Fuzzy Reasoning/Approximate Reasoning/Fuzzy Inference

It is an inference procedure that derives conclusions from a set of fuzzy IF-THEN rules and known facts.

To understand fuzzy reasoning let us first discuss Compositional rule of inference.

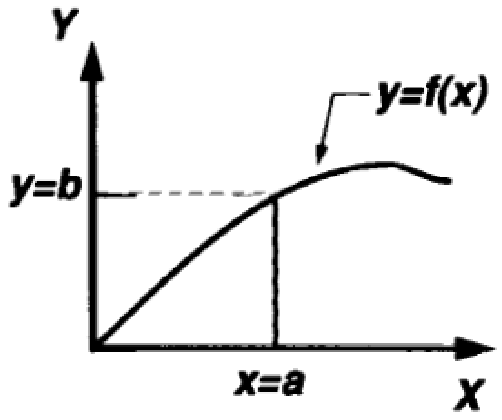
Compositional Rule of Inference

The compositional rule of inference is a generalization of following notion.

a. Derivation of y=b from the x=a and y=f(x) where a and b are points

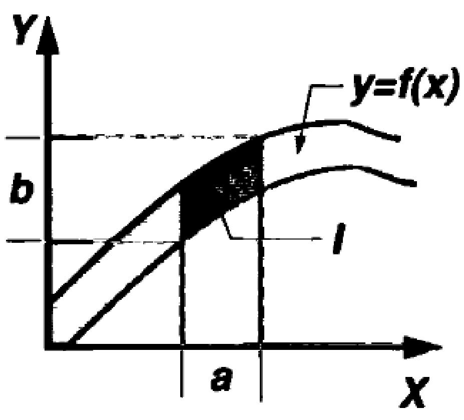
Suppose that we have a curve y=f(x) that represents the relation between in x and y.

When we are given x=a, then from y=f(x) we can infer that y=f(a)=b



b. Derivation of y=b from the x=a and y=f(x) where a, b are intervals and y=f(x) is an interval valued function

A generalization above process would allow ‘a’ to be an interval and f(x) to be an interval valued function.



To find the resulting interval y=b corresponding to the interval x=a, steps are as follows.

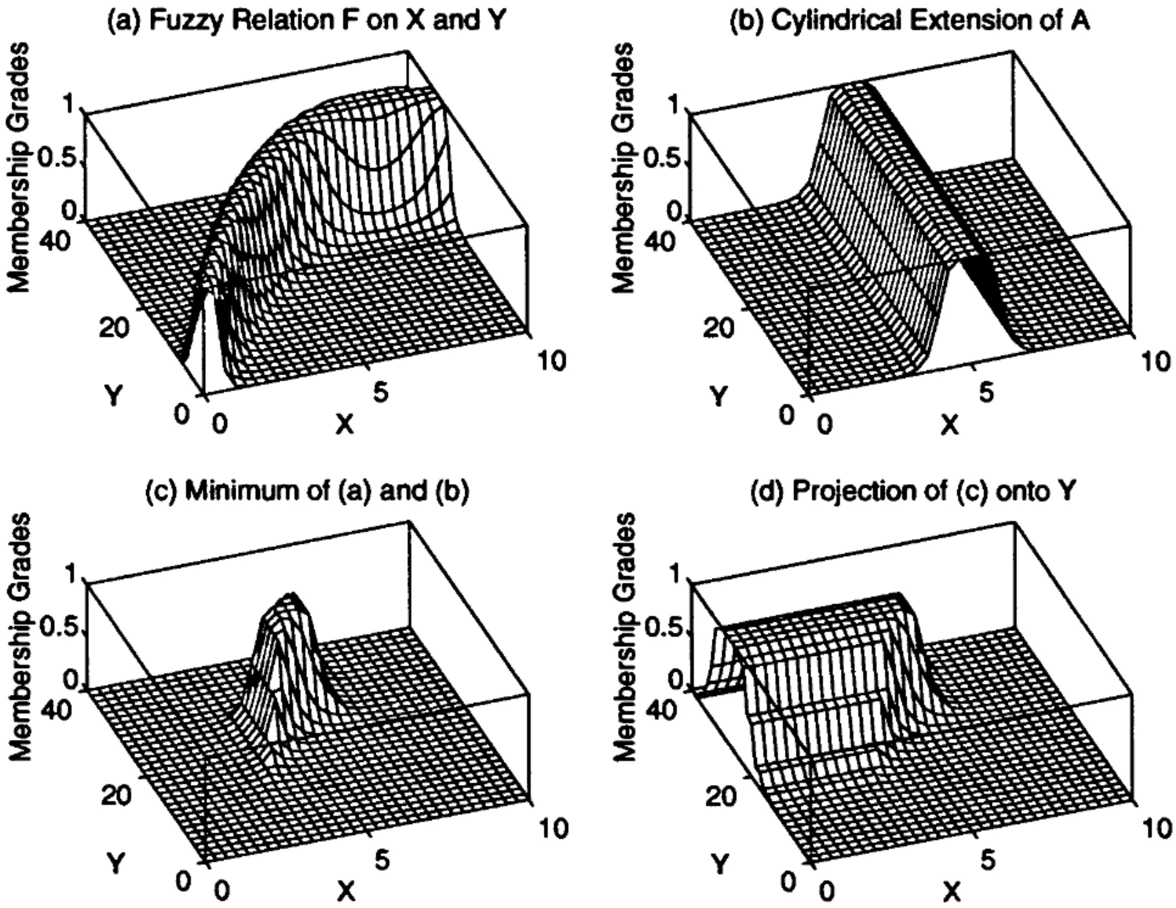
- Construct a cylindrical extension of ‘a’.
- Find the intersection of ‘a’ with the interval-valued curve (I).
- Find projection of I onto the y-axis that yields the interval y=b.

c. Derivation of Fuzzy set B from F and A, where F is fuzzy relation and A is a fuzzy set

Going one step further in our generalization, let F is a fuzzy relation on X x Y and A is a fuzzy set on X.

To infer y as a fuzzy set B on y-axis, steps are:

- Construct a cylindrical extension c(A) with base A.
- Find the intersection of c(A) and F, which forms the region of intersection I.
- Project $c(A) \cap F$ onto the y-axis.



Specifically, let μ_A, $\mu_{c(A)}$, μ_B, and μ_F be the MFs of A, $c(A)$, B, and F, respectively, where $\mu_{c(A)}$ is related to μ_A through

$$\mu_{c(A)}(x, y) = \mu_A(x).$$

Then

$$\begin{aligned} \mu_{c(A)\cap F}(x, y) &= \min[\mu_{c(A)}(x, y), \mu_F(x, y)] \\ &= \min[\mu_A(x), \mu_F(x, y)]. \end{aligned}$$

By projecting $c(A) \cap F$ onto the y-axis, we have

$$\begin{aligned} \mu_B(y) &= \max_x \min[\mu_A(x), \mu_F(x, y)] \\ &= \vee_x[\mu_A(x) \wedge \mu_F(x, y)]. \end{aligned}$$

This formula reduces to the max-min composition of two relation matrices A and F.

A is a unary fuzzy relation and F is a binary fuzzy relation, defined over X and X x Y.

Conventionally, B is represented as

$\boldsymbol{B = A \circ F}$, **Where ° is the composition operator.**

Note# It is interesting to note that the extension principle is in fact a special case of the compositional rule of inference.

Specifically, if y=f(x) is a common crisp one-to-one or many-to-one function, then the derivation of the induced fuzzy set B on Y is exactly what is accomplished by the extension principle.

Using this compositional rule of inference, we can formalize an inference procedure upon a set of fuzzy IF-THEN rules. This inference procedure, generally called approximate reasoning or fuzzy reasoning.