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Sec: ~~CSE-27~~ CI-03

ACTIVITY - 4

1. Ar $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

(a) Let's calculate $\mu_A(x)$ for each $x \in X$:
$\forall\ a = 2,\ b = 5,\ c = 8$

For $x = 0$:-

$\rightarrow \mu_A(0) = 0$ [since $x \leq a$]

Similarly,

$\rightarrow \mu(1) = 0$ ''

$\rightarrow \mu(2) = 0$ ''

$\rightarrow \mu(3) = \frac{3-2}{5-2} = \frac{1}{3} \approx 0.33$

$\rightarrow \mu(4) \approx 0.67$

$\rightarrow \mu(5) = 1$

$\rightarrow \mu(6) = 0.33$

$\rightarrow \mu(7) \simeq \cancel{8}\,0.33$

$\rightarrow \mu(8) = 0$ [since $x = c$]

$\rightarrow \mu(9) = 0$ [since $x \geq c$]

$\rightarrow \mu(10) = 0$ ''

$\therefore$ Membership values for the triangular function:

$\mu_A(x) = \{0, 0, 0, 0.33, 0.67, 1, 0.33, 0.33, 0, 0, 0\}$

(b) Trapezoidal membership f^n

$\forall\ a = 2, b = 4, c = 6, d = 8$

Calculating $\mu_A(x)$ for each $x \in X$:

→ $\mu_A(0) = 0$ (since $x \leq a$)

→ $\mu_A(1) = 0$ "

→ $\mu_A(2) = 0$ "

→ $\mu_A(3) = \frac{3-2}{4-2} = 0.5$

→ $\mu_A(4) = 1$ (top of the trapezoid)

→ $\mu_A(5) = 1$ "

→ $\mu_A(6) = 1$ "

→ $\mu_A(7) = \frac{8-7}{8-6} = 0.5$

→ $\mu_A(8) = 0$ (since $x \geq d$)

→ $\mu_A(9) = 0$ "

→ $\mu_A(10) = 0$ "

∴ Membership values are:

$\mu_A(x) = \{0, 0, 0, 0.5, 1, 1, 1, 0.5, 0, 0, 0\}$

(c) Gaussian Membership function

$\forall$ ~~a=~~ $c = 5$, $\sigma = 2$

Calculating $\mu_A(x)$ for each $x \in X$;

$\rightarrow \mu_A(0) = e^{-1/2\left(\frac{0-5}{2}\right)^2} = e^{-3.125} \simeq 0.0439$

$\rightarrow \mu_A(1) = e^{-1/2\left(\frac{1-5}{2}\right)^2} = e^{-2} \simeq 0.1353$

$\rightarrow \mu_A(2) = e^{-1/2\left(\frac{2-5}{2}\right)^2} = e^{-1.125} \simeq 0.3247$

$\rightarrow \mu_A(3) = e^{-1/2\left(\frac{3-5}{2}\right)^2} = e^{-0.5} \simeq 0.6065$

$\rightarrow \mu_A(4) = e^{-1/2\left(\frac{4-5}{2}\right)^2} = e^{-0.125} \simeq 0.8825$

$\rightarrow \mu_A(5) = e^{-1/2\left(\frac{5-5}{2}\right)^2} = e^{0} = 1$

$\rightarrow \mu_A(6) = e^{-1/2\left(\frac{6-5}{2}\right)^2} = e^{-0.125} \simeq 0.8825$

$\rightarrow \mu_A(7) = e^{-1/2\left(\frac{7-2}{2}\right)^2} = e^{-0.5} \simeq 0.6065$

$\rightarrow \mu_A(8) = e^{-1/2\left(\frac{8-2}{2}\right)^2} = e^{-1.125} \simeq 0.3247$

$\rightarrow \mu_A(9) = e^{-1/2\left(\frac{9-2}{2}\right)^2} = e^{-2} \simeq 0.1353$

$\rightarrow \mu_A(10) = e^{-1/2\left(\frac{10-2}{2}\right)^2} = e^{-3.125} \simeq 0.0439$

$\therefore$ Membership values are:-

$\rightarrow \mu_A(x) = \{0.0439, 0.1353, 0.3247, 0.6065, 0.8825,$
$1, 0.8823, 0.6065, 0.3247, 0.1353,$
$0.0439\}$

(d) Bell Membership Function

$\forall\ c = 5,\ a = 2,\ b = 2$

Formula to be used for $\mu_A(x)$ is

$$\mu_A(x) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}}$$

→ Calculating value of $\mu_A(x)$ for all $x \in X$

→ $\mu_A(0) \approx 0.025$

→ $\mu_A(1) \approx 0.059$

→ $\mu_A(2) \approx 0.165$

→ $\mu_A(3) \approx 0.444$

→ $\mu_A(4) \approx 0.941$

→ $\mu_A(5) \approx 1$

→ $\mu_A(6) \approx 0.941$

→ $\mu_A(7) \approx 0.444$

→ $\mu_A(8) \approx 0.165$

→ $\mu_A(9) \approx 0.059$

→ $\mu_A(10) \approx 0.025$

∴ Membership values are:

$\mu_A(x) = \{0.025, 0.059\ 0.165, 0.444, 0.941, 1, 0.941, 0.444, 0.165, 0.059, 0.025\}$

(e) Sigmoidal Membership Function

$\forall$ c = 5, $\alpha = 0.5$

Formula to be used for $\mu_A(x)$ is

$$\mu_A(x) = \frac{1}{1+e^{-\alpha(x-c)}}$$

Calculating value of $\mu_A(x)$ for all $x \in X$:-

→ $\mu_A(0) \approx 0.075$

→ $\mu_A(1) \approx 0.119$

→ $\mu_A(2) \approx 0.182$

→ $\mu_A(3) \approx 0.269$

→ $\mu_A(4) \approx 0.377$

→ $\mu_A(5) \approx 0.5$

→ $\mu_A(6) \approx 0.622$

→ $\mu_A(7) \approx 0.731$

→ $\mu_A(8) \approx 0.818$

→ $\mu_A(9) \approx 0.881$

→ $\mu_A(10) \approx 0.924$

∴ Membership values are :-

$\mu_A(x) = \{0.075, 0.119, 0.265, 0.377, 0.5, 0.622, 0.731, 0.818, 0.881, 0.924\}$

(f) Singleton Membership Functions

$$\mu_A(x) = \begin{cases} 1 & \text{if } x = 5 \\ 0 & \text{otherwise} \end{cases}$$

$\therefore \mu_A(x) = \{0,0,0,0,0,1,0,0,0,0,0\}$

2. Ans → $X = \{x1, x2\}$

$Y = \{y1, y2, y3\}$

$Z = \{z1, z2, z3\}$

→ Calculating (R∘S) for each element

→ $(R \circ S)_{11} = \max(\min(0.6, 0.7), \min(0.5, 0.1), \min(0.3, 0.4))$

$= \max(0.2, 0.5, 0.3)$

$= 0.5$

Similarly,

→ $(R \circ S)_{12} = 0.5$

→ $(R \circ S)_{13} = 0.5$

→ $(R \circ S)_{21} = 0.5$

→ $(R \circ S)_{22} = 0.7$

→ $(R \circ S)_{23} = 0.6$

$$\therefore R \circ S = \begin{bmatrix} 0.6 & 0.5 & 0.5 \\ 0.3 & 0.7 & 0.6 \end{bmatrix}$$

3. A Cylindrical extension adds a new dimension to a fuzzy set without changing the original membership values. Projection reduces the dimension by ~~reducing~~ taking the membership value over the removed dimension.

→ Projection on X-axis: $\{0.5, 0.8, 0.9, 1.0\}$

→ Projection on Y-axis: $\{0.8, 0.9, 1.0\}$

4. A Applying the mapping f^n for

$$f(x) = x^2 - 2$$

→ for $x = -3$:

$\Rightarrow f(-3) = 7$

∴ Membership value ~~for~~ is 0.1 for $y = 7$.

→ for $x = -2$

$\Rightarrow f(-2) = 2$

∴ Membership value ~~for~~ is 0.2 for $y = -1$

→ for $x = -1$

$\Rightarrow f(-1) = (-1)$

∴ Membership value is 0.8 for $y = -1$

→ for $x = 0$

$\Rightarrow f(0) = -2$

Membership value is 0.4 for $y = -2$

$\rightarrow$ for $x = 3$

$f(3) = 7$

$\therefore$ The membership value is 0.1 for $y = 7$

$\rightarrow$ Construct the transformed Fuzzy set

$\rightarrow$ For $y = 7$

Membership value $= \max(0.1, 0.1) = 0.1$

$\rightarrow$ for $y = 2$

Membership value $= \max(0.2, 0.5) = 0.5$

$\rightarrow$ for $y = -1$

Membership value $= \max(0.8, 0.4) = 0.8$

$\rightarrow$ For $y = -2$

Membership value $= 1$

Thus, transformed fuzzy set is

$$\boxed{A' = 0.1/7 + 0.5/2 + 0.8/-1 + -1/-2}$$

5. Ar (i) $\mu_{\text{young but not too young}}(x) = \mu_{\text{young}}(x) \cdot (1 - \mu_{\text{young}}(x))$

(ii) $\mu_{\text{Extremely young}}(x) = (\mu_{\text{young}}(x))^2$

(iii) $\mu_{\text{not young not and not old}}(x) = \min(1 - \mu_{\text{young}}(x), 1 - \mu_{\text{old}}(x))$

(iv) $\mu_{\text{more or less old}}(x) = e^{-\frac{(x-50)^2}{2(1.5\cdot 1.5)^2}}$

6. Ar → 4 one-dimensional membership function :-

(i) Triangular : Defined by three points, it has peak at a single value

(ii) Trapezoidal : Defined by four points, it has a plateau at the top

(iii) Gaussian : A bell-shaped curve that decays symmetrically

(iv) Sigmoidal : An S-shaped curve, typically used for binary-like membership.

→ <u>T-Norm Operators</u> :-

(i) Minimum : $T(a,b) = \min(a,b)$

(ii) Product : $T(a,b) = a \cdot b$

(iii) Bounded Difference: $T(a,b) = \max(0, a+b-1)$

(iv) Drastic Product ; $T(a,b) = \begin{cases} a & \text{if } b=1 \\ b & \text{if } a=1 \\ 0 & \text{otherwise} \end{cases}$

→ <u>S-Norm Operator</u> :

(i) Maximum: $S(a,b) = \max(a,b)$

(ii) Probabilistic Sum ; $S(a,b) = a+b-a\cdot b$

(iii) Bounded Sum : $S(a,b) = \min(1, a+b)$

(iv) Drastic Sum : $S(a,b) = \begin{cases} 1 & \text{if } a=1 \text{ or } b=1 \\ \max(a,b) & \text{otherwise} \end{cases}$