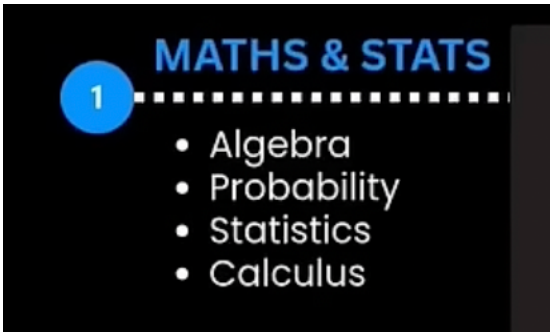
Maths



Algebra

Linear algebra's major goal is to establish systematic techniques for solving systems of linear equations.

$$a_1x_1 + a_2x_2 + \dots + a_nx_n = b$$

$x_1 + x_2 + \dots + x_n$ = Unknown quantities to be found.

$a_1 + a_2 + \dots + a_n$ = coefficients

b = Constant term

Linear equation in one variable

$3x + 5 = 0$, $\quad 98x = 49$

Linear equation in two variable

$y + 7x = 3$, $\quad 3a + 2b = 5$

Linear equation in three variable

$x + y + z = 0$, $\quad a - 3b = c$

* Identifying Linear & Non-linear equations

Equation	Linear / Non Linear
$y = 8x - 9$	Linear
$y = x^2 - 7$	Non-linear, the power of the variable x is 2
$y + 3x - 1 = 0$	Linear
$y^2 - x = 9$	Non-linear, power of the variable y is 2

* Linear Equation Forms

1) Standard Form

a) Formula for one-variable single-line calculations

$$Ax + B = 0$$

A & B = real integers

= variable

x

b) Formula for two-variable single-line calculations

$$Ax + By = C$$

A, B & C = real integers

x, y = variables

Linear Equation

$$\overset{\text{coeff of } x}{\overset{\uparrow}{A}}x + \overset{\text{coeff of } y}{\overset{\uparrow}{B}}y = \underset{\text{constant}}{\underset{\downarrow}{C}}$$

variables

* Linear Equation in slope Intercept Form

A linear equation's slope can be calculated to see how one variable varies in response

* Slope Equation

$mx + b$

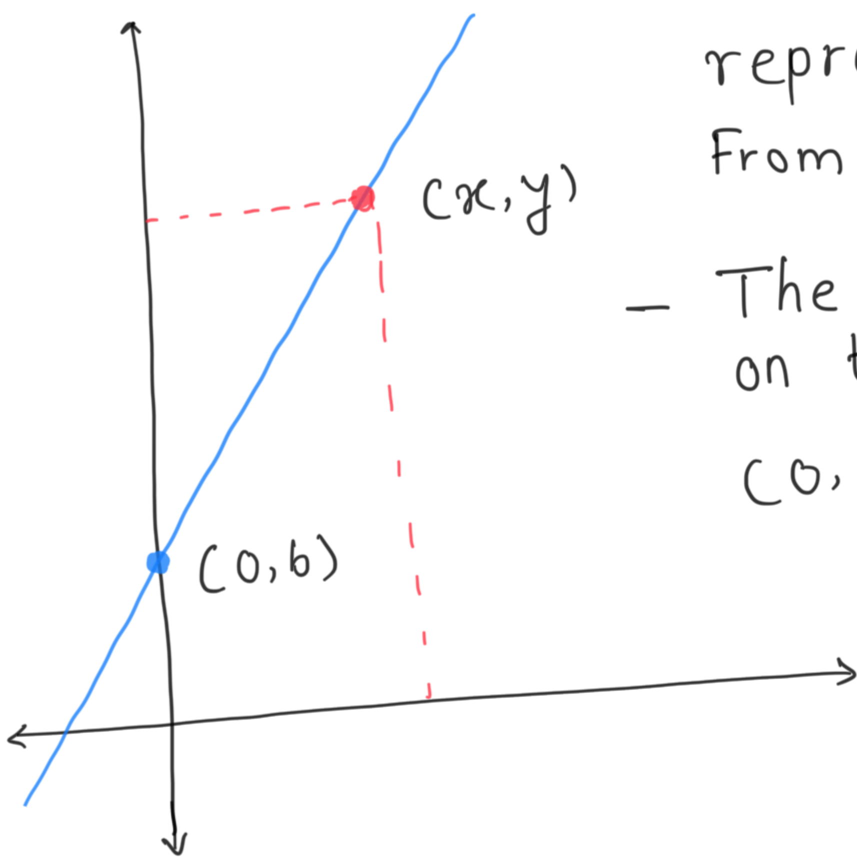
$y = mx + b$

m = slope

b = Intercept

x & y = Line's distance from the x & y-axes resp.

* Linear Equation in slope Intercept Form



- The line's (x, y) pt represents the distance from x & y axis.
- The line intercept on the y axis is at (0, b)

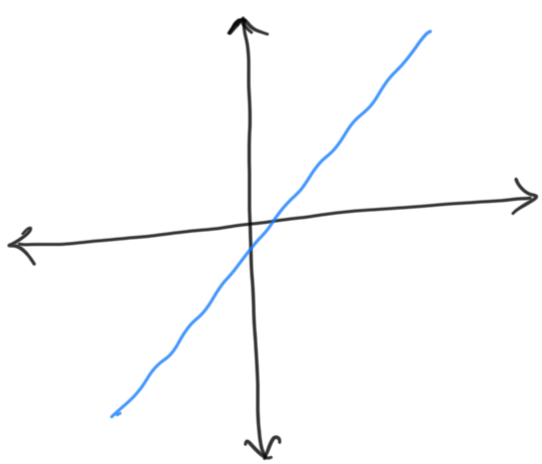
* Linear Equation : Slope

- Slope indicates how steep a line is

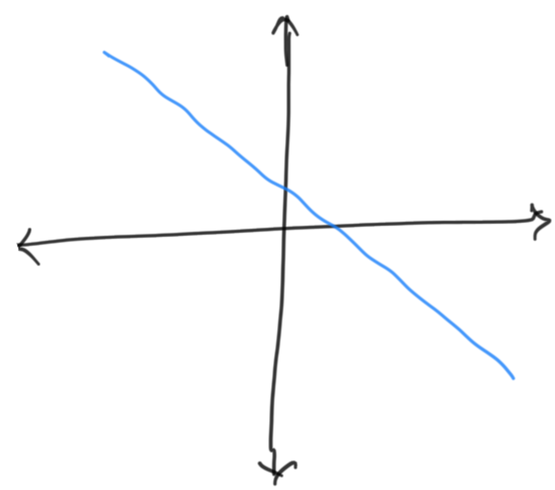
w.r.t y-axis

- When seen from left to right, it indicates whether the line goes up/down.
- slope describes how the independent variable has been charged while the dependent variable is changing.

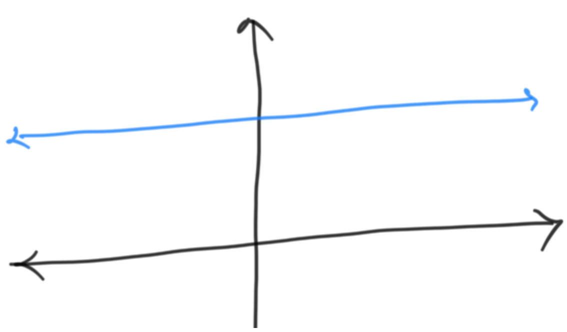
* Types of Slope

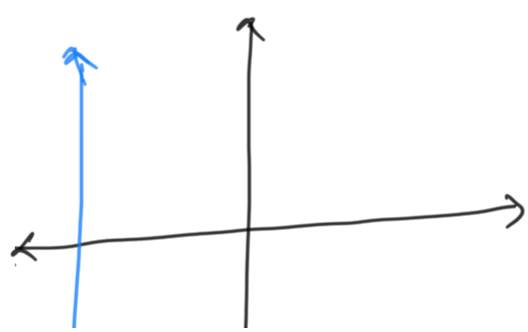


Positive



Negative





Zero

undefined

* Linear Equation in Point slope Form

A straight line is represented in point slope form by its slope & a pt on the line.

Equation

$$y - y_1 = m(x - x_1)$$

Where
(x_1, y_1) are the coordinates of the point

* System of Linear Equation

A system of linear equation is a finite collection of linear eqn

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$$\dots\dots\dots$$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$$

- A linear system can have an unlimited number of solutions, one solution or no solutions at all in case of single linear eq^n

- A consistent linear system has no sol^n
- An inconsistent —— " —— has no sol^n

* Solving Linear system of Equations.

Different methods of solving systems of Linear equation.

1) Graphic Method
2) Substitution Method
3) Linear combination method / Elimination method
4) Matrix method

2) Substitution Method

$$x + y = 9 \quad —— (1)$$

$$2x + y = 10 \quad —— (2)$$

$$y = 9 - x$$

$$2x + (9 - x) = 10$$

$$2x - x + 9 = 10$$

$$x = 1$$

$$\therefore y = 8$$

3) Linear Combination / Elimination Method

$$\begin{array}{rl} & 2x + y = 10 \\ - & x + y = 9 \\ \hline & x = 1 \end{array}$$

$$\therefore y = 8$$

4) Matrix Method

A matrix is a rectangular array or table with rows & columns of numbers, symbols or expressions used to represent a mathemati- l object or an attribute

The matrix's size is expressed as $m \times n$. Where,

m = no. of rows
n = no. of columns

Example

$$A = \begin{bmatrix} 2 & 1 & 3 \\ -1 & 2 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$$

matrix Size

Expressed as $m \times n$

m = no. of rows
n = no. of columns

Notation of matrix

$$\begin{array}{cc} & \overbrace{\begin{array}{cccc} 1 & 2 & \cdots & n \end{array}}^{\text{Column}} \\ 1 & \begin{array}{cccc} a_{11} & a_{12} & \cdots & a_{1n} \end{array} \end{array}$$

$$\text{Rows}\left\{\begin{array}{c} 2\\ 3\\ \vdots\\ m\end{array}\right. \begin{bmatrix} a_{21} & a_{22} & \cdots & a_{2n}\\ a_{31} & a_{32} & \cdots & a_{3n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn}\end{bmatrix} = A_{m\times n}$$

* Forms of Matrix

- An element's entry in the matrix of form a_{ii} is located on the diagonal
- The matrix is termed a square matrix if $n = m$, then no. of columns & rows is equal
- A is called diagonal matrix if $a_{ij} = 0$ where $i \neq j$

* Matrix Operations

1) Addition

Example

$$A = \begin{bmatrix} 22 & 32 \\ 11 & 16 \end{bmatrix} \qquad B = \begin{bmatrix} 13 & 8 \\ 13 & 16 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 22+13 & 32+8 \\ 11+13 & 16+16 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 35 & 40 \\ 24 & 32 \end{bmatrix}$$

Addition Rule

1) Only matrices with same number of rows & columns can be added
2) Matrix addition follows the Commutative property

$$A+B = B+A$$

2) Subtraction

$$A = \begin{bmatrix} 22 & 32 \\ 11 & 16 \end{bmatrix} \qquad B = \begin{bmatrix} 13 & 8 \\ 13 & 16 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 22-13 & 32-8 \\ 11-13 & 16-16 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 24 \\ -2 & 0 \end{bmatrix}$$

Subtraction Rules

1) Only matrix with same number of rows & columns can be subtracted

2) Matrix subtraction does not follow the commutative property

$$A - B \neq B - A$$

3) Multiplication (Product of Matrix)

$$A = \begin{bmatrix} 22 & 32 \\ 11 & 16 \end{bmatrix} \qquad B = \begin{bmatrix} 13 & 8 \\ 13 & 16 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} (22 \times 13) + (32 \times 13) & (22 \times 8) + (32 \times 16) \\ (11 \times 13) + (16 \times 13) & (11 \times 8) + (16 \times 16) \end{bmatrix}$$

The 1st & 2nd rows of A are multiplied with the 1st & 2nd columns of B & added

$$C = \begin{bmatrix} 702 & 688 \\ 351 & 344 \end{bmatrix}$$

* Multiplication Rule

- Let $C = AB$. Use the formula $c_{ik} = \sum_j A_{ij} B_{jk}$ to calculate the value of each member in the $i \times k$ matrix C
- The matrix product AB is defined only when the number of columns in A equals the number of rows in B
- Matrix product BA is defined only when the no of columns in B equals the number of rows in A
- AB is not always equal to BA

4) Transpose

A transpose is a matrix formed by turning all the rows of a given matrix into columns & vice versa.

The transpose of matrix A is denoted as A^T

$$A = \begin{bmatrix} 22 & 32 \\ 11 & 16 \end{bmatrix}, \quad A^T = \begin{bmatrix} 22 & 11 \\ 32 & 16 \end{bmatrix}$$

5) Inverse

If A is non-singular square matrix, there exist a $n \times n$ matrix A^{-1}, known as A's inverse matrix, that satisfies the following property

$$AA^{-1} = A^{-1}A = I$$

where I is identity matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$AB = BA = I_n$$

where,
I_n denotes the $n \times n$ identity matrix

* Special matrix Types

1) Symmetric Matrix

A matrix A is said to be symmetric if $A = A^T$

A Matri...

2) Diagonal Matrix

A matrix D is diagonal only if $D_{ij} = 0$ for all $i \neq j$ $\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3) Identity matrix

The identity matrix is denoted as I_n where

$$A I_n = I_n A = A$$

4) Tensors

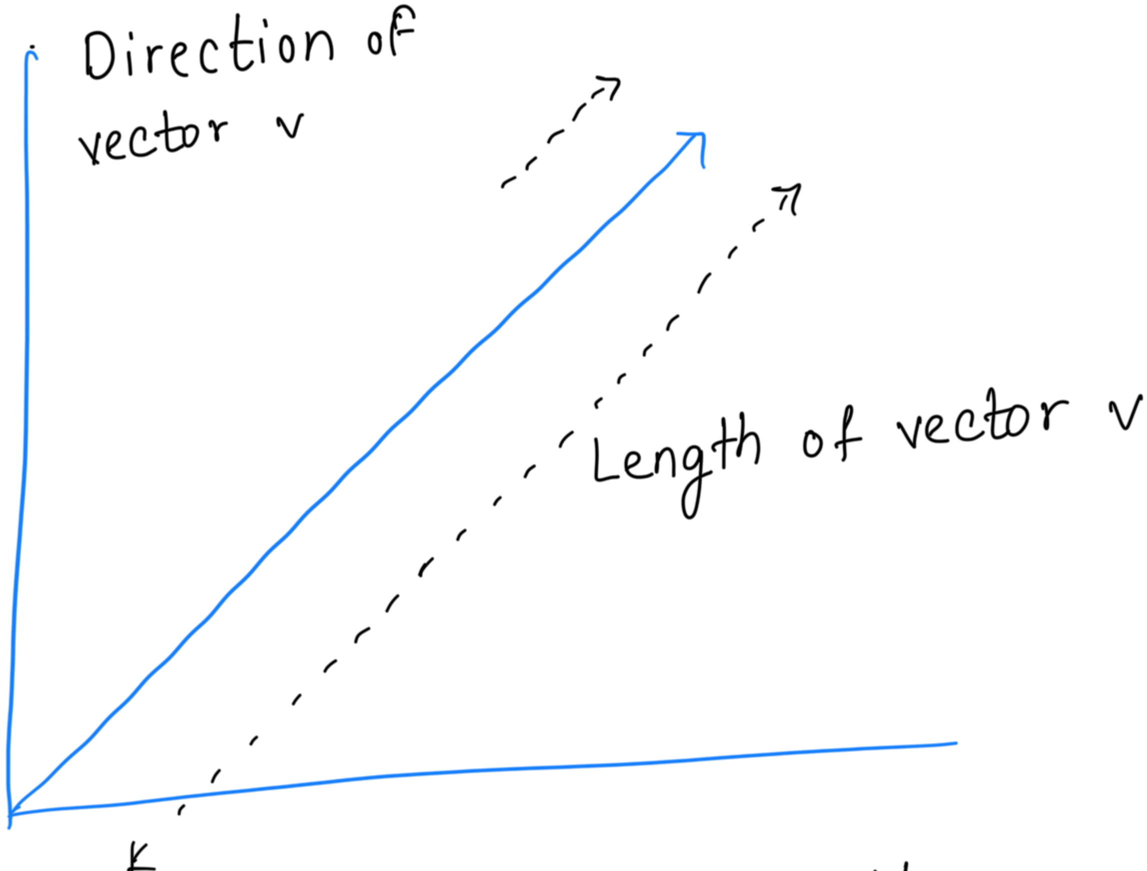
- Tensors are arrays with more than 2 axes
- A tensors can have N dimensions
- $A_{i,j,k}$ is the value at the co-ordinates i, j, k

* Vectors

Object with magnitude & direction is called vectors

... magnitude of vector determines its

The magnitude of
size



- It is represented as a line with an arrow where the length of the line indicates the vector's magnitude & the arrow points in desired direction

- other names for it include Euclidean vector, Geometric vector, spatial vector & simply vector

Its length is indicated by the symbol $||V||$

- Its length is ...
& it begins at origin (0, 0)

* Notation

$$A = a\hat{i} + b\hat{j} + c\hat{k}$$

a, b, c = Numeric values

Unit vectors along x, y, z axes are
$\hat{i}$, $\hat{j}$ & $\hat{k}$ resp