University of Colorado
Boulder

Human-Robot Interaction

Topics: Bayesian HRI

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*Some slides adapted from lecture by Nisar Ahmed

Bayesian Methods

Intuitions

Example 1

Where is this?

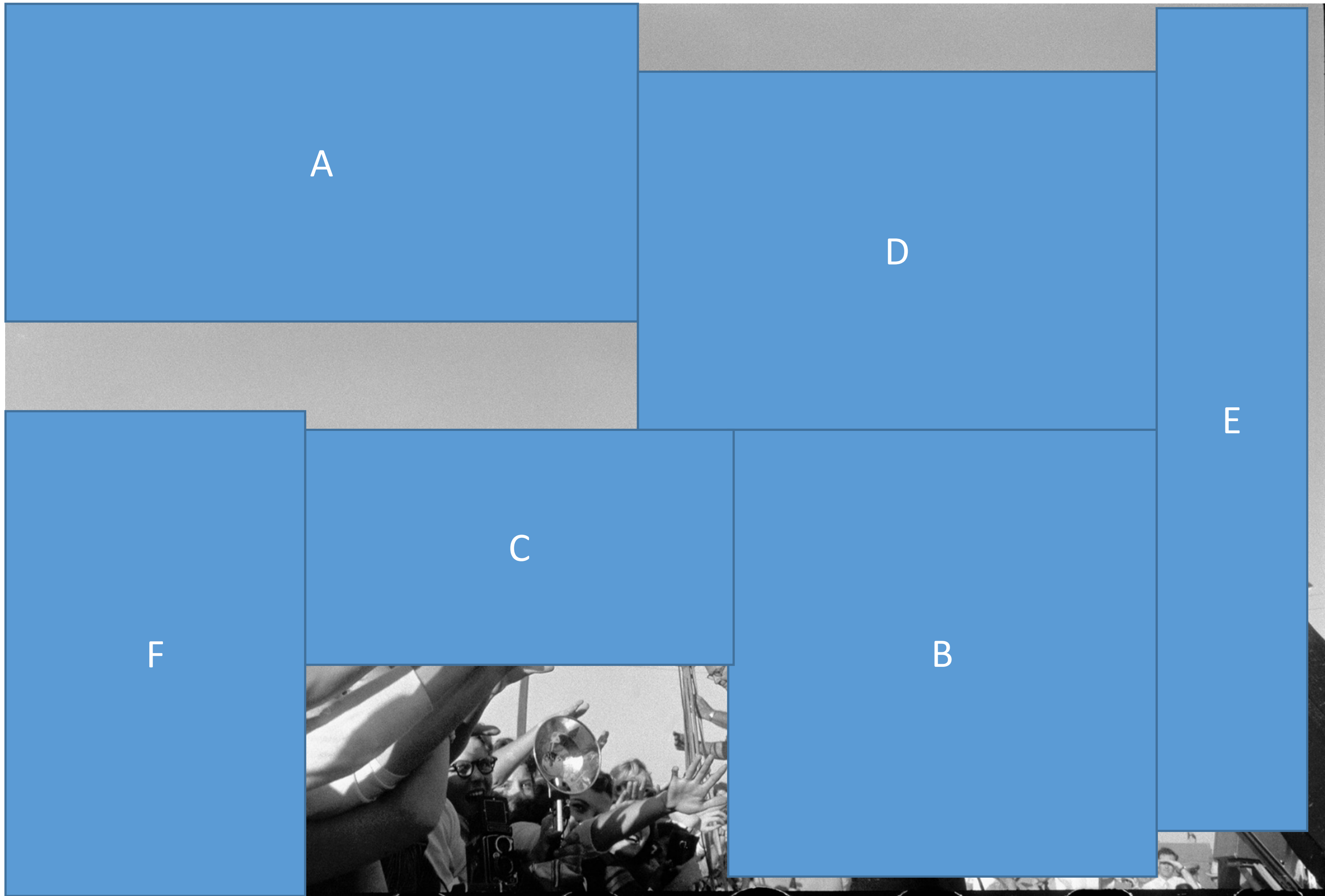
Example 2

What’s happening in this picture?

Guessing wrong costs $10

What order should you remove the boxes

- Each removal costs $2



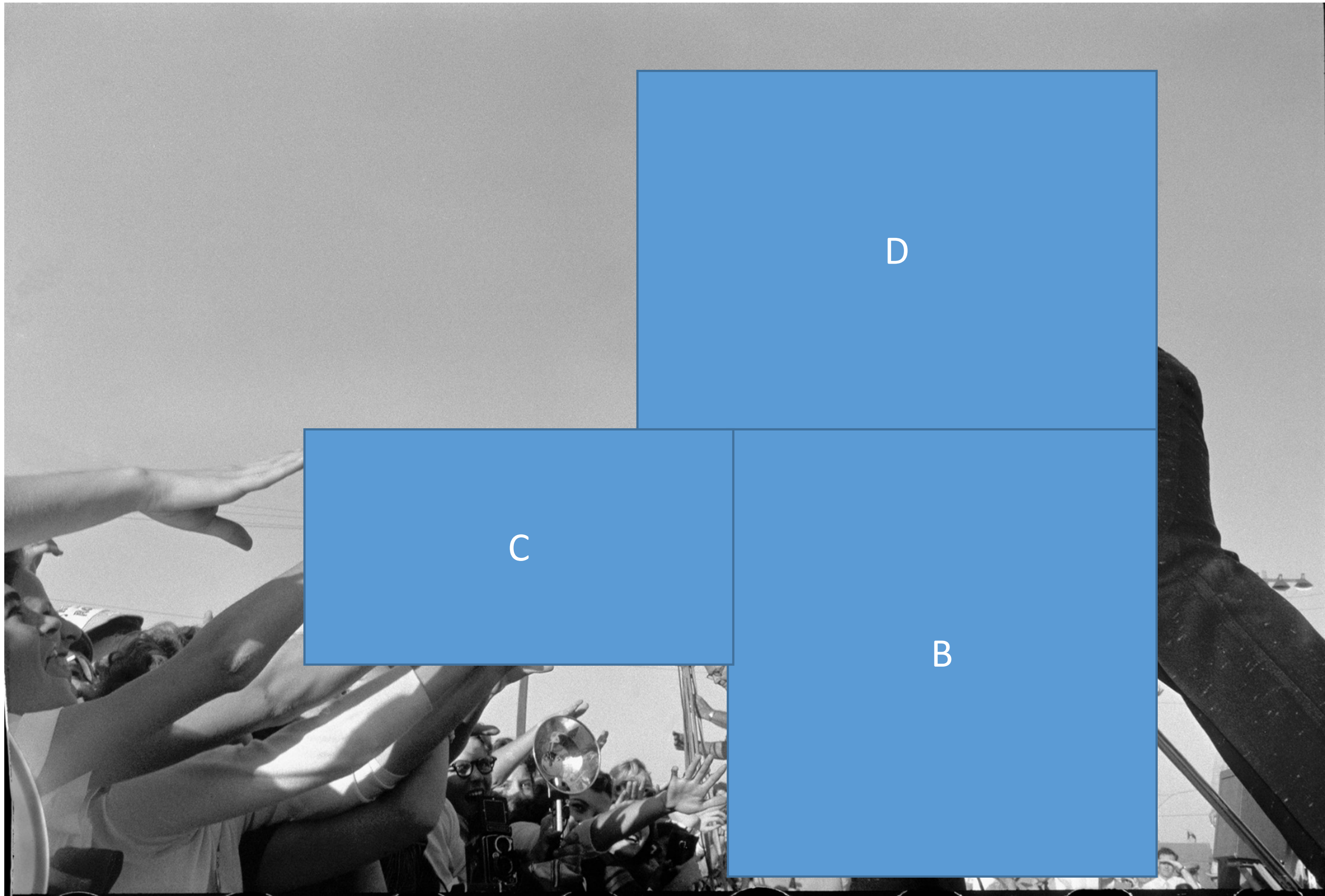
What’s happening in this picture?

Guessing wrong costs $10

What order should you remove the boxes

- Each removal costs $2

Removing A,E,F



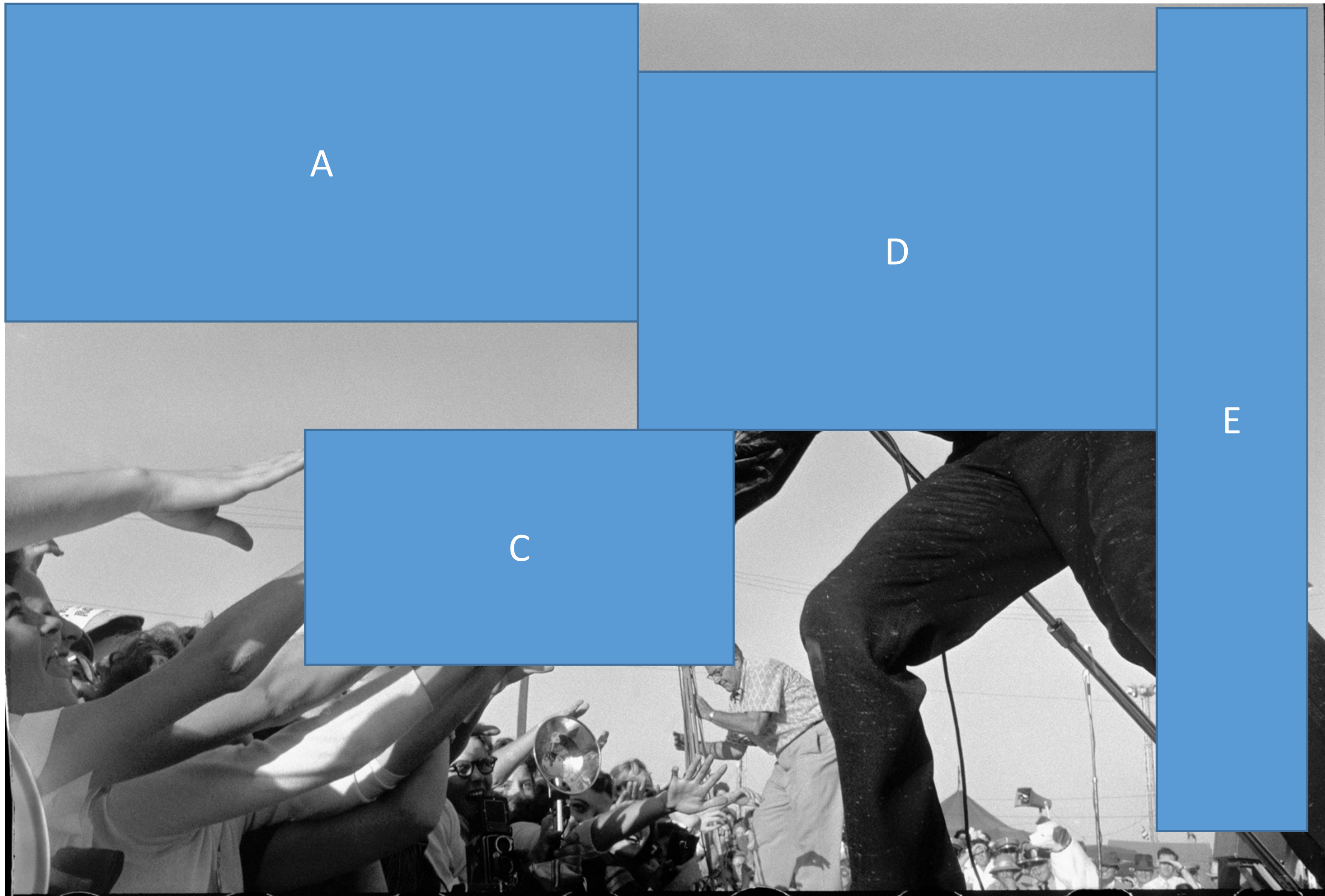
What's happening in this picture?

Guessing wrong costs $10

What order should you remove the boxes

- Each removal costs $2

Removing F,B



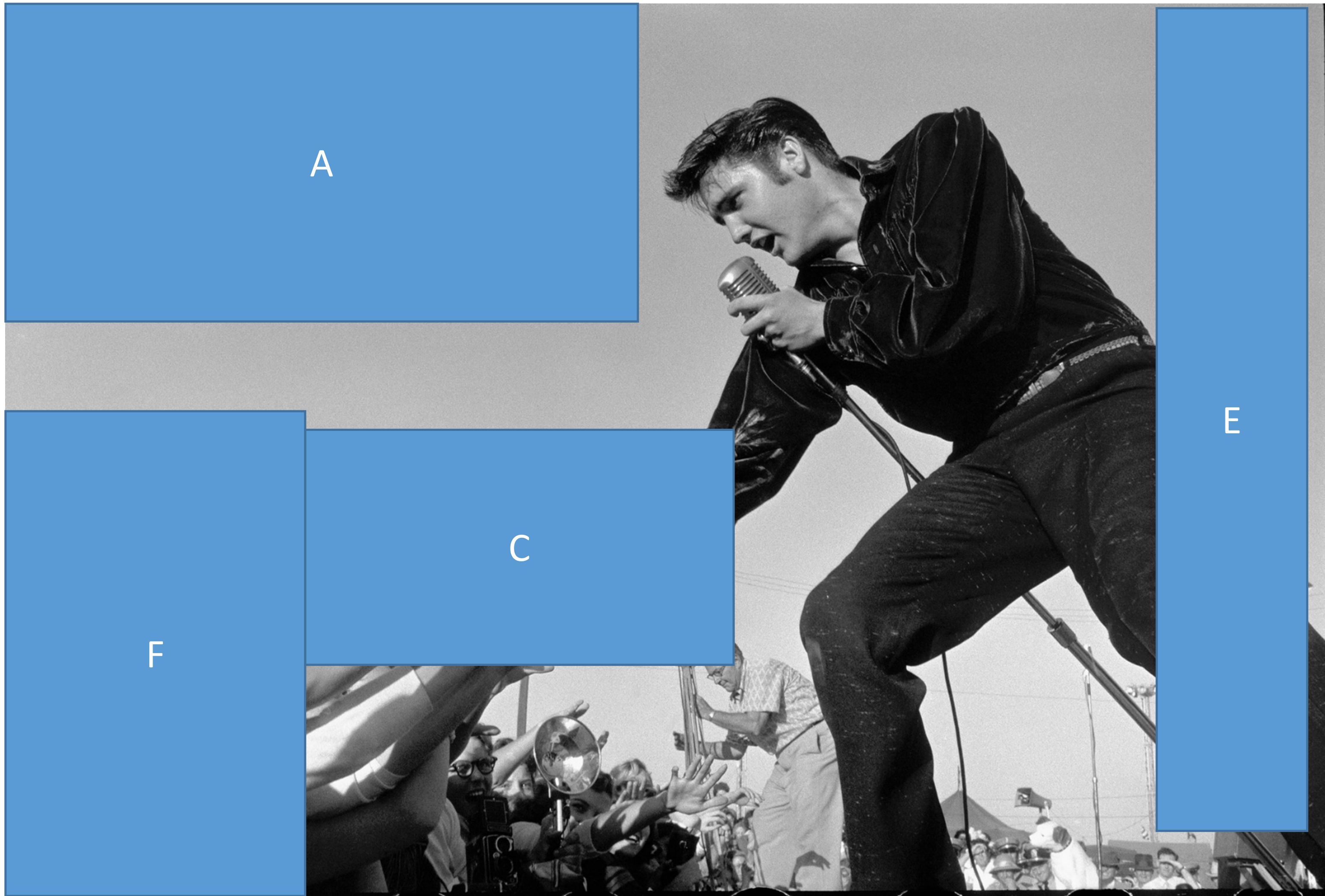
What’s happening in this picture?

Guessing wrong costs $10

What order should you remove the boxes

- Each removal costs $2

Removing B,D



What’s happening in this picture?

Guessing wrong costs $10

What order should you remove the boxes

- Each removal costs $2

The full picture

Example 3

Solve the equation

You know: $X + Y = Z$

X and Y could be any of $\{1,2,3,4,\ldots,10\}$ picked from a hat

A completely trustworthy and reliable person tells you:

$Z = 7$

What are X and Y given this information?

Solve the equation

You know: X + Y = Z

X and Y could be any of {1,2,3,4,…,10} picked from a hat

A completely trustworthy and reliable person tells you…

What are X and Y given this information?

What if they then told you…

How did you try to come up with answers?

Only so many **distinct hypotheses** could explain what’s happening in each case

There is **evidence** in each situation to support/go against **each hypothesis**

To pick **an answer:** look at the hypothesis with the strongest support from the evidence

What if you want to “hedge your bets?”

- Carry **all** hypotheses forward, set yourself up to look for more evidence

Did you treat all possible hypotheses the same before seeing the evidence?

- What did you **expect** to see in the last “where is this” picture before you saw it?
- Did you totally rule out any hypotheses before or after seeing evidence?

You can attack each of these problems with Bayesian reasoning

Some basics

Probabilities

Marginal / Prior probability

The probability of an event occurring: **p(A)**

Doesn't depend (is not conditioned on) any other event

Example 1: probability that a card drawn from a deck is red: p(red) = 0.5

Example 2: probability that a card drawn from a deck is a four: p(four) = 1/13

Joint probability

Probability of event A and B occurring = **p(A ∩ B)**

Example: probability that a card is a four and is red

P(four ∩ red) = 2/52 = 1/26

Conditional probability

Probability of event A occurring given that B occurs = **p(A | B)**

Example: given that you drew a red card, what is the probability that it is a four

P(four | red) = 2/26 = 1/13

Reasoning

Let H represent a possible hypothesis

Suppose we have N distinct hypotheses, so H = 1, or H = 2, or …, or H=N

Let p(H = i) be the **prior probability** of H=i

Prior to seeing any evidence how likely is it that H=i

Let E be a particular piece of evidence that takes on value j

Let p(E=j | H=i) be the **conditional probability of evidence**

If somehow you magically knew that H=i, how likely is it that you would see E=j

A problem…

$p(E=j \mid H=i)$ is the conditional probability of evidence

Example: what is the probability that we see a certain sensor reading *j* given that we know there is a person in front of us?

But we never know for sure the **real value of H**

- Instead, we typically need to calculate $p(H=i \mid E=j)$
- E.g., what is the probability that a person is in front of us, given we have a sensor reading *j*

Bayesian Inference

Want to find conditional probability that H=i is true GIVEN that E=j is known to be true

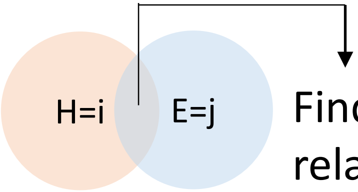
- i.e., invert p(E=j | H=i) to get conditional probability p(H=i | E=j)
- Bayes rule allows us to do this inversion!

Prior — Condition probability of evidence

$$p(H=i|E=j) = \frac{p(H=i) \cdot p(E=j|H=i)}{p(E=j)}$$

Probability of evidence:
Prob. Of seeing E=j no matter what H actually is

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|\bar{A})P(\bar{A})}$$



Find the "size" of overlap area relative to the "size" of E=j

Notes

Probabilities and conditional probabilities over all possible values for H and E must always:

- Be between 0 and 1
- Add up to 1 over all possible values of H and E
- Enforce "**coherence**" of outcomes: things either happen or they don't, but one and only one thing MUST happen
- E.g., X+Y = Z example: Y can't be both 1 and 2 at the same time

Bayes' rule only useful if H and E are **conditionally dependent**

- E tells you nothing about H if they are **conditionally independent**
- I.e., $p(H = i \mid E = j) = p(H)$

Why is this useful?

Sensor observations are noisy, partial, potentially missing

All models are partially wrong and incomplete

Sometimes we have prior knowledge

Probabilities can be objective, subjective, or …(many other flavors)

- Objective: $p(y=3 \mid x=2, z=4)$
- Subjective: p(Tony Romo will retire this week)

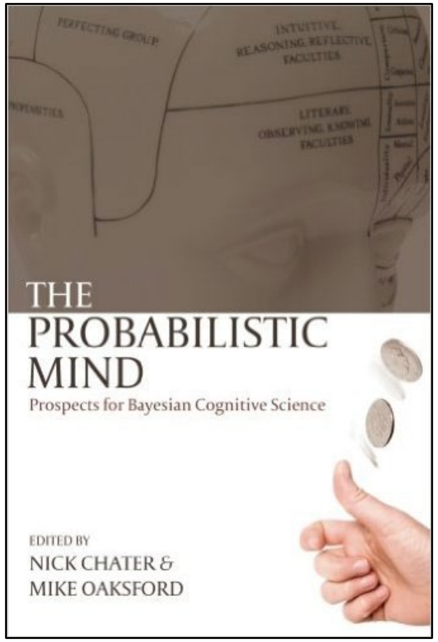
Often easier to specify $p(H)$ and $p(E|H)$ to find $p(H|E)$ than to specify $p(H|E)$ directly

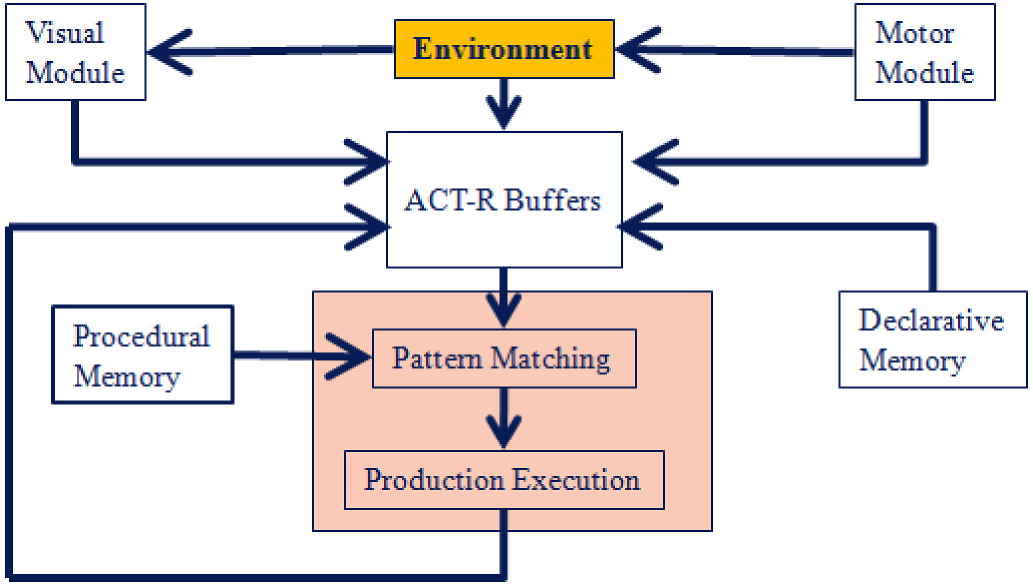
How is this related to how we answered the intro questions?

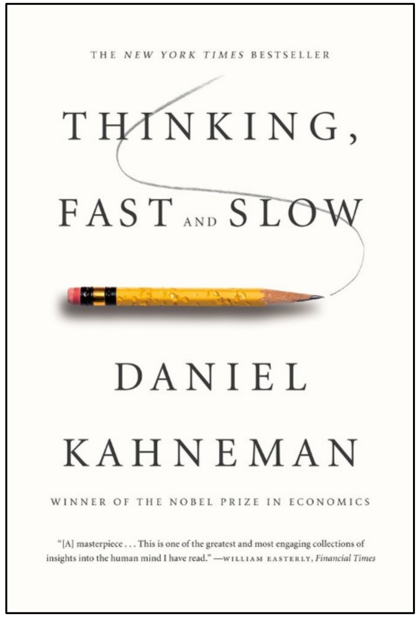
Not Just an Analogy

Closely related to how we (as humans) actually make inferences about the world using incomplete/partial information

Probabilities provide an efficient/flexible way to gauge similarities in complex patterns (though we generally do not keep proper track of probabilities / statistics in our heads!)



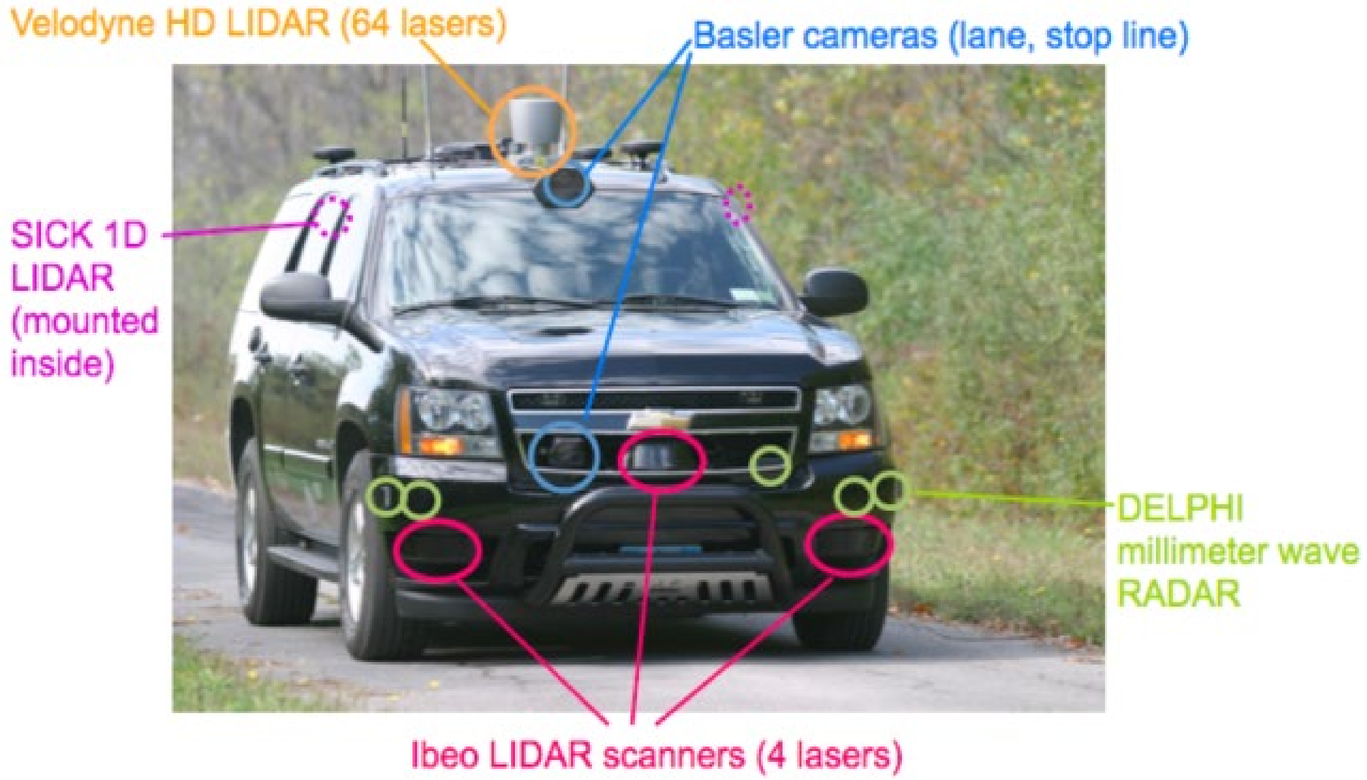




Bayesian Methods for HRI

How an Interactive Autonomous Mobile Robot Sees the World

“Skynet” (Team Cornell’s DARPA Urban Challenge Vehicle)



What inferences does this robot have to make?

Own high-level goals/objectives

-where does passenger/owner want me to go?

-how do I get there? What’s traffic like?

Own low-level/physical state

-position, velocity, acceleration, heading, sensor biases,…

-engine and sensors working? (e.g. GPS, lidar, cameras)

Other vehicles’/objects’ existence, states, intent,…

-are other cars near me? Pedestrians? Bicyclists?

-how many? Do I see them now?

-what are they going to do next?

Where do you get the probabilities from?

What do they mean?

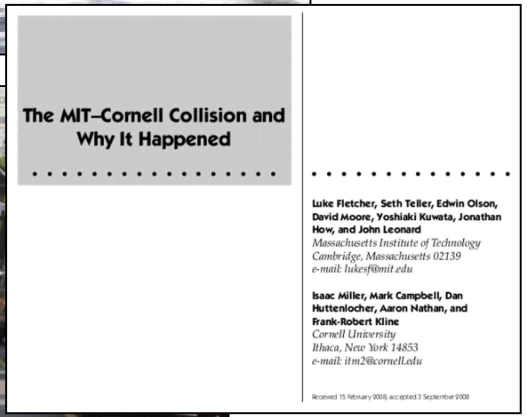
Probabilities provide **cover for our ignorance and laziness** with respect to "all" possible hypotheses in a "universe of discourse" (i.e., **context**)

Learning models from experience, data, and "first principles"

Approximate inference: what if H and E composed of many variables?

Costs and risks for decision making

What if the probability **models themselves are uncertain**?



The MIT–Cornell Collision and Why It Happened

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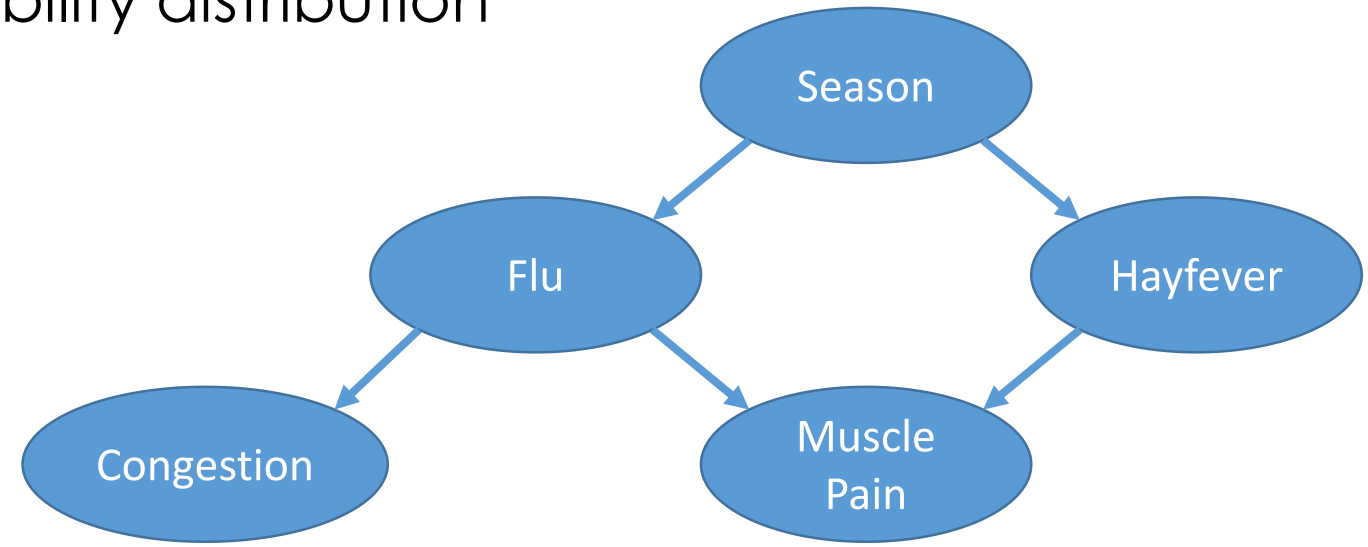
Received 15 February 2008; accepted 3 September 2008

Probabilistic Models

Example: **flu** and **hayfever**

- Not mutually exclusive (could have both, one, or neither)
- **Season** (Spring/Summer/Fall/Winter) is correlated with both
- Symptoms: **congestion** and **muscle pain**
- Total probability space: 2 x 2 x 4 x 2 x 2 = 64 values for joint probability distribution

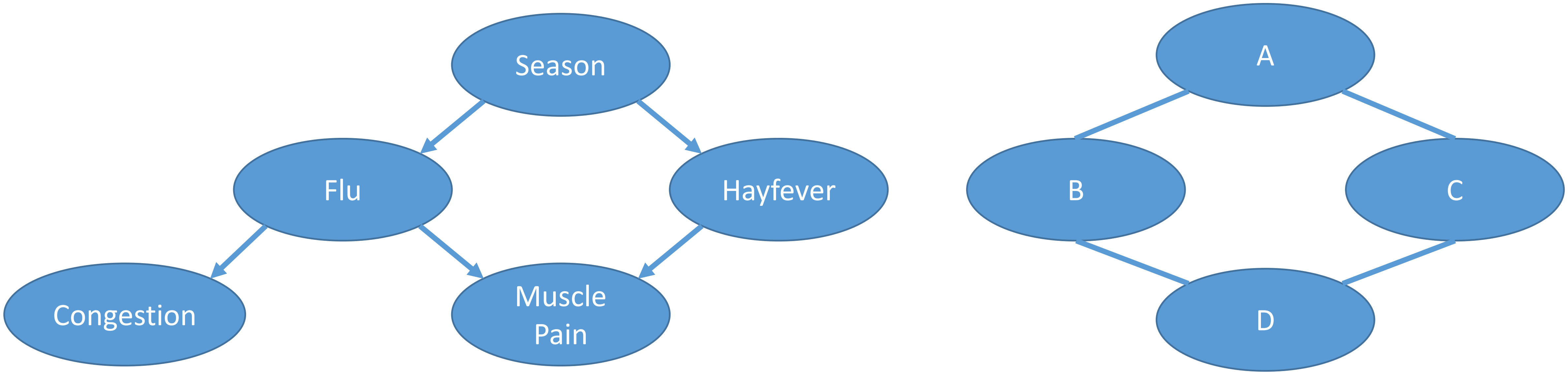


Bayesian vs Markov Networks

Bayesain networks are directed

Markov networks are undirected

Both encode independencies and factorization

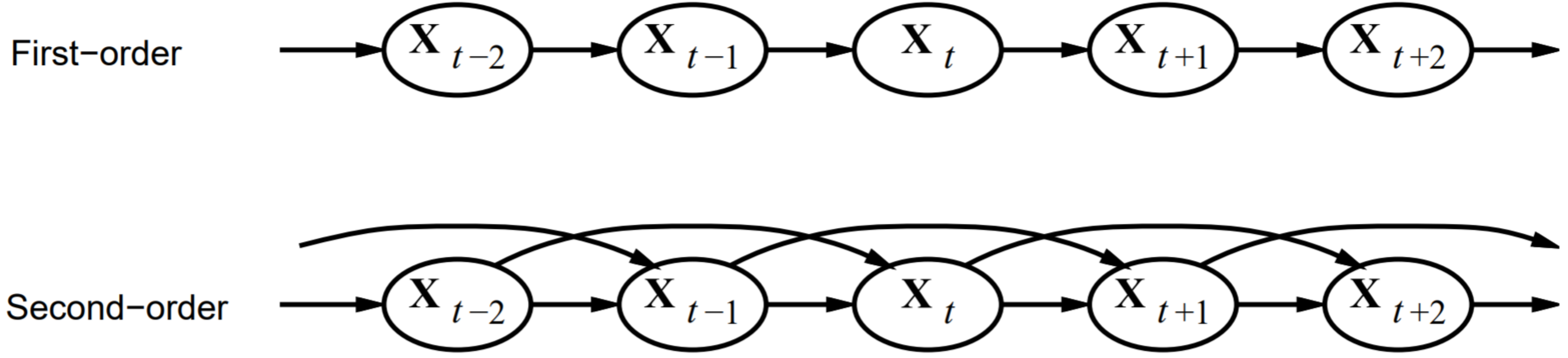


Markov Processes (chains)

Markov assumption: X_t depends on bounded subset of $X_{0:t-1}$

First-order Markov process: $P(X_t|X_{0:t-1}) = P(X_t|X_{t-1})$

Second-order Markov process: $P(X_t|X_{0:t-1}) = P(X_t|X_{t-1}, X_{t-2})$

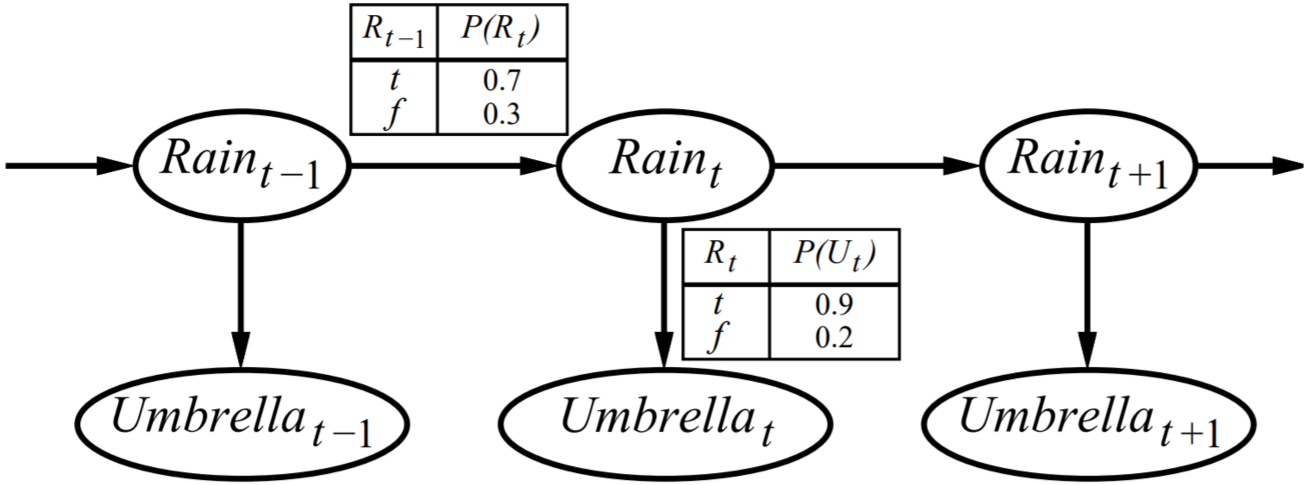


Sensor Markov Assumption

$P(E_t|X_{0:t}, E_{0:t-1}) = P(E_t|X_t)$

- Transition model: $P(X_t|X_{t-1})$
- Sensor model: $P(E_t|X_t)$ fixed for all t

Example:



Notes

Markov assumptions are ways of simplifying our reasoning about high-dimensional probability spaces

- They are not necessarily true in the real world!

Possible fixes

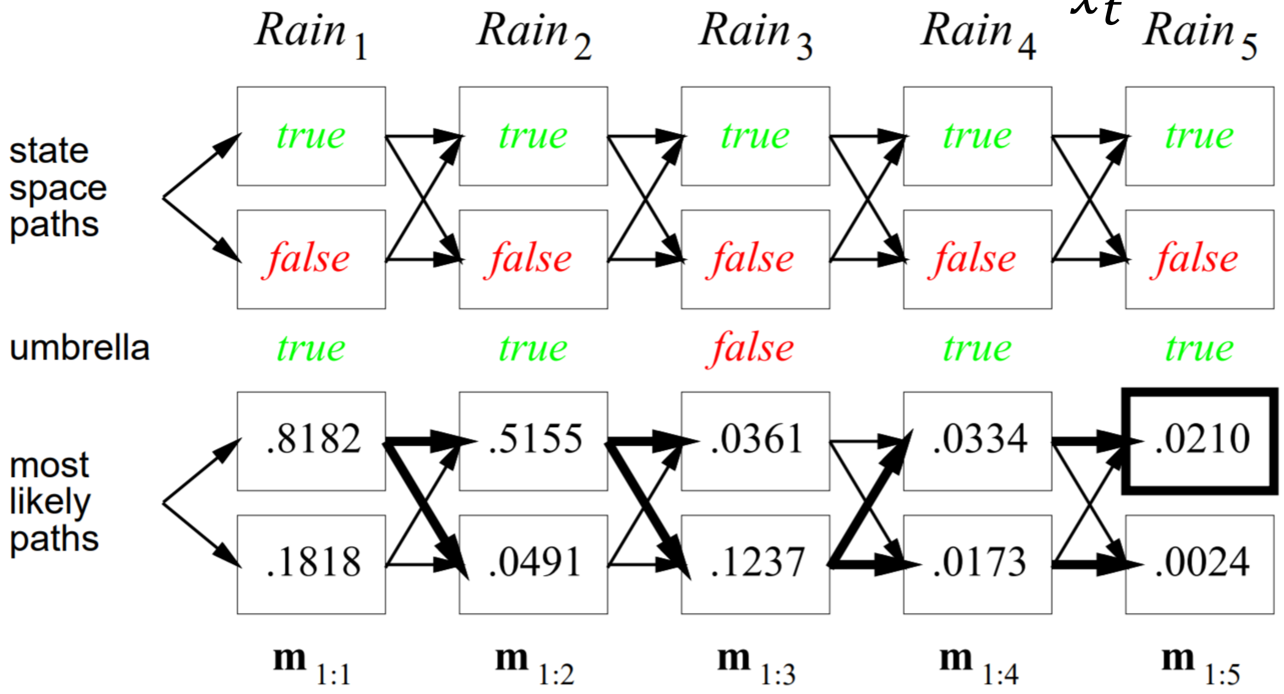
- Increase order of Markov process (third-order, fourth order…etc.)
- Augment state (add additional variables)
  - Example: robot motion estimation – augment $position$ and $velocity$ with $battery_t$

Example Application

Most likely explanation: $\max_{x_{1:t}} P(X_{1:t}|E_{1:t})$

Most likely sequence ≠ sequence of most likely states

Viterbi algorithm: $m_{1:t+1} = P(E_{t+1}|X_{t+1}) \max_{x_t}(P(X_{t+1}|X_t)m_{1:t})$



Hidden Markov Models (HMMs)

HMMs are simple example of state-observation model

- Directed graph where nodes represent system states and edges encode transition model
- Allow for efficient matrix representation

$$HMM = (S, V, B, A, \Pi)$$

N states

$$S = \{s_1, \ldots, s_N\}$$

M symbols

$$V = \{v_1, \ldots, v_M\}$$

Output probability:

$$B = \{b_i(v_k)\} \quad 1 \le i \le N, 1 \le k \le M \quad \sum_{k=1}^{M} b_i(v_k) = 1$$

$b_i(v_k)$: *prob of "generating"* v_k *at* s_i

State transition probability:

$$A = \{a_{ij}\} \quad 1 \le i, j \le N \quad \sum_{j=1}^{N} a_{ij} = 1$$

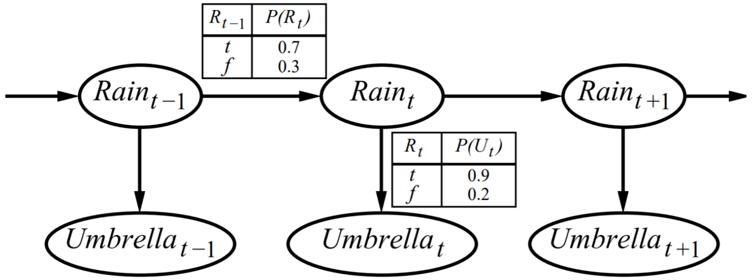
a_{ij} : *prob of going* $s_i \rightarrow s_j$

Initial state probability:

$$\Pi = \{\pi_1, \ldots, \pi_N\} \quad \sum_{i=1}^{N} \pi_i = 1$$

π_i : *prob of starting at state* s_i

HMMs



X_t is a single discrete variable (often E_t / O_t is too)

Domain of X_t is $\{1, \dots, S\}$

Could be higher level variable representing several state variables

Transition matrix: $T_{ij} = P(X_t = j | X_{t-1} = i)$ e.g., $\begin{pmatrix} .7 & .3 \\ .3 & .7 \end{pmatrix}$

Sensor matrix: O_t for each time step, diagonal elements are $P(E_t | X_t = i)$

e.g., for $U_1 = true$, $O_1 = \begin{pmatrix} .9 & 0 \\ 0 & .2 \end{pmatrix}$

Forward and backward messages as column vectors:

$$f_{1:t+1} = \alpha O_{t+1} T^{\tau} f_{1:t}$$
$$b_{k+1:t} = T O_{k+1} b_{k+2:t}$$

Forward-backward algorithm needs time $O(S^2 t)$ and space $O(St)$

HMM Applications

Speech recognition

- Observations: acoustic signals (continuous)
- States: specific positions in specific words

Machine translation

- Observations: words
- States: different translation options

Robot tracking

- Observations: sensor readings (continuous) and or environment features (discrete)
- States: cells (discrete) or map positions (continuous)

Reading 5

Teaming applications involving some collaborative task

Humans can be thought of as “supervisor” or side-by-side worker

A probabilistic approach can help the robot determine what to do

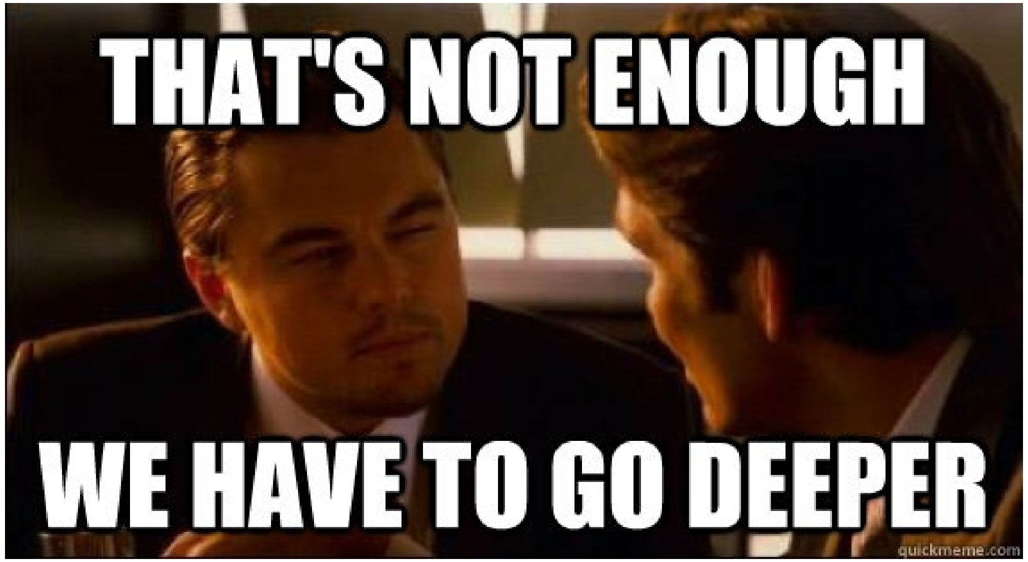
- What might our observations be?
- What might our states be?

Want to go deeper?

https://www.youtube.com/watch?v=ZT3HILfxvps

https://www.youtube.com/watch?v=WPSQfOkb1M8&list=PL50E6E80E8525B59C

…



Next

Reading #6:

Posted on Moodle

Due Wednesday!

Class:

Assignment 2 due Monday!

Project

Continue working with your group to nail down necessary resources

Let me know ASAP if you think you'll need something that none of the labs currently has!

Project updates on March 11

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THANKS!

Professor **Dan Szafir**

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