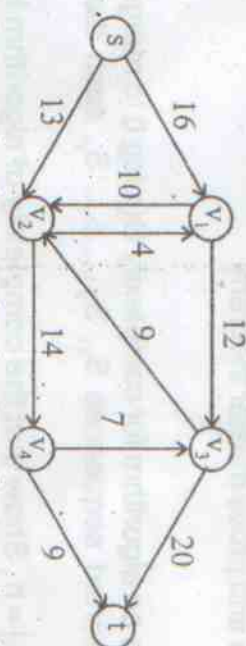


5. a) In the flow network illustrated below each directed edge is labelled with its capacity. Use Ford-Fulkerson algorithm to find the maximum flow. [5]



- b) Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions are (5, 10, 3, 12, 5, 6). [5]
6. a) State and explain quick-sort algorithm. Derive the worst-case time complexity for the same. [5]
- b) Determine the cost and structure of an optimal binary search tree for a set of $n=5$ keys with the following probabilities. [5]

	0	1	2	3	4	5
p_i		0.15	0.10	0.09	0.07	0.14
q_i	0.07	0.05	0.08	0.08	0.07	0.10

7. a) Solve the following linear programming using SIMPLEX Algorithm. [5]
- minimize $x_1 + x_2 + x_3$
- subject to: $2x_1 + 7.5x_2 + 3x_3 \geq 10000$
- $20x_1 + 5x_2 + 10x_3 \geq 30000$
- $x_1, x_2, x_3 \geq 0$
- b) State and explain Heap sort algorithm and find its time complexity. [5]
8. Write short notes (any two) [5 × 2]
- a) Amortized Analysis
- b) Topological Sort
- c) Randomized Hiring Problem



FIRST SEMESTER EXAMINATION-2009

ALGORITHM & COMPLEXITY

[PGCS 101]

Full Marks: 70

Time: 3 Hours

Answer any SIX questions including Question No.1 which is compulsory.

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.

1. a) Consider the following pseudo code: [2 × 10]

```

int n, A[100];
void X ( )
{
    int i;
    for (i = n/2; i ≥ 1; i--)
        Y (i);
    while (n > 1)
        Y (1);
}

```

```

}
void Y (int i)
{
    ...
}

```

let the complexity of 'Y' is $O(n \log_2 n)$. Then the complexity of 'X' will be

(i) $O(n^2)$ (ii) $O(n^2 \log_2 n)$ (iii) $O(n \log_2 n)$ (iv) $O(n)$

- b) In a binary max heap containing 'n' numbers, the smallest element can be found in time

(i) $O(n)$ (ii) $O(n \log_2 n)$ (iii) $O(\log_2 \log_2 n)$ (iv) $O(1)$

c) Solve : $T(n) = T(n-1) + \frac{1}{n}$

d) What is asymptotic notation? Define Big-oh, Big-Theta and Big-Omega notations.

e) Consider the following functions

$$f(n) = 3n^{\sqrt{n}}$$

$$g(n) = 2^{\sqrt{n} \log_2 n}$$

$$h(n) = n!$$

which of the following is true.

(i) $h(n) = O(f(x))$

(ii) $h(n) = O(g(x))$

(iii) $g(n)$ is not $O(f(x))$

(iv) $f(x) = O(g(n))$

f) A sequence of 'n' operations is performed on a data structure. The i^{th} operation costs 'i' if 'i' is an exact power of 2 and 1 otherwise. Use aggregate analysis to determine the amortized cost per operation.

g) Convert the following linear program into standard form.

$$\text{minimize } 2x_1 + 7x_2$$

$$\text{subject to: } x_1 = 7$$

$$3x_1 + x_2 \geq 24$$

$$x_2 \geq 0$$

h) Differentiate between dynamic programming and greedy approach.

i) Differentiate between DFS and BFS.

j) Find the correct matching for the following columns.

- | | |
|-----------------------------|---------------------------|
| (i) All pair shortest path | (i) Greedy |
| (ii) Quick-Sort | (ii) Depth First Search |
| (iii) Minimum Spanning Tree | (iii) Dynamic Programming |
| (iv) Connected component | (iv) Divide and Conquer |

2. a) Solve the following Recurrence Relation using recursion tree.

(2)

T3

$$T(n) = T(n-a) + T(a) + cn$$

where $a \geq 1$, $c > 0$ and a and c are constants.

b) State and prove master theorem.

3. a) Write an algorithm for complexity $O(n \log l)$ to merge the sorted sequences S_1, S_2, \dots, S_p such that

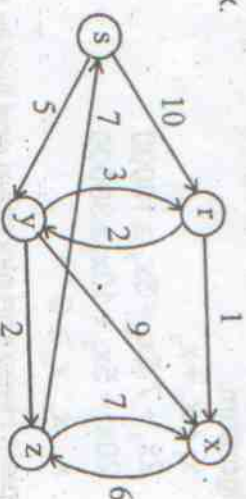
$$\sum_{i=1}^p |S_i| = n, \text{ Show that the complexity of algorithm is } O(n \log l).$$

b) Construct a Hopmann code for the following data.

Character	A	B	C	D	E
Frequency/100 char	0.4	0.1	0.2	0.15	0.15

4. a) Write Kruskal's Algorithm. What are its applications? How it is different from prim's algorithm?

b) Which of the set of shortest path estimates nodes is not obtained as an intermediate on applying Dijkstra's algorithm on the following graph where 's' is the source vertex.



(i) s t x y z
0 ∞ ∞ 5 7

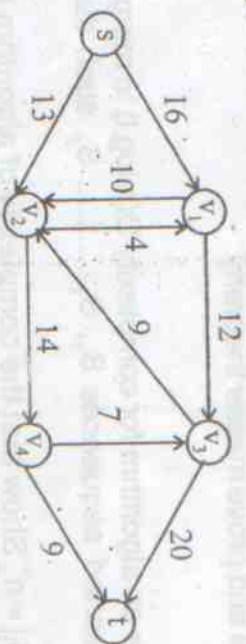
(ii) s t x y z
0 10 ∞ 5 ∞

(iii) s t x y z
0 8 9 5 7

(iv) s t x y z
0 8 13 5 7

(3)

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(1)