It is a social ~~Sign~~ Science

→ Theory of consumer behaviour
→ Nature and scope of engg. economics
→ demand

Interest Rates

→ Simple
→ Compound
→ Nominal :- IR calculated several times in a year more than 1yr.
→ Effective :- Same as nominal but once in a year

Compound IR = $P(1+i)^n$
↳ compound amount factors..

→ Comparision of alternatives
→ Market.
→ Production.
→ Cost.

Deprisiation

→ Straight line method
→ Recycling balance method
→ Sum of the year digit method of depreciation
→ Sinking fund method.

Defination of economics given by Adom Smith

- It is a subject which enquires into the nature and course of the wealth of the nation.

Robbins: He defined economics as a science which studies human behaviour as a relationship b/w ends (unlimed wants) and scares means (shortest money) which have alternative uses.

Engg. Economics

Introduced by L. Grant

Def: It is that branch of economics which deals with method that can enable one to make economic decision twords engg alternatives.

Nature of engg. economics

4 central problem of economic.

i) What to produce
ii) How to produce
iii) For whom to produce
iv) Economic growth problems

Scope of Engg Economics

It deals with theory of consumer behaviour demand elasticity of demand, supply, elasticity of supply equilibrium b/w demand and supply interest rates, compound among factors, comparision of alternatives, revenue, production cost, market depreciation and inflamation.

Demand

It refers to the effective desire to have something backed up by the ability and the willingness to pay for it.

Demand Schedule

It refers to tabular representation of different quantities of commodity demanded at different prices at a given points of time

price of x	Q.D for x
10	5
8	10
6	17
4	22

Where X is any commodity

Types of demand Schedule

i) Individual demand schedule
ii) Market demand schedule.

Individual Demand Schedule

It refers to tabulate representation of different quantities of commodity demanded by an individual consumer at different prices at a given point of time

price of commodity	Demand of Individual commodity
10	10
20	8
30	6
40	4
50	2

Markd Demand Schedule

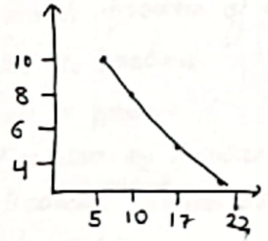
It refers to the tabular representation of different quantities of commodity demanded by different prices at a given point of time

price of X	Q.D. for X by cons. 1	Q.D. for X by consum 2	Q.D. for X by cons. 3	~~Harvest~~ Markd Demand
40	1	3	5	9
30	2	5	9	16
20	3	8	14	25

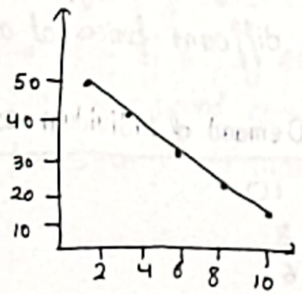
Demand Curve

It refers to the graphical representation of demand schedule

- Dependent Variable X axis
- Independent variable y axis

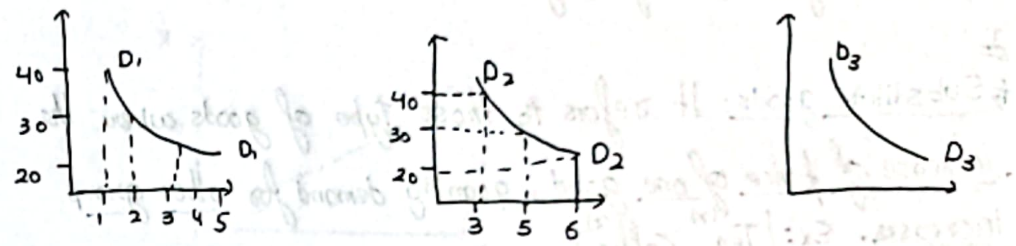


Individual Demand Curve

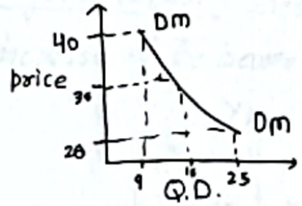


Markd Demand Curve

Individual demand curve of consumer 1



Markd Demand Curve



(★ All diagram in one line)

Q.) From the following individual demand functions find out quantity demand for commodity X by consumer 1 and 2 if price of the commodity X is 5 ruppes per unit and also find out new quantity demand for community x by both the consumer if price of commodity x increases to 7 ruppes per unit.

$Qx_1 = 500 - 2Px_1$

$Qx_2 = 700 - 0.5Px_2$

find out M.D. function and MP with original price and new price.

M.D function $= Qx_1 + Qx_2$

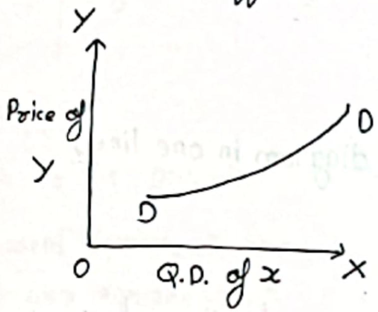
Types of Goods

i.) Substitute goods and complementary goods.

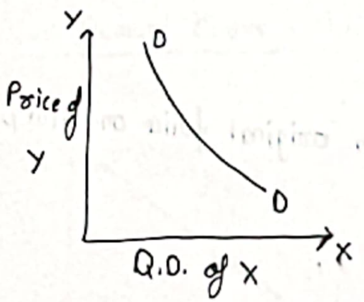
ii.) Normal good and inferior good.

I

Substitute goods: It refers to those type of goods where the increase of price of one good (Y), quantity demand for other group (X) increases. Ex: [Tea, Coffee]

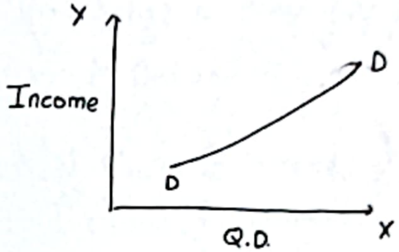


Complementary: It refers to those type of goods where the increase in price of one good (Y), Q.D. for other (X) decreases.
Ex: [Car, Petrol], [Phone, charger].

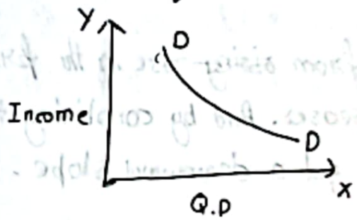


II,

Normal Goods: It refers to those type of goods, whose quantity Demand increases with the increase in income of the consumer.



Inferior Goods: Those goods whose quantity demands ~~..~~ declines with the increase of the income of the consumer.



→ **Law of Demand** (Alfred Marshell): He defined law of Demand as other factors remaining constant (Ceteris Parilus) Q.D. of a commodity increase with a fall in its price and decreases with a rise in it's price.

Assumption of the Law:

i.) Income of the consumer remains constant.

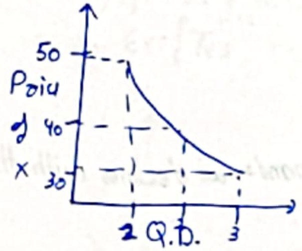
ii.) Prices of relative goods remains constant.

iii.) Taste and Preference of the consumer remains constant.

iv.) Number of consumers in the market does not change.

v.) The good should be a normal good.

Price of X	Q.D. for Com. 1
50	1
~~60~~ 40	2
30	3



Explanation: Here we can see, as from ~~rising~~ rise in the price of commodity 'x', the Q.D. for it decreases. And by combining the points, ~~of the~~ we can see that we get a downward slope.

~~Critism~~ Criticism / Limitation of law of demand / Exception ~~of~~ to the Law of demand:

i.) Giffen Good → Bread meat
ii.) Yeblen Good (Throstein Yeblen) → Diamond (Prestige Value) (rich buy more)
iii.) Speculation
iv.) War and emergency
v.) Other factors

I.) Giffen Good (Robert Giffen):

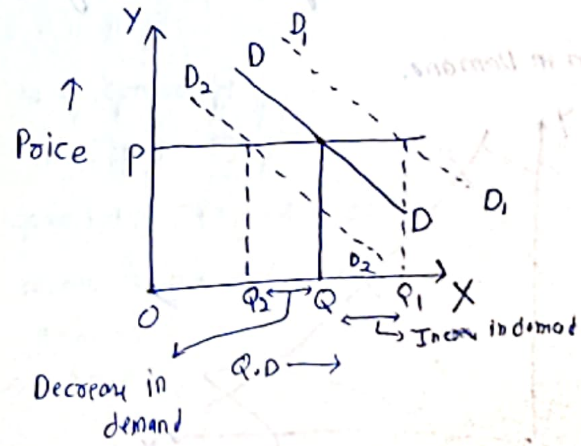
(Qualitative)

Change in Demand: When demand for a commodity changes, due to the change in other factors, price remaining constant, It is called change in Demand

Types of Change in Demand:
i.) Increase in Demand
ii.) Decrease in Demand.

Increase in Demand: When a consumer purchases ~~for~~ more of a commodity than before due to change in other factors, price remaining constant, it is called increase in demand.

Decrease in Demand : When a consumer purchases less of a commodity than before, due to change in other factors, price remaining constant, It is called Decrease in Demand



Change in Q.D: When demand for a commodity changes due to a change in it's price, other's factor remaing constant, it is called Change in Quantity Demand.

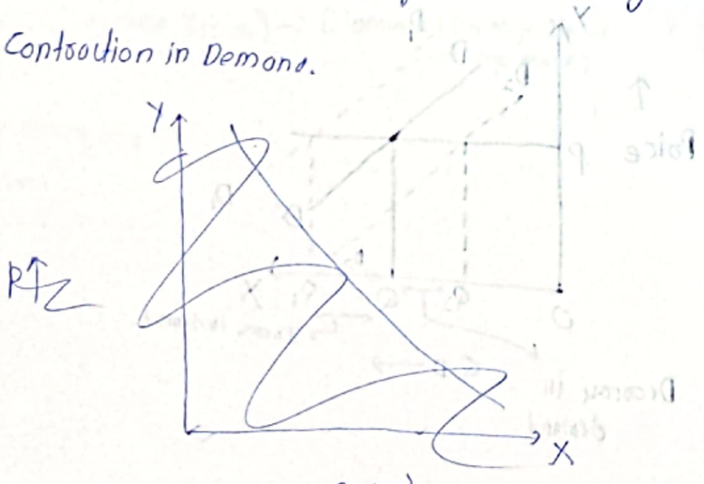
Types of Change in Q.D.:

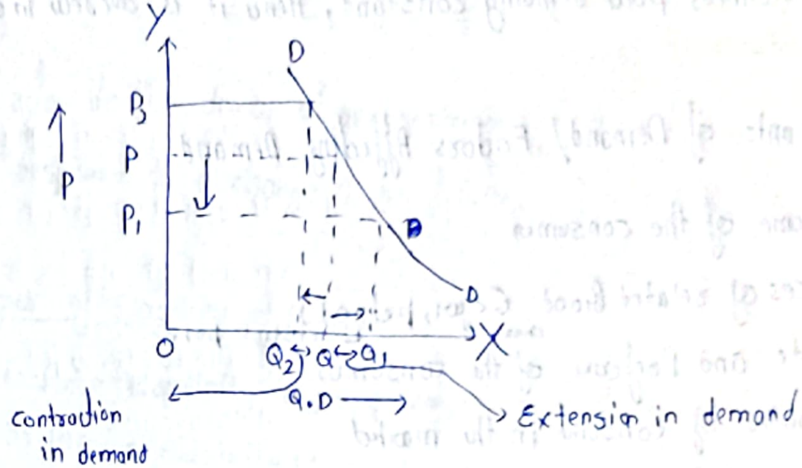
1.) Extension in demand

II.) Contraction

I.) Extension in Demand: When demand for a commodity increases due to a fall ~~#~~ in its price, other factors remaining constant, it is called Extension in Demand.

II.) Contraction in Demand: When demand for a commodity decreases due to a ~~rise~~ a rise in its price, other factors remaining constant, it is called Contraction in Demand.





Change in Q.D.

Q.) From the following Demand function find the Q.D.. if price of the commodity is ₹20 and income of consumer is ₹40,000 per month

$Q = 10{,}000 - 10P + 0.5m$

Find new Q.D for the commodity, if income of the consumer decreases to 35,000 INR per month, price remaining constant and also find what type of change, this change in Quantity implies.

Q → Q.D

P → Price of commodity

m → Income per month

Sol^n^ $Q = 10{,}000 - 10(20) + 20{,}000$

$= 10{,}000 - 200 + 20{,}000$

$= 9{,}800 + 20{,}000$

$Q \rightarrow 29{,}800$

original

$Q = 10{,}000 - 10(20) + 17{,}500$

$= 9{,}800 + 17{,}500$

$Q = 27{,}300.$

Here Q declined, price remains constant, Hence it is decrease in demand.

• Determinants of Demand / Factors Affecting Demand

i.) Income of the consumer

ii.) Prices of related Good (car, petrol) change, constant price

iii) Taste and Preference of the consumer

iv.) Number of consumer in the market

v) Seasonal Demand

vi.) Govt. Policy (Taxation and Subsidy)

vii) Wealth distribution (Equal and Unequal distribution)

viii) Advertisement

~~Elasticit~~

Elasticity of Demand ~~(Quan (Quantity)~~ (Quantitative)

• It refers to the degree of responsiveness of Q.D for a commodity in response to a change in its price.

Types of Elasticity of Demand:

i.) Price elasticity : Degree of change in responsiveness w.r.t price of a good

ii.) Income elasticity : " " " " " Income of a consumer

iii.) Cross elasticity : " " " " " wrt change in price of demand of one commodity to another

I.) Price Elasticity (Elasticity of Demand): Refers to the degree of ~~ch~~ responsiveness of a Q.D. of a commodity in response to a change in it's price.

$$E(e_p) = \frac{\text{Propotional change in Q.D}}{\text{Propotional change in Price.}}$$

coefficient

$$= \frac{\text{Percentage change in Q.D}}{\text{Percentage change in Price.}}$$

Δ → change

Q → Q.D

P → Price

$$= \frac{\frac{\text{Change in QD}}{\text{Original Q.D.}} \times 100}{\frac{\text{change in Price}}{\text{Original Price}} \times 100} = \frac{\frac{\Delta Q}{Q} \times 100}{\frac{\Delta P}{P} \times 100} = \boxed{\frac{\Delta Q}{\Delta P} \times \frac{P}{Q}}$$

for Tabular form

$$\boxed{E(e_p) = \frac{dQ}{dP} \times \frac{P}{Q}}$$ → for functional form.

Q.)

P	Q
100	1000 (Q_1)
200	800 (Q_2)

$E = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$ $\rightarrow \Delta Q = Q_2 - Q_1$, $\Delta P = P_2 - P_1$

$= \frac{-200}{100} \times \frac{100}{1000}$

$= -\frac{1}{5}$

$\Rightarrow |-0.2| \Rightarrow .2$

Q.) If Q.D for a commodity increases from 500 to 700 units due to fall in its price from 80 to 65 INP per unit, find price elasticity (e) of dmn

$e = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$

$\Rightarrow \frac{200}{-15} \times \frac{80}{500}$

$= |-2.1|$

$= 2.1$

Q.) From the following demand function, find (E) elasticity of Demand if price of ~~Pn~~ for commodity X is 200 per unit

$Q_x = 70{,}000 - 15P_x$

$Q_x \rightarrow$ Q.D for X

$P_x \rightarrow$ Price of X for any commodity

Soln

$Q_x = 70{,}000 - 15P_x$ | $\frac{dQ_x}{dP_x} = -15$

$= 70{,}000 - 15(200)$

$\Rightarrow 70{,}000 - 3000$

$Q_x \Rightarrow 67{,}000$

$E \Rightarrow \frac{dQ}{dP} \times \frac{P_x}{Q_x}$

$P_x = 200$

$Q_x = 67{,}000$

$E = -15 \times \frac{200}{67{,}000}$

$E = |-0.0441|$

$E = 0.044$

II) Income elasticity of demand: It refers to the degree of responsiveness of Q.D. for a commodity in response to a change in the income of the consumer.

$$e_y = \frac{\text{Proportionate Change in Q.D.}}{\text{Proportionate change in Income}}$$

$$= \frac{\text{\% change in Q.D}}{\text{\% change in Income}}$$

$$e_y = \frac{\Delta Q}{\Delta Y} \times \frac{Y}{Q}$$

Y: Original Income of consumer

Q: Original Q.D of a commodity

ΔY: change in Income

ΔQ: change in Q.D.

income elasticity of demand $e_y = \frac{dQ}{dY} \times \frac{Y}{Q}$

Q.) A consumer's purchase of a commodity increases from 20 to 23 units due to a rise in his income from 25,000 INR to 30,000 INR per month. Find income elasticity of demand.

$$e_y = \frac{\Delta Q}{\Delta Y} \times \frac{Y}{Q}$$

$$= \frac{3}{5000} \times \frac{25,000}{20}$$

$$= 0.75$$

Q.) From the following Demand function, find Income elasticity of demand if Income of consumer is ~~15~~ 50,000 INR per month.

eg. $Q = 80,000 + 0.6m$

$$e_y = \frac{dQ}{dY} \times \frac{Y}{Q}$$

$Q = \cancel{89,000}\ 1,10,000$

$$e_y = .6 \times \frac{\cancel{15,000}\ \cancel{50,000}}{\cancel{89,000}} \frac{50,000}{1,10,000}$$

$= \cancel{0.101}\ .273$

Q.) From the following Demand function, find price elasticity of demand and Income elasticity of demand if price of the commodity is 30 INR per unit and income of consum is 20,000 INR per month.

$Q = 30,000 - 15P + 0.7Y$

P → Price of comodity

Y → Income

Q = Q.D.

$Q = 30,000 - 15(30) + 0.7(20,000)$

$= 43,550.$

$$e_p = \frac{dQ}{dP} \times \frac{P}{Q}$$

$$= -15 \times \frac{30}{43,550}$$

$= |-0.010|$

$= 0.010$

$$e_y = \frac{dQ}{dY} \times \frac{Y}{Q}$$

$$= .7 \times \frac{20,000}{43,550}$$

$= 0.321$

for Substitute and complementary good. X and Y
+ve relation -ve relation

Cross elasticity of demand : When two goods are so closely related that Q.D for goods X depends on the price of good Y, then cross elasticity of demand is defined as the degree of responsiveness for Q.D. for good 'x' in response to a change in the price of good 'y'.

It is +ve in case of substitute goods and -ve in complementary goods

$$e_{cross} = \frac{\text{Proportional change in Q.D for X}}{\text{Proportional change in Price of Y}}$$

$$e_{cross} = \frac{\Delta Q_x}{\Delta P_y} \times \frac{P_y}{Q_x}$$

$$e_{cross} = \frac{dQ_x}{dP_y} \times \frac{P_y}{Q_x}$$

• If Q.D. for coffee increases from 10,000 units to 15,000 units, find out Cross elasticity of demand b/w Tea and coffee, if price of Tea Increase from 700 to 900 INR per 250 grams pack.

$$\Rightarrow \quad e_{cross} = \frac{15,000 - 10,000}{900 - 700} \times \frac{700}{10,000}$$

$$\Rightarrow \frac{5000}{200}$$

$$\Rightarrow 25 \times \frac{700}{10000}$$

$$= 1.75$$

Q.) From the following Demand fn: find out cross-elasticity of demand b/w Tea and Coffee. If price of Tea is 2000 INR per 500 grams pack

$$Q_c = 60,000 + 0.3 P_t$$

$$= 60,000 + 0.3 (2000)$$

$$= 60,600$$

$$e_{cross} = 0.3 \times \frac{2000}{60,600}$$

$$= 0.009$$

Q.) From the following table. Find out

i.) Cross elasticity of demand b/w 'X' and 'Y', If price of X increases from 200 to 400 per unit.

ii.) Good 'X' and 'Y' are what type of goods on the basis of i.)

iii.) Income elasticity of Demand for good Y if income of the consumer will increase from 40,000 to 55,000 INR per month.

iv.) Good 'y' Is what type of good on the basis of above question

Price of Good X	Q.D for Good Y	Income
200	80	20,000
300	70	40,000
400	65	50,000
500	40	55,000

→ i) $e_c = \dfrac{\Delta Q_Y}{\Delta P_X} \times \dfrac{P_X}{Q_Y}$

$\Rightarrow \dfrac{65-80}{400-200} \times \dfrac{200}{80}$

$\Rightarrow \dfrac{-15}{200} \times \dfrac{200}{80}$

$= -0.187$

ii) ~~Complementary Goods~~ ~~Inferior Goods~~ Supplementary

iii) $e_Y = \dfrac{\Delta Q_Y}{\Delta Y} \times \dfrac{Y}{Q}$

$\Rightarrow \dfrac{40-70}{55000-40000} \times \dfrac{40000}{70}$

$\Rightarrow -1.142$

iv) Inferior good.

→ Degrees of elasticity of Demand

i) Perfectly elastic Demand.

ii) Relatively " " / More elastic demand

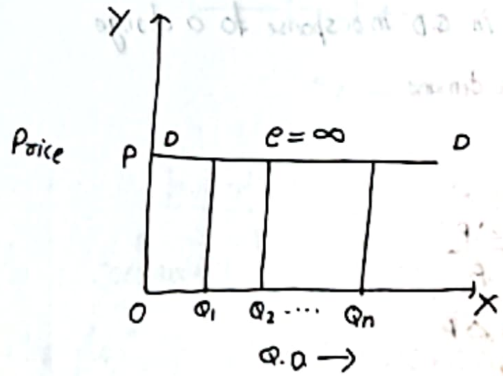
iii) Unitary " "

iv) Relatively inelastic / Less elastic demand

v) Perfectly Inelastic Demand

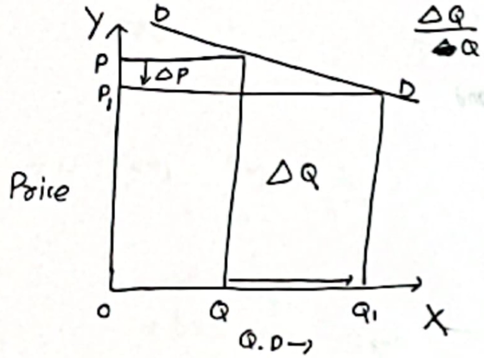
I) Perfectly elastic demand:

· When at a fix price, infinite quantities of commodity are demanded and with a small increase in the price, Q.D falls to zero. It is called perfectly elastic demand.



II.) Relatively Elastic Demand:

→ When there is more than proportionate change in Q.D for a commodity in response to a change in its price. It is called R. E. Demand. Ex → Durable and Luxurious goods.

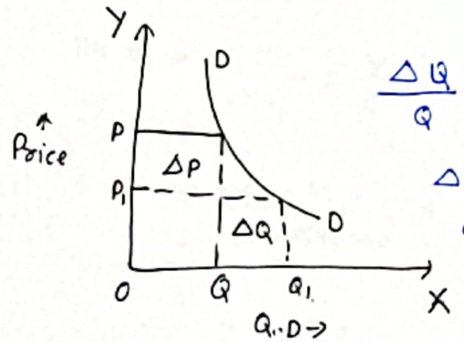


$\frac{\Delta Q}{Q} > \frac{\Delta P}{P}$

$e > 1$

III.) Unitary Elastic Demand.

When there is proportionate change in QD in response to a change in its price, It is called unitary elastic demand.



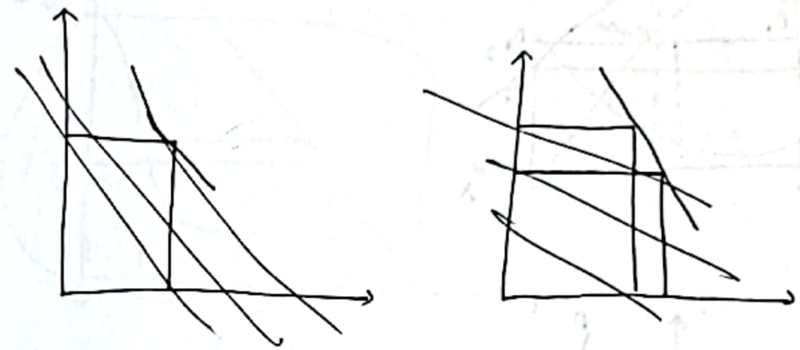
$\frac{\Delta Q}{Q} = \frac{\Delta P}{P}$

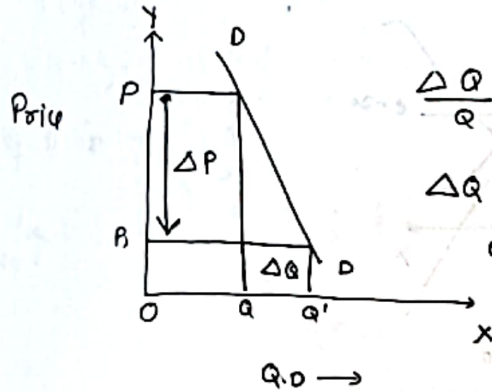
$\Delta Q = \Delta P$

$e = 1$

IV.) Relatively Inelastic Demand.

When there is less than proportionate change of for a QD. of a commodity in response to a change its price, it is relatively inelastic Demand. Ex: Perishable good: meat, fish.





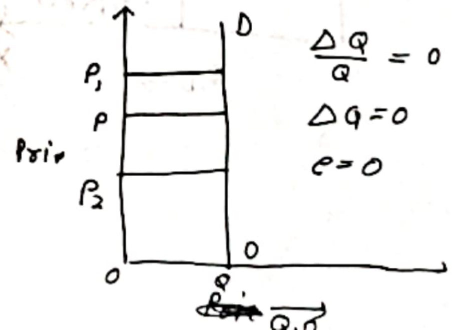
$\frac{\Delta Q}{Q} < \frac{\Delta P}{P}$

$\Delta Q < \Delta P$

$e < 1$

V.) Perfectly Inelastic Demand:

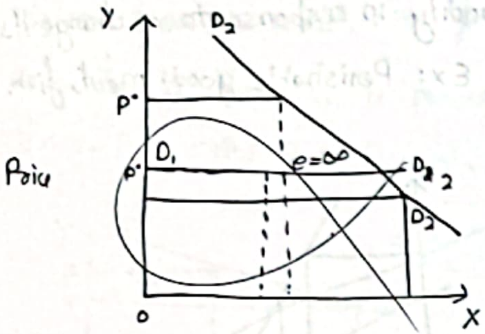
When there is no change in Q.D. for a commodity in response to a change in its price, it is called perfectly inelastic Demand.

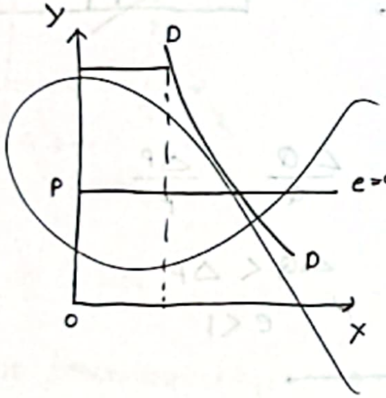


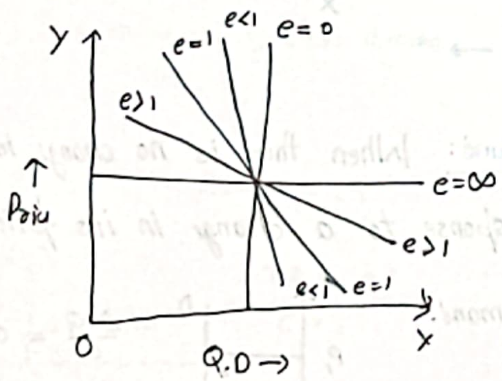
$\frac{\Delta Q}{Q} = 0$

$\Delta Q = 0$

$e = 0$

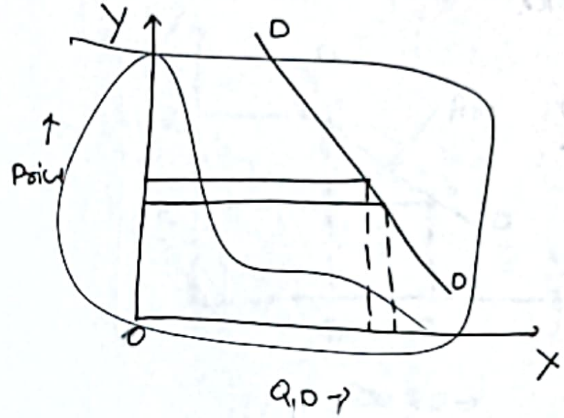




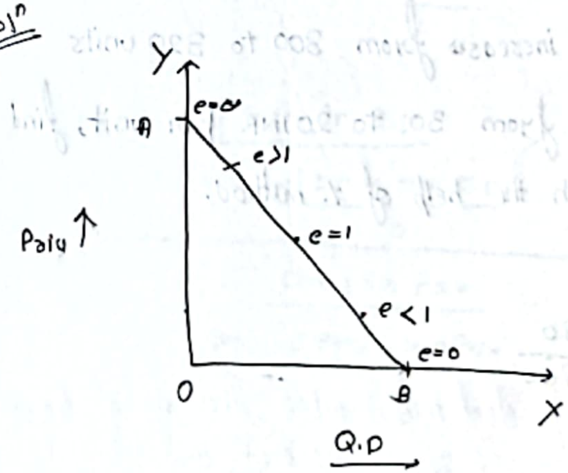


→ All 5 diagrams in one diagram (5 demand curve in 5 degrees)

Show all the 5 degrees of elasticity on a Downward Sloping Straight line demand Curve.



Sol^n



Methods of measuring elasticity of demand.

I.) Percentage Method

II.) Arc method / mid-point method

III.) Total expenditure method / Total outlay method.

I) Percentage Method: It refers to that method where elasticity of demand can be measured on the basis of % change in Q.D. in response to % change in its price.

$$E(P_P) = \frac{\%\text{ change in Q.D}}{\%\text{ change in Price}}$$

$$\Rightarrow \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$$

Q) If Q.D. for a commodity increase from 300 to 320 units due to a fall in its price from 30 to 20 INR per unit, find out elasticity of demand with the help of % method.

$$\Rightarrow \frac{320-300}{20-30} \times \frac{30}{300}$$

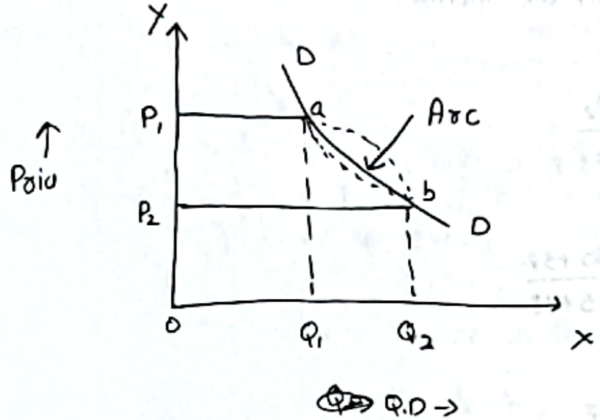
$$\frac{20}{-10} \times \frac{\cancel{30}}{\cancel{300}_{10}}$$

$\Rightarrow -0.2$

$\Rightarrow |-0.2|$

$= 0.2$

II) Arc Method: It refers to that method where elasticity of demand can be measured b/w two separate points.



P = Price

Y = Income

$$e_{arc} = \frac{\dfrac{\text{Change in Q.D}}{\dfrac{\text{Original Q.D + New Q.D}}{2}} \times 100}{\dfrac{\text{Change in Price}}{\dfrac{\text{Original Price + New Price}}{2}} \times 100}$$

$$= \frac{\dfrac{\Delta Q}{\dfrac{Q_1+Q_2}{2}} \times \cancel{100}}{\dfrac{\Delta P}{\dfrac{P_1+P_2}{2}} \times \cancel{100}}$$

arc price elasticity $\Rightarrow \boxed{\frac{\Delta Q}{\Delta P} \times \frac{P_1+P_2}{Q_1+Q_2}}$

for price.

for income

$$\boxed{\frac{\Delta Q}{\Delta Y} \times \frac{Y_1+Y_2}{Q_1+Q_2}}$$

arc income elasticity

Q.) If Q.D. of a commodity, decreases from 55 to 48 units due to a rise in its price from 30 to 37 INR per unit, find elasticity of demand, with arc method

$$e_{arc} = \frac{\Delta Q}{\Delta P} \times \frac{P_1+P_2}{Q_1+Q_2}$$

$$= \frac{48-55}{37-30} \times \frac{30+37}{55+48}$$

$$\Rightarrow \frac{-7}{7} \times \frac{67}{103}$$

$$= -0.65$$

$$|-0.65|$$

$$\Rightarrow .65$$

iii.) ***Total Expenditure***: It refers to that method where elasticity of demand can be measured on the basis of change in total expenditure for a commodity due to a change in its Q.D. in response to a change in the price of the commodity.

$$T.E = P \times Q$$

P → Price of the commodity per unit

Q → No of units of commodity purchase

→ 3 possibilities:

i.) If Total expenditure increases with a fall in the price or decreases with a rise in the price, then price elasticity will be $e>1$.

ii.) If T.E. remains constant with a rise or fall in the price then price elasticity will be $e=1$.

iii.) If T.E. increases with a rise in the price or decreases with a fall in the price, the price elasticity will be $e<1$.

Price	Q.D	T.E ⇒	$e_p = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$
5.0	30	150	—
4.75	40	190	$e_p>1$
4.50	50	225	$e>1$
4.25	60	255	$e>1$
4.00	75	300	$e>1$
3.75	80	300	$e=1$
3.50	83	290.5	$e<1$
3.25	87	282.75	$e<1$

$$\frac{10}{-0.25} \times \frac{5}{30} = -6.6 \to \underline{6.6}$$

Q.) If Q.D. for a commodity declines from 100 to 80 units due to a rise in its price from 8 to 10 INR per unit. Find out elasticity of demand with total expenditure method.

P. Price	Q.D	TE	e_p
8	100	800	—
10	80	800	$e_p=1$

Determinants / Factors affecting elasticity of demand.

i.) Availability of Substitutes
- i.) No Substitute : e = 0
- ii.) Number of " : e > 1

ii.) Nature of the good.
- luxuries : e > 1
- Perishable : e < 1
- Necessary : e = 0

iii.) Alternative Uses
- Number of uses : e > 1
- only one use : e = 0

iv.) Postponement of Consumption
- Can be postpone now : e > 1 (luxurious Good)
- Can't be postpone now : e = 0

v.) Time Period
- Short period : e = 0
- long period : e > 1

vi.) Habit : e = 0

Demand Forecasting

y is dependent

$y = a + bx$

Years	(y) Sales	X	XY
1990	10	-2	
1991	15	-1	
1992	25	0	
1993	36	1	
1994	42	2	
		$\Sigma x =$	

$\Sigma Y = Na + b\Sigma x$

$\Sigma XY = a\Sigma x + b\Sigma x^2$

Q.) From the following table, forecast sales for the year 2021 and 22

Year	(y) Sales ($1000)	X	XY	X^2
2015	20	-2	-40	4
2016	35	-1	-35	1
2017	42	0	0	0
2018	56	1	56	1
2019	63	2	126	4
N = 5	$\Sigma Y = 216$	$\Sigma x = 0$	$\Sigma = 107$	$\Sigma x^2 = 10$

$\Sigma Y = 5a + b\Sigma x$

$216 = 5a$

$a = 43.2$

$\Sigma XY = a\Sigma x + b\Sigma x^2$

$107 = 43.2(0) + b\,10$

$b = 10.7$

$Y = 43.2 + 10.7x$

for 2021, $x = 4$

$Y_{2021} = 43.2 + 0.7(4)$

$= 86$

$Y_{2022} = 43.2 + 0.7(5)$

$= 96.7$

Supply

Stock: Refers to actual amount of production whereas supply refers to the actual amount offered for sale in the market.

Supply Schedule: Refers to the tabular representation of different quantities of a commodity, supplied to the market at different prices.

Price of X (in INR)	Q.S of X (in units)
5	10
10	17
15	19

Types of Supply Schedule:

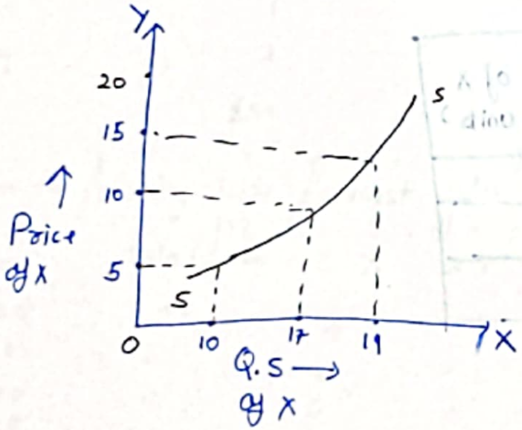
i) Individual Supply Schedule.

ii) Market Supply Schedule.

Individual Supply Schedule: Refers to the tabular representation of different quantities of a commodity supplied to the market by an individual producer at different prices (Draw schedule)

Market Supply Schedule: Refers to the tabular representation of different quantities of a commodity supplied to the market by all the producers at different prices (Draw schedule)

Supply Curve



Law of Supply: It is defined as other factors remaining constant, quantity supply of a commodity increases with a rise in price and decreases with a fall in the price.

→ **Assumptions of the law:**

i) The technique of production remain constant.

ii) Prices of input remain constant.

iii) Prices of other products remain constant

iv) No. of producers in the market remain constant.

v) Prices of other ~~product~~ producer does not change.

(Prepare supply schedule, then supply curve, limitation/critism of law.)

limitation:

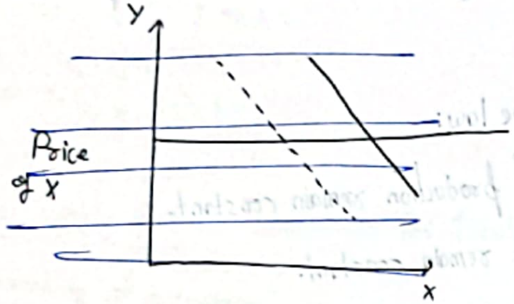
- In case of old coins or rare paintings.
- In case of old stocks.
- If labour price increases.
- When a business person closes his/her bussiness.

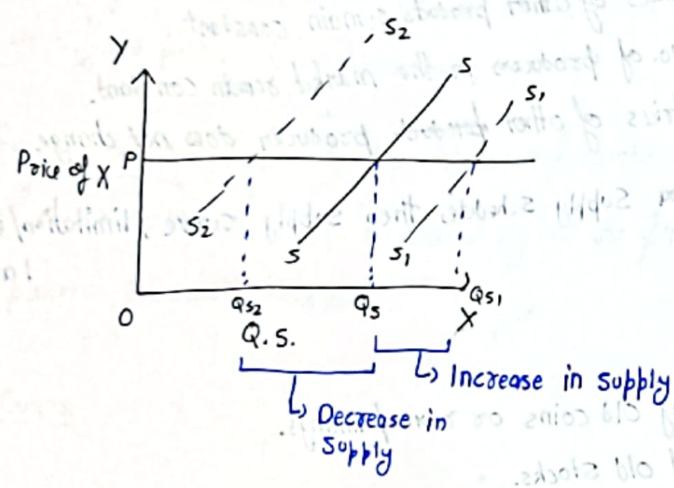
→ **Change in supply:** When supply of a commodity changes due to change in the other factors, price remaining constant, it is called change in supply.

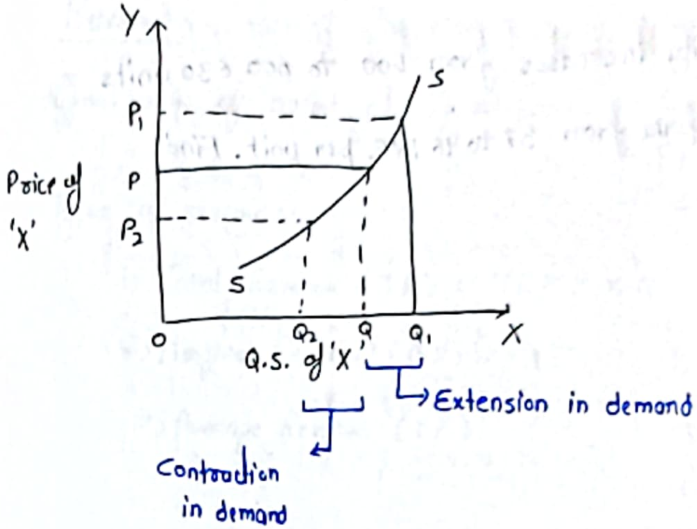
Types: i) Increase in supply

ii) Decrease in supply.

Change in supply curve:







Change in Quantity Supply:

When supply of a commodity changes due to a change in its price, other factor remaining constant, It is called change in Q.S.

Types: i.) Extension in supply.

ii.) Contraction in supply.

Elasticity of Supply: Refers to the degree of responsivness of Q.S. of a commodity in response to a change in its price.

$$e_s = \frac{\text{\% change in Q.S.}}{\text{\% change in Price}}$$

$$= \frac{\Delta Q_s}{\Delta P} \times \frac{P}{Q_s}$$

$$= \frac{dQ_s}{dP} \times \frac{P}{Q_s}$$

Q.) If Q.S. of a commodity increases from 500 to ~~600~~ 630 units due to a rise in its price from 37 to 46 INR per unit. Find elasticity of supply.

$$e_s = \frac{\Delta Q.S}{\Delta P} \times \frac{P}{Q.S}$$

$$= \frac{130}{9} \times \frac{37}{500}$$

$$= \frac{4810}{4500}$$

$$\Rightarrow 1.068$$

→ Degree of elasticity of supply:

i.) Perfectly elastic supply ($e_s = \infty$)

ii.) Relatively " " ($e_s > 1$)

iii.) Unitary " " ($e_s = 1$)

iv.) Relatively inelastic " ($e_s < 1$)

v.) Perfectly " " ($e_s = 0$)

→ Perfectly elastic supply: When price is fixed infinitly

→ Relatively elastic supply:

Revenue: Refers to the income earned by a producer by selling different units of output at different prices.

[P: selling price per unit
Q: No. of units of output sold]

Types of revenue:

i.) Total Revenue (TR) : $TR = P \times Q$

ii.) Marginal Revenue (MR) :

iii.) Average Revenue (AR)

Marginal Revenue: It refers to the net addition to the total revenue by selling one extra unit of output:

$$MR_n = TR_n - TR_{n-1}$$

$$MR = \frac{d(TR)}{dQ}$$

Total Revenue: Total income earned by a producer by selling different unit of output at different prices. $TR = P \times Q$

Average Revenue: It refers to the total income earned ~~ea~~ per unit of output sold.

$$AR = \frac{TR}{Q} \quad -(i)$$

$$TR = AR \times Q$$

$$TR = P \times Q \quad -(ii)$$

$$P = AR$$

Q.) From the following table find total revenue and marginal revenue.

Unit of Output Sold	A.R.	M.R
1	16	16
2	15	14
3	14	12
4	13	10
5	12	8
6	11	6
7	10	4
8	9	2
9	8	0
10	7	-2

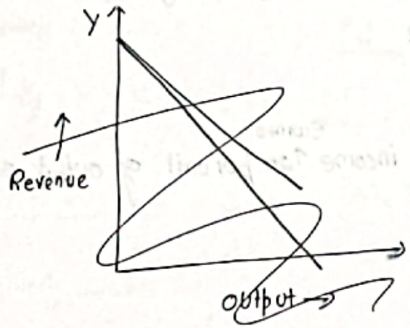
$TR = 16 + 30 + 42 + 52 + 60 + 66 + 70$

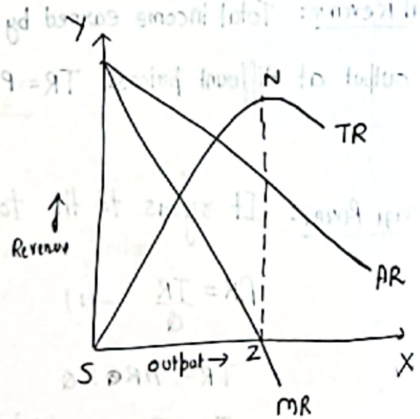
$72 + 72 + 70$

$=$

$MR_n = \frac{\partial(TR)}{\partial Q}$ $TR_n - TR_{n-1}$

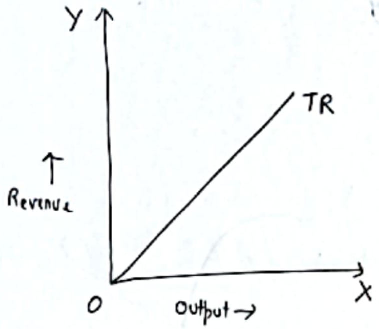
TR is max when MR is zero

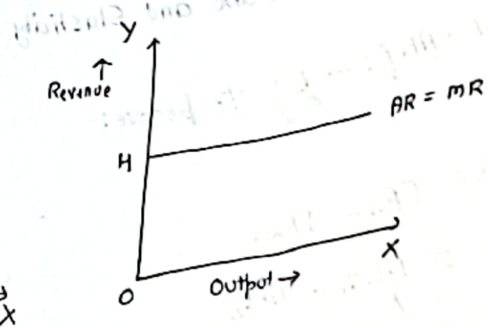




Q.) From the following table find total revenue and ~~Margi~~ marginal Revenue.

Unit of Output Sold	AR	TR	MR
1	30	30	30
2	30	60	30
3	30	90	30
4	30	120	30
5	30	150	30
6	30	180	30
7	30	210	30
		Σ	





Q.) From the following demand function find i.) TR and MR ii.) Price and Quantity when MR=0 iii.) Price and quantity when TR = MAX

$Q = 40,000 - 15P$

i.) $Q = 40,000 - 15P$

$15P = \frac{40,000}{15} - \frac{Q}{15} \Rightarrow 2666 - \frac{Q}{15} \Rightarrow TR = 26666 - \frac{Q^2}{15}$

$MR = 2666 - 0.13Q$

$MR = 0$

$$\frac{2666}{.13} = Q$$

$Q = \cancel{2050}\ 20507.69$

$Q = 20508$

$\cancel{Q} = 40{,}000 - 15P$

$$\frac{20508 - 40{,}000}{-15} = P$$

$\Rightarrow P = \cancel{1298}\ 1298.8$

→ Relationship b/w Revenue and Elasticity

$MR = AR\left(1 - \frac{1}{e}\right)$ to proove:

$MR = TR_n - TR_{n-1}$

$\Rightarrow AR Q_1 - AR Q_2$

$\Rightarrow AR(Q_1 - Q_2)$

$\Rightarrow \left(e = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}\right)$

$\Rightarrow AR\left(\cancel{Q}\ \frac{\Delta Q P}{\Delta P} \times \frac{1}{e_1}\right) - AR\left(\frac{\Delta Q P}{\Delta P} \times \frac{1}{e_2}\right)$ α

$\Rightarrow AR\left(\frac{\Delta Q P}{\Delta P} \times \frac{1}{e_1} -\right.$

$$MR = \frac{d(TR)}{dQ}$$

$$= \frac{d(P \times Q)}{Q}$$

$$\Rightarrow \frac{dP}{dQ} \cdot Q + \frac{dQ}{dQ} \cdot P$$

$$\Rightarrow P + \frac{dP}{dQ} Q$$

$$MR = P\left(1 + \frac{dP}{dQ} \cdot \frac{Q}{P}\right)$$

$$= P\left(1 + \frac{1}{\frac{dQ}{dP}\frac{P}{Q}}\right)$$

$$= P\left[1 + \frac{1}{(-e)}\right] \quad [P = AR]$$

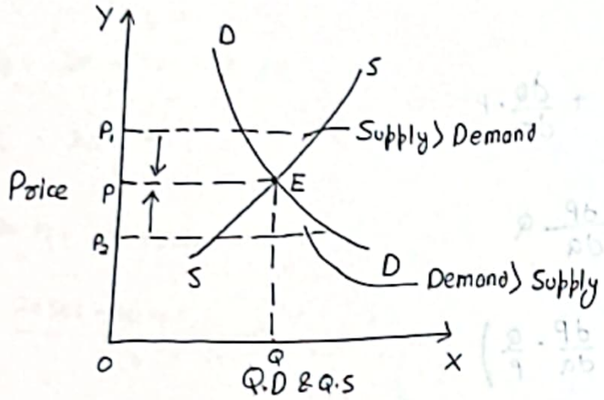
$MR = AR\left(1 - \frac{1}{e}\right)$ proove$\!d$

Q.) Find MR if price of the commodity is 3INR per unit and coefficient of elasticity is 5

$MR = AR\left(1 - \frac{1}{e}\right) \quad [AR = P]$

$= 3\left(1 - \frac{1}{5}\right) \Rightarrow 3 - \frac{3}{5} \Rightarrow \frac{15-3}{5} \Rightarrow 2.4$

Equilibrium b/w demand and Supply : (Y: Independent, X: Dependent)



Q.) From the following Demand and Supply function. Find Equilibrium price and Quantity.

$Q_D = 800 - 15p$

$Q_S = 500 + 5p$

Find new equilibrium price and Quantity if Supply remaning constant Demand decreases to $Q_D = 700 - 10P$

→ $800 - 15p = \cancel{Q_S}\ 500 + 5p$

$300 = 20p$

$p = 15$

$Q \Rightarrow 800 - 15(15)$

$Q \Rightarrow 575$

$Q_D \Rightarrow \cancel{575 = 700 - 10p}$

$\cancel{+125 = +10p}$

$12.5 = p$

ii.) $Q_S = Q_D$

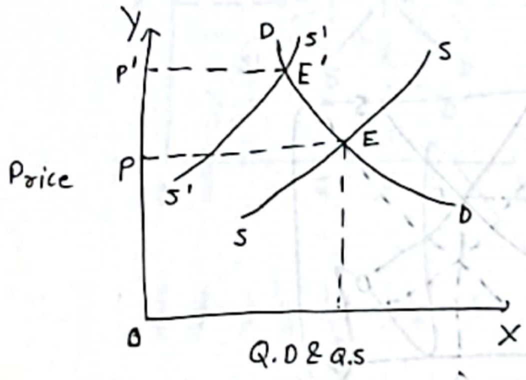
$500 + 5p = 700 - 10P$

$-200 = -15P$

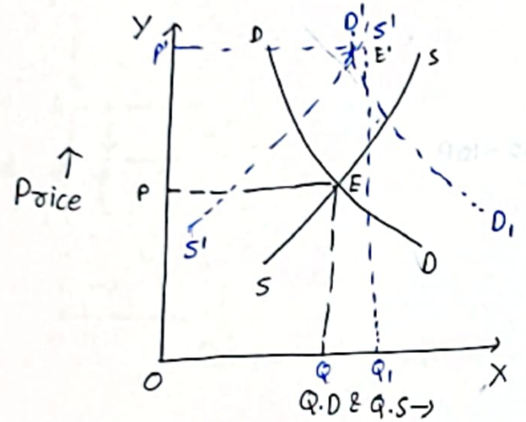
$P = 13.333$

$Q = 566.67$

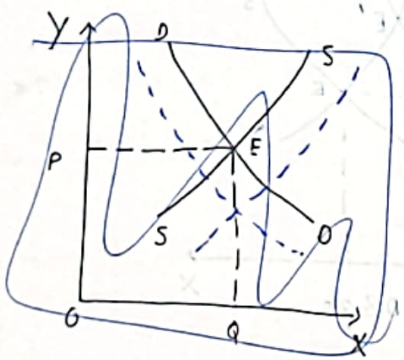
What happens to equilibrium price and Quantity, if demand remaining constant, supply decreases.

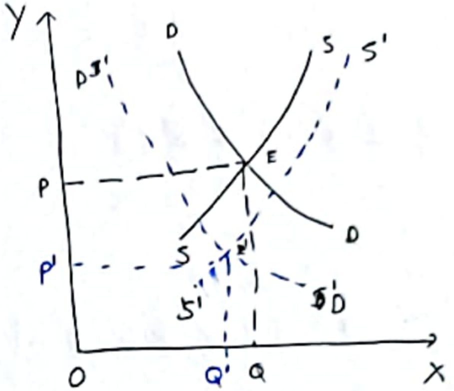


Q.) What happens to equilibrium price and quantity if increase in demand is more than decrease in supply.

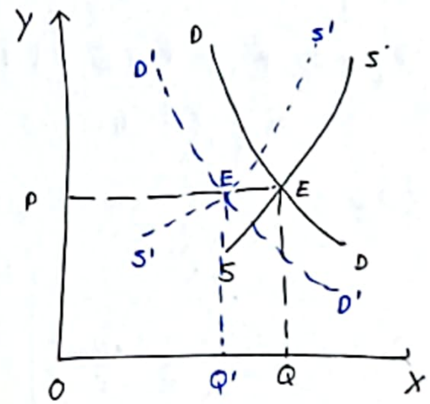


What happens to equilibrium price and quantity if increase in supply is less than decrease in demand.





Q.) What happens to equilibrium price and Quantity if both demand and supply decreases by the same amount.



Vaibhav Sharma

22053123, CSE-27

Q1.) Explain the degrees of elasticity of demand with the help of there respective diagrams.

Q2.) Distinguish b/w % method and Arc method of measuring elasticity of Demand.

Q3.) From the followind demand function, find price elasticity of demand and Income elasticity of demand if price of the commodity is 50INR per unit, and income of the consumer is 30,000 per month

$$Q = 50{,}000 - 5P + 0.7Y$$

Q4.) From the following demand function find

i.) TR function and MR function

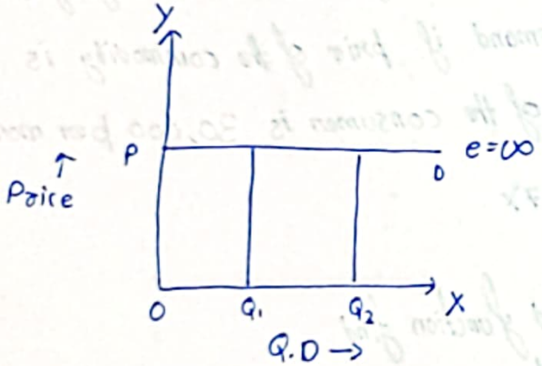
ii.) Price and Quantity for TR to be maximum

$$Q = 80{,}000 - 25P$$

1.) There are 5 degrees of elasticity of demand.

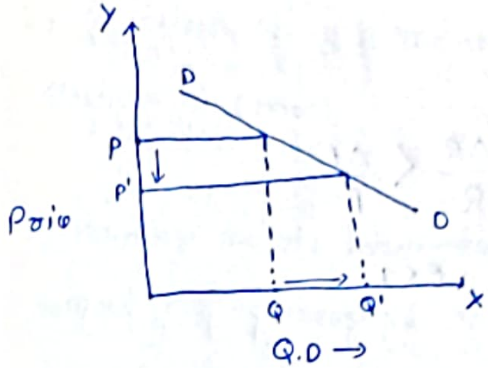
i.) **Perfectly Elastic Demand:**

In perfectly elastic demand, the Q.D of a commodity is infinite with a fixed price. but if there is a change in price of a commodity, Q.D. falls to zero



ii.) **Relatively Elastic Demand:**

In Relative elastic demand, the Q.D. of a commodity is more proportionate with respect to change in price of a commodity i.e, when there is more than proportinate change in Q.D. for a commodity in response to change in price. Ex: Luxurious, Durable goods

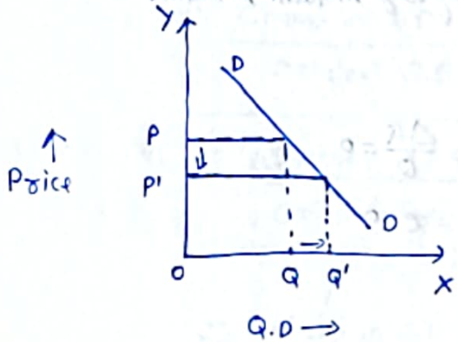


$$\frac{\Delta Q}{Q} > \frac{\Delta P}{P}$$

$e > 1$

iii.) **Unitary Elastic Demand:**

When there is a proportional change in Q.D. for a commodity for a change in price is called Unitary Elastic Demand.



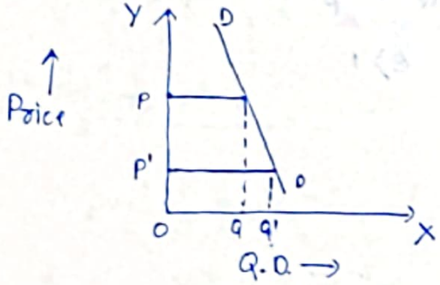
$$\frac{\Delta Q}{Q} = \frac{\Delta P}{P}$$

$e = 1$

iv.) **Relatively Inelastic Demand:**

When there is less than proportional change in Q.D. for a commodity for a change in price is called Relatively Inelastic Demand. Ex: Fish, meat

~~V.) Perfectly Inelastic Demand:~~

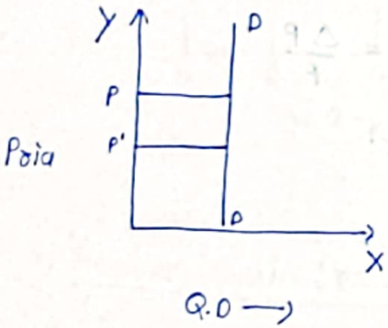


$$\frac{\Delta Q}{Q} < \frac{\Delta P}{P}$$

$e < 1$

V.) Perfectly Inelastic Demand:

When there is no change in Q.D for a commodity with response to change in its price is called perfectly Inelastic Demand.



$$\frac{\Delta Q}{Q} = 0$$

$e = 0$

2.) % method and Arc Methods are ways to compute the elasticity of Demand.

1.) Percentage method: ~~When there~~ Method when elasticity of demand can be measured on the basis of percentage change in Q.D. of a commodity in response to percentage change in its price

$$E(e_p) = \frac{\%\ \text{Change in Q.D}}{\%\ \text{change in Price}}$$

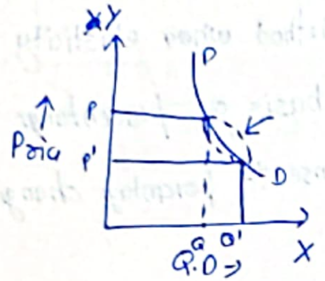
$$= \frac{\frac{\text{Change in Q.D}}{\text{Original Q.D}} \times 100}{\frac{\text{Change in Price}}{\text{Original Price}} \times 100}$$

$$\Rightarrow \frac{\text{change in Q.D}}{\text{Change in Price}} \times \frac{\text{Original Price}}{\text{Original Q.D}}$$

$$\Rightarrow \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$$

[∵ Q → Quantity Demand
P → Price]

ii.) **Arc Method:** It is the method where elasticity of demand can be measured b/w two seperate points



$$e_{arc} = \frac{\dfrac{\text{change in Q.D}}{\dfrac{\text{Original QD} + \text{New Q.D}}{2}} \times 100}{\dfrac{\text{change in Price}}{\dfrac{\text{Original price} + \text{new Price}}{2}} \times 100}$$

$$\Rightarrow \frac{\dfrac{\text{change in Q.D}}{\text{Original QD} + \text{New QD}}}{\dfrac{\text{change in Price}}{\text{Original Price} + \text{New Price}}} \Rightarrow \boxed{\frac{\Delta Q}{\Delta P} \times \frac{P_1 + P_2}{Q_1 + Q_2}}$$

[∴ Q_1 = Quantity Demand of 1
Q_2 = " " of 2
P_1 = Price of commodity 1
P_2 = " " 2]

3.) Given

$$Q = 50,000 - 5p + 0.7Y \quad -(1)$$

price of commodity = 50 INR

Income of consumer = 30,000 per month

Now, putting the values in (1)

$$Q = 50,000 - 5(50) + 0.7(30,000)$$

$$\underline{Q = 70,750}$$

i.) <u>price elasticity of demand</u>

we know

$$e_p = \frac{dQ}{dp} \times \frac{p}{Q}$$

$$= -5 \times \frac{\cancel{30000}\ 50}{70,750}$$

$$= |-0.0031$$

$$= \underline{0.003}$$

ii.) <u>Income elasticity of demand</u>

We know

$$e_y = \frac{dQ}{dY} \times \frac{Y}{Q} \Rightarrow .7 \times \frac{30,000}{70,750}$$

$$= \underline{0.296}$$

Interest Rates:

i.) Simple Interest Rate

ii.) Compound " "

iii.) Nominal " "

iv.) Effective " "

Simple Interest Rate: Refers to that type of Interest rate where principal amount remains same for various years.

$$I = P.i.N$$

$i \rightarrow$ Interest Rate

$N \rightarrow$ No. of Years

$P \rightarrow$ Principal Amount

$I \rightarrow$ Simple Interest Rate

$$F = P + I$$

$$= P + (P.\ i.N)$$

$$F = P(1 + i.N)$$

$F \rightarrow$ Future Value

$P \rightarrow$ Principal

Q.) Find future value of 4 lakh INR at 6% interest rate, after 6 years, with the help of Simple Interest Rate.

$$F = 4,00,000\,(1 + 0.06 \times 6)$$

$= \cancel{4,00,000(1+\ldots)}$

$\cancel{= 4,00,000(}$

$\Rightarrow 5,44,000/-$

Compound Interest Rate: Refers to that type of Interest rate where the principal amount keeps on changing every year.

$$F_n = P(1+i)^n$$

$N \Rightarrow$ number of years

$P \rightarrow$ Principal , $i \rightarrow$ Interest rate

Q.) If A person deposits 7 lakh INR in a bank at 9.5% interest rate compounded anually, find out the maturity amount of his account after 10 years.

$$F_n = 7,00,000\,(1 + 0.095)^{10}$$

$$= 17,34,759.339/-$$

Nominal Interest Rate: Refers to that type of interest rate where interest rate is calculated several times in a year, i.e, Quaterly, monthly, half-yearly/ Semi-anually and each day to find out compound Interest rate.

i.) Quaterly: $F_4 = P\left(1+\frac{i}{4}\right)^{4N}$

ii.) Monthly: $F_{12} = P\left(1+\frac{i}{12}\right)^{12N}$

iii.) Half yearly / Semi-Annually = $F_2 = P\left(1+\frac{i}{2}\right)^{2N}$

iv.) Each Day: $F_{365} = P\left(1+\frac{i}{365}\right)^{365N}$ [not including leap year]

Q.) find future value of 5 lakh INR at 4.5% interest rate after 7 years if the compounding is Monthly.

$$F_{12} = 5,00,000\left(1+\frac{0.045}{12}\right)^{12(7)}$$

$$= 6,84,726.12/-$$

Effective Interest Rate: Refers to the ratio of interest charged for 1 year to the principal amount. $(N=1)$

$$i_{eff} = \frac{F-P}{P} \qquad (i_{eff} \rightarrow \text{Interest Rate})$$

Q.) Find effective Interest Rate of 8,00,000 INR at 9% interest rate if the compounding is quaterly.

$$F_4 = P\left(1+\frac{i}{4}\right)^{4N}$$

$$\cancel{F_4} = 8,00,000\left(1+\frac{0.09}{4}\right)^4$$

$$= 8,74,466.65/-$$

$$i_{eff} = \frac{8,74,466.65 - 8,00,000}{8,00,000} \qquad \left(i_{eff} = \frac{F-P}{P}\right)$$

$$= 0.093$$

$$= \underline{9.3\%}$$

q.) If Principal amount is not given:

$$i_{eff} = \left(1 + \frac{r}{m}\right)^m - 1$$

$r \to$ Nominal Interest Rate

$m \to$ No. of times interest Rate is calculated in a year

-or

No. of compounding

q.) If a credit amount charges 19% interest rate, find out effective interest rate if the compounding is half-yearly.

$$i_{eff} = \left(1 + \frac{0.19}{2}\right)^2 - 1$$

$= 0.199$

$= 19.9\%$

Utility

Refers to the want satisfying capacity of the commodity.

It can be +ve or -ve.

- When utility has -ve impact on health and society, it is -ve utility. Ex- Smoking
- When utility has +ve impact on health and society, it is +ve utility.

Types of Utility:

i) Total Utility (TU): Refers to the total satisfaction that a consumer can get by consuming various units of a commodity.

ii) Marginal Utility (MU): Refers to the net addition to the total utility by consuming 1 extra unit of a commodity.

$$\text{Marginal Utility } (MU_n) = TU_n - TU_{n-1}$$

X → commodity 'x'

$$MU_x = \frac{d(TU)}{dx}$$

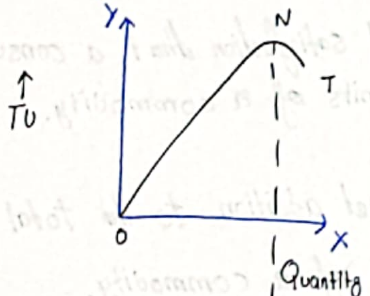
Y → commodity 'y'

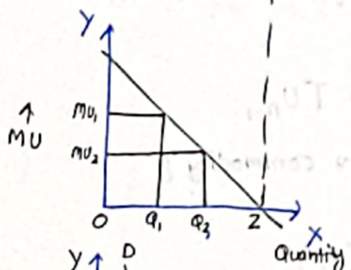
$$MU_y = \frac{d(TU)}{dy}$$

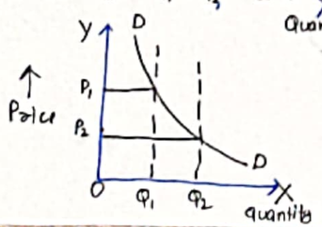
Q.) From the following table find out marginal utility

Units of Commodity Consumed	Total Utility	Marginal Utility
1	10	–
2	18	→8
3	24	→6
4	28	→4
5	30	→2
6	30	→0
7	26	-4

→ Total utility is max when marginal utility is zero.







Theory of consumer behaviour.

1.) ~~Indiffrunce~~ Indifference Curve (IC)

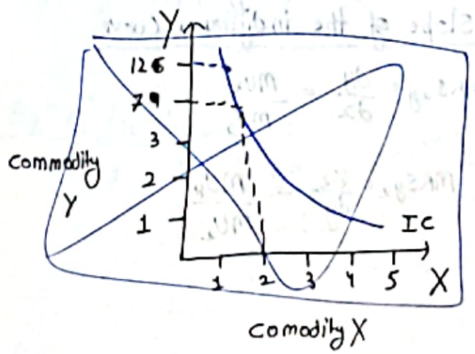
2.) Marginal rate of Substitution (MRS)

3.) Budget line

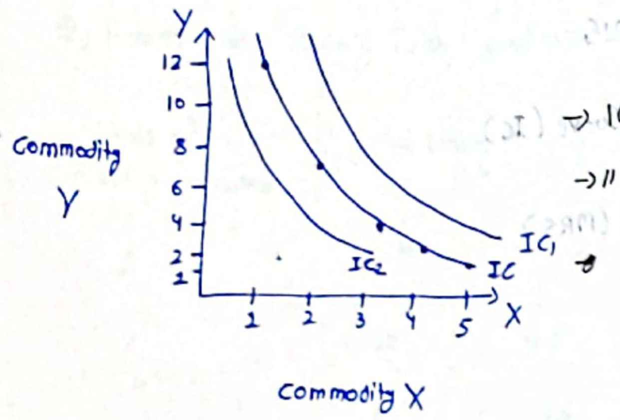
I.) Indifference Curve:

It refers to those curves which shows various combination of two commodities that give equal level of satisfaction to the consumer.

Combination	X	Y
A	1	12
B	2	7
C	3	3
D	4	2
E	5	1



(Not straight line.)



→ IC increases, right shift

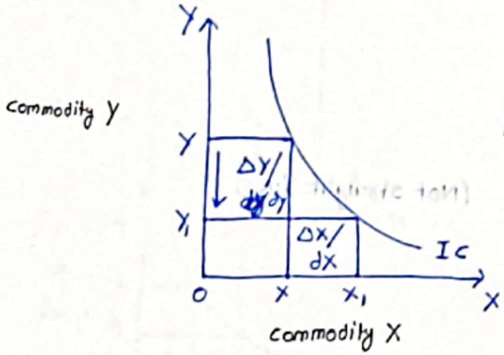
→ " decreases, left shift

II) Marginal Rate of Substitution (MRS):

It refers to the rate at which number of units of 1 commodity substituted to have 1 more unit of ~~co~~ another commodity.

i) MRS_{xy}: Refers to the rate at which the number of unit of commodity Y substituted to have one more unit of commodity X.

ii) MRS_{yx} : Refers to the rate at which the number of unit of commodity X substituted to have one more unit of commodity Y.



Slope of the indifference curve

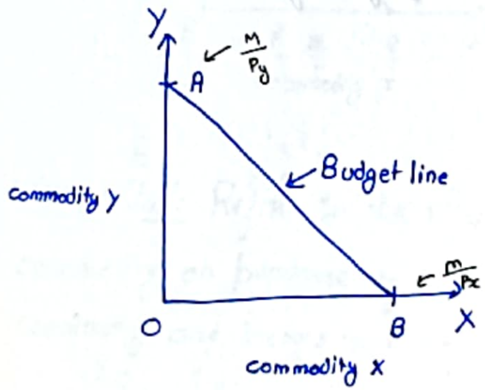
$$MRS_{xy} = \frac{dy}{dx} = \frac{MU_x}{MU_y}$$

$$MRS_{yx} = \frac{dx}{dy} = \frac{MU_y}{MU_x}$$

Q.) From the following table find out ~~MRS~~ MRS_{xy}, MRS_{yx}.

Combination	X	Y	MRS_{xy}	MRS_{yx}
A	1	12		
B	2	7	|-5|	|-0.2|
C	3	3	-4	-0.25
D	4	2	-1	-1
E	5	1	-1	-1

· Budget Line : Refers to the line that shows various combination of two commodities that a consumer can purchase with a given level of income.



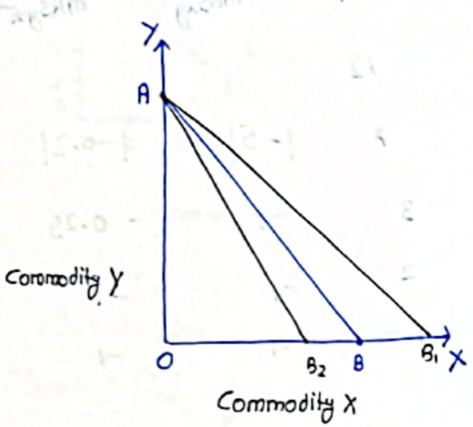
Slope of the budget line:

$$\frac{OA}{OB} = \frac{\frac{m}{P_y}}{\frac{m}{P_x}} \Rightarrow \frac{P_x}{P_y}$$

Eqⁿ of the budget line:

$$M = \underset{\text{Price}}{P_x}\ \underset{\text{No. of unit of commodity } x}{q_x} + P_y q_y$$

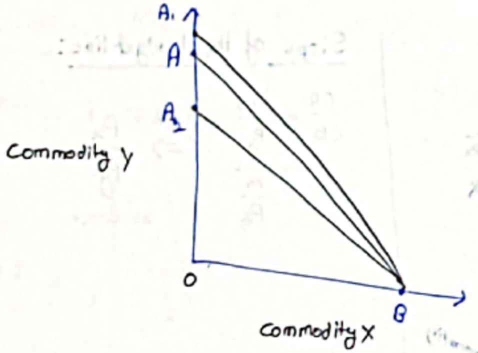
Shift in the budget line if purchase of commodity x changes, y and income remaining constant.



$B_1 \rightarrow$ Decrease in price of X, increase in ~~pu~~ Demand.

$B_2 \rightarrow$ Increase " " " " , decrease in Demand.

• Shift in the budget line if purchase of commodity y changes, x and income remaining constant.



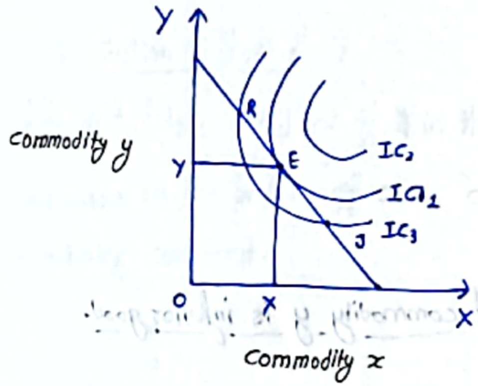
• Condition for a consumer to be in equilibrium.

i) Slope of the indifference curve is equal to slope of the budget line.

ii) The indifference curve should be convex at equilibrium point.

$$MRS_{xy} = \frac{dy}{dx} = \frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

$$\Rightarrow \frac{MU_x}{P_x} = \frac{MU_y}{P_y}$$ $\rightarrow$ Optimum condition bundle for x and y.

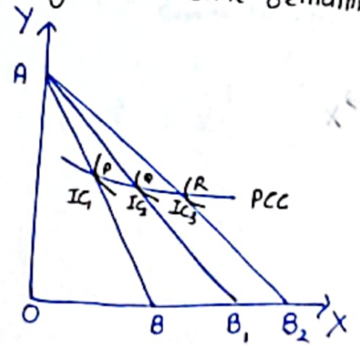


→ maximum level of satisfaction

• E → lying on the maximum possible indifference curve. IC2

• OX, OY → optimal combination of commodity of X, Y for consumer to be in Equilibrium

• **Price Effect:** Refers to the effect of change in the price of one commodity on purchase of commodities, price of ~~an~~ another commodity and income remaining constant.

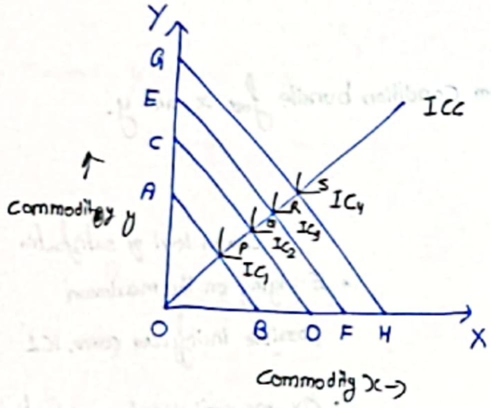


Joining P, Q, R → Price Consumption Curve.

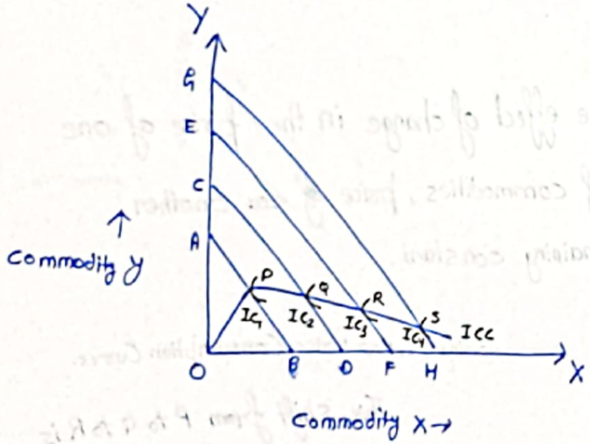
This shift from P to Q to R is Price Effect

· ~~Ico~~ Income Effect:

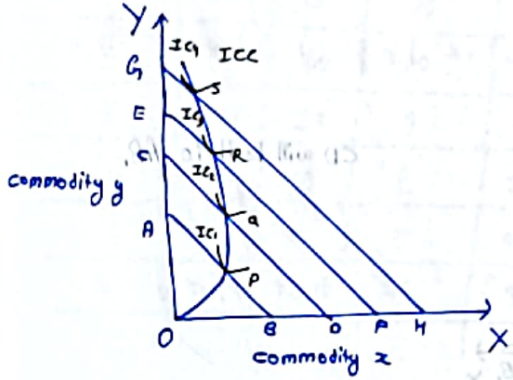
Refers to the effect of change of income of the consumer on purchase of commodities, price of both commodities remaining constant.



· ~~Inco~~ Income Consumption curve if commodity y is inferior good.

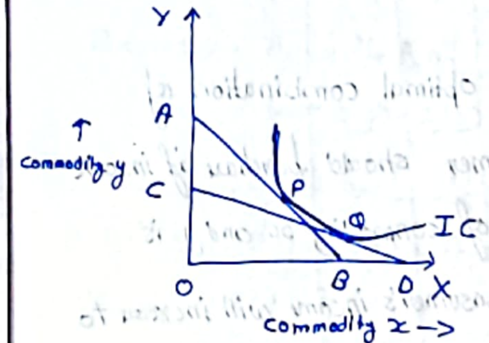


· Income Consumption Curve if commodity x is an inferior good.

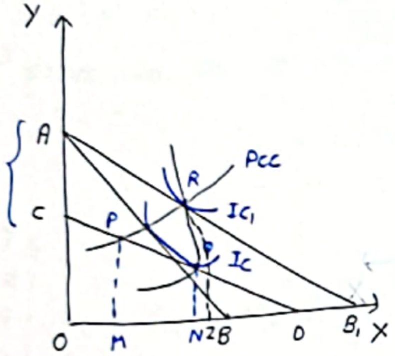


· Substitution Effect:

Refers to the effect of fall in the price of one commodity and increase in the price of other commodity, income of the consumer remaining constant.



· Relationship among price effect, income effect, Substitution effect.



CD will be ∥ to AB.

$MZ = PE$

$NZ = IE$

$MN = SE$

$MZ = MN + NZ$

$PE = SE + IE$

Q.) From the following table find out optimal combination of commodity x and y, that a consumer should purchase if income of the consumer is 10 INR and price of commodity x and y is 3 INR and 2 INR respectively. If the consumer's income will increase to 18, find out the no. of unit of commodity x and y, consumer should purchase to maximize utility

Quantity (in units)	Commodity x			Commodity y		
	T.U.	M.U.	MU Per Rs ($\frac{MU}{P_x}$)	T.U.	M.U.	MU Per Rs
1	45	45	15	40	40	20
2	75	30	10	60	20	10
3	102	27	9	72	12	6
4	120	18	6	82	10	5
5	135	15	5	90	8	4
6	144.99	19.99	3.33	92	2	1

$\frac{MU_x}{P_x} = 15$

$MU_x = 15 \times 3 \Rightarrow 45$

$MU = T_n - T_{n-1}$

Now, A/Q, income of the consumer = 10 INR

Budget line $= P_x \times Q_x + P_y Q_y$

$10 = 3(2) + 2(2)$

at

$18 = 3.(4) + 2(3)$ ✓ → Consumer satisfaction will be maximum by purchasing 4 unit of x and 3 unit of y given

$= 3(2) + 2(6)$

Q.) If a consumer has 22INR to spend on ~~both~~ good a and b, whose prices are 2INR each, find out

i.) How many units of a and b should be purchased for maximum utility of the consumer

ii.) If Income of the consumer increases to 28INR, find optimal consumption bundle of good a and b for maximum satisfaction of Consumer.

Q.) What is the marginal utility per INR spent on 4^{th} unit of good 'a' and 6 unit of good 'b'.

Quantity for good 'a'	T.U	M.U	M.U Per INR	Quantity for good 'b'	T.U.	M.U.	M.U. Per INR
1	10	10	5	1	16	16	8
2	19	9	4.5	2	30	14	7
3	27	8	4	3	42	12	6
4	34	7	3.5	4	52	10	5
5	40	6	3	5	60	8	4
6	45	5	2.5	6	66	6	3
7	49	4	2	7	70	4	2

$22 = 2(7) + 2(4)$ ✗

$22 = 2(6) + 2(5)$ ✗

~~22 = 2(7) + 2(2)~~

~~22 = 2(3)~~

$22 = 2(5) + 2(6)$ ✓

$28 = 2(\ ) + 2($

φ

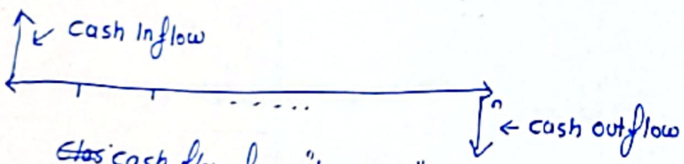
Time value of Money.

Refers to the value of money at a particular time period.

- Value of money in present is greater than value of money in future.

Cash flow diagram: Refers to graphical representation of cash inflow and outflow among a timeline.

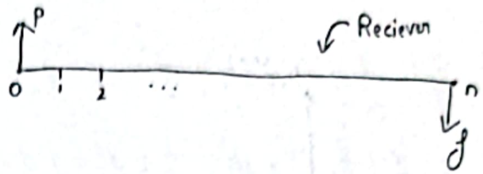
Timeline: Horizontal Scale that shows us time period.

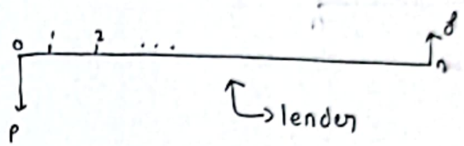


~~Clas~~ cash flow from "borrower" point of view

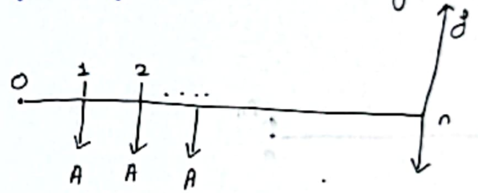
- **Types of cashflow Diagram:**
  - i) Single Payment Cashflow
  - ii) Uniform payment series
  - iii) Linear Gradient Series
  - iv) Geometry Gradient Series.
  - v) ~~Ir~~ Irregular Payment series.

I.) Single Payment Cashflow:

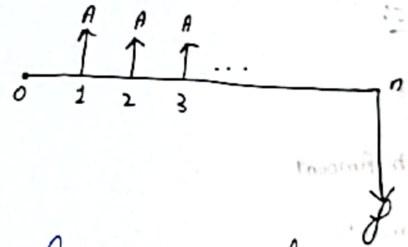




II.) Uniform Payment Series: (Starts from 1)



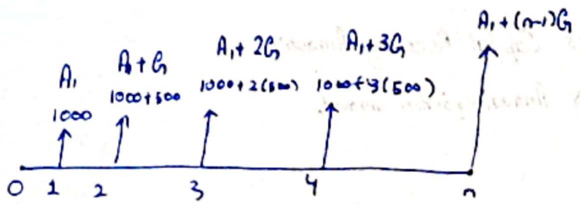
Annuity: Equal Payment at regular Interval



III.) linear Gradient Series:

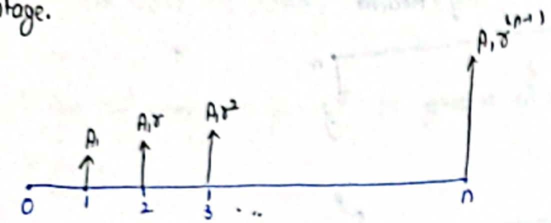
Refers to the series of cashflow, increasing or decreasing by a fixed amount.

$A_1 = 1000$
$G = 500$

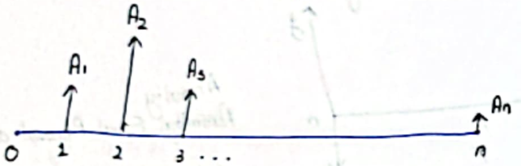


IV.) Geometric Gradient Series:

Refers to series of cashflow increasing or decreasing by a fixed percentage.



V.) Irregular Payment Series:



• Types of Compound Amount Factors:

i.) Single Payment Compound Amount

ii.) Single Payment Present worth Amount

iii.) Equal " Series Compound Amount

iv.) " " " Sinking Fund

V.) " " " Present worth amount.

VI.) Equal-Payment Series Capital Recovery Amount.

VII.) Linear gradient series Annual equivalent amount.

I.) Single Payment Compound Amount:

↳ Going to calculate future value.

• Here the objective is to find future value of a present sum after n^{th} year, compounded at an Interest rate 'i'.

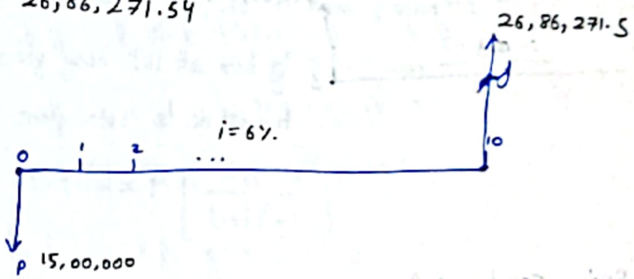
$$F = P(1+i)^n$$ → Single Payment Compound Amount.

Q.) A person deposits 15 lakh INR in a bank for 10 years. Find the maturity amount of his account if interest rate is 6% compounded anually.

$$F = P(1+i)^n$$

$$= 15,00,000\,(1+0.06)^{10}$$

$$= 26,86,271.54$$



II.) Single Payment Present worth Amount.

Here the objective is to find the present value of a future sum after n^{th} year, compounded at an interest rate 'i'.

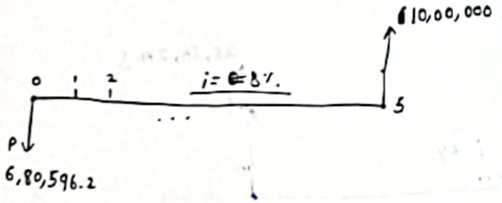
$P = \frac{F}{(1+i)^n}$ → Single payment present worth amount.

Q.) A person needs 10 lakh INR after 5 years. Find how much money the person has to deposit now to get 10 lakh after 5 years. If Interest rate is 8% compounded Anually

$$P = \frac{F}{(1+i)^n}$$

$$P = \frac{10,00,000}{(1+0.08)^5}$$

$$= 6,80,596.20 \text{ INR}$$



· Equal payment Series Compound Amount.

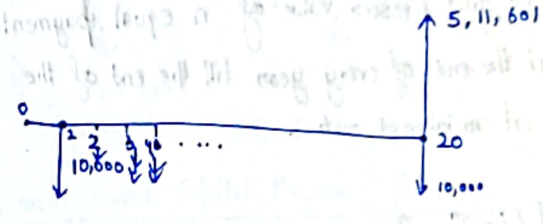
Here the objective is to find future value of n equal payments that is to be made at the end of every year, till the end of n^{th} year, Compounded at an interest rate i

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

Q.) A person invest an equal amount of 10,000 INR at the end of every year for 20 years, find the maturity amount of his account if interest rate is 9% compounded anually

$$\Rightarrow F = 10000\left(\frac{(0.09+1)^{20} - 1}{0.09}\right)$$

$$= 511601.1964$$



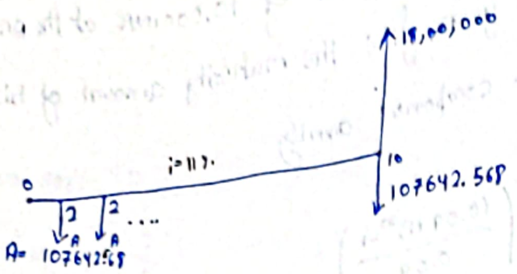
· Equal payment Series sinking fund.

Here the objective is to find n equal payments that is to be collected at the end of every year till the end of the n^{th} year to realise a future sum after n^{th} year compounded at an interest rate i.

$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

Q.) A person needs 18,00,000 INR after 10 years. Find out how much equal amount of money the person has to deposit at the end of every year for 10 years if interest rate is 11% compounded anually.

$$A = 18,00,000\left[\frac{0.11}{(1+0.11)^{10} - 1}\right] \Rightarrow A = 107642.5688$$



Equal-Payment Series present worth amount.

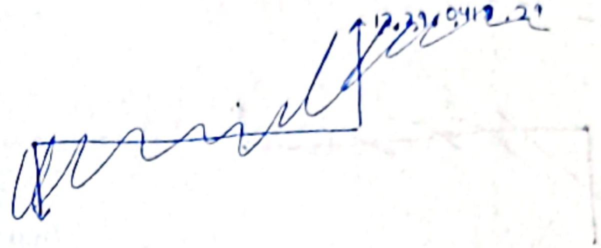
Here the objective is to find present value of n equal payments that is to be made at the end of every year till the end of the n^{th} year compounded at an interest rate i.

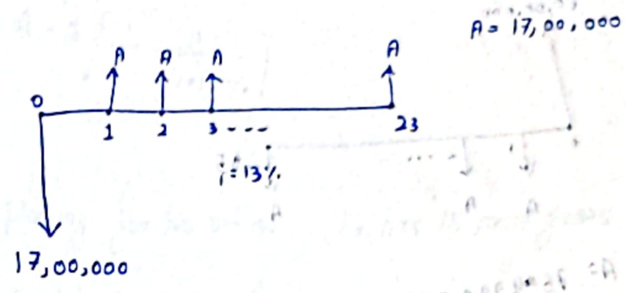
$$P = A\left[\frac{(i+1)^n - 1}{i(1+i)^n}\right]$$

Q) A company wants to set up a reserve which will help it to have an ~~eq~~ annual equivalent amount of 17 lakh INR for next 23 years towards its employee welfare majors. The reserve is assumed to grow at the rate of 13% compounded anually. Find single payment that must be made as the reserve amount now.

$$P = 17{,}00{,}000\left[\frac{(0.13+1)^{23} - 1}{(0.13+1)^{23}\, 0.13}\right]$$

$P = 12290418.29$





Equal payment series Capital Recovery Amount.

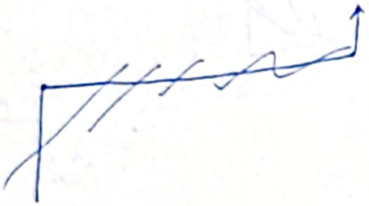
Here the objective is to find ~~present~~ n equal payments that is to be recovered at the end of everyyear, till the end of n^{th} year for a loan i.e sanctioned now, compounded at an iterset rate i.

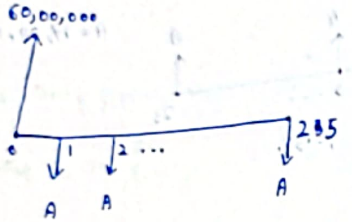
$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

Q) A company has taken a loan of 60,00,000 INR find out the installement amount that the company has to pay at 12% interest rate compounded anually if the no. of installement is 25.

$$A = 60{,}00{,}000\left[\frac{0.12\,(1+0.12)^{25}}{(1+0.12)^{25} - 1}\right]$$

$= 7{,}64{,}999.8189$

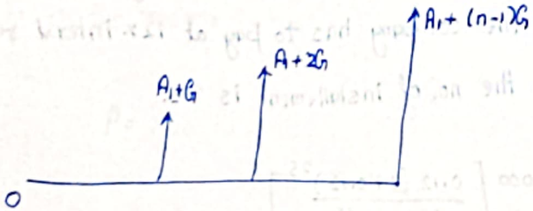




$A = 7{,}6^{\bullet}4{,}999.8181$

• Linear gradient series Annual equivalent amount

• Here the objective is to find annual equivalent amount (A), with a series of equal amount, starting in the first year (A_1), with a fixed amount increasing or decreasing (G) at the end of every year, till the end of the n^{th} year, following at the first year compounded at an interest rate i.



Increasing series:

$$A = A_1 + G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

Decreasing Series:

$$A = A_1 - G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

Q.) A person is planning for his retired life, he has 15 more years of service, he would like to deposit 10 % of his salary, which is 5000 INR, starting at the end of first year, with an annual increase of ~~100~~ 1000 thereafter for the next 14 years. If the interest rate is 10% compounded anually find out the total amount of the above series after 15 years.

Future Value

$A_1 = 5000$

$G = 1{,}000$

for G = 14 yr

Total amount = 15 yr

$$A = 5000 + 1000\left(\frac{1}{.1} - \frac{15}{(1+.1)^{15} - 1}\right)$$

$A = 4994.7210 \Rightarrow$ ~~2278.93~~ $10{,}278.93$

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 4994.7210\left[\frac{(1+.10)^{15} - 1}{.10}\right] = 3{,}26{,}587.226$$

$\boxed{F = 10{,}57{,}464}$ ~~$F = 10{,}278.93$~~

Comparision of Alternatives

i) Present worth method

ii) Future Worth Method

iii) Annual worth ~~Method~~ Method

iv) Rate of return method

v) Cost-Benefit Analysis

vi) Pay-back period ~~of~~ method.

• Present worth Method

I) In case of One Project:

$NPW(i\%) = PW(B) - \cancel{P(w)}\, PW(C)$

$NPW \rightarrow$ ~~New~~ Net Present worth

$B \rightarrow$ Benefit

$C \rightarrow$ Cost

• If $NPW(i\%) > 0$, Project will be Selected

• If $NPW(i\%) < 0$, " " " Rejected

• If $NPW(i\%) = 0$, " may or may not be selected

II) In case of mutually exclusive projects. (more than 1 Project)

i) Revenue Based Method.

ii) Cost Based Method.

• Revenue Based Method:

Refers to that method where all types of benefit where all types of benefits that is profit, income or earning and ~~salary~~ salvage value will be assigned with a +ve sign and all types of the cost that is spending, expenditure, payment and investment will be assigned ~~two~~ with -ve sign

• Cost Based Method.

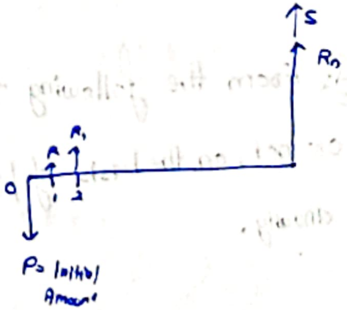
Refers to that method where all types of benefit will be assigned with -ve sign and all types of cost will be assigned with +ve sign

→ Revenue Based Method:

$F \rightarrow$ convert F to P

$$= -P + R_1\left[\frac{1}{(1+i)^1}\right] + R_2\left[\frac{1}{(1+i)^2}\right] \dots + R_n\left[\frac{1}{(1+i)^n}\right] + S\left[\frac{1}{(1+i)^n}\right]$$

$F \rightarrow$ convert to P



• Equal payment Series:

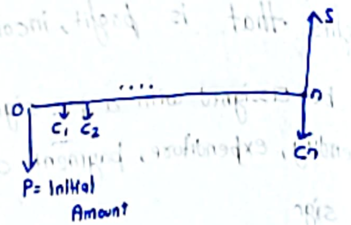
$$\Rightarrow -P + R\left[\frac{(1+i)^n-1}{i(1+i)^n}\right] + S\left[\frac{1}{(1+i)^n}\right]$$

⇒ Cost Based Method:

a) Different Series

~~Ans~~

$$NEw(i\%) = P + C_1\left[\frac{1}{(1+i)^1}\right] + C_2\left[\frac{1}{(1+i)^2}\right] + \dots + C_n\left[\frac{1}{(1+i)^n}\right] - S\left[\frac{1}{(1+i)^n}\right]$$



b) Equal payment series

$$NEw(i\%) = \cancel{P + C\left[\frac{1}{}\right]}\; P + C\left[\frac{(1+i)^n-1}{i(1+i)^n}\right] - S\left[\frac{1}{(1+i)^n}\right]$$

Q) From the following table find out, project will be selected or not, on the basis of present worth method, if i=14% compounded anually,

Year End	Cash flow
0	-80,000 → cash outflow
1	25,000 → cash inflow
2	37,000 ....
3	40,000 ...
4	48,000 ..

every year cash inflow → ~~present~~ Revenu ~~worth~~ Based metho

just to show cash outflow

$$= -P + R_1\left[\frac{1}{(1+i)^1}\right] + \dots + S\left[\frac{1}{(1+i)^n}\right] \qquad S=0$$

$$= -80,000 + 25,000\left[\frac{1}{(1+.14)^1}\right] + \dots + 48,000\left[\frac{1}{(1+.14)^4}\right]$$

NPW(1%) = ~~185818.8371~~ 25818.837

~~NPW(1%)=~~

NPW(14%) = -80,00,000 + ...

= 25818.837

NPW(14%) ≥ 0 selected.

Q.) From the following table find out, project is financially feasible or not on the basis of present worth method if $i = 12\%$.

Year End	Cashflow
0	-90,000
1	40,000
2	40,000
3	⋮
4	
5	
6	
7	
8	
⋮	⋮
20	~~20~~40,000

$$\Rightarrow -P + R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] + S\left[\frac{1}{(1+i)^n}\right] \quad S = 0$$

$$\Rightarrow -90{,}000 + 40000\left[\frac{(1+.12)^{20} - 1}{.12(1+.12)^{20}}\right]$$

$= 2{,}08{,}777.745$ as $NPW(12\%) \geq 0$ ↳ feasible

Q.) From the following table find out which technology will be selected on the basis of present worth method if $i = 16\%$ compounded anually.

Technology	Initial outlay	Annual Income	Life in year
1	10,00,000	5,00,000	15
2	18,00,000	7,00,000	15
3	16,00,000	6,00,000	15

Technology 1:

$$-P + R\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] + S\left[\frac{1}{(1+i)^n}\right] \quad S = 0$$

$= 1787728.081$

Technology 2:

$= 2102819.314$

Technology 3 $= 1745273.698$

Tehnology 2 will be seleted.

Q.) From the following table find out which machine will be selected on the basis of present worth if $i = 17\%$ compounded anually.

Machine	Initial Cost	Service life (in Years)	Annual operation and maintain	Salv
A	6,00,000	20	35,000	15,000
B	7,00,000	20	40,000	12,000

$$P + C\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] - S\left[\frac{1}{(1+i)^n}\right]$$

A ⇒ 796322.6635 → cost less ⇒ machine will be selecte

B ⇒ 924591.3388 → cost high

→ Future Worth method:

· In case of 1 project.

$NFW(i\%) = FW(B) - FW(C)$

If. $NFW(i\%) > 0$, Project will be selected

if $NFW(i\%) < 0$, " " " rejected.

if $NFW(i\%) = 0$, " may or may not be Selected.

· In case of mutually exclusive projects

a) Revenue Based Method

i) Different Series

$$NFW(i\%) = -P(1+i)^n + R_1(1+i)^{n-1} + R_2(1+i)^{n-2} + \dots + R_n + S$$

ii) Equal-Payment Series

$$NFW(i\%) = -P(1+i)^n + R\left[\frac{(1+i)^n - 1}{i}\right] + S$$

b) Cost Based Method

i) Different Series

$$NFW(i\%) = P(1+i)^n + C_1(1+i)^{n-1} + C_2(1+i)^{n-2} + \dots + C_n - S$$

ii) Equal Payment Series

$$NFW(i\%) = P(1+i)^n + C\left[\frac{(1+i)^n - 1}{i}\right] - S$$

Q.) From the following table find out which alternative will be selected on the basis of future worth if $i = 13\%$ compounded anually

Particulars	Alternative A	Alternative B
initial Cost	4,00,000	6,00,000
Uniform annual benefit	64,000	96,000
life in years	15	15

for A $\boxed{85009.57333}$ for B $\boxed{127514.36}$ ✓

Q.) From the following table find out which mac[illegible] be selected on the basis of future worth method if $i = 11\%$.

Particulars	Machine 1	Machine 2	Machine 3
• Initial Investment	80,00,000	70,00,000	90,00,000
• life in years	17	17	17
• Annual operation and maintaine	8,00,000	9,00,000	8,50,000
• Salvage	5,00,000	4,00,000	7,00,000

$$NFW(i\%) = P(1+i)^n + C\left[\frac{(1+i)^n - 1}{i}\right] - S$$

NFW(.11)% = I.) 82261415.91

NFW(.11%) = II.) 80916407.48 ✓

NFW(.11%) = III.) 90181550.76

Q.) Which alternative will be selected on the basis of future worth method if $i = 12\%$ compounded anually

Particulars	Alternative 1	Alternative 2
• First Cost	15,00,000	20,00,000
• Annual Property tax	70,000	90,000
• Annual Income	5,00,000	7,00,000
• life in years	15	15
• Net annual Incom	~~4,30,0~~ 4,30,00	6,10,000

$$NFW(i\%) = -P(1+i)^n + R\left[\frac{(1+i)^n - 1}{i}\right] + S$$

1: 7819928.665

2: 11793499.42 ✓

Annual Worth Method / Annual Equivalent Method:

In case of one project:

$$NAW(i\%) = AW(B) - AW(C)$$

- if $NAW(i\%) > 0$, Project will be selected
- " " < 0, " " not be "
- " " $= 0$, Project may or may not be selected.

In case of mutually exclusive project / more than 1

a.) Equal-Payment series:

i.) Revenue Based Method.

$$NAW(i\%) = -P\left[\frac{i(1+i)^n}{(1+i)^n-1}\right] + \overset{A}{R} + S\left[\frac{i}{(1+i)^n-1}\right]$$

ii.) Cost Based Method:

$$NAW(i\%) = P\left[\frac{i(1+i)^n}{(1+i)^n-1}\right] + C - S\left[\frac{i}{(1+i)^n-1}\right]$$

b.) Different Series:

$$NAW(i\%) = NPW\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$$

Q.) From the following table, find out which technology will be selected on the basis of present worth method if $i = 18\%$.

Particular	Tech A	Tech B
First Cost	5,00,000	7,00,000
End of Year	Cashinflow Year End	Cashinflow Year End
1	10,000	15,000
2	20,000	30,000
3	30,000	0
4	45,000	0

(Tech A: 4 yrs; Tech B: 2 yrs; Prev. present worth)

•) Different Series:

$$NAW(i\%) = NPW\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$$

Tech A

$$NPW(i\%) = -P + R_1\left[\frac{1}{(1+i)^1}\right] + R_2\left[\frac{1}{(1+i)^2}\right] + R_3\left[\frac{1}{(1+i)^3}\right] + R_4\left[\frac{1}{(1+i)^4}\right]$$

$$= -435692.3016 \Rightarrow -4,35,692.309\left[\frac{i(1+i)^n}{(1+i)^n-1}\right]$$

$$= -1,61,963.69$$

Tech B

$$NPW(i\%) = -665742.6027$$

$\Downarrow$

$-\cancel{2,47,482.22}$ $-4,25,220.1835$

Q.) From the following table find out which [illegible] selected on the basis of annual equivalent method if i = 20% compounded anually

Machine	Down Payment	Yearly equal Installment	No. Inst.
1	5,00,000	2,00,000	15
2	4,00,000	3,00,000	15
3	6,00,000	1,50,000	15

Name: Vaibhav Sharma, Roll: 22053123

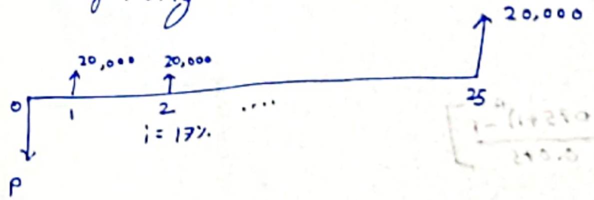
Q.) A person needs 7,00,000 INR after 3 years find out how much money the person has to deposit now to get 7,00,000 INR after 3 years if the Interest rate is 6% compounded anually

Q2.) A person invest an equal amount of 25,000 INR at the end of every year, starting from the end of next year. Find maturity amount of his account after 12 years, if the interest rate is 7.5% compounded anually

Q3.) A company has taken a loan of 20,00,000 INR. Find out the installment amount the company has to pay if interest rate is 12% compounded anually and no. of Installement is 10.

Q4.) A person plans to invest an equal amount of 30,000 INR starting in the first year with an annual decrease of 500 INR for next 10 years. Find out total amount of the above series after 11 years if the interest rate is 10% compounded anually

Q.) Solve the following



1.) $F = 7,00,000 INR$

$n = 3$

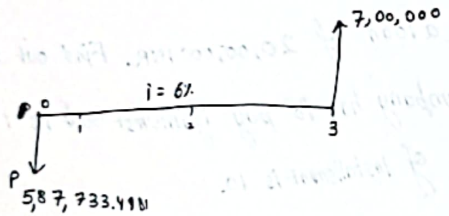
interest $= 6\%$

$P = ?$

$$P = \frac{F}{(1+i)^n}$$

$$= \frac{7,00,000}{(1+.06)^3}$$

$$P = 587733.4981 \text{ INR}$$



2.) $A = 25,000 INR$

$n = 12$ years

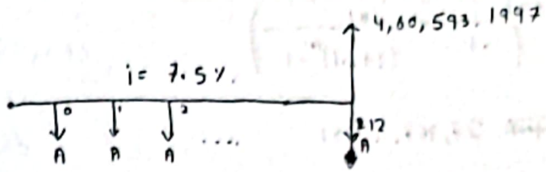
$i = 7.5\%$

$F = ?$

$$F = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$= 25,000\left[\frac{(0.075+1)^{12} - 1}{0.075}\right]$$

$$F = 460593.1997 \text{ INR}$$



$A = 25,000$

3.) $P = 20,00,000 INR$

$i = 12\%$

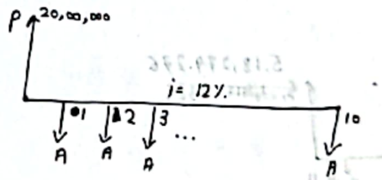
$n = 10$

$A = ?$

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

$$= 20,00,000\left[\frac{.12(1+.12)^{10}}{(.12+1)^{10} - 1}\right]$$

$$A = 353963.3283 \text{ INR}$$



$A = 353963.3283$

4.) $A_1 = 30,000 INR$

$G = 500$

$n = 10$

$i = 10\%$

$$A = A_1 - G\left[\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right]$$

$A = 30{,}000 - 500\left(\frac{1}{.1} - \frac{10}{(1+.1)^{10}-1}\right)$

$A = \cancel{28{,}137.269\,\text{INR}}\ 27{,}967.97281$

Now, A/G

Total amount after 11 years

$\Rightarrow A = \cancel{28{,}137.269}\ 27{,}967.97281$

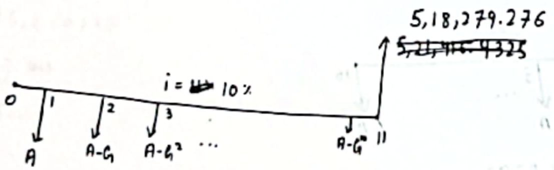
$i = 10\%$, $n = 11$

$F = ?$

$$F = A\left[\frac{(i+1)^n - 1}{i}\right]$$

$$= \cancel{28{,}137.269}\ 27{,}967.972\left[\frac{(.1+1)^{11} - 1}{.1}\right]$$

$F = \cancel{5{,}21{,}112.4325\,\text{INR}}$ 518279.276 INR



5) ~~P=?~~ From the following diagram, we can deduce that:

$A = 20{,}000$

$i = 17\%$

$n = 25$

$P = ?$

$$P = A\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$= 20{,}000\left[\frac{(.17+1)^{25} - 1}{.17(1+.17)^{25}}\right]$$

$P = 1{,}15{,}324.6722$