DIFFERENTIAL EVOLUTION (DE) : METAHEURISTIC OPTIMIZATION ALGORITHM

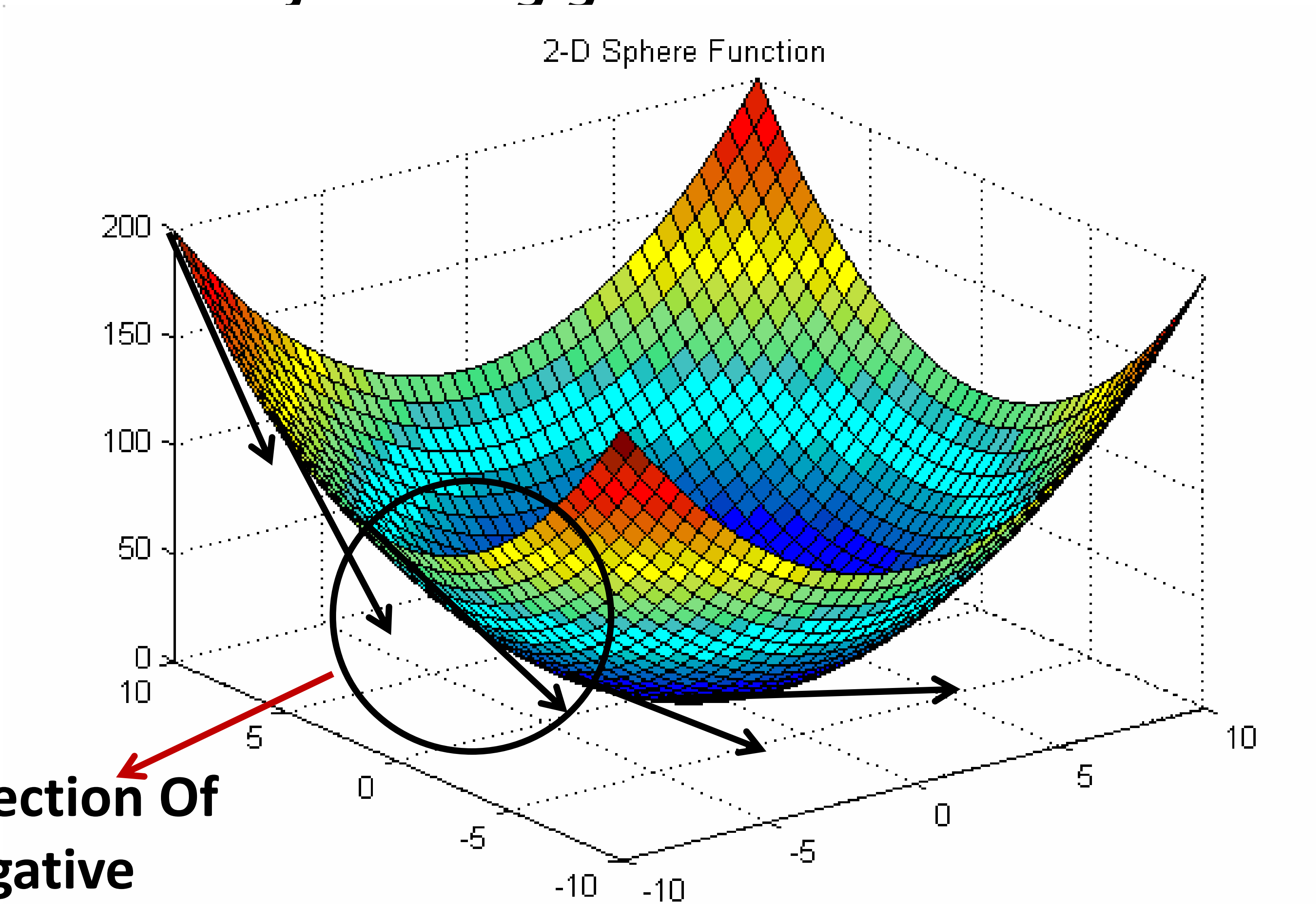
Meta-heuristics

- **A metaheuristic is a heuristic method for solving a very general class of computational problems.**

- **Usually heuristics themselves are used to solve computational problems in the hope of obtaining a more efficient or more robust procedures.**

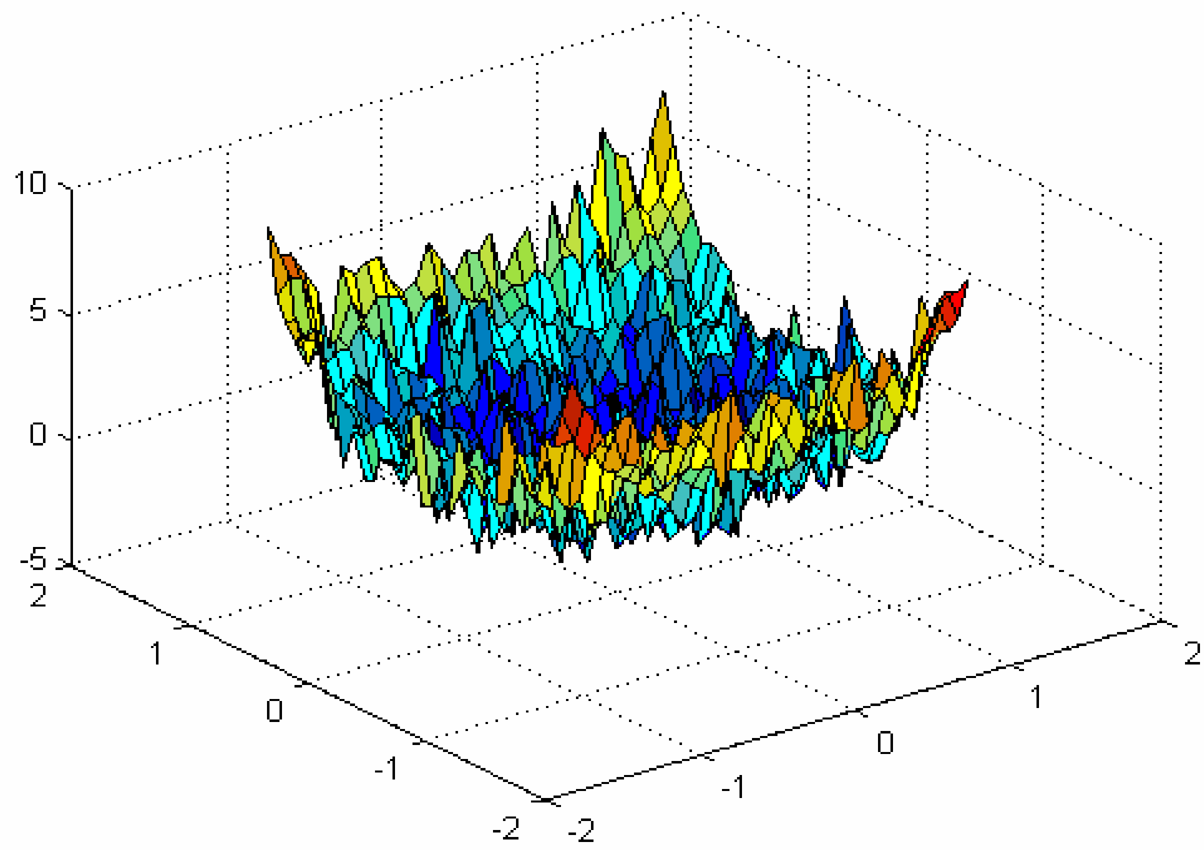
- **The name combines the Greek prefix "meta“ ("beyond", here in the sense of "higher level") and "heuristic"("to find").**

How a single agent can find global optima by following gradient descent?



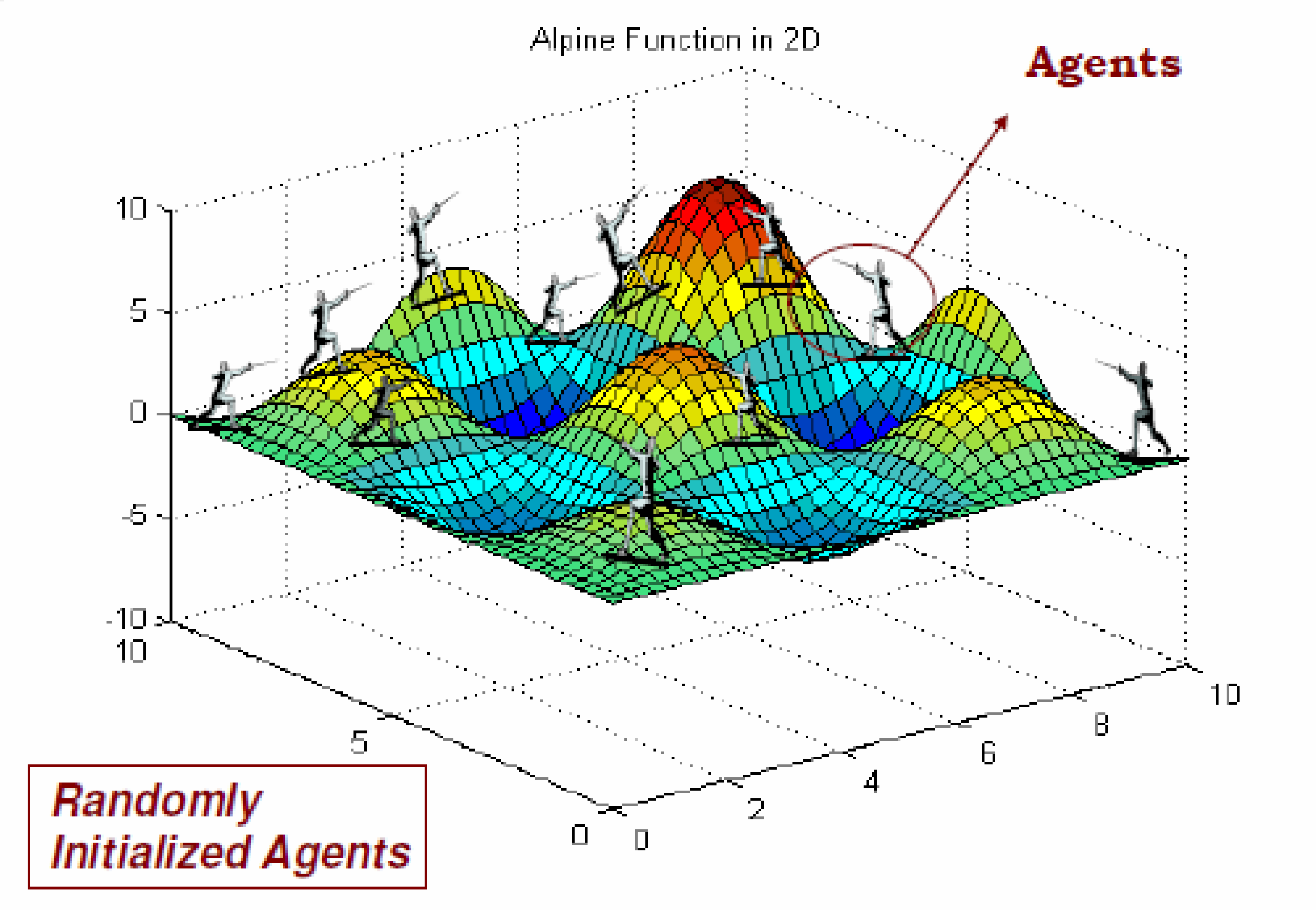
Direction Of Negative gradient

But What about these multi-modal, noisy and even discontinuous functions?

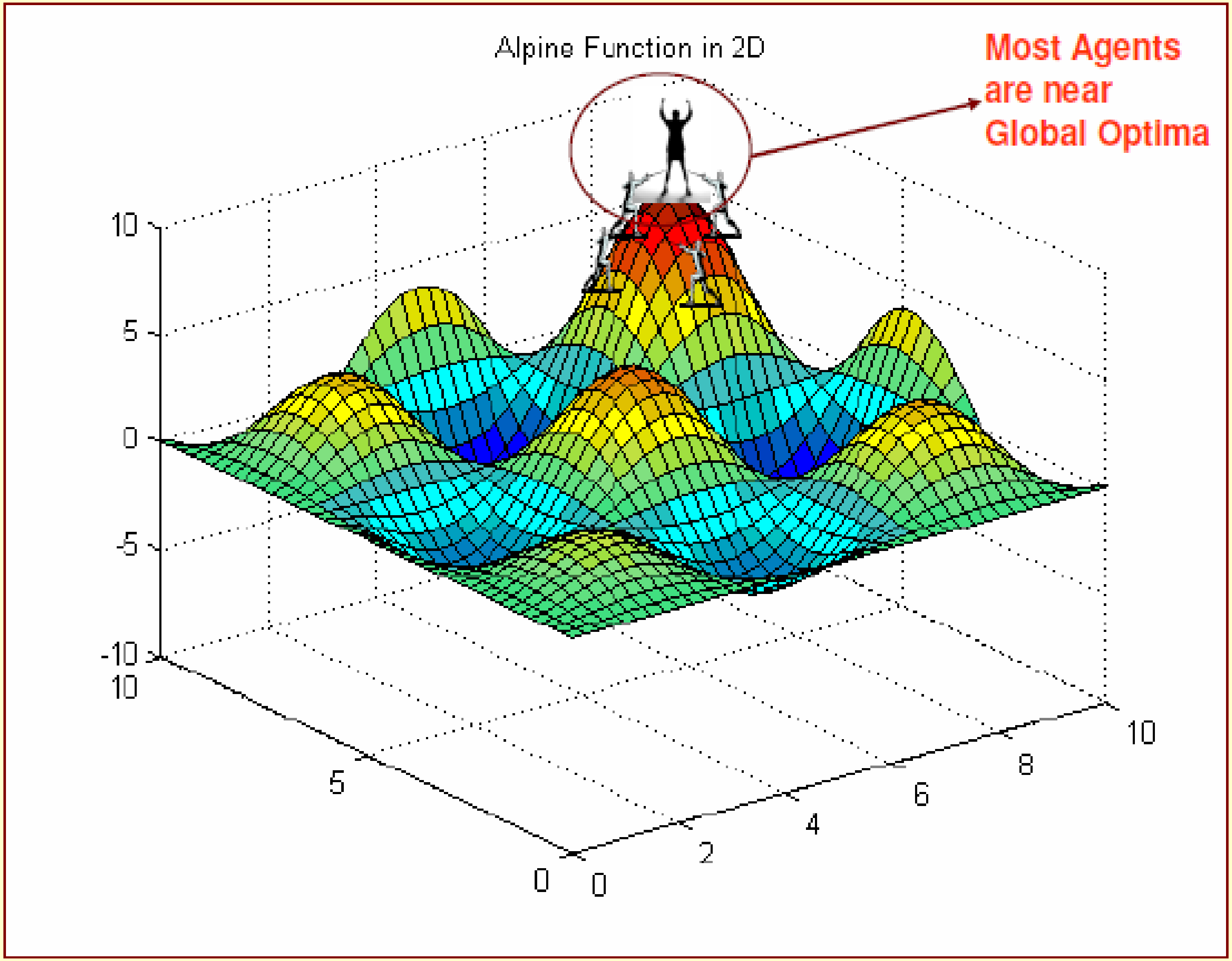


Gradient based methods get trapped in a local minima or the Function itself may be non differentiable.

Multi-Agent Optimization in Continuous Space



After Convergence



Differential Evolution

- **A stochastic population-based algorithm for continuous function optimization (Storn and Price, 1995)**
- **Continually exhibited remarkable performance in competitions on different kinds of optimization problems like dynamic, multi-objective, constrained, and multi-modal problems held under IEEE congress on Evolutionary Computation (CEC) conference series.**
- **DE is an Evolutionary Algorithm.**

Evolution

- The **processes** that have transformed life on earth from it's **earliest forms** to the vast **diversity** that characterizes it today.
- A **change** in the **genes!!!!!!!!**
- **Example:**

A giraffe acquired its long neck because its ancestor stretched higher and higher into the trees to reach leaves, and that the animal's increasingly lengthened neck was passed on to its offspring.

Charles Darwin

- **Wrote in 1859**: **"On the Origin of Species by Means of Natural Selection"**

- **Two main points:**

1. **Species were not created in their present form, but evolved from ancestral species.**

2. **Proposed a mechanism for evolution:**

   NATURAL SELECTION

Natural Selection

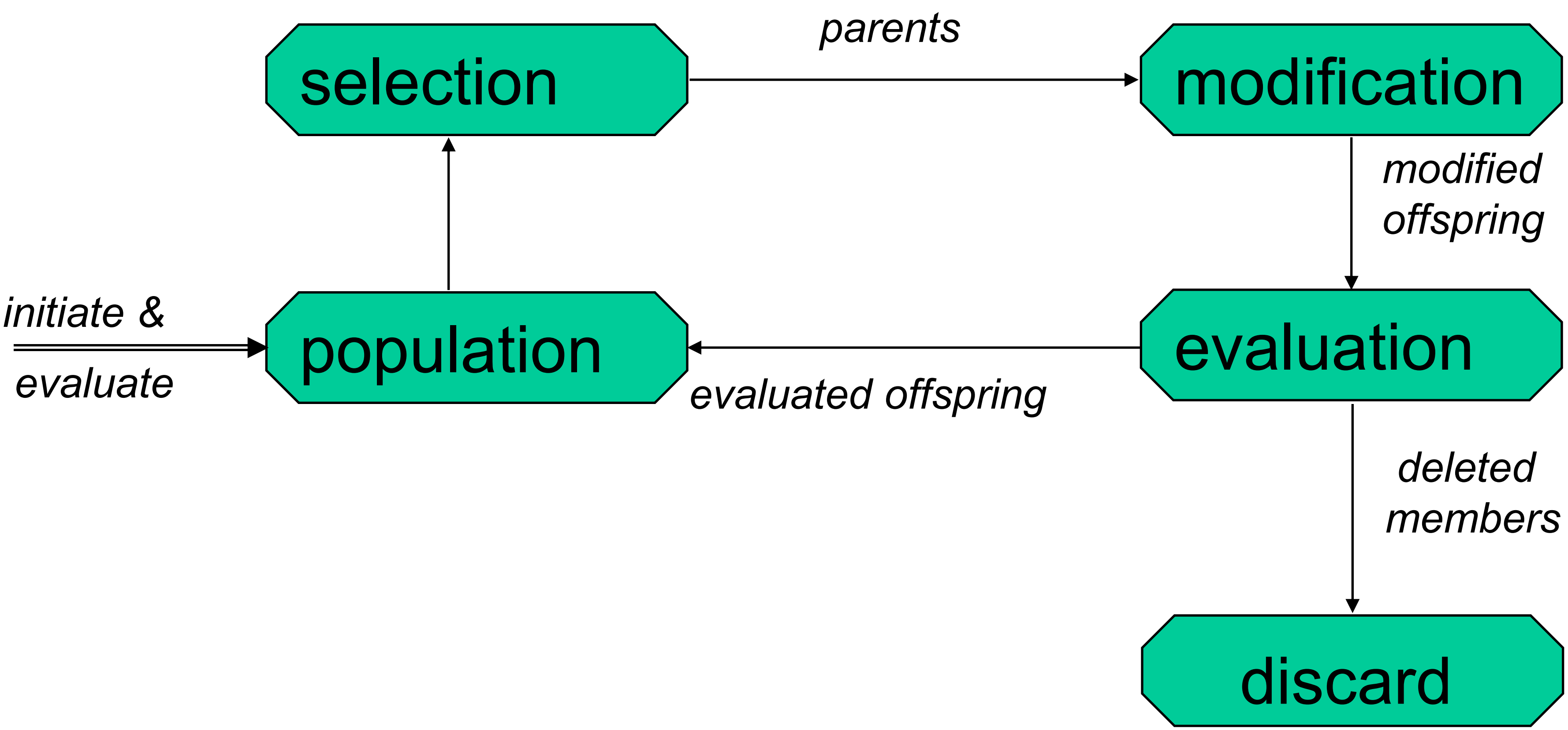
- **Individuals with favorable traits are more likely to leave more offspring better suited for their environment.**
- **Also known as “Differential Reproduction”.**
- The **science** of **genetic change** in population.

Population

- A localized group of **individuals** belonging to the **same species**.

- A group of **populations** whose **individuals** have the potential to **interbreed** and produce **viable** offspring.

The Evolutionary Cycle



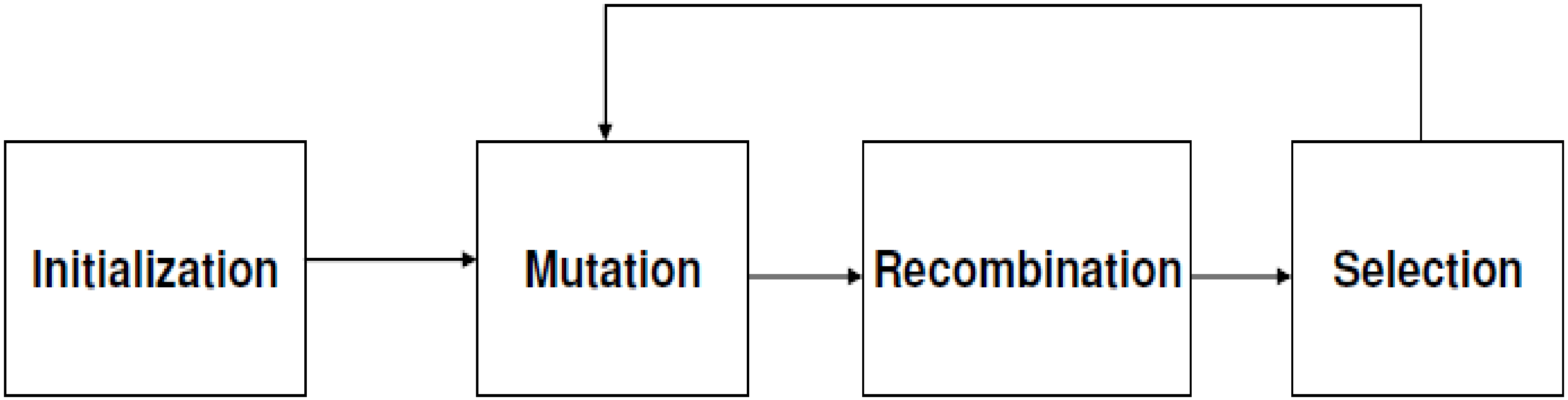
The Main Evolutionary Computing Metaphor

EVOLUTION		**PROBLEM SOLVING**
Environment	$\longleftrightarrow$	Problem
Individual	$\longleftrightarrow$	Candidate Solution
Fitness	$\longleftrightarrow$	Quality

Fitness $\rightarrow$ chances for survival and reproduction

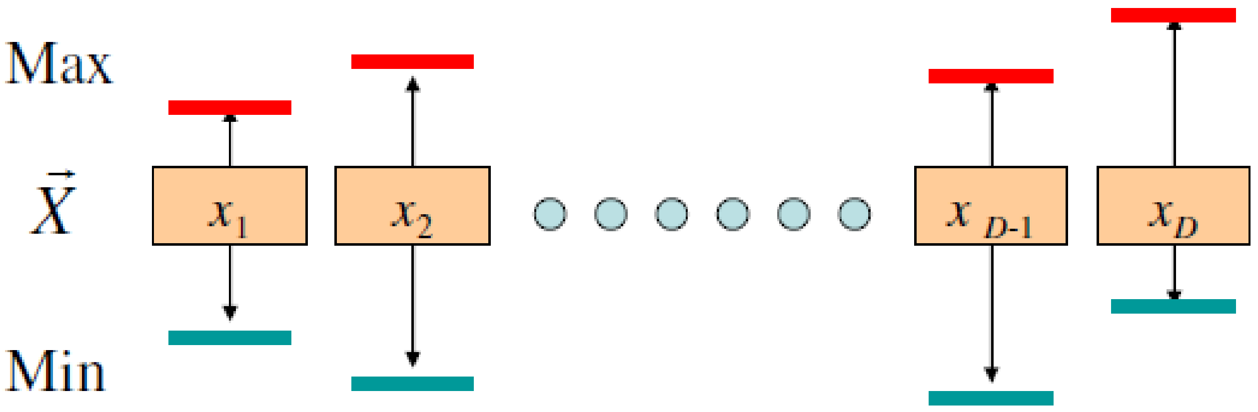
Quality $\rightarrow$ chance for seeding new solutions

DE Based Evolutionary Algorithm



Basic steps of an Evolutionary Algorithm

Representation

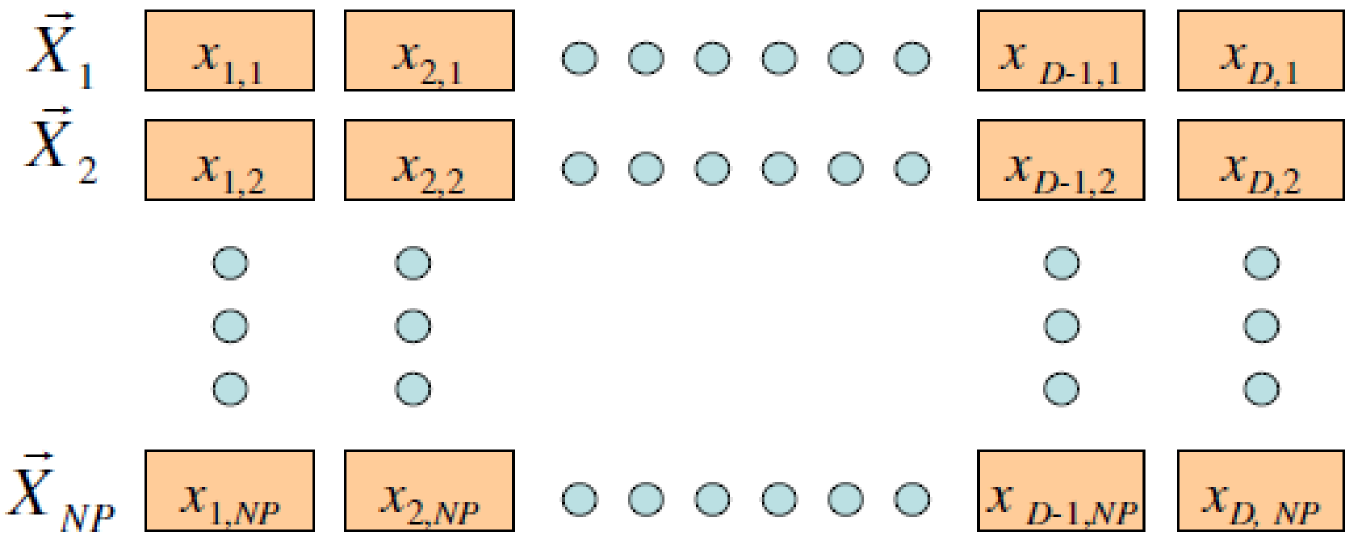


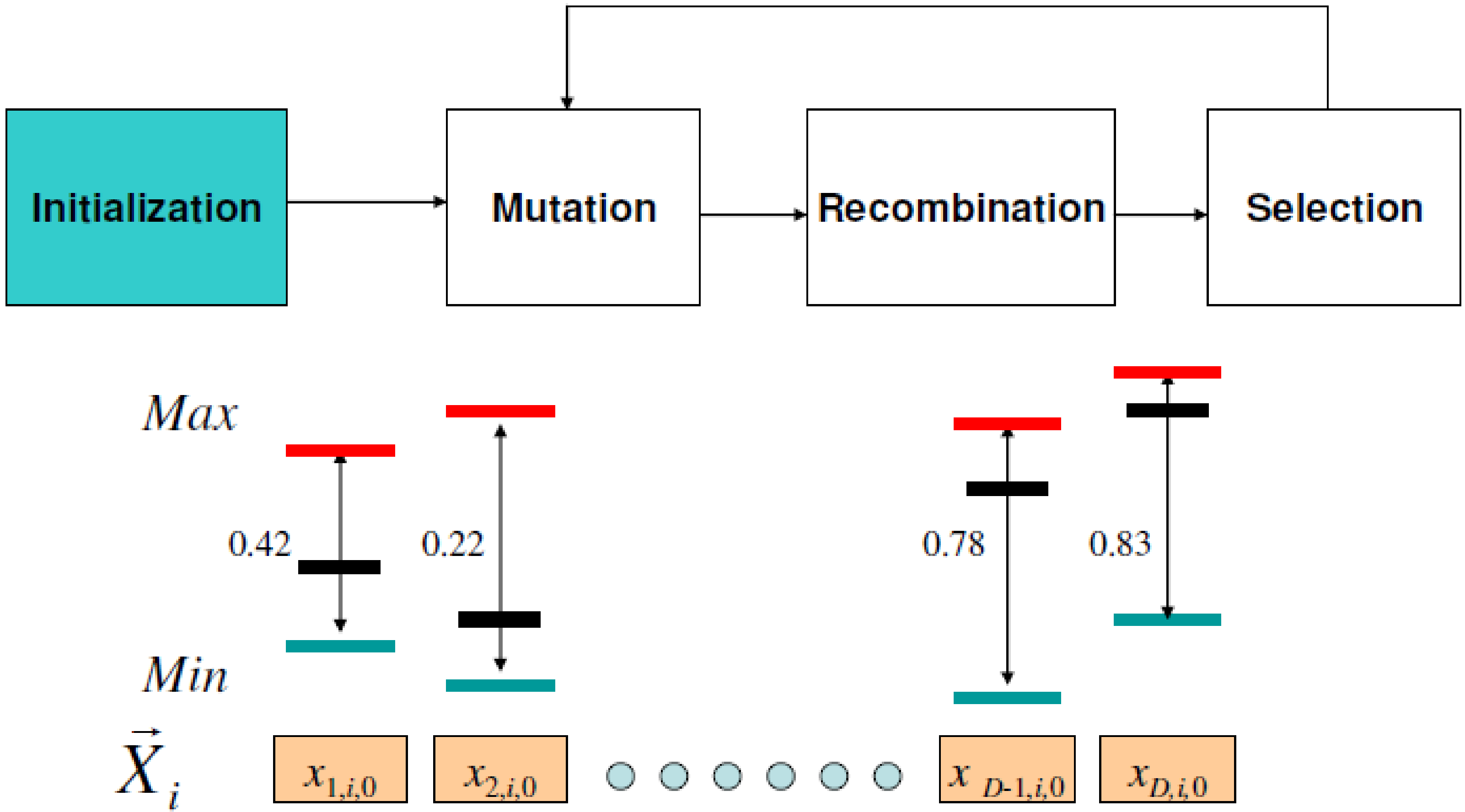
Solutions are represented as vectors of size *D* with each value taken from some domain.

May wish to constrain the values taken in each domain above and below.

Maintain Population - *NP*

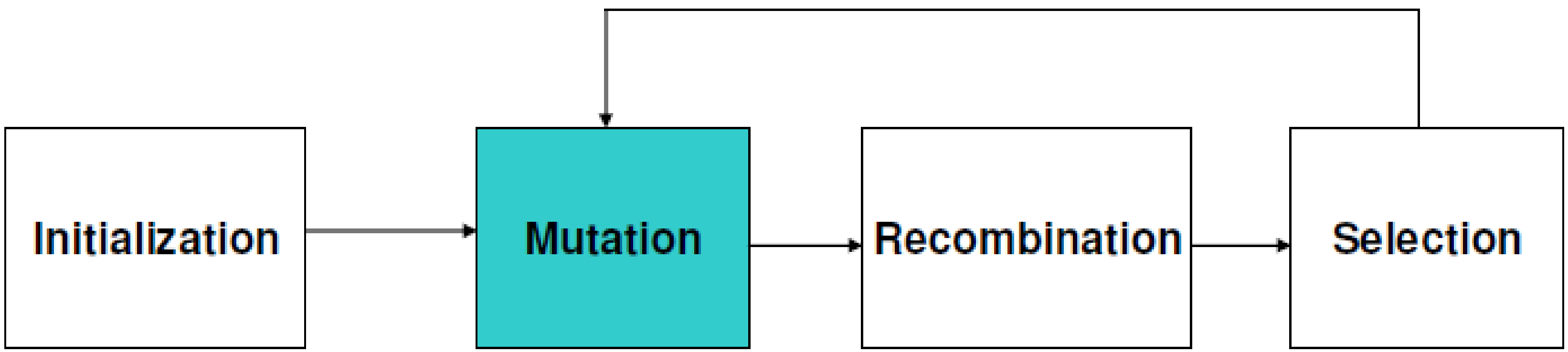
We will maintain a population of size *NP*





$$x_{j,i,0} = x_{j,\min} + rand_{i,j}[0,1] \cdot (x_{j,\max} - x_{j,\min})$$

Different $rand_{i,j}[0,1]$ values are instantiated for each i and j.



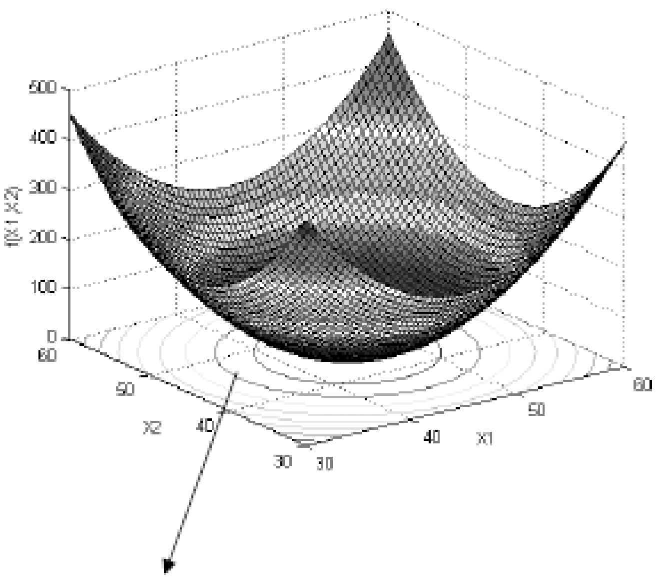
- For each vector select three other parameter vectors randomly.

- Add the weighted difference of two of the parameter vectors to the third to form a donor vector (most commonly seen form of DE-mutation):

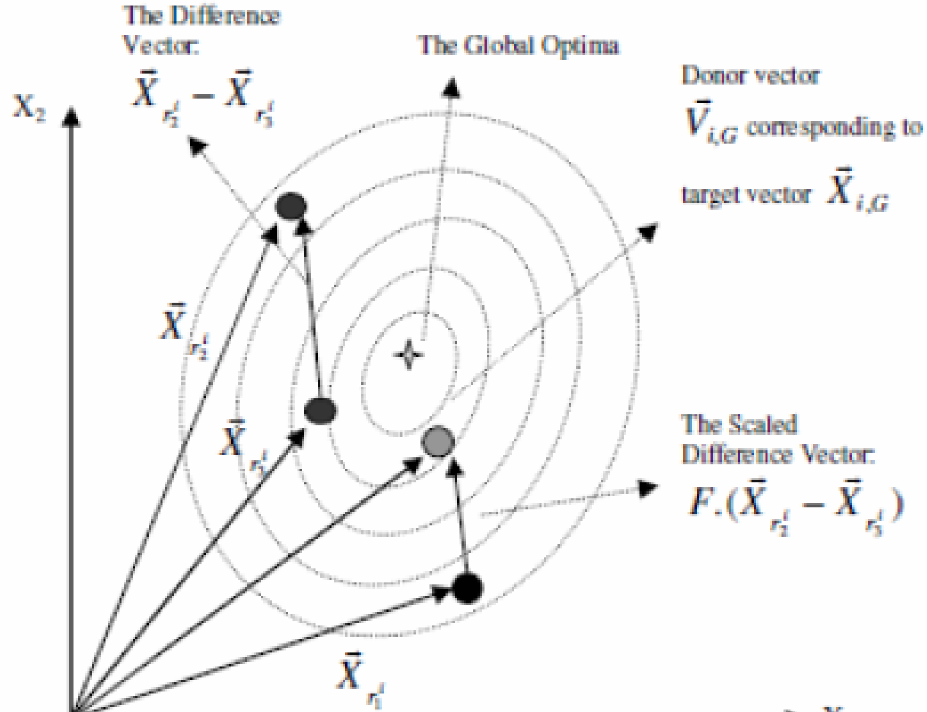
$$\vec{V}_{i,G} = \vec{X}_{r_1^i,G} + F \cdot (\vec{X}_{r_2^i,G} - \vec{X}_{r_3^i,G}).$$

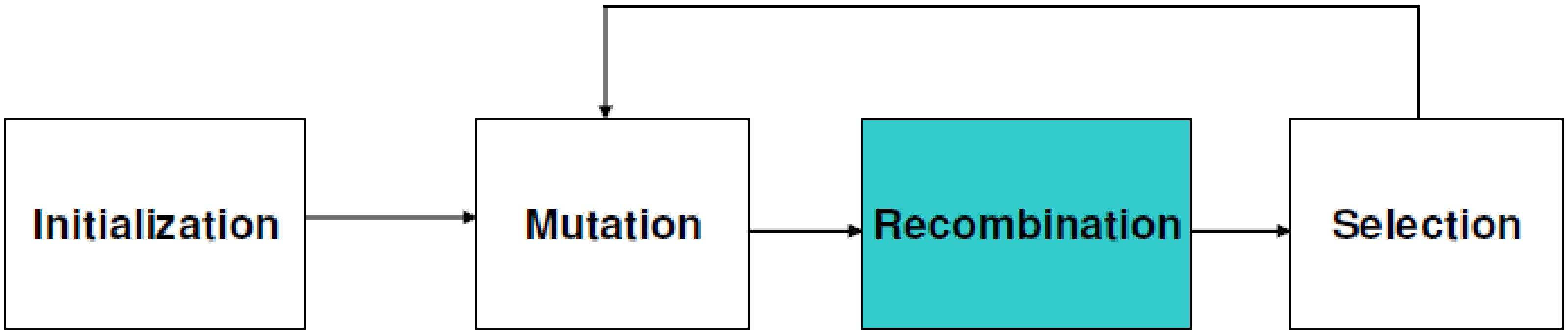
- The scaling factor *F* is a constant from (0, 2)

Example of formation of donor vector over two-dimensional constant cost contours



Constant cost contours of Sphere function



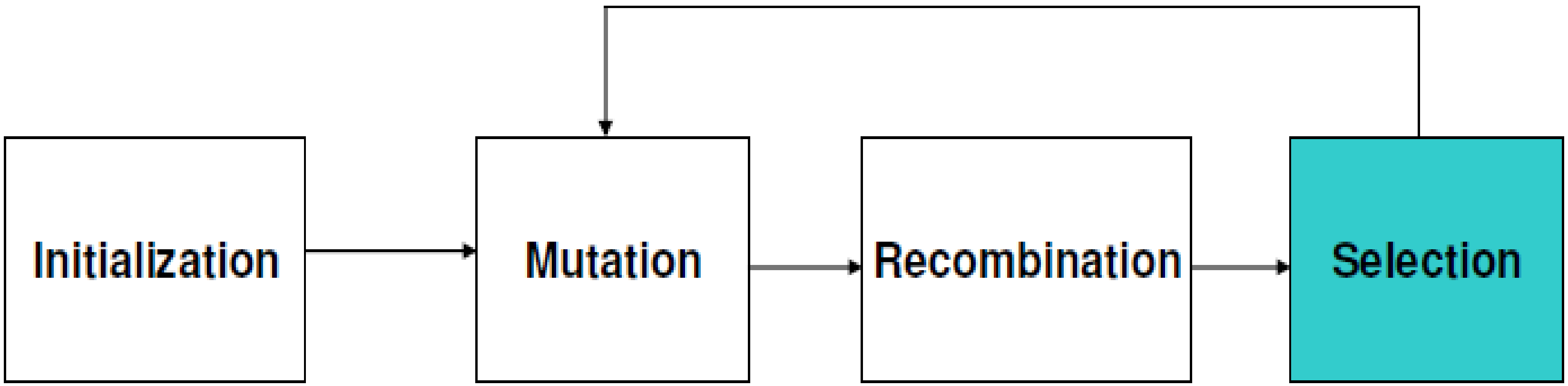


Binomial (Uniform) Crossover:

Components of the donor vector enter into the trial offspring vector in the following way:

Let j_{rand} be a randomly chosen integer between $1,\ldots,D$.

$$u_{j,i,G} = \begin{cases} v_{j,i,G}, & \text{if } (rand_{i,j}[0,1) \le Cr \text{ or } j = j_{rand}) \\ x_{j,i,G}, & \text{otherwise,} \end{cases}$$



- "Survival of the fittest" principle in selection: The trial offspring vector is compared with the target vector and that on with a better fitness is admitted to the next generation.

$$\vec{X}_{i,G+1} = \vec{U}_{i,G}, \quad \text{if } f(\vec{U}_{i,G}) \le f(\vec{X}_{i,G})$$

$$= \vec{X}_{i,G}, \quad \text{if } f(\vec{U}_{i,G}) > f(\vec{X}_{i,G})$$

An Example of Optimization by DE

Consider the following two-dimensional function

$f\,(x,\,y) = x^2 + y^2$ ***The minima is at (0, 0)***

Let's start with a population of 5 candidate solutions randomly initiated in the range (-10, 10)

$X_{1,0} = [2, -1]$ $X_{2,0} = [6, 1]$ $X_{3,0} = [-3, 5]$ $X_{4,0} = [-2, 6]$
$X_{5,0} = [6, -7]$

For the first vector X_1, randomly select three other vectors say X_2, X_4 and X_5

Now form the donor vector as, $V_{1,0} = X_{2,0} + F.\,(X_{4,0} - X_{5,0})$

$$V_{1,0} = \begin{bmatrix} 6 \\ 1 \end{bmatrix} + 0.8 \times \left\{ \begin{bmatrix} -2 \\ 6 \end{bmatrix} - \begin{bmatrix} 6 \\ -7 \end{bmatrix} \right\} = \begin{bmatrix} -0.4 \\ 10.4 \end{bmatrix}$$

Now we form the trial offspring vector by exchanging components of $V_{1,0}$ with the target vector $X_{1,0}$

Let *rand*[0, 1) = 0.6. If we set *Cr* = 0.9, since 0.6 < 0.9, $u_{1,1,0} = V_{1,1,0} = -0.4$

Again next time let *rand*[0, 1) = 0.95 > *Cr*

Hence $u_{1,2,0} = x_{1,2,0} = -1$

So, finally the offspring is $U_{1,0} = \begin{bmatrix} -0.4 \\ -1 \end{bmatrix}$

Fitness of parent:

$f(2, -1) = 2^2 + (-1)^2 = 5$

Fitness of offspring

$f(-0.4, -1) = (-0.4)^2 + (-1)^2 = 1.16$

Hence the parent is replaced by offspring at *G* = 1

Population at $G = 0$	Fitness at $G = 0$	Donor vector at $G = 0$	Offspring Vector at $G = 0$	Fitness of offspring at $G = 1$	Evolved population at $G = 1$
$X_{1,0}$ = [2,-1]	5	$V_{1,0}$ =[-0.4,10.4]	$U_{1,0}$ =[-0.4,-1]	1.16	$X_{1,1}$ =[-0.4,-1]
$X_{2,0}$= [6, 1]	37	$V_{2,0}$ =[1.2, -0.2]	$U_{2,0}$ =[1.2, 1]	2.44	$X_{2,1}$ =[1.2, 1]
$X_{3,0}$= [-3, 5]	34	$V_{3,0}$ =[-4.4, -0.2]	$U_{3,0}$ =[-4.4, -0.2]	19.4	$X_{3,1}$ =[-4.4, -0.2]
$X_{4,0}$= [-2, 6]	40	$V_{4,0}$ =[9.2, -4.2]	$U_{4,0}$ =[9.2, 6]	120.64	$X_{4,1}$ =[-2, 6]
$X_{5,0}$= [6, 7]	85	$V_{5,0}$ =[5.2, 0.2]	$U_{5,0}$ =[6, 0.2]	36.04	$X_{5,1}$ =[6, 0.2]