Computational Intelligence (CI)

Fuzzy Complement Operation

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Fuzzy Complement

A fuzzy complement operator is a continuous function **N: [0, 1] → [0, 1],** which meets the following axiomatic requirements.

$$N(0) = 1, \quad N(1) = 0 \qquad (Boundary)$$
$$N(a) \geq N(b) \quad if\ a \leq b \qquad (Monotonicity)$$

Any function satisfying these requirements form the general class of fuzzy complement.

Optional requirement

$$N\big(N(a)\big) = a \qquad (Involution\ Property)$$

Classical Fuzzy Complement

$$N(a) = 1 - a$$

$N(0) = 1 - 0 = 1$

$N(1) = 1 - 1 = 0$

Boundary property satisfied

$N(0.2) = 1 - 0.2 = 0.8$

$N(0.7) = 1 - 0.7 = 0.3$

Monotonicity property satisfied

0.2 < 0.7 whereas N(0.2) > N (0.7)

Sugeno's Fuzzy Complement

One class of fuzzy complements is Sugeno's complement, defined by

$$N_s(a) = \frac{1-a}{1+sa}$$ Where parameter s > -1

For each value of **s,** we obtain a particular fuzzy complement operator.

$N_0(a)$ is same as classical complement.

$N_1(0) = (1-0)/1 = 1$

$N_1(1) = 0$

} Boundary property satisfied

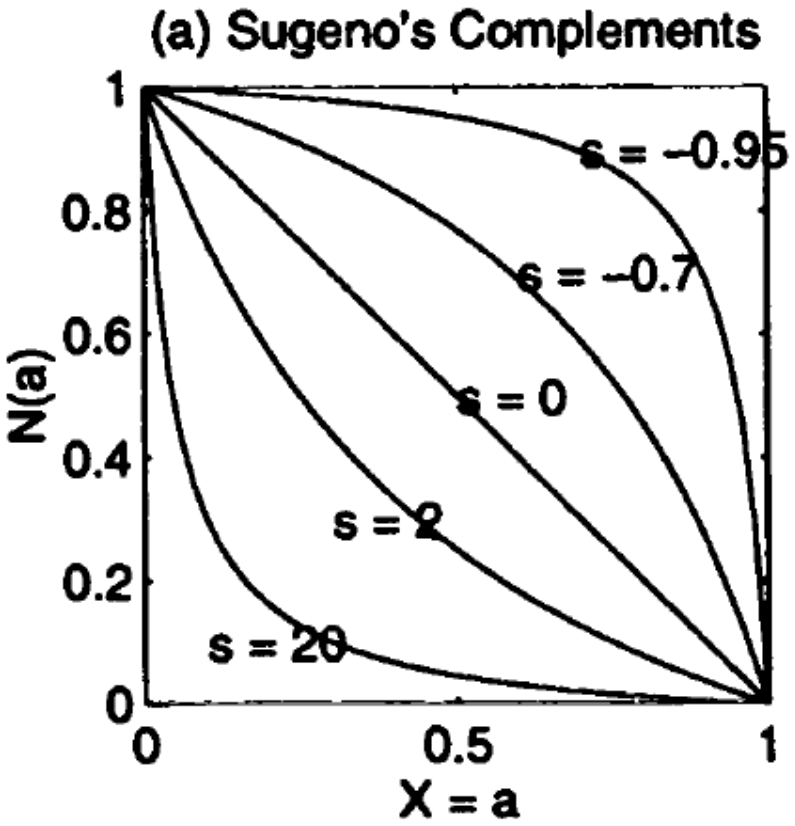
$N_1(0.2) = (1\text{-}0.2)./(1+1*0.2) = 0.667$

$N_1(0.7) = (1\text{-}0.7)./(1+1*0.7) = 0.1765$

} Monotonicity property satisfied

$0.2 < 0.7$ whereas $N_1(0.2) > N_1(0.7)$

Sugeno's Fuzzy Complement



Yager's Fuzzy Complement

Another class of fuzzy complements is Yager's complement, defined by

$$N_w(a) = (1 - a^w)^{\frac{1}{w}}$$ Where parameter w > 0

For each value of **w,** we obtain a particular fuzzy complement operator.

$N_1(a)$ is same as classical complement.

$N_2(0) = (1 - 0.\text{^}2).\text{^}(1/2) = 1$

$N_2(1) = (1\text{-}1.\text{^}2).\text{^}(1/2) = 0$

} Boundary property satisfied

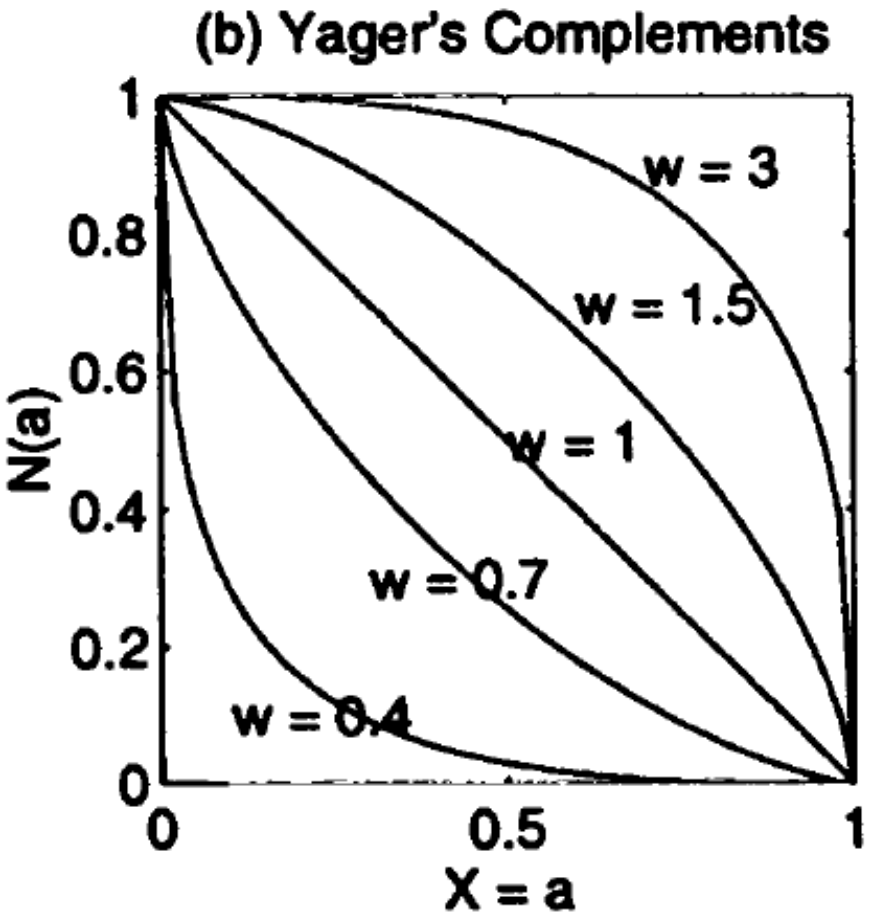
$N_2(0.2) = (1\text{-}0.2.\text{^}2).\text{^}(1/2) = 0.9798$

$N_2(0.7) = (1\text{-}0.7.\text{^}2).\text{^}(1/2) = 0.7141$

} Monotonicity property satisfied

$0.2 < 0.7$ whereas $N_2(0.2) > N_2(0.7)$

Yager's Fuzzy Complement



Fuzzy Complement (Matlab)

Task: Write script to implement following fuzzy complement operations on continuous membership functions and visualize them for different parameter values.

- Classical fuzzy complement
- Sugeno's fuzzy complement
- Yager's fuzzy complement

```
a = (0:0.02:1)';
c = 1 - a;

subplot(331)
plot(a,c)

axis('square');
title('Classical Complements');
xlabel('X = a');
ylabel('N(a)');
```

```
s =   20; c1 = (1-a)./(1+s*a);
s =    2; c2 = (1-a)./(1+s*a);
s =    0; c3 = (1-a)./(1+s*a);
s = -0.7; c4 = (1-a)./(1+s*a);
s = -0.95; c5 = (1-a)./(1+s*a);
complement_all = [c1 c2 c3 c4 c5];
subplot(332)
plot(a, complement_all);
text(0.1, 0.1, 's = 20');
text(0.3, 0.3, 's = 2');
text(0.5, 0.5, 's = 0');
text(0.6, 0.7, 's = -0.7');
text(0.7, 0.9, 's = -0.95');
```

Fuzzy Complement

Task: Show that Sugeno's complement satisfy involution property.

$N_0(a) = 1 - a$

Involution

$N_0(N_0(a)) = N_0(1-a) = 1 - (1-a) = a$

$N_2(a) = \frac{1-a}{1+2a}$

$N_2(N_2(a)) = a$

$= \frac{1 - \left(\frac{1-a}{1+2a}\right)}{1 + 2 * \left(\frac{1-a}{1+2a}\right)} =$

Thank you