’Blind’ Iterative SAFT Reconstruction for Manually Acquired Ultrasonic Measurement Data in Nondestructive Testing

CSP Advanced Research Project WS19/20

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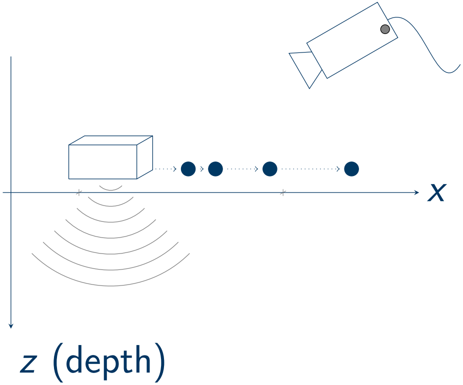
Measurement Assistance System

Features:

- Position recognition
- Data recording
- Data visualization
- Post-processing

Problem: Observation errors

e.g. tracking error



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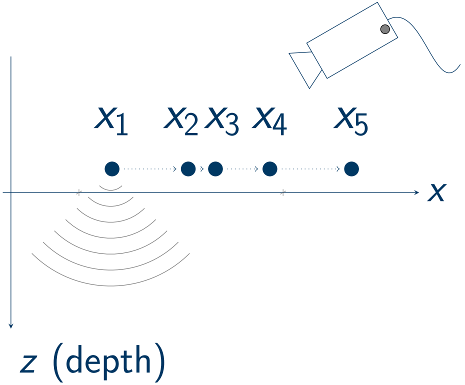

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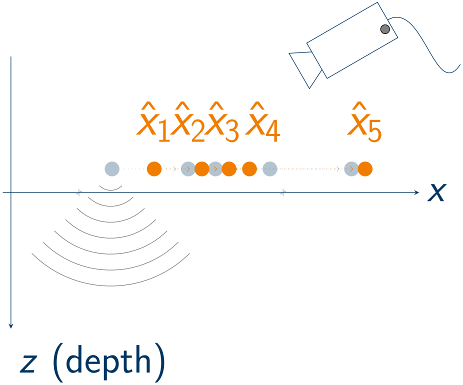
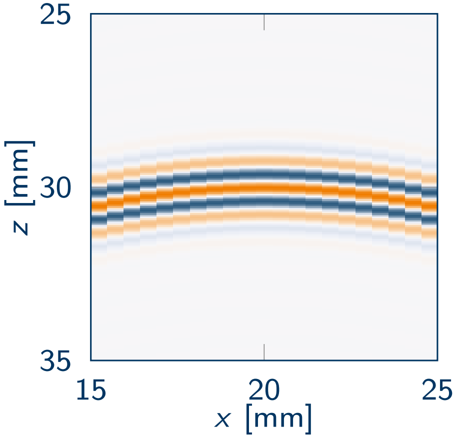
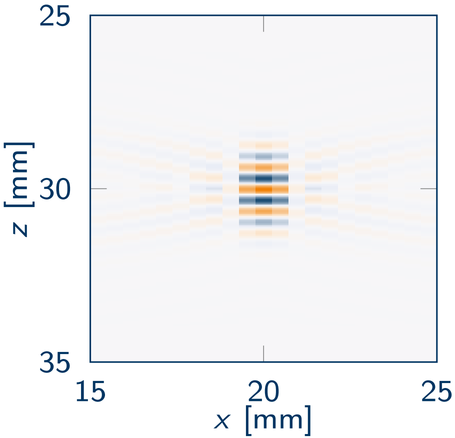


Impact of Positional Inaccuracy

Measurement data



Reconstruction (no error)



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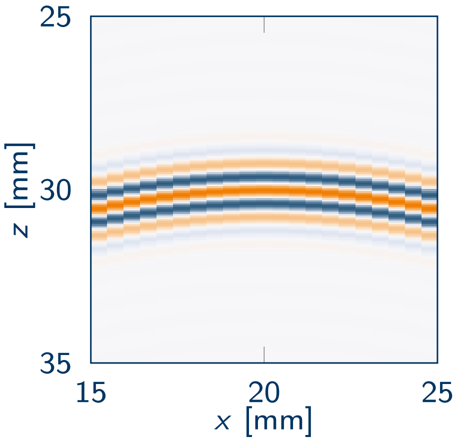
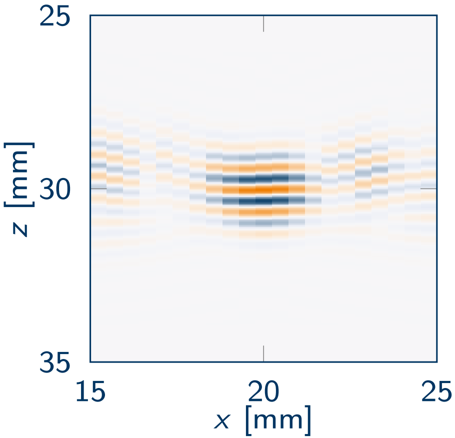


Reconstruction (with error)



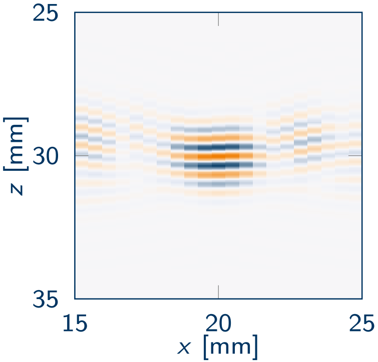
Objective and Contributions

Objective

- Reduce error-induced artefacts in SAFT reconstructions

Contributions

- Data model considering the positional inaccuracy
- Preprocessing method to estimate and correct positional error



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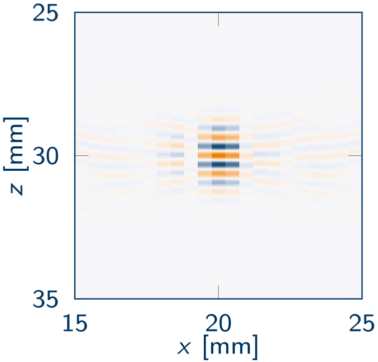

Blind Error Correction (BEC)

Data model based on spatial approximation

- Signal source positions
- Tracking error

Preprocessing in 2 steps

(1) Estimate the signal source positions
Known: data structure
$\rightarrow$ Robust polynomial regression

(2) Estimate and correct the tracking error
$\rightarrow$ Nonlinear programming

BEC Performance Analysis

Simulation studies

- Error tolerance
- Impact of the ROI depth

Scenario and Assumptions

- Linear contact scanning (0.5 mm grids)
- One point source in ROI
- Tracking error $= -\lambda \dots + \lambda$

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Evaluation methods

- Normalized squared error
- *Generalized Contrast-to-Noise Ratio* (gCNR)
- *Array Performance Indicator* (API)

BEC Performance Analysis

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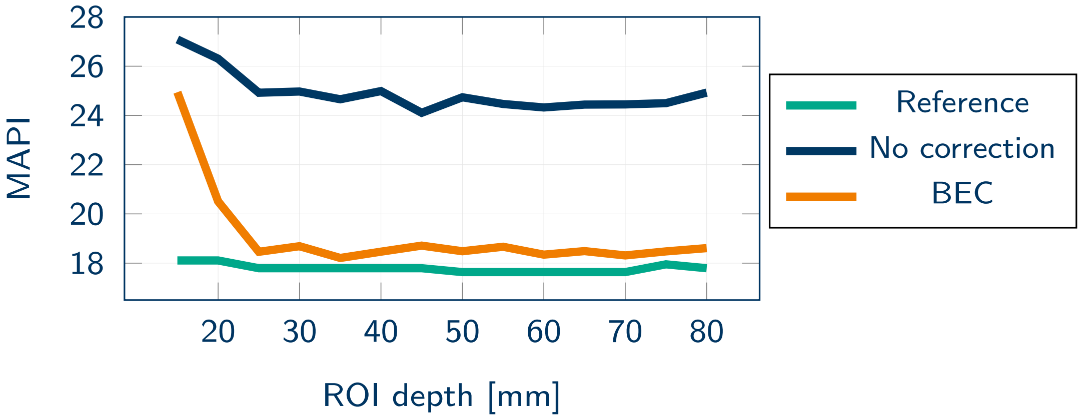
- Normalized squared error
- *Generalized Contrast-to-Noise Ratio* (gCNR)
- *Array Performance Indicator* (API)

$$\text{API} = \text{area} > \epsilon \text{ (normalized with } \lambda^2\text{)}$$
$$\Rightarrow \text{smaller API} \mathrel{\hat{=}} \text{better resolution}$$

Impact of the ROI Depth



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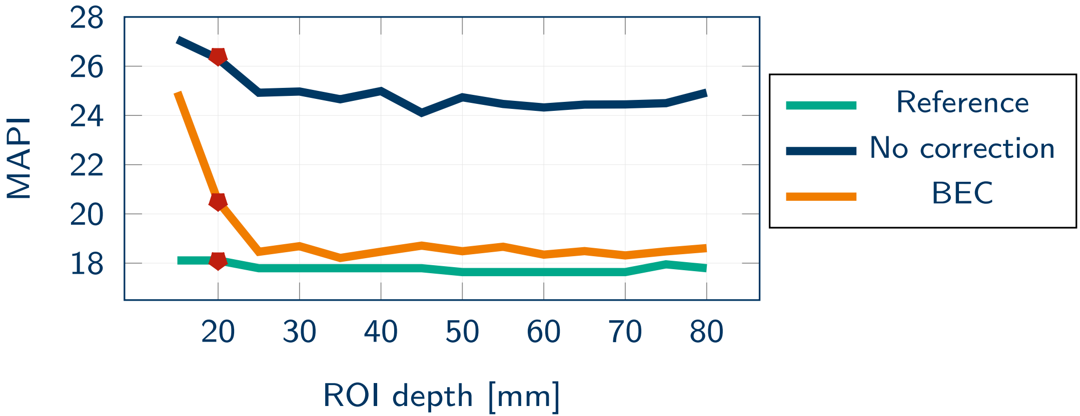

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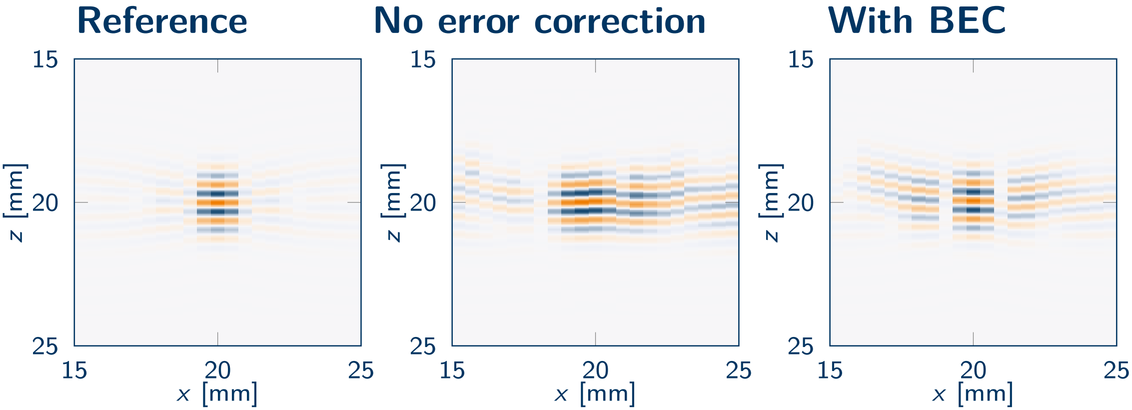

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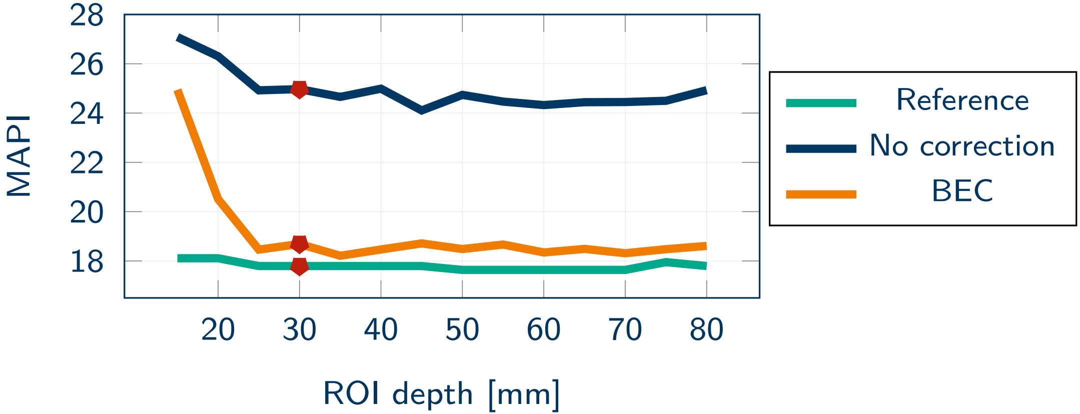

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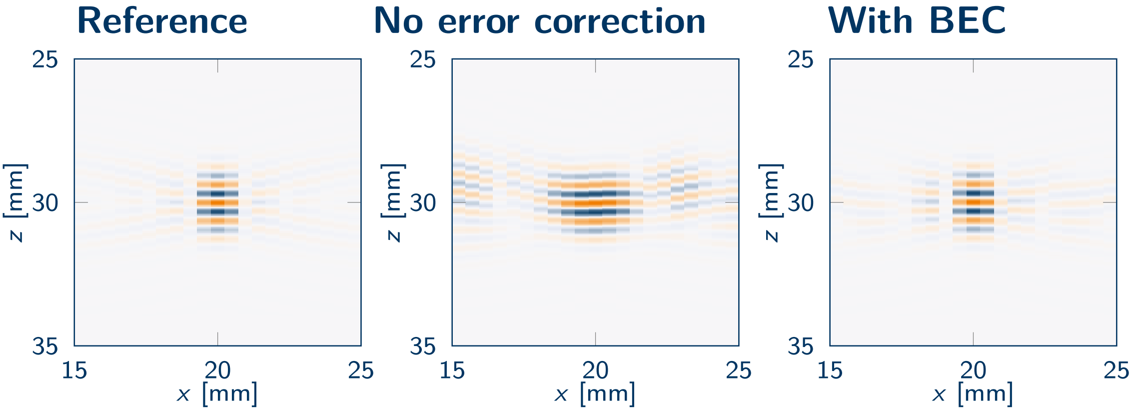

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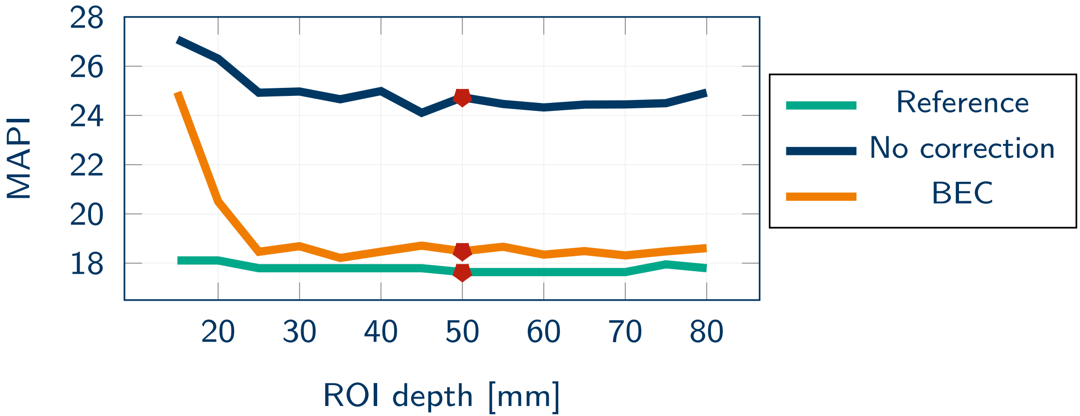

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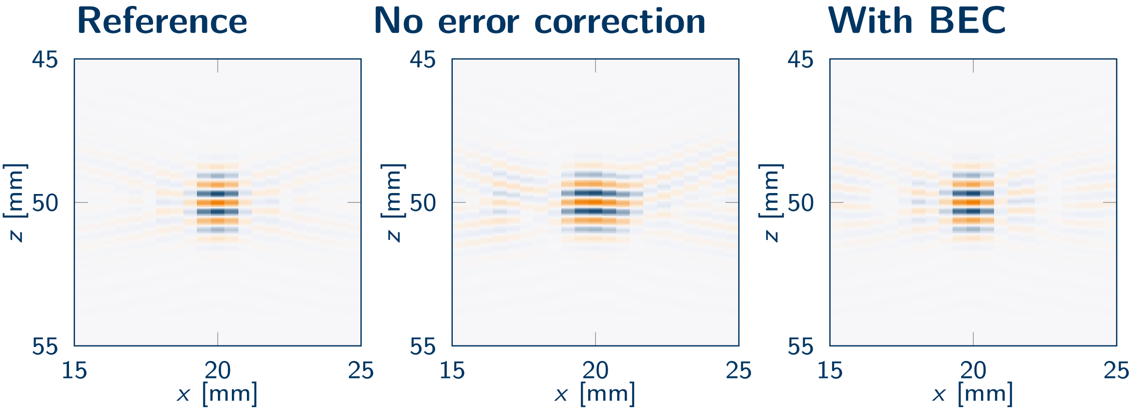


Conclusion

BEC $\Rightarrow$ artefacts reduction

Region	Near surface	Middle	Deep
Resolution	Error susceptive	Good	Very good

Future Work

- Extension to 3D and/or *gridless* cases
- Error correction via minimax estimator

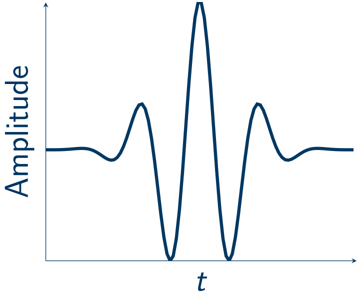
Backup

Parameters w.r.t. Test Object

Parameter	Value
Material	Aluminium
ROI (L $\times$ H)	40 mm $\times$ 100 mm
Speed of sound c_0	$6300\,\mathrm{m\,s^{-1}}$
Sampling frequency f_S	80 MHz
$\mathrm{dt} = \frac{1}{f_s}$	12.5 ns
Sampling distance, surface (dx)	0.5 mm
Sampling distance, depth (dz)	39.375 µm

Parameters w.r.t. Pulse

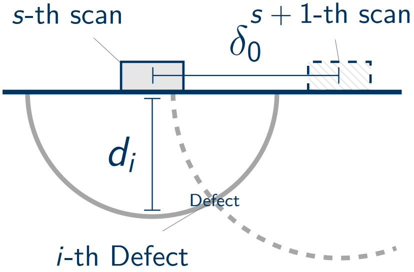
Parameter	Value
Model	Gaussian (Gabor)
Carrier frequency f_c	5 MHz
Wavelength λ	1.26 mm
α	20 $(\text{MHz})^2$
Beam spread	30°



Post-processing Method

Synthetic Aperture Focusing Technique (SAFT)

- Superposition according to propagation time delay
- Spatial sampling of the specimen



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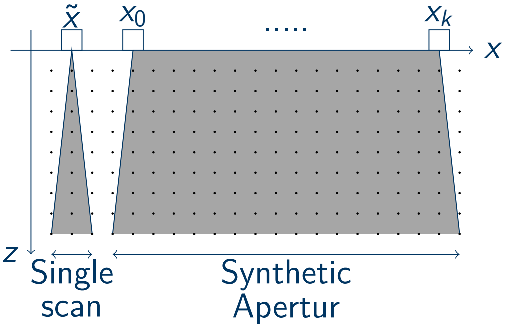
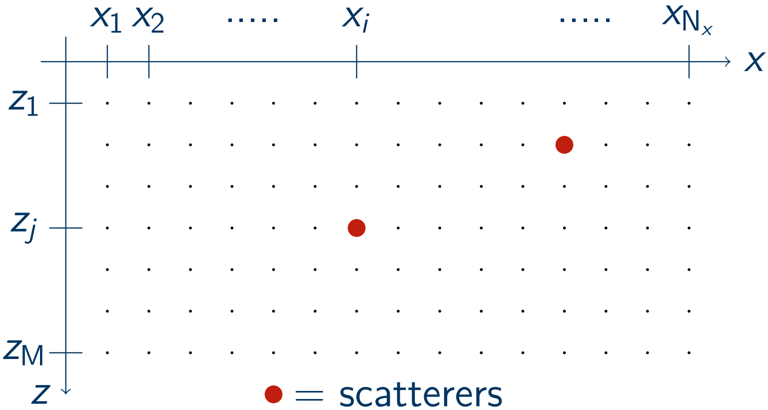


Mathematical Model

defect map for ROI $= \mathrm{M} \times \mathrm{N}_x$



Mathematical Model

Transform: *defect map* $\rightarrow$ single A-Scan at $x_k \in \mathbb{R}$

$$\boldsymbol{a}_k = \boldsymbol{H}_k \boldsymbol{b} + \boldsymbol{n} \in \mathbb{R}^{\mathrm{M}}$$

- $\boldsymbol{a}_k$: measured A-Scan at x_k (M)
- $\boldsymbol{H}_k$: SAFT matrix at x_k ($\mathrm{M} \times \mathrm{L} = \mathrm{M} \times \mathrm{M}\,\mathrm{N}_x$)
  Containing pulse information for $\boldsymbol{s}_l = (x_i, z_j)$, $l = 1...\mathrm{L}$
- $\boldsymbol{b}$: vectorized *defect map* ($\mathrm{L} = \mathrm{M}\,\mathrm{N}_x$)
- $\boldsymbol{n}$: noise (M)

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Transform: *defect map* $\rightarrow$ single A-Scan at $x_k \in \mathbb{R}$

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SAFT matrix at x_k

$$\boldsymbol{H}_k = \begin{bmatrix} \boldsymbol{h}_k^{(1)} & \boldsymbol{h}_k^{(2)} & \cdots & \boldsymbol{h}_k^{(\mathrm{L})} \end{bmatrix} \in \mathbb{R}^{\mathrm{M} \times \mathrm{L}}$$

Mathematical Model

Transform: *defect map* $\rightarrow$ K A-Scans at $\boldsymbol{x} \in \mathbb{R}^{\mathrm{K}}$

$$\mathrm{vec}\{\boldsymbol{A}\} = \boldsymbol{H}\boldsymbol{b} + \mathrm{vec}\{\boldsymbol{N}\} \in \mathbb{R}^{\mathrm{M\,K}}$$

- $\boldsymbol{A}$: measured A-Scans at $\boldsymbol{x}$ ($\mathrm{M} \times \mathrm{K}$)
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A-Scans collected at $\boldsymbol{x}$

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \cdots & \boldsymbol{a}_{\mathrm{K}} \end{bmatrix} \in \mathbb{R}^{\mathrm{M} \times \mathrm{K}}$$

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SAFT matrix at $\boldsymbol{x}$

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Spatial Approximation

Tracked position: $\hat{x}_k = x_k + \Delta x_k \in \mathbb{R}$

Data model

$$\boldsymbol{a}_k = \boldsymbol{H}_k \boldsymbol{b} + \boldsymbol{n} \in \mathbb{R}^{\mathrm{M}}$$

Approximation

$$\boldsymbol{H}_k \approx \hat{\boldsymbol{H}}_k - \hat{\boldsymbol{J}}_k \Delta x_k \in \mathbb{R}^{\mathrm{M} \times \mathrm{L}}$$

- $\hat{\boldsymbol{H}}_k$: SAFT matrix at $\hat{x}_k$ ($\mathrm{M} \times \mathrm{L}$)
- $\hat{\boldsymbol{J}}_k$: Derivative of $\hat{\boldsymbol{H}}_k$ w.r.t. x ($\mathrm{M} \times \mathrm{L}$)

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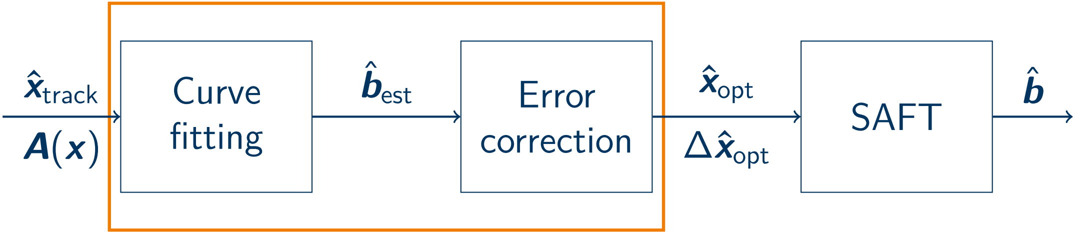
- $\hat{\boldsymbol{H}}$: SAFT matrix at $\hat{\boldsymbol{x}}$ ($\mathrm{M\,K} \times \mathrm{L}$)
- $\hat{\boldsymbol{J}}$: Derivative of $\hat{\boldsymbol{H}}$ w.r.t. $\boldsymbol{x}$ ($\mathrm{M\,K} \times \mathrm{L}$)
- $\boldsymbol{E}$: Error matrix $= \mathrm{diag}\{\Delta\boldsymbol{x}\} \otimes \boldsymbol{I}_{\mathrm{M}}$ ($\mathrm{M\,K} \times \mathrm{M\,K}$)

Proposed Preprocessing

$$\min_{\Delta \boldsymbol{x}} \min_{\boldsymbol{b}} \left\| \operatorname{vec}\{\boldsymbol{A}\} - \left(\hat{\boldsymbol{H}} + \boldsymbol{E}\hat{\boldsymbol{J}}\right) \boldsymbol{b} \right\|_2^2$$



Curve Fitting

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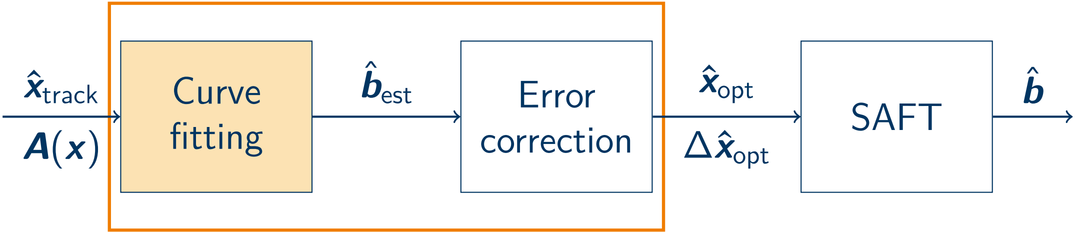
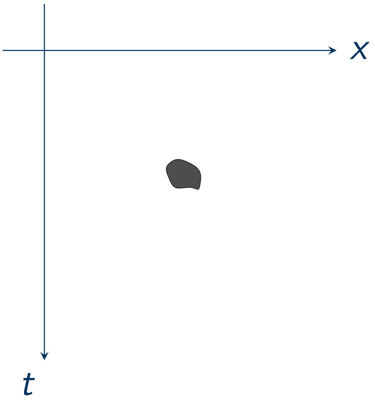


Curve Fitting

B-Scan

- Defect map $\boldsymbol{b} \mathrel{\hat{=}} (x_\mathrm{d}, z_\mathrm{d})$
- Scan positions $\boldsymbol{x} \in \mathbb{R}^K$
- Peak positions $\boldsymbol{z} \in \mathbb{R}^K$
  $\mathrel{\hat{=}}$ Time-of-Flight
- Curvature

$\Rightarrow \approx$ **Parabola**



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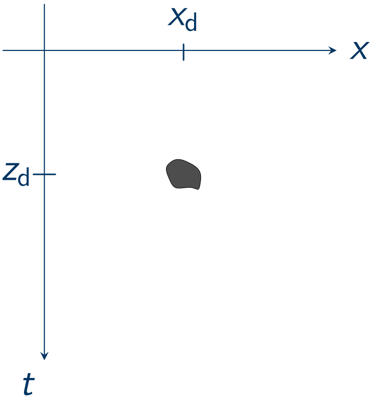

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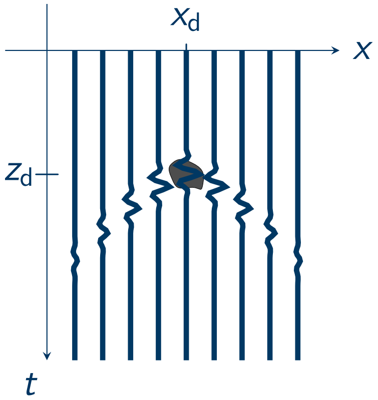


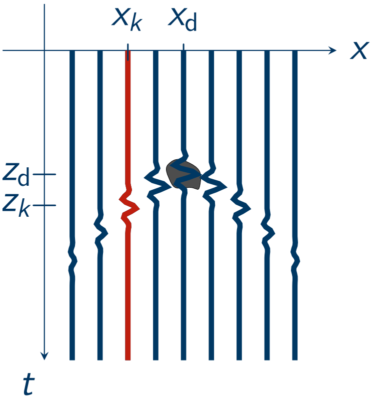
Curve Fitting

Parabola approximation: k-th scan

$$z_k \approx u_0 + u_1 \cdot x_k + u_2 \cdot x_k^2$$

$$\Rightarrow \quad x_d = -\frac{u_1}{2u_2}$$

$$z_d = u_0 - \frac{u_1^2}{4u_2}$$



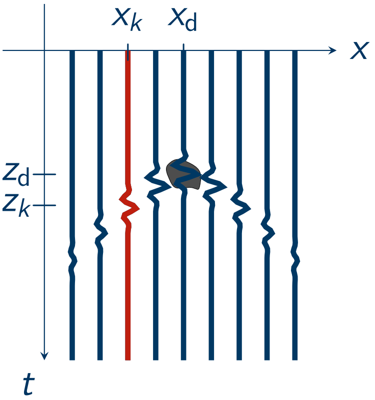
Curve Fitting

Parabola approximation: k-th scan

$$z_k \approx u_0 + u_1 \cdot x_k + u_2 \cdot x_k^2$$

K **scan positions**:

$$\underbrace{\begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_K \end{bmatrix}}_{\boldsymbol{z}} \approx \underbrace{\begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ \vdots & \vdots & \vdots \\ 1 & x_K & x_K^2 \end{bmatrix}}_{\boldsymbol{X}} \cdot \underbrace{\begin{bmatrix} u_0 \\ u_1 \\ u_2 \end{bmatrix}}_{\boldsymbol{u}}$$



Polynomial Regression via TLS

Approximation

$$\begin{bmatrix} z_1^2 \\ z_2^2 \\ \vdots \\ z_K^2 \end{bmatrix} \approx \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ \vdots & \vdots & \vdots \\ 1 & x_K & x_K^2 \end{bmatrix} \cdot \begin{bmatrix} u_0 \\ u_1 \\ u_2 \end{bmatrix}$$

$$\boldsymbol{z} \qquad \boldsymbol{X} \qquad \boldsymbol{u}$$

Errors in $\boldsymbol{z}$ and $\boldsymbol{X}$

- $\Delta\boldsymbol{z}$: Measurement noise, quantization error etc
- $\Delta\boldsymbol{X}$:Tracking error

$\Rightarrow$ **independent errors**

Polynomial Regression via TLS

Approximation

$$\underset{K}{\boldsymbol{z}} \approx \underset{K \times 3}{\boldsymbol{X}} \cdot \underset{3}{\boldsymbol{u}}$$

Polynomial Regression via TLS

Incorporating errors

$$\underset{K}{\boldsymbol{z} + \Delta \boldsymbol{z}} = \underset{K \times 3}{(\boldsymbol{X} + \Delta \boldsymbol{X})} \cdot \underset{3}{\boldsymbol{u}}$$

Polynomial Regression via TLS

Incorporating errors

$$\underbrace{\boldsymbol{z} + \Delta \boldsymbol{z}}_{K} = \underbrace{(\boldsymbol{X} + \Delta \boldsymbol{X})}_{K \times 3} \cdot \underbrace{\boldsymbol{u}}_{3}$$

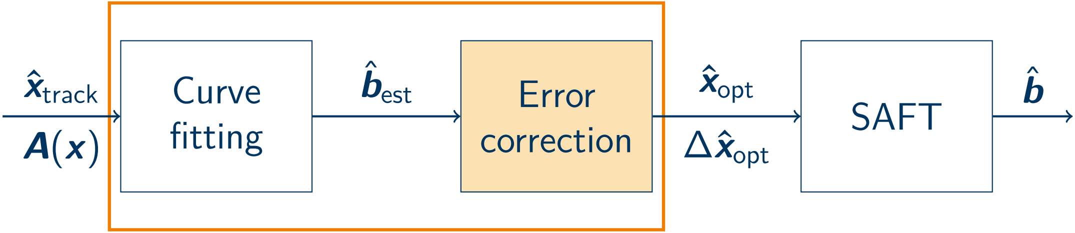
$\Rightarrow$ Total least squares

$$\min \|[\Delta \boldsymbol{X} | \Delta \boldsymbol{z}]\|_F^2 \quad \text{s.t.} \quad \boldsymbol{z} + \Delta \boldsymbol{z} = [\boldsymbol{X} + \Delta \boldsymbol{X}] \cdot \boldsymbol{u}$$

$$\Rightarrow \boldsymbol{u} \Rightarrow \hat{\boldsymbol{b}}_{\text{est}}$$

Iterative Error Correction

$$\min_{\Delta \boldsymbol{x}} \min_{\boldsymbol{b}} \left\| \mathrm{vec}\{\boldsymbol{A}\} - \left(\hat{\boldsymbol{H}} + \boldsymbol{E}\hat{\boldsymbol{J}}\right) \boldsymbol{b} \right\|_2^2$$



Iterative Error Correction

Taylor approximation

$$\mathrm{vec}\{\boldsymbol{A}\} \approx \left(\hat{\boldsymbol{H}} + \boldsymbol{E}\hat{\boldsymbol{J}}\right) \boldsymbol{b}$$

Cost function (2nd step)

$$\min_{\Delta \boldsymbol{x}} \left\| \mathrm{vec}\{\boldsymbol{A}\} - \left(\hat{\boldsymbol{H}} + \boldsymbol{E}\hat{\boldsymbol{J}}\right) \hat{\boldsymbol{b}}_{\mathrm{est}} \right\|_2^2$$

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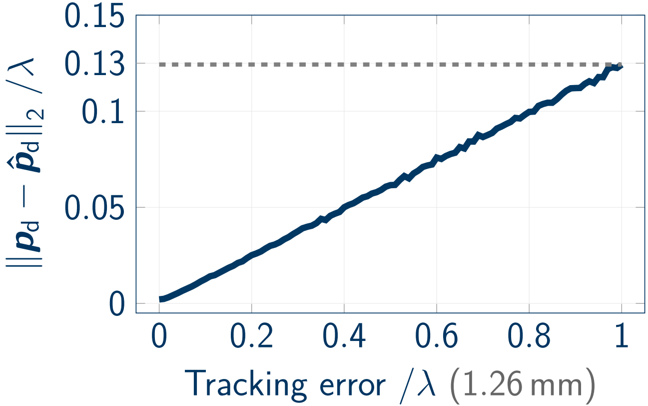
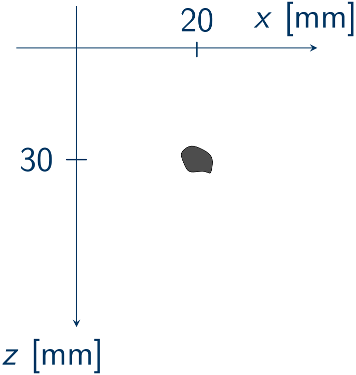
$\Rightarrow$ Nonlinear programming (e.g. Newton method)

Signal Source Estimation

Estimation error





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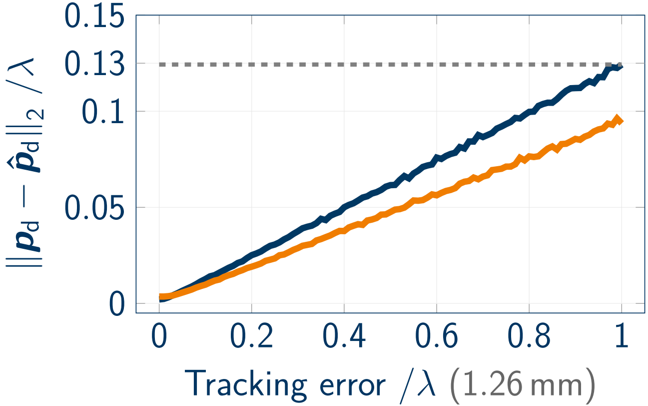
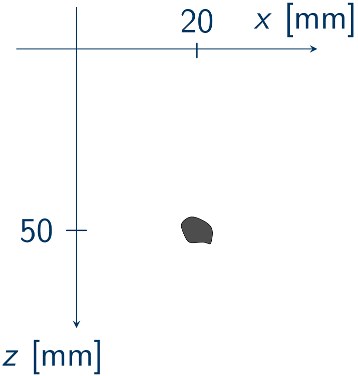




Metrics: SE†

$$\mathrm{SE}^\dagger = \frac{\|\gamma\hat{\boldsymbol{a}} - \boldsymbol{a}\|_2}{\|\boldsymbol{a}\|_2}$$

γ: normalization factor

$$\gamma = \frac{\boldsymbol{a}^\mathsf{T} \cdot \hat{\boldsymbol{a}}}{\hat{\boldsymbol{a}}^\mathsf{T} \cdot \hat{\boldsymbol{a}}}.$$

Metrics: API

$$\text{API} = \frac{A_\epsilon}{\lambda^2}$$

Metrics: gCNR

$$\mathrm{OVL} = \int \min\{p_i(x), p_o(x)\}\,\mathrm{dx}$$

where

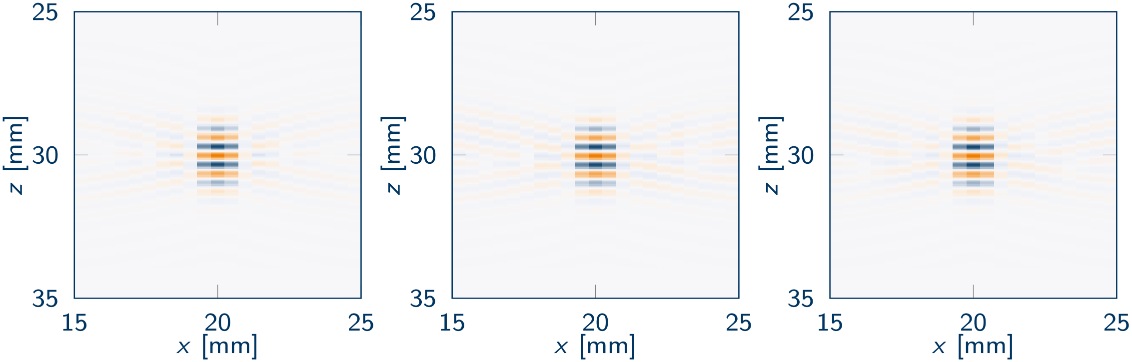
- $p_i(x)$: p.d.f for inside the target area
- $p_o(x)$: p.d.f for outside the target area

$$\mathrm{gCNR} = 1 - \mathrm{OVL}$$

Error Tolerance: Images

Error $= 0.4\lambda\ (\lambda = 1.26\,\mathrm{mm})$

Reference — **No error correction** — **With BEC**



Error Tolerance: Images

Error $= 0.8\lambda$ $(\lambda = 1.26\,\mathrm{mm})$

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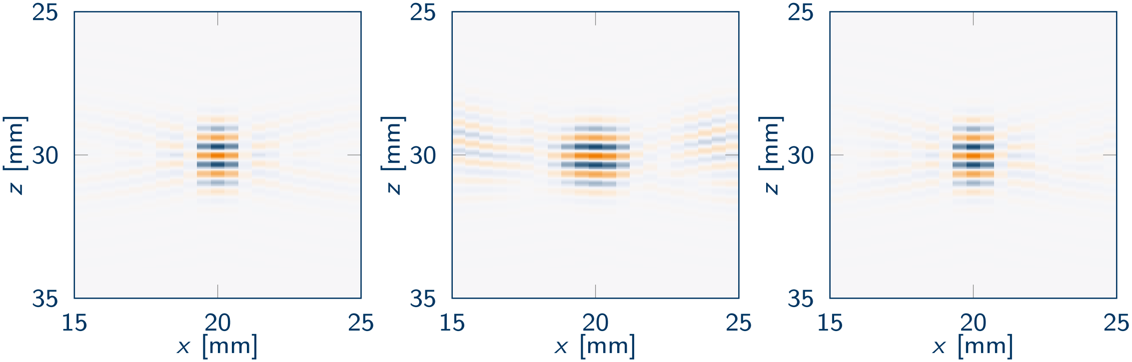
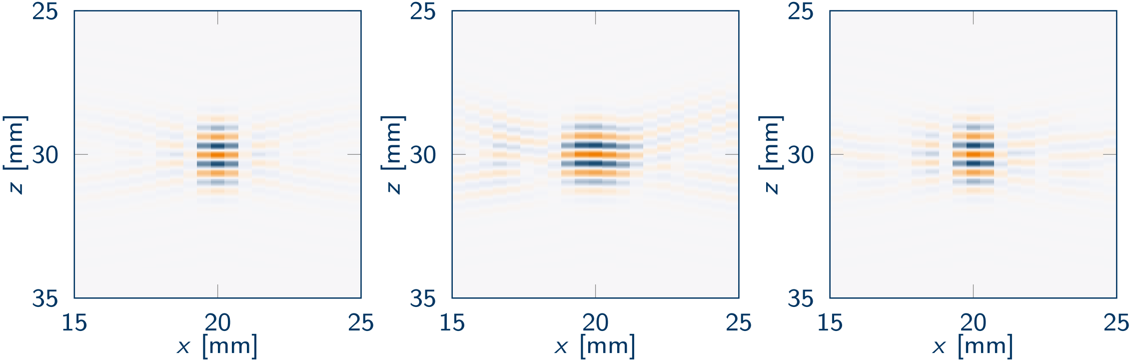


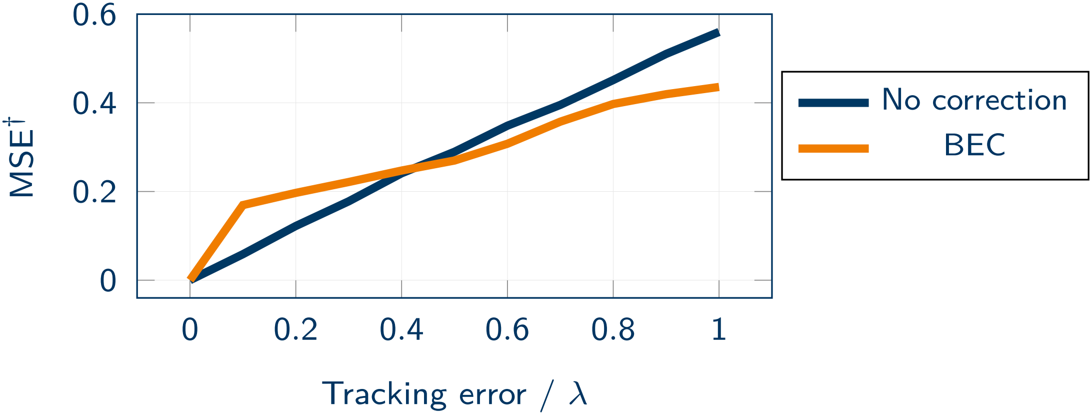
Error Tolerance: Images

Error $= 1\lambda \; (\lambda = 1.26\,\mathrm{mm})$

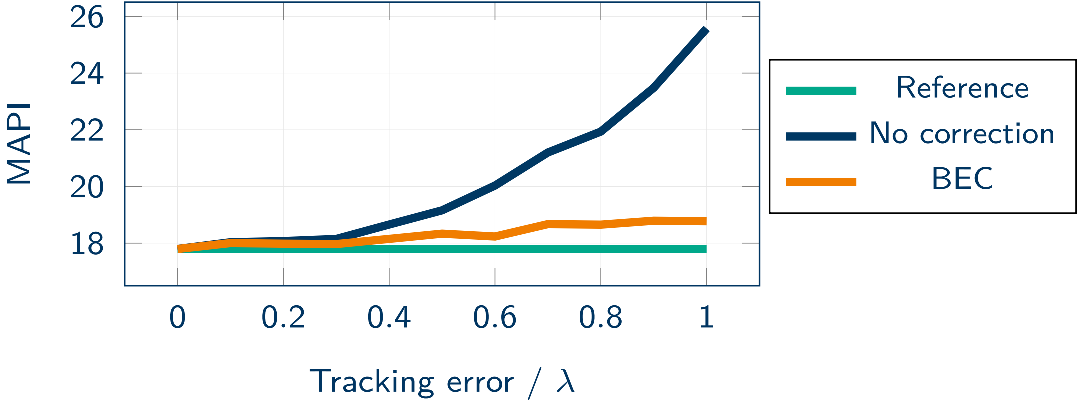
Reference | **No error correction** | **With BEC**



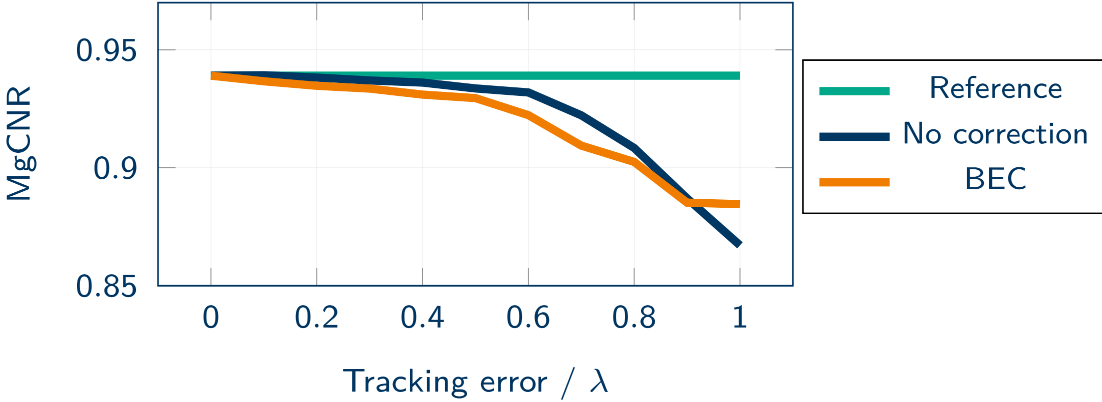
Error tolerance Results: SE†



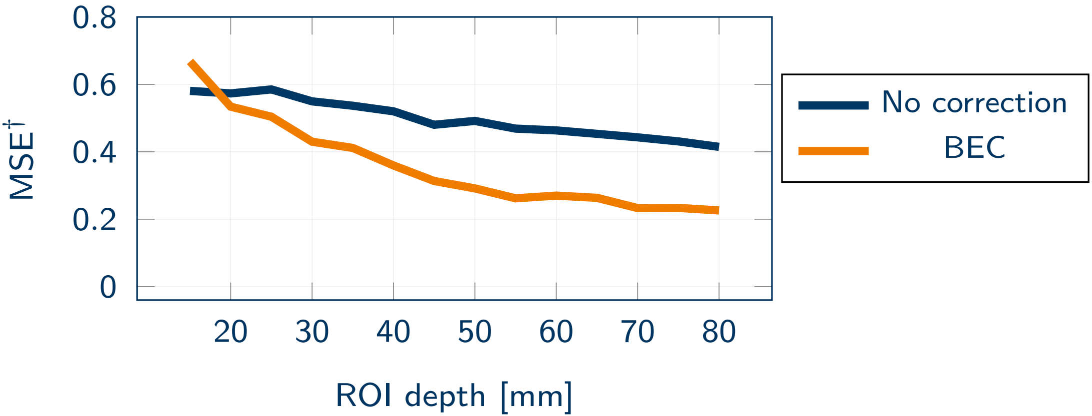
Error tolerance Results: API



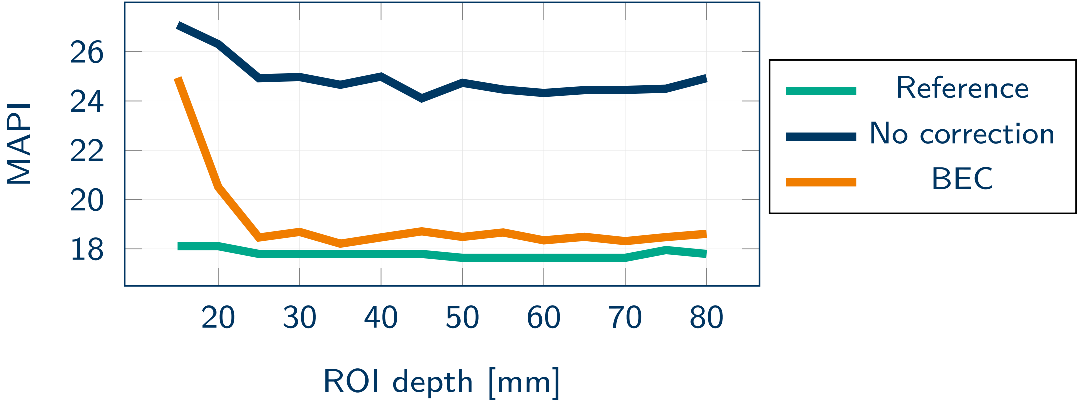
Error tolerance Results: gCNR



Impact of the ROI Depth: $SE^{\dagger}$



Impact of the ROI Depth: API



Impact of the ROI Depth: gCNR