Computational Intelligence (CI)

Fuzzy Proposition

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Boolean Logic vs. Multi-valued Logic

- A proposition is the basic building block of logic. It is defined as **a declarative sentence that is either True or False, but not both**.

- In Crisp logic: every proposition is either TRUE or FALSE.

- The classical two-valued logic can be extended to multi-valued logic.

- Example: Consider a three valued logic

  - True (1)
  - False (0)
  - Intermediate (1/2)

Boolean Logic vs. Multi-valued Logic

Different operations with three valued logic

a	b	∧ AND	∨ OR	¬a NOT	⟹ IMPLICATION	= EQUAL
0	0	0	0	1	1	1
0	½	0	½	1	1	½
0	1	0	1	1	1	0
½	0	0	½	½	½	½
½	½	½	½	½	½	1
½	1	½	1	½	1	½
1	0	0	1	0	0	0
1	½	½	1	0	½	½
1	1	1	1	0	1	1

Multi-valued Logic

Symbol	Connective	Usage	Definition
¬	NOT	¬P	$1 - T(P)$
V	OR	PVQ	$max\{T(P), T(Q)\}$
Λ	AND	$P\Lambda Q$	$min\{T(P), T(Q)\}$
⟹	IMPLICATION	$(P \Rightarrow Q) or (\neg PVQ)$	$\max\{(1 - T(P)), T(Q)\}$
=	EQUALITY	$(P = Q) or [(P \Rightarrow Q)\Lambda(Q \Rightarrow P)]$	$1 - \lvert T(P) - T(Q)\rvert$

Fuzzy Proposition

Example 1

P : Ram is honest

1. T(P) = 0.0 : Absolutely false
2. T(P) = 0.2 : Partially false
3. T(P) = 0.4 : May be false or not false
4. T(P) = 0.6 : May be true or not true
5. T(P) = 0.8 : Partially true
6. T(P) = 1.0 : Absolutely true.

Fuzzy Proposition

Example 2

P : Mary is efficient ; T(P) = 0.8
Q : Ram is efficient ; T(Q) = 0.6

- **Mary is not efficient.**
$$T(\neg P) = 1 - T(P) = 0.2$$

- **Mary is efficient and so is Ram.**
$$T(P \wedge Q) = min\{T(P), T(Q)\} = 0.6$$

- **Either Mary or Ram is efficient**
$$T(P \vee Q) = max\{T(P), T(Q)\} = 0.8$$

- **If Mary is efficient then so is Ram**
$$T(P \implies Q) = \max\{1 - T(P), T(Q)\} = 0.6$$

Fuzzy Proposition

- The fundamental difference between crisp proposition and fuzzy proposition is in the **range of their truth values**.

- The degree of truth of each fuzzy proposition is expressed by value in the interval **[0,1] both inclusive**.

- We define fuzzy proposition as follows:

  P: x is intelligent. Where intelligent is a fuzzy set.

Thank you