Chapter 8
Median Location Problem

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The median problem is considered as the main problems identified with the location-allocation problems (see Chap. 5). These problems are intended to find the median points among the candidate points, so that the sum of costs can be minimized through this target function. These kinds of problems include the establishment of the public services including schools, hospitals, firefighting, Ambulance, technical audit stations of cars, and etc. The target function in the median problems is of the minisum kind. In fact in these problems we try to quantify the sum of distances (costs).

We call the first algorithm to be considered the Chinese algorithm. Apparently, the problem that motivated the development of the Chinese algorithm was the location of a threshing floor, used to separate the wheat from the chaff after a wheat harvest. In this case the tree network represents a road network, with the vertices being the location of wheat fields. The weight for each vertex represents the amount of wheat to be transported to and from the field and the threshing floor. Locating the threshing floor at the 1-median causes the total cost of transporting the wheat to and from the threshing floor to be minimized (Francis et al. 1992).

In median problems, there is the location finding in the kind of network or graph. The discussion of median problems is focused on graph theory and cannot be beyond the network or graph.

Fermat (seventeenth century) proposed one problem in order to minimize the sum of distances. In this problem, one rectangular was considered (three points on the plane) and the aim was to find a point among three points, so that the sum of distances among the chosen point could be minimized through three angles of the rectangular. The problem was developed by Alfred Weber and again was introduced in twentieth century. In this problem, Weber called them as the clients demand through applying the weight on the angles points of rectangular, and called the median point as the servicing point. He defined the servicing point among three pointof clients demand, so that the sum of distances can be minimized. This problem is called as the first problem of the location-Allocation (see Chap. 5). Later the problem proposed by Weber was introduced for multiple servicing states (multiple facilities) and developed more than three points. The problem introduced by Weber was connection problem. N can be achieved through choosing the median points on apexes points of the graph or the nodes of network. This problem was quite

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similar to the candidate of the correlated problem proposed by Weber. Again the problem was developed by Hakimi (1964), and through application of the weight on the graph, he began to find the P point on it in order to minimize the sum of the weighted distance from the points.

8.1 Classification

8.1.1 1-Median

The median problems are intended to find the location of one facility on the network, so that the total cost can be minimum.

8.1.2 P-Median

The P-median problems it is a disconnected problem and we can choose the candidate points through it.

8.1.3 An Example

Now, with an example, we want showing that how we can change continues problem to discrete problem and find feasible solution. Assume in the city we want to find some location for change store. This problem is a continuous problem. For changing this problem into a discrete problem, we can divide the city into smaller regions. These regions can be municipal regions or any other region. In every region, we determine the demand for chain stores. If we divide the city into five regions, the continuous map of city with its demands will be changed into a problem with five demand points. Determining the center of every region and putting whole of the demand on that point, the problem changes into network. For determining the center of every region we can use the weight of every region, for example the center of region can be in the east or west of region of course the location of facility is only on nodes. Consequently, we face with discrete network. Figure 8.1 is the schematic representation of this network.

In this network, the nodes are considered among two points of supply. The supply can be in one of the following forms:

- *Static demand*. That is the demand defined in a fixed or definite form
- *Probable demand*. The demand which involves a variable with a gravity function
- *Dynamic demand*. The demand of network (defined in a function of time.

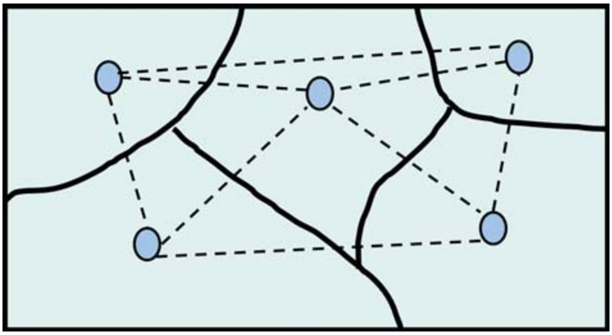

Fig. 8.1 Distance–cost relationship in three conditions

Since in modeling design, the demand of network in real problems depends on the population, we can argue that the dynamic demand is more correct and true than any other kind of demands. These suggestions can lead us to the proper answers.

8.2 Mathematical Models

8.2.1 Classic Model

The median problems are intended to find the location of P facility on the network, so that the total cost can be at minisum. The cost means the cost of providing services from node i to the nearest node established there. This cost depends on factors like distance between the node of i and servicing node and volume of demands of in node. The Classic Median Problem has some assumption as follows:

- Linear relationship between cost and distance
- Good facilitated
- Infinite time horizon
- Infinite facility capacity
- Don't have an initial setup cost
- Exogenous problem
- Same facilities
- Stationary facilities
- Constant node's demand
- Discrete problem

In this problem;

X_{ij}: is equal to 1 if the demand of node i is covered by the facility that has been setup at node j, otherwise is 0
Y_j: is equal to 1 if a facility is setup at node j, otherwise is 0
d_{ij}: The distance between the node of i demand and candidate node to establish facility j (d_{ij} is zero if $i = j$).

P: The number of facilities to be established
h_i: Demand of i node
n: Number of nodes

This model was proposed by ReVelle and Swain (1970);

$$\text{Min} \sum_i \sum_j h_i d_{ij} X_{ij} \qquad i, j = 1, 2, ..., n. \tag{8.1}$$

Subject to

$$\sum_j X_{ij} = 1 \qquad \forall i, \tag{8.2}$$

$$\sum_j Y_j = P, \tag{8.3}$$

$$X_{ij} \leq Y_j \qquad \forall i, j, \tag{8.4}$$

$$X_{ij}, Y_j \in \{0, 1\} \quad \forall i, j. \tag{8.5}$$

In this problem, the aim is to minimize the total cost needed to satisfy the needs of nodes (to minimize sum of demand-distance). Equation (8.2) states that all demands should and each node is serviced by just one facility (demand limitation). Equation (8.3) states that there is an endogenous problem and proves the exact P establishment. Equations (8.4) states that open facilities can meet the demands. Thus the demand of i node can be provided with facility established in j node ($X_{ij} = 1$) through one facility in j node ($Y_j = 1$).

8.2.2 Capacitated Plant Location Problem Model (CPLPM)

In this part, we introduce the capacitated plant location problem (CPLP) with multiple facilities in the same site (CPLPM), a special case of the classical CPLP where several facilities can be opened in the same site. Applications of the CPLPM arise in a number of contexts, such as the location of polling stations. Although the CPLPM can be modeled and solved as a standard CPLP, this approach usually performs very poorly. CPLP is a classical combinatorial optimization problem (Bramel et al. 1998). This location problem is of utmost importance for many public and private organizations and its aim is determining a set of capacitated facilities (warehouses, plants, polling stations, etc.) in such a way that the sum of facility construction costs and transportation costs is minimized. Unlike other problem we allow multiple facilities in the same site. The CPLP is strongly NP-hard (Mirchandani and Francis 1990) and has been extensively studied in clustering and location theory. As a result, an overabundance of solution approaches has been proposed in the past decades. Exact algorithms have been developed, among others, by Christo Edes and Beasley (1983), Leung and Magnanti (1989), whereas heuristics have been investigated by

Van Roy (1985), Beasley (1988), ChristoEdes and Beasley (1983), GeoIrion and McBride (1978), Guinard and Kim (1987), Jacobsen (1983), Khumawala (1974). A systematic comparison of heuristics and relaxations for the capacitated plant location problem is provided by Cornuejols et al. (1991) Based on both a theoretical analysis and extensive computational results; they suggest the use of a Lagrangian heuristic to solve large instances of the CPLP.

In this problem;

U: The set of potential facilities,
V: The set of customers,
d_j: The demand of customer j $(j \in V)$, where $d_j > 0$,
q_i: The capacity of facility i $(i \in U)$, where $q_i > 0$,
c_{ij}: The cost of supplying all the demand of customer j $(j \in V)$ from facility i $(i \in U)$,
f_i: The fixed cost associated with opening facility i $(i \in U)$, where $f_i > 0$,
p: The desired number of open facilities (also referred to as medians),
y_i: A binary decision variable, which takes the value 1, if facility i $(i \in U)$ is open, 0 otherwise,
x_{ij}: A continuous decision variable, corresponding to the fraction of the demand of customer j $(j \in V)$ supplied from facility i $(i \in U)$.

Then CPLP can be formulated as a mixed integer linear programming problem as follows:

$$\text{(CPLP) Min} \sum_{i \in U} \sum_{j \in V} c_{ij} x_i + \sum_{i \in U} f_i y_i. \tag{8.6}$$

Subject to

$$\sum_{i \in U} x_{ij} = 1, \qquad i \in V, \tag{8.7}$$

$$\sum_{j \in V} d_j x_{ij} \leq q_i y_{i,} \qquad i \in U, \tag{8.8}$$

$$\sum_{i \in U} y_i = p, \tag{8.9}$$

$$x_{ij} \geq 0, \qquad i \in U, j \in V, \tag{8.10}$$

$$y_i \in \{0, 1\}, \qquad i \in U. \tag{8.11}$$

The objective function (8.6) expresses the minimization of the total costs. Equations (8.7) ensure that the demand of each customer is satisfied. Equations (8.8) establish the connection between (x_{ij}) and (y_i) variables. They state that no customer can be supplied from a closed facility and the total demand supplied from each open facility does not exceed the capacity of the facility. Equation (8.9) establishes that the number of open facilities is p. Equations (8.10) provide lower bounds on the (x_{ij}) variables. It is worth noting that (8.7)–(8.10) imply $x_{ij} \leq 1$ $(i \in U; j \in V)$. Finally, (8.11) are the integrality constraints.

In the CPLPM, U represents a set of areas where facilities can be located. Such set is partitioned into n nonempty U_1‘…’U_n, each corresponding to a site where one or more areas are available. Consequently $f_i = f'_i$, if i; $i \in U_K$ $(k \in \{1, \ldots, n\})$, and $C_{ij} = C'_{ij}$ if $i \in U_K$ $(k \in \{1, \ldots, n\})$ and $j,\ j' \in V$.

8.2.3 Capacitated P-median Problem (Lorenaa 2004)

If no fixed costs are associated to the potential facilities, then the CPLP is called the capacitated p-median problem (CPMP).

Where $N = \{1, \ldots, n\}$ is the index set of entities to allocate and also of possible medians with $x_{ij} = 1$ if entity i is allocated to median j, and $x_{ij} = 0$, otherwise; $x_{jj} = 1$ if median j is selected and $x_{jj} = 0$, otherwise.

The CPMP model can be formulated in two ways. The first is the following binary integer-programming problem (P):

$$\text{Min} \sum_{i \in N} \sum_{j \in N} c_{ij} x_{ij}. \tag{8.12}$$

Subject to

$$\sum_{i \in N} x_{ij} = 1, \quad i \in N, \tag{8.13}$$

$$\sum_{j \in N} x_{jj} = P, \tag{8.14}$$

$$\sum_{i \in N} d_i x_{jj} \le q_i x_{jj,} \quad j \in N, \tag{8.15}$$

$$x_{ij} \in \{0, 1\}, \quad i \in N, j \in N. \tag{8.16}$$

Equations (8.13)–(8.14) impose that each entity is allocated to only one median. Equations (8.15) imposes that a total median capacity must be respected, and (8.16) provide the integer conditions.

The CPMP problem can also be modeled as the following set partitioning problem with a cardinality constraint (SPP).This is the formulation found in Minoux. The same formulation can be obtained from the problem P by applying the Dantzing–Wolfe decomposition.

Where $S = \{S_1, S_2, \ldots, S_m\}$, is a set of subsets of N;

$A = [a_{ik}]_{n \times m}$, is a matrix with

$$a_{ik} = \begin{cases} 1 & \text{if } i \in S_{K'} \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{(SPP) Min} \sum_{k=1}^{m} c_k x_k. \tag{8.17}$$

Subject to

$$\sum_{k=1}^{m} A_k x_k = 1, \tag{8.18}$$

$$\sum_{k=1}^{m} x_k = P, \tag{8.19}$$

$$x_{ij} \in \{0, 1\}. \tag{8.20}$$

Equations (8.12)–(8.13) are conserved and respectively updated to (18) and (20), according the Dantzig–Wolfe decomposition process.

If S is the set of all subsets of N, the formulation can give an optimal solution to the CPMP. However, the number of subsets may be huge, and a partial set of columns can be considered instead.

The SPP defined above is also known as the restricted master problem in the column generation context.

8.3 Solution Techniques

Some kinds of solution for solving the p-median problems are as follows:

- Exact methods
- Heuristic algorithm
- Metaheuristic algorithm

The complete accounting, heuristic and met heuristic algorithm are one of the first technique that are used them for solving the median problem.

Teitz and Bart (1968) proposed two innovated algorithm to solve the P-median problems through studying the complete number algorithm of Hakimi. This algorithm is based on choosing a primary series of nodes and then exchanging its members with other nodes of the network in order to improve the target function. Jarvinen et al. (1972) proposed a branch and bound algorithm to solve P-median problem. Meanwhile, Egbo, Samelson and colleagues proposed a branch and bound algorithm used to determine its limit through method of logerangean reparation (Narula et al. 1968). Neebe (1978) defined the transportation P-median problem in which the number of supply points was limited to P. Also he proposed a branch and bound algorithm for this problem in which the logerangean reparation method was used to determine its limit. Kariv and Hakimi (1979) proved to find the P-median in a network was to find NP-Hard, and to determine 1-median for a tree was possible in $O(n^2)$ stage. Galvao (1980) determined the dual form of P-median problem, and obtained a low limit for the problem using the solution by an innovative method. This limit has been used in a branch and bound algorithm and applied to solve the problems with 30 nodes and desirable P numbers.

8.3.1 Lemma

The Weber's problem was developed by Hakimi (1964), and through application of the weight on the graph, he began to find the P point on it in order to minimize the sum of the weighted distance from the points. To solve the problem, there was no limitation far the places of the points, and they could be placed in any points of the graph. Though the points were not located exactly on the apexes in the resulted solutions, Hakimi (1964) showed that there could be always a series apexes P to minimize the target functions. Thus, through the constraint of finding the solution just on the apexes, again Hakimi (1964) considered this disconnected problem as the candidate of Weber's problem. In the median problems, the solution consisting of the apexed P is called the P-median. Also, these problems are to be studied and defined just over the graph or networks. Hakimi (1964) generalized the concept of 1-median to multi-median. He used the concept to determine how to distribute the switching centers in the Tele-communication network. Later the P-median problems were considered as the inseparable part of the location theory and were called as a main problem.

8.3.2 Solving 1-Median Problem Algorithm on Tree (Goldman 1971)

- Put for all nodes $\hat{h}_i = \mathrm{h_i}$
- Choice one node beside the i if it is $\hat{h}_i \geq \frac{\sum h_i}{2}$, put the facility in the node and go to step 3.
- Otherwise add W_i to w_k. k is the only node that it has intersection with inside i node.
- Calculate the objective function.

In the algorithm, time of solving the algorithm increases by developing the number of nodes linearly, because any node will consider its application just one time $O(n^2 p^2)$. Kariv and Hakimi (1979) submitted one algorithm for finding the place of facility P on a tree networks with (n)nodes.

According to previous, without considering the type of network that can be a tree network or general, if at least half of the application of all network appear on a node, we can get at least one best solution by putting facility on that node. In tree network, we will lead to getting place of best facility through using the node on top of the tree and mixing those with other connected nodes. This subject is not possible in general networks. For better understanding of this subject consider Fig. 8.2. These different conditions that all tree nodes can be placed on best settlement are changed according to the length of relating vectors.

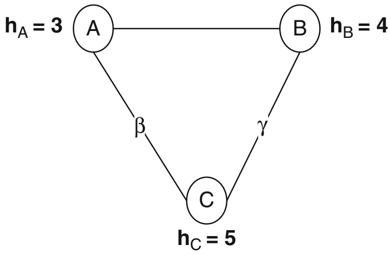

Fig. 8.2 Instance of general network

8.3.3 Exact Methods

In solving median problem, mathematic models are used by applying the integer programs. For solving this model of linear integer program, some ways are submitted that will solve based on relieving model of original model and by the dual theory and relieving Lagrangian method.

8.3.3.1 Complete Accounting (Teitz and Bart 1968)

First, for solving the median problem, we should consider a condition that a facility may be put on a network. In this method, by counting all conditions we will simply get the best result simply. Especially, we know this characteristic of median problems that one best result is obtained by putting the facilities on nodes. In order to calculate the amount of objective function, we can use this condition:

$$Z_j = \sum_j h_i d_{ij}. \tag{8.21}$$

When we put the facility on j node, we can calculate the amount of Z_j for all nodes and choose the least one for the best result. In this phrase, just one facility has located from p-facility that should be located and is left p-1 facilities.

The number of forward solution is calculated with this equation:

$$\binom{N}{P} = \frac{N!}{P!(N-P)!}. \tag{8.22}$$

8.3.4 Heuristic Algorithms

Just as mentioned, solving the median problem in time dimension imitate just one polynomial, but it had been shown by Kariv and Hakimi (1979) that if this subject be in a general network, it is a NP-complete. So some struggles have drawn for finding heuristic solutions for solving these problems. The important facts about these facility algorithms are that how much these are good or possible answers.

Although these algorithms result in real world problems very well, they don not guarantee achieving the optimum or approximating it. In solving some problems, these solutions may be optimum or near the best. In other cases, these solution may be very far. The first Heuristic algorithm for solving the median problems:

- Greedy-adding algorithm
- Alternate algorithm
- Vertex substitutions algorithm

According to these tree algorithms, innumerable algorithms and new techniques are made for solving the median problems that are based on those three algorithms. From these three basic algorithms, the vertex substitution algorithm is more general in comparison to the other two algorithms and up to the present time, it was one of the mostly used algorithms for solving median problems. Another heuristic algorithm is the branch and bound algorithm. (Heuristic branch and bound algorithm is not the same as one of the linear-program problems techniques named branch and bound.)

According to a classification, algorithms are classified into two groups of reply maker algorithms and recovering reply algorithms.

Myopic algorithm is one of the reply maker algorithms. This algorithm tries to get one primary and possible solution for median problems. The algorithm is similar to Greedy adding algorithm that is applied for maximum covering problems.

Both exchange and neighborhood search algorithms are kinds of recovering reply algorithm. These algorithms are not able to get the primary reply for problems, but they use them to improve reply that how got from maker algorithms. Two algorithms are similar to both Greedy adding and maximum heuristic algorithms for maximum covering and substitution problems. When Lagrangian discounting method is used besides one or more heuristic algorithm that will be introduced this section, they will get the best or near the best reply.

8.3.5 Metaheuristic Algorithms

Meta heuristic algorithms for solving the median problem:

- Genetic algorithm
- Concentration algorithm
- Neural network
- Tabu search algorithm
- Simulated annealing

In recent decades metaheuristic algorithms are submitted for solving these kinds of problems. Among these algorithms we can mention the genetic algorithm, neighborhood search algorithm, tabu search algorithm, concentration algorithm, simulated annealing algorithm and neural network. Also there are some other algorithms for obtaining one reply with acceptable quality.

Metaheuristic methods are usually used for too many nodes (100, 200, 300, 400, 500, 600, 700, 800 and 900).

8.4 Comparison of Methods

In this section, all achieved results that have been concluded from a 12-nodes network are mentioned. Here there is no solution way and just we consider their comparisons results.

- The solutions that have been achieved by using the myopic algorithms, in the most cases, appears almost in 30% error of amount of Lagrange objective function.
- In each stage, all solution that has been achieved from exchange algorithms method is always for the each number of facilities better than solution by using the neighborhood search algorithms.
- It is better in some situations that neighborhood search algorithms applied after setting each facility at network in each stage, and in some other situations, it is better to apply the neighborhood search algorithms after setting all facilities.
- We should consider that when the number of settled facilities will be average figure, the differences between all achieved results differ from various algorithms together and with Lagrange algorithm method.
- When the number of the settled facilities is few, it seams that all algorithms work better. For instance, when the number of the settled facilities is more, all results that have been achieved from algorithms are so good and algorithms work so well. But when the number of the settled facilities is neither few nor more, the problem is so difficult.
- Exchange algorithm is the best method for this network, considering that exchanging appears after putting all facilities by using the myopic algorithm.

8.5 Studying Statically the Methods for Solutions of Median Problem (Reese 2005)

The aim here is to study the methods used to solve the median problems and the years of proposing them which were classified for 1963–2005 periods. The following factors were selected among other sources:

- Those focused directly on median problems,
- Those involved minisum target function,
- Those which define median problems on the graph or network,
- Sum of answers limited to searching on nodes,
- Those that did not have primary cost of establishment,
- Those belonging to median problems with limited capacity or unlimited facilities.

Also we avoided the resources with one of the following conditions:

- They had minimax target function,
- Study establishment of median problems in a connected space,
- There was a probable state demand or cost,
- There was a multi-purpose target function,
- Considering dislocation of facilities in a time horizon.

The following classification obtained considering the proposed solutions.

8.5.1 Classification of Solving Methods by Period

The number of presented paper in this area before 1970 were 7, between 1970 and 1974 were 9, between 1975 and 1979 were 12, between 1980 and 1984 were 9, between 1985 and 1989 were 6, between 1990 and 1994 were 11, between 1995 and 1999 were 24 and between 2000 and 2005 the number of paper have the remarkable growth so that it reaches 42 paper.

It should also be noted that the methods obtained and used in the last 10 years are so higher than the methods used in 1963–1994.

8.5.2 Classification of Different Solving Methods

According to the researches between 1965 and 2005 the LP Relaxation was used more than other methods, after that respectively Vertex substitution, approximation algorithm, genetic algorithm (GA), IP formulation were used, graph theory and surrogate relaxation are in the same level, and other methods were assigned less than five cases.

8.6 Case Study

In this section we will introduce some real-world case studies related to p-median problem:

8.6.1 Post Center Locations (Alba and Dominquez 2006)

In Australia in order to determine ten post centers among 200 centers, 10-median problem formulated and finally ten cities including Sidney, Melborn and Adlid, etc. were selected as post centers.

8.6.2 Entrance Exam Facilities (Correa et al. 2004)

PARANA State University (UFPR) in Curitiba in Brazil used the median P model to determine the location of facilities concerning M.A entrance exam in 2001. The aim was to appropriate 19,710 candidate students to the facilities located nearer to their homes as possible.

It was determined that 26 facilities needed to meet the demands of 19,710 students among 43 candidate facilities. Finally the number of candidate solutions should be 421 billion.

$$\binom{43}{26} = 421,171,648,758.$$

The problem was solved by a genetic algorithm.

8.6.3 Polling Station Location (Ghiani et al. 2002)

Polling station location problem in Italy: Number of polling stations was determined according to the number of resident voters in five Italian municipalities (data: November 2000).

This study of the CPLPM was motivated by a polling station location problem in an Italian municipality. In Italy, the following binding obligations must be taken into account:

- The number of polling stations is fixed for each municipality and calculated according to the number of resident voters the number of voters assigned to each polling station may not exceed given lower and upper bounds;
- The suitability of the potential sites is established by specific safety measures on the accessibility and typology of the buildings (for example, in some countries, only public buildings, such as schools, are eligible).

8.6.3.1 Formulating the Problem as a CPLPM

U: The set of areas where polling stations can be located.
P: Number of polling stations to be activated.
V: The set of streets having at least one resident voter
d_j: The number of voters in street j $(j \in V)$, where $d_j > 0$.
q_i: The upper limit on the total number of voters assigned to a polling station located in area i $(i \in U)$; q_i may be the same for all areas, i.e.

$$q_i = q = \sum_{j \in V} \frac{d_j}{p}; i \in U. \tag{8.23}$$

c_{ij}: The transportation cost incurred if all voters resident in street j $(j \in V)$ are assigned to a polling station in area i $(i \in U)$; c_{ij} can be chosen, for simplicity, equal to $d_j{}^* S_{ij}$ j, where S_{ij} is the walking distance between area i and the "centre of gravity" of street j; $f_i = f$, for all areas. Consequently, the contribution of the fixed costs to the objective function (6) can be excluded.

8.6.3.2 Computational Results

As far as the experimental phase is concerned, we have solved first the problem of the optimal location of polling stations in the municipality of Castrovillari. Town located in Southern Italy with a population, dated back to November 2000, of 15,709 voters, located in 351 different streets spread in a wide area. The solution actually adopted by the local government is based on the assignment of all voters resident in the same street to the same polling station, chosen among the nearest ones. As a result, the walking distance covered by all voters is about 10,731 km, and the number of voters for each polling station was reported. It was observed that, with respect to the ideal capacity, the fourth polling station has 18% of voters less, whereas the 18th polling station has 21% of voters more. The feasible solution found by the heuristic is well balanced and has a cost 37.6% less than that of the solution currently adopted by the municipality of Castrovillari. Computational results show that the average deviation of the heuristic solution over the lower bound is less than 2%.

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