HW3_SR_442

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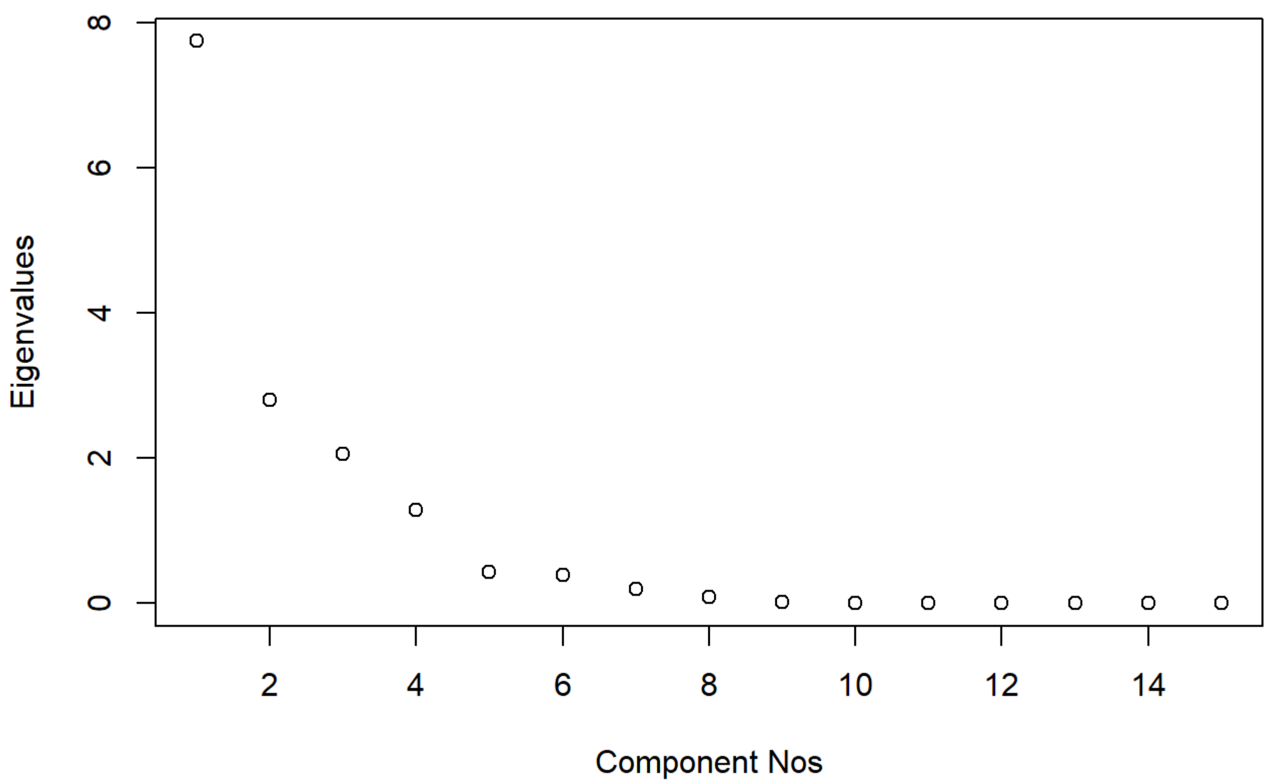
1. Build brand maps for car brands using Cars_Data.xlsx. The client's brand is Infinity.

```
setwd("C:/Users/ranad/Desktop/Winter 2022/442/Homework 3/")
rm(list=ls(all=TRUE))

cars <- read.csv("Cars_Data.csv", header=T)

y <-  cars[,17]
x <-  as.matrix(cars[,2:16])
data <- cbind(y,x)
cor_mat = cor(x)

out1 <-  eigen(cor_mat) # eigen decomposition of correlation matrix
va <-  out1$values          # eigenvalues
ve <-  out1$vectors     # eigenvector
```

2. Determine how many factors to retain?

```
# scree plot
plot(va, ylab = "Eigenvalues", xlab = "Component Nos")
```




```
ego <- va[va > 1]                        # eigenvalues > 1
nn <- nrow(as.matrix(ego))               # number of factors to retain

print(paste("Factors to retain : ", nn))
```

```
## [1] "Factors to retain :  4"
```

3. Assign names to the retained factors (you may need to flip the factors and then assign names)

```
out2 <- ve[,1:nn]                           # eigenvectors associated with the retained factors
out3 <- ifelse(abs(out2) < 0.3, 0, out2)        # ignore small values < 0.3

rownames(out3) <- c("Attractive", "Quiet", "Unreliable", "Poorly.Built", "Interesting", "Sporty", "Uncomfortable", "Roomy",
                    "Easy.Service", "Prestige", "Common", "Economical", "Successful", "AvantGarde", "Poor.Value")

z = x %*% out3          # Component Scores; coordinates of the brands in the map

pref_reg = lm(y ~ z)        # Preference Regression to estimate how benefits drive overall preferences = f(benefits)

summary(pref_reg)
```

```
##
## Call:
## lm(formula = y ~ z)
##
## Residuals:
##        1        2        3        4        5        6        7        8
## -0.41399  0.20488  0.21196  0.17883 -0.17872  0.29476  0.07001 -0.13008
##        9       10
## -0.11977 -0.11788
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  2.18458    0.43161   5.061 0.003895 **
## z1          -0.68944    0.07258  -9.499 0.000219 ***
## z2          -0.28516    0.11920  -2.392 0.062209 .
## z3           0.46138    0.11900   3.877 0.011675 *
## z4           0.21219    0.21267   0.998 0.364205
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.303 on 5 degrees of freedom
## Multiple R-squared:  0.9742, Adjusted R-squared:  0.9536
## F-statistic: 47.21 on 4 and 5 DF,  p-value: 0.0003671
```

```
# Since estimates should be positive, flipping the eigenvectors for Z1 and Z2

out4  <- out3

out4[,1] <- (-1)*out4[,1]
out4[,2] <- (-1)*out4[,2]

z = x %*% out4

pref_reg = lm(y ~ z)

summary(pref_reg)
```

```
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## Call:
## lm(formula = y ~ z)
##
## Residuals:
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```

```
# eliminating Z4 as is it not significant

out4 <- out4[,1:3]

out4
```

```
##                     [,1]       [,2]       [,3]
## Attractive     0.3262814  0.0000000  0.0000000
## Quiet          0.3173818  0.0000000  0.0000000
## Unreliable     0.0000000  0.4011014  0.0000000
## Poorly.Built  -0.3319244  0.0000000  0.0000000
## Interesting    0.0000000  0.0000000 -0.4290683
## Sporty         0.0000000 -0.4288285  0.0000000
## Uncomfortable  0.0000000  0.0000000  0.0000000
## Roomy          0.0000000  0.3789204  0.0000000
## Easy.Service   0.0000000 -0.4279673  0.0000000
## Prestige       0.3322855  0.0000000  0.0000000
## Common         0.0000000  0.0000000  0.0000000
## Economical     0.0000000  0.0000000  0.6642165
## Successful     0.3155394  0.0000000  0.0000000
## AvantGarde     0.0000000  0.0000000  0.0000000
## Poor.Value     0.0000000  0.0000000 -0.4710589
```

Names

Z1 - Premium
Z2 - Unreliable
Z3 - Value for Money

4. Explain iso-preference line and its difference from the regression line

All brands on the iso-preference line have the same preference i.e. all brands satisfy the preference.
In case of regression line, the all brands on the same line won’t have the same outcome. It will depend on y-axis.

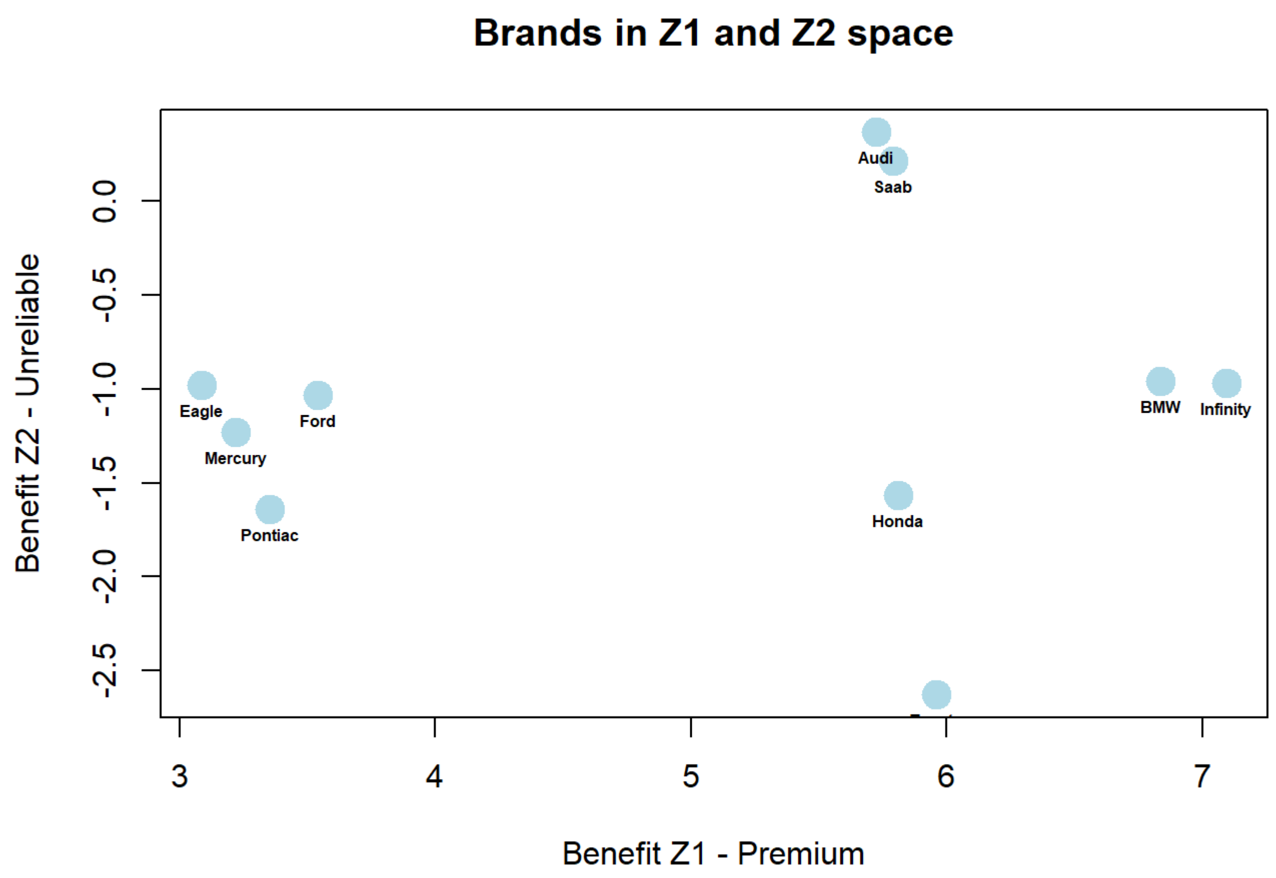
5. Explain what is an ideal vector and why it indicates the direction of increasing preferences?

Ideal vector is the line perpendicular to iso-preference line. It indicates the direction in which the brand should move to differentiate itself from it's nearest competitors.
Assuming that all betas are positive, brands to the right of iso-preference line are more preferred and ideal vector is perpendicular to the line so it indicates the direction of increasing preferences.
6. Compute the angles of iso-preference line and ideal vector arrow

Brand Maps

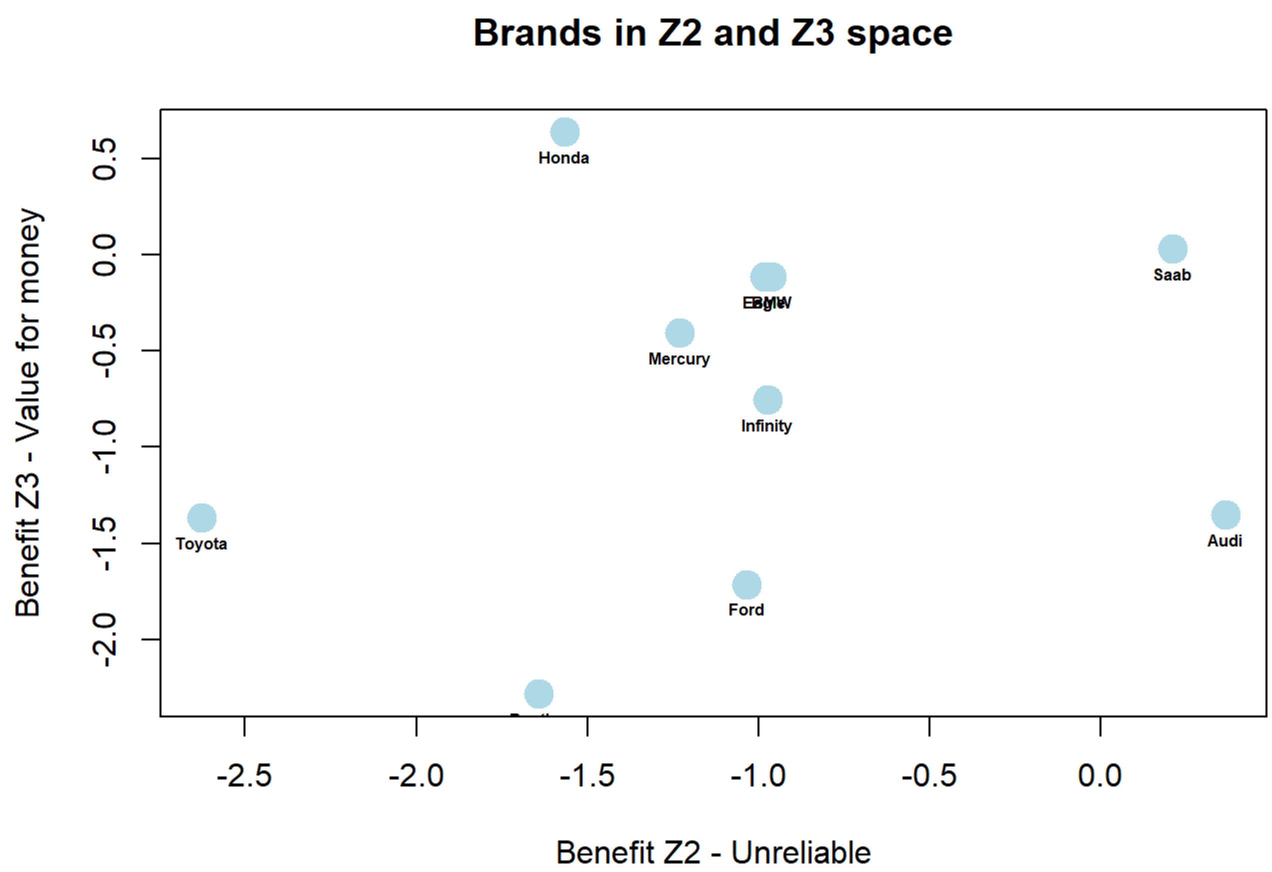
Z1, Z2

```
Z1 = z[,1]
Z2 = z[,2]
z.out = cbind(Z1, Z2)
rownames(z.out) = cars[,1]

plot(Z1, Z2, main = "Brands in Z1 and Z2 space", xlab = "Benefit Z1 - Premium", ylab = "Benefit Z2 - Unreliable", col = "lig
htblue", pch = 19, cex = 2) # Brand Map in Z1-Z2 space
text(z.out, labels = row.names(z.out), font = 2, cex = 0.5, pos = 1)                        # Labeling brands
```



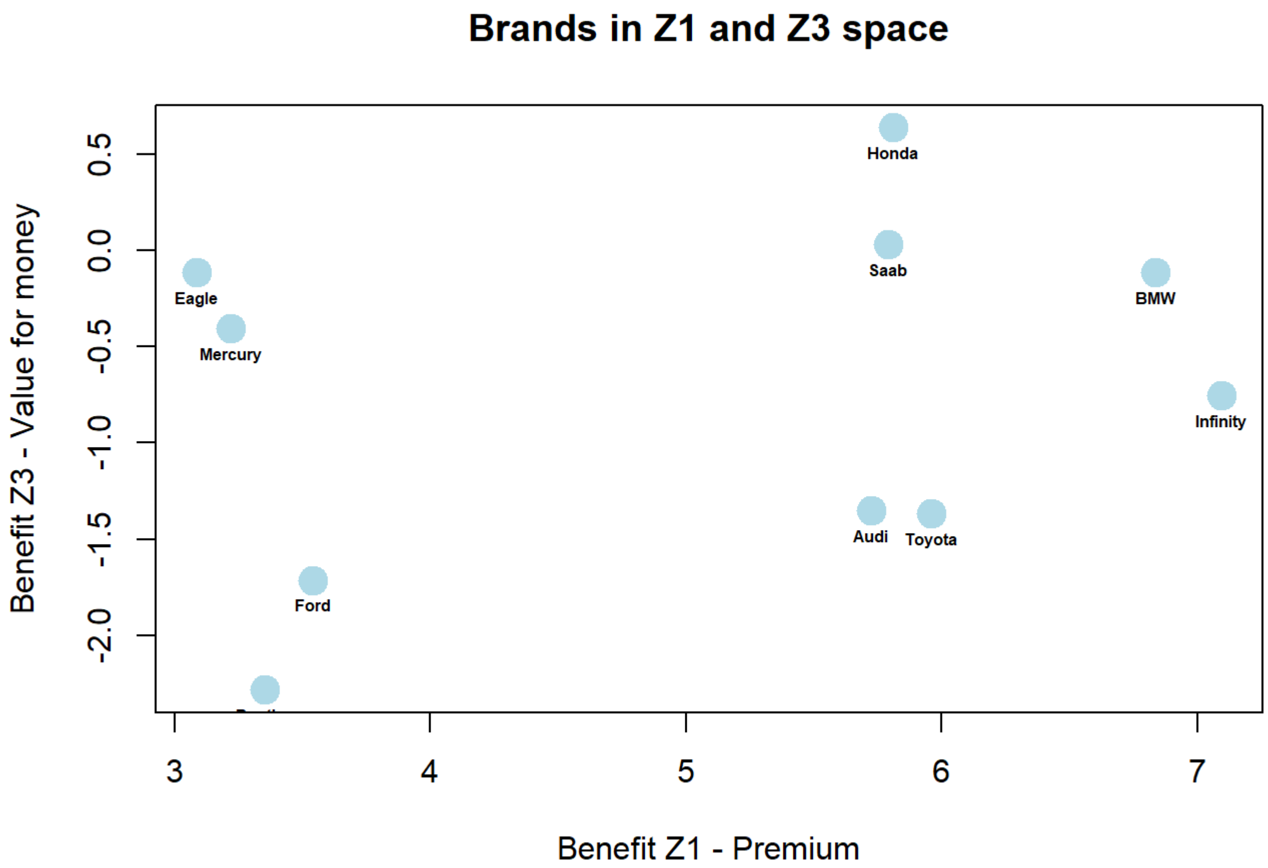
Z2, Z3

```
Z3 = z[,3]
z1.out = cbind(Z2, Z3)
rownames(z1.out) = cars[,1]

plot(Z2, Z3, main = "Brands in Z2 and Z3 space", xlab = "Benefit Z2 - Unreliable", ylab = "Benefit Z3 - Value for money", co
l = "lightblue", pch = 19, cex = 2) # Brand Map in Z2-Z3 space
text(z1.out, labels = row.names(z1.out), font = 2, cex = 0.5, pos = 1)                    # Labeling brands
```



Z1, Z3

```
z2.out = cbind(Z1, Z3)
rownames(z2.out) = cars[,1]

plot(Z1, Z3, main = "Brands in Z1 and Z3 space", xlab = "Benefit Z1 - Premium", ylab = "Benefit Z3 - Value for money", col =
"lightblue", pch = 19, cex = 2) # Brand Map in Z1-Z2 space
text(z2.out, labels = row.names(z2.out), font = 2, cex = 0.5, pos = 1)                    # labeling brands
```




```
# Slopes of iso-preference and ideal vector
```

For Z1 and Z2

```
b1 = as.vector(coef(pref_reg)[2])
b2 = as.vector(coef(pref_reg)[3])
slope.iso.preference = - b1/b2
slope.ideal.vector = b2/b1

# Angles of iso-preference and ideal vector

angle.iso.preference = atan(slope.iso.preference)*180/pi
angle.ideal.vector = atan(slope.ideal.vector)*180/pi

print(paste("angle.iso.preference : ", angle.iso.preference))
```

```
## [1] "angle.iso.preference :  -67.5297112315307"
```

```
print(paste("angle.ideal.vector : ", angle.ideal.vector))
```

```
## [1] "angle.ideal.vector :  22.4702887684693"
```

For Z2 and Z3

```
b3 = as.vector(coef(pref_reg)[4])
slope.iso.preference = - b2/b3
slope.ideal.vector = b3/b2

# Angles of iso-preference and ideal vector

angle.iso.preference = atan(slope.iso.preference)*180/pi
angle.ideal.vector = atan(slope.ideal.vector)*180/pi

print(paste("angle.iso.preference : ", angle.iso.preference))
```

```
## [1] "angle.iso.preference :  -31.7182997006912"
```

```
print(paste("angle.ideal.vector : ", angle.ideal.vector))
```

```
## [1] "angle.ideal.vector :  58.2817002993088"
```

For Z1 and Z3

```
slope.iso.preference = - b1/b3
slope.ideal.vector = b3/b1

# Angles of iso-preference and ideal vector

angle.iso.preference = atan(slope.iso.preference)*180/pi
angle.ideal.vector = atan(slope.ideal.vector)*180/pi

print(paste("angle.iso.preference : ", angle.iso.preference))
```

```
## [1] "angle.iso.preference :  -56.2092740146157"
```

```
print(paste("angle.ideal.vector : ", angle.ideal.vector))
```

```
## [1] "angle.ideal.vector :  33.7907259853843"
```

7. Find 95% confidence interval for the angle of the ideal vector. Use data bootstrap method we learnt in Class 3.

```
angle <- list()
r <- nrow(data)

set.seed(876)

# Do Data Bootstrap 1000 times to get 95% CI for R^2
for(i in 1:1000) {
    tryCatch({

              data.star <- data[sample(r, r, replace = T),]     # create (y*, x*) by resampling rows in original data matrix
            ystar <- data.star[,1]
            xstar <- data.star[,2:16]
            cor_mat = cor(xstar)
            o1 <-  eigen(cor_mat)   # eigen decomposition of correlation matrix
            va <-  o1$values            # eigenvalues
            ve <-  o1$vectors
            ego <- va[va > 1]                          # eigenvalues > 1
            n <- nrow(as.matrix(ego))
            o2 <- ve[,1:n]                         # eigenvectors associated with the retained factors
            o3 <- ifelse(abs(o2) < 0.3, 0, o2)      # ignore small values < 0.3
            rownames(o3) <- c("Attractive", "Quiet", "Unreliable", "Poorly.Built", "Interesting", "Sporty", "Uncomfortable", 
"Roomy",
                                "Easy.Service", "Prestige", "Common", "Economical", "Successful", "AvantGarde", "Poor.Value"
)
            zstar = xstar %*% o3            # Component Scores; coordinates of the brands in the map
            pref_reg_star = lm(ystar ~ zstar)       # Preference Regression to estimate how benefits drive overall preferenc
es = f(benefits)

            # flipping o3 if coefficients are negative
            for (j in 2:length(coef(pref_reg_star))){
              if(coef(pref_reg_star)[j] < 0){
                o3[,j-1] = (-1)*o3[,j-1]
              }
            }
            # new model
            zstar = xstar %*% o3
            pref_reg_star = lm(ystar ~ zstar)

            # finding significant coefficients
```

```
            cf <- data.frame(summary(pref_reg_star)$coef) # collecting coefficients
            cf <- cf[-1,] # removing intercept
            a <- cf[cf[,4] <= .05, 1] # taking significant coefficients only

            # making sure that there are more than 1 significant variable
            if(length(a) > 1){
              b <- t(combn(a, 2)) # creating pairs
                c <- c()

                # calculating angle
                for (k in 1:nrow(b)) {
                   c <- c(c, atan(b[k,2]/b[k,1])*180/pi)
                }

                # saving the output
                angle[[i]] <- c
            }
      }, error=function(e){})
}
```

```
## Warning in summary.lm(pref_reg_star): essentially perfect fit: summary may be
## unreliable

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```

```
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## unreliable
```

```
# removing nulls and converting it to data frame

df <- plyr::ldply(angle, rbind)

df <- df[,colSums(is.na(df))<nrow(df)]
```

printing the confidence intervals for each pair of signficant betas.

```
for (i in 1:ncol(df)){
  print(paste("For pair : ", i))
  print(quantile(df[,i], probs = c(0.025, 0.975), na.rm = TRUE))
  print(paste("mean : ", mean(df[,i], na.rm = TRUE)))
}
```

```
## [1] "For pair :  1"
##     2.5%    97.5% 
##  4.45337 50.68285 
## [1] "mean :  25.5694442549705"
## [1] "For pair :  2"
##      2.5%     97.5% 
##  3.921029 54.509593 
## [1] "mean :  27.981349621021"
## [1] "For pair :  3"
##      2.5%     97.5% 
##  7.146867 75.734755 
## [1] "mean :  46.0685047089444"
## [1] "For pair :  4"
##      2.5%     97.5% 
##  5.413237 83.393219 
## [1] "mean :  45.7872411258584"
## [1] "For pair :  5"
##      2.5%     97.5% 
##  4.482864 83.213689 
## [1] "mean :  44.1959489869967"
## [1] "For pair :  6"
##      2.5%     97.5% 
##  4.095385 81.598240 
## [1] "mean :  43.723438541603"
```

8. Recommend to Infinity's managers what they should do to improve their product design
We got three brand maps -
1. Premium vs unreliable - In this brand map, BMW is infinity's close competitor - with ideal vector at 22 degrees which leans towards premium feature.
Thus, infinity should work on premium cars which are attractive and well built.
2. unreliable vs value for money - In this brand map, Mercury, BMW, Eagle are infinity's close competitor - with ideal vector at 58 degrees which leans towards value for money feature.
Thus infinity should focus on economical and good value cars.
3. Premium vs value for money - In this brand map, BMW is infinity's close competitor - with ideal vector at 33 degrees which leans towards premium feature.
Thus, infinity should work on premium cars which are attractive and well built.