Iterative SAFT Reconstruction for Manually Acquired Ultrasonic Measurement Data in Nondestructive Testing

CSP Research Project 19SS

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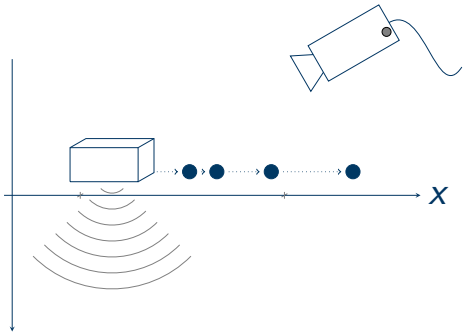
Technische Universität Ilmenau

Measurement Assistance System

Features:

- Position recognition
- Data recording
- Data visualization
- Post-processing

Problem: Systematic errors
e.g. tracking error



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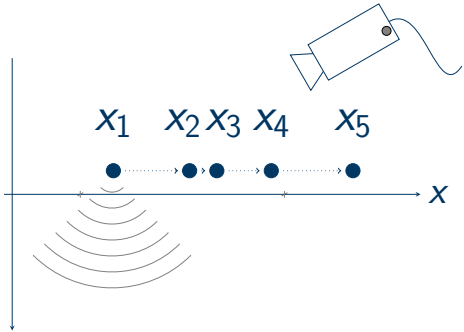

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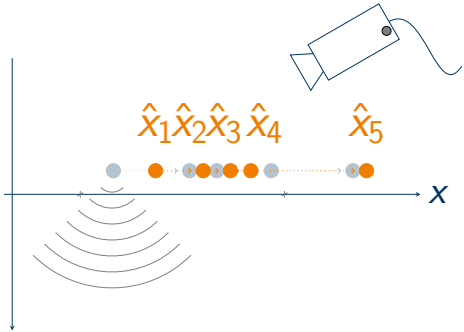
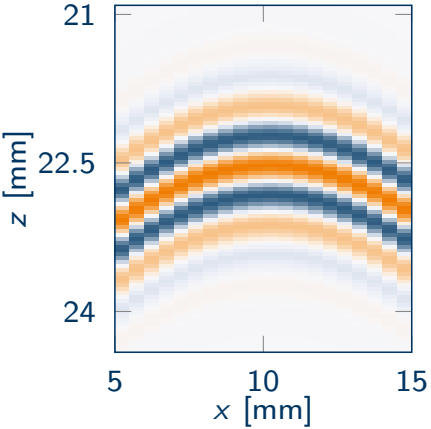


Impact of Position-Inaccuracy

Correct positions



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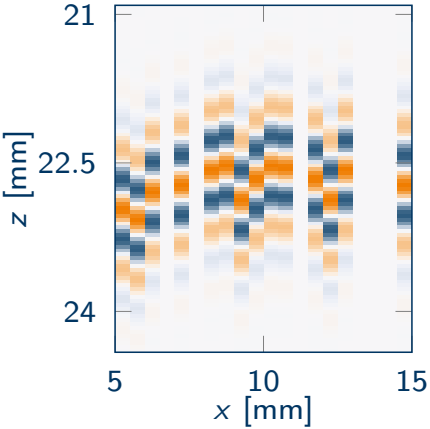

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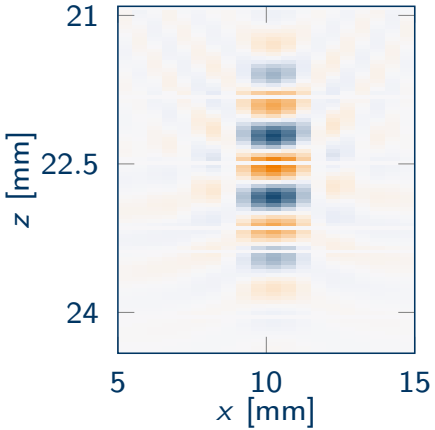

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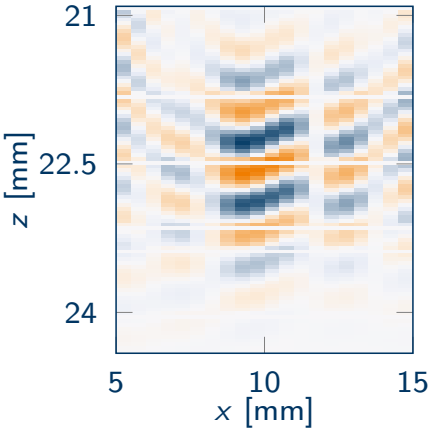


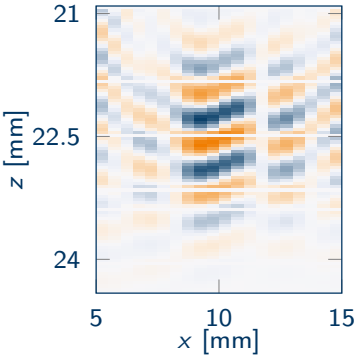
Objective and Contributions

Objective

- Artefacts reduction

Contributions

- Spatial approximation of the reconstruction matrix
- Iterative method to correct the positional information



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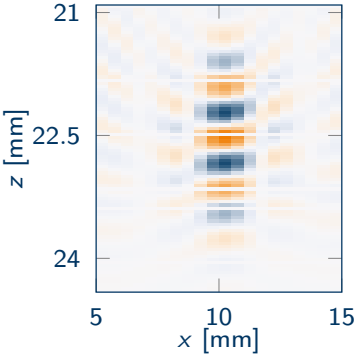


Iterative Matrix Improvement

Spatial approximation

- 1st order Taylor approximation

Iterative position correction

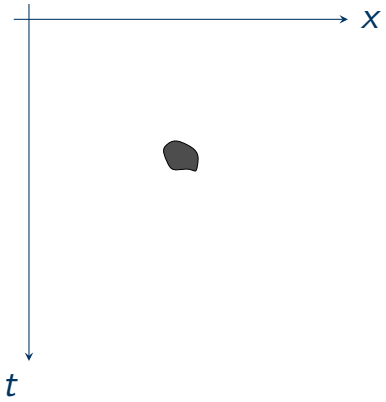
- Solving the least squares problem
- Update the positional information $\hat{x} \rightarrow \hat{x}_{\text{opt}}$

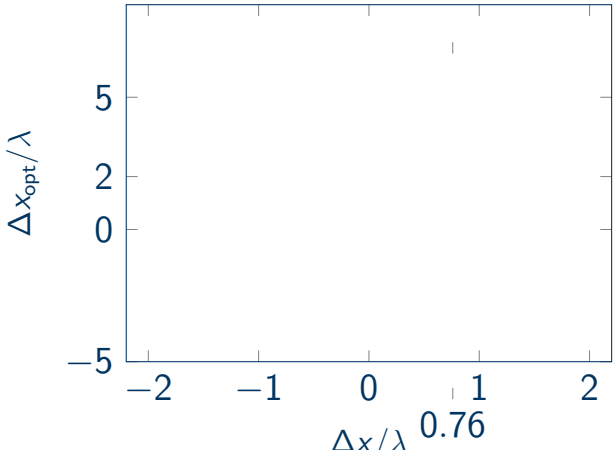
Evaluation criterion

- Error correction $\Delta x_{\text{opt}} = x - \hat{x}_{\text{opt}}$
- Approximation quality $\text{SE}^{\dagger} = 0 \ldots 1$

Results: Error Correction

Variables: $|s_x - x|$, Δx

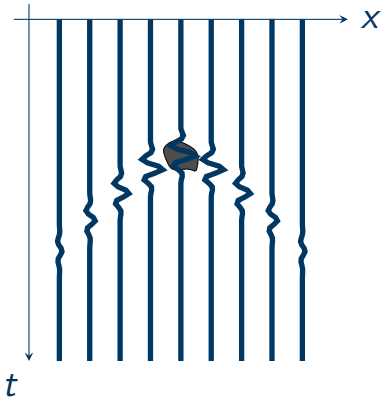


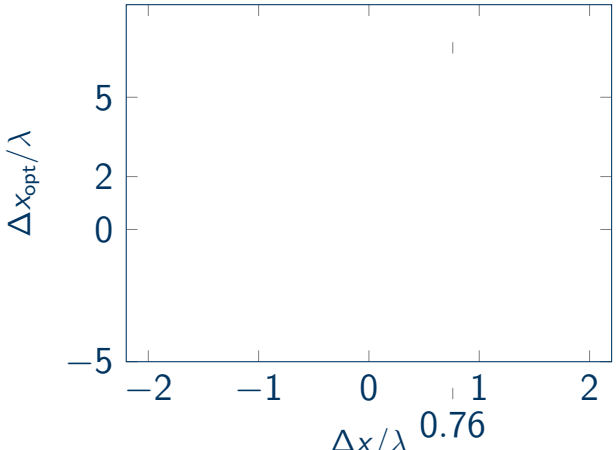


Wavelength $\lambda = 1.26\,\text{mm}$

Results: Error Correction

Variables: $|s_x - x|$, Δx

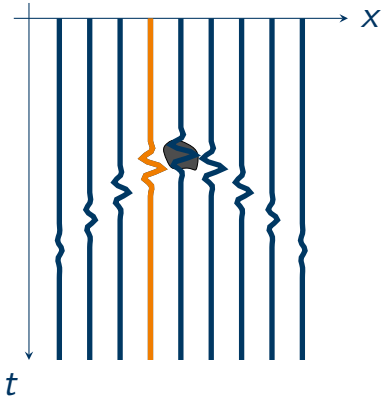


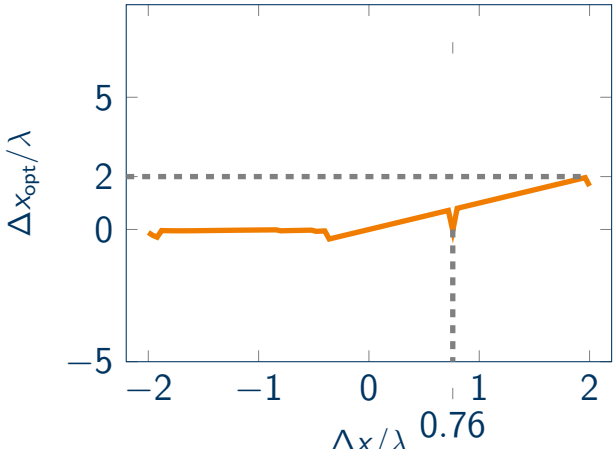


Wavelength $\lambda = 1.26\,\text{mm}$

Results: Error Correction

$|s_x - x| = 1\,\text{mm} \mathrel{\hat{=}} 0.8\lambda$

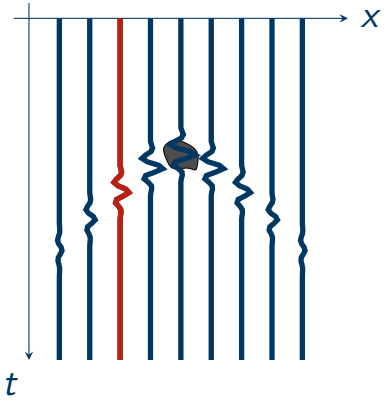


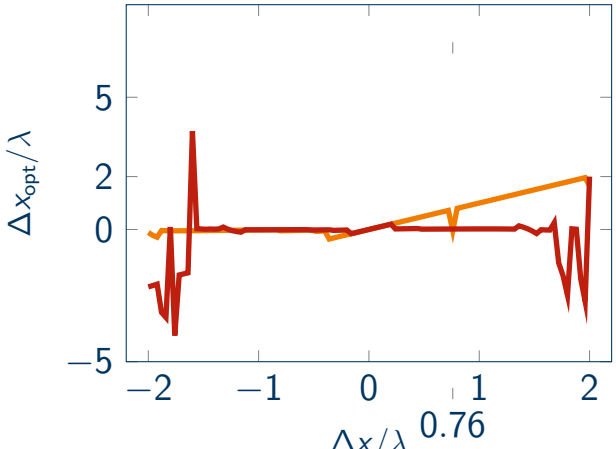


Wavelength $\lambda = 1.26\,\text{mm}$

Results: Error Correction

$|s_x - x| = 2.5\,\text{mm} \mathrel{\hat{=}} 1.98\lambda$

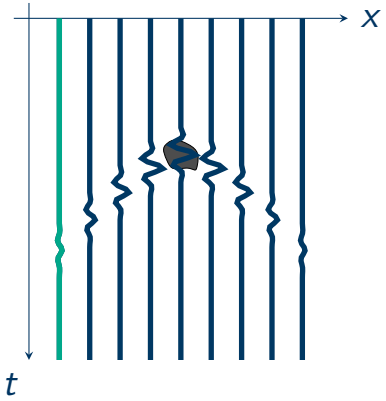


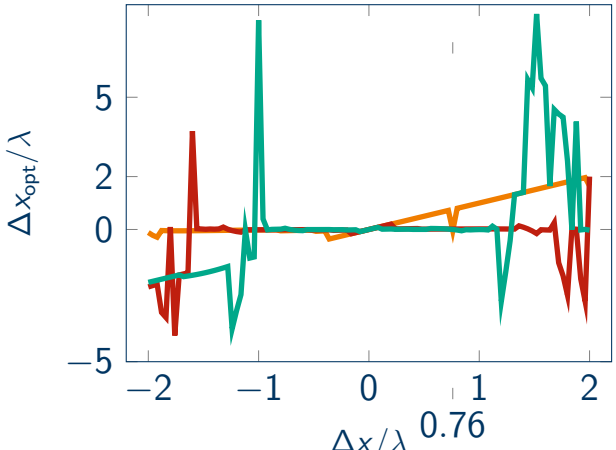


Wavelength $\lambda = 1.26\,\text{mm}$

Results: Error Correction

$|s_x - x| = 5\,\text{mm}$





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Results: A-Scan Approximation

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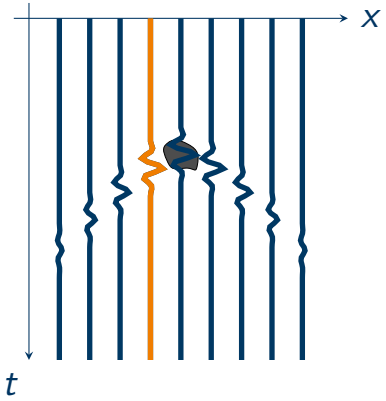

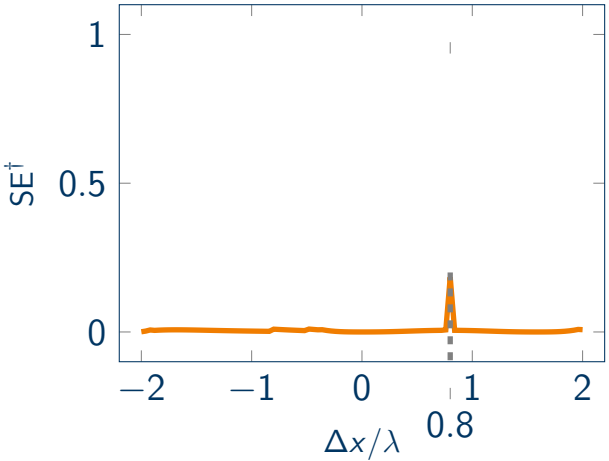

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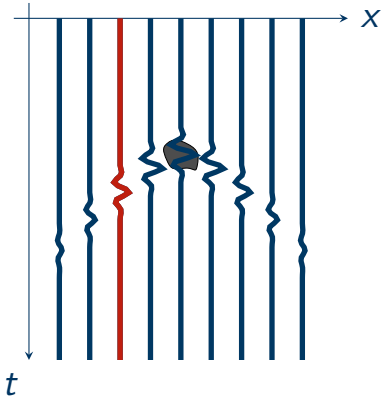

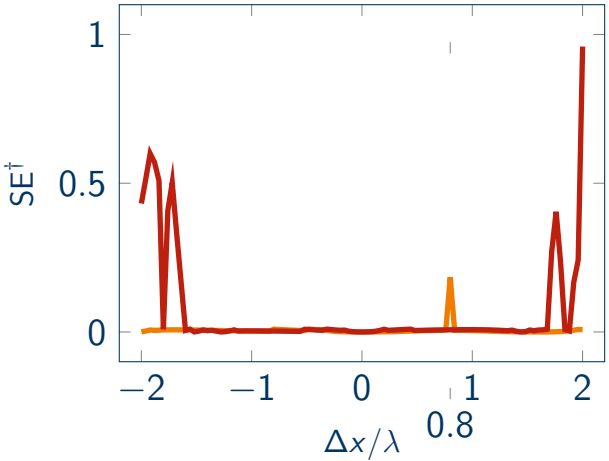

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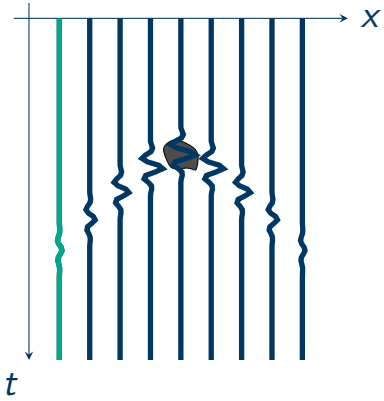
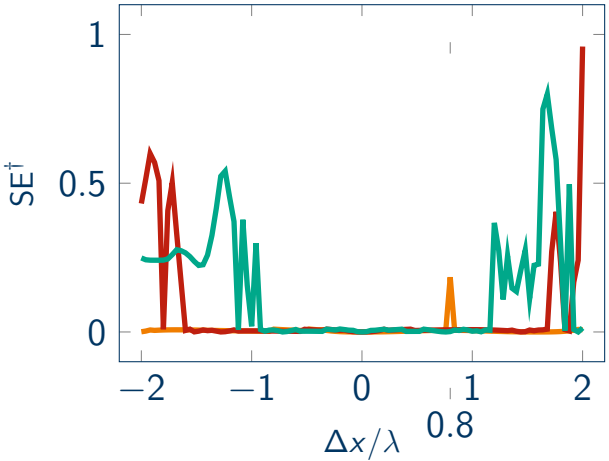
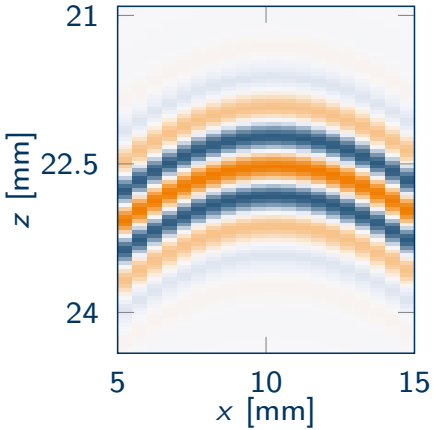




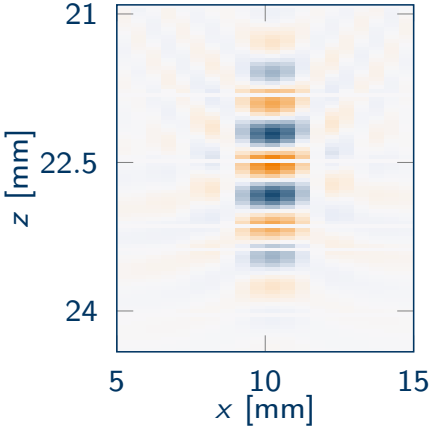
Wavelength $\lambda = 1.26\,\text{mm}$

Results: Reconstruction

No tracking error

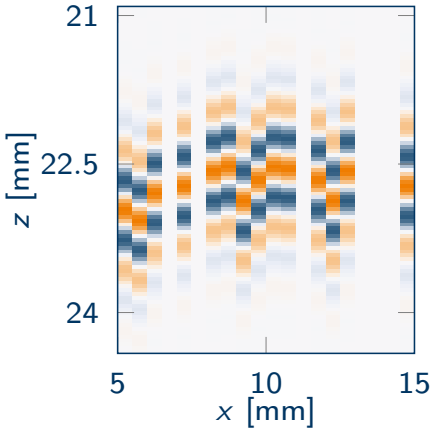


Reconstruction



Results: Reconstruction

With $\Delta x = 1.26\,\mathrm{mm}$



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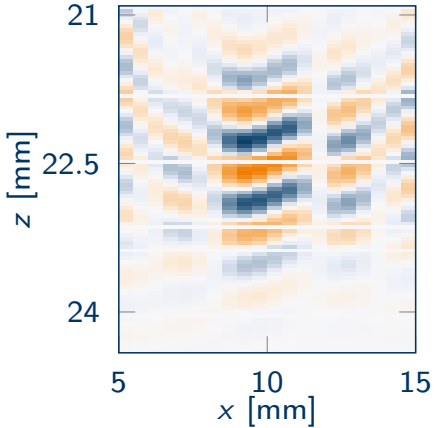
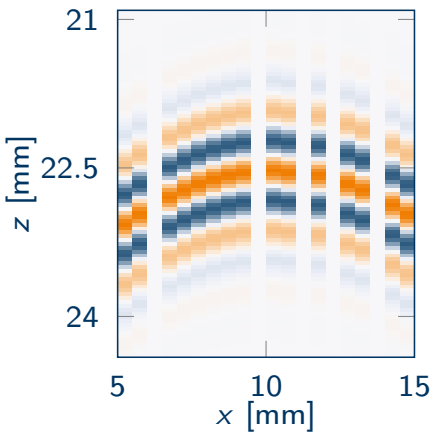


Results: Reconstruction

After position correction



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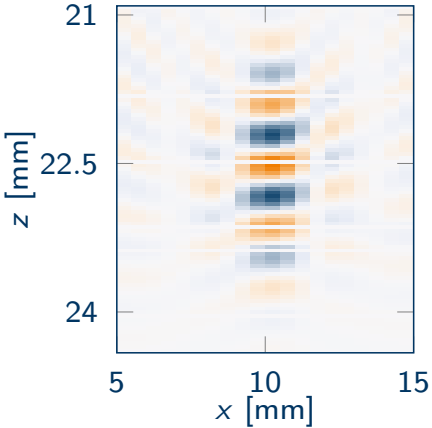


Conclusion

$\lvert s_x - x \rvert$	Position	A-Scan	SAFT Reconstruction
Very close	✗	✓	* Less significant * Occasional position correction
In-between	✓	✓	* Crucial * Apply position correction
Far apart	≈	≈	* Out of *sight* in real measurements * Apply a spatial filter

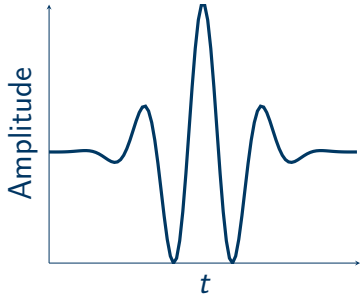
Appendix

Parameters w.r.t. Test Object

Parameter	Value
Material	Aluminium
Dimension (L × H) $N_x \times N_z$	20 mm × 35 mm 40 × 880
Speed of sound c_0	$6300\,\mathrm{m\,s^{-1}}$
Sampling frequency f_S	80 MHz
$\mathrm{dt} = \frac{1}{f_s}$	12.5 ns
Sampling distance, surface (dx)	0.5 mm
Sampling distance, depth (dz)	39.375 µm

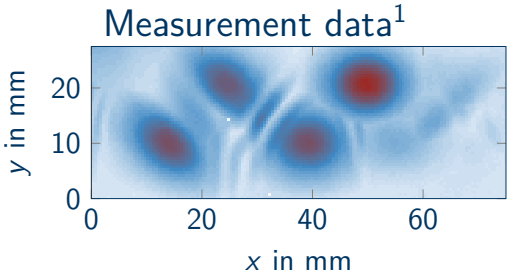
Parameters w.r.t. Pulse

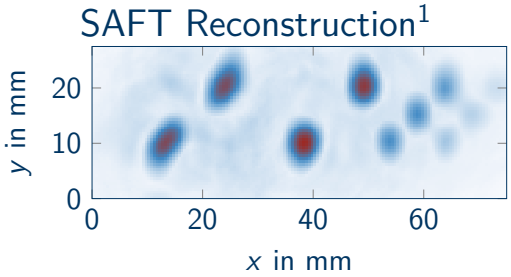
Parameter	Value
Model	Gaussian (Gabor)
Carrier frequency f_c	5 MHz
Wavelength λ	1.26 mm
α	20 $(\text{MHz})^2$



Reconstructing UT Data

Automatic measurement



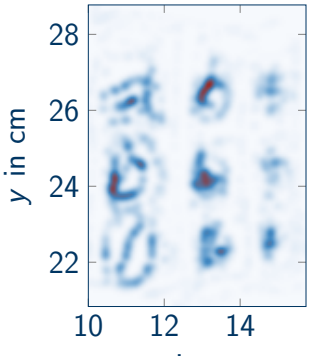


[1] F. Krieg et al., SAFT processing for manually acquired ultrasonic measurement data with 3D SmartInspect, *SHM-NDT*, 2018

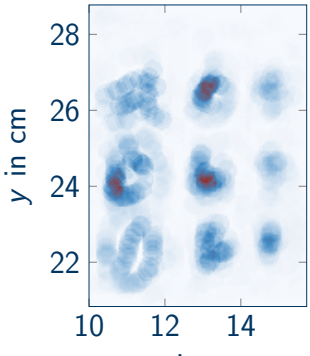
Reconstructing UT Data

Manual measurement

Measurement data[1]



SAFT Reconstruction[1]



Impact of Systematic Errors

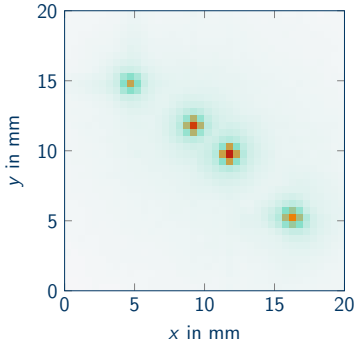
Vertical shift

- High impact
- No accessible information
  $\rightarrow$ only estimation through data

Positional inaccuracy

- Less impact
- Critical when sparsely scanned

Equidistant 1600 positions
Without systematic errors



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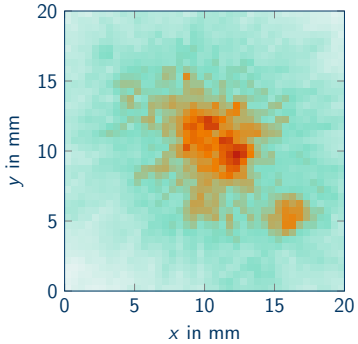
Equidistant 1600 positions
Vertical shift = 0.3 mm (0.24λ)



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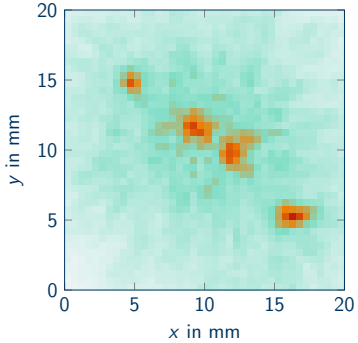
1600 random positions
Positional error = 2 mm (1.6λ)



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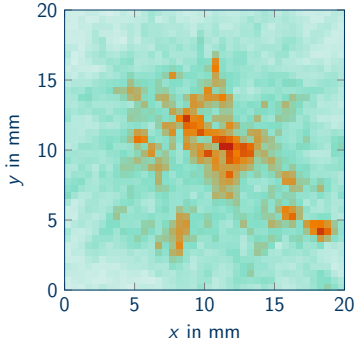
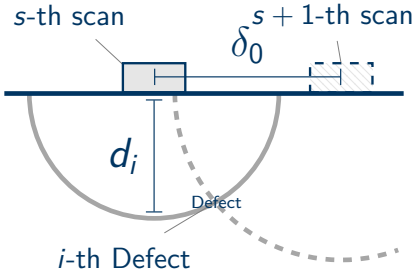


Post-processing Method

Synthetic Aperture Focusing Technique (SAFT)

- Superposition according to propagation time delay
- Spatial sampling of the specimen



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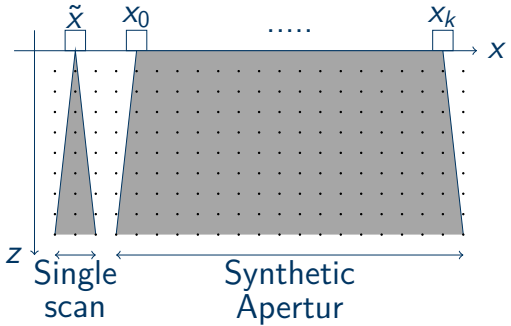
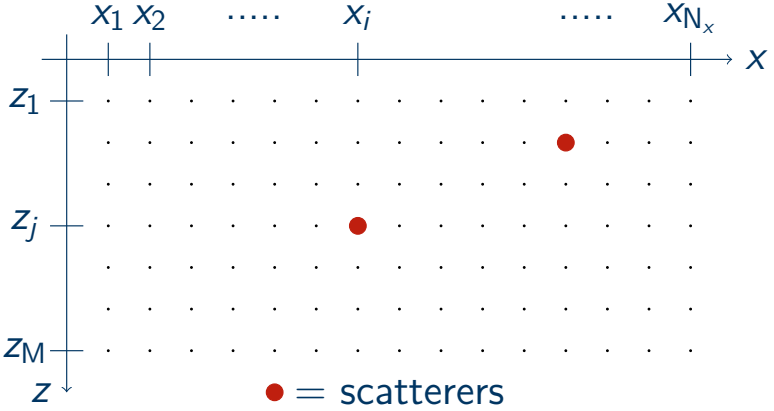


SAFT as Linear Transform

defect map for ROI $= \mathrm{M} \times \mathrm{N}_x$



SAFT as Linear Transform

Transform: *defect map* $\rightarrow$ A-Scan

$$\boldsymbol{a}(\boldsymbol{p}) = \boldsymbol{H}(\boldsymbol{p}) \cdot \boldsymbol{b} + \boldsymbol{n}$$

- $\boldsymbol{a}(\boldsymbol{p})$: measured A-Scan at $\boldsymbol{p}$ (M)
- $\boldsymbol{H}(\boldsymbol{p})$: SAFT matrix at $\boldsymbol{p}$ ($\mathrm{M} \times \mathrm{L} = \mathrm{M} \times \mathrm{M}\,\mathrm{N}_x$)
  Containing pulse information for $\boldsymbol{s}_l = (x_i, z_j)$, $l = 1 \ldots \mathrm{L}$
- $\boldsymbol{b}$: vectorized *defect map* ($\mathrm{L} = \mathrm{M}\,\mathrm{N}_x$)
- $\boldsymbol{n}$: noise (M)

SAFT as Linear Transform

Transform: *defect map* $\rightarrow$ A-Scan

$$\hat{\boldsymbol{a}}(\boldsymbol{p}) = \boldsymbol{H}(\boldsymbol{p}) \cdot \boldsymbol{b}$$

- $\hat{\boldsymbol{a}}(\boldsymbol{p})$: modeled A-Scan at $\boldsymbol{p}$ (M)
- $\boldsymbol{H}(\boldsymbol{p})$: SAFT matrix at $\boldsymbol{p}$ (M $\times$ L)
  Containing pulse information for $\boldsymbol{s}_l = (x_i, z_j)$, $l = 1...\mathrm{L}$
- $\boldsymbol{b}$: vectorized *defect map* (L)

SAFT as Linear Transform

Transform: *defect map* $\rightarrow$ A-Scan

$$\boldsymbol{h}_l(\boldsymbol{p}) = \boldsymbol{H}(\boldsymbol{p}) \cdot \boldsymbol{b}^{(l)}$$

- $\boldsymbol{h}_l(\boldsymbol{p})$: l-th column vector of $\boldsymbol{H}(\boldsymbol{p})$ (M)
- $\boldsymbol{H}(\boldsymbol{p})$: SAFT matrix at $\boldsymbol{p}$ (M $\times$ L)
  Containing pulse information for $\boldsymbol{s}_l = (x_i, z_j)$, $l = 1...\mathrm{L}$
- $\boldsymbol{b}^{(l)}$: vectorized *defect map* (L)
  Containing single non-zero element in l-th row

Spatial Approximation of SAFT Matri

Transform: *defect map* $\rightarrow$ A-Scan

$$\boldsymbol{h}_l(\boldsymbol{p}) = \boldsymbol{H}(\boldsymbol{p}) \cdot \boldsymbol{b}^{(l)}$$

$$\mathrm{M} \qquad \mathrm{M} \times \mathrm{L} \quad \mathrm{L}$$

Goal: spatial approximation of SAFT matrix

$$\boldsymbol{H}(\boldsymbol{p}) \approx f\left(\boldsymbol{H}(\boldsymbol{p} + \Delta\boldsymbol{p}); \boldsymbol{H}'(\boldsymbol{p} + \Delta\boldsymbol{p}); \Delta\boldsymbol{p}\right)$$

Spatial Approximation of SAFT Matri

Transform: *defect map* $\rightarrow$ A-Scan

$$\underset{\mathrm{M}}{\boldsymbol{h}_l(\boldsymbol{p})} = \underset{\mathrm{M}\times\mathrm{L}}{\boldsymbol{H}(\boldsymbol{p})} \cdot \underset{\mathrm{L}}{\boldsymbol{b}^{(l)}}$$

Transform: scan position $\rightarrow$ A-Scan

$$\underset{\mathrm{M}}{\boldsymbol{h}_l(\boldsymbol{p})} \approx \underset{\mathrm{M}}{\boldsymbol{h}_l(\boldsymbol{p}+\Delta\boldsymbol{p})} - \underset{\mathrm{M}\times\mathrm{K}}{\boldsymbol{J}_{\boldsymbol{h}_l}(\boldsymbol{p}+\Delta\boldsymbol{p})} \cdot \underset{\mathrm{K}}{\Delta\boldsymbol{p}}$$

Spatial Approximation of SAFT Matri

Transform: *defect map* $\rightarrow$ A-Scan

$$\underset{\mathrm{M}}{\boldsymbol{h}_l(\boldsymbol{p})} = \underset{\mathrm{M}\times\mathrm{L}}{\boldsymbol{H}(\boldsymbol{p})} \cdot \underset{\mathrm{L}}{\boldsymbol{b}^{(l)}}$$

Transform: scan position $\rightarrow$ A-Scan

$$\underset{\mathrm{M}}{\boldsymbol{h}_l(\boldsymbol{p})} \approx \underset{\mathrm{M}}{\boldsymbol{h}_l(\boldsymbol{p}+\Delta\boldsymbol{p})} - \underset{\mathrm{M}\,\mathrm{L}\times\mathrm{K}}{\mathrm{vec}^{-1}_{\mathrm{M,L}}\{\boldsymbol{J}(\boldsymbol{p}+\Delta\boldsymbol{p})\cdot\Delta\boldsymbol{p}\}} \cdot \underset{\mathrm{L}}{\boldsymbol{b}^{(l)}}$$

Spatial Approximation of SAFT Matri

Transform: *defect map* $\rightarrow$ A-Scan

$$\underset{\mathrm{M}}{\boldsymbol{h}_l(\boldsymbol{p})} = \underset{\mathrm{M}\times\mathrm{L}}{\boldsymbol{H}(\boldsymbol{p})} \cdot \underset{\mathrm{L}}{\boldsymbol{b}^{(l)}}$$

Transform: scan position $\rightarrow$ A-Scan

$$\underset{\mathrm{M}\times\mathrm{L}}{\boldsymbol{H}(\boldsymbol{p})} \cdot \underset{\mathrm{L}}{\boldsymbol{b}_l} \approx \Big[\underset{\mathrm{M}\times\mathrm{L}}{\boldsymbol{H}(\boldsymbol{p}+\Delta\boldsymbol{p})} - \underset{\mathrm{ML}\times\mathrm{K}}{\mathrm{vec}^{-1}_{\mathrm{M,L}}\{\boldsymbol{J}(\boldsymbol{p}+\Delta\boldsymbol{p})\cdot\Delta\boldsymbol{p}\}}\Big] \cdot \underset{\mathrm{K}}{\boldsymbol{b}^{(l)}}$$

Spatial Approximation of SAFT Matri

Transform: *defect map* $\rightarrow$ A-Scan

$$\underset{\mathrm{M}}{\boldsymbol{h}_l(\boldsymbol{p})} = \underset{\mathrm{M}\times\mathrm{L}}{\boldsymbol{H}(\boldsymbol{p})} \cdot \underset{\mathrm{L}}{\boldsymbol{b}^{(l)}}$$

Spatial approximation of SAFT matrix

$$\underset{\mathrm{M}\times\mathrm{L}}{\boldsymbol{H}(\boldsymbol{p})} \approx \underset{\mathrm{M}\times\mathrm{L}}{\boldsymbol{H}(\boldsymbol{p}+\Delta\boldsymbol{p})} - \underset{\mathrm{M,L}}{\mathrm{vec}^{-1}}\{\underset{\mathrm{ML}\times\mathrm{K}}{\boldsymbol{J}(\boldsymbol{p}+\Delta\boldsymbol{p})} \cdot \underset{\mathrm{K}}{\Delta\boldsymbol{p}}\}$$

Spatial Approximation of SAFT Matri

Transform: *defect map* $\rightarrow$ A-Scan

$$\underset{\mathrm{M}}{\boldsymbol{h}_l(\boldsymbol{p})} = \underset{\mathrm{M}\times\mathrm{L}}{\boldsymbol{H}(\boldsymbol{p})} \cdot \underset{\mathrm{L}}{\boldsymbol{b}^{(l)}}$$

Spatial approximation of SAFT matrix $\boldsymbol{p} = [x, 0]$

$$\underset{\mathrm{M}\times\mathrm{L}}{\boldsymbol{H}(x)} \approx \underset{\mathrm{M}\times\mathrm{L}}{\boldsymbol{H}(x+\Delta x)} - \underset{\mathrm{M}\,\mathrm{L}\times 1}{\mathrm{vec}^{-1}_{\mathrm{M,L}}\{\boldsymbol{J}(x+\Delta x)\}} \cdot \underset{1}{\Delta x}$$

Spatial Approximation of SAFT Matri

Transform: *defect map* $\rightarrow$ A-Scan

$$\underset{\mathrm{M}}{\boldsymbol{h}_l(\boldsymbol{p})} = \underset{\mathrm{M}\times\mathrm{L}}{\boldsymbol{H}(\boldsymbol{p})} \cdot \underset{\mathrm{L}}{\boldsymbol{b}^{(l)}}$$

Spatial approximation of SAFT matrix $\boldsymbol{p} = [x, 0]$

$$\underset{\mathrm{M}\times\mathrm{L}}{\boldsymbol{H}(x)} \approx \underset{\mathrm{M}\times\mathrm{L}}{\boldsymbol{H}(x+\Delta x)} - \underset{\mathrm{M}\times\mathrm{L}}{\boldsymbol{H}'(x+\Delta x)} \cdot \underset{1}{\Delta x}$$

Inverse vec Operator

$$\mathrm{vec}^{-1}_{\mathrm{M,L}}\{\boldsymbol{J}(\boldsymbol{p}+\Delta\boldsymbol{p})\cdot\Delta\boldsymbol{p}\} = \left[(\mathrm{vec}\{\boldsymbol{I}_{\mathrm{L}}\}^{\mathrm{T}} \otimes \boldsymbol{I}_{\mathrm{M}}\right]\cdot[\boldsymbol{I}_{\mathrm{L}} \otimes (\boldsymbol{J}(\boldsymbol{p}+\Delta\boldsymbol{p})\cdot\Delta\boldsymbol{p})]$$

Iterative Matrix Improvement

1st order Taylor approximation of A-Scan $\boldsymbol{p} = [x, 0]$

$$\boldsymbol{h}_l(x) \approx \boldsymbol{h}_l(x + \Delta x) - \boldsymbol{h}'_l(x + \Delta x) \cdot \Delta x$$

Problems:

- Acquisition of Δx
- Limited validity range

$\Rightarrow$ iterative estimation and correction of Δx

$$\min_{\Delta x} \|\boldsymbol{h}_l(x + \Delta x) - \boldsymbol{h}_l(x) - \boldsymbol{h}'_l(x + \Delta x) \cdot \Delta x\|_2$$

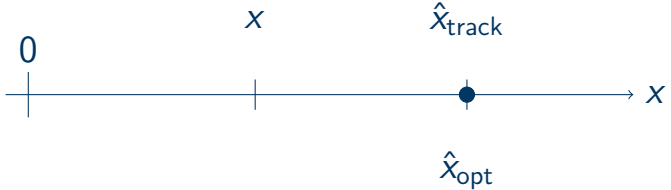
Iterative Matrix Improvement

Iterative estimation and correction of Δx

(1) Calculate $\boldsymbol{h}_I(\hat{x}_{\text{opt}})$
(2) Solve the least square problem and obtain Δx_{est}
(3) Update $\hat{x}_{\text{opt}} = \hat{x}_{\text{opt}} - \Delta x_{\text{est}}$
** Break, when $\|\boldsymbol{h}_I(x) - \boldsymbol{h}_{I,\text{opt}}(\hat{x}_{\text{opt}}; \Delta x_{\text{opt}})\|_2 \leq$ target

Iterative Matrix Improvement

Iterative estimation and correction of Δx



(1) Calculate $\boldsymbol{h}_I(\hat{x}_{\text{opt}})$
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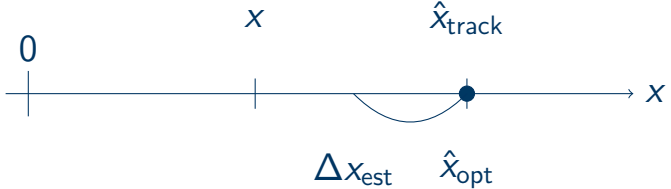
Iterative Matrix Improvement

Iterative estimation and correction of Δx



(1) Calculate $\boldsymbol{h}_I(\hat{x}_{\text{opt}})$
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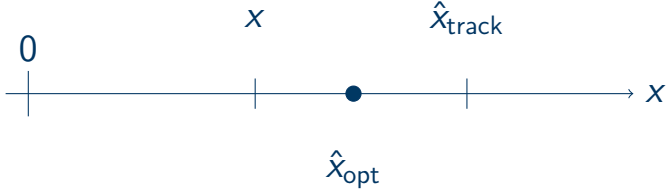
Iterative Matrix Improvement

Iterative estimation and correction of Δx



(1) Calculate $\boldsymbol{h}_l(\hat{x}_{\text{opt}})$
(2) Solve the least square problem and obtain Δx_{est}
(3) Update $\hat{x}_{\text{opt}} = \hat{x}_{\text{opt}} - \Delta x_{\text{est}}$
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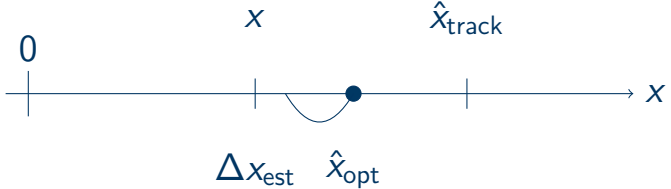
Iterative Matrix Improvement

Iterative estimation and correction of Δx



(1) Calculate $\boldsymbol{h}_l(\hat{x}_{\text{opt}})$
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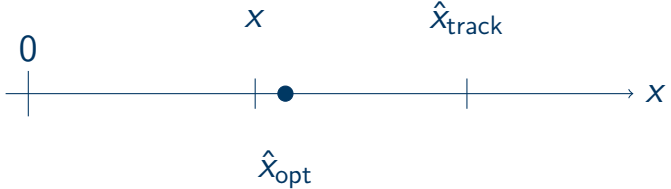
Iterative Matrix Improvement

Iterative estimation and correction of Δx



(1) Calculate $\boldsymbol{h}_l(\hat{x}_{\text{opt}})$
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** Break, when $\|\boldsymbol{h}_l(x) - \boldsymbol{h}_{l,\text{opt}}(\hat{x}_{\text{opt}}; \Delta x_{\text{opt}})\|_2 \leq$ target

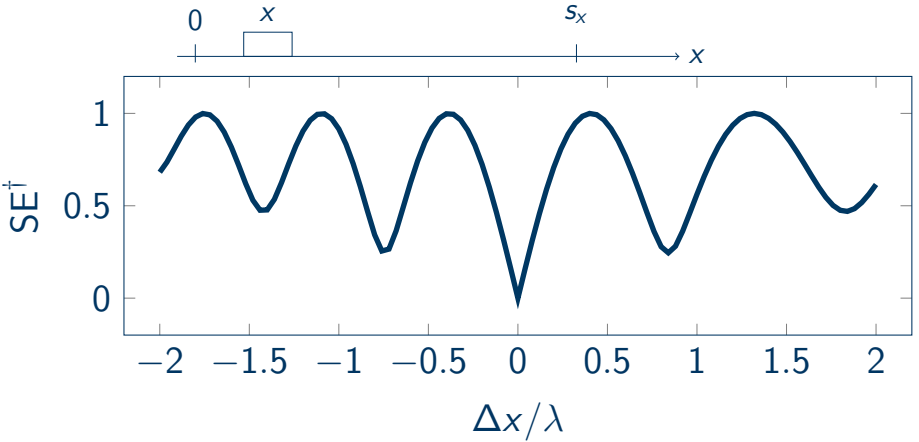
Normalized Squared Error $\mathrm{SE}^{\dagger}$

$$\mathrm{SE}^{\dagger} = \frac{\|\gamma\tilde{\boldsymbol{a}} - \boldsymbol{a}\|_2}{\|\boldsymbol{a}\|_2}$$

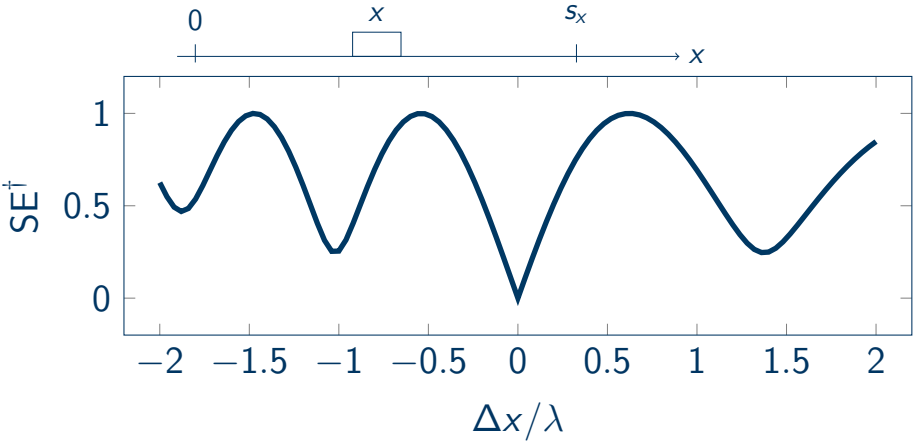
γ: normalization factor

$$\gamma = \frac{\boldsymbol{a}^{\mathsf{T}} \cdot \tilde{\boldsymbol{a}}}{\tilde{\boldsymbol{a}}^{\mathsf{T}} \cdot \tilde{\boldsymbol{a}}}.$$

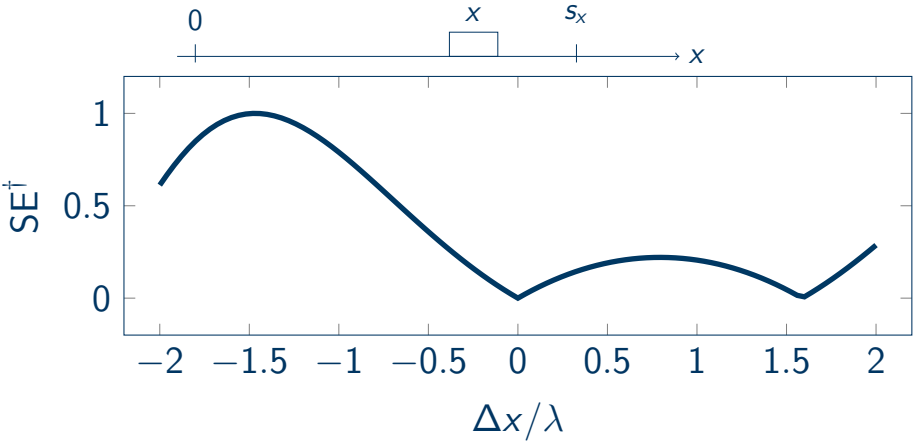
Effect of Error on A-Scan Modeling



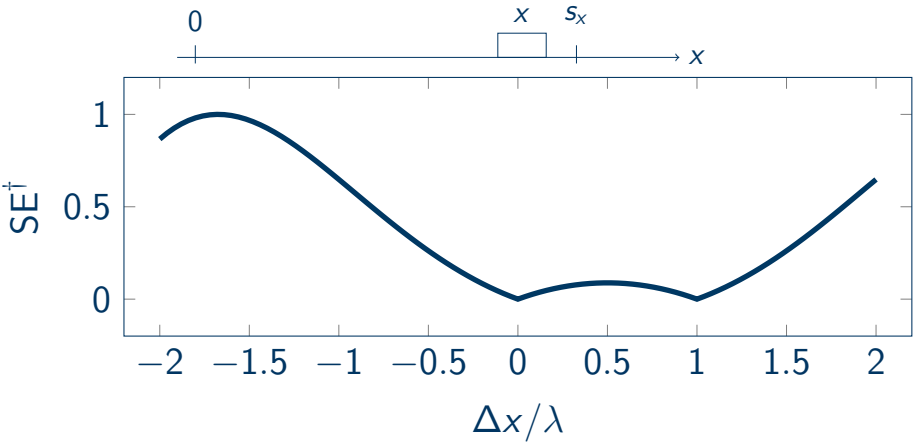
Effect of Error on A-Scan Modeling



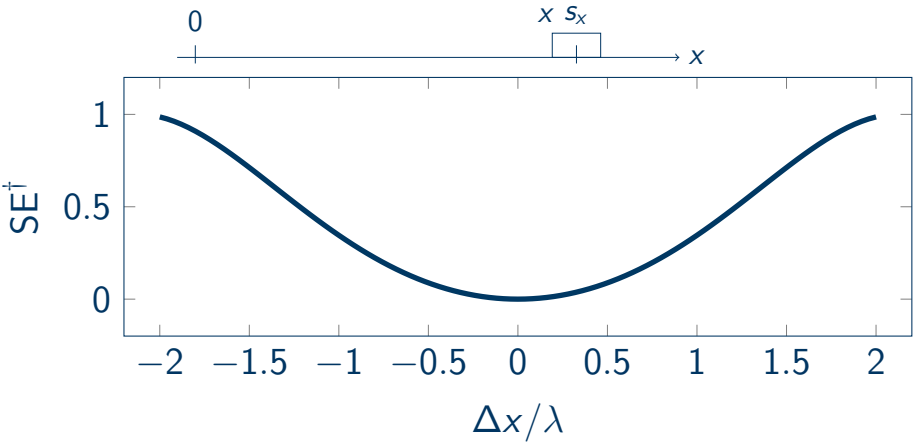
Effect of Error on A-Scan Modeling



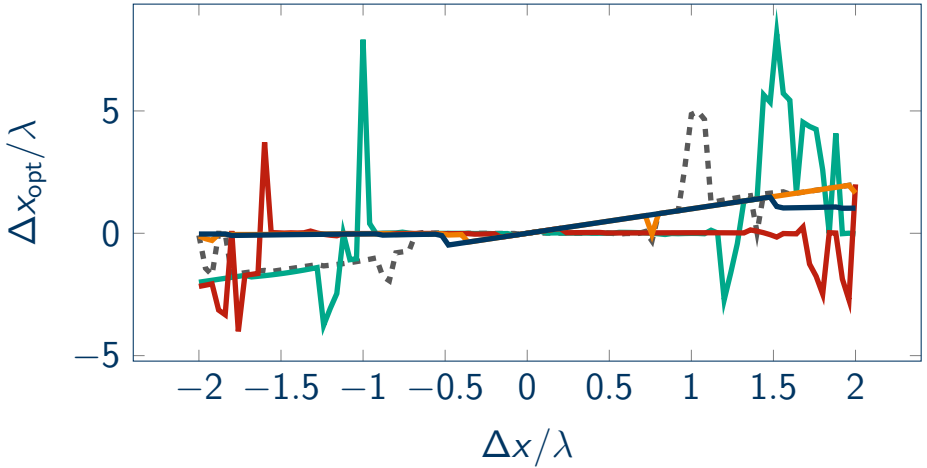
Effect of Error on A-Scan Modeling



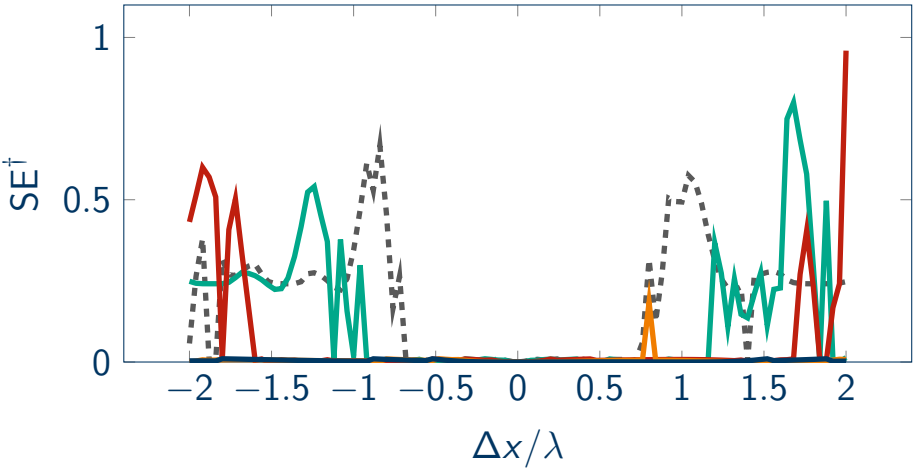
Effect of Error on A-Scan Modeling



Results: Error Correction



Results: A-Scan Approximation



SAFT reconstruction

FWM: *defect map* $\rightarrow$ A-Scan

$$\hat{\boldsymbol{a}}(\boldsymbol{p}) = \boldsymbol{H}(\boldsymbol{p}) \cdot \boldsymbol{b}$$

$$\mathrm{M} \qquad \mathrm{M} \times \mathrm{L} \qquad \mathrm{L}$$

FWM: *defect map* $\rightarrow$ all A-Scans

$$\mathrm{vec}\{\boldsymbol{A}\} = \boldsymbol{\mathcal{H}} \cdot \boldsymbol{b}$$

$$\mathrm{L} \qquad \mathrm{L} \times \mathrm{L} \qquad \mathrm{L}$$

SAFT reconstruction

FWM: *defect map* $\rightarrow$ all A-Scans

$$\mathrm{vec}\{\boldsymbol{A}\} = \boldsymbol{\mathcal{H}} \cdot \boldsymbol{b}$$

$$\mathrm{L} \qquad \mathrm{L} \times \mathrm{L} \quad \mathrm{L}$$

Reconstruction: all A-Scans $\rightarrow$ reconstructed *defect map*

$$\tilde{\boldsymbol{b}} = \boldsymbol{\mathcal{H}}^{\mathsf{T}} \cdot \mathrm{vec}\{\boldsymbol{A}\}$$

$$\mathrm{L} \quad \mathrm{L} \times \mathrm{L} \quad \mathrm{L}$$