Data Driven Hybrid Algorithms for Preprocessing of Manually Acquired Ultrasound NDT Data

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Fraunhofer IZFP

Assisted Manual Ultrasonic Testing

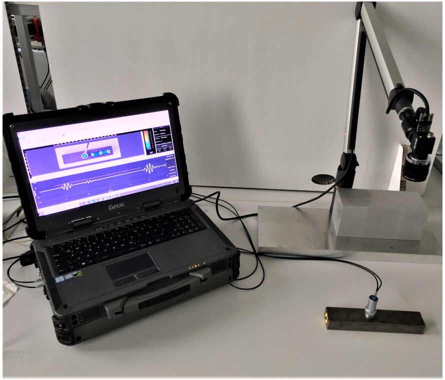
Conventional manual UT

Source: Quality Magazine

Problem: reliability

Assisted Manual Ultrasonic Testing

3D SmartInspect [1]



Problem: reliability

→ Assistance system

- Position recognition
- Data recording
- Data visualization
- Visual feedback
- Post-processing

[1] A. Omira, *Real-time Reconstruction of Manually Measured Compressed Ultrasound Synthetic Aperture Measurement Data*, 2021

Assisted Manual Ultrasonic Testing

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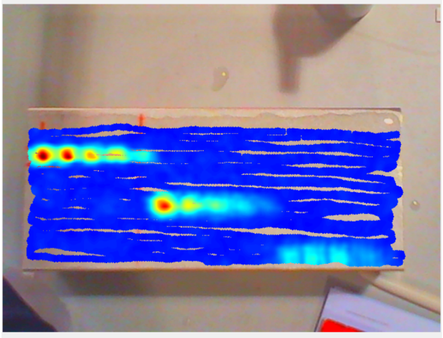

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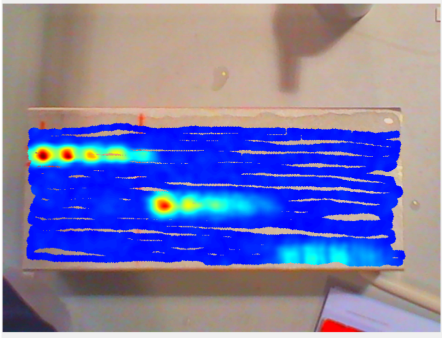

Problem: reliability

→ Assistance system
- Position recognition
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- Post-processing

→ Degraded resolution

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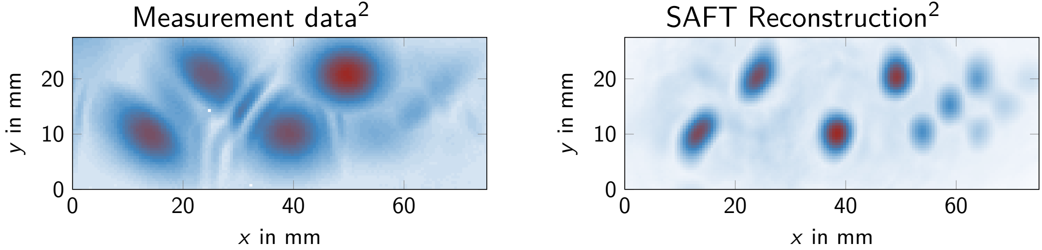
Motivation: Image Quality Improvement

Automatic measurement



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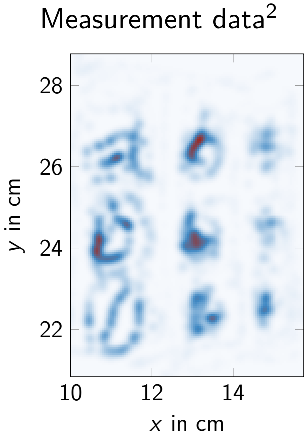
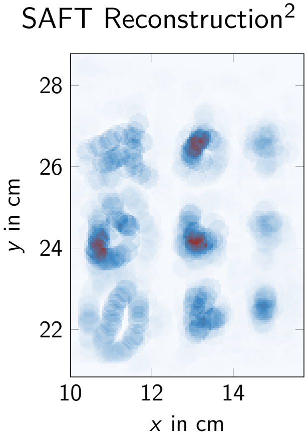
[2]F. Krieg et al., SAFT processing for manually acquired ultrasonic measurement data with 3D SmartInspect, *SHM-NDT*, 2018

Motivation: Image Quality Improvement

Manual measurement

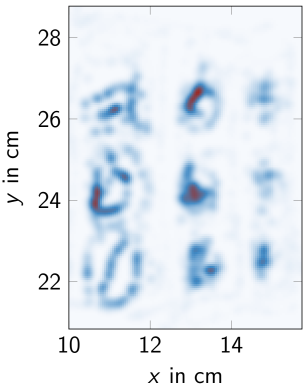




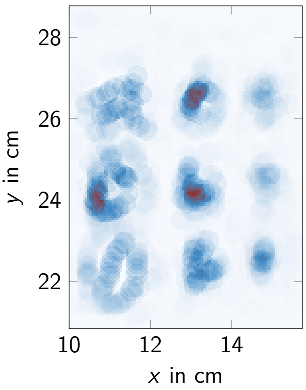
Motivation: Image Quality Improvement

Manual measurement ∵ **accumulation of random factors**

Measurement data[2]

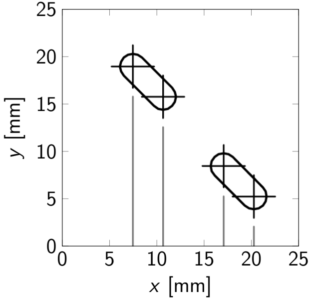


SAFT Reconstruction[2]



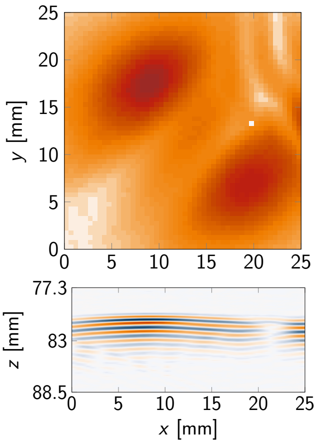
Impact of Missing Data

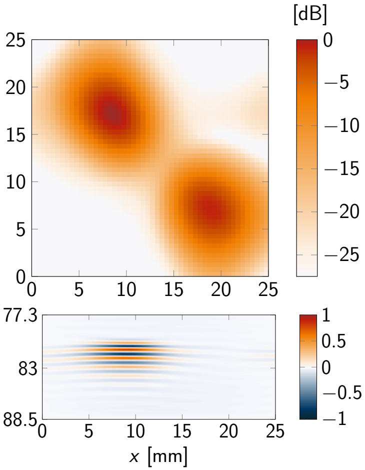
ROI



Impact of Missing Data

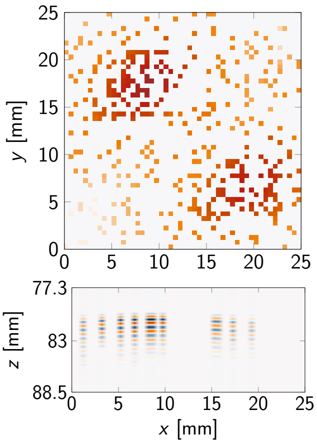
Full sampling → Reconstruction

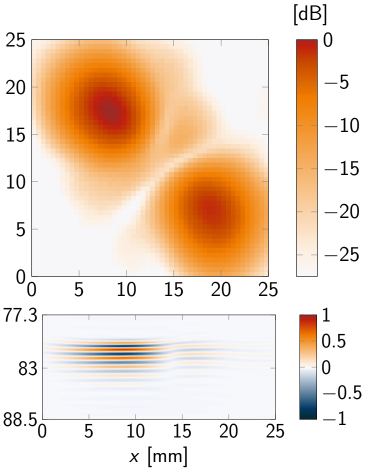




Impact of Missing Data

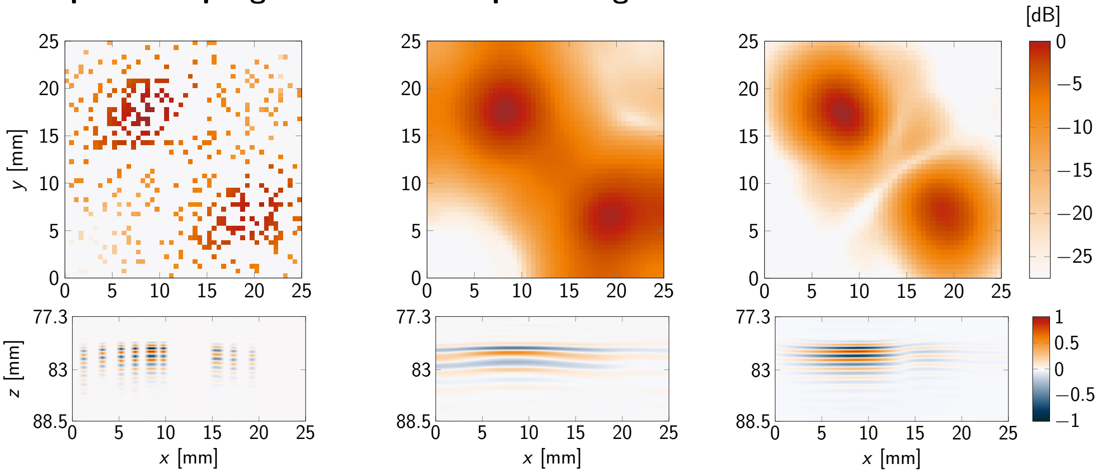
Sparse sampling → **Reconstruction**





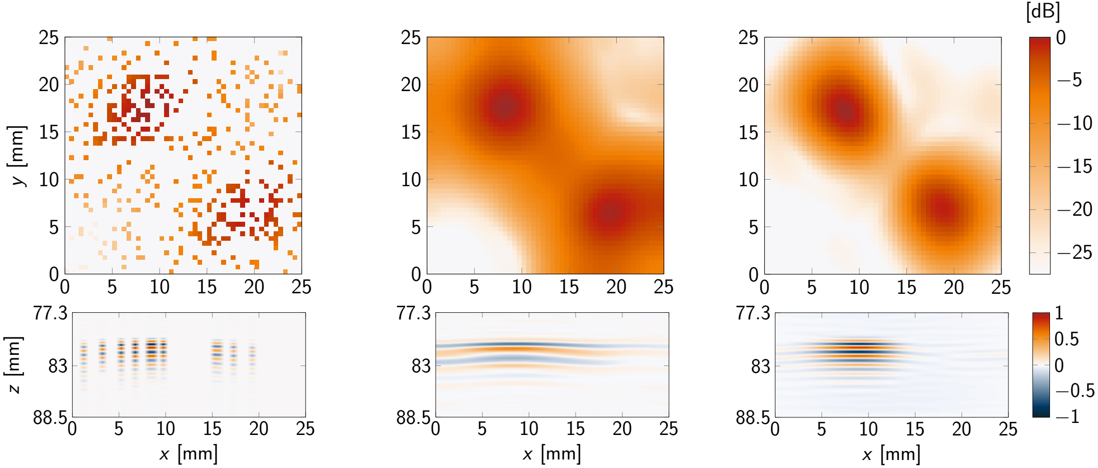
Solution: Interpolation as Preprocessing

Sparse sampling → **Preprocessing** **Reconstruction**



Solution: Interpolation as Preprocessing

Sparse sampling → Preprocessing → Reconstruction

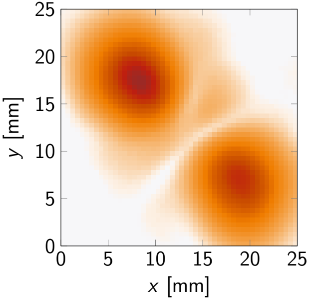


Objectives and Contributions

Objectives

- Artefacts reduction in reconstructions
  $\rightarrow$ Interpolate missing data as preprocessing
  $\hat{=}$ Interpolation of nonlinear spatio-temporal data
- Fast yet interpretable method

Sparse sampling



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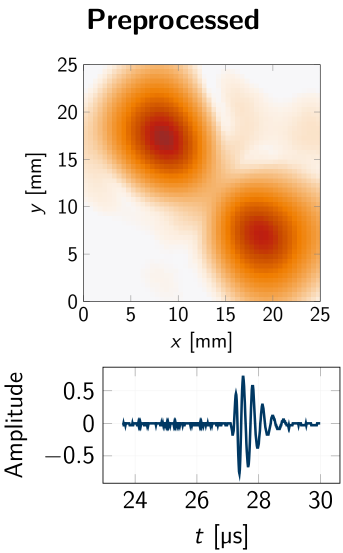

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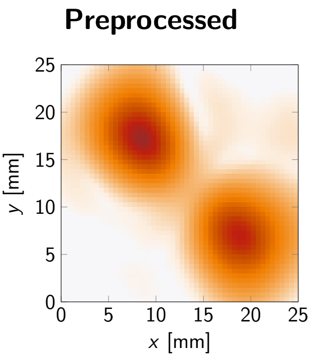
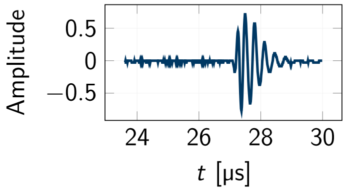
- Artefacts reduction in reconstructions
  → Interpolate missing data as preprocessing
  ≙ Interpolation of nonlinear spatio-temporal data
- Fast yet interpretable method

Solutions

- Spatial statistical interpolation
- Batch-wise interpolation
- Perform in space-frequency domain
- Incorporate DNN for fast on-site execution

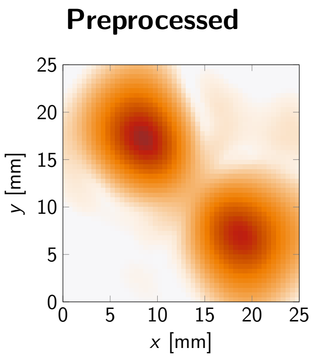


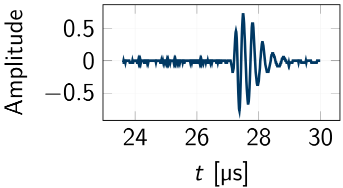


Objectives and Contributions

Contributions

- Spatial statistical modeling of UT data in space-frequency (SF) domain
- Develop a hybrid interpolation scheme in SF-domain
  - (i) Interpolation via a MMSE estimator
    $\rightarrow$ **SF-Kriging**
  - (ii) Estimation of spatial statistics via DNN
    $\hat{=}$ Vector-valued regression
    $\rightarrow$ **FVnet**
- Feedback feature for experimental design





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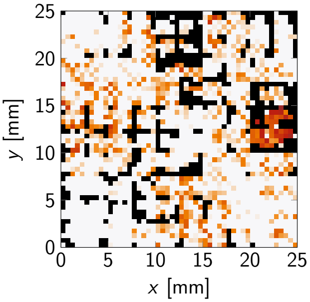
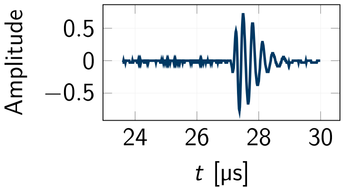
Feedback





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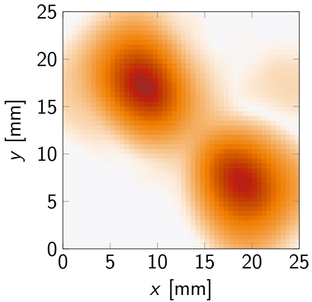
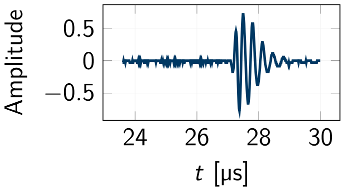
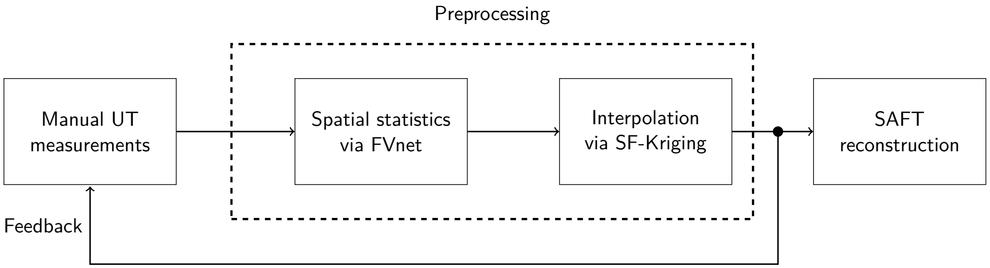
Resmpl. + preproc.

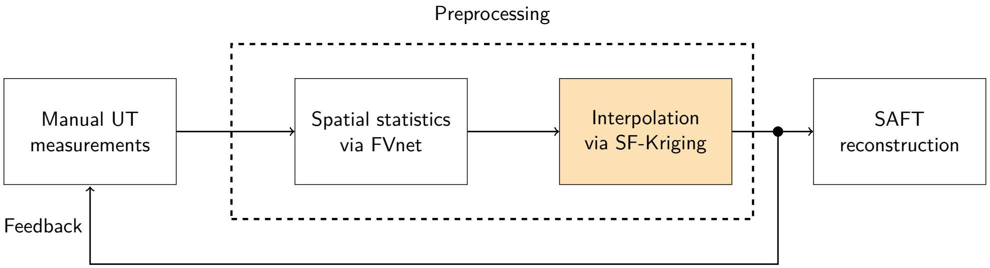




Preprocessing Scheme



ST-Interpolation in SF-domain



ST-Interpolation in SF-domain

ST-domain: temp. correlation → vector-valued pred.

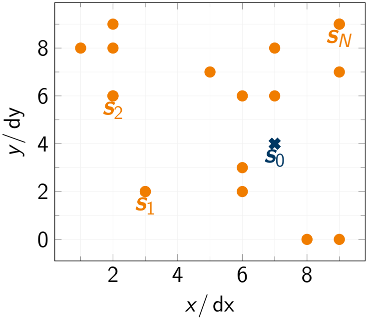
Sought:

$$\boldsymbol{a}_{\boldsymbol{s}_0} \in \mathbb{R}^M$$

Given:

$$\boldsymbol{A}_S = \begin{bmatrix} \boldsymbol{a}_{\boldsymbol{s}_1} & \boldsymbol{a}_{\boldsymbol{s}_2} & \cdots & \boldsymbol{a}_{\boldsymbol{s}_N} \end{bmatrix} \in \mathbb{R}^{M \times N}$$

Samp. and pred. positions



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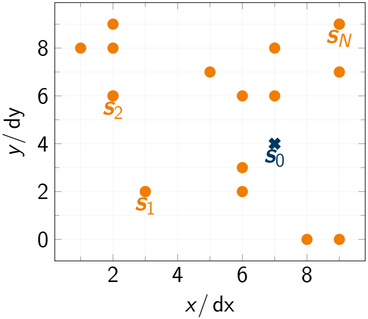
SF-domain: orthogonal Fourier bases

⇒ individual prediction for a single frequency

$$\boldsymbol{p}_{\boldsymbol{s}_0} = \boldsymbol{F}_M \boldsymbol{a}_{\boldsymbol{s}_0} \in \mathbb{C}^M$$

Samp. and pred. positions



ST-Interpolation in SF-domain

SF-domain: $\rightarrow$ set of scalar-valued pred. $\forall \omega_m$

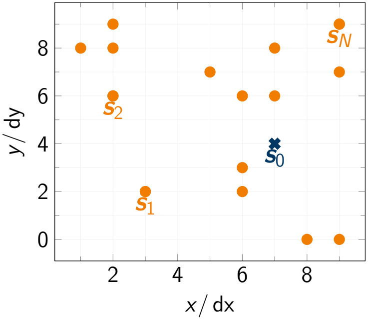
Sought:

$$p_{\boldsymbol{s}_0 m} \in \mathbb{C}$$

Given:

$$\pi_m^S = \begin{bmatrix} p_{\boldsymbol{s}_1 m} & p_{\boldsymbol{s}_2 m} & \cdots & p_{\boldsymbol{s}_N m} \end{bmatrix}^\mathsf{T} \in \mathbb{C}^M$$

Samp. and pred. positions



Spatial Statistical Approach: SF-Kriging

Assumptions: **intrinsic stationarity**

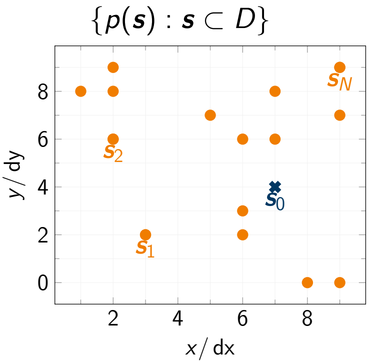
$\hat{=}$ Stationary assumptions for the increments

(1) Mean of the increments is 0

$$\mathrm{E}\left\{p(\boldsymbol{s}) - p(\boldsymbol{s}+\boldsymbol{h})\right\} = 0$$

(2) Variance of the increments is shift invariant
$\rightarrow$ function of the spatial lag

$$\mathrm{Var}\left\{p(\boldsymbol{s}) - p(\boldsymbol{s}+\boldsymbol{h})\right\} := 2\gamma(\boldsymbol{h})$$



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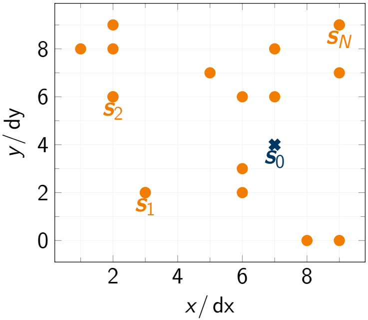
$2\gamma(\boldsymbol{h})$ = **Frequency variogram (FV)**
(nonnegative, real-valued)

$\{p(\boldsymbol{s}) : \boldsymbol{s} \subset D\}$



Spatial Statistical Approach: SF-Kriging

Linear unbiased MMSE predictor (Kriging):

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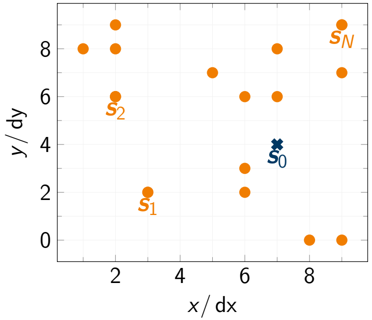

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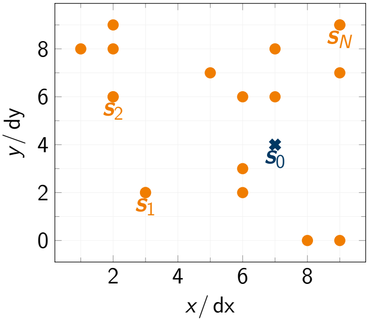

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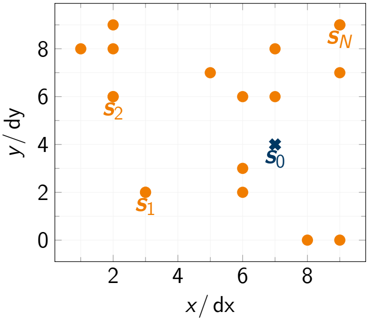

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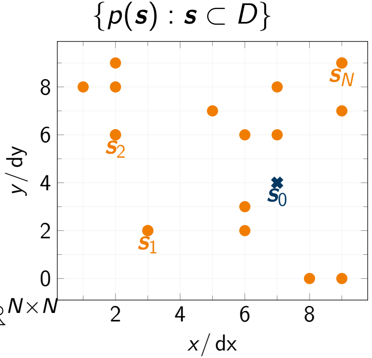
Between the samples:

$$\boldsymbol{\Gamma}_m = \begin{bmatrix} \mathrm{Var}\{p_{\boldsymbol{s}_1 m} - p_{\boldsymbol{s}_1 m}\} & \cdots & \mathrm{Var}\{p_{\boldsymbol{s}_N m} - p_{\boldsymbol{s}_1 m}\} \\ \vdots & \ddots & \vdots \\ \mathrm{Var}\{p_{\boldsymbol{s}_1 m} - p_{\boldsymbol{s}_N m}\} & \cdots & \mathrm{Var}\{p_{\boldsymbol{s}_N m} - p_{\boldsymbol{s}_N m}\} \end{bmatrix} \in \mathbb{R}^{N \times N}$$



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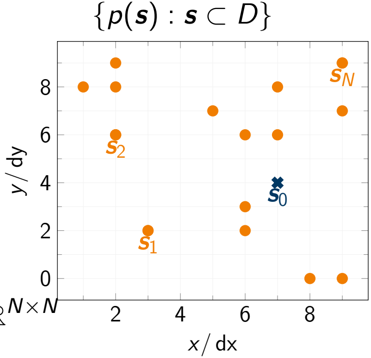

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Between $p_{\boldsymbol{s}_0 m}$ and the samples:

$$\boldsymbol{\upsilon}_m = \begin{bmatrix} \mathrm{Var}\{p_{\boldsymbol{s}_0 m} - p_{\boldsymbol{s}_1 m}\} & \cdots & \mathrm{Var}\{p_{\boldsymbol{s}_0 m} - p_{\boldsymbol{s}_N m}\} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^N$$

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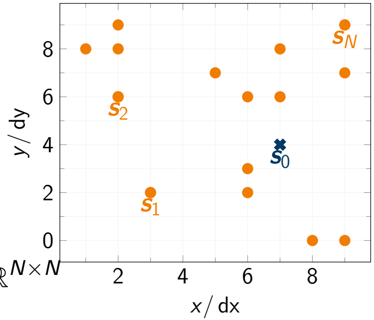

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Between $p_{\boldsymbol{s}_0 m}$ and the samples: $\rightarrow$ unknown

$$\boldsymbol{\upsilon}_m = \begin{bmatrix} \mathrm{Var}\left\{p_{\boldsymbol{s}_0 m} - p_{\boldsymbol{s}_1 m}\right\} & \cdots & \mathrm{Var}\left\{p_{\boldsymbol{s}_0 m} - p_{\boldsymbol{s}_N m}\right\} \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^N$$

Spatial Statistical Approach: SF-Kriging

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$$\hat{p}_{s_0 m} = \boldsymbol{w}_m^{\mathrm{H}} \boldsymbol{\pi}_m^S$$

Weights: 2nd order statistics = FVs

$$\boldsymbol{w}_m = \boldsymbol{\Gamma}_m^{-1} \boldsymbol{\upsilon}_m \in \mathbb{R}^N$$

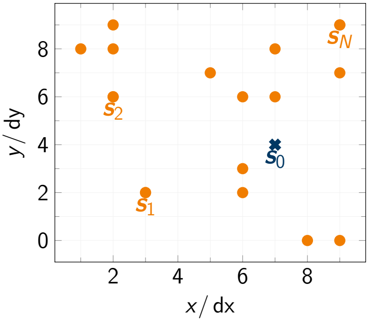
Between the samples:

$$\boldsymbol{\Gamma}_m = \begin{bmatrix} \gamma(\boldsymbol{s}_1 - \boldsymbol{s}_1) & \cdots & \gamma(\boldsymbol{s}_N - \boldsymbol{s}_1) \\ \vdots & \ddots & \vdots \\ \gamma(\boldsymbol{s}_1 - \boldsymbol{s}_N) & \cdots & \gamma(\boldsymbol{s}_N - \boldsymbol{s}_N) \end{bmatrix} \in \mathbb{R}^{N \times N}$$

Between $p_{s_0 m}$ and the samples:

$$\boldsymbol{\upsilon}_m = \begin{bmatrix} \gamma(\boldsymbol{s}_0 - \boldsymbol{s}_1) & \cdots & \gamma(\boldsymbol{s}_0 - \boldsymbol{s}_N) \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^N$$

$\{p(\boldsymbol{s}) : \boldsymbol{s} \subset D\}$



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Between $p_{\boldsymbol{s}_0 m}$ and the samples:

$$\boldsymbol{\upsilon}_m = \begin{bmatrix} \gamma(\boldsymbol{s}_0 - \boldsymbol{s}_1) & \cdots & \gamma(\boldsymbol{s}_0 - \boldsymbol{s}_N) \end{bmatrix}^{\mathrm{T}} \in \mathbb{R}^N$$

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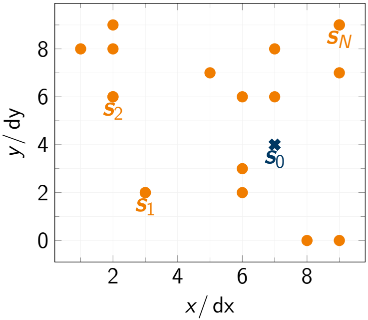
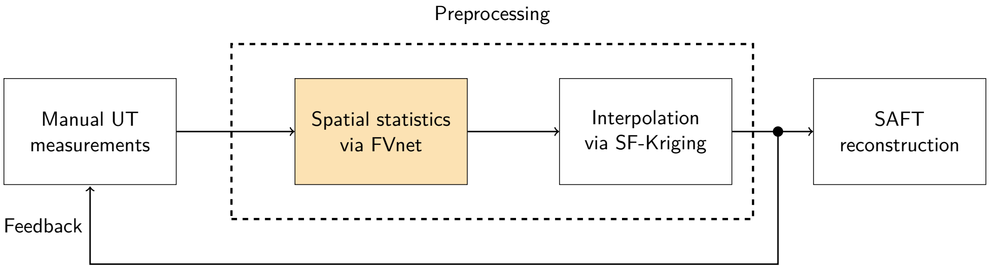


$\Rightarrow$ **Estimate of $\gamma_m(\boldsymbol{h})$**

Frequency Variogram Estimation



Frequency Variogram Estimation

(I) **Nonparametric estimation**

(II) **Parametric estimation**

Frequency Variogram Estimation

(I) Nonparametric estimation

- Method-of-moments estimate
- $(-)$ requires adequate samples
  $\because$ some lags may be missing

(II) Parametric estimation

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- Modeling based on SPDEs
- Parameter estimation for each lag and frequency
  $\rightarrow$ not suitable for online applications

Frequency Variogram Estimation

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  ∵ some lags may be missing

(II) Parametric estimation

- Modeling based on SPDEs
- Parameter estimation for each lag and frequency
  → not suitable for online applications

(III) Data driven approach via DNN (FVnet)

- Property: lattice data within a small batch
  → Vector-valued lags and frequency variograms
- Method-of-moments estimates are available
  ⇒ Vector-valued regression problem

Hybrid Preprocessing: Performance

Simulation studies

- Batch interpolation
- Reconstruction of subsampled data

Hybrid Preprocessing: Performance

Simulation studies

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Hybrid Preprocessing: Performance

Simulation studies

- Batch interpolation
- Reconstruction of subsampled data

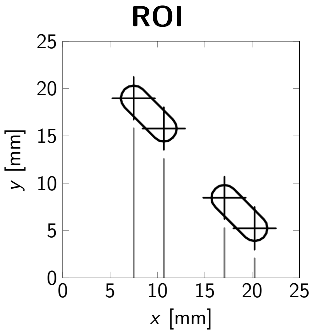
Purpose

- Resolution of reconstructed images
- Effectiveness of experimental design

Hybrid Preprocessing: Performance

Scenario and Assumptions

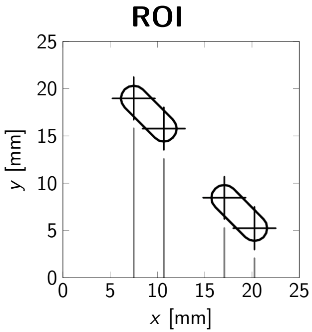
- Base data = densely scanned measurements (0.5 mm grids $\hat{=}$ 8.7 samples $/\lambda^2$)
- ROI = 25 mm × 25 mm
- Subsampling in ROI: 5 . . . 17% ($\hat{=}$ 0.44 . . . 1.48 samples $/\lambda^2$)
- Moving window interpolation (50% overlap)
- Single batch = 5 mm × 5 mm



Hybrid Preprocessing: Performance

Comparison

- Reference
  = Reconstruction of fully sampled data
- Inverse Distance Weighting (IDW)



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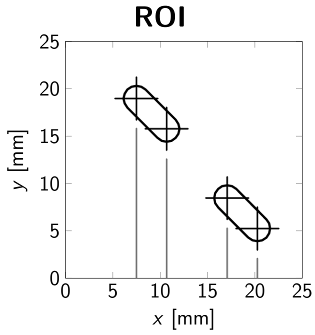
Evaluation metrics

- Normalized squared error
- *Generalized Contrast-to-Noise Ratio* (gCNR)



Hybrid Preprocessing: Performance

Comparison

- Reference
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Evaluation metrics

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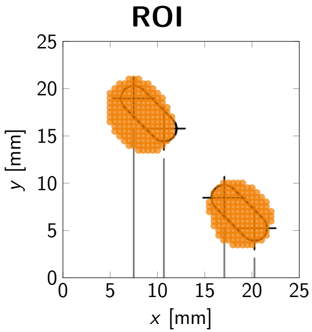
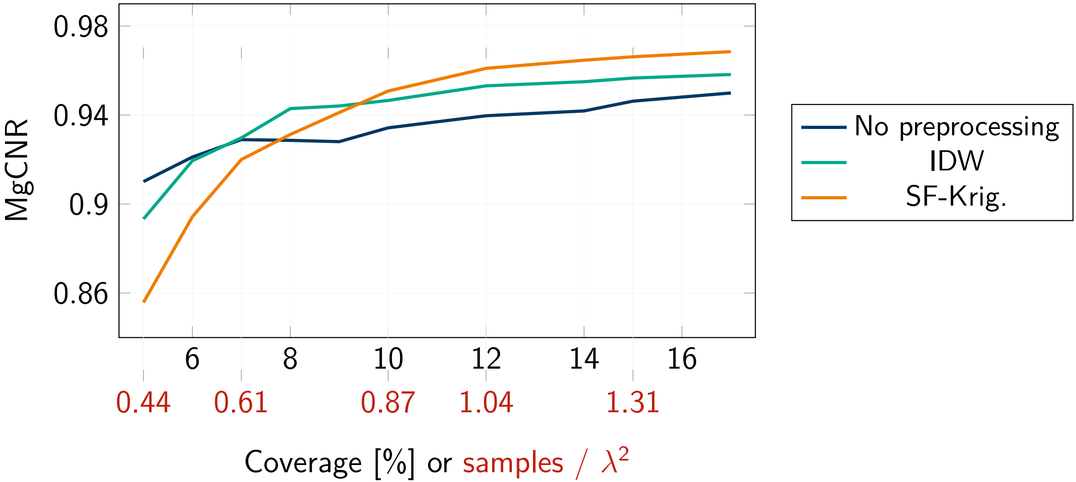
- *Generalized Contrast-to-Noise Ratio* (gCNR)

  gCNR = 1 - P(false detection)

  $\Rightarrow$ higher gCNR $\hat{=}$ better resolution



Performance Evaluation for Varying Coverage



Experimental Design: Strategic Resampling

Feedback for experimental design

- (1) Initial sampling ($= N$)
- (2) Interpolation via SF-Kriging
- (3) Variance map as feedback
- (4) Prioritized resampling
  $+$ random resampling
  ($= 2N$)

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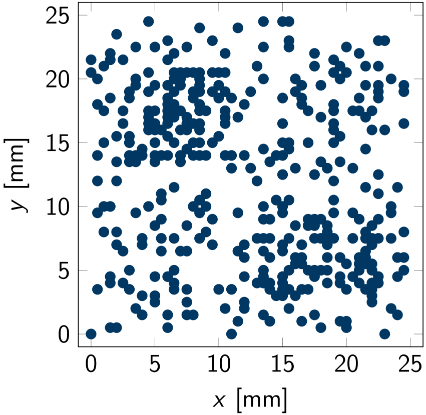

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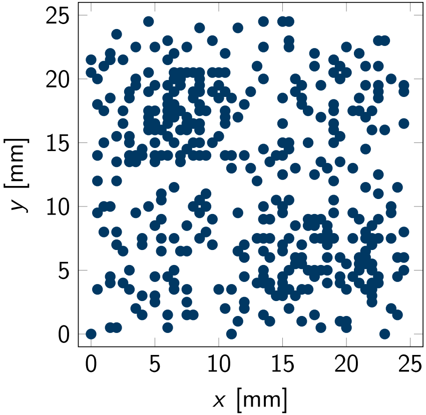

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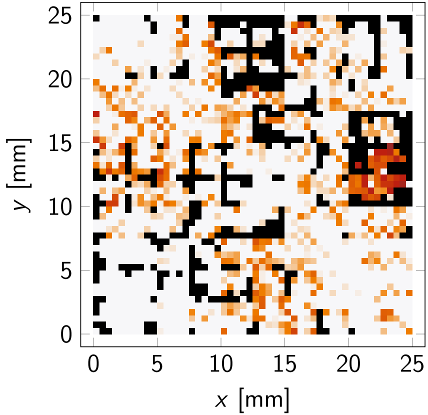

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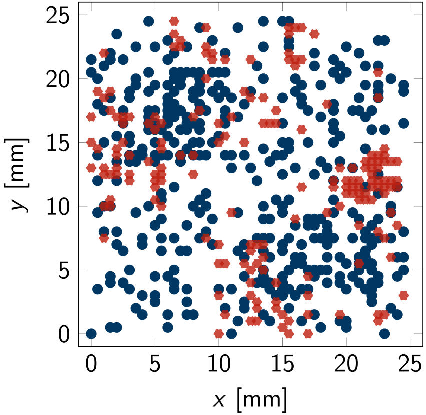

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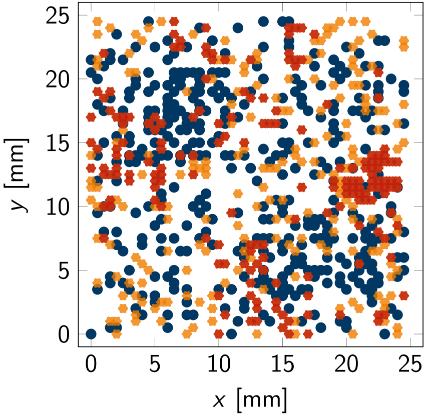

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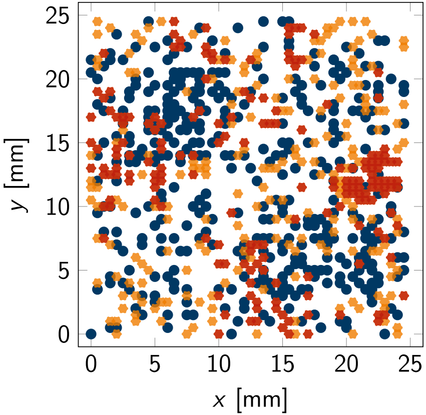
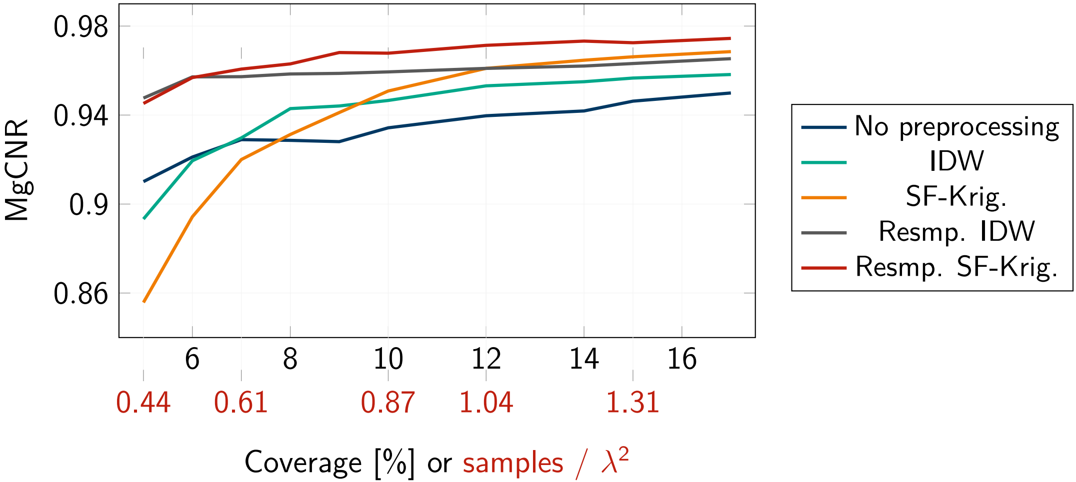
  ($= 2N$)

$\rightarrow$ Effect on reconstruction resolution



Effectiveness of Strategic Resampling



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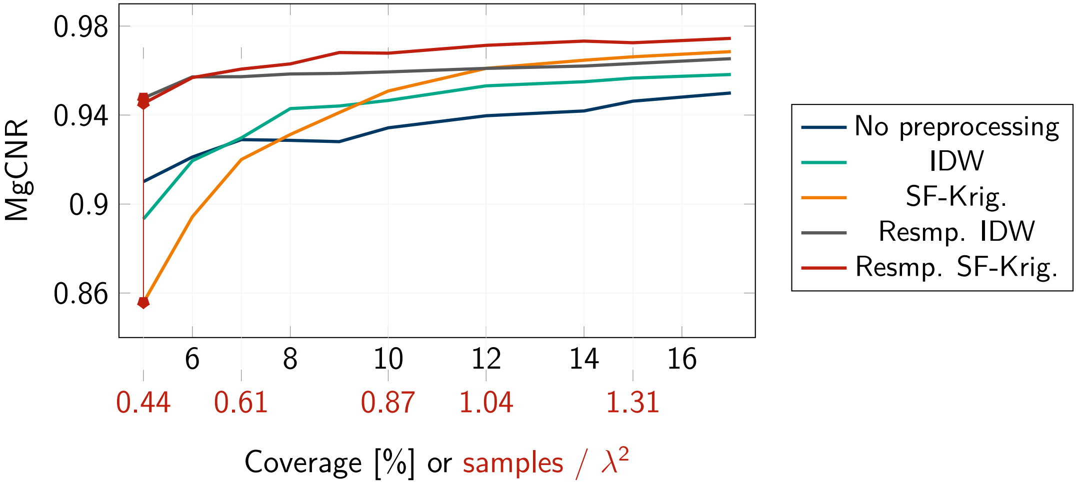
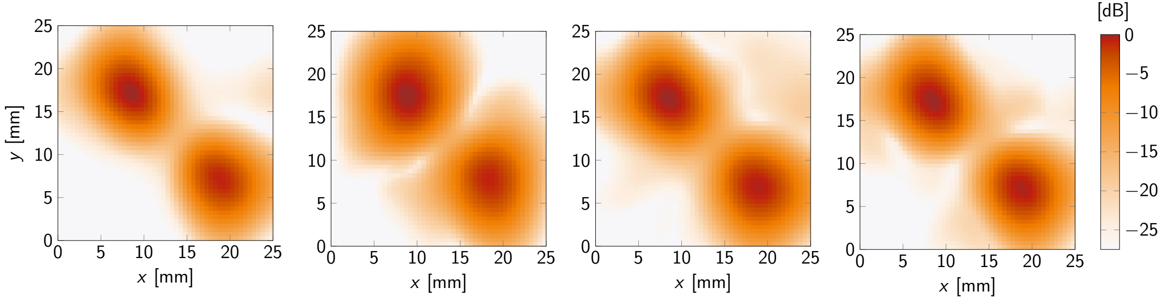


Effectiveness of Strategic Resampling

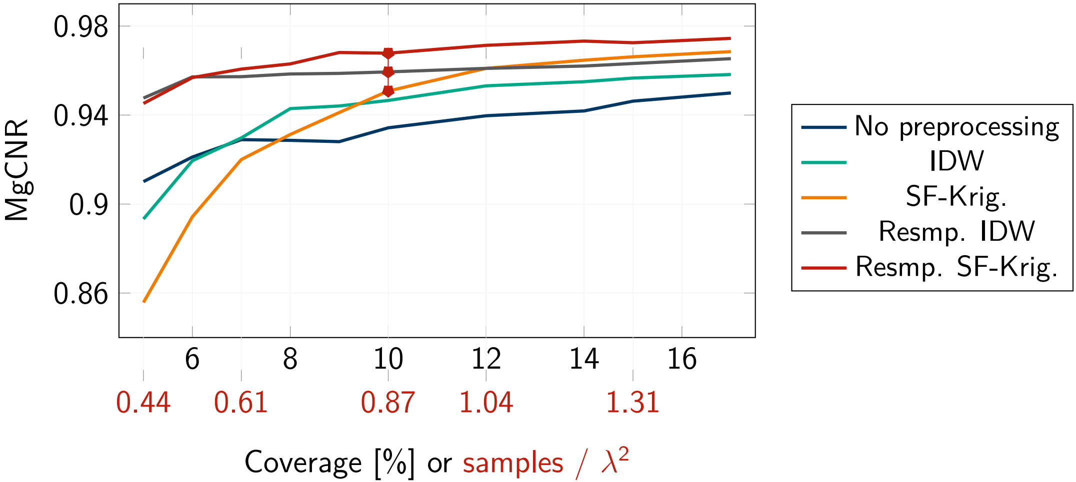
$N = 0.44$ samples $/\lambda^2$

Reference **SF-Krig.** **Resmp. IDW** **Resmp. SF-Krig.**



1 **0.893** **0.953** **0.966**

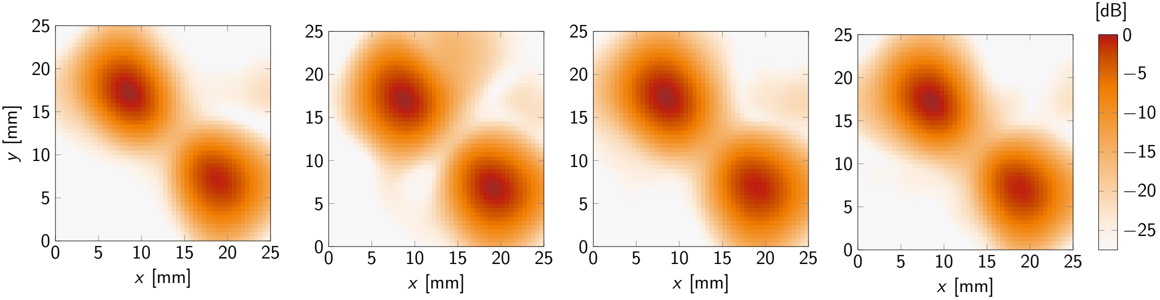
Effectiveness of Strategic Resampling



Effectiveness of Strategic Resampling

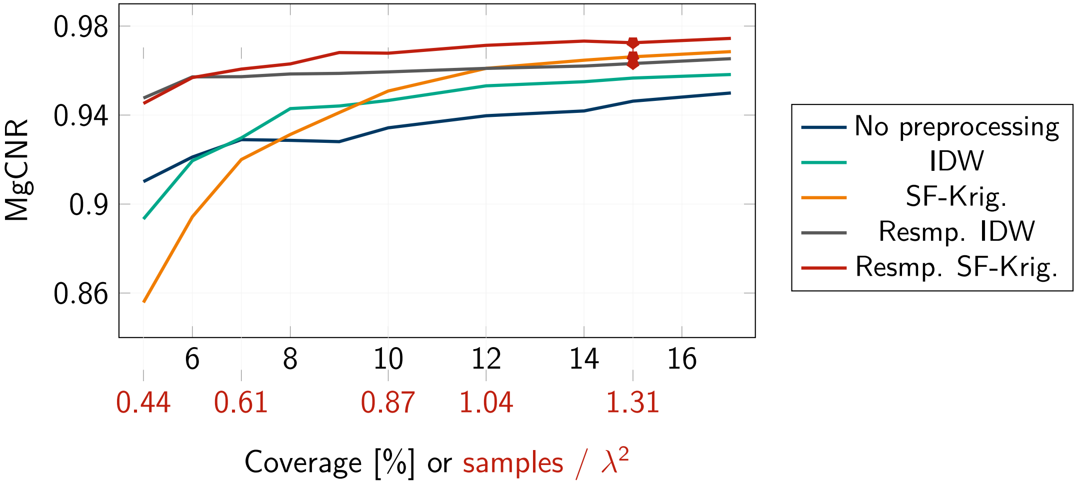
$N = 0.87$ samples $/\lambda^2$

Reference | **SF-Krig.** | **Resmp. IDW** | **Resmp. SF-Krig.**



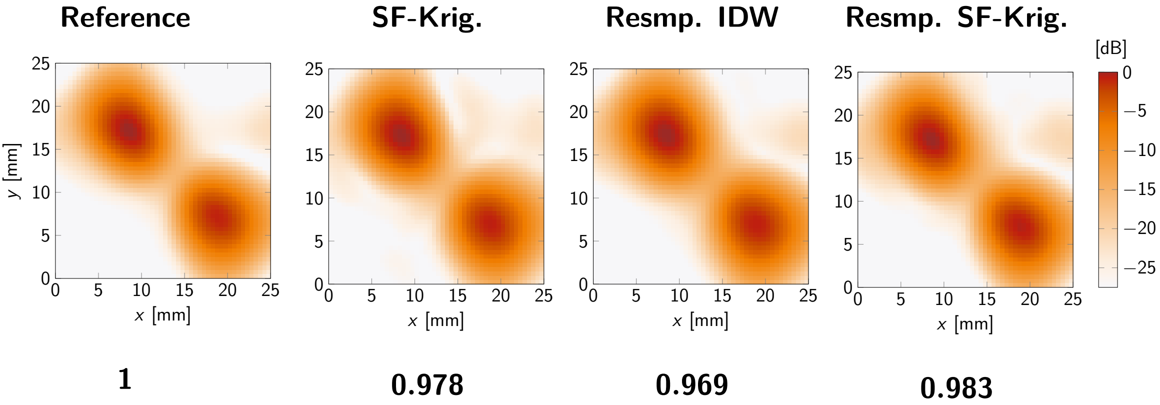
1 | **0.959** | **0.964** | **0.973**

Effectiveness of Strategic Resampling



Effectiveness of Strategic Resampling

$N = 1.31$ samples $/\lambda^2$



Conclusion (1)

Research problem and objectives

- Direct SAFT reconstruction of manaual UT data
  $\rightarrow$ Degraded resolution
  $\because$ Accumulation of random factors
- Sparse and/or irregular spatial subsampling of UT data
  $\rightarrow$ Spatial aliasing in SAFT reconstructions
  $\Rightarrow$ Appearance of artefacts
- Goal = artefacts reduction by preventing spatial aliasing
  $\rightarrow$ Interpolate missing UT data
  $\triangleq$ Interpolation of nonlinear spatio-temporal data
- Fast yet interpretable method

Conclusion (2)

Achievements and contributions

- Extensive study on spatial statistics
- Spatial statistical modeling of UT data in space-frequency domain
- Problem formulation for a hybrid approach
  (i) Space-frequency domain Interpolation via SF-Kriging
  (ii) Estimation of second order spatial statistics via FVnet
    $\triangleq$ Vector-valued regression
    $\rightarrow$ Applicable to other types of spatio-temporal lattice data
- Extension of point SF-Kriging to multi-point ones
- Establishing a preprocessing scheme with a feedback feature

Conclusion (3)

Findings

Samples $/\lambda^2$	$\ldots 0.61$	$0.61 \ldots 0.87$	$0.87 \ldots$
FVnet SF-Krig.	X	✔	✔✔✔
Resamp. + SF-Krig.	$\approx$	✔✔✔	✔

Conclusion (3)

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FVnet SF-Krig.	X	✔	✔✔✔
Resamp. + SF-Krig.	$\approx$	✔✔✔	✔

Future work

- Incorporate the neighboring batch information in FVnet
- Parameteric estimation of frequency variogram via DNN
- Extension to a progressive approach

Backup

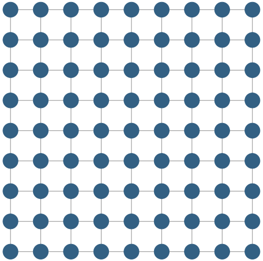
Accumulation of randomness

System inaccuracy

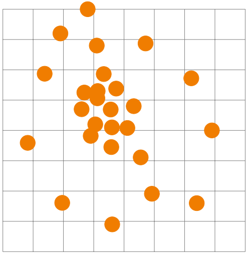
- Positional inaccuracy
- Inconsistent coupling

Path selection

- Spatial undersampling



Automatic



Manual

Accumulation of randomness

System inaccuracy

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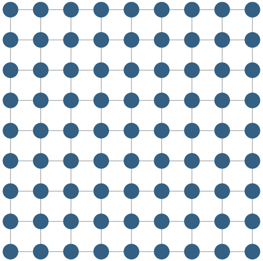
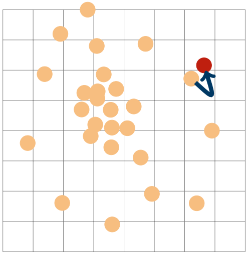


Accurate



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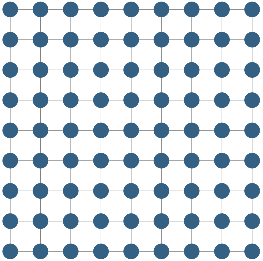
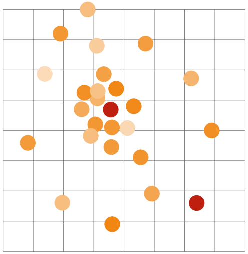


Constant



Inconsistent

Accumulation of randomness

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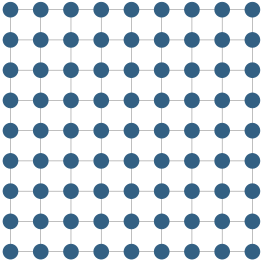
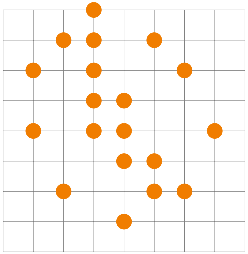


Full



Incomplete

Accumulation of randomness

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Path selection

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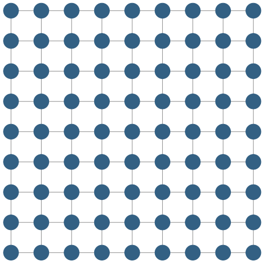

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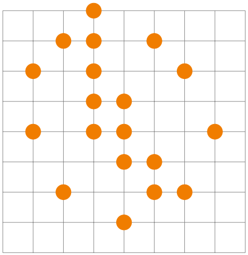
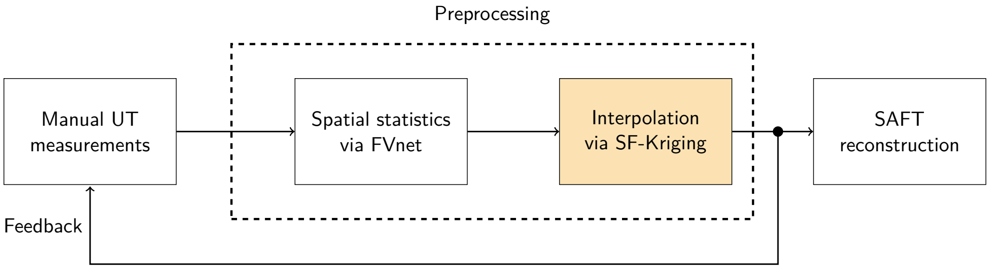


Incomplete

ST-Interpolation in SF-domain



ST-Interpolation in SF-domain

ST-domain: temp. correlation $\rightarrow$ vector-valued pred.

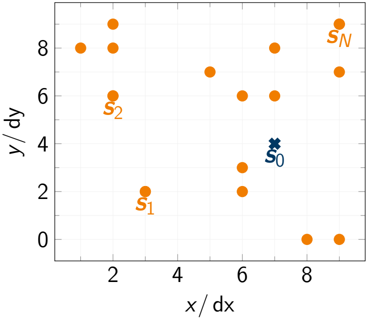
Sought:

$$\boldsymbol{a}_0 \in \mathbb{R}^M$$

Given:

$$\boldsymbol{A}_S = \begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \cdots & \boldsymbol{a}_N \end{bmatrix} \in \mathbb{R}^{M \times N}$$

Samp. and pred. positions



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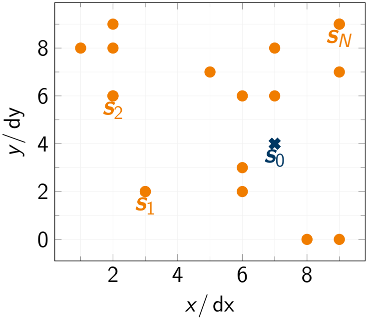
SF-domain: orthogonal Fourier bases

⇒ individual prediction for a single frequency

$$\boldsymbol{p}_0 = \boldsymbol{F}_M \boldsymbol{a}_0 \in \mathbb{C}^M$$

Samp. and pred. positions



ST-Interpolation in SF-domain

SF-domain: $\rightarrow$ set of scalar-valued pred. $\forall \omega_m$

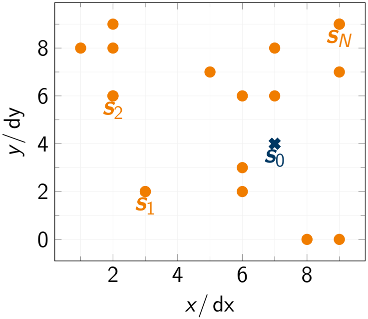
Sought:

$$p_{0m} \in \mathbb{C}$$

Given:

$$\boldsymbol{\pi}_m^S = \begin{bmatrix} p_{1m} & p_{2m} & \cdots & p_{Nm} \end{bmatrix}^\mathsf{T} \in \mathbb{C}^M$$

Samp. and pred. positions



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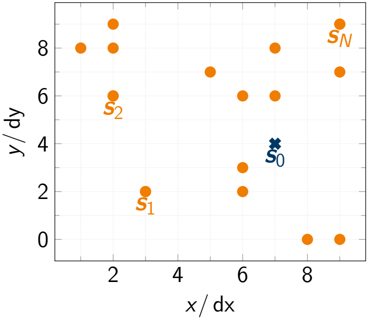
$\Rightarrow$ **Optimal prediction**:

$$\hat{p}_{0m} = \mathrm{E}\left\{ p_{0m} \mid \pi_m^S \right\}$$

Samp. and pred. positions



Spatial Statistical Approach: SF-Kriging

Assumpations: **intrinsic stationarity**

→ closer points ≙ similar values

(1) Mean of the increments is 0

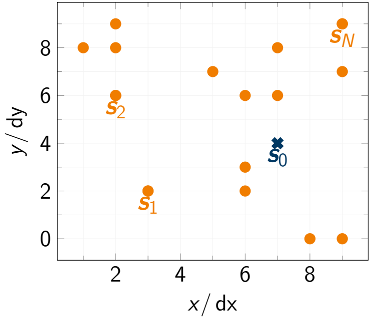
$$\mathrm{E}\left\{p(\boldsymbol{s}) - p(\boldsymbol{s}+\boldsymbol{h})\right\} = 0$$

(2) Variance of the increments is shift invariant

→ function of the spatial lag

$$\mathrm{Var}\left\{p(\boldsymbol{s}) - p(\boldsymbol{s}+\boldsymbol{h})\right\} := 2\gamma(\boldsymbol{h})$$

$\{p(\boldsymbol{s}) : \boldsymbol{s} \subset D\}$



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$\rightarrow$ function of the spatial lag

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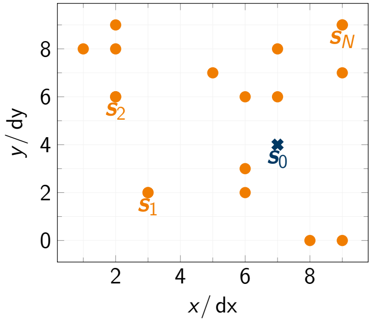
$2\gamma(\boldsymbol{h})$ = **Frequency variogram (FV)**

$\{p(\boldsymbol{s}) : \boldsymbol{s} \subset D\}$



Spatial Statistical Approach: SF-Kriging

Linear predictor:

$$\hat{p}_{0m} = \boldsymbol{w}_m^{\mathrm{H}} \boldsymbol{\pi}_m^S$$

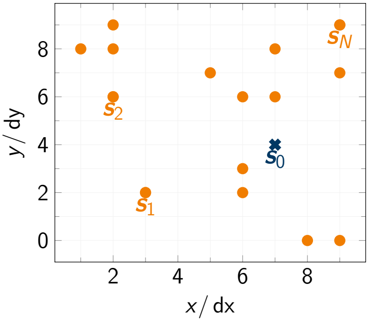
Linear unbiased MMSE predictor (Kriging):

$$\min_{\boldsymbol{w}_m} f(\boldsymbol{w}_m) \text{ s.t. } \sum_{i=1}^{N} w_i = 1$$

$$\begin{aligned} f(\boldsymbol{w}_m) &= \mathrm{E}\left\{ \left| p_{0m} - \hat{p}_{0m} \right|^2 \right\} \\ &= \mathrm{Var}\left\{ p_{0m} - \hat{p}_{0m} \right\} \\ &= -\sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \gamma_m(\boldsymbol{h}_{ij}) + 2\sum_{i=1}^{N} w_i \gamma_m(\boldsymbol{h}_{0i}) \end{aligned}$$

with $\boldsymbol{h}_{ij} = \boldsymbol{s}_i - \boldsymbol{s}_j$ and $\boldsymbol{h}_{0i} = \boldsymbol{s}_0 - \boldsymbol{s}_i$

$\{p(\boldsymbol{s}) : \boldsymbol{s} \subset D\}$



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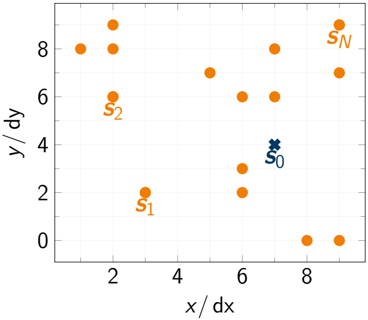
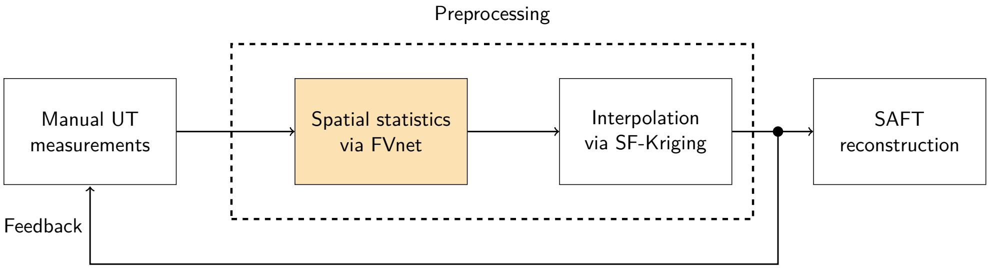


$\rightarrow$ **Estimate of $\gamma_m(\boldsymbol{h})$**

FVnet: Estimation of Spatial Statistics



FVnet: Estimation of Spatial Statistics

Properties and assumptions:

$\rightarrow$ within a small batch

- Intrinsic stationary
- Variability in x and y is negligible
  $\rightarrow \gamma_m(\boldsymbol{h}) = \gamma_m(h)$
- Lattice data
  $\rightarrow$ known vector-valued lags $\in \mathbb{R}^{N_h}$
- Vector-valued FV $\boldsymbol{\gamma_m} \in \mathbb{R}^{N_h}$
- There are certain structures in FVs
  $\rightarrow$ similar within the neighboring bins

FVnet: Estimation of Spatial Statistics

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$\rightarrow$ within a small batch

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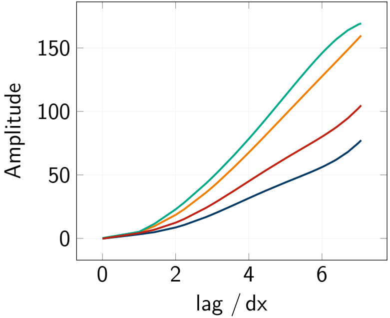
- Variability in x and y is negligible
  $\rightarrow \gamma_m(\boldsymbol{h}) = \gamma_m(h)$
- Lattice data
  $\rightarrow$ known vector-valued lags $\in \mathbb{R}^{N_h}$
- Vector-valued FV $\boldsymbol{\gamma_m} \in \mathbb{R}^{N_h}$
- There are certain structures in FVs
  $\rightarrow$ similar within the neighboring bins

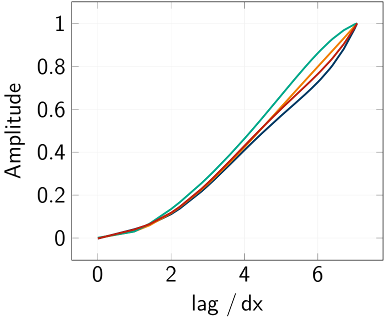


FVnet: Estimation of Spatial Statistics

Properties and assumptions:

$\rightarrow$ within a small batch

- Intrinsic stationary
- Variability in x and y is negligible
  $\rightarrow \gamma_m(\boldsymbol{h}) = \gamma_m(h)$
- Lattice data
  $\rightarrow$ known vector-valued lags $\in \mathbb{R}^{N_h}$
- Vector-valued FV $\boldsymbol{\gamma_m} \in \mathbb{R}^{N_h}$
- There are certain structures in FVs
  $\rightarrow$ similar within the neighboring bins



FVnet: Estimation of Spatial Statistics

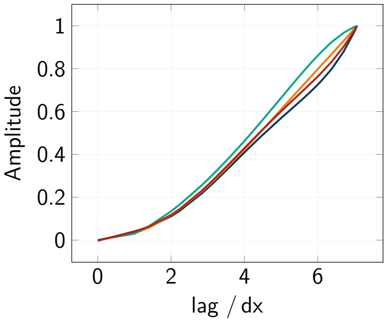
Problem formulation for DNN

$\Rightarrow$ Estimate the structure of $\boldsymbol{\gamma_m} \in \mathbb{R}^{N_h}$

$\triangleq$ Vector-valued regression problem

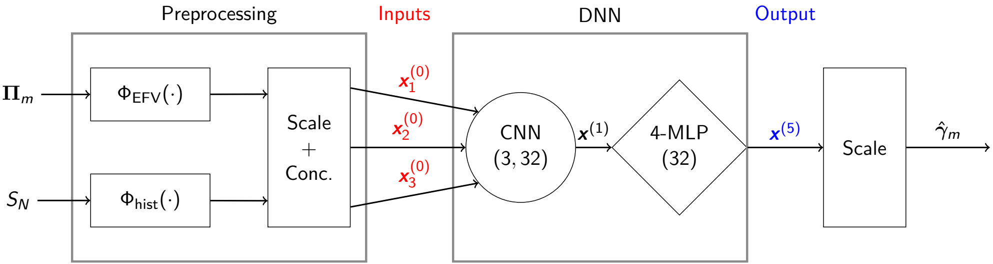
Network inputs

- Input 1 = Fourier coefficients of 3 bins
  $\boldsymbol{\Pi}_m = [\boldsymbol{\pi}_{m-1}\ \boldsymbol{\pi}_{m-1}\ \boldsymbol{\pi}_{m-1}] \in \mathbb{C}^{N\times 3}$
- Input 2 = Scan positions



FVnet: Estimation of Spatial Statistics

FVnet



(i) $\Phi_{\text{EFV}} : \mathbb{C}^{M\times 3} \mapsto \mathbb{R}^{N_h \times 3}$

Fourier coeffs. $\mapsto$ smoothed & normalized method-of-moments estimate of the FVs

(ii) $\Phi_{\text{hist}} : \mathbb{R}^{N\times 2} \mapsto \mathbb{R}^{N_h}$

Sampling positions $\mapsto$ distribution of the available lags

Parameters: MUSE

	Parameter	Value/range
Specimen	Material	Steel
	Speed of sound c_0	5900 m s
	Dimension (L × D × H)	200 mm × 140 mm × 90 mm
UT Probe	Manufacturer	KARL DEUTSCH
	Model	P 1462.1
	Diameter (transducer element)	10 mm
	Center frequency f_C	4.0 ± 0.4 MHz
	Bandwidth (−6 dB)	2.6 ± 0.5 MHz
Measurements	Wavelength λ	≈ 1.475 mm
	Spacing along x-axis dx	0.5 mm
	Spacing along y-axis dy	0.5 mm
	Sampling frquency f_S	80 MHz
	Temporal interval dt	12.5 ns
	Spacing along z-axis dz	36.875 µm

Parameters: SF-Kriging

	Parameter	Value/range
For all schemes	Batch size ($N_x \times N_y \times N_z$)	$10 \times 10 \times 512$
	Maximal lag h_{max}	$5\sqrt{2}\,\text{dx} \approx 7.071\,\text{dx}$
For SF-Kriging	Minimal numer of points	5
	Weights regularization λ	1.0

Parameters: FWM Constant

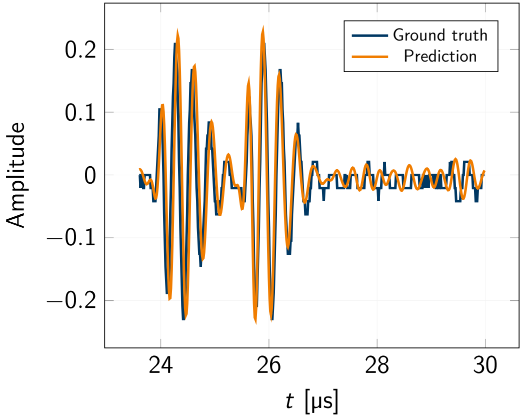
	Parameter	Value/range
Specimen / Measurements	Material	Steel
	Speed of sound c_0	5900 m s
	Dimension (L $\times$ D $\times$ H)	45 mm $\times$ 45 mm $\times$ 90 mm
	Spacing along x-axis dx	0.5 mm
	Spacing along y-axis dy	0.5 mm
	Sampling frquency f_S	80 MHz
	Temporal interval dt	12.5 ns
	Spacing along z-axis dz	36.875 μm
UT pulse	Transducer beam spread	25°
	Pulse model	Gabor
	SNR	20 dB

Parameters: FWM Variables

	Parameter	Value/range
Synthetic data	ROI (I) $(30 \times 30 \times 512)$	$x = 10 \ldots\ 35\,\mathrm{mm}$ $y = 10 \ldots\ 35\,\mathrm{mm}$ $z = 24 \ldots\ 43\,\mathrm{mm}$
	ROI (II) $(30 \times 30 \times 512)$	$x = 10 \ldots\ 35\,\mathrm{mm}$ $y = 10 \ldots\ 35\,\mathrm{mm}$ $z = 33 \ldots\ 52\,\mathrm{mm}$
Scatterer configuration	Number of scatterers in ROI	[2, 5, 10]
	Reflectivity of each scatterer	$0.1 \ldots 1$
UT pulse	Center frequency f_C	$3.36 \pm 0.34\,\mathrm{MHz}$
	Bandwidth $(-6\,\mathrm{dB})$	$1.0 \pm 0.1\,\mathrm{MHz}$
	Wavelength λ	$\approx 1.756\,\mathrm{mm}$
Spatial subsampling	Block size $(N_x \times N_y \times N_z)$	$10 \times 10 \times 512$
	Number of scans / block	$10 \ldots 100$
	Scan distribution	Uniform or random walk

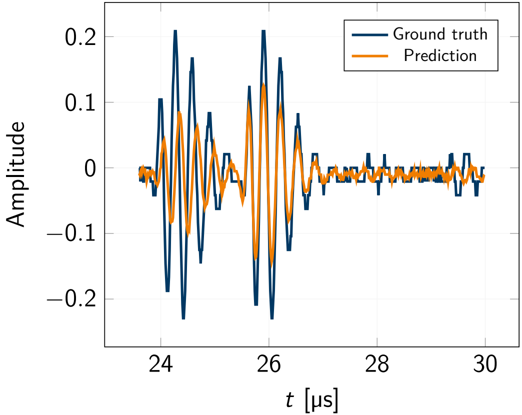
Results: Batch Wise Interpolation

FVnet + SF-Kriging

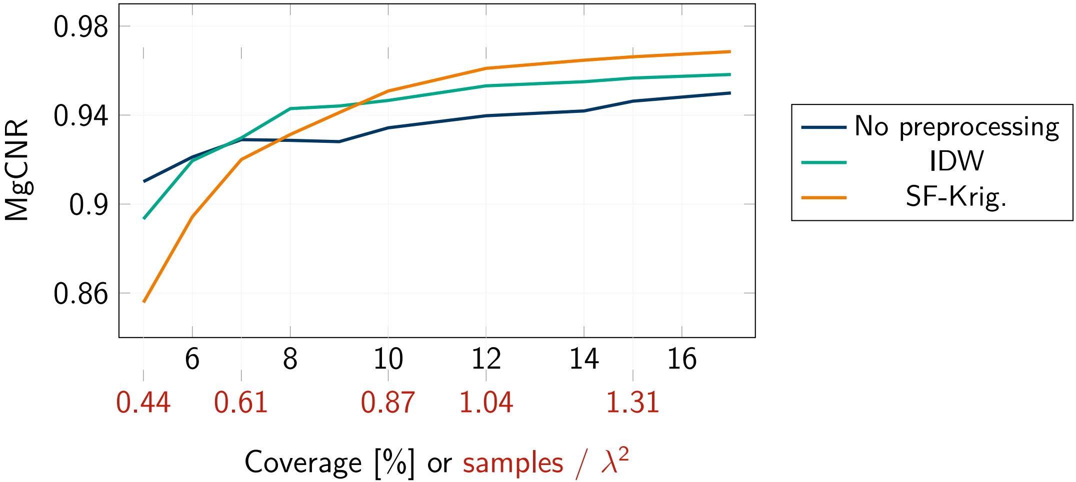


Results: Batch Wise Interpolation

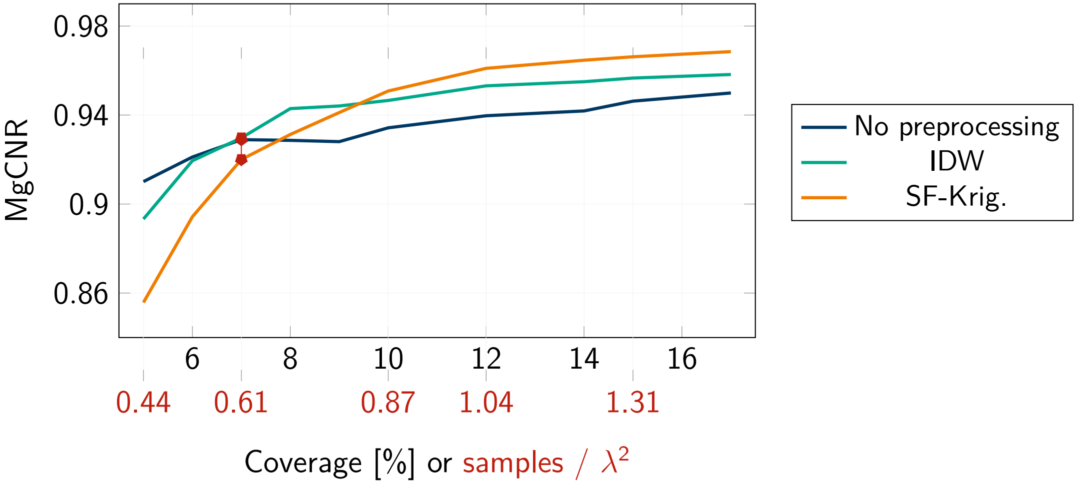
IDW



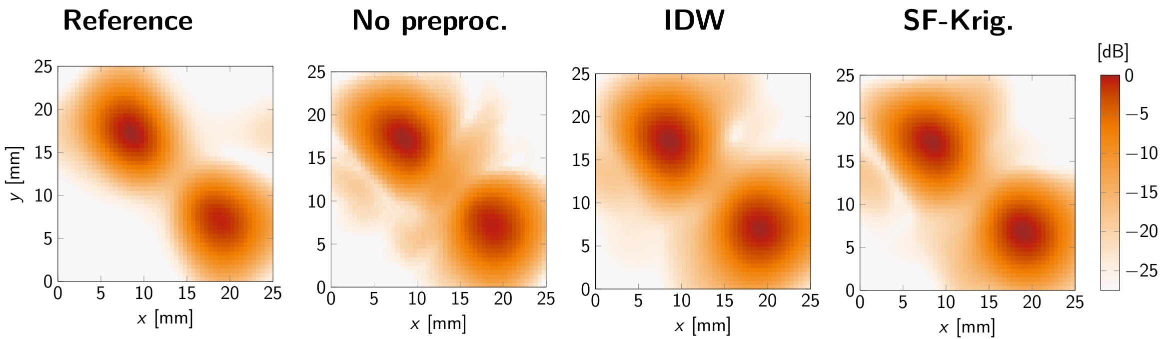
Performance Evaluation for Varying Coverage



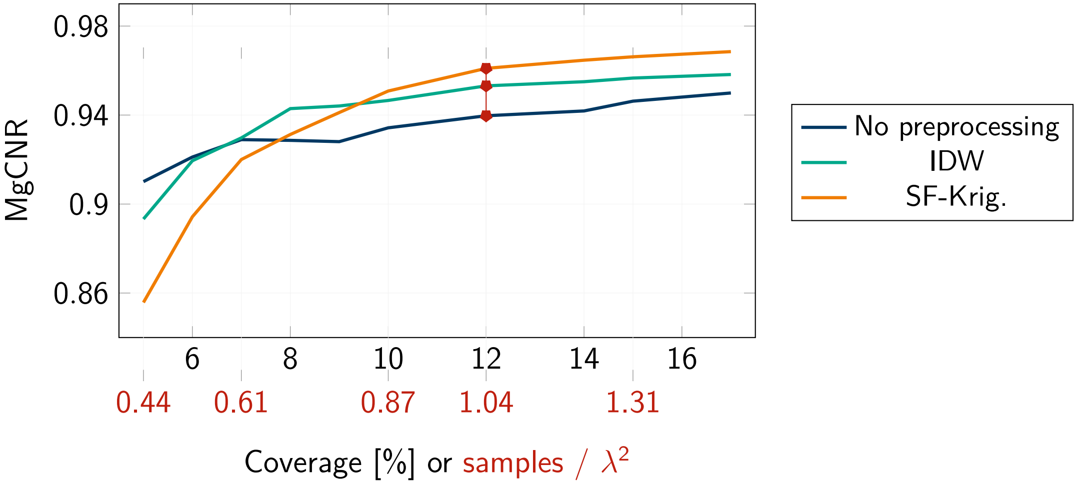
Performance Evaluation for Varying Coverage



Performance Evaluation for Varying Coverage



Performance Evaluation for Varying Coverage



Performance Evaluation for Varying Coverage