NUMERICAL PROBLEMS

Problem 1. *A firm is producing output using labour and capital in such quantities that marginal product of labour is 15, and marginal product of capital is 8. The wage rate of labour is Rs 3 and price of capital is Rs 2. Is the firm using efficient factor combination for production? If not, what it should do to achieve economic efficiency ?*

Solution :

Efficiency condition for factor use (that is, optimal factor combination) requires that the following condition should be fulfilled:

$$\frac{MP_L}{w} = \frac{MP_K}{r}$$

Now, $$\frac{MP_L}{w} = \frac{15}{3} \text{ and } \frac{MP_K}{r} = \frac{8}{2}$$

Now, $$\frac{15}{3} > \frac{8}{2}$$

The given factor combination cannot therefore be efficient or optimal factor combination because the firm is getting more output from a rupee spent on labour than on capital. To achieve economic efficiency in the use of resources and to maximise profits the firm should substitute labour for capital

so that $\frac{MP_L}{w}$ becomes equal to $\frac{MP_K}{r}$.

Problem 2. *The wage rate of labour is Rs 6 and price of raw materials is Rs 2. The marginal product of labour is 16 while the marginal product of raw materials is 4. Can a firm operating under these conditions be maximising profits ?*

Solution :

Profit maximisation is achieved when resources are efficiently used. Efficiency in resource use requires the following condition :

$$\frac{MP_L}{w} = \frac{MP_{RM}}{P_{RM}}$$

$$\frac{MP_L}{w} = \frac{16}{6}, \frac{MP_{RM}}{P_{RM}} = \frac{4}{2}$$

$$\frac{16}{6} > \frac{4}{2} \text{ or } \frac{MP_L}{w} > \frac{MP_{RM}}{P_{RM}}$$

Thus, the firm will not be maximising profits. To maximise profits, it should substitute labour for raw materials

Problem 3. *A firm reports that marginal product of labour is 5 and marginal rate of technical substitution of labour for capital is 2. What is the marginal product of capital?*

Solution :

$$MRTS_{LK} = \frac{MP_L}{MP_K} \qquad 2 = \frac{5}{MP_K}$$

$$MP_K = \frac{5}{2} = 2.5$$

If the value of L and $\frac{w}{r}$ are known, we can find out the value of Q.

Numerical Problem : *Given the production function $Q = 100\, K^{0.5}\, L^{0.5}$. Determine the optimal input combination for producing 1444 units of output if wage rate of labour (w) is Rs. 30 and price per unit of capital (r) is Rs. 40. What is the minimum cost of production ?*

Solution :

The given production function is

$$Q = 100K^{0.5} . L^{0.5}$$

$$MP_L = \frac{\partial Q}{\partial L} = 100 \times 0.5\, K^{0.5}\, L^{-0.5}$$

$$= 50K^{0.5}L^{-0.5}$$

$$MP_K = \frac{\partial Q}{\partial K} = 100 \times 0.5K^{-0.5}L^{0.5}$$

$$= 50\, K^{-0.5}\, L^{0.5}$$

$$\frac{MP_L}{MP_K} = \frac{50K^{0.5}L^{-0.5}}{50K^{-0.5}\, L^{0.5}} = \frac{K}{L}$$

In equilibrium $MRS_{LK} = \frac{MP_L}{MP_K} = \frac{w}{r}$ Thus, in optimal input combination, $\frac{K}{L} = \frac{w}{r}$

or

$$K = \frac{w}{r} L.$$

To obtain the value of L we substitue $K = \frac{w}{r}L$. in the production function with $Q = 1444$ units. Thus

$$1444 = 100\, K^{0.5}\, L^{0.5}$$

$$= 100 \left(\frac{w}{r} L\right)^{0.5} L^{0.5}$$

$$1444 = 100\, L \left(\frac{w}{r}\right)^{0.5}$$

Substituting $w = 30$ and $r = 40$

$$1444 = 100L \times \left(\frac{30}{40}\right)^{0.5}$$

$$1444 = 100\, L \times (0.75)^{0.5}$$

$$= 100\, L \times 0.866 = 86.6L$$

$$L = \frac{1444}{86.6} = 16.67$$

Now, using the equation for expansion path ($K = \frac{w}{r} L$) we can obtain the value of K by substituting the values of L, w and r. Thus

$$K = \frac{30}{40} \times 16.67$$

$$= 0.75 \times 16.67$$

$$= 12.5$$

Thus optimum combination of inputs consists of 16.67 units of labour and 12.5 units of capital. This will ensure minimum possible cost for producing 1444 units of output.

In order to determine this minimum cost we substitute the optimum values of L and K obtained above and the given prices of labour and capital (*i.e.*, w and r) in the cost function. Thus

$$C = wL + rK$$

$$= 30 \times 16.67 + 40 \times 12.50$$

$$= 500 + 500$$

$$= \text{Rs. } 1000$$

Thus minimum cost of producing 1444 units of output is Rs. 1000.

Numerical Problem 2

Given : $Q = 100K^{0.5} L^{0.5}$, C = Rs. 1200, $w = 30$ and $r = 40$. Determine the quantity of labour and capital that the firm should use in order to maximise output. What is this level of output ?

Solution. The problem of constrained maximisation is :

$$\text{Maximise } Q = 100\, K^{\frac{1}{2}}\, L^{\frac{1}{2}}$$

Subject to cost constraint : $1200 = 30\, L + 40\, K$

$$MP_K = \frac{\partial Q}{\partial K} = \frac{1}{2}\, 100\, K^{-1/2}\, L^{1/2}$$

$$MP_L = \frac{\partial Q}{\partial L} = \frac{1}{2}\ 100\ K^{1/2}\ L^{-1/2}$$

$$\frac{MP_K}{MP_L} = \frac{50K^{-1/2}\ L^{1/2}}{50K^{1/2}\ L^{-1/2}} = \frac{L}{K} \quad ...(1)$$

For output maximisation

$$\frac{MP_K}{MP_L} = \frac{r}{w} \quad ...(2)$$

Substituting $\frac{MP_K}{MP_L} = \frac{L}{K}$ and $w = 30$ and $r = 40$ in equation (2) we have

$$\frac{L}{K} = \frac{4}{3} \text{ or } L = \frac{4K}{3} \quad ...(3)$$

Putting the value of $L = \frac{4}{3}K$ in cost-constraint equation we get

$$1200 = wL + rK$$

$$= 30 \times \frac{4K}{3} + 40K = 80\ K$$

$$K = \frac{1200}{80} = 15$$

Substituting $K = 15$ in the cost-constraint equation we have

$$1200 = 30 \times L + 40 \times 15$$

$$30L + 600 = 1200$$

$$L = \frac{1200 - 600}{30} = 20$$

Thus output-maximising amounts of capital and labour are 15 and 20 respectively. To get the level of output produced we substitute these amounts of capital and labour in the given production function

$$Q = 100K^{1/2}\ L^{1/2}$$

$$Q = 100\sqrt{15}\ \sqrt{20}$$

$$= 100\sqrt{300}$$

$$= 100 \times 17.32$$

$$= 1732$$