Unit IV: Economic Appraisal Techniques
Net Present Value (NPV), Internal Rate of Return (IRR), Cost Benefit analysis. Depreciation calculation; Meaning and Definition, Methods.

N.B. Exclude the pay-back period criteria for this examination

Economic Appraisal Techniques-Pay-Back Period criteria, Net Present Value (NPV), Internal Rate of Return (IRR) comparison with MARR, Cost- Benefit analysis, Numerical Examples
DEPRECIATION CALCULATION: Meaning and Definition **Methods:** Straight Line Method, Declining Balance method, Sum-of-years digit method and Sinking Fund Method (Methods to be explained with illustrations)

1. Payback period criteria

Payback period is the time in which the initial outlay of an investment is expected to be recovered through the cash inflows generated by the investment.

The formula to calculate the payback period of an investment depends on whether the periodic cash inflows from the project are **even** or **uneven**.

Case 1: Even cash inflows

$$Payback\ Period = \frac{Initial\ Investment}{Net\ Cash\ Flow\ per\ Period}$$

Example 1:

Company C is planning to undertake a project requiring initial investment of $105 million. The project is expected to generate $25 million per year in net cash flows for 7 years. Calculate the payback period of the project.

Solution:
Payback Period = Initial Investment ÷ Annual Cash Flow = $105M ÷ $25M = 4.2 years

Example 2:

The Delta company is planning to purchase a machine which would cost $25,000 and have a useful life of 10 years with zero salvage value. The expected annual cash inflow of the machine is $10,000. Find the viability in acquiring this machine.

Payback period = $25,000/$10,000 = 2.5 years

The purchase of this machine is desirable because its payback period is 2.5 years which is shorter than the maximum payback period of the company.

Case 2: Uneven cash inflows

$$Payback\ Period = A + \frac{B}{C}$$

Where:
A is the last period number with a negative cumulative cash flow; **B** is the absolute value (i.e. value without negative sign) of cumulative net cash flow at the end of the period A; and **C** is the total cash inflow during the period following period A

Years before full recovery
Unrecovered cost at start of the year
Cash flow during the year

Example 2:

An investment of $200,000 is expected to generate the following cash inflows in six years:

Year 1: $70,000	**Year 2:** $60,000	**Year 3:** $55,000
Year 4: $40,000	**Year 5:** $30,000	**Year 6:** $25,000

Required: Compute payback period of the investment. Should the investment be made if management wants to recover the initial investment in 3 years or less?

Solution:

Initial investment = $200000						
Year	1	2	3	4	5	6
Cash Inflow	$70000	$60000	$55000	$40000	$30000	$25000
Cumulative cash inflow	$70000	$130000	$185000	$225000	$255000	$280000

Payback period = 3 + (15,000*/40,000) = 3 + 0.375 = 3.375 Years

*Unrecovered investment at start of 4th year:
= Initial cost – Cumulative cash inflow at the end of 3rd year = $200000 – $185000 = $15000

The payback period for this project is 3.375 years which is longer than the maximum desired payback period of the management (3 years). The investment in this project is therefore not desirable.

2. Net Present Value (NPV)

Net Present Value = Present Value – Initial Cost

Year	Project A			Project B		
	Net Cash Income (Rs.)	Discount factor (@7%)	PV (Rs.)	Net Cash Income (Rs.)	Discount factor (@7%)	PV (Rs.)
1	4000	$=\frac{4000}{(1.07)^1}$	3,740	8000	$=\frac{8000}{(1.07)^1}$	7,480
2	4000	$=\frac{4000}{(1.07)^2}$	3,492	6000	$=\frac{6000}{(1.07)^2}$	5,238
3	4000	$=\frac{4000}{(1.07)^3}$	3,264	2000	$=\frac{2000}{(1.07)^3}$	1,632
4	8000	$=\frac{8000}{(1.07)^4}$	6,104	2000	$=\frac{2000}{(1.07)^4}$	1,526
5	2000	$=\frac{2000}{(1.07)^5}$	1,426	2000	$=\frac{2000}{(1.07)^5}$	1,426
6				2000	$=\frac{2000}{(1.07)^6}$	1,332
7				2000	$=\frac{2000}{(1.07)^7}$	1,246
8				2000	$=\frac{2000}{(1.07)^8}$	1,164
Total Present Value (Rs.)			18,026			21,044
Initial Cost (Rs.)			20,000			20,000
Net Present Value (Rs.)			-1,974			1,044

3. Internal Rate of Return (IRR)

A company is trying to diversify its business in a new product line. The life of the project is 10 years with no salvage value at the end of its life. The initial outlay of the project is Rs.2,000,000. The annual net profit is Rs.350,000. Find the rate of return for the new business.

Solution

$$(P/A, i, n) = \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

Life of the product line(n) = 10 years
Initial outlay = Rs.2,000,000
Annual net profit = Rs.350,000
Scrap value after 10 years = 0

If i = 8% PW (8%) = -2,000,000 + 350,000 (P/A, 8%, 10)
= -2,000,000 + 350,000 (6.710) = Rs. 3,48,500

If i = 10% PW (10%) = -2,000,000 + 350,000 (P/A, 10%, 10)
= -2,000,000 + 350,000 (6.1446) = Rs.150,610

If i = 12% PW (12%) = -2,000,000 + 350,000 (P/A, 12%, 10)
= -2,000,000 + 350,000 (5.6502) = Rs. –22,430

IRR = 10% + [150,610 – 0 / 150,610 – (–22,430)] * 2% = **11.74%**

IRR ≥ MARR => Accept the proposal or else reject

4. Cost Benefit analysis or Benefit Cost analysis

$$\text{BC ratio} = \frac{\text{Equivalent benefits}}{\text{Equivalent costs}}$$

$$\text{BC ratio} = \frac{B_P}{P + C_P} = \frac{B_F}{P_F + C_F} = \frac{B_A}{P_A + C} \quad \text{or} \quad \text{BC ratio} = \frac{B_P}{P + C_P} + \frac{B_F}{P_F + C_F} + \frac{B_A}{P_A + C}$$

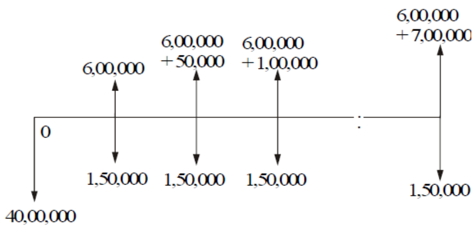
B_P = present worth of the total benefits
B_F = future worth of the total benefits
B_A = annual equivalent of the total benefits
P = initial investment
P_F = future worth of the initial investment
P_A = annual equivalent of the initial investment
C = yearly cost of operation and maintenance
C_P = present worth of yearly cost of operation and maintenance
C_F = future worth of yearly cost of operation and maintenance

Example 1:

In a particular locality of a state, the vehicle users take a roundabout route to reach certain places because of the presence of a river. This results in excessive travel time and increased fuel cost. So, the state government is planning to construct a bridge across the river. The estimated initial investment for constructing the bridge is Rs.40,00,000. The estimated life of the bridge is 15 years. The annual operation and maintenance cost is Rs.1,50,000. The value of fuel savings due to the construction of the bridge is Rs.6,00,000 in the first year and it increases by Rs.50,000 every year thereafter till the end of the life of the bridge. Check whether the project is justified based on BC ratio by assuming an interest rate of 12%, compounded annually.

Solution 1:

Initial investment = Rs. 40,00,000
Annual operation and maintenance = Rs. 1,50,000
Annual fuel savings during the first year = Rs. 6,00,000
Equal increment in fuel savings in the following years = Rs. 50,000
Life of the project = 15 years
Interest rate = 12%



Total present worth of costs **(or Equivalent costs)** = Initial investment (P) + Present worth of annual operating and maintenance cost (C_P) = $P + C_P$

= Rs. 40,00,000 + 1,50,000 × (P/A, 12%, 15)

$$\boxed{(P/A, 12\%, 15) = \frac{(1+i)^n - 1}{i\,(1+i)^n}}$$

= Rs. 40,00,000 + 1,50,000 × 6.8109
= Rs. 50,21,635

Total present worth of fuel savings (B_P):

A_1 = Rs. 6,00,000
G = Rs. 50,000
n = 15 years
i = 12%

Annual equivalent fuel savings (A) $= A_1 + G\,(A/G, 12\%, 15)$
$= 6{,}00{,}000 + 50{,}000\,(4.9803)$
$=$ Rs. 8,49,015

$$(A/G, 12\%, 15) = \frac{(1+i)^n - in - 1}{i\,(1+i)^n - i}$$

Present worth of the fuel savings (B_P) **(or Equivalent benefits)** $= A\,(P/A, 12\%, 15)$
$= 8{,}49{,}015\,(6.8109)$
$=$ Rs. 57,82,556

BC ratio $= \frac{B_p}{P+C_p} = \frac{57{,}82{,}556}{50{,}21{,}635} = 1.1515$

Since the BC ratio is more than 1, the project is economically reasonable.

Example 2:

Two mutually exclusive projects are being considered for investment. Project A_1 requires an initial outlay of Rs.30,00,000 with net receipts estimated as Rs.9,00,000 per year for the next five years. The initial outlay for the project A_2 is Rs.60,00,000, and net receipts have been estimated at Rs.15,00,000 per year for the next seven years. There is no salvage value associated with either of the projects. Using the benefit cost ratio, which project would you select? Assume an interest rate of 10%.

Solution 2:

Project A_1

Initial cost (P) = Rs. 30,00,000
Net benefits/year (B) = Rs. 9,00,000
Life (n) = 5 years
Annual equivalent of initial cost $= P \times (A/P, 10\%, 5)$
$= 30{,}00{,}000 \times (0.2638)$
$=$ Rs.7,91,400

$$(A/P, 10\%, 5) = \frac{i\,(1+i)^n}{(1+i)^n - 1}$$

Benefit-Cost ratio = Annual equivalent benefit / Annual equivalent cost
= 9,00,000/7,91,400 = **1.137**

Project A_2

Initial cost (P) = Rs. 60,00,000
Net benefits/year (B) = Rs. 15,00,000
Life (n) = 7 years
Annual equivalent of initial cost $= P \times (A/P, 10\%, 7)$
$= 60{,}00{,}000 \times (0.2054)$
$=$ Rs. 12,32,400

BC ratio = Annual equivalent benefit / Annual equivalent cost
= 15,00,000/12,32,400 = **1.217**

Since, **1.217 > 1.137** Project A_2 will be selected.

Depreciation calculation: ***See the slides***

Meaning and Methods of Accounting Depreciation

Meaning

Depreciation is the decrease in the value of physical properties with the passage of *time* and *use*.

- It can be defined in three senses:
  - **Physical** Depreciation: Due to physical decay
  - **Economic** Depreciation: Loss of value of an asset based on technology, ownership, rights, etc.
  - **Accounting** Depreciation: Estimated value of fall in the worth of an asset

Causes of Depreciation

1. Physical depreciation
2. Functional depreciation
3. Technological depreciation
4. Accident (Calamities, Fire, Water)
5. Depletion
6. Monetary depreciation
7. Time factor
8. Deferred maintenance

Methods of Accounting Depreciation

1. Straight line method of depreciation
2. Declining balance method of depreciation
3. Sum of the years-digits method of depreciation
4. Sinking-fund method of depreciation

5. Service output method of depreciation (N/A)

Straight Line Method of Depreciation

In this method of depreciation, a fixed sum is charged as the depreciation amount throughout the lifetime of an asset such that the accumulated sum at the end of the life of the asset is exactly equal to the purchase value of the asset.

Formula:

$$D_t = (P-S)/n$$

$$B_t = B_{t-1} - D_t = P - t[(P-S)/n]$$

D_t = Depreciation amount for the period t.

B_t = Book Value of the asset at the end of the period t.

P = Purchase Price or First Cost of the Asset

S = Salvage Value of the asset

n = Life of the asset

Example-1 (Straight Line Method of Depreciation)

Company has purchased an equipment whose cost is Rs.1,00,000 with an estimated life of 8 years. The salvage value is Rs.20,000. Determine the depreciation charge and book value at the end of various years using the straight line method of depreciation.

P = Rs. 1,00,000 , S= Rs. 20,000 , n = 8 years

$D_t = (P – S)/n = (1,00,000 – 20,000)/8 =$ Rs. 10,000

$B_t = B_{t-1} – D_t = P – t[(P-S)/n]$

The value of D_t is same for all the years but, B_t is different for each year.

Year (t)	0	1	2	3	4	5	6	7	8
$\mathbf{D_t}$	0	10000	10000	10000	10000	10000	10000	10000	10000
$\mathbf{B_t}$	100000	90000	80000	70000	60000	50000	40000	30000	20000

Exercise 1:

Consider Last Example and compute the depreciation and the book value for period 5.

Declining Balance Method of Depreciation

Constant percentage of the book value of the previous period of the asset will be charged as the depreciation amount for the current period.

Formula:

$$D_t = K \times B_{t-1}$$

$$B_t = B_{t-1} – D_t = B_{t-1} – K \times B_{t-1} = (1\text{-}K) \times B_{t-1}$$

The Formula for depreciation and book value in terms of P are as follows:

$$D_t = K(1\text{-}K)^{t-1} \times P$$

$$B_t = (1\text{-}K)^t \times P$$

Where;

K = a fixed percentage

Example -2 (Declining Balance Method)

Ref. Example-1: First cost is Rs.100000; Estimated life is 8 years. The salvage value is Rs.20000. Compute the depreciation by if K is 0.2.

$$P = \text{Rs. } 1{,}00{,}000 \text{ ; } S = \text{Rs. } 20{,}000 \text{ ; } n = 8 \text{ years ; } K = 0.2$$

$$D_t = K \times B_{t-1}$$

$$B_t = B_{t-1} - D_t = B_{t-1} - K \times B_{t-1} = (1-K) \times B_{t-1}$$

$$D_t = K(1-K)^{t-1} \times P$$

$$B_t = (1-K)^t \times P$$

t	0	1	2	3	4	5	6	7	8
$\mathbf{D_t}$	0	20000	16000	12800	10240	8192	6553.6	5242.88	4194.3
$\mathbf{B_t}$	100000	80000	64000	51200	40960	32768	26214.4	20971.5	16777.2

Exercise 2: *Ref. Example-1* calculate the depreciation and book value for period 5.

$$D_t = K(1-K)^{t-1} \times P$$
$$D_5 = 0.2(1-0.2)^4 \times 1{,}00{,}000$$
$$= \text{Rs. } 8{,}192$$

$$B_t = (1-K)^t \times P$$
$$B_5 = (1-0.2)^5 \times 1{,}00{,}000$$
$$= \text{Rs. } 32{,}768$$

Sum-of-the-Years-Digits Method of Depreciation

The book value decreases at a decreasing rate. Asset has a life of 8 years hence, the *sum of years* = n (n + 1) / 2 = **36**

The rate of depreciation charged in first year is maximum & it decreases thereafter as follows: 8/36, 7/36, 6/36, 5/36, 4/36, 3/36, 2/36, and 1/36.

Formula:

For any year, depreciation is calculated by multiplying the corresponding rate of depreciation with (P – S).

$$D_t = \text{Rate}\,(P - S)$$

$$B_t = B_{t-1} - D_t$$

The formulae for D_t and B_t for a specific year t are as follows:

$$D_t = \frac{n-t+1}{n(n+1)/2}(P - S)$$

$$B_t = (P - S)\frac{n-t}{n}\frac{n-t+1}{n+1} + S$$

Example-3 (Sum-of-the-years-digits Method)

Ref. Example-1: First cost is Rs.100000; Estimated life is 8 years. The salvage value is Rs.20000.

P = Rs. 1,00,000 ; S = Rs. 20,000 ; n = 8 years ; K = 0.2

t	0	1	2	3	4	5	6	7	8
D_t	0	17777.78	15555.56	13333.33	11111.11	8888.889	6666.667	4444.444	2222.222
B_t	100000	82222.22	66666.67	53333.33	42222.22	33333.33	26666.67	22222.22	20000

Exercise 2: *Ref. Example-1* calculate the depreciation and book value for period 5.

$$D_t = \frac{n-t+1}{n(n+1)/2}(P-S)$$

$$D_5 = \frac{8-5+1}{8(8+1)/2}(1{,}00{,}000 - 20{,}000) = 8888.889$$

$$B_t = (P-S)\frac{n-t}{n} \times \frac{n-t+1}{n+1} + S$$

$$B_5 = (1{,}00{,}000 - 20{,}000)\ \frac{8-5}{8}\ \frac{8-5+1}{8+1} + 20{,}000$$

$$= 80{,}000 \times (3/8) \times (4/9) + 20{,}000$$

$$= \text{Rs. } 33{,}333.33$$

Sinking Fund Method of Depreciation

Book value decreases at increasing rates with respect to life of the asset.

- The loss in value of the asset (P – S) is made available in the form of cumulative depreciation amount at the end of the life of the asset by setting up an equal depreciation amount (A) at the end of each period during the lifetime of the asset.

  Annual Equivalent Amount (A) = (P – S) [A/F, i, n]

- The fixed sum depreciated at the end of every time period earns an interest at the rate of i% compounded annually, and hence the actual depreciation amount will be in the increasing manner with respect to the time period.

  $D_t = (P - S)\,(A/F, i, n)\,(F/P, i, t - 1)$

Formula

$D_t = (P – S) \times (A/F, i, n) \times (F/P, i, t – 1)$

$B_t = P – (P – S) \times (A/F, i, n) \times (F/A, i, t)$

Where; $(A/F, i, n) = \frac{i}{(1+i)^n - 1}$; $(F/A, i, t) = \frac{(1+i)^t - 1}{i}$

and $(F/P, i, t\text{-}1) = (1 + i)^{t-1}$

Example 4: *Ref. Example 1* calculate the depreciation with an interest rate of 12%, compounded annually.

P = Rs. 1,00,000; S = Rs. 20,000; n = 8 years; i = 12%

$A = (P – S) \frac{i}{(1+i)^n - 1}$ = (1,00,000 – 20,000) 0.0813 = Rs. 6,504

a fixed amount of Rs. 6,504 is depreciated at the end of every year from the earning of the asset. The depreciated amount will earn interest for the remaining period of life of the asset at an interest rate of 12%, compounded annually.

Depreciation at the end of year 1 (D1) = Rs. 6,504.

Depreciation at the end of year 2 (D2) = 6,504 + 6,504 x 0.12 = Rs. 7,284.48

Depreciation at the end of the year 3 (D3)

= 6,504 + (6,504 + 7,284.48) x 0.12 = Rs. 8,158.62

Depreciation at the end of year 4 (D4)

= 6,504 + (6,504 + 7,284.48 + 8,158.62) x 0.12 = Rs. 9,137.65

End of year t	*Fixed depreciation* (Rs.)	*Net depreciation* D_t (Rs.)	*Book value* B_t (Rs.)
0	6,504	–	1,00,000.00
1	6,504	6,504.00	93,496.00
2	6,504	7,284.48	86,211.52
3	6,504	8,158.62	78,052.90
4	6,504	9,137.65	68,915.25
5	6,504	10,234.17	58,681.08
6	6,504	11,462.27	47,218.81
7	6,504	12,837.74	34,381.07
8	6,504	14,378.27	20,002.80

Exercise 3:

Ref. Example 1: Compute D5 and B7 using the sinking fund method of depreciation with an interest rate of 12%, compounded annually.

$$Dt = (P - S)\frac{i}{(1+i)^n - 1} \times (1 + i)^{t-1}$$

D5 = (1,00,000 – 20,000) x 0.0813 x 1.574 = Rs. 10,237.30

$$Bt = P - (P - S) \times \frac{i}{(1+i)^n - 1} \times \frac{(1+i)^t - 1}{i}$$

B7 = 1,00,000 – (1,00,000 – 20,000) × 0.0813 × 10.089

= 34,381.10