Theory of Production

- **The theory of production brings out the nature and the extent of relationship between output and the factors of production and the cause of different kind of input-output relationships.**

- **Business managers have to take production decisions under two time frames – short-run and long-run. The short-run refers to time period during which supply of some inputs – specifically capital—remains constant and the supply of other factors, particularly of labour, are variable.**

- **The long-run refers to the time period during which all inputs – labour, capital, raw materials and technology – are supposed to be variable factors.**

Meaning of Production

- In general sense of the term, 'production' means transforming inputs (labour, capital, raw materials, time, etc.) into an output with value added.

- This concept of production is however limited to only 'manufacturing'. In economic sense, the term 'production' means a process by which resources (men, material, time, etc.) are transformed into a different form of product with greater utility.

- In other words, a process by which men, material, capital and time are converted into value added products is called production.

Input and Output

- An input is any thing that is used in the process of production of some thing. In the words of Baumol, "An input is simply any thing which the firm buys for use in its production or other processes".

- Production of different kinds of things requires different kinds of inputs, known also as 'factors of production', generally classified as

(i) land,

(ii) labour,

(i) capital,

(ii) raw materials,

(iii) time, and

(iv) entrepreneurship.

PRODUCTION FUNCTION

- **A production function states the technological relationship between inputs and output in the form of an equation, a table or a graph.**

- **In its general form, it specifies the inputs required for the production of a commodity or service. In its specific form, it states the extent of quantitative relationships between inputs and output.**

- **Besides, the production function represents the technology of a firm or of an industry. For example, suppose production of a product, say X, depends on labour (L) and capital (K), then production function is expressed in equation form as:**

$$Qx = f(L, K)$$

- **The actual production function is generally very complex. It includes a wide range of inputs, viz.,**

(i) land and building;

(ii) labour including manual labour, engineering staff and production manager,

(iii) capital,

(iv) raw material,

(v) time, and

(vi) technology.

All these variables enter the actual production function of a firm. The long run production function is generally expressed as

$$Q = f(LB, L, K, M, T, t)$$

where LB = land and building, L = labour, K = capital, M = raw materials, T = technology and t = time.

Product/Output Concepts

- Total Product: It gives maximum of output that can be produced at different levels of one input, assuming that the other input is fixed at a particular level.

Marginal Product: Change in the output resulting from a very small change in one factor input , keeping the other factor inputs constant. $MP_L = \Delta Q / \Delta L$

- Average Product: Total production for per unit of Variable input. $AP_L = Q/L$

THE LAWS OF PRODUCTION: *Laws of Variable Proportions*

- **The short-run laws of production state the input-output relations between output and one variable input (labour), other inputs (especially, capital) held constant.**

- **The laws of production under these conditions are called the 'Laws of Variable Proportions' or the 'Laws of Returns to a Variable Input'.**

- **The laws of returns to variable input can be states as when more and more of a variable input (Labour) is used with a given quantity of fixed inputs (Capital), the total output may initially increase at increasing rate, but the output eventually increases at diminishing rates.**

Law of Variable Proportions

- 'How Total, Average & Marginal Product/Output is affected by change in one input keeping other inputs constant' is the essence of the Law of Variable Proportions.
- "As proportion of one factor in a combination of factors is increased, Total, Marginal & Average Output/Product will increase then after a point, first Marginal then Average and at last the Total Output/Product will diminish".
- Applicable in short run.

- **Assumptions. The laws of returns to variable input, i.e., labour, is based on the following assumptions:**

(i) Labour is the variable input, capital remaining constant;

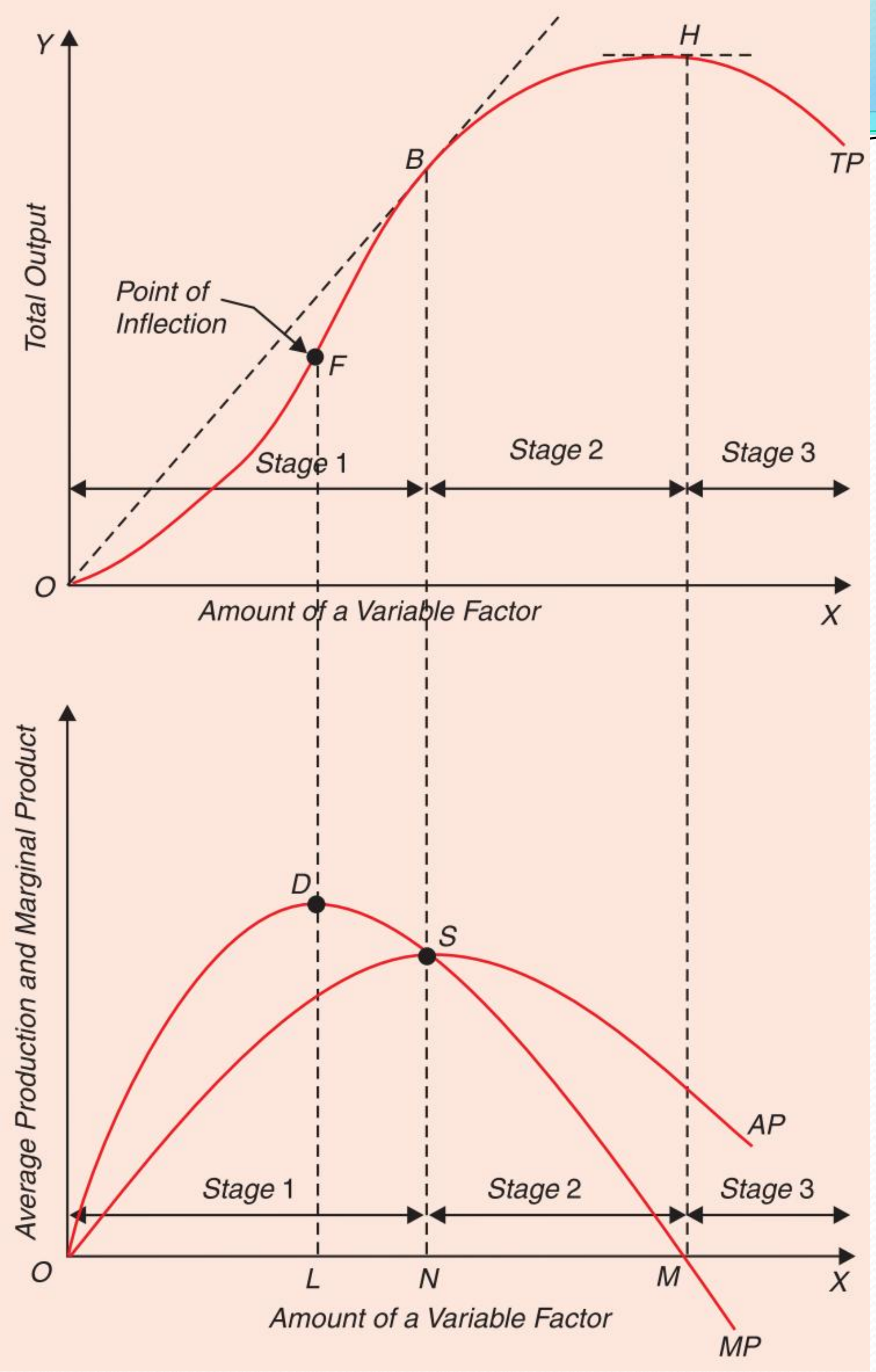
(ii) Labour is homogeneous;

(iii) The state of technology is given; and

(iv) Input prices remain constant.

Table: TP, MP, & AP calculation

Labour	TP	MP(ΔQ/ΔL)	AP(Q/L)
1	80	80	80
2	170	90	85
3	270	100	90
4	368	98	92
5	430	62	86
6	480	50	80
7	504	24	72
8	504	0	63
9	495	-9	55
10	480	-15	48



Y
H
B
TP
Total Output
Point of
Inflection
F
Stage 1
Stage 2
Stage 3
O
Amount of a Variable Factor
X
Average Production and Marginal Product
D
S
AP
Stage 1
Stage 2
Stage 3
O
L
N
M
X
Amount of a Variable Factor
MP

The Three Stages of Production

Stage I: Stage of Increasing Average Returns: (Increasing Return)

•AP is increasing and the MP is greater than the AP. Up to point B on the TP curve Stage I exist.
•AP is increasing, but MP is increasing first up to point A then decreasing.

Stage II: Stage of Decreasing Average Returns (Decreasing Return)

•Both AP and MP is decreasing. But MP is positive.
•The portion of TP curve between B and C represents this stage.

Stage III: Stage of Absolute Decreasing Average Returns (Negetive return)

•TP is diminishing and the MP is negative.
•The portion of TP curve which lies to the right of point C represents this stage.

In which stage would the rational producer like to operate?

In Stage I, MP and AP both are rising, and the MP is more than AP.

- A given increase in variable factor leads to a more than proportionate increase in the output.
- The producer is not making the best possible use of the fixed factor. A particular portion of fixed factor remains unutilized.

In Stage III, MP of variable factor is negative and the TP is also decreasing.

In Stage II, MP and AP both are falling and MP though positive, is less than AP.

- There is less than proportionate change in output due to change in labour force .
- Hence at this stage the producer will employ the variable factor in such a manner that the utilization of fixed factor is most efficient.

Thus Stage-II is the efficient stage and is a logical eventuality. That's why the Law of Variable Proportions is otherwise called as the Law of Diminishing Returns

Problem on Law of Variable Proportions

- Consider the following short-run production function (where X is the variable input and Q is the output).

$Q = 8X^2 - 0.5X^3$

Find out:

i) The Marginal Product function (MPx) and Average Product function (APx).

ii) The value of X that maximises output and the value of X at which AP is maximum.

Problem on Law of Variable Proportions

- Consider the following short-run production function (where L is Labour, K is Capital and Q is the output).

$Q = 10L^2K^2 - 0.2L^3K^3$

The value of K is given as 100 and it is fixed.

Find out:

i) The value of L that maximises the Average Product.

ii) The value of L that maximises Total Product.

iii) The value of L that maximises the Marginal Product (or corresponds to the Inflexion Point on TP curve)

THE LAWS OF LONG-RUN PRODUCTION: THE LAWS OF RETURNS TO SCALE

- **The long-term laws of production, i.e., the nature of relationship between inputs and output under the condition that both the inputs, capital and labour, are variable factors.**

- **In the long-run, supply of both the inputs is supposed to be elastic and, therefore, firms can use larger quantities of both labour and capital. With larger employment of capital and labour, the scale of production increases.**

- **The nature of changing relationship between changing scale of inputs and output is also refered to the laws of returns to scale.**

Isoquant: The Tool of Analyses

- The term ‘isoquant’ has been derived from the Greek word iso meaning ‘equal’ and Latin word quantus meaning ‘quantity’.

- The ‘isoquant curve’ is, therefore, also known as ‘Equal Product Curve’ and ‘Production Indifference Curve’.

- An isoquant curve can be defined as the locus of points representing various combinations of two inputs—capital and labour—yielding the same level of output.

- An ‘isoquant curve’ is analogous to consumer ‘indifference curve’, with two points of distinction:

(a) an indifference curve is constructed of two consumer goods while an isoquant curve is constructed of two production inputs (labour and capital), and

(b) an indifference curve represents a subjective level of utility whereas an isoquant represents the actual quantity of output of a commodity.

- Isoquant curves are drawn on the basis of the following assumptions:

(i) Only two inputs – labour (L) and capital (K)—are used to produce commodity (X);

(ii) Both L and K and product X are perfectly divisible;

(iii) The two inputs—L and K—can be substituted for each other but at a diminishing rate as they are imperfect substitutes; and

(iv) The technology of production is variable.

Marginal Rate of Technical Substitution (MRTS)

- The MRTS is a very important concept used in determining the shape and properties of the isoquants and also in analyzing the production with two variable inputs.

- In simple words, MRTS is the rate at which a marginal unit of one input is so substituted for a marginal unit of another input that the total output of the commodity remains constant.

- The MRTS gives the slope of the isoquant at different levels of input combin

$$MRTS = \frac{K_c - K_p}{L_c - L_p} = \frac{-\Delta K}{\Delta L} = \text{Slope of the isoquant}$$

Properties of Isoquants

The properties of isoquants are discussed below with respect to production inputs – labour and capital.

(i) Isoquants have a negative slope: The isoquants have a negative slope in their economic region.

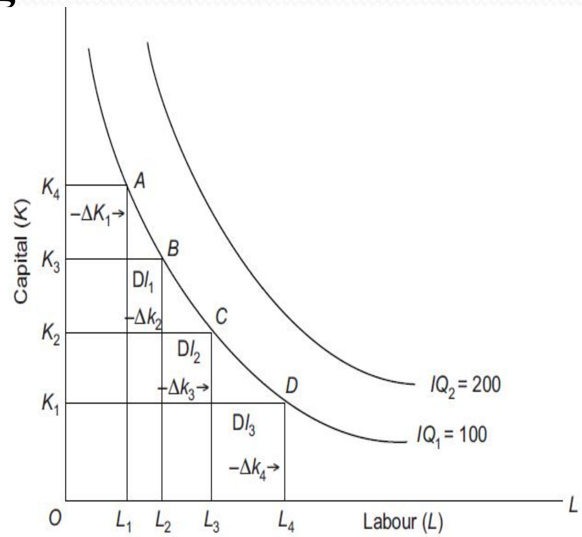


Fig. 10.4 *Curvilinear Isoquants*

Given the isoquants, consider the movement from point *A* to point *B* on isoquant IQ_1. As discussed above, *MRTS* between points *A* and *B* can be measured as

$$MRTS = K_3 - K_4 / L_2 - L_1$$
$$= -\Delta K_1 / \Delta L_1$$

(ii) Isoquants are convex to origin:

Isoquants are convex to origin in the sense that the curves tend to bend towards the point of origin.

(iii) Isoquants are neither intersecting and nor tangential:

Another important feature of isoquants as a tool of analysis is that they do not intersect nor are the isoquants tangent to one another. If isoquants intersect or are tangent, the laws of production get violated as it leads to two untenable facts:

(a) given the technology, a combination of two inputs, can produce two different quantities - larger and smaller, and

(b) a given quantity of a commodity can be produced with a smaller and a larger combination of inputs.

THE THEORIES OF LONG-TERM PRODUCTION: THE LAWS OF RETURNS TO SCALE

- **The laws of returns to scale explain the kind of change in output in response to a proportional and simultaneous change in inputs. Increasing inputs proportionately and simultaneously is, in fact, an expansion of the scale of production.**
- **When a firm expands its scale of production, i.e., it increases both the inputs in a certain proportion, then there are three technical possibilities of increase in production:**

(i) Total output may increase more than proportionately; Increasing Return to scale

(ii) Total output may increase proportionately; Constant Return to scale

(iii) Total output may increase less than proportionately. Decreasing Return to Scale

1. The Law of Increasing Returns to Scale

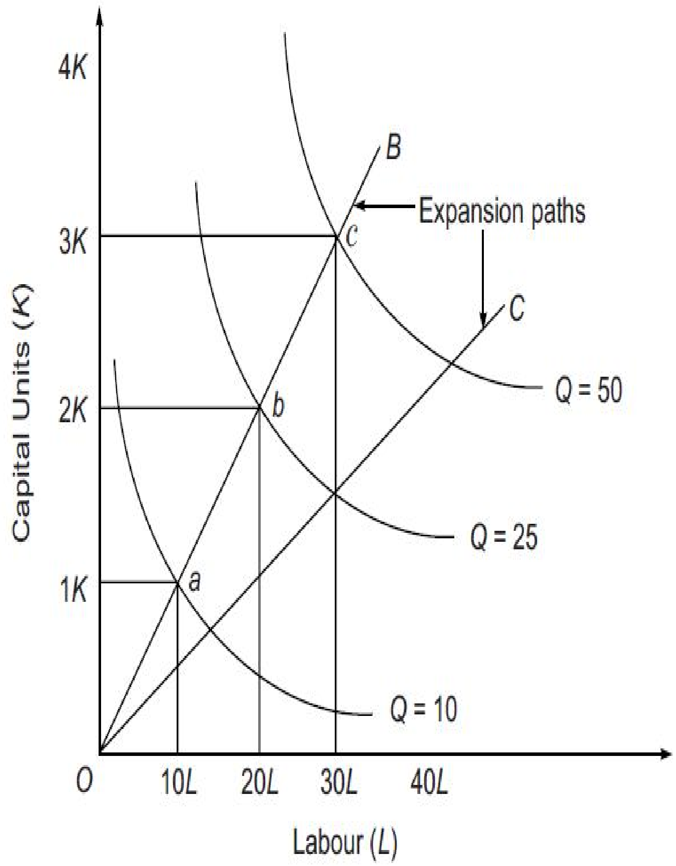
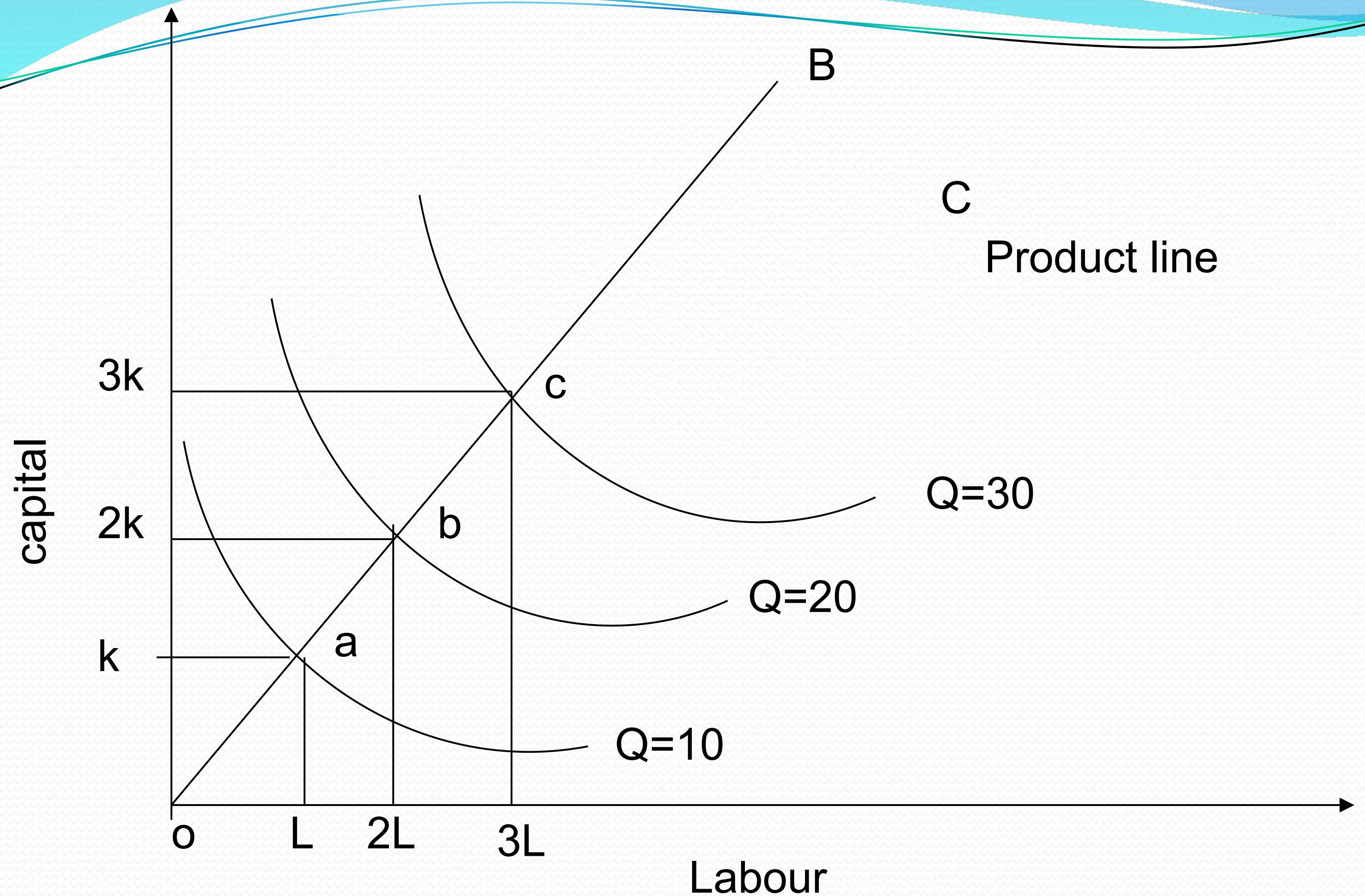


Fig. 10.8 *Increasing Returns to Scale*

- **As a result of doubling the inputs, output is more than doubled: it increases from 10 to 25 units, i.e., an increase of more than double.**
- **Similarly, the movement from point b to point c indicates 50% increase in inputs as a result of which the output increases from 25 units to 50 units, i.e., by 100%.**
- **Clearly, output increases more than the proportionate increase in inputs. This kind of relationship between the inputs and output exemplifies the law of increasing returns to scale.**

2.CONSTANT RETURN TO SCALE:

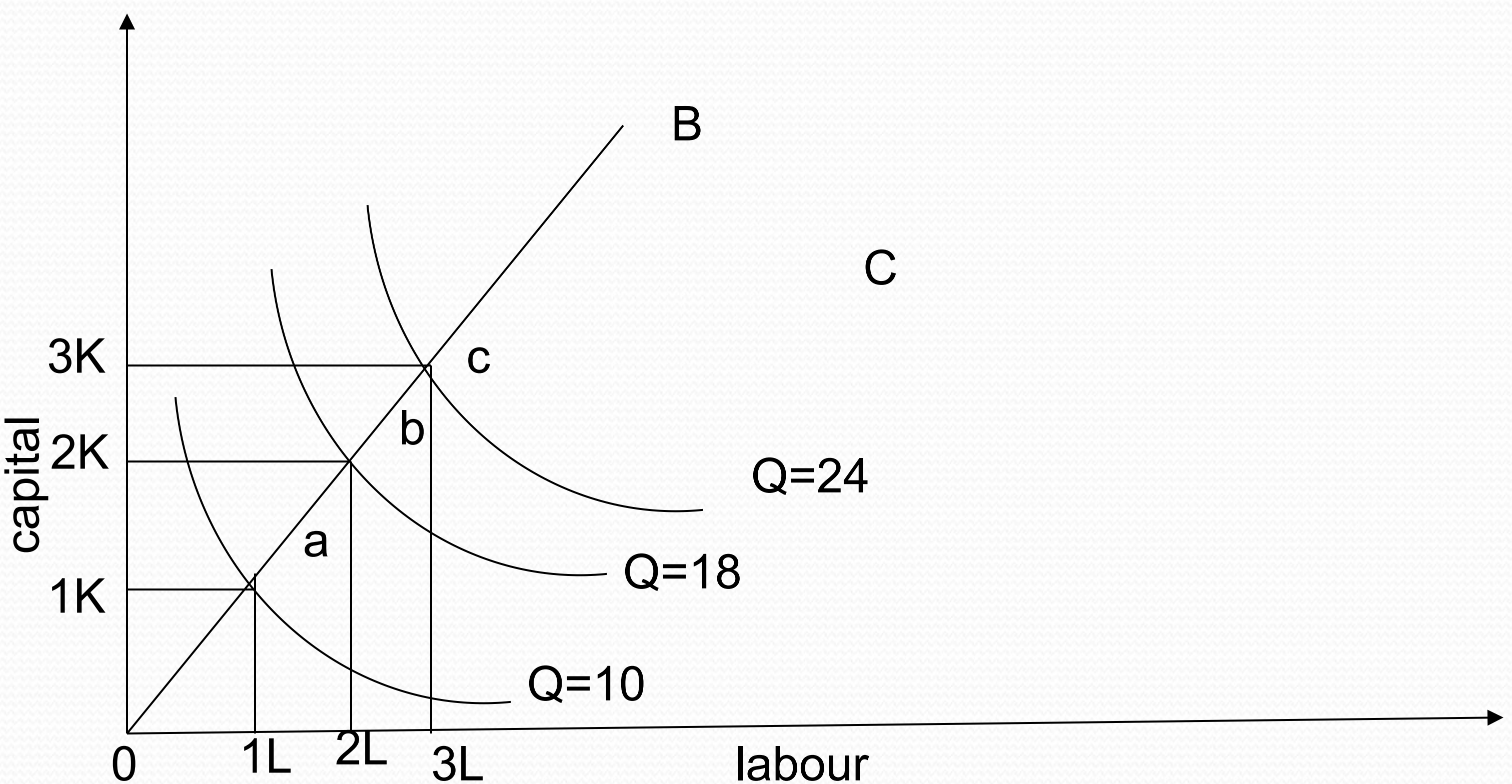
If we increase all factors in a given proportion and the output increases in a same proportion ,return scale are said to be the constant. so constant return to scale means that with the increase in the scale or the amount of all factors leads to a proportionate increase in output ,i.e. doubling of all inputs doubles the output.



constant return to scale:
oa =ab= bc

DECREASING RETURN TO SCALE:

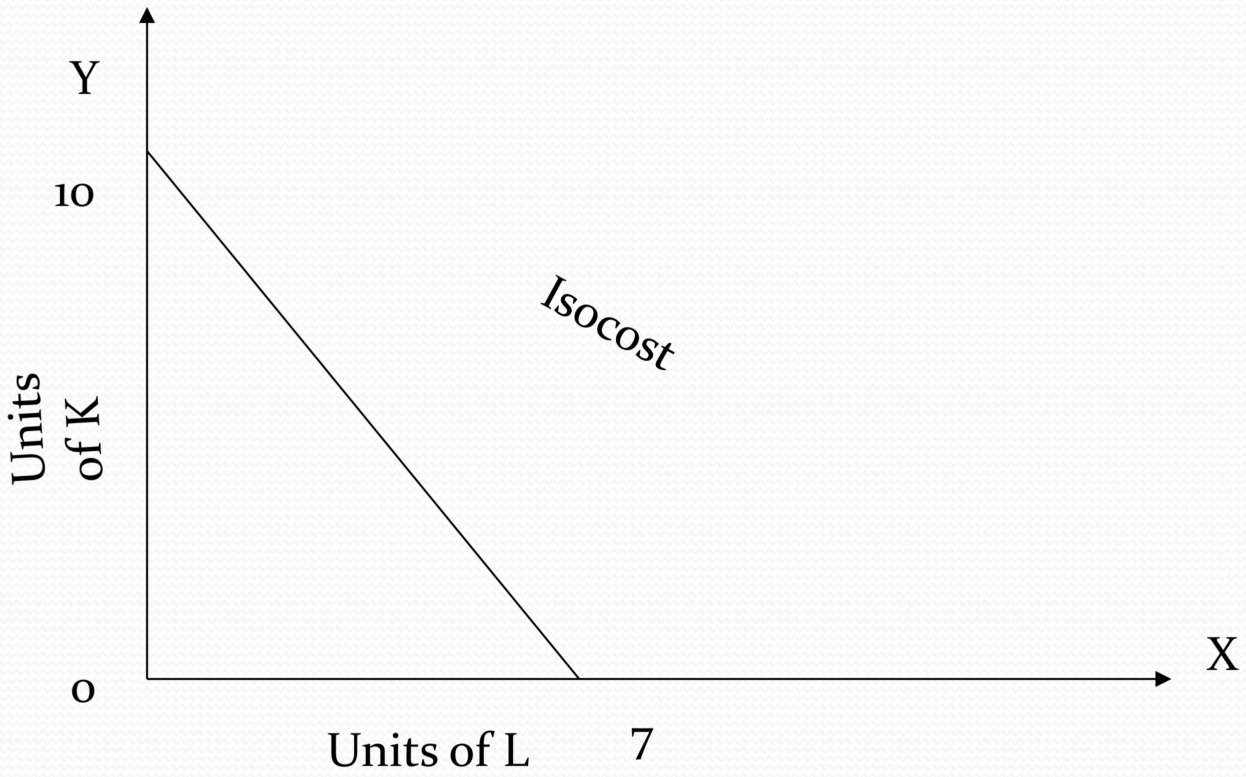
When output increases in a smaller proportion than the increase in all inputs i.e. called decreasing return to scale, when inputs are doubled and output is less than doubled then decreasing return to scale is in operation



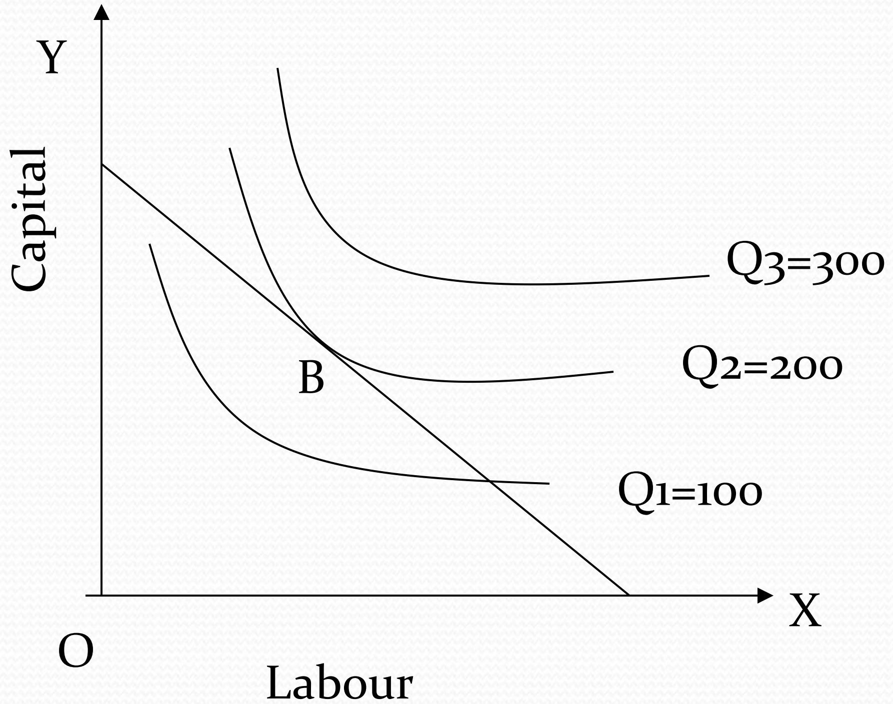
Iso-Cost line

- Assume that labour costs Rs.10 per unit and capital, Rs. 7 per unit.
- Suppose the company has a budget of Rs. 70.
- It can buy 7 units of labour (with no capital), or 10 units of K (with no labour), or some in-between combination.
- *By joining the two extreme points we get an isocost line*

Iso-Cost line



Producer's Equlibrium Point



Producer has a constraint- namely, budget.

Producer attains equilibrium when he reaches highest attainable level of output.

The Factors Behind Increasing Returns to Scale

- **The factors that lead to increasing returns to scale are known as internal economies of scale. There are at least three plausible factors causing increasing returns to scale.**

(i) Technical and managerial indivisibilities:

- **Certain inputs, particularly mechanical equipments and managers, used in the process of production are available in a given size.**
- **Such inputs cannot be divided into parts to match with small scale of production.**

(ii) Higher degree of specialization:

- **Another factor causing increasing returns to scale is the automatic improvement in the degree of specialization of both labour and machinery, which becomes possible with increase in scale of production.**

- **The use of specialized labour suitable to a particular job and of a composite machinery increases productivity of both labour and capital per unit of inputs.**

(iii) Dimensional relations:

- **Increasing returns to scale is also a matter of dimensional improvement. For example, when the length and breadth of a room (15′ × 10′ = 150 sq. ft.) are doubled, then the size of the room is more than doubled: it increases to 30′ × 20′ = 600 sq. ft.**

- **When diameter of a pipe is doubled, the flow of water is more than doubled. In accordance with this dimensional relationship, when the labour and capital are doubled, the output is more than doubled and so on.**

LAWS OF RETURNS TO SCALE VIA PRODUCTION FUNCTION

The laws of returns to scale may be explained more precisely by applying production function. Let us assume a production function involving two variable inputs (K and L) and one commodity X. The production function may then be expressed as:

$$Qx = f(K, L) \ldots$$

where Qx denotes the quantity of commodity X.

Cobb-Douglas Production Function— The Multiplicative Power Function

The most popular production function of this category is 'Cobb-Douglas Production Function of the form:

$$Q = AK^{a} L^{b}$$

where A is a positive constant; a and b are positive fractions; and b = 1 – a. The Cobb-Douglas production function is often used in its following form.

$$Q = AK^{a} L^{1-a}$$

Cobb-Douglas Production Function

Cobb-Douglas Proiduction Function is

$Q=AK^{\alpha}L^{\beta}$

Where A, α and β are constants, Q is Output and K & L are inputs (factors)

$\alpha + \beta$ indicates returns to scale.

If $\alpha + \beta = 1 \rightarrow$ Constant Returns to Scale

If $\alpha + \beta > 1 \rightarrow$ Increasing Returns to Scale

If $\alpha + \beta < 1 \rightarrow$ Decreasing Returns to Scale

Producer's Equlibrium through Cobb-Douglas Production Function

- A Producer remains in equilibrium when Iso-Quant and Iso-Cost Line becomes tangent to each other. It means, the condition of Producer's equilibrium is
- $MP_L/MP_K = P_L/P_K$

  $MP_L = \partial Q/\partial L$

  $MP_K = \partial Q/\partial K$

The equation obtained from the equilibrium condition, i. e., $MP_L/MP_K = P_L/P_K$ is also the equation of Expansion Path

- Expansion Path: Locus of Producer's equilibrium points emerged due to change in the producer's budget (resources disposable for production), otherthnigs remaining constant.

Question 1.1

A manufacturing firm faces the following short-run production function

$$Q = 6L^2 - 0.4L^3$$

(i) Find the Labour (L) unit beyond which the Marginal Product of Labour (MP_L) starts falling.

(ii) What is the Labour (L) unit after which Average Product (AP_L) remains higher than the Marginal Product (MP_L).

(iii) Find the Labour (L) unit beyond which the producer will not apply any more labour.

Question 1.2

(i) Decide the return to scale from the following input-output relation

$$Q = 0.5\ KL$$

(ii) In a certain production system output changes more than proportionately than the change in inputs. Draw a correct diagram with the help of Isoquants to explain it.

Question 1.3

In the long run the firm chooses the least cost combination of Labour and Capital to produce a desired output.

(i) Illustrate this with a proper diagram.

(ii) In a production process a firm is using labour and capital in such quantities that the Marginal Product of Labour (MP_L) is 20 and Marginal Product of Capital (MP_K) is 15. The price of Labour is \$6 and that of the Capital is \$5. Is the firm using optimal combination of inputs? How?

Question 2.1

An Auto manufacturing company faces the following production function in the short-run

$Q = 50L^2 - L^3$

where

Q = output

L = units of labour

(i) Find the labour units employed corresponding to the point of inflexion.

(ii) Find the labour units employed at the point before which Marginal Product (MP_L) remains higher than the Average Product (AP_L).

(iii) Find the labour units to be employed when MP_L is zero. What is the maximum output?

Question 2.2

The production function of a firm is

$Q = L^{.75}K^{.25}$

Q = output

L = labour

K = capital

Given the wage (w) = Rs.150 per unit and the rent (r) = Rs.50 per unit.

(i) Find whether the firm will use equal amounts of labour and capital at the producer's equilibrium point.

(ii) If the firm's ISO COST line is

$20000 = 150L + 50K$

What unit of labour and capital it uses? With the help of a figure show this equilibrium condition of the firm.

Question 3.1

Explain the law of variable proportion with suitable diagrams.

Question 3.2

From the following table find out $MRTS_{LK}$ and $MRTS_{KL}$

Combination	Labour(L)	Capital(K)
A	10	20
B	15	19
C	19	18
D	22	17

Question 3.3

Distinguish between increasing returns to scale and constant returns to scale.

Question 4.1

Wear and run shoe company manufactures shoe for exports. The shortrun production function faced by the company is

$$Q = 15L^2 - L^3$$

(i) Find the labour (L) to be employed beyond which Average Product (AP_L) will decline.

(ii) Find the labour (L) to be employed beyond which Marginal Product (MP_L) will be negative.

Question 4.2

The Average Production of Labour (AP_L) is given by the equation.

$$AP_L = 200 + 1000L - 10L^2$$

(i) At what unit of Labour Marginal Production of Labour (MP_L) is maximum?

(ii) Verify that AP_L is maximum at a labour unit that is higher than the labour unit where MP_L is maximum.

Question 4.3

A firm has the following short run production function where the only variable input is Labour(L). The output(Q) function is $Q=9L^2-0.5L^3$.

(i) Find the labour to be employed at the end of Stage-I.

(ii) Find the labour to be employed at the end of Stage-II.

Question 5.1

Delisha, an entrepreneur is facing the following short-run production function

$Q = f(L)$

$Q = 100L + 500L^2 – 5L^3$

where Q = output / L = units of Labour

(i) Beyond what labour unit the Total Product of Labour will increase at a decreasing rate?

Find the Labour unit after which Delisha will enter into the stage of Decreasing Return in the short run.

Thank You

Question 1.1:- Answer

$Q = 6L^2 - 0.4L^3$

(i) $MP_L = 12L - 1.2L^2$

$$\frac{dMP_L}{dL} = 0 \qquad \Rightarrow 12 - 2.4L = 0$$

$$\Rightarrow L = 5$$

(ii) $AP_L = 6L - 0.4L^2$

$$\frac{dAP_L}{dL} = 0 \Rightarrow 6 - 0.8L = 0 \Rightarrow L = 7.5$$

(iii) $MP_L = 0 \Rightarrow 12L - 1.2L^2$

$$\Rightarrow L = 10$$

Question 1.2:- Answer

(i) Q = 0.5KL shows Increasing Return to Scale (IRS)

(ii) It is IRS. Draw a suitable diagram with the help of isoquants and explain the IRS.

Question 1.3: Answer

Illustrate the least cost combination of labour and capital to produce a desired output with proper diagram.

(ii) $MP_L = 20$ $MP_K = 15$

$W = 6$, $r = 5$

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

Here $\frac{20}{6} > \frac{15}{5}$

The firm is not using optimal combination of inputs.

Question 2.1: Answer

(i) $MP_L = \frac{dQ}{dL} = 100L - 3L^2$ $\quad \frac{dMP_L}{dL} = 0 \Rightarrow 100 - 6L = 0$

$\Rightarrow L = 16.66$

(ii) $AP_L = 50L - L^2$ $\quad \frac{dAP_L}{dL} = 0 \Rightarrow 50 - 2L = 0$

$\Rightarrow L = 25$

(iii) $MP_L = 100L - 3L^2$ $\quad MP_L = 0 \Rightarrow 100L - 3L^2 = 0$

$\Rightarrow L(100 - 3L) = 0$

$\Rightarrow L = 33.3333333$

Maximum output

$Q = 50L^2 - L^3$

$= 55,555.5554 - 37037.0369$

$=18518.5185$ units

Question 2.2:- Answer

(i) $Q = L^{.75}K^{.25}$

$w = 150 \quad r = 50$

$$MP_L = \frac{\partial Q}{\partial L} = .75L^{-.25}K^{.25}$$

$$MP_K = \frac{\partial Q}{\partial K} = .25L^{.75}K^{-.75}$$

$$\frac{MP_L}{MP_K} = \frac{150}{50}$$

$$\Rightarrow \frac{3K}{L} = 3$$

$$\Rightarrow K = L$$

The firm uses same amount of L & K.

(ii) $20000 = 150L + 50K$

$L = 100, K = 100$

$\Rightarrow$ Diagram for the equilibrium condition.

Question 3.2:- Answer

Combination	Labour(L)	Capital(K)	$MRTS_{LK}$	$MRTS_{KL}$
A	10	20	-	-
B	15	19	1/5	5/1
C	19	18	1/4	4/1
D	22	17	1/3	3/1

Question 4.2:- Answer

$AP_L = 200 + 1000L – 10L^2$

$TP_L = 200L + 1000L^2 – 10L^3$

(i) $MP_L = \dfrac{dTP_L}{dL} = 200 + 2000L – 30L^2$

$\dfrac{dMP_L}{dL} = 0$

$\Rightarrow$ 2000 – 60L = 0

$\Rightarrow$ L = 33.33 units (lower labour units)

(ii) $\dfrac{dAP_L}{dL} = 0$

$\Rightarrow$ 1000 – 20L = 0

$\Rightarrow$ L = 50 units (higher labour units)

Question 4.3:- Answer

$Q = 9L^2 – 0.5L^3$

(i) $AP_L = 9L – 0.5L^2$

$$\frac{dAP_L}{dL} = 0$$

$\Rightarrow$ $9 – L = 0$

$\Rightarrow$ $L = 9$ units

(ii) $MP_L = 18L – 1.5L^2 = 0$

$\Rightarrow$ $L = 12$ units

Question 4.1:- Answer

$Q = 15L^2 – L^3$

$AP_L = 15L – L^2$

$$\frac{dAP}{dL} = 0 \Rightarrow 15 - 2L = 0$$

$\Rightarrow$ $L = 7.5$ units

(ii) $MP_L = 30L – 3L^2 = 0$

$\Rightarrow$ $L(30 – 3L) = 0$

$\Rightarrow$ $L = 10$ units

Question 5.1:- Answer

$Q = 100L + 500L^2 - 5L^3$

(i) $MP_L = 100 + 1000L - 15L^2$

$$\frac{dMP_L}{dL} = 100 - 30L = 0$$

$\Rightarrow L = 33.33$

(ii) $AP_L = 100 + 500L - 5L^2$

$$\frac{dAP_L}{dL} = 500 - 10L = 0$$

$\Rightarrow L = 50$