

Brand Maps & PCA

1. Build brand maps for car brands for the client's brand - Infinity.

```
rm(list=ls(all=TRUE))

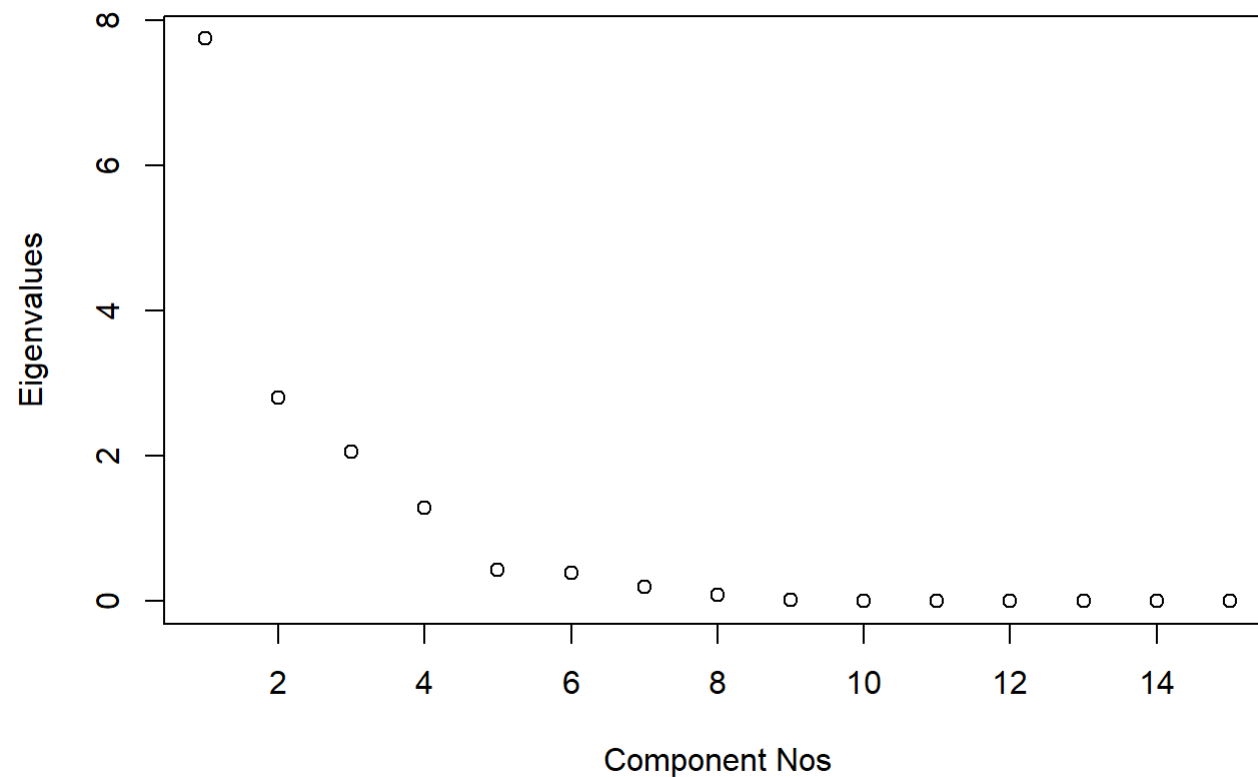
cars <- read.csv("Cars_Data.csv", header=T)

y <- cars[,17]
x <- as.matrix(cars[,2:16])
data <- cbind(y,x)
cor_mat = cor(x)

out1 <- eigen(cor_mat) # eigen decomposition of correlation matrix
va <- out1$values      # eigenvalues
ve <- out1$vectors     # eigenvector
```

2. Determine how many factors to retain?

```
# scree plot
plot(va, ylab = "Eigenvalues", xlab = "Component Nos")
```



```
ego <- va[va > 1]                # eigenvalues > 1
nn <- nrow(as.matrix(ego))        # number of factors to retain

print(paste("Factors to retain : ", nn))
```

```
## [1] "Factors to retain : 4"
```

3. Assign names to the retained factors

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```

out2 <- ve[,1:nn]                # eigenvectors associated with the retained factors
out3 <- ifelse(abs(out2) < 0.3, 0, out2)    # ignore small values < 0.3

rownames(out3) <- c("Attractive", "Quiet", "Unreliable", "Poorly.Built", "Interesting", "Sporty", "Uncomfortable", "Roomy",
                    "Easy.Service", "Prestige", "Common", "Economical", "Successful", "AvantGarde", "Poor.Value")

z = x %*% out3                    # Component Scores; coordinates of the brands in the map

pref_reg = lm(y ~ z)              # Preference Regression to estimate how benefits drive overall preferences = f(benefits)

summary(pref_reg)

```

```

##
## Call:
## lm(formula = y ~ z)
##
## Residuals:
##      1      2      3      4      5      6      7      8
## -0.41399  0.20488  0.21196  0.17883 -0.17872  0.29476  0.07001 -0.13008
##      9     10
## -0.11977 -0.11788
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   2.18458    0.43161   5.061 0.003895 **
## z1            -0.68944    0.07258  -9.499 0.000219 ***
## z2            -0.28516    0.11920  -2.392 0.062209 .
## z3             0.46138    0.11900   3.877 0.011675 *
## z4             0.21219    0.21267   0.998 0.364205
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.303 on 5 degrees of freedom
## Multiple R-squared:  0.9742, Adjusted R-squared:  0.9536
## F-statistic: 47.21 on 4 and 5 DF,  p-value: 0.0003671

```

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```
# Since estimates should be positive, flipping the eigenvectors for Z1 and Z2
```

```
out4 <- out3
```

```
out4[,1] <- (-1)*out4[,1]
```

```
out4[,2] <- (-1)*out4[,2]
```

```
z = x %*% out4
```

```
pref_reg = lm(y ~ z)
```

```
summary(pref_reg)
```

```
##
## Call:
## lm(formula = y ~ z)
##
## Residuals:
```

	1	2	3	4	5	6	7	8
##	-0.41399	0.20488	0.21196	0.17883	-0.17872	0.29476	0.07001	-0.13008

```
##      9      10
## -0.11977 -0.11788
##
## Coefficients:
```

	Estimate	Std. Error	t value	Pr(> t)
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## z3	0.46138	0.11900	3.877	0.011675 *
## z4	0.21219	0.21267	0.998	0.364205

```
## ---
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```

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```
# eliminating Z4 as is it not significant
```

```
out4 <- out4[,1:3]
```

```
out4
```

```
##           [,1]      [,2]      [,3]
## Attractive  0.3262814  0.0000000  0.0000000
## Quiet       0.3173818  0.0000000  0.0000000
## Unreliable  0.0000000  0.4011014  0.0000000
## Poorly.Built -0.3319244  0.0000000  0.0000000
## Interesting 0.0000000  0.0000000 -0.4290683
## Sporty      0.0000000 -0.4288285  0.0000000
## Uncomfortable 0.0000000  0.0000000  0.0000000
## Roomy       0.0000000  0.3789204  0.0000000
## Easy.Service 0.0000000 -0.4279673  0.0000000
## Prestige    0.3322855  0.0000000  0.0000000
## Common      0.0000000  0.0000000  0.0000000
## Economical  0.0000000  0.0000000  0.6642165
## Successful  0.3155394  0.0000000  0.0000000
## AvantGarde  0.0000000  0.0000000  0.0000000
## Poor.Value  0.0000000  0.0000000 -0.4710589
```

```
# Names
```

```
Z1 - Premium
```

```
Z2 - Unreliable
```

```
Z3 - Value for Money
```

4. Compute the angles of iso-preference line and ideal vector arrow

```
# Brand Maps
```

```
# Z1, Z2
```

```
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```

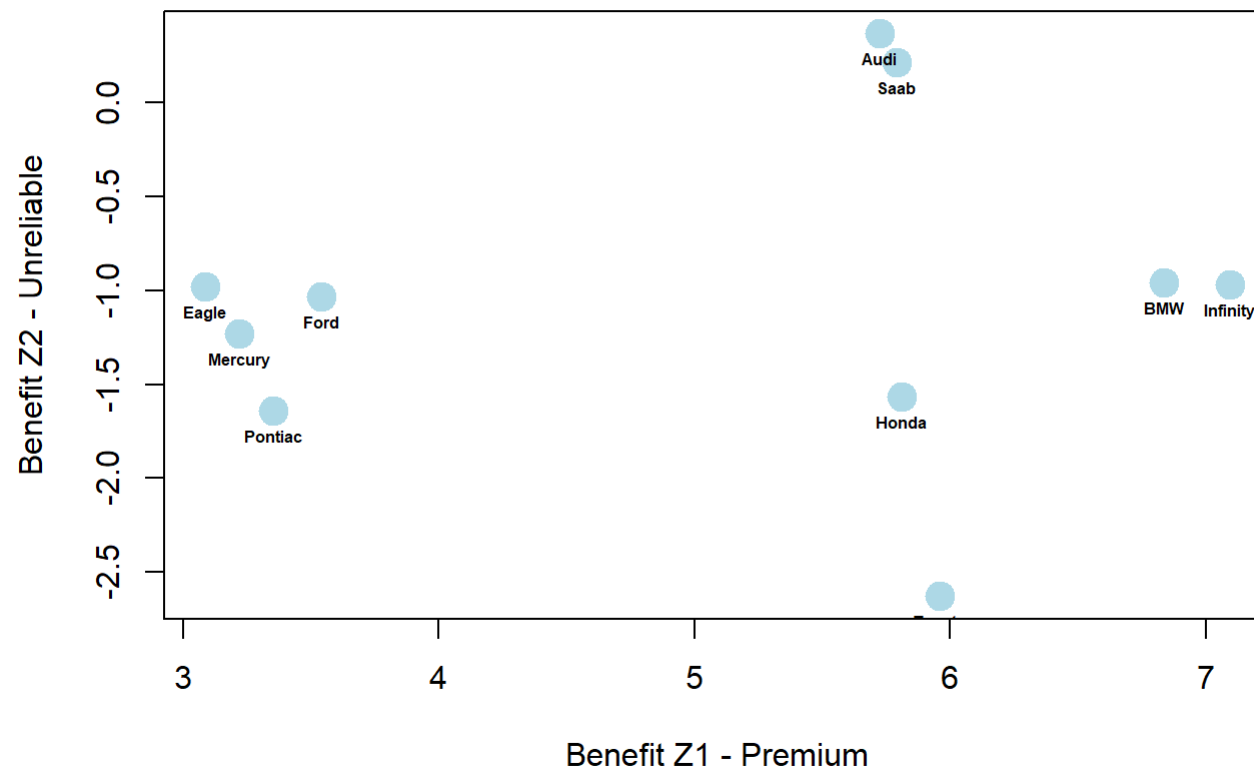
```

Z1 = z[,1]
Z2 = z[,2]
z.out = cbind(Z1, Z2)
rownames(z.out) = cars[,1]

plot(Z1, Z2, main = "Brands in Z1 and Z2 space", xlab = "Benefit Z1 - Premium", ylab = "Benefit Z2 - Unreliable", col = "lightblue", pch = 19, cex = 2) # Brand Map in Z1-Z2 space
text(z.out, labels = row.names(z.out), font = 2, cex = 0.5, pos = 1) # Labeling brands

```

Brands in Z1 and Z2 space



Z2, Z3

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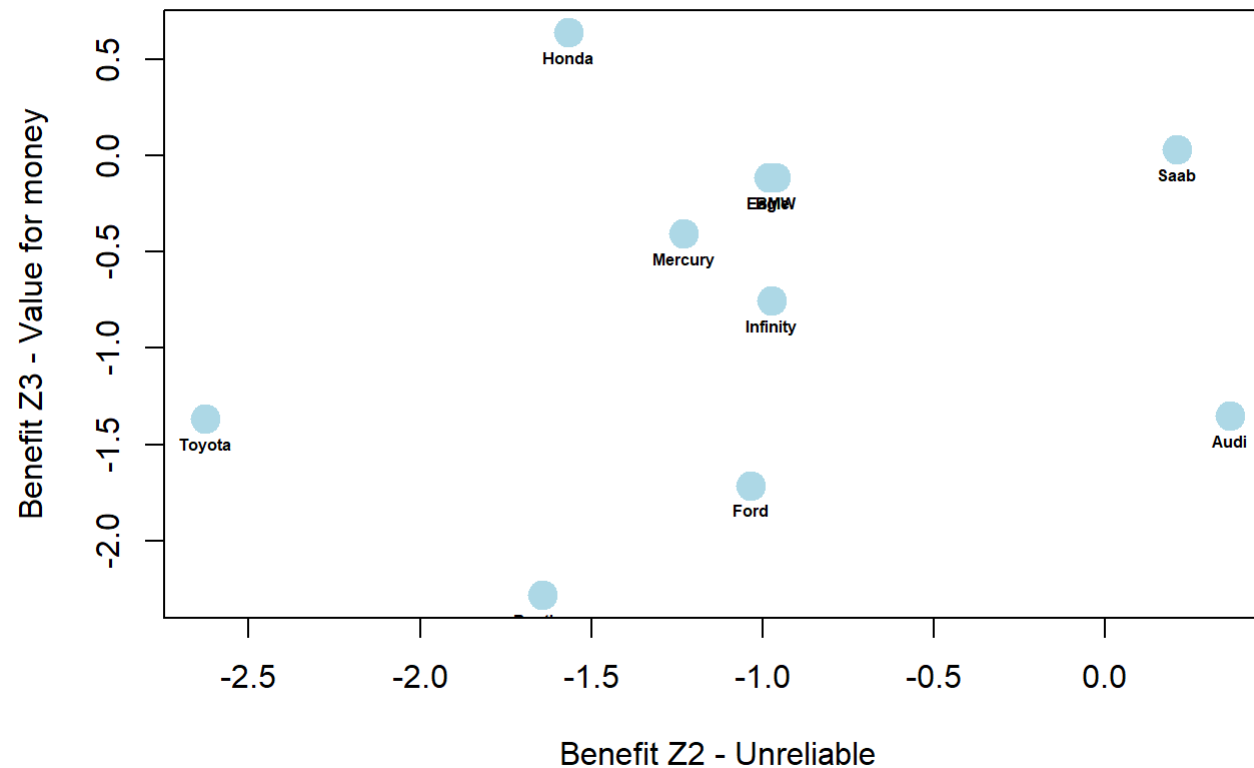
```

Z3 = z[,3]
z1.out = cbind(Z2, Z3)
rownames(z1.out) = cars[,1]

plot(Z2, Z3, main = "Brands in Z2 and Z3 space", xlab = "Benefit Z2 - Unreliable", ylab = "Benefit Z3 - Value for money", col = "lightblue", pch = 19, cex = 2) # Brand Map in Z2-Z3 space
text(z1.out, labels = row.names(z1.out), font = 2, cex = 0.5, pos = 1) # Labeling brands

```

Brands in Z2 and Z3 space



Z1, Z3

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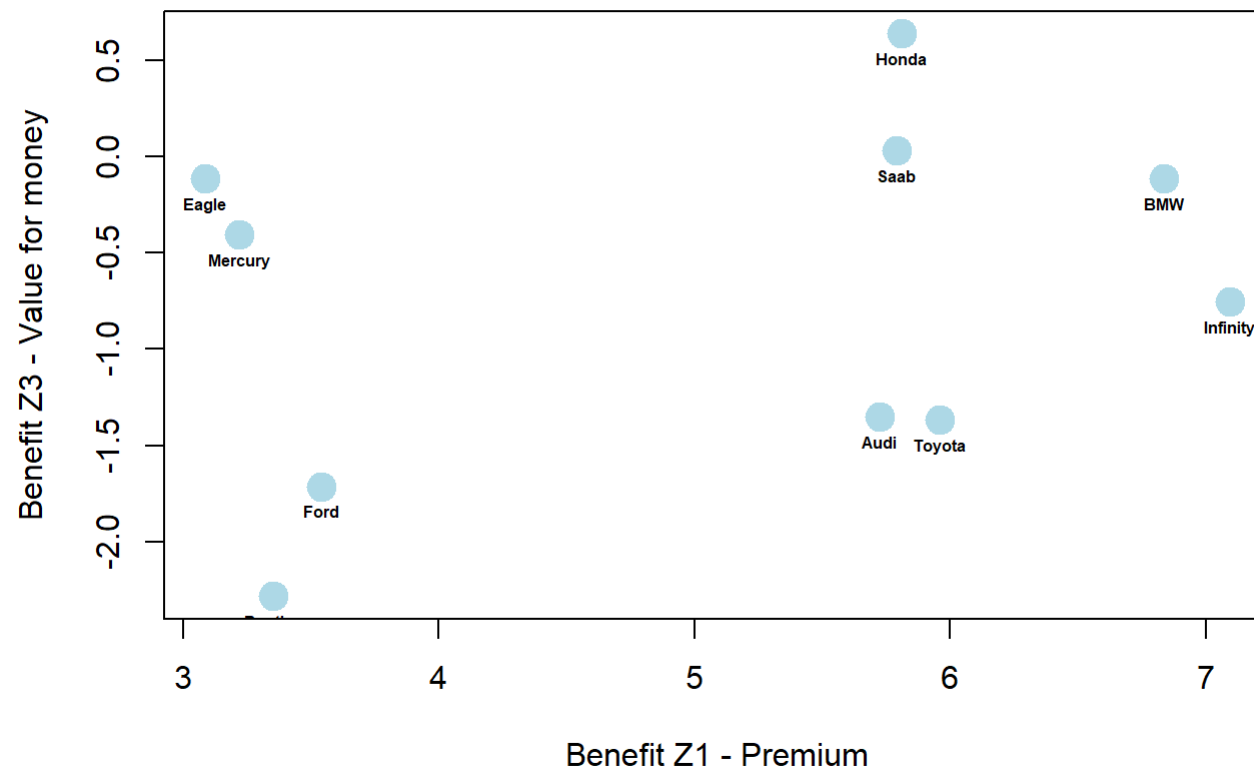
```

z2.out = cbind(Z1, Z3)
rownames(z2.out) = cars[,1]

plot(Z1, Z3, main = "Brands in Z1 and Z3 space", xlab = "Benefit Z1 - Premium", ylab = "Benefit Z3 - Value for money", col =
"lightblue", pch = 19, cex = 2) # Brand Map in Z1-Z2 space
text(z2.out, labels = row.names(z2.out), font = 2, cex = 0.5, pos = 1) # Labeling brands

```

Brands in Z1 and Z3 space



Slopes of iso-preference and ideal vector


```

b1 = as.vector(coef(pref_reg)[2])
b2 = as.vector(coef(pref_reg)[3])
slope.iso.preference = - b1/b2
slope.ideal.vector = b2/b1

# Angles of iso-preference and ideal vector

angle.iso.preference = atan(slope.iso.preference)*180/pi
angle.ideal.vector = atan(slope.ideal.vector)*180/pi

print(paste("angle.iso.preference : ", angle.iso.preference))

```

```
## [1] "angle.iso.preference : -67.5297112315307"
```

```
print(paste("angle.ideal.vector : ", angle.ideal.vector))
```

```
## [1] "angle.ideal.vector : 22.4702887684693"
```

For Z2 and Z3

```

b3 = as.vector(coef(pref_reg)[4])
slope.iso.preference = - b2/b3
slope.ideal.vector = b3/b2

# Angles of iso-preference and ideal vector

angle.iso.preference = atan(slope.iso.preference)*180/pi
angle.ideal.vector = atan(slope.ideal.vector)*180/pi

print(paste("angle.iso.preference : ", angle.iso.preference))

```

```
## [1] "angle.iso.preference : -31.7182997006912"
```

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```
print(paste("angle.ideal.vector : ", angle.ideal.vector))
```

```
## [1] "angle.ideal.vector : 58.2817002993088"
```

For Z1 and Z3

```
slope.iso.preference = - b1/b3  
slope.ideal.vector = b3/b1  
  
# Angles of iso-preference and ideal vector  
  
angle.iso.preference = atan(slope.iso.preference)*180/pi  
angle.ideal.vector = atan(slope.ideal.vector)*180/pi  
  
print(paste("angle.iso.preference : ", angle.iso.preference))
```

```
## [1] "angle.iso.preference : -56.2092740146157"
```

```
print(paste("angle.ideal.vector : ", angle.ideal.vector))
```

```
## [1] "angle.ideal.vector : 33.7907259853843"
```

5. Find 95% confidence interval for the angle of the ideal vector using data bootstrap method.

```

angle <- list()
r <- nrow(data)

set.seed(876)

# Do Data Bootstrap 1000 times to get 95% CI for R^2
for(i in 1:1000) {
  tryCatch({

    data.star <- data[sample(r, r, replace = T),]    # create (y*, x*) by resampling rows in original data matrix
    ystar <- data.star[,1]
    xstar <- data.star[,2:16]
    cor_mat = cor(xstar)
    o1 <- eigen(cor_mat)    # eigen decomposition of correlation matrix
    va <- o1$values          # eigenvalues
    ve <- o1$vectors
    ego <- va[va > 1]          # eigenvalues > 1
    n <- nrow(as.matrix(ego))
    o2 <- ve[,1:n]            # eigenvectors associated with the retained factors
    o3 <- ifelse(abs(o2) < 0.3, 0, o2)    # ignore small values < 0.3
    rownames(o3) <- c("Attractive", "Quiet", "Unreliable", "Poorly.Built", "Interesting", "Sporty", "Uncomfortable",
"Roomy",
                        "Easy.Service", "Prestige", "Common", "Economical", "Successful", "AvantGarde", "Poor.Value"
)

    zstar = xstar %*% o3          # Component Scores; coordinates of the brands in the map
    pref_reg_star = lm(ystar ~ zstar)    # Preference Regression to estimate how benefits drive overall preferenc
es = f(benefits)

    # flipping o3 if coefficients are negative
    for (j in 2:length(coef(pref_reg_star))){
      if(coef(pref_reg_star)[j] < 0){
        o3[,j-1] = (-1)*o3[,j-1]
      }
    }
    # new model
    zstar = xstar %*% o3
    pref_reg_star = lm(ystar ~ zstar)

    # finding significant coefficients

```

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```
cf <- data.frame(summary(pref_reg_star)$coef) # collecting coefficients
cf <- cf[-1,] # removing intercept
a <- cf[cf[,4] <= .05, 1] # taking significant coefficients only

# making sure that there are more than 1 significant variable
if(length(a) > 1){
  b <- t(combn(a, 2)) # creating pairs
  c <- c()

  # calculating angle
  for (k in 1:nrow(b)) {
    c <- c(c, atan(b[k,2]/b[k,1])*180/pi)
  }

  # saving the output
  angle[[i]] <- c
}
}, error=function(e){})
}
```

```
## Warning in summary.lm(pref_reg_star): essentially perfect fit: summary may be
## unreliable

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```

```
# removing nulls and converting it to data frame
```

```
df <- plyr::ldply(angle, rbind)
```

```
df <- df[,colSums(is.na(df))<nrow(df)]
```

```
# printing the confidence intervals for each pair of significant betas.
```

```
for (i in 1:ncol(df)){
  print(paste("For pair : ", i))
  print(quantile(df[,i], probs = c(0.025, 0.975), na.rm = TRUE))
  print(paste("mean : ", mean(df[,i], na.rm = TRUE)))
}
```

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```

## [1] "For pair : 1"
##      2.5%      97.5%
##  4.45337 50.68285
## [1] "mean : 25.5694442549705"
## [1] "For pair : 2"
##      2.5%      97.5%
##  3.921029 54.509593
## [1] "mean : 27.981349621021"
## [1] "For pair : 3"
##      2.5%      97.5%
##  7.146867 75.734755
## [1] "mean : 46.0685047089444"
## [1] "For pair : 4"
##      2.5%      97.5%
##  5.413237 83.393219
## [1] "mean : 45.7872411258584"
## [1] "For pair : 5"
##      2.5%      97.5%
##  4.482864 83.213689
## [1] "mean : 44.1959489869967"
## [1] "For pair : 6"
##      2.5%      97.5%
##  4.095385 81.598240
## [1] "mean : 43.723438541603"

```

6. Recommend to Infinity's managers what they should do to improve their product design

We got three brand maps -

1. Premium vs unreliable - In this brand map, BMW is infinity's close competitor - with ideal vector at 22 degrees which leans towards premium feature.

Thus, infinity should work on premium cars which are attractive and well built.

2. unreliable vs value for money - In this brand map, Mercury, BMW, Eagle are infinity's close competitor - with ideal vector at 58 degrees which leans towards value for money feature.

Thus infinity should focus on economical and good value cars.

3. Premium vs value for money - In this brand map, BMW is infinity's close competitor - with ideal vector at 33 degrees which leans towards premium feature.

Thus, infinity should work on premium cars which are attractive and well built.

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