*Semester: V (Regular)*
**Sub & Code: DAA, CS-3001**
**Branch (s): CSE & IT**

# AUTUMN MID SEMESTER EXAMINATION-2015

# Design & Analysis of algorithm

## [ CS-3001]

**Full Marks: 25** **Time: 2 Hours**

***Answer any four questions including question No.1 which is compulsory.***
***The figures in the margin indicate full marks.***
***Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.***

**DAA MID-SEM SOLUTION & EVALUATION SCHEME**

Q1 Answer the following questions: (2 x 5)

a) Consider the following C function.

```
int fun(int n)
{
int i, j, p=0;
for(i = 1; i < n; ++i)
for(j = n; j > 1; j = j/2)
++p;
return p;
}
```

What is most closely approximates value returned by the function fun?

**Scheme:**
Correct answer : 2 Mark
Wrong answer, but explanation approaches to answer : Step Marking

**Answer:**
$(n-1)\log_2 n \Rightarrow n \log_2 n$

The outer loop i body will execute (n-1) times. For each outer i iteration the inner j loop will execute k=logn times ( $2^k \le n$) means k times the value of p will be incremented by 1.

b) Rank the following functions by order of growth in increasing sequence?
$\log n$, $\log \sqrt{n}$, $\sqrt{n}$, $n \log \sqrt{n}$, $n!$, $2^n$

**Scheme:**
Arrangement of given functions with correct sequence (only answer) : 2 Mark

**Answer:**
The functions by order of growth in increasing sequence
$\log \sqrt{n}$, $\log n$, $\sqrt{n}$, $n \log \sqrt{n}$, $2^n$, $n!$

c) Can the master method be applied to solve the following recurrence?

$T(n) = 4T(n/2) + n^2 \log n$

Justify your answer.

**Scheme:**
Correct answer with justification: 2 Mark
Wrong answer, but explanation approaches to answer : Step Marking

**Answer:**
Master theorem can not be applied to solve the given recurrence.
**Explanation**
Solving by master theorem
**Step-1:** Guess for which case of master theorem

Given a=4, b=2, $f(n)= n^2 \log n$

$n^{\log_b a}= n^{\log_2 4}=n^2$

Comparing $n^{\log_b a}$ with f(n), we found f(n) is asymptotically faster growing function. So we guess the solution may exists in case-3.

**Step-2: Conformation test for case-3**
If case-3, then $f(n)= \Omega(n^{\log_b a+\epsilon})$

- ⇨ $f(n) >= cn^{\log_b a+\epsilon})$
- ⇨ $n^2 \log n >= cn^{2+\epsilon}$
- ⇨ $n^2 \log n >= cn^{2+\epsilon}$

As no value of $\epsilon$ >0 exists that makes the above inequality valid, so master theorem can not be applied to solve the given recurrence.

d) Write the merge sort procedure (only) which divides the array into two parts such that first part contains elements twice of second part. Also derive the time complexity of that merge sort.

**Scheme:**
Writing the algorithm with partitioning index q as mentioned below : 1 Mark
Time complexity derivation: 1 Mark

**Answer:**
Let p and r are the lower and upper index of the array. q is the partitioning point that divides the array into two parts such that first part contains elements twice of second part. First Part Second Part

p q r

So, q-p+1=2(r-q+1)=> q=(2r+p-1)/3

```
MERGE-SORT(A,p,r)
{
   q ← (2r+p-1)/3
   MERGE-SORT(A,p,q)
   MERGE-SORT(A,q+1,r)
   MERGE(A,p,q,r)
}
```

The Recurrence relation of the above algorithm is
$T(n) = T(n/3) + T(2n/3) + O(n)$
Solving the unbalanced partitioning we will get the solution as $\mathbf{T(n) = nlog_{3/2}n}$

e) Match the following Algorithms and its recurrences

| | |
|---|---|
| Bubble-sort | $T(n)=T(n/2)+ \Theta(1)$ |
| Quick-sort | $T(n)=T(n-1)+ \Theta(n)$ |
| Merge-sort | $T(n)=T(k) + T(n-k)+\Theta(n)$ |
| Binary-Search | $T(n)=2T(n/2)+ \Theta(n)$ |

**Scheme:**

Each correct matching : 0.5 Mark

**Answer:**

| | |
|---|---|
| Bubble-sort | $T(n)=T(n-1)+ \Theta(n)$ |
| Quick-sort | $T(n)=T(k) + T(n-k)+\Theta(n)$ |
| Merge-sort | $T(n)=2T(n/2)+ \Theta(n)$ |
| Binary-Search | $T(n)=T(n/2)+ \Theta(1)$ |

Q2 a) What is the significance of asymptotic notations? Define different asymptotic notations used in algorithm analysis. (2.5)

**Scheme:**
Significance of asymptotic notation : 0.5 Mark
Correct Definition of asymptotic notations => 2 Marks

**Answer:**

- We often want to know a quantity approximately, instead of exactly, in order to compare one algorithm with another, we introduce some terminology that enables us to make meaningful (but inexactness) statements about the time and space complexity of an algorithm, they are known as **asymptotic notations**.

- **Significance of Asymptotic Notations**
    - Asymptotic Notations are used to describe the running time of an algorithm in a meaningful way, is defined in terms of functions whose domains are the set of natural numbers.
    - These notations refer to how the problem scales as the problem gets larger.
    - The asymptotic run time of an algorithm gives a simple and machine independent, characterization of its complexity.
    - The notations works well to compare algorithm efficiencies because we want to say that the growth of effort of a given algorithm approximates the shape of a standard function.

- **Definitions of different asymptotic notations used in algorithm analysis**

  There are five different notations commonly used in algorithm analysis. They are

  i) O - Notation (Big-Oh Notation) : Asymptotic upper bound

  ii) Ω - Notation (Big-Omega Notation) : Asymptotic lower bound

  iii) Θ - Notation (Theta Notation) : Asymptotic tight bound

  iv) o – Notation (Little-oh Notation) : Upper bound that is not asymptotically tight

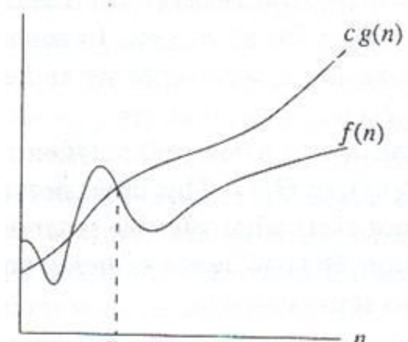
  v) ω – Notation (Little-omega Notation): Lower bound that is not asymptotically tight

**i) O - Notation (Big-Oh Notation**

- It represents the upper bound of the resources required to solve a problem.(worst case running time)
- **Definition:** Formally it is defined as

  For any two functions f(n) and g(n), which are non-negative for all n ≥ 0, f(n) is said to be g(n), f(n) = O(g(n)), if there exists two positive constants c and $n_0$ such that

  $$0 \le f(n) \le c\ g(n) \text{ for all } n \ge n_0$$
- Less formally, this means that for all sufficiently big n, the running time of the algorithm is less than g(n) multiplied by some constant. For all values n to the right of $n_0$, the value of the function f(n) is on or below g(n).

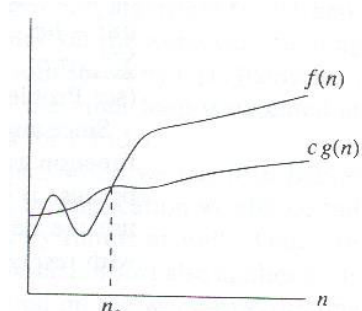


$f(n) = O(g(n))$

**ii) Ω - Notation (Big-Omega Notation)**

- **Definition:** For any two functions f(n) and g(n), which are non-negative for all n ≥ 0, f(n) is said to be g(n), f(n) = Ω (g(n)), if there exists two positive constants c and n0 such that

  $$0 \le cg(n) \le f(n) \text{ for all } n \ge n0$$

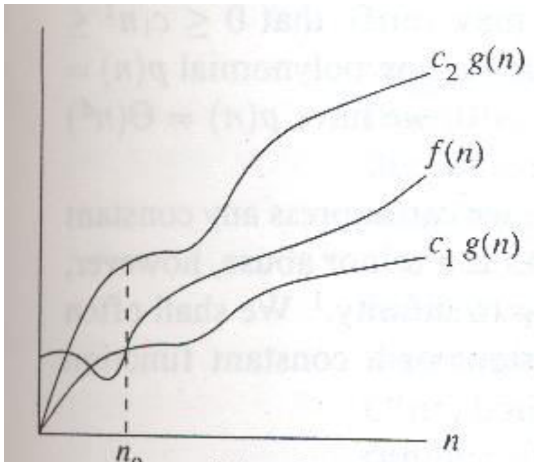


$f(n) = \Omega(g(n))$

**iii) Θ - Notation (Theta Notation)**

- **Definition:** For any two functions f(n) and g(n), which are non-negative for all $n \geq 0$, f(n) is said to be g(n), $f(n) = \Theta(g(n))$, if there exists positive constants c1, c2 and n0 such that

$$0 \leq c1g(n) \leq f(n) \leq c2g(n) \text{ for all } n \geq n0$$




$f(n) = \Theta(g(n))$

**iv) o – Notation (Little-oh Notation)**

For any two functions f(n) and g(n), which are non-negative for all $n \geq 0$, f(n) is said to be lillte oh of g(n) => $f(n) = o(g(n))$, for any positive constant $c > 0$, if there exists a positive constant $c > 0$ such that $0 \leq f(n) < c\, g(n)$ for all $n \geq n_0$.

**v) ω – Notation**

For any two functions f(n) and g(n), which are non-negative for all $n \geq 0$, f(n) is said to be lillte omega of g(n) => $f(n) = \omega(g(n))$, for any positive constant $c > 0$, if there exists a positive constant $c > 0$ such that $0 \leq c\, g(n) < f(n)$ for all $n \geq n_0$.

b) Solve the following recurrence. (2.5)

$T(n) = 4T(n/2) + n^2$ where $n>1$ and $T(1)=1$

**Scheme:**

Correct Answer: 0.5 Mark

Correct Answer with explanation :2 Mark

**Answer:**

Solving the recurrence $T(n) = 4T(n/2) + n^2$ where $n>1$ and $T(1)=1$, by master theorem.

**Step-1:** Given a=4, b=2, $f(n)= n^2$

$n^{\log_b a}= n^{\log_2 4}= n^2$

Comparing both $n^{\log_b a}$ and f(n), we found both are same. So as per Master theorem we guess the solution may exist in case-2

**Step-2:** Confirmation test for case-2

If case-2, then $f(n)=\theta(n^{\log_b a})$

⇨ $c1n^{\log_b a} <= f(n) <= c2n^{\log_b a}$

⇨ $c1n^2 <= n^2 <= c2n^2$

The above inequality is valid for some constant c1=1,c2=2, $n_0$=1., so case-2 is confirmed. As per case-2 the solution is

$$\mathbf{T(n)= \theta(n^{\log_b a} \log_2 n)= \theta(n^2 \log_2 n)}$$

Q3 a) Write the PARTITION() algorithm of Quick Sort and describe step by step how you would get the pass1 result by taking last element as pivot on the following data. (3)

8, 2, 1, 5, 6, 1, 3, 7, 4, 9, 5

Derive the average case time complexity of quick sort.

**Scheme:**

PARTITION Algorithm: 1 Mark

Representation of intermediate steps of pass-1 : 1 Mark

Average case time complexity derivation : 1 Mark

**Answer:**

**PARTITION Algorithm:**

```
PARTITION(A,p,r)
{
    x ← A[r]
    i ← p-1
    for j ← p to r-1
    {
       if A[j] ≤ x
       {
           i ← i+1
           A[i] ↔A[j]
        }
    }
     i ← i+1
     A[i] ↔A[r]
     return i
}
```

**<u>Representation of intermediate steps of pass-1</u>**

Given Data: 8, 2, 1, 5, 6, 1, 3, 7, 4, 9, 5

| i | j=p | | | | | | | | | | r |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 8 | 2 | 1 | 5 | 6 | 1 | 3 | 7 | 4 | 9 | $\boxed{5}$ |

| i p | j | | | | | | | | | r |
|---|---|---|---|---|---|---|---|---|---|---|
| **8** | **2** | 1 | 5 | 6 | 1 | 3 | 7 | 4 | 9 | $\boxed{5}$ |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| i p | | j | | | | | | | | r |
| 2 | 8 | 1 | 5 | 6 | 1 | 3 | 7 | 4 | 9 | 5 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| p | i | j | | | | | | | | r |
| 2 | **8** | **1** | 5 | 6 | 1 | 3 | 7 | 4 | 9 | 5 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| p | i | | j | | | | | | | r |
| 2 | 1 | 8 | 5 | 6 | 1 | 3 | 7 | 4 | 9 | 5 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| p | | i | j | | | | | | | r |
| 2 | 1 | **8** | **5** | 6 | 1 | 3 | 7 | 4 | 9 | 5 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| p | | i | | j | | | | | | r |
| 2 | 1 | 5 | 8 | 6 | 1 | 3 | 7 | 4 | 9 | 5 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| p | | | i | | j | | | | | r |
| 2 | 1 | 5 | **8** | 6 | **1** | 3 | 7 | 4 | 9 | 5 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| p | | | i | | | j | | | | r |
| 2 | 1 | 5 | 1 | 6 | 8 | 3 | 7 | 4 | 9 | 5 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| p | | | | i | | j | | | | r |
| 2 | 1 | 5 | 1 | **6** | 8 | **3** | 7 | 4 | 9 | 5 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| p | | | | i | | | j | | | r |
| 2 | 1 | 5 | 1 | 3 | 8 | 6 | 7 | 4 | 9 | 5 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| p | | | | | i | | | j | | r |
| 2 | 1 | 5 | 1 | 3 | **8** | 6 | 7 | **4** | 9 | 5 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| p | | | | | i | | | | j | r |
| 2 | 1 | 5 | 1 | 3 | 4 | 6 | 7 | 8 | 9 | 5 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| p | | | | | | i | | | | r j |
| 2 | 1 | 5 | 1 | 3 | 4 | **6** | 7 | 8 | 9 | **5** |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| p | | | | | | i | | | | r j |
| 2 | 1 | 5 | 1 | 3 | 4 | **5** | 7 | 8 | 9 | **6** |

**Average case time complexity derivation :**

Average case time complexity of quick sort is $O(n \log_2 n)$

b) Given 10 activities, A=< a1, a2,….,an > along with their start time (si) and finish time (fi) as Si=< 1, 2, 4, 3, 7, 7, 8, 9, 11, 12 > and fi=< 3, 5, 4, 7, 5, 9, 14, 18, 10, 12 > and that requires the exclusive use of a common stage for scheduling these activities. Use an efficient method that computes a schedule with largest number of activities on that stage. (2)

**Scheme:**
Some values are incorrectly written. Identifying these values and correct the same by taking some assumptions ( reverse the timings etc) : 1 Mark
Sorting the activities in their increasing order of their finishing time and finding out the solution : 1 Mark

**Answer:**
The activities in their increasing order of their finishing time (Assuming the starting time is less than finishing time in case any incorrect value is written, reverse the timings)

| Activities ($a_i$) | Starting Time (si) | Finishing Time (fi) | Accept/ Reject |
|---|---|---|---|
| a1 | 1 | 3 | √ |
| a3 | 4 | 5 | √ |
| a2 | 2 | 5 | X |
| a4 | 3 | 7 | X |
| a5 | 5 | 7 | √ |
| a6 | 7 | 9 | √ |
| a9 | 10 | 11 | √ |
| a10 | 12 | 12 | √ |
| a7 | 8 | 14 | X |
| a8 | 9 | 19 | X |

**The solution is <a1,a3,a5,a6,a9,a10>**

Q4 a) Find an optimal solution to the knapsack instance n=7, W=15. (v1, v2, v3, v4, v5, v6, v7) = (10, 5, 15, 7, 6, 18, 3) and (w1, w2, w3, w4, w5, w6, w7) = (2, 3, 5, 7, 1, 4, 1), where n is the number of items, W is the knapsack capacity that thief can carry, $v_i$ stands for value or profit $w_i$ stands for weight of the $i^{th}$ element. (2)

**Scheme:**
Finding value per weight of each items and arrange them in decreasing order: 1Mark
Finding the solution vector or the optimal solution : 1 Mark

**Answer:**

**Step-1 (Finding value per weight of each items)**

| tem/ Object (i) | Value/Profit ($v_i$) | Weight ($w_i$) | Value per weight ($v_i/w_i$) |
|---|---|---|---|
| 1 | 10 | 2 | 5 |
| 2 | 5 | 3 | 1.66 |
| 3 | 15 | 5 | 3 |
| 4 | 7 | 7 | 1 |
| 5 | 6 | 1 | 6 |
| 6 | 18 | 4 | 4.5 |
| 7 | 3 | 1 | 3 |

**Step-2 (sorting the items in decreasing order of their value per weight)**
**U=W=15**

| Item/ Object (i) | Value/Profit ($v_i$) | Weight ($w_i$) | **Value per weight ($v_i/w_i$)** | Weight taken | $x_i$ |
|---|---|---|---|---|---|
| 5 | 6 | 1 | **6** | $1 \leq 15$<br>U=15-1=14 | 1 |
| 1 | 10 | 2 | **5** | $2 \leq 14$<br>U=14-2=12 | 1 |
| 6 | 18 | 4 | **4.5** | $4 \leq 12$<br>U=12-4=8 | 1 |
| 3 | 15 | 5 | **3** | $5 \leq 8$<br>U=8-5=3 | 1 |
| 7 | 3 | 1 | **3** | $1 \leq 3$<br>U=3-1=2 | 1 |
| 2 | 5 | 3 | **1.66** | 3 >2<br>Fraction=2/3 | 2/3 |
| 4 | 7 | 7 | **1** | - | 0 |

The **solution vector or the optimal solution** is
$(x_1, x_2, x_3, x_4, x_5, x_6, x_7) = (1, 2/3, 1, 0, 1, 1, 1)$
Profit earned = $\sum_{i=1}^{7} v_i x_i$
= 1x10 + 2/3 x 5 + 1x15 + 0x7 + 1x6 + 1x18 +1x3
= 10 + 3.33 + 15 + 0 + 6 + 18 + 3 =55.33

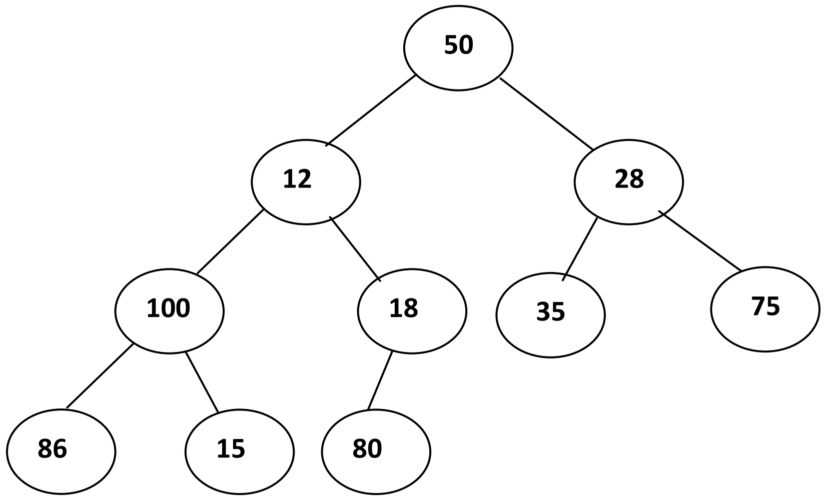
b) Sort the following data in descending order by using heap sort technique. (2)
50, 12, 28, 100, 18, 35, 75, 86, 15, 80

**Scheme:**

Showing the intermediate steps to build a Min Heap : 1 Mark
Showing some initial steps and drawing the final figure to sort by heap sort : 1 Mark

**Answer:**

**Step-0:** Constructing the heap structure with the data given as follows
50, 12, 28, 100, 18, 35, 75, 86, 15, 80




**Step-1:** Building a min-heap by applying MIN-HEAPIFY from index of element 18 down to the index of the root element.

c) Consider a complete binary tree where the left and the right subtrees of the root are max-heaps. What is upper bound to convert the tree onto a heap? (1)

**Scheme:**

Only answer : 1 Mark

**Answer:**

$O(\log_2 n)$

**Explanation**

In the worst case applying MAX-HEAPIFY(1) will terminate at leaf that visits the heights of the tree.

Q5 a) Write an algorithm to find out two elements from an array having non-negative numbers such that the difference should be maximum. (3)

**Scheme:**

Correct algorithm : 3 Mark

**Answer:**

```
MAX-TWO-ARRAY(A, p, r)
{
  max=min=a[p]
  for i ← p+1 to r
  {
     if a[i]> max
        max ← a[i]
     if a[i] < min
        min ← a[i]
  }
  return(max, min)
}
```

c) An unordered list contains n distinct elements. What is the minimum number of comparisons required to find an element in this list that is neither maximum nor minimum? (2)

A. O(1)   B. O(log n)   C. O( n log n)   D. O(n)

**Scheme:**

Correct option : 2 Marks

**Answer:**

Option-A

**Explantion**

Comparing first three elements to get max and min. The max and min may be the final max or min value. But the middle element that is in between max and min is the element that is neither max nor min.

So to get this we require two comparisons that is O(1) constant time.

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