price and quantity after the change in price as the basis of measurement, there will not be any significant difference in the coefficient of elasticity.

Some Numerical Problems of Price Elasticity of Demand

Let us solve some numerical problems of price elasticity of demand (both point and arc) by percentage method.

Problem 1. *Suppose the price of a commodity falls from Rs. 6 to Rs. 4 per unit and due to this quantity demanded of the commodity increases from 80 units to 120 units. Find out the price elasticity of demand.*

Solution : Change in quantity demand $(Q_2 - Q_1) = 120 - 80$

$$\frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$$

Percentage change in quantity demanded $= \dfrac{Q_2 - Q_1}{\frac{Q_2 + Q_1}{2}} \times 100$

$$= \frac{40}{\frac{200}{2}} \times 100$$

$$= 40$$

Change in price $= P_2 - P_1 = 4 - 6 = -2$

% Change in price $= \dfrac{P_2 - P_1}{\frac{P_2 + P_1}{2}} \times 100 = \dfrac{-2}{\frac{10}{2}} \times 100$

$$= -40$$

Price elasticity of demand $= \dfrac{\text{\% change in quantity demanded}}{\text{\% Change in price}}$

$$= \frac{40}{-40} = -1$$

We ignore the minus sign. Therefore, price elasticity of demand is equal to one.

Problem 2. *A consumer purchases 80 units of a commodity when its price is Re. 1 per unit and purchases 48 units when its price rises to Rs. 2 per unit. What is the price elasticity of demand for the commodity?*

Solution : It should be noted that the change in price from Re. 1 to Rs. 2 in this case is very large (*i.e.*, 100%). Therefore, to calculate the elasticity coefficient in this case midpoint elasticity formula should be used.

Change is price $(\Delta p) = \text{Rs. } 2 - 1 = 1$

Average of the original and subsequent prices $= \dfrac{p_1 + p_2}{2}$

$$= \frac{1+2}{2} = \frac{3}{2} = 1.5$$

Change in quantity demanded (Δq) = 80 − 48 = 32

Average of the original and subsequent quantities $= \frac{q_1 + q_2}{2} = \frac{80+48}{2} = \frac{128}{2} = 64$

$$e_p = \frac{\Delta q}{\frac{q_1+q_2}{2}} \div \frac{\Delta p}{\frac{p_1+p_2}{2}}$$

$$= \frac{32}{64} \div \frac{1}{1.5}$$

$$= \frac{32}{64} \times \frac{1.5}{1} = \frac{1}{2} \times \frac{15}{10} = \frac{3}{4} = 0.75$$

Thus, the price elasticity of demand obtained is equal to 0.75.

Problem 3. *Suppose a seller of a textile cloth wants to lower the price of its cloth form Rs. 150 per metre to Rs. 142.5 per metre. If its present sales are 2000 metres per month and further it is estimated that its price elasticity of demand for the product is equal to 0.7. Show*

(a) Whether or not his total revenue will increase as a result of his decision to lower the price; and

(b) Calculate the exact magnitude of its new total revenue.

Solution (a) Price elasticity $= \frac{\Delta q}{\Delta p} \cdot \frac{p}{q}$

$$p = \text{Rs. } 150$$

$$q = 2000 \text{ metres}$$

$$\Delta p = 150 - 142.5 = 7.5$$

$$e_p = 0.7$$

$$\Delta q = ?$$

Substituting the values of p, q., Δp and e_p in the price elasticity formula we have

$$0.7 = \frac{\Delta q}{7.5} \cdot \frac{150}{2000}$$

$$\Delta q = \frac{0.7 \times 7.5 \times 2000}{150} = 70$$

Since the price has fallen the quantity demanded will increase by 70 metres. So the new quantity demanded will be 2000 + 70 = 2070.

(b) Total Revenue before reduction in price = 2000 × 150 = Rs. 3,00,000

Total revenue after price reduction = 2070 × 142.5 = 2,94,975

Thus with reduction in price his total revenue has decreased.

Finding Price Elasticity from Demand Function:

their income ... goods with high income elasticity of demand.

Some Numerical Problems on Income Elasticity

Problem 1. If a consumer's daily income rises from Rs. 300 to Rs. 350, his purchase of a good X increases from 25 units per day to 35 units, find income elasticity of demand for X.

Solution. Change in quantity demand $(\Delta Q) = (Q_2 - Q_1) = 35 - 25 = 10$

Change in income $(\Delta M) = M_2 - M_1 = 350 - 300 = 50$

$$e_i = \frac{\%\ \text{Change in quantity demanded}}{\%\ \text{Change in price}}$$

$$e_i = \frac{\Delta Q}{\Delta M} \times \frac{M_2 + M_1}{Q_2 + Q_1}$$

$$= \frac{10}{50} \times \frac{350 + 300}{25 + 35}$$

$$= \frac{10}{50} \times \frac{650}{60} = 2.17$$

Income elasticity of demand in this case is 2.17.

Problem 2. *Suppose demand for cars in Bombay as a function of income is given by the following equation :*

$$Q = 20{,}000 + 5M$$

where Q is quantity demanded, M is per capita level of income in rupees.

Find out income elasticity of demand when per capita annual income in Bombay is Rs 15,000.

Solution.

$$\text{Income elasticity } (e_i) = \frac{\Delta Q}{\Delta M} \cdot \frac{M}{Q}$$

In order to obtain income elasticity, we have to first find out quantity demanded (Q) at income level of Rs 15,000. Thus.

$$Q = 20{,}000 + 5 \times 15{,}000 = 95{,}000$$

It will be seen from the given income demand function that coefficient of income (M) is equal to 5. This implies that $\frac{\Delta Q}{\Delta M} = 5$. With this information we can calculate income elasticity.

$$e_i = \frac{\Delta Q}{\Delta M} \times \frac{M}{Q} = 5 \times \frac{15,000}{95,000} = 0.8$$

Problem 3. *The following demand function for readymade trousers has been estimated*

$$Q = 2,000 + 15Y - 5.5P$$

where Y is income in thousands of rupees, Q is the quantity demanded in units and P is the price per unit.

(a) *When P = Rs 150 and Y = 15 thousand rupees, determine the following :*

1. *Price elasticity of demand*
2. *Income elasticity of demand*

(b) *Determine what effect a rise in price would have on total revenue.*

(c) *Asses how sale of trousers would change during a period of rising incomes.*

Solution

(a) Coefficient of P, *i.e.*, $\frac{\Delta Q}{\Delta P} = 5.5$

Price elasticity of demand $\frac{\Delta Q}{\Delta P} \cdot \frac{P}{Q} = 5.5 \times \frac{150}{Q}$

Let us first find out the quantity demanded (*i.e.*, Q) at the given income (Y = 15 thousands) and given price (P = Rs 150 per unit). Substituting the values of income and price in the given demand function, we have :

$$\begin{aligned} Q &= 2,000 + 15 \times 15 - 5.5 \times 150 \\ &= 2,000 + 225 - 825 = 1,400 \end{aligned}$$

Thus, $e_p = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q} = 5.5 \times \frac{150}{1400} = \frac{82.5}{140} = 0.59$

Income elasticity $= \frac{\Delta Q}{\Delta Y} \times \frac{Y}{Q}$

$$\frac{\Delta Q}{\Delta Y} = 15, \ Q = 1400, \ Y = 15 \text{ thousand rupees}$$

$$e_i = 15 \times \frac{15}{1400} = \frac{9}{56} = 0.16$$

(b) Since price elasticity of demand for trousers is less than one, rise in price would cause increase in total revenue.

(c) Since income elasticity of demand for trousers is less than one, trousers are a necessity and therefore the increase in income of the people will lead to less than a proportionate increase in their sales.

there is 5 per cent rise in price. We therefore predict increase in revenue.

3. *For the following demand functions, determine whether demand is elastic, inelastic or unitary elastic at the given price:*

(i) $Q = 100 - 4P$ *and the given* P = *Rs 20*

(ii) $Q = 1500 - 20P$ *and the given* P = *Rs 5*

(iii) $P = 50 - 0.1Q$ *and the given* P = *Rs 20*

Solution. (*i*) $Q = 100 - 4P$ where P = Rs 20

In this demand function the derivative $\frac{dQ}{dP} = 4.$

Substituting the value of P in this demand function (*i*)

$$Q = 100 - (4 \times 20) = 20$$

$$e_p = \frac{dQ}{dP} \times \frac{P}{Q} = 4 \times \frac{20}{20} = 4.$$

Since $e_p > 1$, demand is elastic.

(*ii*) $Q = 1500 - 20P$ where P = Rs 5

In this demand function equation, the derivative $\frac{dQ}{dP} = 20$

Substituting the value of P in this demand function

$$Q = 1500 - (20 \times 5) = 1400$$

$$e_p = \frac{dQ}{dP} \times \frac{P}{Q} = 20 \times \frac{5}{1400} = \frac{5}{70} = 0.07$$

Since $e_p < 1$, demand is inelastic.

(*iii*) $P = 50 - 0.1Q$ where P = Rs 20

Let us first express this demand function in terms of quantity demanded as a function of price.

$$P = 50 - 0.1Q$$

. Therefore,

$$\text{Coefficient of corss elasticity of demand of } X \text{ for } Y = \frac{\text{Proportionate change in the quantity demanded of } X}{\text{Proportionate change in the price of good } Y}$$

$$\text{or, } e_c = \frac{\dfrac{\Delta q_x}{q_x}}{\dfrac{\Delta p_y}{P_y}} = \frac{\Delta q_x}{q_x} \div \frac{\Delta p_y}{p_y}$$

$$= \frac{\Delta q_x}{q_x} \times \frac{p_y}{\Delta p_y}$$

$$= \frac{\Delta q_x}{\Delta p_y} \times \frac{p_y}{q_x}$$

where e_c stands for cross elasticity of demand of X for Y.

q_x stands for the original quantity demanded of good X

Δq_x stands for change in quantity demanded of good X

p_y stands for the original price of good Y

Δp_y stands for a small change in the price of good Y

When change in price is large, we should use midpoint method for estimating cross elasticity of demand. We can write midpoint formula for measuring cross elasticity of demand as

$$e_c = \frac{q_{x2} - q_{x1}}{\frac{q_{x2} + q_{x1}}{2}} \div \frac{p_{y2} - p_{y1}}{\frac{p_{y2} + p_{y1}}{2}}$$

Numerical Problems

Problem 1. *If price of coffee rises from Rs. 45 per 250 grams pack to Rs. 55 per 250 grams pack and as a result the consumers demand for tea increases from 600 packs to 800 packs of 250 grams, then find the cross elasticity of demand of tea for coffee.*

Solution:

We use midpoint method to estimate cross elasticity of demand.

Change in quantity demanded of tea = $q_{t2} - q_{t1} = 800 - 600$

Change in price of Coffee = $P_{C2} - P_{C1} = 55 - 45$

Substituting the values of the various variables in the cross eleasticity formula we have

$$\text{Gross elasticity of demand} = \frac{800-600}{\frac{800+600}{2}} \div \frac{55-45}{\frac{55+45}{2}} = \frac{200}{700} \times \frac{50}{10} = \frac{10}{7} = 1.43$$

Problem 2. *Suppose the following demand function for coffee in terms of price of tea is given. Find out the cross elasticity of demand when price of tea rises from Rs 50 per 250 grams pack to Rs 55 per 250 grams pack.*

$$Q_c = 100 + 2.5P_t$$

where Q_c is the quantity demand of coffee in terms of packs of 250 grams and P_t is the price of tea per 250 grams pack.

Solution. The positive sign of the coefficient of P_t shows that rise in price of tea will cause an increase in quantity demanded of coffee. This implies that tea and coffee are substitutes.

The demand function equation implies that coefficient $\frac{dQ_c}{dP_t} = 2.5$.

In order to determine cross elasticity of demand between tea and coffee, we first find out quantity demanded of coffee when price of tea is Rs 50 per 250 grams pack. Thus,

$$Q_c = 100 + 2.5 \times 50 = 225$$

$$\text{Cross elasticity,} \quad e_c = \frac{dQ_c}{dP_t} \times \frac{P_t}{Q_c}$$

$$= 2.5 \times \frac{50}{225} = \frac{125}{225} = 0.51.$$

Cross Elasticity of Demand : Substitutes and Complements