow$ Weight of Quantities

Alligations

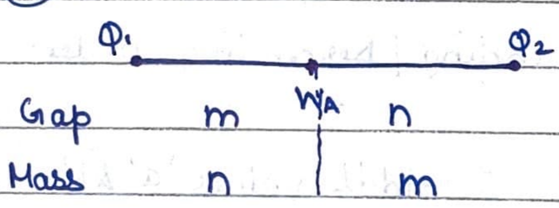
①



$Q_1, Q_2 \rightarrow$ 2 Quantities in same measurement

$M \rightarrow$ Mixture that has the same measurement as Q_1 & Q_2

②



$$\text{Gap} \propto \frac{1}{\text{mass}}$$

Time & Work

Work done = Efficiency (Rate) × Time

↓ (Distance) ↓ (Speed) ↓ (Time)

Fraction Method

A → 10 days to complete the work
B → 15 days to complete the work
How many days can A & B take to finish the work?

$$\frac{1}{10} + \frac{1}{15} = \frac{5}{30} = \frac{1}{6}$$ th work in 1 day

1 work → 6 days

LCM Method

A → 10 days & B → 15 days to complete the work. How many days will be required by A & B to finish the work?

LCM(10, 15) = 30 = Work done

$S_A = 3$ u/day , $S_B = 2$ u/day
Total 'S' = 5 u/day

30 = 5 × T
T = 6 days

DATE
PAGE

Concept of Negative Work

A → 15 mins to fill the tank, B → 20 mins to fill the tank, C → 30 mins to empty the tank. How much time will it take to fill the tank?

$LCM(15, 20, 30) = 60 = W$

$S_A = 4\, l/min$, $S_B = 3\, l/min$, $S_C = -2\, l/min$

Total 'S' = $5\, l/min$

$60 = 5 \times T$

$T = 12$ mins.

Time, Speed & Distance

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

Conversion → $1\, Km/h = \frac{5}{18}\, m/s$

→ $1\, m/s = \frac{18}{5}\, Km/h$

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}}$$

- t = Same, Avg. Speed $= \frac{S_1 + S_2}{2}$ (A.M.)
- S = Same, Avg. Speed $= S$
- d = Same, Avg. Speed $= \frac{2S_1S_2}{S_1 + S_2}$ (H.M.)

Ratios To Remember

1) d = constant

$$S \propto \frac{1}{t} \Rightarrow \frac{S_1}{S_2} = \frac{t_2}{t_1}$$

2) t = constant (Start at same time & meet at one point)

$S \propto d$

$$\frac{S_1}{S_2} = \frac{d_1}{d_2}$$

Relative Speed

1) Objects moving in different directions

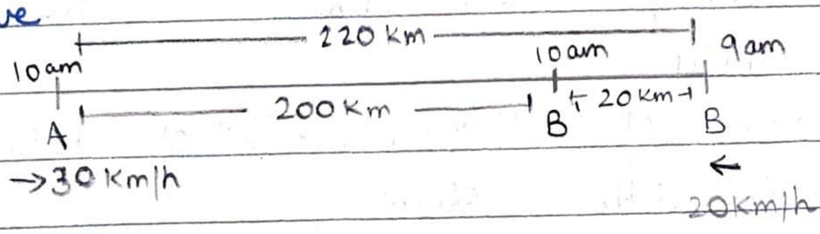
→ Starting Simultaneously

- $S_{rel} = S_A + S_B$ (Relative Speed)
- t → constant
- Gap → Distance b/w 2 objects at same time

$$T_m = \frac{\text{Gap}}{S_{rel}}$$

T_m → Meeting Time

→ Starting Seperately : Make the time same, solve like above



B → B' (Time → 10 am)

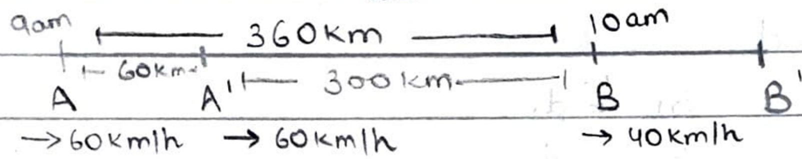
$$T_m = \frac{200}{50} = 4 \text{ hrs}$$

2) Objects moving in Same Direction

→ $S_{rel} = S_A - S_B \quad (S_A > S_B)$

→ t → constant

$$T_m = \frac{Gap}{S_{Rel}}$$



$$T_m = \frac{300}{20} = 15 \text{ hrs}$$

Applications of TSD (Train)

Situation	Relative Distance	Speed
I) Stationary Object	'L' of train	'S' of train
→ Train + Objects of negligible width		
→ Train + Object of Substantial width	'L' of train + 'W' of Object	'S' of train
II) Moving Object		
→ Train + Object of negligible width	'L' of train	Relative Speed
→ Train + Object of substantial width	'L' of train + 'W' of Object	Relative Speed

Applications of TSD (Boats & Streams)

Speed of boat in still water = a
Speed of river | Stream | Current = b
S_u → Upstream Speed = a − b
S_d → Downstream Speed = a + b

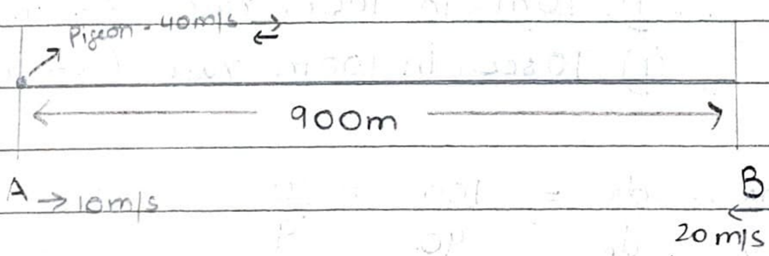
NOTE

* Use relative ~~time~~ Speed if finding the meeting time
* Use Speed of river | Upstream | Downstream if distance, Speed or time from one object is to be measured

Forward & Backward Distance

If an object moves from one object to another & keeps repeating the same till the two objects finally meet.

Eg −



$T_m(A, B) = \frac{900}{30}$

$= 30$ sec

Total distance travelled by the pigeon = S × t
= 40 × 30 = 1200m

∴ Forward + Backward dist. of pigeon = 1200
F + B = 1200 —— ①

F − B = Distance travelled by A
= S × t
= 10 × 30 = 300m
F − B = 300 —— ②

from ① & ②,

F = 750m , B = 450m

Linear Race

① Time constant ⇒ $S \propto d$

Eg - A beats B by 10m in a 100m race

$$\frac{S_A}{S_B} = \frac{d_A}{d_B} = \frac{100}{90} = \frac{10}{9}$$

② Distance Constant ⇒ $S \propto \frac{1}{t}$

Eg - A beats B by 10sec in 100m race

$$\frac{S_A}{S_B} = \frac{t_B}{t_A} = \frac{t+10}{t}$$

③ Head Start

Eg - A gives B a head start of -

i) 10m in 100m race ($t \rightarrow$ constant)

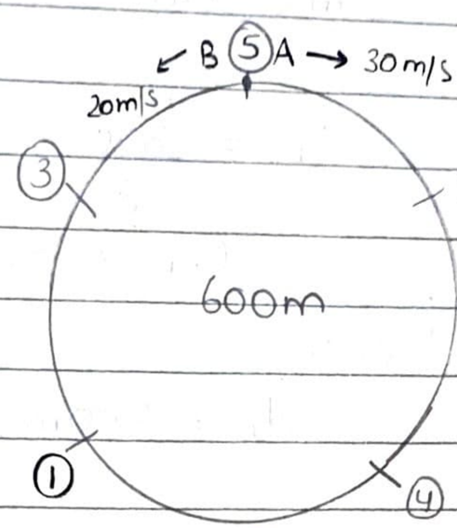
ii) 10sec in 100m race ($d \rightarrow$ constant)

i) $\frac{S_A}{S_B} = \frac{d_A}{d_B} = \frac{100}{90} = \frac{10}{9}$

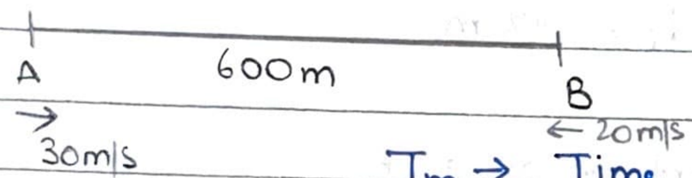
ii) $\frac{S_A}{S_B} = \frac{t+10}{t}$

Circular Races

I) Opposite Directions



The circular track can be converted to a linear track



$T_m \rightarrow$ Time taken for 1^{st} meeting

2^{nd} & 3^{rd} meeting will have Same time as time taken for 1^{st} meeting

Eg) $T_1 = \frac{600}{50} = 12$ sec $\quad (S_{Rel} = S_A + S_B)$

$\therefore T_2 = 12$ sec , $T_3 = 12$ sec

At T_1, $d_A = 360m$ & $d_B = 240m$

For distances for $T_2, T_3 \ldots\ldots$

$\frac{S_A}{S_B} = \frac{3}{2} \Rightarrow$ Distinct points $= 3 + 2 = 5$

$\therefore$ Every point will be 120m away from it's adjacent point.

d_A at $T_2 = 720m \; (600 + 120)m \Rightarrow$ 2

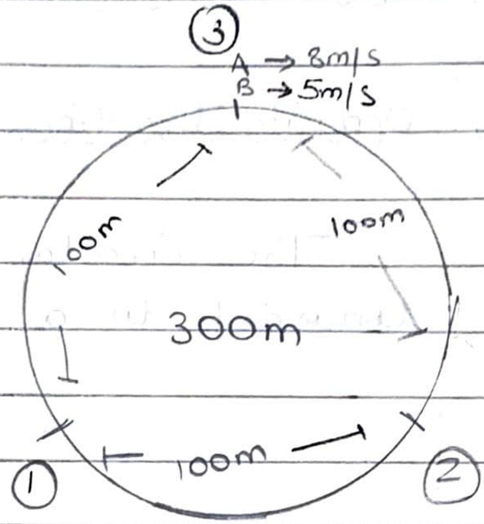
d_A at $T_3 = 1080m \; (600 + 480)m \Rightarrow$ 3

d_A at $T_4 = 1440m \; (600 + 600 + 240)m \Rightarrow$ 4

d_A at $T_5 = 1800 \; (600 + 600 + 600)\,m \Rightarrow$ 5

T_6 onwards the Cycle will be repeated

II) Same Direction



$$T_1 = \frac{300}{3} = 100\,sec$$

$(S_{rel} = S_A - S_B)$

At T_1,

$d_A = 800m, \; d_B = 500m$

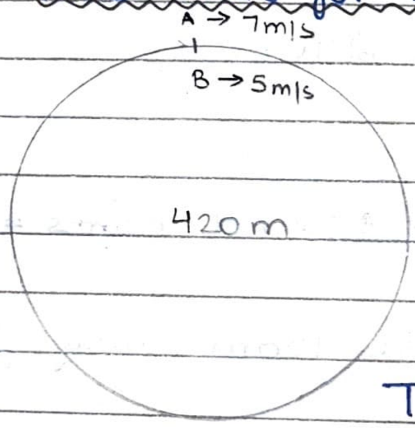
Distinct point $\Rightarrow \frac{S_A}{S_B} = \frac{8}{5}$

$\Rightarrow 8 - 5 = 3$

At T_2, $d_A = (300 + 300 + 300 + 300 + 300 + 100)m \Rightarrow ②$

T_3, $d_A = (300 \times 8)m \Rightarrow ③$

Time taken for 1st meeting at Starting point



$$T_A\,(1\ lap) = \frac{420}{7} = 60\,sec$$

$$T_B\,(1\ lap) = \frac{420}{5} = 84\,sec$$

$$T_m = LCM\,(T_A, T_B)$$
$$= LCM(60, 84)$$
$$= 420\,sec$$

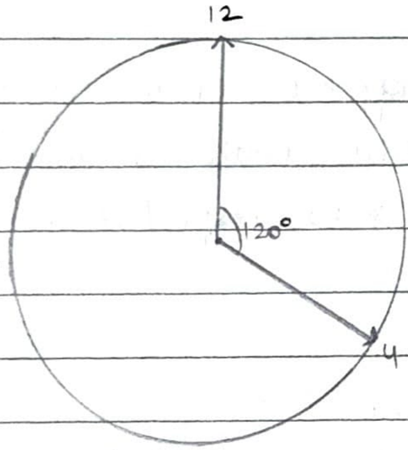
* For combinations of 3 or more people, take LCM of 2 people at a time for 2 pairs & then find the LCM of the obtained results

Clocks

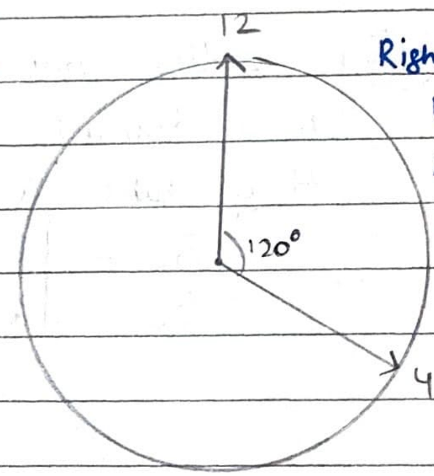
Speed of minute hand (per hour) $= \frac{360^\circ}{60\,min} = 6^\circ/min$

Speed of hour hand (per hour) $= \frac{30^\circ}{60\,min} = \frac{1}{2}{}^\circ/min$

Eg) 1) coincide b/w 4 & 5 'O' Clock



2) Right angles b/w 4 & 5 'O' clock



1) $d =$ angle subtended $= 120m$

$$T_m = \frac{120}{6-1/2} = \frac{120}{11} \times 2 = \frac{240}{11} = 21.81 \text{ mins}$$

2) ~~d = 120m~~

Case I $\rightarrow$ $d = 90m$ ($\angle = 90^\circ$)

$$T_m = \frac{90}{6-1/2} = \frac{180}{11} = 16.36 \text{ mins}$$

Case II $\rightarrow$ $d = (120+90)m = 210m$ [$\angle$ Subtended from 4 to form 90° (towards the right)]

$$T_m = \frac{210}{6-1/2} = \frac{420}{11} = 38.18 \text{ mins}$$

Wine Left Formula

$$\left(\frac{a-b}{a}\right)^n = \frac{\text{Wine}}{\text{Mixture}}$$

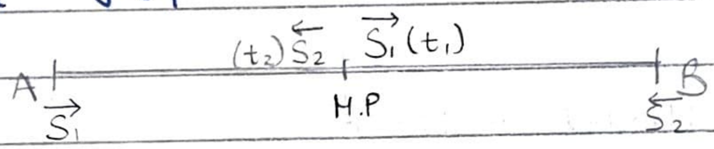
$a \rightarrow$ Mixture

$b \rightarrow$ Removed (wine)

$n \rightarrow$ no. of times process is repeated

2 bodies start from opposite ends at the same time & move towards each other with speeds S_1 & S_2. After meeting they take time t_1 & t_2 respectively to reach the dest.

$$\frac{S_1}{S_2} = \sqrt{\frac{t_2}{t_1}}$$



Effective rate of interest when multiple times compounding is done

$$r_e = \left(1 + \frac{r}{K \times 100}\right)^K - 1$$

$r_e \rightarrow$ effective rate of interest

$r \rightarrow$ rate of interest given

$K \rightarrow$ no. of times compounding is done in an year