# KIIT UNIVERSITY, BHUBANESWAR

## FIFTH SEMESTER EXAMINATION-2011

**Applicable for branches----CS/IT----------**

**Name of the Paper----------Design and Analysis of Algorithms----------**

**Paper code----------CS-602----------**

**Full Marks----------60----------**

***Answer any SIX questions including Question No.1 which is compulsory.***
***The figures in the margin indicate full marks.***
***Candidates are required to give their answers in their own words as far as practicable and all parts of a question should be answered at one place only.***

Instruction regarding answering questions

| Q. No 1. | | Marks [2×10] |
|---|---|---|
| (a) | $f(n)=\Theta(g(n))$ implies<br>A. $0 \le c_1\, g(n) \le f(n) \le c_2\, g(n)$ ∀ $n \ge n_0$ , where $c_1$, $c_2$, and $n_0$ are positive integers<br>B. $0 \le c_1\, g(n) \le f(n) \le c_2\, g(n)$ ∀ $n \ge n_0$ , for some positive constants $c_1$, $c_2$, and $n_0$<br>C. $0 \le c_1\, g(n) \le f(n) \le c_2 g(n)$ ∀ $n \ge n_0$ , for any positive constants $c_1$, $c_2$, and $n_0$<br>D. None of these | |
| (b) | Find the Theta ($\Theta$) notation for the following function<br>$f(n) = \log(n!)$ | |
| (c) | What is the content of the given array (15, 27, 8, 37, 10, 50, 54, 31, 85, 5) after 4 iterations of Insertion-sort? | |
| (d) | An input comprises a sorted list of *n* integers with many duplications such that the number of distinct integers in the sequence is O (*log n*), what is the time complexity to find an element in the list? | |
| (e) | What is the time complexity of inserting '$\log_2 n$' number of elements into a binary heap of size n? | |

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| (f) | Given 11 activities, A=<$A_1$, $A_2$,….,$A_{11}$> along with their start time ($s_i$) and finish time ($f_i$) as $S_i$=<1, 2, 3, 5, 7, 8, 9, 10, 12, 12, 14> and $f_i$=<3, 5, 4, 7, 10, 9, 11, 11, 12, 14, 15>. Compute a schedule where largest number of activities takes place. | |
| (g) | Which of the following is the correct ordering for optimal storage on tape problem, If there are 5 programs of length $l_1$=7, $l_2$=10, $l_3$= 5, $l_4$=3, and $l_5$= 4 and a tape T?<br>A. $l_4, l_2, l_3, l_5, l_1$ B. $l_4, l_5, l_3, l_1, l_2$ C. $l_4, l_5, l_1, l_3, l_2$<br>D. $l_2, l_4, l_3, l_5, l_1$ | |
| (h) | An adjacency-matrix representation may be preferred in implementing<br>A. Kruskal's Algorithm B. Prim's Algorithm<br>C. Dijkstra's Algorithm. D. Floyd-Warshall's Algorithm | |
| (i) | Differentiate between Divide-And-Conquer and Dynamic Programming approach. | |
| (j) | Define NP-Completeness and give two examples of NP-complete problems. | [2×4] |
| 2. | | |
| (a) | Write the recurrence and solve it for the following code segment.<br>Int BTECH(int n)<br>{<br>if (n== 1)<br>return(1);<br>else<br>return(BTECH(n-1) + BTECH(n-1));<br>} | |
| (b) | State and explain Master's theorem. Solve the following recurrence using master's method.<br>T(n)=9T(n/3) + n | |

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| 3. | | [2×4] |
| (a) | Consider a set $S$ of $n \geq 2$ distinct numbers given in unsorted order and, $x$ and $y$ are two distinct numbers in the set $S$. Write an $O(n)$ time recursive algorithm to determine $x, y \in S$ such that $\|x-y\| \geq \|w-z\|$ for all $w, z \in S$. Also write the recurrence of the above procedure and show that its complexity is $O$(n). | |
| (b) | Write a Quick-Sort algorithm in which the PARTITION procedure always produces a n/4 and 3n/4 proportional split of the array. Write the recurrence of the above Quick-Sort algorithm and find its time complexity. | |
| 4. | | [2×4] |
| (a) | Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions are <5, 7, 10, 4, and 6>. | |
| (b) | A robot arm is employed to tighten the nuts on some piece of machinery on an assembly line. The arm will start from its initial position i.e. the first nut to be tightened, successively move to each of the remaining nuts, and return to the initial position. The arm movement time from one nut i to other j is a variable, denoted by $c_{ij}$ and can be expressed in the matrix form given below. Find a minimum-cost tour of the robot arm that will minimize the time needed to complete its tasks. $\begin{bmatrix} 0 & 11 & 16 & 21 \\ 7 & 0 & 10 & 11 \\ 8 & 14 & 0 & 13 \\ 9 & 9 & 10 & 0 \end{bmatrix}$ | |
| 5. | | [2×4] |
| (a) | Write HUFFMAN'S algorithm and obtain a set of optimal Huffman codes for the messages ($M_1$ ….$M_5$) with relative frequencies f1: 12, $f_2$: 20, $f_3$: 10, $f_4$: 35, $f_5$: 23. Draw the decode tree for this set of codes. | |
| (b) | Apply prim's algorithm to find minimum cost spanning tree of the following graph by taking 'a' as the root vertex. | |

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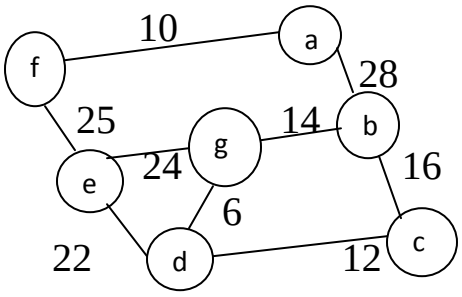
| |  | [2×4] |
| 6. | | |
| (a) | Write an algorithm for Building a min-heap and find its worst-case time complexity. Build a min heap considering the following array elements and also show the heap data structure at each step. A[ ] = {10, 7, 14, 6, 5, 12, 7, 4, 9, 1}. | |
| (b) | Find out the time complexity of a "BINARY SEARCH" algorithm that always splits an array into two sets, one of which is twice the size of the other. Also write the algorithm for above procedure. How does this algorithm compare with standard BINARY SEARCH? | [2×4] |
| 7. | | |
| (a) | Write a procedure for union operation between two sets using a disjoint-set forest. | |
| (b) | Apply backtracking to solve the following instance of the sum of subset problem. S= {2, 3, 4, 5} and m=11. Also write the algorithm for sum of subset problem. | [2×4] |
| 8. | | |
| (a) | Write the algorithm for All-Pairs Shortest path and find out its time complexity. | |
| (b) | Prove that if any NP-compete problem is polynomial time solvable, then P=NP. Equivalently, if any problem in NP is not polynomial time solvable, then no NP-complete problem is polynomial time solvable. | |