Rate of Return method

- Types of return:
  - ~~i) Minimum Acceptable Rate of Return~~
  - i) Minimum Attractive Rate of Return /

    Minimum Acceptable Rate of Return (MARR)
  - ii) Net Present Value (NPV)
  - iii) Internal Rate of Return (IRR)

1.) Minimum Attractive Rate of Return

Refers to the lower limit of project acceptablity beyound which if rate of return falls, project will be rejected.

II.) Net present Value (Same as Present worth)

Refers to the sum of all the ~~pres~~ present values from a future stream of benefit during the lifespan of the project.

III.) Internal Rate of Return (IRR)

Refers to that Rate of Return which equates present ~~value~~ worth of benefit to present ~~value~~ worth of cost.

$\therefore PW(B) = PW(C)$

$NPW = 0$

ex: $i = 12\%$

$P = 10,00,000$

$R = 80,000$

suppose, $NPW(12\%) = 0$

$12\% = IRR$

∴ if NPW != 0

IRR = Interest Rate of last +ve result + Diff. b/w two interest rate $\left[\frac{+ve\ value - Salvage\ 1\%}{+ve\ value - (-ve\ value)}\right]$

i.) if 1st try = +ve,

take interest rate more

if 1st try = -ve,

take interest rate less

ex: MARR = 10%

$P = 5,00,000$

$NPW(10\%) = 6000$

$NPW(12\%) = 2000$

$NPW(15\%) = -1000$

$$IRR = 12\% + 3\%\left[\frac{2000 - 0}{2000 - (-1000)}\right]$$

$$= 12\% + 3\%\left[\frac{2000 - 0}{2000 + 1000}\right]$$

Selection or Rejection of project

i) if IRR > MARR : Project Selected

ii) if MARR > IRR : Project Rejected

iii) if MARR = IRR : Project may or may not be selected.

Q.) From the following information, find out the investor should go with the new busisness or not with the help of Rate of Return, if MARR = 10 %

Year	Cashflow
0	−1,00,000
1	30,000
2	30,000
3	30,000
4	30,000
5	30,000

$$\Rightarrow -P + R\left[\frac{(1+i)^n-1}{i(1+i)^n}\right] + S\left[\frac{1}{(1+i)^n}\right]$$

$$\Rightarrow -1,00,000 + 30,000\left[\frac{(1+.1)^5-1}{(1+.1).1}\right] = \cancel{13,000}$$

13,723.6030

at $i = 16\% \Rightarrow -ve$

NPW(10%) = 13,723.6030

(11%) = 10,876.91

(12%) = 8143.28

(13%) = 2992.42

$\boxed{(15\%) = 564.6526 \quad (16\%) = -1771.191}$

Now,

$$IRR = 15\% + 1\%\left[\frac{564.656 - 0}{564.656 - (-1771.191)}\right]$$

$$= 15\% + 1\%\left[\frac{564.659 - 0}{564.656 + 1771.191}\right]$$

$$= 15.24\%$$

as IRR > MARR, Yes

Q.) A company is trying to diversify its business in a new product line, life of the project is 10 years with no salvage value at the end of its life. Initial outlay of project is 20,00,000 INR with annual equal return of ~~300,t~~ 3,50,000 INR. Find Rate of Return for new business.

Since MARR is not given, take any +ve i, for which it is final

for $i = 10\%$.

$$= -P + R\left[\frac{(i+1)^n - 1}{i(i+1)^n}\right]$$

for $i = 11\% = 61231.203$

$i = 12\% = -22421.940$

$$\Rightarrow 11\% + 1\%\left[\frac{61231^{.203} - 0}{61231 + 22421.940}\right]$$

$i = 0.117$

$= 11.7\%$

Q.) From the following table find out rate of return for all alternative and find out which alternative will be selected on the basis of RoR method, if MARR = 12%.

Particulars	Alt. A1	Alt. A2	Alt. A3
Investment	1,50,000	2,10,000	2,55,000
annual net Income	45,570	58,260	69,000
life in years	5	5	5

Alt. 1

MARR = 12%

$$= -P + R\left[\frac{(i+1)^n - 1}{i(i+1)^n}\right]$$

$$= -1,50,000 + 45,570\left[\frac{(0.12+1)^5 - 1}{(0.12)(0.12+1)^5}\right]$$

$= 14269.651$

at $= 14\% \Rightarrow 6445.494$

$15\% = 2757.707$

$16\% = -790.438$

$$IRR = 15\% + 1\%\left[\frac{14269.651}{14269.651 + 790.438}\right]$$

$= .529$

$$= 15\% + 1\%\left[\frac{2757.707}{2757.707 + 790.437}\right]$$

$= 0.157$

Alt 2 12%

$$-2{,}10{,}000 + 58260\left[\frac{(1+0.12)^5-1}{0.12(1+0.12)^5}\right]$$

$= 14.261$

13% = -5086.106

$$IRR = 12\% + 1\%\left[\frac{14.261}{14.261+5086.106}\right]$$

$= 0.12$

Alt 3 12%

IRR = 11%

A_1 will be selected as RoR is more

Benefit cost Analysis

. Benefit cost Ratio (B/C)

$$\frac{B}{C} = \frac{PW(B)}{PW(C)} = \frac{\text{Present value Benefit}}{\text{Present value of Cost}}$$

. Selection or Rejection of the project

If $\frac{B}{C} > 1$: Selected.

if $\frac{B}{C} < 1$: Rejected

if $\frac{B}{C} = 1$: may or may not be selected.

Q) From the following table find out which project will be selected on the basis of benefit cost analysis.

Project	PW(B)	PW(C)
1	60,00,000	40,00,000
2	80,00,000	20,00,000
3	90,00,000	35,00,000

$\left(\frac{B}{C}\right)_1 = \frac{PW(B)}{PW(C)} = 1.5$

" = 4

" = 2.57

1 = 1.5

2 = 4

3 = 2.57 | 42857143

2 will be selected.

Q.) In a particular locality of ~~space~~ state, the vehicle users take a round about route to reach certain places because of presence of a river. It results in excessive travel time and increased fuel cost. So, the state govt. is planning to construct a bridge across the river. The estimated initial investment for constructing the bridge is 40,00,000. Estimated life of bridge is 15 years, annual maintainance and operation cost is 1,50,000 INR. The value of fuel savings due to construction of bridge is 6,00,000 INR in 1st year and increases by 50,000 every year thereafter till the end of the life of the bridge; check whether the project is justified based on B/C ratio by assuming an interest rate of 12% compounded anually.

$A_1 = 6,00,000$
$G = 50,000$

$$PW(C) = P + C\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

if initial and cost given

$$A = A_1 + G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

$$PW(B) = AW(B)\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$A_1 = 6,00,000 + 50,000\left[\frac{1}{.12} - \frac{15}{(1+.12)^{15} - 1}\right]$$

$= 849015.168$ INR

$$PW(B) = 8,49,015.168\left[\frac{(1+.12)^{15} - 1}{(0.12)(0.12+1)^{15}}\right]$$

$= 5,782,527.266$

$$PW(C) = 40,000,000 + 150,000\left[\frac{(1+.12)^{15} - 1}{.12(1+.12)^{15}}\right]$$

$= 5021629.673$

$$\Rightarrow \frac{PW(B)}{PW(C)} = \frac{57,82,527.266}{50,21,629.673}$$

$= 1.15$.

Project justified as B/C is > 1

Q) The state govt. is planning a hydro electric project in addition to the production of electric power the project will provide flood control, irrigation and recreation. Estimated benefit and cost given as follow.

Initial cost = 8,00,00,000 (C)

Annual Power sales = 60,00,000 (B)

Annual flood control saving = 30,00,000 (B)

Annual Irrigation Benefit = 50,00,000 (B)

Annual Recreation Benefit = 20,00,000 (B)

Annual operating and maintain cost = 30,00,000 (C)

life of project = 50 Years

i = 15 %

total annual Benefit, then convert to P

Total annual benefit = 16,000,000

$$P(W(C)) = 8,00,00,000 + 30,00,000 \left[\frac{(1+.15)^{50}-1}{.15(1+.15)^{50}} \right]$$

$$= \cancel{85,998,154.398} \ 99\,981\,543.983$$

$$P(W(B)) = 16,00,00,000 \left[\frac{(1+.15)^{50}-1}{(1+.15)^{50} .15} \right]$$

$$= 106\,568\,234.576$$

$$B/C = \frac{106\,568\,234.576}{99\,981\,543.983} = 1.06$$

$= > 1$ project will be selected

Q.) Two mutually exclusive projects are being consider for investment. Project A1 requires an initial outlay of 30,00,000 INR with net ~~recieves~~ recipts of 9,00,000 INR per year, for the next 5 years. The initial outlay ~~fro~~ for project A2 is 60,00,000 INR with net recipts of 15,00,000 INR per year for next 7 years. Using B/c ratio find out which project will be selected if i = 10% compounded anually

A1

~~PW(C) = 30,00,000~~

Cost = 30,00,000 INR (already in Present)

Benefit = 9,00,000 INR → convert to (P)

$$PW(B) = 9,00,000 \left[\frac{(0.1+1)^5 - 1}{(.1+1)^5 \cdot .1}\right]$$

= 3411708.092

B/C =

B/C = 1.137

A2

Cost = 60,00,000 INR

$$PW(B) = P\ 15,00,000 \left[\frac{(.1+1)^5 - 1}{(.1+1)^5 \cdot 1}\right]$$

= 7302628.226

B/C) = 1.2

Pay-Back Period Method

i) Equal Return : Gets equal amount of return every year.

$$P = \frac{I}{C}$$

I → Initial Investment

C → CashInflow (Yearly equal)

P → Pay back year

Q) An investor has invested 50,00,000 INR. Find out how many years it'll take for investor to get back his money with the help of Equal Return method, if the estimated return is 4,00,000 INR every year.

$$P = \frac{I}{C} = \frac{50,00,000}{4,00,000} = 12.5 \text{ years} = 13 \text{ years}$$

ii) In case of different amount of ~~retun~~ return.

$$\frac{\text{The remaining amount}}{\text{Next CashInflow}} \times 12$$

Investment = 5,00,000

ex: 1 1,00,000

2 2,50,000 → 3,50,000 (1,50,000 remain)

3 4,20,000 → 7,70,000

4 5,00,000

$$\Rightarrow \frac{1,50,000}{4,20,0000} \times 12 \Rightarrow 4.28$$

⇒ 4.3

⇒ 4 years 4 months

Q.) One Investor has invested 10,00,000 INR on a project, the investor wants to purchase a machine for which there are two machine available in the market. find out which machine will be selected with the help of payback period method.

End of Year	Cashinflow from machine 1	Years Required	Cashinflow from machine 2	Years Required
1	1,00,000	1	2,00,000	1
			4,00,000	1
2	3,00,000	1		
			4,50,000	1
3	5,00,000	1		
			5,60,000	
4	7,00,000			
5	8,00,000		7,00,000	
Total				

Machine 1

$\frac{1,00,000}{7,00,000} \times 12 = 1.71$ month $= 3$ year 2 month

Machine 2

$\frac{4,00,000}{4,50,000} \times 12 = 10.66 \Rightarrow$ 2 year 11 month

As machine 2 requires lesser time than machine 1, machine 2 will be selected.

Depreciation

Refers to the loss of value in any asset due to its constant use.

Types of method

i.) Straight line method.

ii.) Declining balance method.

iii.) Sum-of-the-year digit method.

iv.) Sinking fund method.

1.) Straight line Method

Refers to that method of depreciation, where a fixed rate of depreciation is charged directly on the initial value of the asset for various years

i.) Depreciation amount per year (D)

$$D = \frac{I - S}{N}$$

I = Initial value of the asset / Purchase price

S = Salvage value / Reduced value of asset

N ⇒ life of asset in Years.

ii.) Rate of Depreciation (d):

$$d = \frac{D}{I} \times 100$$

iii) Book Value (B_{yr} / B_t) :

$$B_t = I - (t \times D)$$

t = No. of years the asset has been used.

or

$$B_t = B_{t-1} - D_t$$

Declining Blance Method:

It refers to that method of depriciation where a fixed rate of depriciation will be charged on the decling balance everyyear.

$$D_t = K \times B_{t-1}$$

$K \to$ A fixed rate of depreciation

$$B_t = B_{t-1} - D_t$$

Q.1) If a machine has been purchased at 90,000 INR with estimated service life of 10 years. find out depreciation amount and Book Value for varius years if rate of depreciation is 30%, with the help of Decling-Balance Method

$I = 90,000$ $\quad D = \frac{I-S}{N} = \frac{90,000}{10} = 9,000$

$N = 10$

$K = 30\%$,

(Next Page)

End of Year	D_t	B_t
0	—	90,000
1	[illegible]	
2	[illegible]	
3	[illegible]	
4	[illegible]	
5	[illegible]	
6	[illegible]	
7	[illegible]	
8	5.586	
9	1.7[illegible]	
10	0.579	

Q.) If an equipment has been purchased at 80,000 INR with estimated salvage value of 8,000 INR at the end of its service life of 10 years, find out rate of Depreciation and Book value of its? The asset after 5 years, with the help of straight line method.

$I = 80,000$

$S = 8,000$

$N = 10$

$$D = \frac{I-S}{N} = \frac{80,000-8000}{10} \Rightarrow 7,200/-$$

$$d = \frac{D}{I} \times 100$$

$$= \frac{7,200}{80,000} \times 100 = 9\%$$

$$B_t = I - (t \times D)$$

$$= 80,000 - (5 \times 7,200)$$

$$= 44,000$$

Q.) If initial value of a machine is 50,000 INR with estimated Salvage value of 5,000INR at the end of its service life of 8 years find out Depprociation amount and Book value for every year of machine, using Straight line method

End of Year	D	B_t
0	—	50,000
1	5625	44375
2	5625	38750
3	5625	33125
4	5625	27500
5	5625	21875
6	5625	16250
7	5625	10625
8	5625	5000

$B_t = B_{t-1} - D$

(all methods, last book value = Salvage value except, declining

$$D = \frac{50,000 - 5,000}{8} = 5625$$

Qs Q1.)

End of Year	D_t	B_t
0	—	90,000
1	27,000	63,000
2	18,900	44,100
3	13,230	30870
4	~~900~~ 9261	21,609
5	6482	15,0126.3
6	4537.889	10,588.41
7	3176.523	7441.887
8	2222	5,188.321
9	1556.496	63631.824
10	1089.54	25 42177

→ Sum of the year digit Method

i.) Sum of the year = $\frac{N(N+1)}{2}$

ii.) $D_t = \cancel{\text{Rate}} \; \text{Rate}\,(I - S)$

$\text{Rate} = \frac{\text{Rank}}{\text{Sum of the Year}}$

lowest rank=1

if total year=5, highest rank=5

since for 1st year, value = highest, rank=5. for 2nd, Rank=4

iii.) $B_t = B_{t-1} - D_t$

Q.) If an equipment has been purchased with 1 lakh INR with estimated salvage of 20,000 at the end of its service life of 8 years, find out depreciation amount and Book-value, using Sum of the year's digit method of depreciation

$I = 1{,}00{,}000$ INR

$S = 20{,}000$

$N = 8$

$D_t = Rate(I - S)$

$Rate = \frac{Rank}{\text{Sum of year}}$

Sum of the year $= \frac{8(8+1)}{2} = 36$

$B_t = B_{t-1} - D_t$

End of Year	Rank	D_t	B_t
0	–	–	1,00,000
1	8	17777.778	82,222.222
2	7	15555.556	66,666.668
3	6	13333.333	53,333.333
4	5	11111.111	42,222.224
5	4	8888.888	33,333.336
6	3	6666.667	26,666.67
7	2	4444.444	22,222.226
8	1	2222.222	20,000.004

Sinking Fund Method

i.) $A = (I - S)\left[\frac{i}{(1+i)^n - 1}\right]$

ii.) $D_t = A(1+i)^{t-1}$

iii.) $B_t = B_{t-1} - D_t$

Q.) If an asset has been purchased at 90,000 INR with estimated salvage value of 5,000 INR at the end of it's service life of 10 years. Find out depreciation amount and book value of asset every year with the help of sinking fund, when $i = 12\%$.

$\Rightarrow$ ~~End of Year D_t B_t~~

$A = \cancel{15044.7213}$ 4843.6539

End of Year	A	D_t	B_t
0	4843.6539	~~4324.691~~	90,000
1	"	4324.691	~~85567.308~~ 85,156.340
2	"	50424.81	79,731.454
3	"	6075.871	73,655.575
4	"	6804.984	66,850.59
5	"	7621.583	59,229.007
6	"	8536.173	50,692.834
7	"	9560.513	41,132.32
8	"	10707.775	30,132.545
9	"	11992.708	18,434.837
10	"	13431.83	5000.003

- Production

Refers to the process of physical transformation of input × output.

Production Function:

It refers to the functional relationship b/w input and output.

Let Q = Output

~~L~~ N = Land

L = Labour

K = Capital

$Q = f(N, L, K)$, if N inputs then $Q = f(x_1, x_2, x_3 \ldots x_n)$

i) Law of variable production / Short-run theory of production with one variable input.

ii) Law of Returns to scale / ~~So~~ long-run production theory

iii) Short-run production function with two variable input.

Factors of production

i) Fixed Factor.

ii) Variable Factor. (No. of labour etc)

- Fixed Factor

Factors which remains fixed. ex: Land, Building, Machine etc

· It refers to those factors which can't be changed during the process of production. Ex: Big mahine, Plant size, land, Big Building.

- Variable Factor

· It refers to those factors which can be changed during the process of production. Ex: Labour, Raw material.

· Production Time Period.

i) Short ~~Ter~~ Period / Short Run

ii) long Period / long Run (Nothing is fixed)

- Short Period

It refers to that time period of production, where fixed factors can't be changed but variable factors can be changed

- **Long Period:** Refers to that time period of production. where nothing is called fixed. All factors are variable factors.

3 concepts of Production

i) Total Product (TP)

ii) Marginal Product (MP)

iii) Average Product (AP)

- Total Product (TP):

- It refers to the total amount of production with a given amount of variable factors.

- Marginal Product (MP):

- It refers to the net addition to the total product by implying one extra unit of input.

$$\boxed{MP_n = TP_n - TP_{n-1}}$$

ex: $MP_L = \frac{d(TP)}{dL} = \frac{dQ}{dL}$

L -> ~~Land~~ Labour

K -> capital

ex: $MP_K = \frac{d(TP)}{dK} = \frac{dQ}{dK}$

$$\boxed{MP_x = \frac{d(TP)}{dx}}$$

x -> input

- Average Product (AP):

- It refers to the total production per unit of input used.

$$AP_L = \frac{TP}{L} = \frac{Q}{L}$$

$$AP_K = \frac{TP}{K} = \frac{Q}{K}$$

Law of Variable Production (G. Stigler)

It is defined as "equal increment of one input are added, the i/p of other productive services being held constant, the resulting increment of product beyound a certain point decrease, i.e, marginal product will diminish."

- Assumption of the Law

i) Rate of technology remains constant

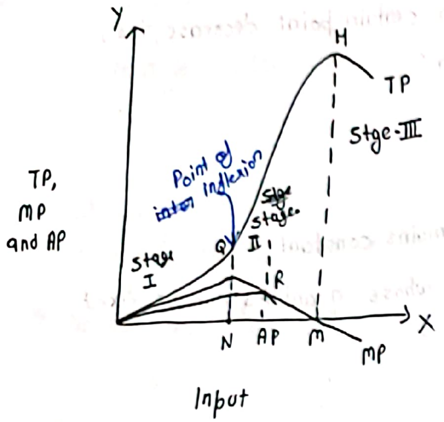
ii) There must be ~~same~~ some inputs whose quantities can be fixed

	Units of Labour used (L)	Total Product (Q)	MP	AP	Stage of operation
under utilization of Fixed Factors	1	80	–	80	Stage-I Increasing return
	2	170	90	85	
	3	270	100	90	
Full utilization of Fixed Factors (optimum stage of production)	4	368	98	92	Stage-II Diminishing return
	5	430	62	86	
	6	480	50	80	
	7	504	24	72	
	8	504	0	63	
	9	490	-14	54.444	Stage-III Negative return
	10	485	-5	48.5	

Stage I: Increasing return: In this stage, as MP increases, TP also increases at a incresing rate

Stage I: Diminishing return: In this stage, as MP increases, TP decreases at a diminishing rate

Q.) What is the optimal stage of production? (Last Page)



Q.) From the following table find out MP, AP and also stage of production

Unit of Labour L	TP	MP	AP	Stage
1	30	–	30	Stage I
2	80	50	40	
3	140	60	46.666	
4	190	50	47.5	Stage II
5	190	0	38	
6	180	-10	30	Stage III

Q.) From the following Short run production fun^c

i.) MP Fun^c

ii.) AP Fun^c

iii.) The value of L at which the o/p will be max.

iv.) " " " " " " " AP will be max.

$Q = 8L^2 - 0.4L^3$

i.) $MP_L = \frac{dQ}{dL} = 16L - 1.2L^2$

ii.) $AP_L = \frac{Q}{L} = 8L - 0.4L^2$

iii.) $MP_L = 0$

$16L - 1.2L^2 = 0$

$L = 16.67$

$L \simeq 17$

iv.) $MP = AP$

$16L - 1.2L^2 = 8L - 0.4L^2$

$\boxed{L = 10}$

or

$\frac{d(AP_L)}{dL} = 0 \Rightarrow 8 - 0.8L = 0$

$\boxed{L = 10}$

$\frac{d(MP_L)}{dL} = 0$

$\cancel{8L - 0.8}$

Q.) From the following short run production fun^c

i.) MP Fun^c

ii.) AP Fun^c

iii.) At what level of X, output will be max, and at what level of X, AP and MP will be max

$Q = 6x^2 - 0.5x^3$

i) $MP: \frac{dQ}{dx} = 12x - 1.5x^2$

ii) $AP: \frac{Q}{x} = 6x - 0.5x^2$

iii.) Output will be max when $MP = 0$

$12x - 1.5x^2 = 0$

$x = 8$

AP is max when

$MP = AP$ / $\frac{dAP}{dx} = 0$

$\Rightarrow 12x - 1.5x^2 = 6x - 0.5x^2$

$\Rightarrow 6x = x^2$, $x = 6$

MP max when $\frac{dMP}{dx} = 0$

$12 - 3x = 0$

$12 = 3x$

$x = 4$, at this level of x, MP is max.

• Law of Returns to Scale. (Ex of short run production).

- It returns to the long run homogeneous production func, which disscusses the relation b/w output and all the variable input.

$Q = f(L, K)$

if we change production by K times,

$Q^* = f(LK, KK)$

• Types of returns to Scale

i) Increasing Returns to Scale

ii) Constant Returns to Scale

iii) Decreasing Returns to Scale

- Increasing Returns to Scale:

When the rate of change in output is more than the rate of change in input or, doubling of input results in more than doubling of output, it is called increasing returns to scale

- Constant Returns to Scale:

When the rate of change in output is equal to the rate of change in input or doubling of inputs result in doubling of output, it is called constant returns to scale.

- Decreasing Returns to Scale:

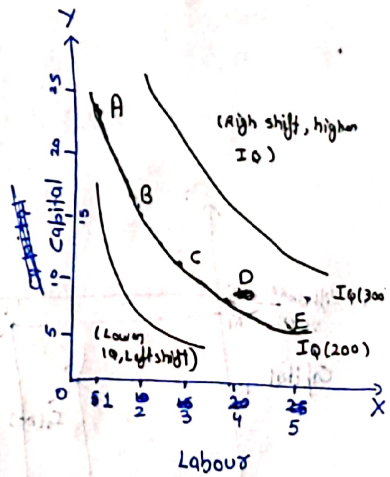
When the rate of change in output is less than the rate of change in input or doubling of input results in less than doubling of output, it is called decreasing returns to scale.

1.) Iso quant / Iso- Product Curve:

It refers to those curves, which shows various combination of two factors, producing equal level of output.

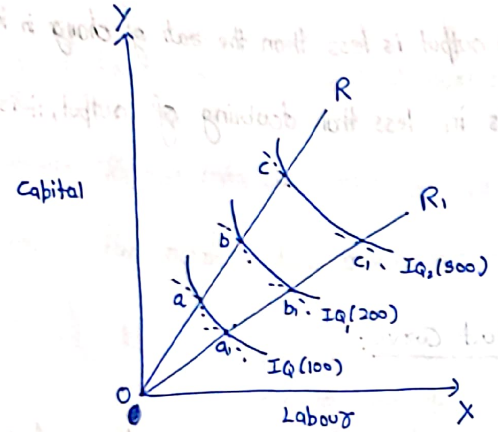
Combination	L	K	Q
A	1	22	200
B	2	17	200
C	3	13	200
D	4	10	200
E	5	8	200

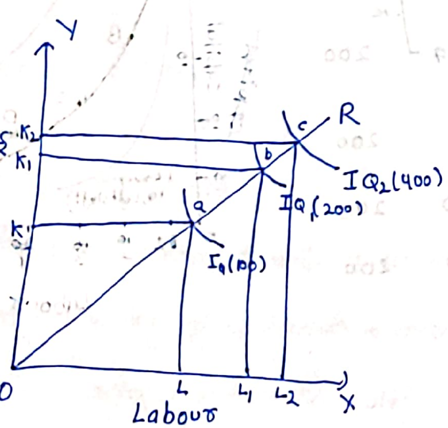
+ve -ve



11.) ~~Isoclient~~: Iso-cline

Refers to the line that shows us locus of various points on different iso-quants.

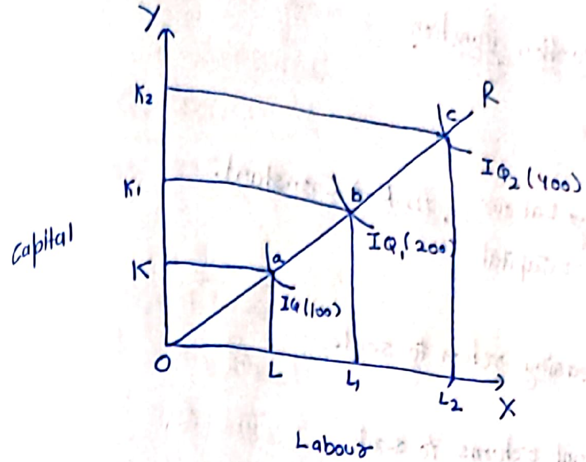




different beless

→ Because rate of change of output is more than rate of input, the distance b/w iso points declines.

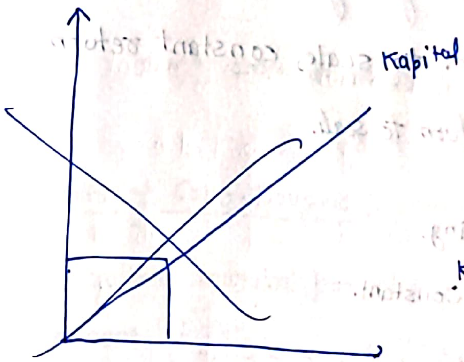
↳ Increasing Return to Scale

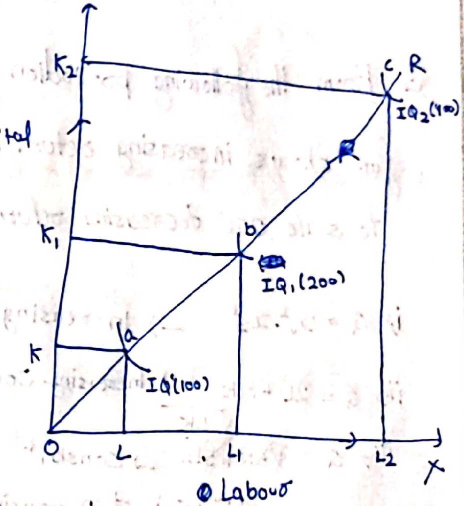


→ rate of change in output is equal to rate of change in input.

$oa = ab = bc$

Constants returns to Scale.





Decreasing Return to Scale

rate of change in output is less than rate of change of in input.

$oa < ab < bc$

→ Cobb-Douglas production function.

$Q = A \cdot L^{\alpha} \cdot K^{\beta}$

$Q \to$ Output, $L \to$ Labour, $\alpha, \beta \to$ Constant.

$A \to$ constant, $K \to$ Capital

If $\alpha+\beta > 1$, increasing return to scale

if $\alpha+\beta = 1$, constant returns to scale.

if $\alpha+\beta < 1$, Decreasing returns to scale.

Q) From the following production func. find out which prodn func. shows increasing return to scale, constant return to scale and decreasing return to scale.

i.) $Q = x_1^2 \cdot x_2^3$ → Increasing.

ii.) $Q = 2L + 3K$ → ~~Increasing~~ Constant.

iii.) $Q = \sqrt{LK}$ ~~$\sqrt{6K+5L}$~~ → Constant

iv.) $Q = 0.5\, L^{0.2} \cdot K^{0.1}$ → Decreasing

I.) $Q = x_1^2 \cdot x_2^3$, let chang input by λ.

$Q_1 = (x_1\lambda)^2 \cdot (x_2\lambda)^3$

$= x_1^2 \cdot \lambda^2 \cdot x_2^3 \cdot \lambda^3$

$\Rightarrow \lambda^5(x_1^2 \cdot x_2^3) \to Q_1 = \lambda^5 Q \; (Q = x_1^2 x_2^3)$

as rate of output is more than rate of change in input., increasing.

II.) $Q_1 = 2\lambda L + 3\lambda K$

$= 2L + 3K(\lambda)$

$Q = Q(\lambda)$

as rate of change in input is rate of change in output.

III.) $Q_1 = \sqrt{L\lambda K\lambda}$

$= \sqrt{\lambda^2 KL}$

$\Rightarrow \lambda\sqrt{KL}$

$Q_1 = \lambda Q$ → rate of change in input is rate of change in output

IV.) $Q_1 = 0.5\, L^{0.2}\lambda^{0.2} \cdot K^{0.1}\lambda^{0.1}$

$= \lambda^{0.3}(Q)$

rate of change in ~~in~~ output is less than rate of change in input.

- Properties of Cobb-Douglas production func.

I.) The ratio of marginal product and average product gives its exponent. : $\frac{MP_L}{AP_L} = \alpha$, $\frac{MP_K}{AP_K} = \beta$

II.) The Elasticity of output is equal to the respective exponents.

I.) $Q = A \cdot L^{\alpha} \cdot K^{\beta}$

$\frac{dQ}{dL} = \alpha \cdot A L^{\alpha-1} \cdot K^{\beta}$

$= \alpha \cdot \frac{A L^{\alpha} \cdot K^{\beta}}{L}$

$= \alpha \cdot \frac{Q}{L}$

$MP = \alpha \cdot AP_L$

$\frac{MP_L}{AP_L} = \alpha$ proved

$$Q = A \cdot L^{\alpha} \cdot K^{\beta}$$

$$\frac{dQ}{dK} = \beta \cdot A \cdot L^{\alpha} \cdot K^{\beta-1}$$

$$\frac{dQ}{dK} = \beta \cdot \frac{A \cdot L^{\alpha} \cdot K^{\beta}}{K}$$

$$\frac{dQ}{dK} = \beta \cdot \frac{Q}{K}$$

$$MP_K = \beta \cdot AP_K$$

$$\frac{MP_K}{AP_K} = \beta$$ Prooved.

II)

$$E_L = \frac{\text{Proportionate change in Output}}{\text{Proportionate change in Input.}}$$

$$= \frac{\frac{dQ}{Q} \times 100}{\frac{dL}{L} \times 100}$$

$$= \frac{dQ}{dL} \times \frac{L}{Q}$$

$$E_L = MP_L \times \frac{1}{AP_L} = \frac{MP_L}{AP_L} = \alpha$$

$\Rightarrow$ $E_L = \alpha$ (Prooved)

$$E_K = \frac{\text{Proportionate change in Output}}{\text{Proportionate change in Input}}$$

$$= \frac{\frac{dQ}{Q} \times 100}{\frac{dK}{K} \times 100}$$

$$= \frac{dQ}{dK} \times \frac{K}{Q}$$

$$= MP_K \times \frac{1}{AP_K}$$

$$= \frac{MP_K}{AP_K}$$

$E_K = \beta$ Prooved.

Q.) From the following, find out marginal production func and short run production if fixed quantity of capital is 10000 units.

$$Q = L^{0.75} K^{0.25}$$

$\Rightarrow$

$$\frac{dQ}{dK} = MP_K = 0.25\, L^{0.75} K^{-.75} \quad \Big| \quad \frac{dQ}{dL} = 0.75\, L^{-.25} K^{0.25}$$

Short run production f^n:

$$Q = L^{0.75} (10000)^{0.25}$$

$$Q = 10\, L^{0.75}$$

Q.) From the production func following, find out MP fun', AP fun' and also find out elasticity of output wrt Inputs.

$Q = 1.50 L^{0.75} K^{0.25}$

$\Rightarrow MP_K = \frac{dQ}{dK} = 0.375(L^{0.75})(K^{-.75})$

$MP_L = \frac{dQ}{dL} = 1.125(L^{-.25})(K^{.25})$

$\Rightarrow AP_L = \frac{Q}{L} = 1.50(L^{-.25})(K^{0.25})$

$AP_K = \frac{Q}{K} = 1.50(L^{0.75})(K^{-.75})$

$\rightarrow$ Elasticity of output:

Labour = 0.75

Kapital = 0.25

wrt, Labour

$E_L = \frac{dQ}{dL} \times \frac{1}{AP_L}$

$= \frac{MP_L}{AQ_L} = 0.75$

$E_K = \frac{MP_K}{AQ_K} = 0.25$

Producer's Equilibrium

I.) Iso quant / Iso-Product Curves.

II.) Marginal Rate of Technical Substitution (MRTS).

III.) Iso-Cost line.

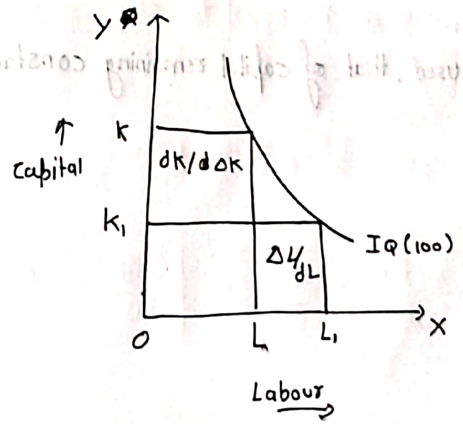
II.) MRTS:

The rate at which, number of unit of 1 factor substituted to have one more unit of another factor.

i.) $MRTS_{LK}$: Referes to the rate at which the number of unit of capital substituted to have one more unit of labour.

ii.) $MRTS_{KL}$: Refers to the rate at which the number of unit substituted to have one more unit of capital.

- Slope of the iso-quant.



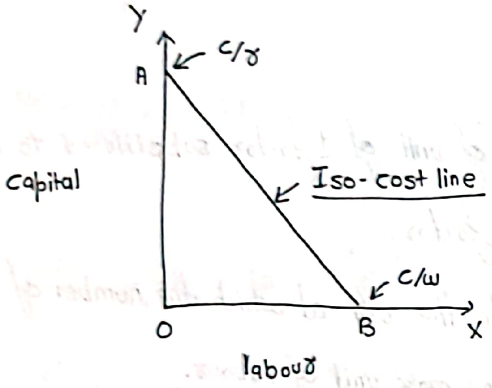
Slope:

$MRTS_{LK} = \frac{dK}{dL} = \frac{MP_L}{MP_K}$

$MRTS_{KL} = \frac{dL}{dK} = \frac{MP_K}{MP_L}$

III) Iso-Cost line:

It refers to the line, that shows us various combination of two factors, that a producer can buy with a given level of expenditure.



Slope of the iso-cost line:

$$\frac{OA}{OB} = \frac{C/r}{C/w} = \frac{w}{r}$$

Eq^n of the iso-cost line

$$C = wL + rK$$

C → cost of Production

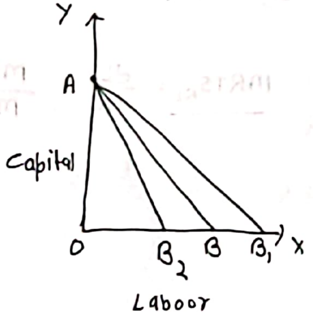
w → wage rate of the labour

r → Price of capital per unit

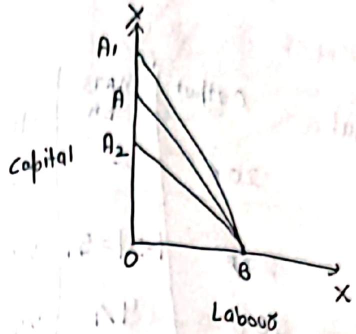
K → No. of unit of capital used

- Shift in the iso-cost line:

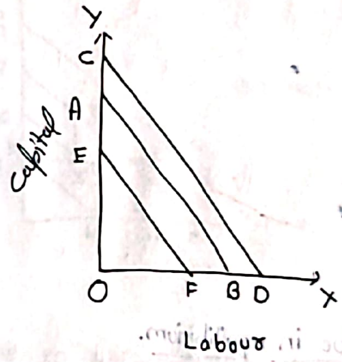
If more units of labour used, that of capital remaining constant.



- Shift in iso-cost line, if units of capital used changes, that of labour remaining constant.



- Shift in the iso-cost line, if number of units of labour used and capital used changes.



Q.) From the following find out $MRTS_{LK}$, $MRTS_{KL}$

Combination	labour	Capital	Output	$MRTS_{LK}$	$_{KL}$
A	1	22	200	–	–
B	2	17	200	$\lvert -5 \rvert = 5/1$	1/5
C	3	13	200	4/1	1/4
D	4	11	200	2/1	1/2
E	5	10	200	1/1	1/1

$$MRTS_{LK} = \frac{\Delta K}{\Delta L} = \frac{MP_L}{MP_K}$$

$$MRTS_{KL} = \frac{\Delta L}{\Delta K} = \frac{MP_K}{MP_L}$$

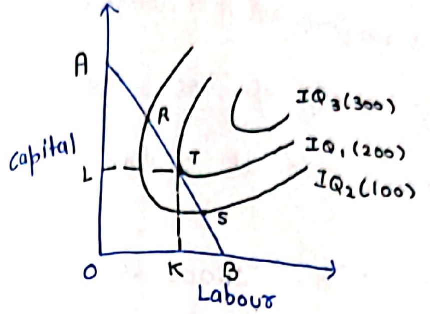
→ Condition for a producer to be in equilibrium.

i) Slope of the iso-quant should be equal to slope of the iso-cost line.

ii) The isoquant should be convex at equilibrium point.

$$\boxed{MRTS_{LK} = \frac{dK}{dL} = \frac{MP_L}{MP_K} = \frac{w}{r}} \Rightarrow \boxed{\frac{MP_L}{w} = \frac{MP_K}{r}}$$

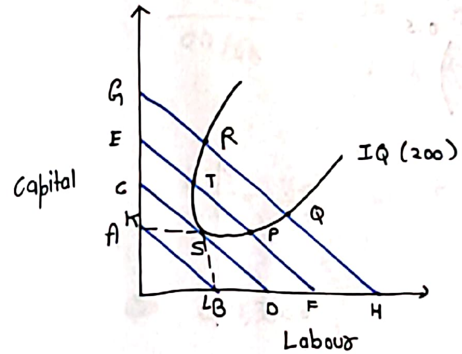
· Profit Maximization



out of point R, T, S as point T is lying on maximum IQ point (200), output ↑, profit will be maximum

OL, OK are optimal input combination.

· Cost Minimisation



CD

S lying on the lowest possible IQ point, point S gives ↑ minimum cost.

Q.) From the following short run production funᶜ, find out optimal input combination, for producing 1500 unit of output if the wage-rate of the labour is 30 INR and price of capital per unit is 40 INR. What is the minimum cost.

$w = 30$, $r = 40$

$$Q = 100 \cdot L^{0.5} \cdot K^{0.5}$$

Condⁿ for Optimal Input Combination: $\frac{MP_L}{MP_K} = \frac{w}{r}$

$MP_L = \frac{dQ}{dL} = 50 L^{-.5} K^{0.5}$

$MP_K = \frac{dQ}{dK} = 50 K^{-0.5} L^{0.5}$

$\Rightarrow \dfrac{50 L^{-0.5} x^{0.5}}{50 x^{-0.5} L^{0.5}} = \dfrac{30}{40}$

$= \dfrac{K}{L} = \dfrac{30}{40}$

$\Rightarrow \dfrac{K}{L} = \dfrac{3}{4}$

$\Rightarrow K = \dfrac{3}{4} L$

$Q = 100 . L^{0.5} . K^{0.5}$

$1500 = 100 . L^{0.5} . \left(\dfrac{3}{4} L\right)^{0.5}$

$\therefore L = 17.320$

$\boxed{L = 17}$

$K = \dfrac{3}{4} L$

$= 12.75$

$\boxed{K = 13}$

→ Eqn of 150 cost line =

$C = wL + rK$

$C = 1030$ ans

Q.) From the following production func find Quantity of labour and capital, that the company should use in order to maximise output and also findout if cost of production is 1,300 INR wagerate is 30 and price of capital = 40.

$Q = 100 . L^{0.5} . K^{0.5}$

→ Optimal input combination = $\dfrac{MP_L}{MP_K} = \dfrac{w}{r}$

$\dfrac{dQ/dL}{dQ/dK} = \dfrac{50 L^{-0.5} K^{0.5}}{50 K^{-0.5} L^{0.5}} = \dfrac{K}{L}$

$\dfrac{K}{L} = \dfrac{30}{40} \Rightarrow K = \dfrac{3}{4} L$

$C = 1{,}300$

$1300 = wL + rK$

$1300 = 30L + \overset{10}{40} \dfrac{3}{4} L$

$1300 = 30L + 30L$

$1300 = 60L$

$L = 21.6$

$\boxed{L = 22.}$

$K = \dfrac{3}{4} L$

$\boxed{K = 16.5 \Rightarrow 17}$

$Q = 100 . L^{0.5} . K^{0.5}$

$Q = 1933.90$

$Q = 1934$

$AC(ATC) = \frac{TC}{Q}$

$TC = TFC + TVC$

$AFC = \frac{TFC}{Q}$

$AVC = \frac{TVC}{Q}$

$AC = AFC + AVC$

~~$MC = TC_n - TC_{n-1}$~~

$MC_n = \cancel{TFC} + TVC_n - \cancel{TFC} - TVC_{n-1}$

$$\boxed{MC_n = TVC_n - TVC_{n-1}}$$

• **Break-Even Analysis:**

It is that analysis which analyzes the behaviour of total revenue and ~~total~~ total cost as the level of output changes

$TR = P \times Q$

$TC = TFC + TVC$

$TVC = AVC \times Q$

- **Break-even Point**

Refers to the point where $TR = TC$

if π is profit / loss, formula of profit is $= TR - TC$

(if +ve, profit
-ve, loss)

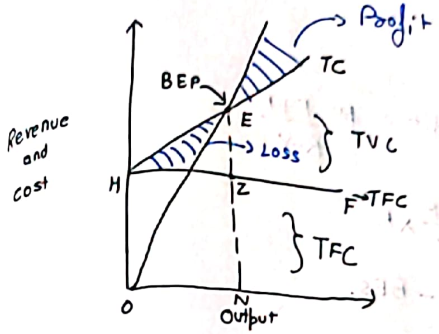
$TR = TC$

$\pi = 0$

- **methods of break-even analysis.**

i.) Graphical Method.

ii.) Algebric Method.



ON → Break-even level of output

$TFC = ZN$

$TC = EN$

$TVC = EZ \; \boxed{(TC - TFC)}$

↳ $TVC = TC - TFC$

• Selling price per unit $= R$

Variable cost per unit $= V$

Total Fixed Cost $= F$

No. of units of output $= N$

At, BEP, $TR = TC$ —(1)

$TR = P \times Q$

$TR = R \times N$

$TC = \cancel{F + V.N}$ $TFC + TVC$

$= F \times V.N$

$R \times N = F + V.N$

$F = RN - VN$

$F = N(R - V)$

$$\boxed{N = \frac{F}{R-V}}$$ → Break-Even level of output.

Formula for Break-Even Analysis

i) BEP in (units) ⇒ $N = \frac{F}{R-V}$

ii) BES / BEP (in Sales) = $\frac{F}{R-V} \times R$

iii) Contribution per unit (C) = $R-V$

iv) Profit = Total Contribution − F

v) Total Contribution = F + P

vi) Units Required to have a target profit = $\frac{F+P}{R-V}$

vii) Sales " " " " " " = $\frac{F+P}{R-V} \times R$

viii) Margin of safety = Actual Sales − BES.

ix) Profit / value Ratio = $\frac{\text{Contribution Per unit}}{\text{Selling price per unit}} \times 100$

$= \frac{R-V}{R} \times 100$

x) Break-even-sales = $\frac{F}{R-V} \times R$

$= \frac{F}{P/V \text{ ratio}}$ (if R and V not given)

xi) Sales required to have a target profit = $\frac{F+P}{P/V \text{ ratio}}$

xii) Margin of Safety = $\frac{\text{Profit}}{P/V \text{ ratio}}$

~~xiii) Contribution per unit = $\frac{R-V}{R}$ = change~~

xiii) P/V ratio = $\frac{\text{change In Profit}}{\text{change In Sales}} \times 100$

xiv) Fixed Cost = (Sales × P/V ratio) − Profit

xv) Variable Cost = (1 − P/V ratio) × Sales.

Q.) From the following, Calculate BEP and P/V ration

Selling price per unit = 30 INR (R)

Variable Cost per unit = 20 INR (V)

Total Fixed Cost = 20,000 INR (F)

$$BEP = \frac{F}{R-V} = \frac{20,000}{30-20}$$

$= 2000$ unit

$$BEP (sales) = \frac{F}{R-V} \times R$$

$= 2,000 \times 30$

$= 60,000$ INR/-

$$P/V \text{ ratio} = \frac{R-V}{R} \times 100$$

$= \frac{10}{30} \times 100 = 33.33\%$

Q.) From the following, P/V ratio, Fixed Cost, Variable cost in 2010, BES and Sales required to have a target profit of 20,000 INR

Years	Sales	Cost	Profit
2010	1,20,000	1,11,000	9,000
2011	1,40,000	1,27,000	13,000

$$P/V \text{ ratio} = \frac{\text{change in profit}}{\text{change in Sales}} \times 100$$

$$= \frac{(1,40,000 - 1,27,000) - (1,20,000 - 1,11,000)}{1,40,000 - 1,20,000} \times 100$$

$$= \underline{20\%}$$

Profit = TR − TC

Fixed Cost = (Sales × P/V ratio) − Profit → can take for 2010 and
= (1,40,000 × .20) − 13,000
= 15,000

Variable Cost (2010) = 96,000 | → (1 − 0.2) 1,20,000

Sales required to have target = 1,75,000 | $\left(\frac{F+P}{P/V}\right) = \frac{15,000 + 20,000}{.2}$

Break-even Sales = $\frac{F}{P/V \text{ ratio}} = \frac{15,000}{0.2}$

= 75,000/-

Q) From the following info. find P/V ratios, BEP, Profit when output is 50,000 INR.

Fixed Cost = 1,20,000
Variable Cost per unit = 3 INR
Selling Price per unit = 7 INR

$$\frac{\quad}{R} \times 100$$

$$= \frac{7-3}{7} \times 100$$

$$= 57.143\%$$

$$\underline{BEP} = \frac{F}{R-V} = \frac{1,20,000}{7-3} = 30,000$$

Units for target profit = $50,000 = \frac{F+P}{R-V}$

$$50,000 = \frac{1,20,000 + P}{7-3}$$

P = 80,000

Inflamation

· Whenever there is a rapid continuous and substantial rise in which reduces value of money, it is called inflation

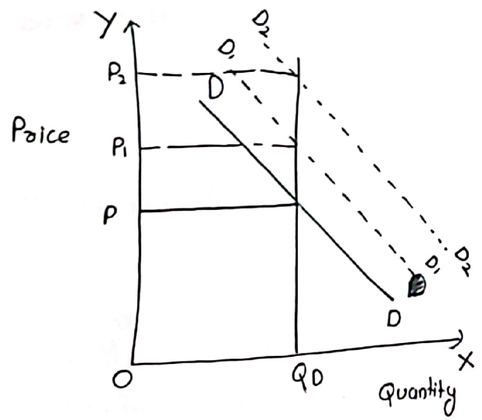
Types of Inflation:

1.) Demand-~~Full~~ Pull inflation.

II.) Cost-push inflation.

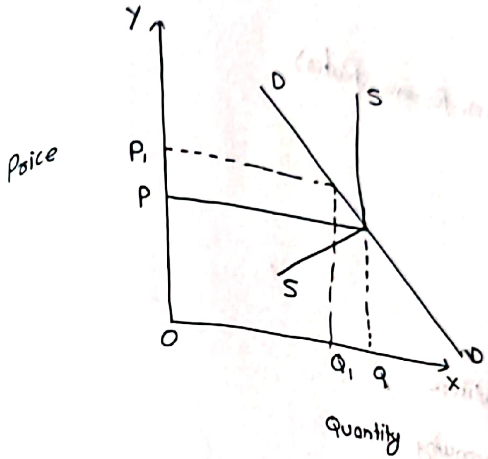
I.) Demand-Pull Inflation:

When aggregate demand for a commodity is more than supply of goods and services of an economy, ~~it~~ results in gap in ~~the~~ a gap of an economy, which pulls the prize up.



II.) Cost-Push ~~Inflat~~ Inflation

When cost of productn increases instead of increasing the price because of the risk of reduction in the Q.D. The producer reduces the supply of goods and services to the market, which push the prize up.



(Explain in exam)

Causes of Inflation

Demand-side inflation

- Increase in money supply → people get more money, more they buy, ~~the~~ more demand, increase in inflation.
- Decrease in tax
- Increase in govtn expenditure
- Increase in export
- Increase in population
- Black money

~~Increase~~

Supply-side inflation

- Shortage in factor of production
- Trade union
- Hoarding
- war and emergency

{- Natural Calamitis

Supply-side Inflation.

{Explanation is recorded in the phone.}

Factors of Control of Inflation

i) Monetary Policy

ii) Fiscal Policy

i) a) Increase in bank Rate

b) Increase in CRR (Cash-Reserve-Ratio)

c) Credit Control

4) Sale of Govt. Security

ii) a) Increse in tax

b) Decrease in Govt. expenditure

c) Increase in Public ~~Bor~~ Borrowing

· Quantitative Methods of Controlling Inflation by Reserve Bank of India

i) Bank Rate

ii) Open Market operation.

iii) CRR

iv) Credit Control

~~iii)~~