



## FIFTH SEMESTER EXAMINATION-2012

### DESIGN & ANALYSIS OF ALGORITHMS

[ CS 502 ]

Full Marks: 60

Time: 3 Hours

*Answer any SIX questions including Question No.1 which is compulsory.*

*The figures in the margin indicate full marks.*

*Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.*

1. a) Where in a max-heap of size  $n$  might the smallest  $[2 \times 10]$  element reside, assuming that all elements are distinct? What is the maximum number of comparison required to find the smallest element in a max-heap?
- b) Solve the recurrence  $T(n) = 4T(n/2) + n$
- c)  $f(n) = O(g(n))$  implies  $0 \leq f(n) \leq cg(n)$  for all  $n \geq n_0$  and
  - A. For some  $c$  and any  $n_0$  are +ve integers
  - B. For some  $c$  and some  $n_0$  are +ve integers
  - C. For any  $c$  and some  $n_0$  are +ve integers
  - D. For any  $c$  and any  $n_0$  are +ve integers
- d) What is content of the given array  $\langle 6, 4, 10, 5, 7, 1, 8, 4 \rangle$  after 4-iterations of Insertion-sort?
- e) Maximum number of comparisons required to find any element in a given array of length  $n$  is 4. Each element is distinct and occurs equal number of times in that array. Determine the number of distinct elements present in that array.

(1)

- f) Consider the items for  $n=3$ , weights ( $w_1, w_2, w_3$ ) = (2, 4, 5), profits ( $v_1, v_2, v_3$ ) = (10, 12, 15) and a Knapsack of capacity  $W=7$ . Find the optimal solution for the fractional knapsack problem.
- g) Compare and contrast BFS and DFS.
- h) What are the similarities and dissimilarities between Divide-and-Conquer strategy and Dynamic-Programming strategy?
- i) Using the linked-list representation of disjoint sets and the weighted-union heuristic, a sequence of  $n$  MAKE-SET, UNION and FIND-SET operations takes \_\_\_\_\_ time.  
(A)  $O(n^2)$  (B)  $O(n \log n)$  (C)  $O(n)$  (D)  $O(1)$
- j) Write an algorithm to find out the maximum element of a given array of size  $n$  using divide-and-conquer strategy.
2. a) State master method and use the method to give tight asymptotic bounds for the following recurrences. [4]
- i.  $T(n) = 4T(n/2) + n^2$   
ii.  $T(n) = 4T(n/2) + n^3$
- b) Discuss different asymptotic notations used in algorithm analysis. [4]
- Is  $n^{\log n} + (\log n)^n + 2^{n \log n} = \theta(n^n)$ ?
3. a) Describe a  $\Theta(n \log n)$  time algorithm that, given a set  $X$  of  $n$  integers and another integer  $y$ , determines whether or not there exist two elements in  $X$  whose sum is exactly  $y$ . [5]



b) Give two examples where greedy strategy cannot give optimal solution. [3]

4. a) Find an optimal number of Scalar multiplication required of a MATRIX-CHAIN product whose sequence of dimensions is  $\langle 5, 10, 3, 12, 5, 50, 6 \rangle$ . [5]

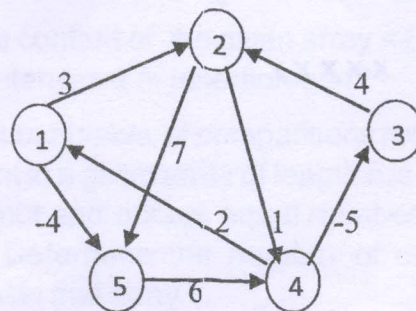
b) Determine Longest-Common-Subsequence of the sequences  $\langle 1, 0, 0, 1, 0, 1, 0, 1 \rangle$  and  $\langle 0, 1, 0, 1, 1, 0, 1, 1, 0 \rangle$ . [3]

5. a) [5]

	a	b	c	d
a	0	10	15	20
b	5	0	9	10
c	6	13	0	12
d	8	8	9	0

Find a tour of minimum cost for the above graph using Dynamic-Programming technique which start and end at vertex a.

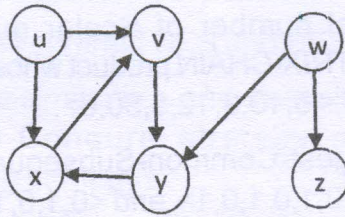
b) Explain Floyd-Warshall algorithm. Run the algorithm for the following Graph. [3]



(3)

6. a) Let  $w = \{5, 7, 10, 12, 15, 18, 20\}$  and  $m=35$ . Find all possible subsets of  $w$  that sum to  $m$ . Draw the portion of the state space tree that is generated. [4]

b) [4]



Show all the steps to find Depth first forest for the above graph.

7. a) Define P, NP and NPC class of problems. [4]  
b) Explain with an example, how Travelling-Salesman Problem can be solved using Approximation algorithm. [4]

8. a) What is the optimal huffman code for the following set of frequencies  $\langle a:1, b:2, c:15, d:10, e:20, f:35, g:17 \rangle$ ? [4]  
b) Explain the operation of HEAPSORT on the array  $A = \langle 5, 13, 2, 25, 7, 17, 20, 8, 4 \rangle$ . [4]

What is the running time of HEAPSORT on an array  $A$  of length  $n$  that is already sorted in increasing order?

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