though they are inefficient in terms of cost of production. Finally, survival technique does not permit us to measure the degree of economies or diseconomies of scale.

NUMERICAL PROBLEMS ON COST FUNCTIONS

Problem 1. *Suppose a firm faces a cost function of $C = 8 + 4q + q^2$*

(i) *What is the firm's fixed cost ?*

(ii) *Derive an expression for the firm's average variable cost and marginal cost.*

Solution. (i) As fixed cost of the firm does not vary with output, the term in the given cost function which has no output (q) term will be the fixed cost. From the given cost function it is evident that fixed cost is 8.

(ii) Total variable cost (TVC) $= TC - TFC$

$$= (8 + 4q + q^2) - 8 = 4q + q^2$$

and

$$AVC = \frac{TVC}{q} = \frac{4q + q^2}{q} = 4 + q$$

Marginal cost is the first derivative of total cost function or total variable cost function.

Thus, $$MC = \frac{\Delta TVC}{\Delta q} = 4 + 2q$$

Problem 2. *A biscuit producing company has the following variable cost function:*

$$TVC = 200Q + 9Q^2 + 0.25Q^3$$

If the company's fixed costs are equal to Rs. 150 lakhs find out :

(a) *total cost function*

(b) *marginal cost function*

(c) *average variable cost function*

(d) *average total cost function*

(e) *at what output levels average variable cost and marginal cost will be minimum.*

Solution. Since total cost is the sum of total fixed cost and total variable cost ($TC = TFC + TVC$), we get the total cost function as under:

$$TC = 150 + 200Q - 9Q^2 + 0.25Q^3$$

To determine the marginal cost we take the first derivative of the total variable cost function with respect to output Q. Thus,

$$MC = \frac{d\,(TC)}{dQ} = 200 - 18Q + 0.75Q^2$$

To derive the average total cost and average variable cost we divide the respective total costs by the output level.

$$AC = \frac{TC}{Q} = \frac{150}{Q} + \frac{200Q}{Q} - \frac{9Q^2}{Q} + \frac{0.25Q^3}{Q}$$

$$= \frac{TC}{Q} = \frac{150}{Q} + 200 - 9Q + 0.25Q^2$$

and

$$AVC = \frac{TVC}{Q} = 200 - 9Q + 0.25Q^2$$

It is also useful to know at what level of output, average variable cost takes on its minimum value. To determine the level of output at which average variable cost is minimum, we have to take first derivative of the average variable cost (AVC) function and set this derivative equal to zero.

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Thus, taking the first derivative of AVC function ($AVC = 200 - 9Q + 0.25Q^2$), we have :

$$\frac{d(AVC)}{dQ} = -9 + 0.50Q$$

Setting it equal to zero we have

$$-9 + 0.50Q = 0$$
$$0.50Q = 9$$
$$\frac{1}{2}Q = 9$$
$$Q = 18$$

Thus, at output level equal to 18, average variable cost will be minimum.

Output at which MC Function is Minimum

$$MC = 200 - 18Q + 0.75Q^2$$

To find the output level at which MC is minimum, we have to set the first derivative of MC function equal to zero. The first derivative of MC function is

$$\frac{d(MC)}{dQ} = -18 + 1.50\,Q$$

Setting $\frac{d(MC)}{dQ}$ equal to zero, we have :

$$-18 + 1.50\,Q = 0$$
$$1.50Q = 18$$
$$Q = -18 \times \frac{10}{15} = 12$$

Thus, at output level 12, MC is minimum.

It is thus clear from above that marginal cost takes on the minimum value at an output level smaller than that at which AVC is minimum.

Problem 3. *A firm producing hockey sticks has a production function given by $Q = 2\sqrt{KL}$. In the short run, the firm's amount of capital equipment is fixed at $K = 100$. The rental rate for K is Re. 1 and the wage rate is Rs. 4.*

(i) *Calculate the firm's short run total and average costs.*

(ii) *What are STC, SAC and SMC for producing 25 sticks.*

Solution. The given production function of the firm is

$$Q = 2\sqrt{KL}$$

With $K = 100$ in the short run, the short-run production function is

$$Q = 2\sqrt{100L} = 2 \times 10\sqrt{L} = 20\sqrt{L}$$

Cost,

$$C = \omega L + rK$$

Given that

$$w = 4 \text{ and } r = 1$$
$$C = 4L + 1K$$

With the given

$$K = 100$$
$$C = 4L + 100 \quad \ldots(1)$$

The short-run production function when $K = 100$ as obtained above is:

$$Q = 20\sqrt{L}$$

Taking square of both sides we have

$$Q^2 = 400\,L$$

or

$$\frac{Q^2}{400} = L. \quad \ldots(2)$$

Substituting (2) in (1) we have

$$C = 100 + 4.\frac{Q^2}{400}$$

$$C = 100 + \frac{Q^2}{100} \quad \ldots(3)$$

The above equation (3) represents the short-run total cost function.

To get the short-run average cost function we divide the short-run total cost function in (3) by output (Q). Thus,

$$SAC = \frac{100 + \frac{Q^2}{100}}{Q} = \frac{100}{Q} + \frac{Q}{100}$$

Short-run Marginal Cost Function :

Short-run marginal cost function can be obtained by taking the first derivative of the short-run total cost function.

Short-run total cost function as found above is

$$C = 100 + \frac{Q^2}{100}$$

$$SMC = \frac{dC}{dQ} = \frac{2Q}{100} = \frac{Q}{50}$$

(*ii*) If output of hockey sticks = 25, then

$$STC = 100 + \frac{(25)^2}{100} = 100 + \frac{625}{100}$$

$$STC = 106.25$$

$$SAC = \frac{STC}{Q} = \frac{106.25}{25} = 4.25$$

$$SMC = \frac{Q}{50} = \frac{25}{50} = 0.5$$

Problem 4. *If $Q = A(KL)^{0.5}$, what is short-run cost function when $K = 100$? What is MC function ?*

Solution. With $K = 100$, the short-run production function can be written as

$$Q = A\,(100\,L)^{0.5} = 10A\,(L)^{0.5}$$

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Squaring both sides we have

$$Q^2 = 100A^2L$$

Now, the short-run cost function is

$$C = TFC + TVC \quad ...(i)$$

$$TFC = Kr = 100r \text{ and } TVC = wL$$

where r is the rental price of capital and w is wage rate of labour and given $K = 100$

Therefore

$$C = 100r + wL \quad ...(ii)$$

From equation (*i*) we have

$$L = \frac{Q^2}{100A^2}$$

Substituting the value of L in (*ii*) we get the following short-run cost function :

$$C = 100r + w.\frac{Q^2}{100A^2} \quad ...(iii)$$

Note that total variable cost function is $w\frac{Q^2}{100A^2}$

Differentiating the total variable cost (*TVC*) function with respect to output (*Q*) we have the following marginal cost function :

$$MC = \frac{dTVC}{dQ} = \frac{2wQ}{100A^2}$$

QUESTIONS FOR REVIEW

1. Distinguish between economic costs and accounting costs. Which should be taken into account for calculating the economic profits of the firm?
2. What is the difference between explicit costs and implicit costs? Should both be considered for optimal business decision-making by the firm?
3. Explain the concepts of total fixed cost, total variable costs and total costs. How are they related to each other? Illustrate them through curves. Is the distinction between the fixed costs and variable costs relevant in the long run ?
4. Explain the following concepts of cost:
   (a) Average fixed cost (*AFC*)
   (b) Average variable cost (*AVC*)
   (c) Average total cost (*ATC*)
   (d) Marginal cost (*MC*)
   Why does *ATC* curve reach its lowest point after the *AVC* curve? Why does the *MC* curve inte...

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