SPRING END SEMESTER EXAMINATION-2020

4^{th} Semester B.Tech & B.Tech Dual Degree

DESIGN & ANALYSIS OF ALGORITHMS

[CS-2008]

(For 2018 & Previous Admitted Batches)

Time: 3 Hours — Full Marks: 50/60

Answer any SIX questions.
Question paper consists of four sections-A, B, C, D.
Section A is compulsory.
Attempt minimum one question each from Sections B, C, D.
The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

SECTION-A

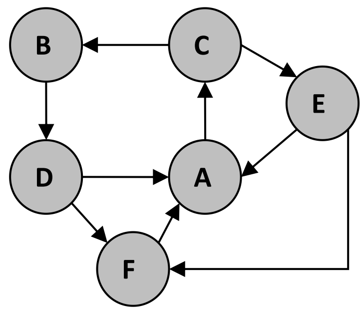
1.			$[1 \times 10]$/ $[2 \times 10]$

	(a)	Consider the following function that accepts a positive integer n as input.
		int fun(int n)
		{
		while (n > 1)
		{
		n=n+1
		while (n%2==0)
		{
		n=n/2
		}
		}
		}
		Determine a tight bound on the running time of the above function.	

	(b)	Is n! = $O(n^n)$? Justify your answer.	

KIIT-DU/2020/SOT/Spring End Semester Examination-2020

	(c)	The insertion sort and merge sort are implemented on the same machine for comparision purpose. For inputs of size n, insertion sort runs in $8n^2$ steps, while merge sort runs in $64n \log_2 n$ steps. For which values of n does insertion sort beat merge sort?	
	(d)	How will you check for a graph's acyclicity with DFS and BFS?	
	(e)	Given a big array, how to efficiently find k'th largest element in it?	
	(f)	Compare insertion sort with quick sort.	
	(g)	Choose the correct answer. Given a sorted array of integers, what can be the minimum worst case time complexity to find ceiling of a number x in a given array (x may or may not be an element in the array)? The Ceiling of an element x is the smallest element present in array which is greater than or equal to x. Ceiling is not present if x is greater than the maximum element present in array. For example, if the given array is {10, 20, 30, 40, 50, 60} and x = 42, then output should be 50. i) O(loglog n) ii) O(n log n) iii) O(log n) iii) O(n^2) Justify your answer.	
	(h)	What can be the best data structure to be used to find 10 maximum numbers from a big file containing billions of numbers? What is the worst case time complexity of this problem w.r.t the data structure used.	
	(i)	Define optimal storage on tapes problem.	
	(j)	Match the following: (P) Prim's algorithm for minimum spanning tree — (i) Backtracking (Q) Floyd-Warshall algorithm for all pairs shortest paths — (ii) Greedy method (R) Merge sort — (iii) Dynamic programming (S) Sum of Subset — (iv) Divide and conquer	

		problem	
		SECTION-B	
2.	(a)	Write a Θ(nlogn) algorithm to determine whether or not the elements of an array are unique. Analyze its overall-time complexity.	[4]
	(b)	Find a solution to the recurrence $T(n) = 2T(n/4) + \Theta(n)$, $T(1)=1$	[4]
3.	(a)	Consider the following graph:  a) Compute the DFS tree and draw the tree edges, forward edges, back edges and cross edges. b) Write the order in which the vertices were reached for the first (i.e.pushed into the stack) c) Write the order in which the vertices became dead ends (i.e. popped from the stack)	[4]
	(b)	Describe a Θ(n log n) time algorithm that, given a set S of n integers and another integer x, determines whether or not there exist two elements in S whose sum is exactly x.	[4]
		SECTION-C	
4.	(a)	Use a dynamic programming algorithm to find the Longest Common Subsequence between the following two sequences: X = ababaabaa Y = aababaab	[4]
	(b)	Find an optimal solution to the knapsack instance n=7, W=15. (v1, v2, v3, v4, v5, v6, v7) = (5, 15, 10, 7, 6, 20, 3) and	[4]

KIIT-DU/2020/SOT/Spring End Semester Examination-2020

(w1, w2, w3, w4, w5, w6, w7) = (2, 3, 5, 6, 1, 4, 1), where n is the number of items, W is the knapsack capacity that thief can carry, v_i stands for value or profit w_i stands for weight of the i^{th} element.

5. (a) The operation HEAP-DELETE(A, i) deletes the item in node i from heap A. Give an implementation of HEAP-DELETE that runs in O(log n) time for an n-element max-heap. [4]

(b) State and explain union by rank and find-path compression algorithms of Dis-joint Data Structures with suitable examples. [4]

6. (a) Construct a Huffman code for the following data (show all the steps): [4]

Symbol	A	B	C	D
Frequency	0.1	0.2	0.4	0.3

How many bits are needed to encode a string containing 10 A's, 5 B's, 15 C's and 2 D's using this code. Compare this code with another code where each character is encoded with fixed two bits. Which code is better?

(b) Assume that you are given a chain of matrices < A1 A2 A3 A4 >, with dimensions 2x3, 3x4, 4x6 and 6x5 respectively. Compute the optimal number of multiplications required to calculate the chain product and also indicate what the optimal order of multiplication should be using parentheses. [4]

SECTION-D

7. (a) We are given 10 tasks $T_1, T_2, T_3, \ldots, T_9, T_{10}$. The execution of each task requires one unit of time. We can execute one task at a time. Each task T_i has a profit P_i and a deadline D_i. Profit P_i is earned if the task is completed before the end of the D_i unit of time. [4]

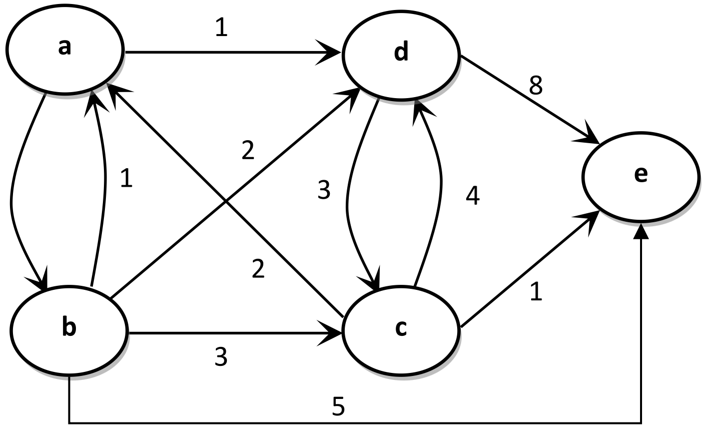
Task	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}
P_i	20	21	22	27	34	12	28	16	26	30
D_i	4	5	3	2	4	5	2	1	3	6

a) Schedule the set of tasks.

b) In the above problem, what is the maximum profit earned?

(b) A sequence of n numbers (positive or negative) $x_1, x_2, x_3, \ldots x_n$. are given. Write an algorithm to select a subset of these numbers of maximum total sum, subject to the constraint that two adjacent elements are not selected (that is, if you pick x_i then you cannot pick either x_{i-1} or x_{i+1}). [4]

8. (a) Use suitable shortest path algorithm to find out shortest path from vertex 'a' to all other vertices. Show all the steps. [4]



(b) Define and Describe P, NP and NPC class of problems through suitable examples. [4]

*****

KIIT-DU/2020/SOT/Spring End Semester Examination-2020