

# Algorithmic Human-Robot Interaction

---

Developmental Milestones/Project Inspiration  
**Task Planning IV: Planning and Search – Detailed Dive**

CSCI 7000

Prof. Brad Hayes

Computer Science Department  
University of Colorado Boulder

# Final Projects

**Behavior coaching using reward saliency and models of how people consider reward**

Shruthi Sukumar and Lakham Kamireddy

**Navigation assistant for vision-impaired: path planning, scene analysis, verbal summarization of scene/map**

Shivendra Agrawal, Dhanendra Soni, Ian Loefgren

**Conversational planner to augment plan cost function through visualization**

Matthew Luebbers and Shohei Wakayama

**Learning motion planning/grasp preferences from human feedback  
(e.g., don't move sharp end of scissors next to people)**

Jack Kawell and Nishank Sharma

**Sim2Real via AR or Pantomime**

Ryan Leonard, Karthik Palavalli, Ashwin Vasan, Chandan Naik

# Thursday's Papers (2/14):

**Probabilistically Safe Robot Planning with Confidence-Based Human Predictions**

Jaime Fisac et al.

PRO presenter:

CON presenter:

**An Implemented Theory of Mind to Improve Human-Robot Shared Plans Execution**

Sandra Devin and Rachid Alami

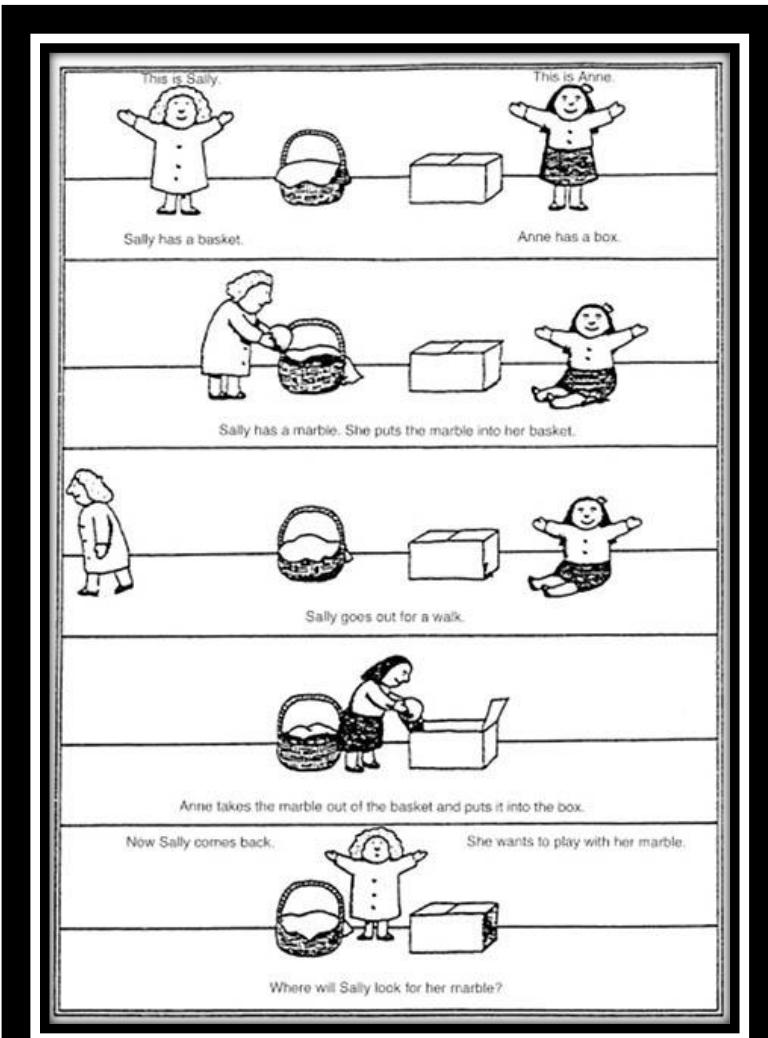
PRO presenter: Matthew Luebbers

CON presenter: Shohei Wakayama

# Project Inspiration

Some developmental milestones worth observing...

# Sally-Anne Test of False Belief



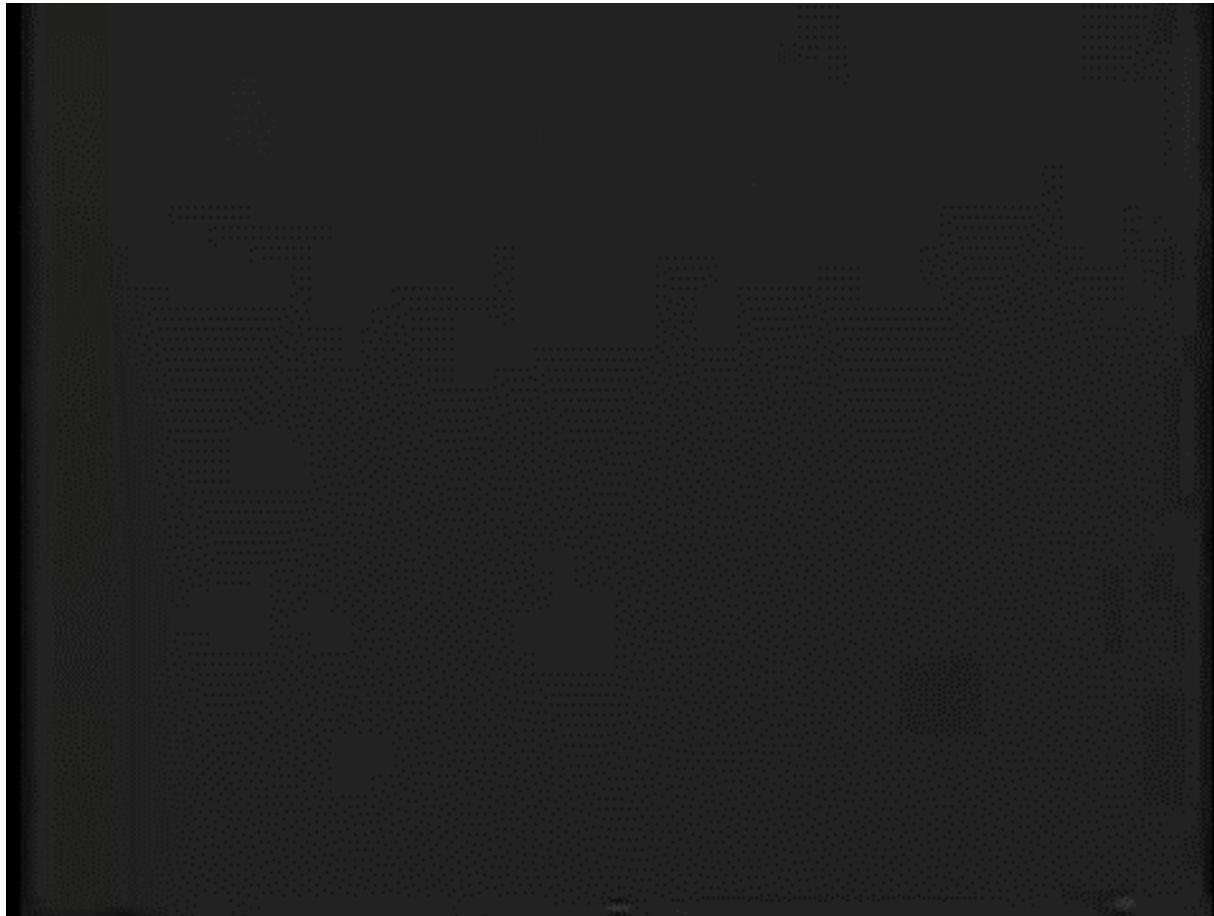
Children under the age of four do not do well on this test!

# Altruism in Children



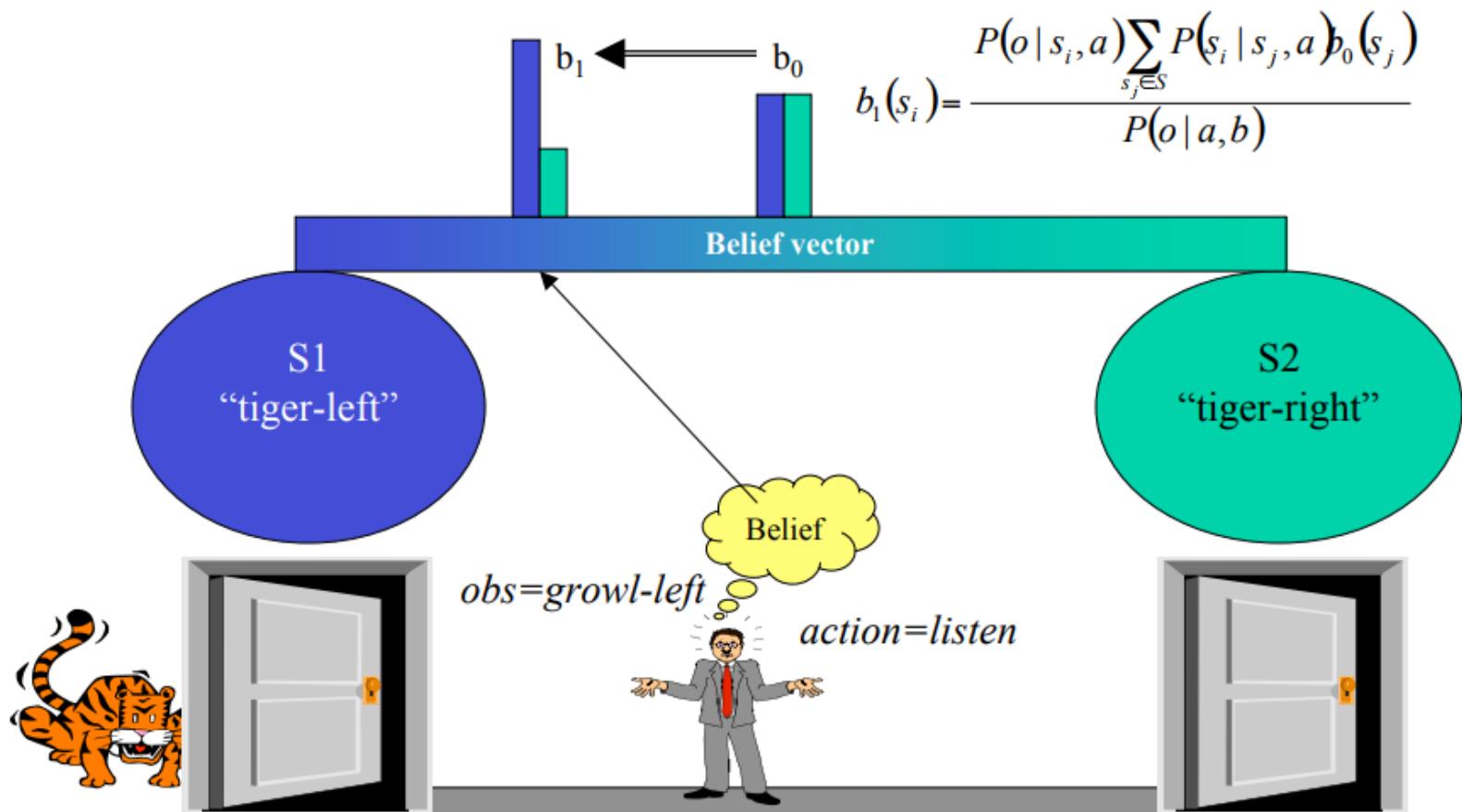
© Warneken & Tomasello

# Heider and Simmel Animation



Last Time...

# POMDP: The Tiger Problem

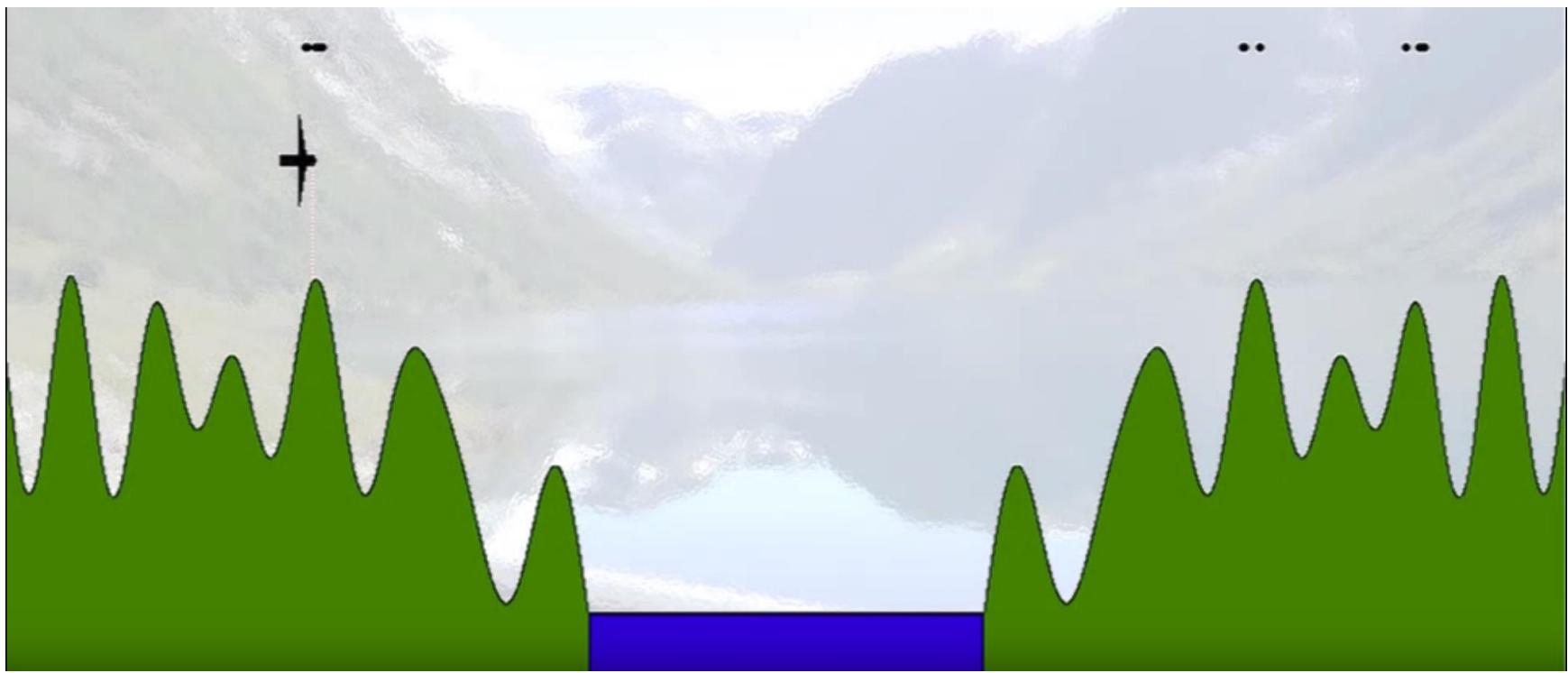


# Particle Filter: Exhaust/Resample

- **Latent variable:** Horizontal aircraft position
- **Observable variable:** Vertical aircraft position (altitude)
- **Known information:** Airspeed, Map
- **Goal:** Use repeated observations to find true horizontal position

**Main Idea:**

Generate lots of hypotheses  
and let the observations  
determine their likelihood.



# Markov Model Chart

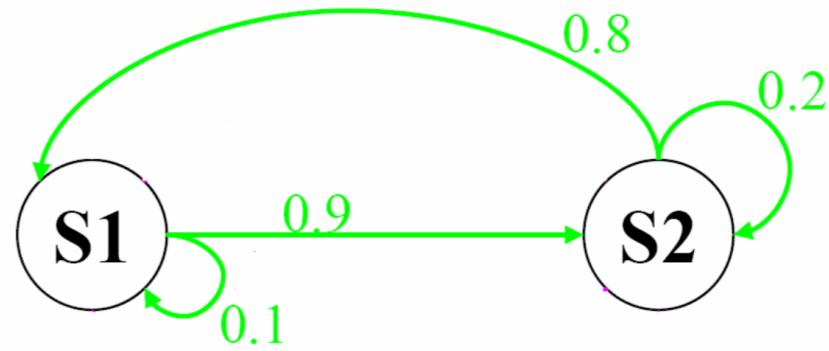
**Are the states  
completely  
observable?**

**Do we have control over the state transitions?  
(Are we picking which actions are executed)**

	<b>NO</b>	<b>YES</b>
<b>YES</b>	Markov Chain	MDP
<b>NO</b>	HMM	POMDP

# Markov Chains

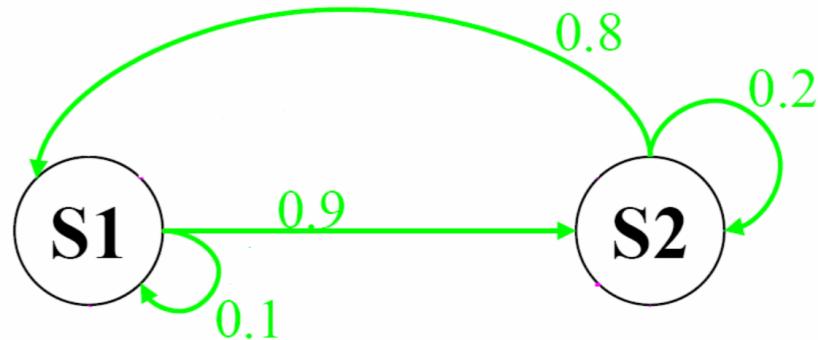
- Finite number of discrete states
- Probabilistic transitions between states
- Next state determined only by the current state
  - This is the Markov property



Rewards:  $S1 = 10, S2 = 0$

# Hidden Markov Model

- Finite number of discrete states
- Probabilistic transitions between states
- Next state determined only by the current state
- We're unsure which state we're in
  - The current state emits an observation

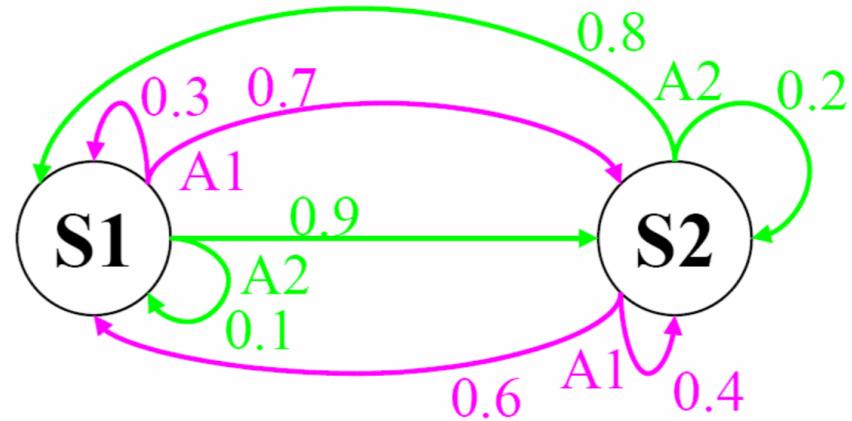


Rewards:  $S1 = 10, S2 = 0$

Do not know state:  
S1 emits O1 with prob 0.75  
S2 emits O2 with prob 0.75

# Markov Decision Process

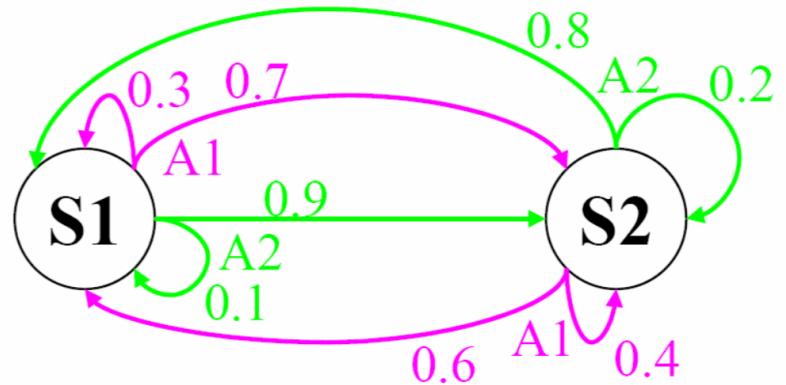
- Finite number of discrete states
- Probabilistic transitions between states **and** controllable actions in each state
- Next state determined only by the current state **and** current action
  - This is still the Markov property



Rewards:  $S1 = 10, S2 = 0$

# Partially Observable Markov Decision Process

- Finite number of discrete states
- Probabilistic transitions between states and controllable actions
- Next state determined only by the current state and current action
- We're unsure which state we're in
  - The current state emits observations



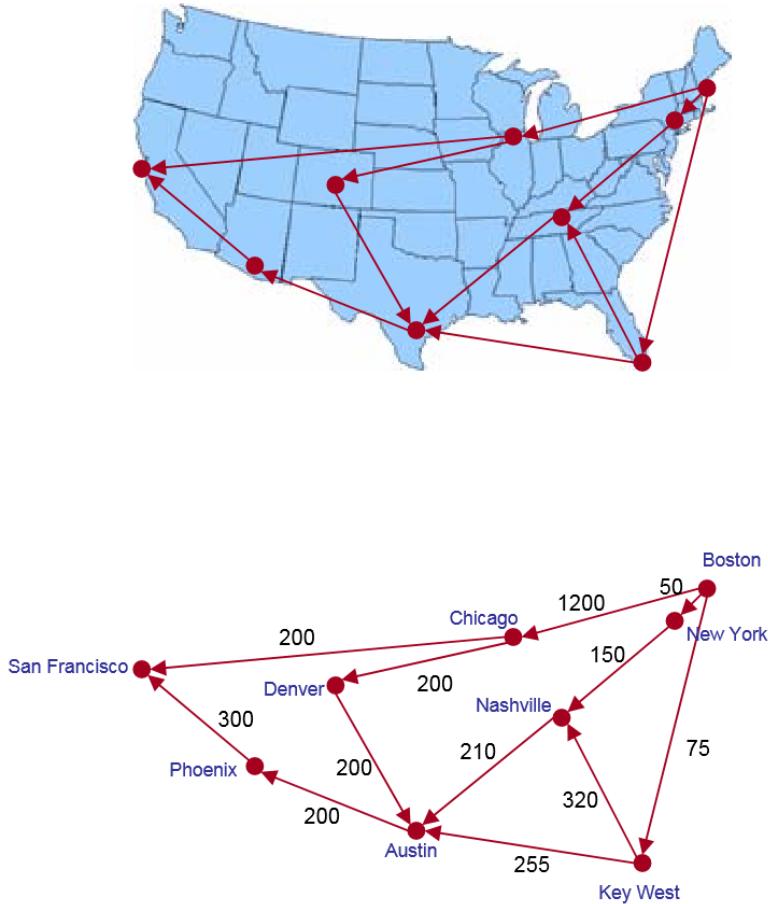
Rewards: S1 = 10, S2 = 0

Do not know state:  
S1 emits O1 with prob 0.75  
S2 emits O2 with prob 0.75

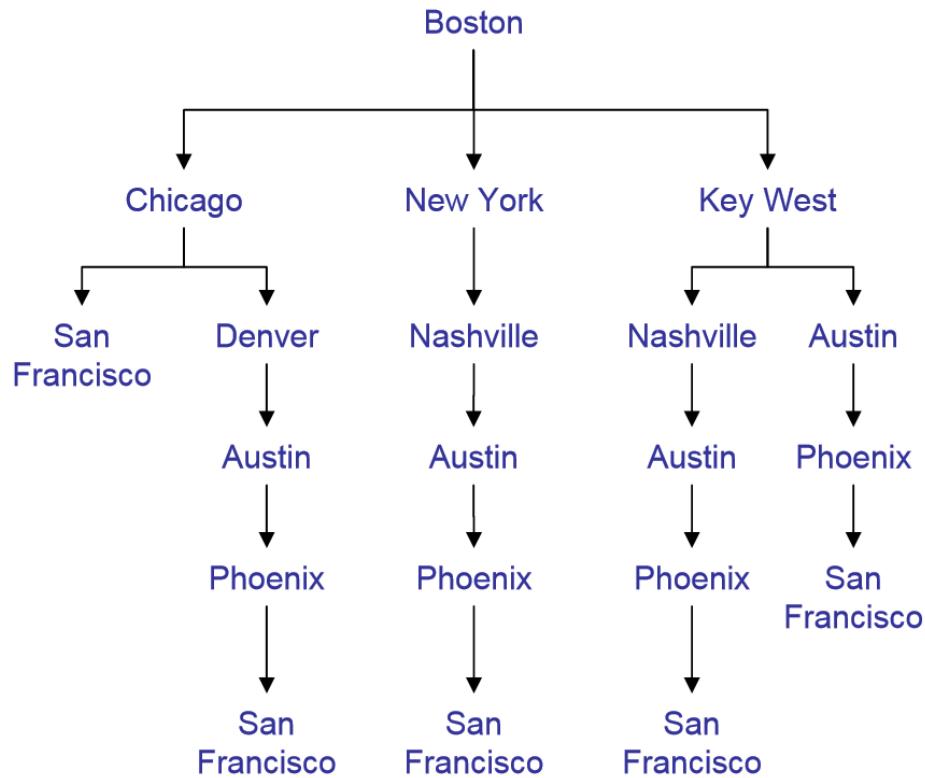


# Planning Deep Dive

# Search as a Problem-Solving Technique



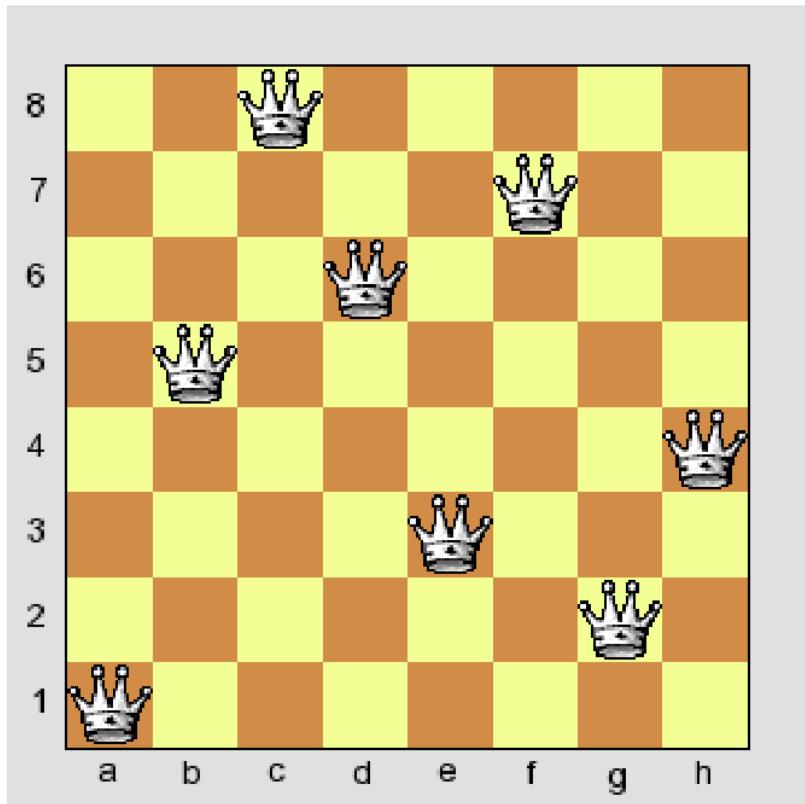
Depth
0
1
2
3
4
5



Branching Factor  $b=3$

# Problem Formulation Matters!

## The 8 Queens Problem



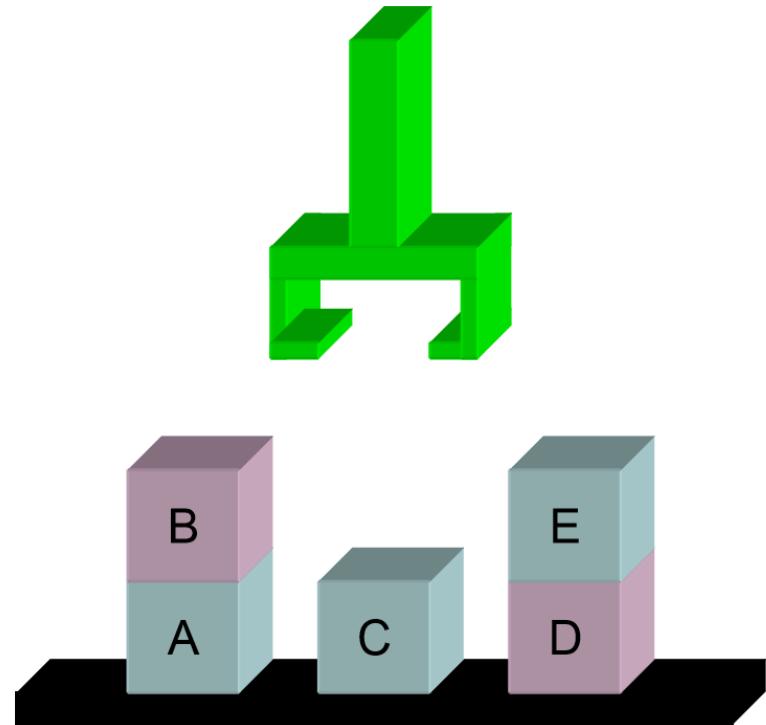
- Formulation #1:
  - Place a queen on any open square
  - Repeat until all queens are placed
  - State space of  $\frac{64!}{56!} = 1.78 * 10^{14}$
- Formulation #2:
  - Place a queen on any row 1 square
  - Place a queen on any row 2 square
  - State space of  $8^8 = 1.68 * 10^7$
- Formulation #3:
  - Place a queen on any row 1 square
  - Place a queen on any row 2 square not sharing a column...
  - State space of  $8! = 40,320$

# STRIPS: Language for Classical Planning

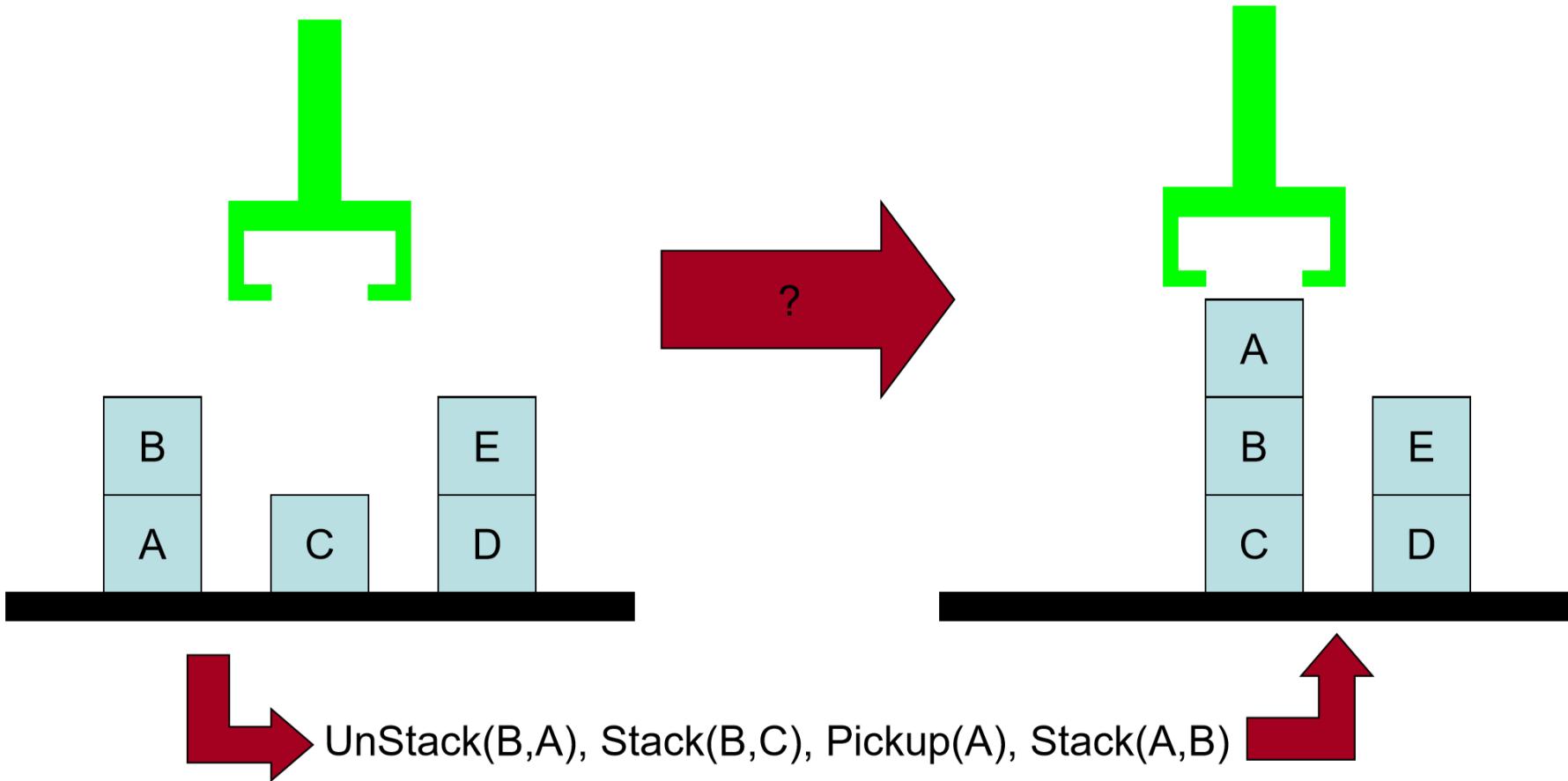
- A **problem** in Strips is a tuple  $P = \langle F, O, I, G \rangle$ :
  - ▷  $F$  stands for set of all **atoms** (boolean vars)
  - ▷  $O$  stands for set of all **operators** (actions)
  - ▷  $I \subseteq F$  stands for **initial situation**
  - ▷  $G \subseteq F$  stands for **goal situation**
- Operators  $o \in O$  **represented** by
  - ▷ the **Add** list  $Add(o) \subseteq F$
  - ▷ the **Delete** list  $Del(o) \subseteq F$
  - ▷ the **Precondition** list  $Pre(o) \subseteq F$
- Pickup(X)
  - P:  $\text{grip}(\emptyset) \wedge \text{clear}(X) \wedge \text{ontable}(X)$
  - A:  $\text{grip}(X)$
  - D:  $\text{onTable}(X) \wedge \text{grip}(\emptyset)$

# Blocks World

- Robot Gripper
- Square objects on a perfect, flat table
- Predicates to describe world:
  - $\text{On}(x,y)$
  - $\text{TopClear}(x)$
  - $\text{Grip}(x)$

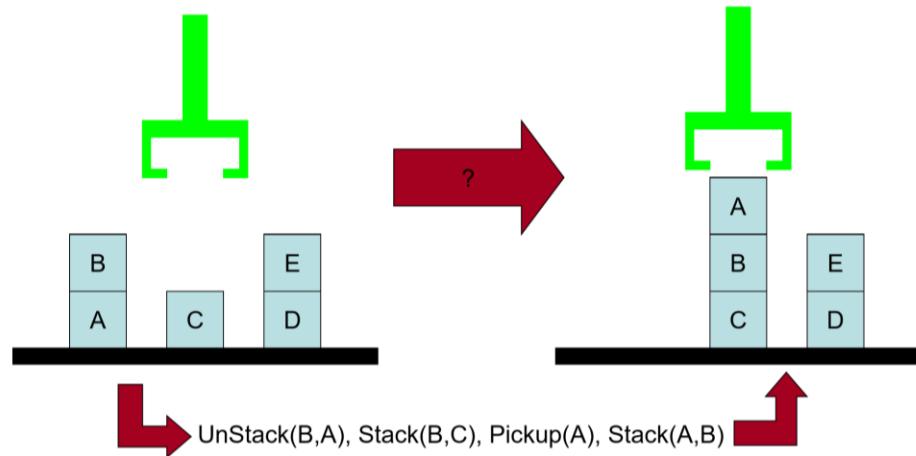


# Planning in Blocks World

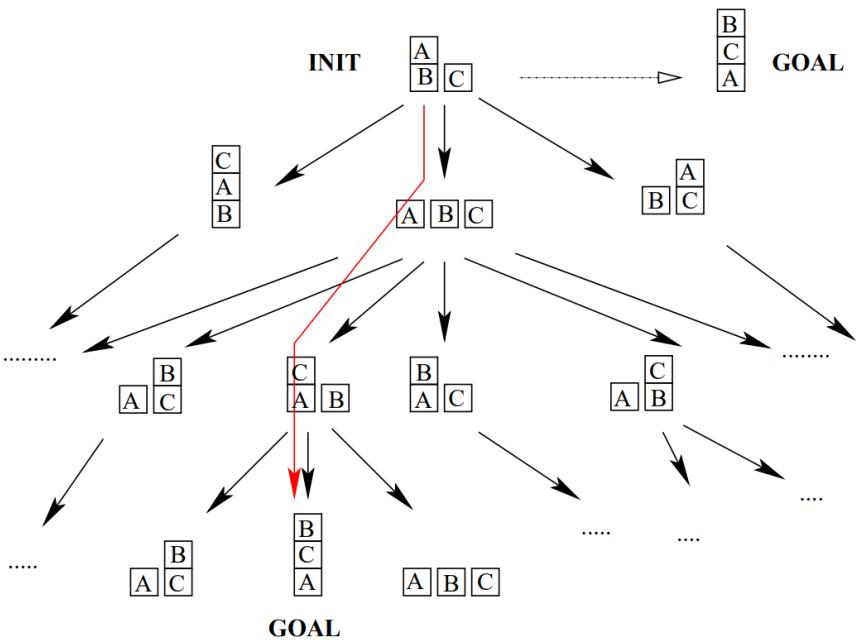


# Blocks World

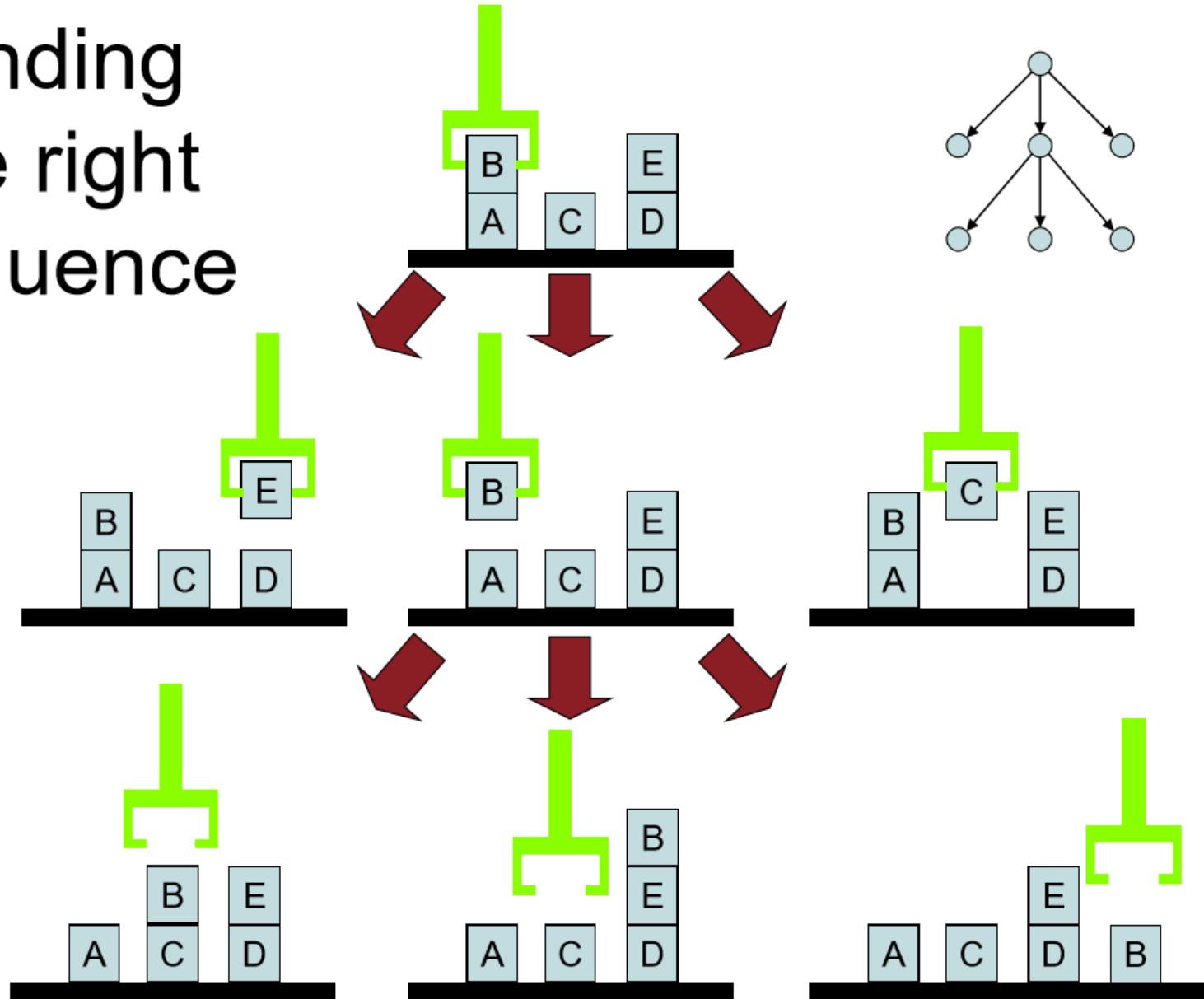
Given actions that move 'clear' block to the table or onto another 'clear' block,  
**find a plan to achieve the goal**



How can we find a path in a graph whose size is exponential in the number of blocks (parameterized actions)?

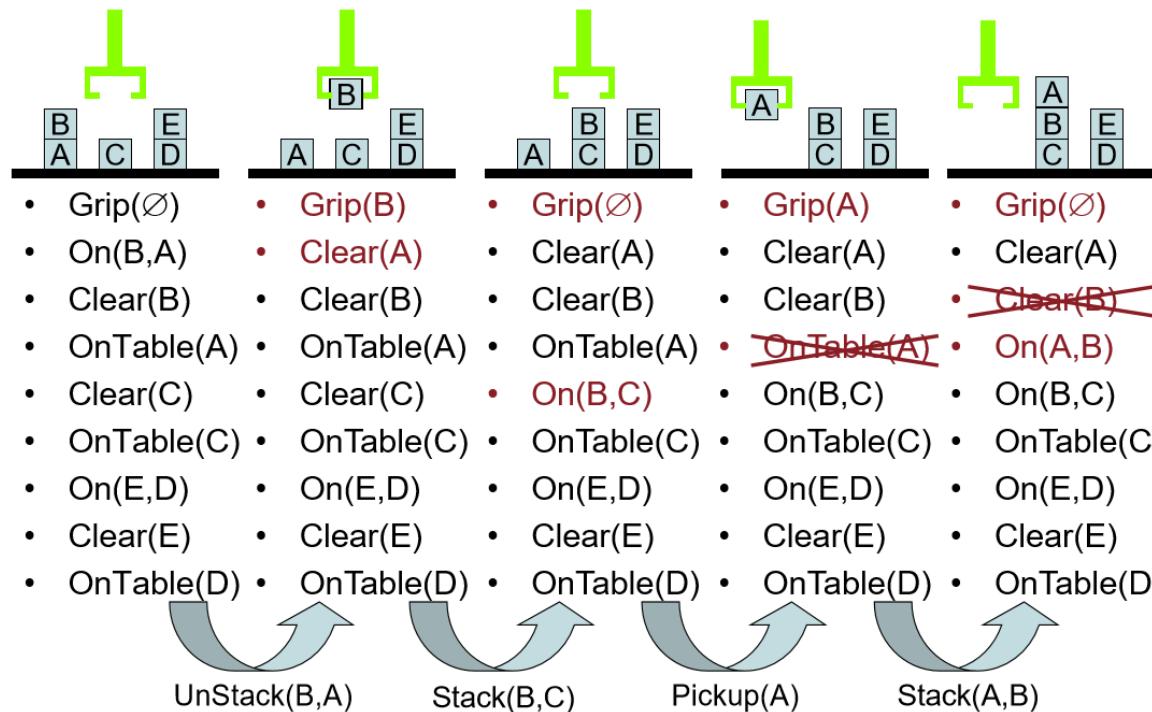


# Finding the right Sequence



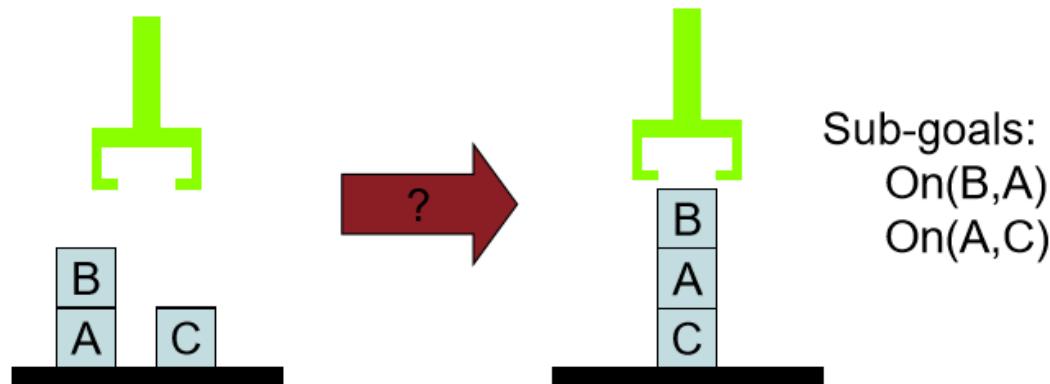
- Pickup(X)
    - P: grip( $\emptyset$ )  $\wedge$  clear(X)  $\wedge$  onTable(X)
    - A: grip(X)
    - D: onTable(X)  $\wedge$  grip( $\emptyset$ )
  - PutDown(x)
    - P: grip(X)
    - A: onTable(X)  $\wedge$  grip( $\emptyset$ )  $\wedge$  clear(X)
    - D: grip(X)
  - Stack(X,Y)
    - P: clear(Y)  $\wedge$  grip(X)
    - A: on(X,Y)  $\wedge$  grip( $\emptyset$ )  $\wedge$  clear(X)
    - D: clear(Y)  $\wedge$  grip(X)
  - UnStack(X,Y)
    - P: clear(X)  $\wedge$  grip( $\emptyset$ )  $\wedge$  on(X,Y)
    - A: grip(X)  $\wedge$  clear(Y)
    - D: grip( $\emptyset$ )  $\wedge$  on(X,Y)
- 

## Update of Knowledge

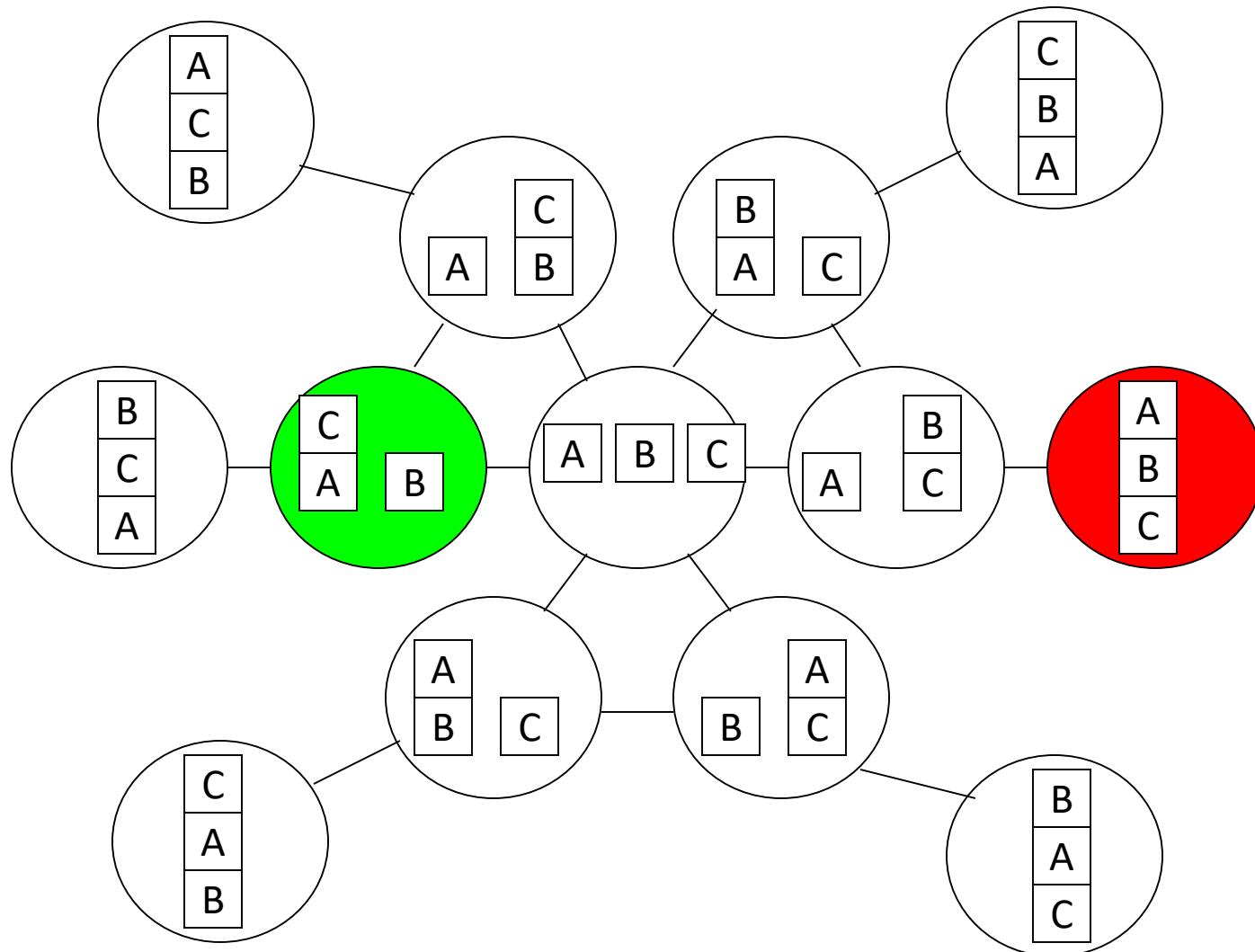


# (Minor) Problems

- Exponential sequence of moves
- Does the tree have a fixed depth?
  - Cycles
- Incompatible sub-goals



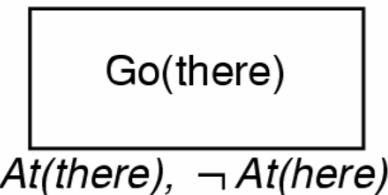
# Using A\* in Planning



$h(n)$  = "# of goals remaining to be satisfied"    $g(n)$  = "# of steps so far"

# STRIPS Operators Revisited

*At(here), Path(here, there)*



- Operators are triplets of descriptors
  - A set of **P**reconditions
  - A set of **A**dditions
  - A set of **D**eletions
- Graphical Representation

# Predicates

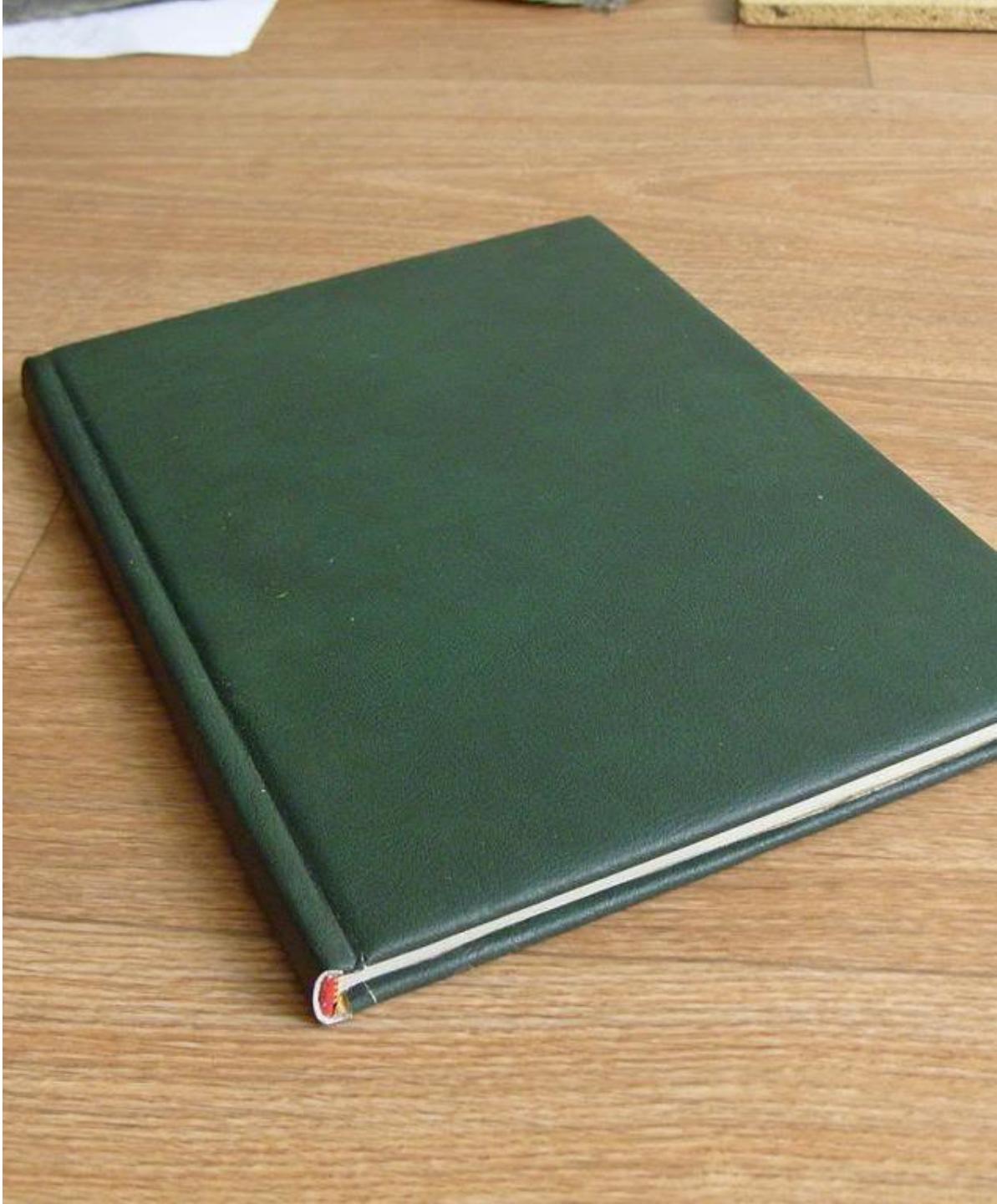
Two types of predicates:

Direct measurements of state

- $At(object, x, y, z)$ :  
 $At(book\_0, 0, 0, 1.6)$

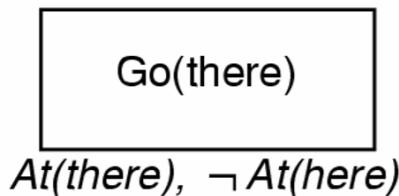
Indirect from state variable

- $On\_top(object, object)$ :  
 $On\_top(book, table)$



# STRIPS Operators Revisited

*At(here), Path(here, there)*



- Operators are triplets of descriptors

- A set of **Preconditions**

- A set of **Additions**

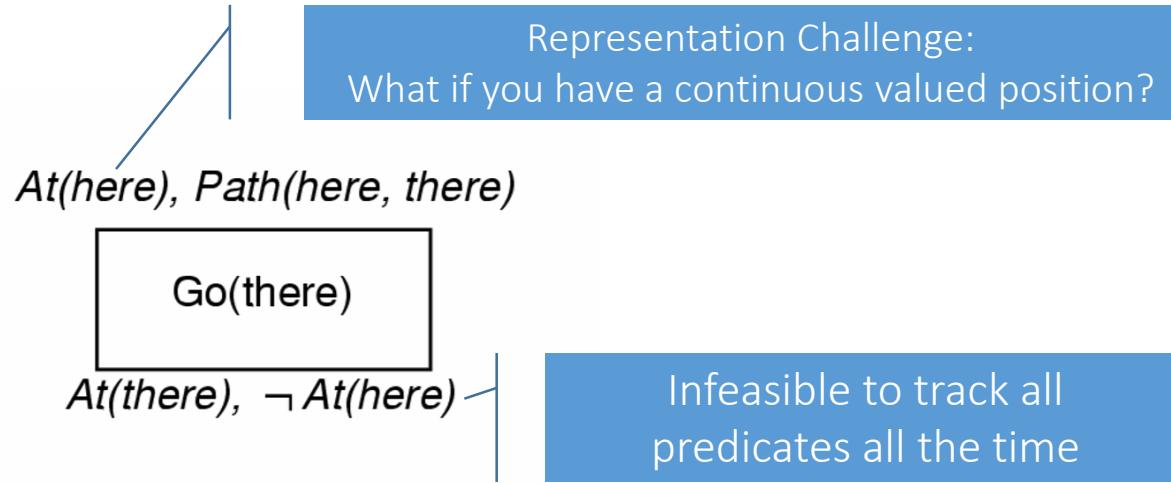
- A set of **Deletions**

Set predicate values in ranges  
[-1,0] or (0,1)

Predicates are ternary, 0 is “doesn’t exist”

- Graphical Representation

# STRIPS Operators Revisited



- Operators are triplets of descriptors
  - A set of **Preconditions**
  - A set of **Additions**
  - A set of **Deletions**
- Graphical Representation

# Lifted vs. Grounded Planning

Plan in this space

Execute in this space

Predicate Representation

On(a,b)  
Moving(d)  
Clear(Lhand)

1.0  
1.0  
0  
0  
0  
-1.0  
-1.0  
1.0

Mapping Function

World State Vector

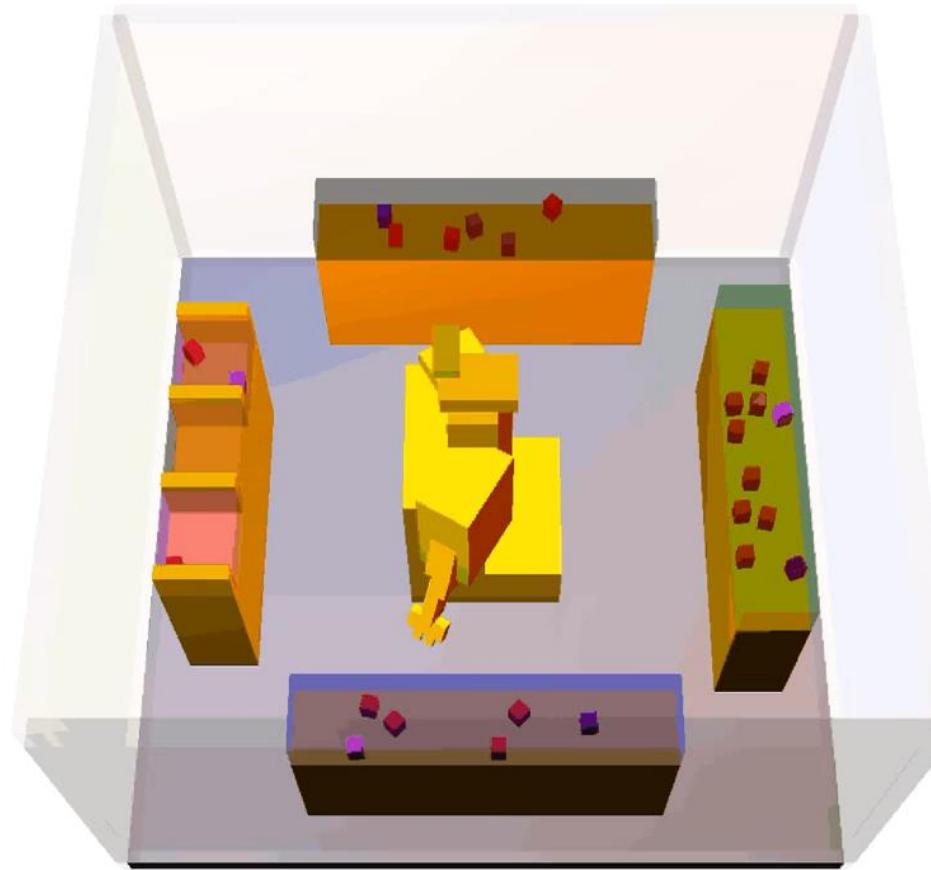
0.124  
543.1  
491.3  
-50.11  
194.2  
...  
...  
...

Task  
Planning

Compression  
Function

Motion  
Planning

# Choosing the right granularity is not always easy



(source: FFRob -- <http://web.mit.edu/caelan/www/research/ffrob/>)



## Important Details

- **Everything** must map back to motor commands!
- The real world has annoying physics details that are difficult to efficiently model
- Can't run through state space with something like RRT:
  - A dynamics model probably won't help you make much progress
  - State space still probably too large to make meaningful advances

How do I make a plan for a problem like this?

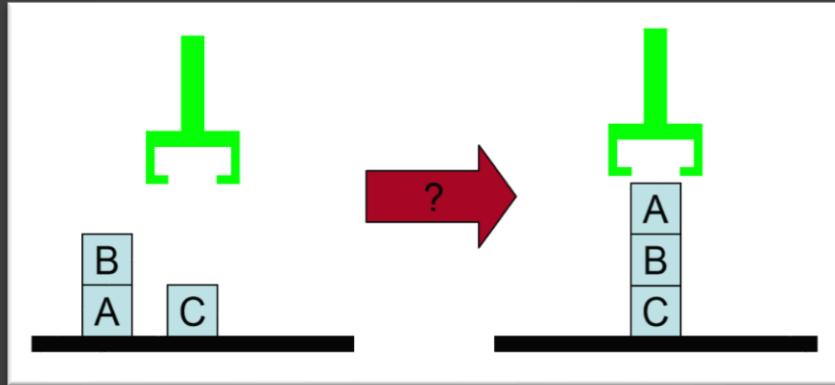


## Strategies

- Breadth-first search with sampled action parameterizations
- Pre-determined action parameterizations
  - e.g., Place(object,x,y,z)
- Plan for abstract ‘areas’
  - **Lazy Grounding:**  
Sample parameterizations and add that action to plan if a valid parameterization is found
  - **Greedy Grounding:**  
Sample parameterizations and add that specific parameterization to the plan once a valid one is found

# Non-Interleaved Planning

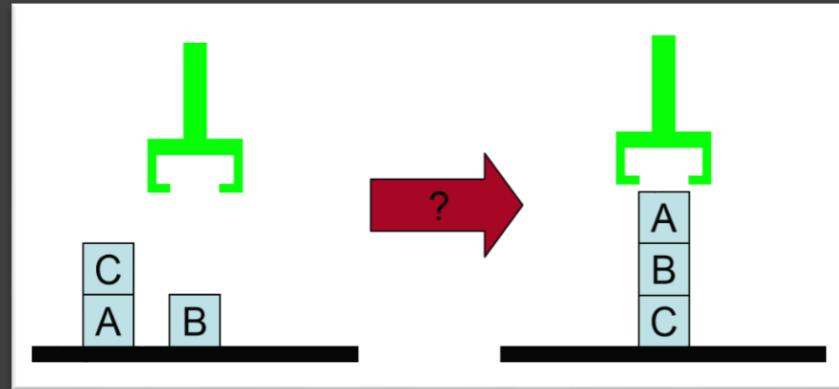
- Most parts of the world are independent, facilitating divide-and-conquer strategies
  - Plan to achieve individual/subsets of final goal state's predicate objectives



- Goal:  $\text{On}(B,C) \wedge \text{On}(A,B)$
- Achieve goal #1, then achieve goal #2

## Problems with Non-interleaved Planners: Sussman Anomaly

- Switch the location of B & C in the initial state
- Same goal:  $\text{On}(B,C) \wedge \text{On}(A,B)$



- Achieving goals sequentially doesn't work:  
 $\text{On}(B,C)$  is violated when achieving  $\text{On}(A,B)$ .

# Creating a Lifted Plan

Naïve strategy:

- Breadth-first search over actions
- Pre-determined max search depth

Better strategy:

- Employ planning heuristics to inform action selection and generate sub-goal orderings

# Key Problem: Branching Factor

A linear increase in search depth  
requires an exponential increase in  
visited states



Heuristics are required to help guide  
action selection:



Our search algorithm needs a  
notion of direction!



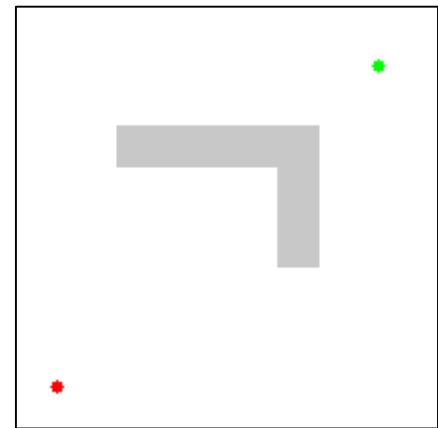
# A\* Search Algorithm

Adds heuristic to Dijkstra's Algorithm  
to bias exploration.

Finds shortest path between  
two nodes in a graph

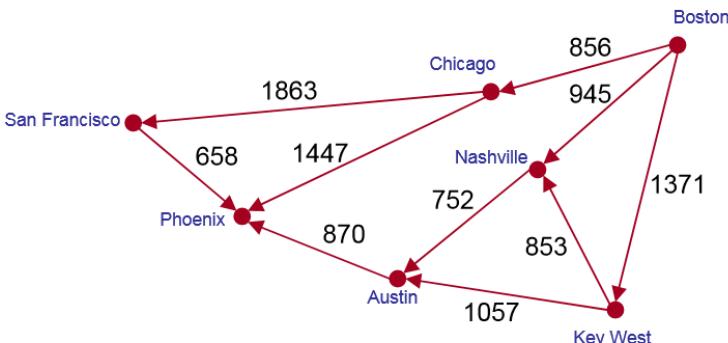
$$f(n) = \frac{\text{Insight}}{g(n)} + h(n)$$

$f(n)$  = Cost so far + Est. cost to go



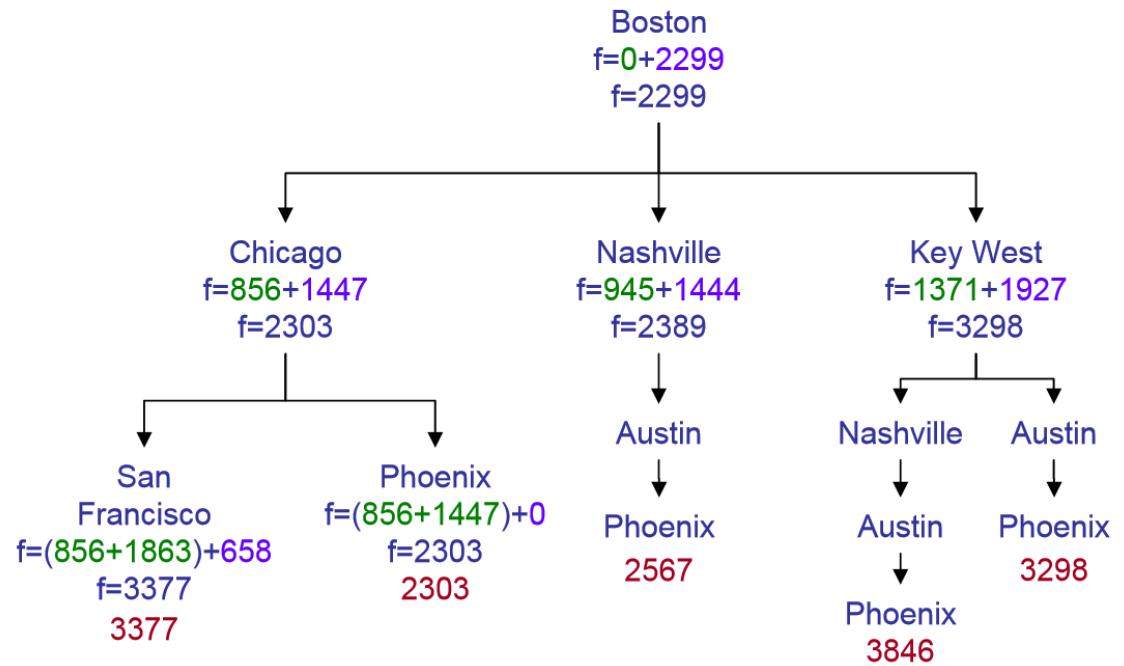
Admissible Heuristic: An underestimate of the shortest possible path from node n to the goal.

# A\* Search Algorithm



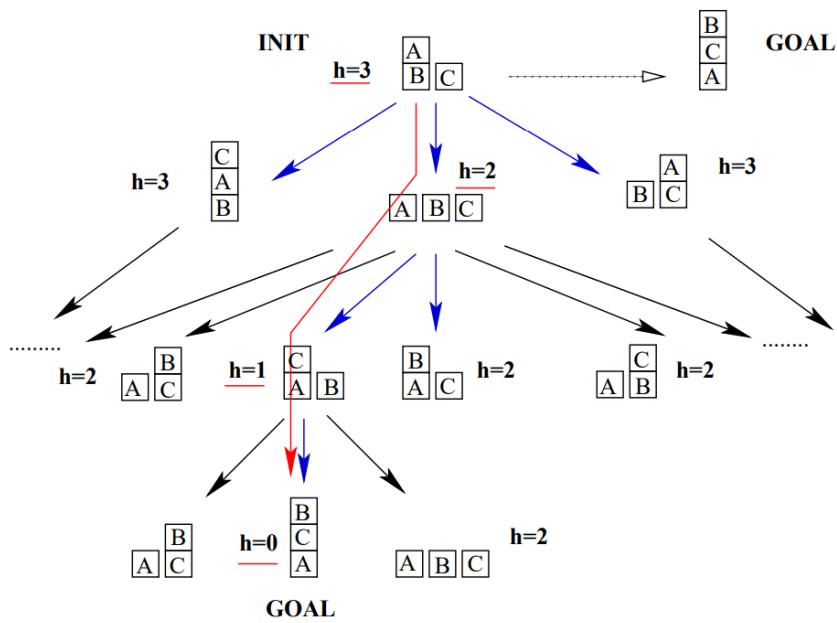
	Distance to Phoenix
Boston	2299
Chicago	1447
Nashville	1444
Key West	1927
Austin	870
San Francisco	658

- Combine Greedy search with Uniform Cost Search
- Minimize the total path cost ( $f$ ) = **actual path so far (g)** + **estimate of future path to goal (h)**

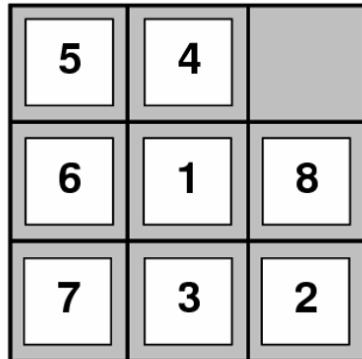


# Developing Heuristics

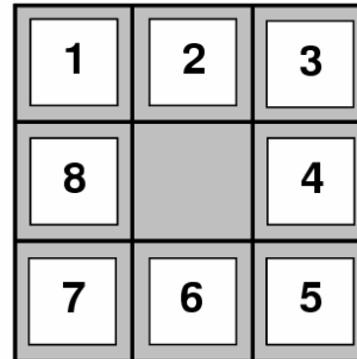
- **Heuristic evaluations  $h(s)$  provide ‘sense-of-direction’**
- Derived efficiently in a **domain-independent** fashion from **relaxations** where effects made **monotonic** (delete relaxation).
- **“Fast-Forward” Heuristic:** Remove all deletions from operators



# Heuristic Selection



Start State



Goal State

- Must be admissible (never over-estimate)
- Heuristics for the 8-Puzzle

# Heuristic Selection Effects

d	Search Cost			Effective Branching Factor		
	IDS	A*( $h_1$ )	A*( $h_2$ )	IDS	A*( $h_1$ )	A*( $h_2$ )
2	10	6	6	2.45	1.79	1.79
4	112	13	12	2.87	1.48	1.45
6	680	20	18	2.73	1.34	1.30
8	6384	39	25	2.80	1.33	1.24
10	47127	93	39	2.79	1.38	1.22
12	364404	227	73	2.78	1.42	1.24
14	3473941	539	113	2.83	1.44	1.23
16	–	1301	211	–	1.45	1.25
18	–	3056	363	–	1.46	1.26
20	–	7276	676	–	1.47	1.27
22	–	18094	1219	–	1.48	1.28
24	–	39135	1641	–	1.48	1.26

- Compare iterative-deepening with A\* using  $h_1$  (# misplaced tiles) and  $h_2$  (city block distance)
- Effective branching factor  $b^*$ 
  - Number of expanded nodes =  $1 + b^* + (b^*)^2 + \dots + (b^*)^{\text{depth}}$
  - $b^*$  remains relatively constant across many measurements

# Planning Heuristics

- **FastForward Heuristic**
  - Remove deletions from all operators and run planner
  - Resulting plan gives a heuristic for # steps from current state to goal
  - FF Plan can provide ordering constraints on actions, even within partially specified plans
- **Learned Cost Function**
  - Learning Real-Time A\* (LRTA\*)
  - Update heuristic dynamically: Value function  $V(s)$
  - Connection back to reinforcement learning:  **$V(s)$  and  $Q(s,a)$**  for search heuristic

# FastForward Heuristic

- Modifying the formulation to be easier is called a **relaxation** of the original problem.
- **FastForward** uses a **delete-relaxation** to transform the original problem  $P$  into  $P^+$  where the operators (actions) have no deletions
- Solving  $P^+$  optimally should be much easier than solving  $P$  optimally!
  - It isn't. These are both NP-hard ☹
  - But finding **any solution** is a lot easier!

# Learning Real-Time A\*

- Hill-climbing algorithm where the **best child node** is selected and others **discarded**, and the heuristic is updated dynamically.

Cost paid  
already

Cost to  
goal

- A\* Cost Formula:  $f(n) = g(n) + h(n)$ 
  - Replace  $h(n)$  with  $V(s')$
  - Replace  $g(n)$  with  $Q(s, a)$  where  $a$  has high  $T(s, a, s')$
- $Q(s,a) = c(s,a) + V(s')$

# Admissible Heuristics and Pruning Strategies

Admissible heuristics from delete-relaxation:

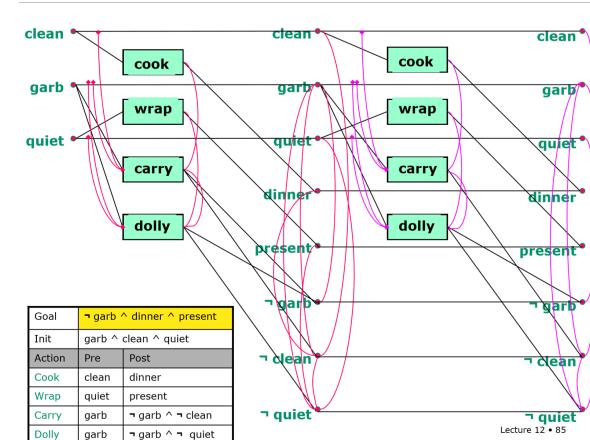
- $h_{max} = i$  iff goal  $g$  is reachable in  $i$  steps from  $s$ 
  - Admissible because of GRAPHPLAN Algorithm
- $h_{FF}$  = number of **different** actions in  $\pi(g)$

Search Pruning Strategies:

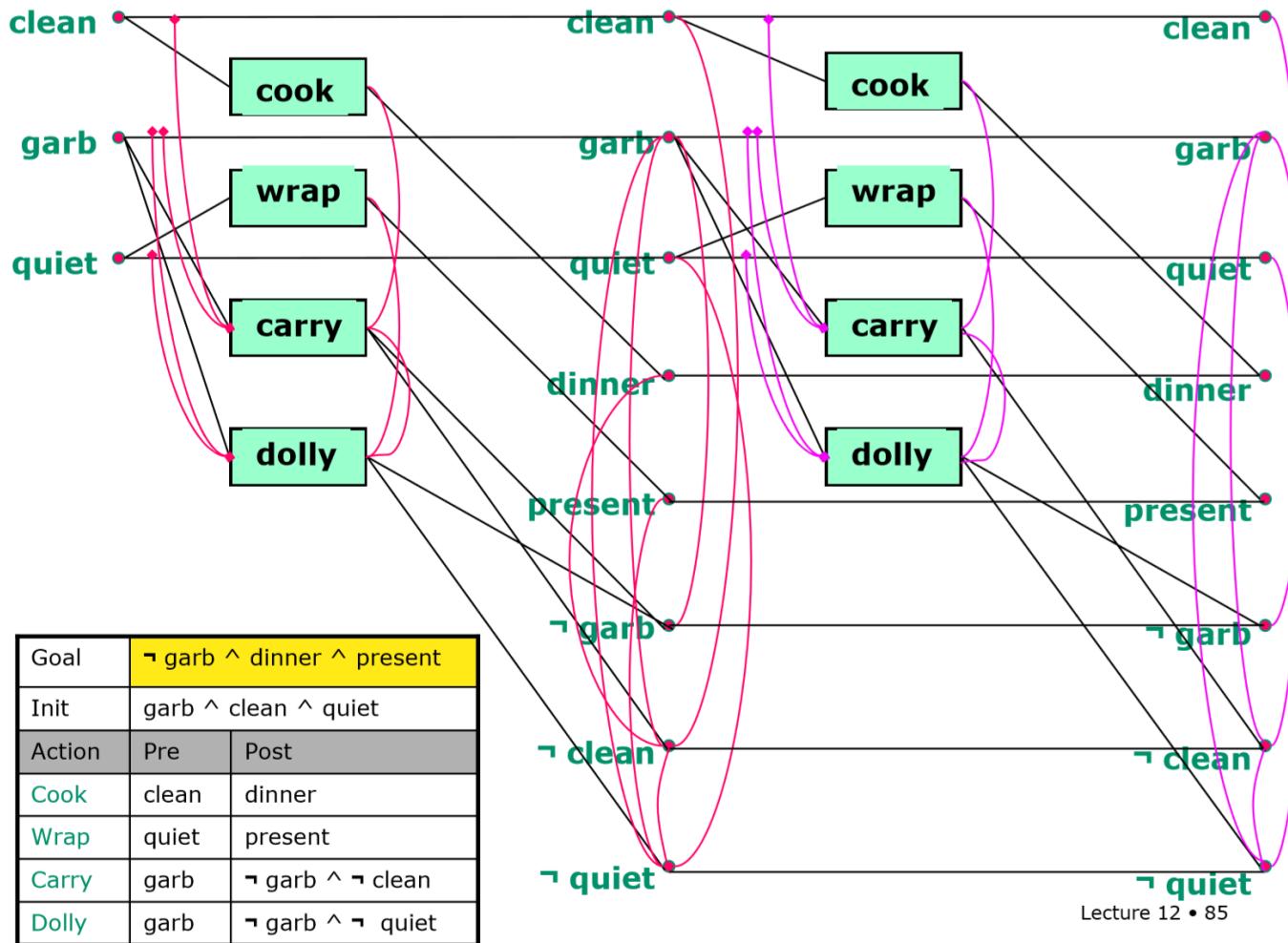
- Novelty: Assign each state  $novelty(s) = i$  if  $i$  predicates take on a value never seen in path from  $s_0$  to  $s$
- IW-search (Iterative Width): Ignore any nodes at the search frontier with  $novelty(s) > i$ .
  - Call planner for increasing  $i$  until problem solved or  $i$  exceeds number of variables in the problem.

# GRAPHPLAN for $h_{max}$

- Alternating layers of State and Action in leveled graph
- Preconditions connect predicates in layer  $2t$  to actions in layer  $2t + 1$
- Effects connect actions to predicates in next layer
- All predicates in layer  $2t$  connected to themselves in layer  $2(t + 1)$  by “maintenance actions”
- First layer where goal predicates exist is lower bound
- Need to compute MUTEX actions and predicates



# GRAPHPLAN Graph



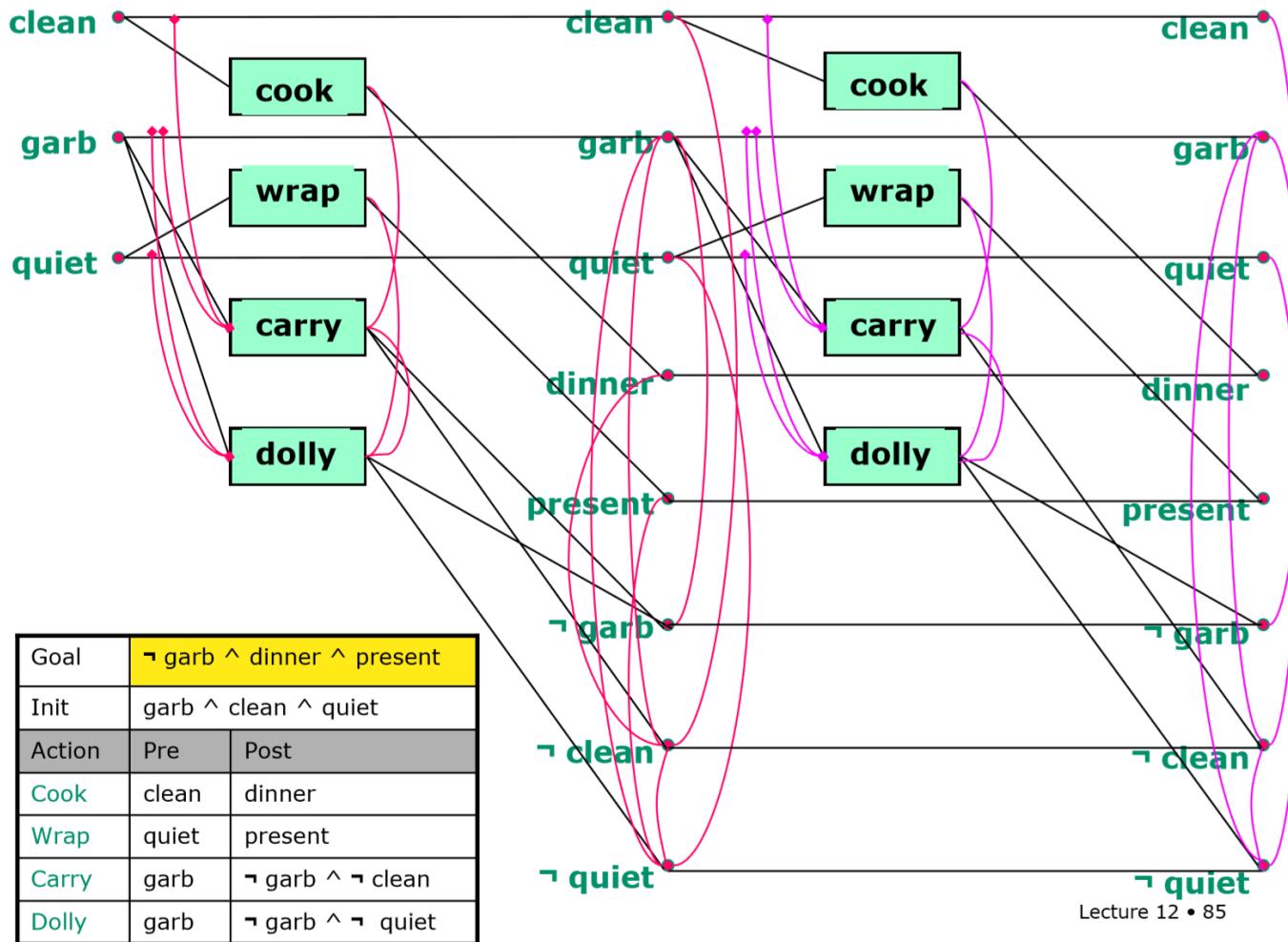
# GRAPHPLAN MUTEX

- Actions
  - **Inconsistent:** Effect of one action is negation of other's effect
  - **Interference:** Action deletes precondition of other
  - **Competing Needs:** Two actions have mutex preconditions at previous level
- Predicates
  - **Negation:** Two predicates are negations of each other
  - **Inconsistent Support:** All means of achieving the pair are pairwise mutex

# Sample Problem: Birthday Dinner

- Goal: ~Garbage & Dinner & Present
- Init: Garbage & Clean & Quiet
- Actions
  - Cook
    - Pre: clean
    - Effect: dinner
  - Wrap
    - Pre: quiet
    - Effect: present
  - Carry
    - Pre: garbage
    - Effect: ~Garbage & ~Clean
  - Dolly
    - Pre: garbage
    - Effect: ~garbage & ~quiet

# GRAPHPLAN Graph



# GRAPHPLAN Graph Solution

