

## It is a social ~~Sign~~ Science

- Theory of consumer behaviour
- Nature and scope of engg. economics

→ demand

## Interest Rates

- Simple
- Compound

- Nominal :- IR calculated several times in a year more than 1yo.
- Effective :- Same as nominal but once in a year

$$\text{Compound IR} = P(1+i)^n$$

↳ compound amount factors.

- Comparison of alternatives
- Market.
- Production.
- Cost.

## Depreciation

- Straight line method
- Recycling balance method
- Sum of the year digit method of depreciation
- Sinking fund method.

## Definition of economics given by Adam Smith

• It is a subject which enquires into the nature and course of the wealth of the nation.

Robbins: He defined economics as a science which studies human behaviour as a relationship b/w ends (unlimited wants) and scarce means which have alternative uses.

## Engg. Economics

Introduced by L. Gossen

Def: It is that branch of economics which deals with method that can enable one to make economic decision towards engg alternatives.

## Nature of engg. economics

4 central problem of economics.

i) What to produce

ii) How to produce

iii) For whom to produce

iv) Economic growth problems

## Scope of Engg. Economics

It deals with theory of consumer behaviour, demand elasticity of demand, supply, elasticity of supply, equilibrium b/w demand and supply, interest rates, compound among factors, composition of alternatives, revenue, production cost, market depreciation and inflation.

## Demand

It refers to the effective desire to have something backed up by the ability and the willingness to pay for it.

## Demand Schedule

It refers to tabular representation of different quantities of commodity demanded at different prices at a given point of time.

price of x	Q.O for x
10	5
8	10
6	17
4	22

Where x is any commodity

## Types of demand Schedule

i) Individual demand schedule

ii) Market demand schedule.

## Individual Demand Schedule

It refers to tabular representation of different quantities of commodity demanded by an individual consumer at different prices at a given point of time.

price of commodity	Demand of Individual Commodity
10	10
20	8
30	6
40	4
50	2

## Market Demand Schedule

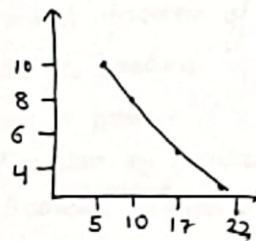
It refers to the tabular representation of different quantities of commodity demanded by different prices at a given point of time.

price of X	Q.D. for X by cons. 1	Q.D. for X by consumer 2	Q.D. for X by consumer 3	Market Demand
40	1	3	5	9
30	2	5	9	16
20	3	8	14	25

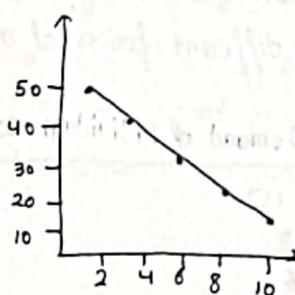
## Demand Curve

It refers to the graphical representation of demand schedule.

- Dependent Variable X axis
- Independent variable Y axis



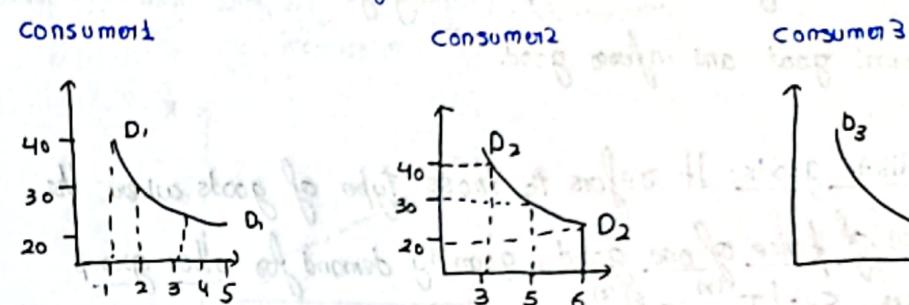
## Individual Demand Curve



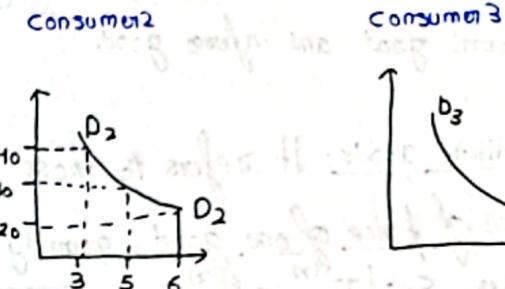
## Market Demand Curve

Individual demand curve of consumer 1

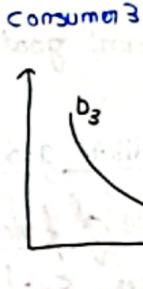
consumer 1



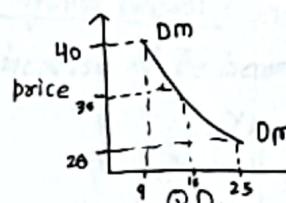
consumer 2



consumer 3



## Market Demand Curve



(★ All diagram in one line)

Q. From the following individual demand functions find out quantity demanded for commodity X by consumer 1 and 2 if price of the commodity X is 5 rupees per unit and also find out new quantity demanded for commodity X by both the consumers if price of commodity X increases to 7 rupees per unit.

$$Q_{X_1} = 500 - 2P_X$$

$$Q_{X_2} = 700 - 0.5P_X$$

Find out M.D. function and MP with original price and new price.

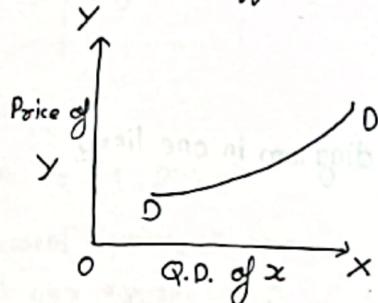
$$M.D \text{ function} = Q_{X_1} + Q_{X_2}$$

## Types of Goods

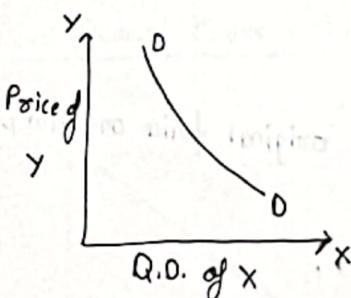
- i) Substitute goods and complementary goods.
- ii) Normal good and inferior good.

I

Substitute goods: It refers to those type of goods where the increase of price of one good, quantity demand for other group increases. Ex: [Tea<sup>(1)</sup>, Coffee<sup>(2)</sup>]



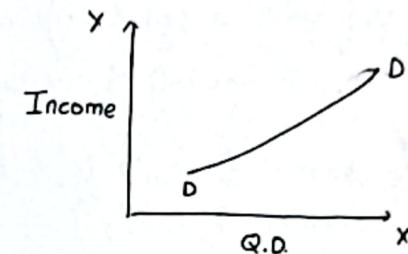
Complementary: It refers to those type of goods where the increase in price of one good, Q.D. for other decreases. Ex: [Car<sup>(1)</sup>, Petrol<sup>(2)</sup>], [Phone, charger].



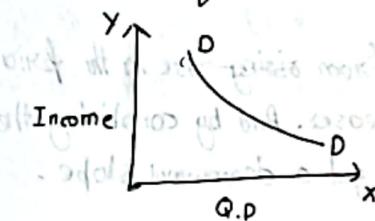
II,

Normal Goods: It refers to those type of goods, whose quantity Demand increases with the income of the consumer.

increase in



Inferior Goods: Those goods whose quantity demand ~~will~~ declines with the increase of the income of the consumer.

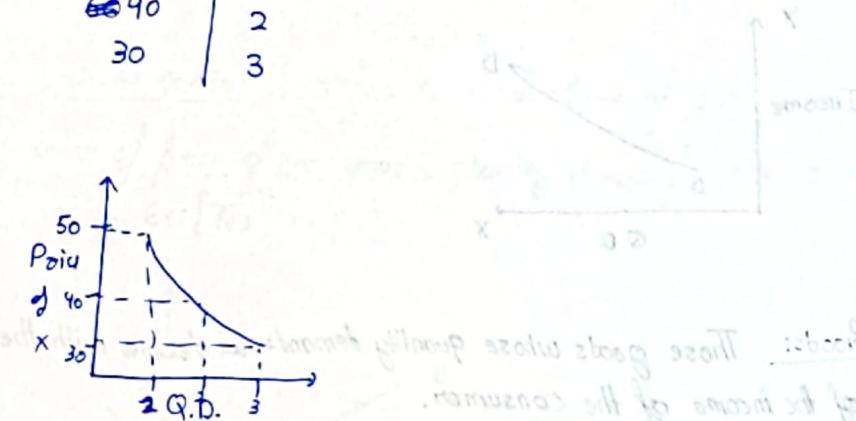


→ Law of Demand (Alfred Marshall): He defined law of Demand as other factors remaining constant (ceteris Paribus) Q.D. of a commodity increases with a fall in its price and decreases with a rise in its price.

Assumption of the Law:

- i) Income of the consumer remains constant.
- ii) Prices of relative goods remains constant.
- iii) Taste and Preference of the consumer remains constant.
- iv) Number of consumers in the market does not change.
- v) The good should be a normal good.

Price of X	Q.D. for Commodity 1
50	1
40	2
30	3



Explanation: Here we can see, as from rising rise in the price of commodity 'X', the Q.D. for it decreases. And by combining the points, ~~of the~~ we can see that we get a downward slope.

Criticism / Limitation of law of demand / Exception of to the Law of demand:

- i) Giffen Good  $\rightarrow$  Bread meat.
- ii) Veblen Good (Thorstein Veblen)  $\rightarrow$  Diamond (Prestige Value)
- iii) Speculation
- iv) War and emergency
- v) Other factors

### I) Giffen Good (Robert Giffen):

(Sub. opp. Qualitative)

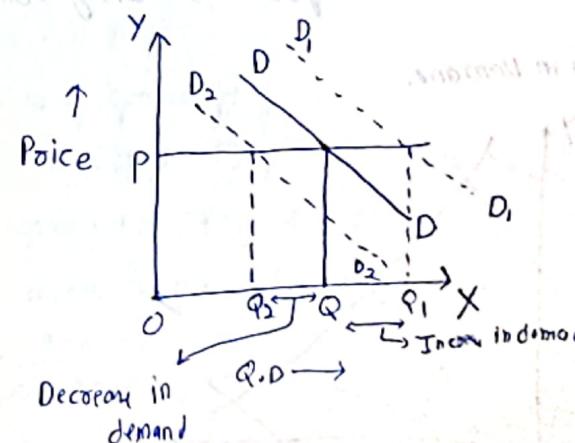
Change in Demand: When demand for a commodity changes, due to the change in other factors, price remaining constant, it is called change in Demand.

#### Types of Change in Demand:

- i.) Increase in Demand
- ii.) Decrease in Demand.

Increase in Demand: When a consumer purchases ~~for~~ more of a commodity than before due to change in other factors, price remaining constant, it is called increase in demand.

Decrease in Demand: When a consumer purchases less of a commodity than before, due to change in other factors, price remaining constant, it is called decrease in demand.



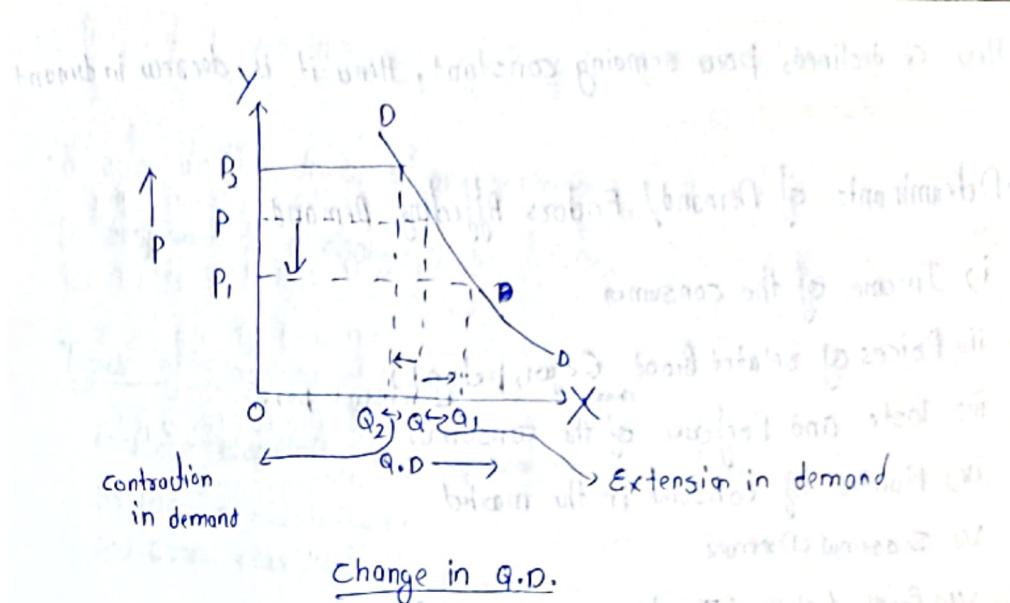
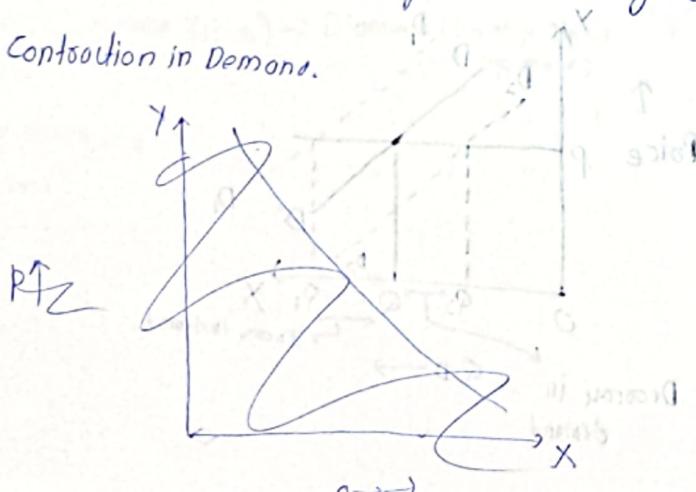
Change in Q.D.: When demand for a commodity changes due to a change in its price, other factors remaining constant, it is called Change in Quantity Demand.

### Types of Change in Q.D.:

- I) Extension in demand
- II) Contraction

I) Extension in Demand: When demand for a commodity increases due to a fall in its price, other factors remaining constant, it is called Extension in Demand.

II) Contraction in Demand: When demand for a commodity decreases due to a rise in its price, other factors remaining constant, it is called Contraction in Demand.



### Change in Q.D.

Q.) From the following Demand function find the Q.D. if price of the commodity is ₹20 and income of consumer is ₹4,000 per month

$$Q = 10,000 - 10P + 0.5m$$

Find new Q.D. for the commodity, if income of the consumer decreases to 35,000 INR per month, price remaining constant and also find what type of change this change in quantity implies.

$$Q \rightarrow Q.D.$$

P → Price of commodity

m → Income per month

$$\begin{aligned} Q &= 10,000 - 10(20) + 20,000 \\ &= 10,000 - 200 + 20,000 \\ &= 9,800 + 20,000 \\ Q &\rightarrow 29,800 \end{aligned}$$

original

$$\begin{aligned} Q &= 10,000 - 10(20) + 17,500 \\ &= 9,800 + 17,500 \\ Q &= 27,300 \end{aligned}$$

Here Q declined, price remaining constant, hence it is decrease in demand

## Determinants of Demand/ Factors Affecting Demand

- i) Income of the consumer
  - ii) Prices of related Good ( $C_{x_1}, p_{x_2}$ )  
constant pair
  - iii) Taste and Preferences of the consumer
  - iv) Number of consumers in the market
  - v) Seasonal Demand
  - vi) Govt. Policy (Taxation and Subsidy)
  - vii) Wealth distribution (Equal and Unequal distribution)
  - viii) Advertisement

## Elasticity

Elasticity of Demand ~~from~~ (Quantity).  
(Quantitative)

- It refers to the degree of responsiveness of Q.D for a commodity in response to a change in its price.

## Types of Elasticity of Demand:



I, Price Elasticity: Refers to the degree of responsiveness of a Q.D. of a commodity in response to a change in its price.

$$E(e_p) = \frac{\text{Proportional change in Q.D}}{\text{Coefficient Proportional change in Price.}}$$

$$= \frac{\text{Percentage change in Q.D}}{\text{Percentage change in Price}}$$

$$= \frac{\text{Change in QD}}{\text{Original Q.D.}} \times 100$$

$$\frac{\frac{\Delta Q}{Q} \times 100}{\frac{\Delta P}{P} \times 100} = \left| \frac{\frac{\Delta Q}{\Delta P} \times \frac{P}{Q}}{} \right|$$

for Tabular form

$$E(\ell_p) = \frac{dQ}{dP} \times \frac{P}{Q} \quad \rightarrow \text{for functional form.}$$

Q. P & Q demand function to find

100	1000	(Q <sub>1</sub> )
200	800	(Q <sub>2</sub> )

Given

$$E = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q} \rightarrow \Delta Q = Q_2 - Q_1 \\ \Delta P = P_2 - P_1$$

$$= \frac{-200}{100} \times \frac{100}{1000}$$

$$= -\frac{1}{5} \\ \Rightarrow 1 - 0.2 = \underline{\underline{0.2}}$$

Q. If Q.D for a commodity increases from 500 to 700 units due to fall in its price from 80 to 65 INR per unit, find price elasticity of demand (e)

$$e = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$$

$$\Rightarrow \frac{200}{-15} \times \frac{80}{500}$$

$$= 1 - 2.11$$

$$= \underline{\underline{2.1}}$$

(E) elasticity of Demand if price of

Q. From the following demand function, find E. for commodity X is 200 per unit

$$Q_x = 70,000 - 15P_x$$

$$Q_x \rightarrow Q.D \text{ for } X$$

$$P_x \rightarrow \text{Price of } X \text{ for any commodity}$$

so

$$Q_x = 70,000 - 15P_x \quad | \quad \frac{dQ_x}{dP_x} = -15$$

$$= 70,000 - 15(200)$$

$$\Rightarrow 70,000 - 3000$$

$$Q_x \rightarrow 67,000$$

units of X

$$E \rightarrow \frac{\Delta Q}{\Delta P} \times \frac{P}{Q} \rightarrow \frac{dQ}{dP} \times \frac{P_x}{Q_x}$$

$$P_x = 200$$

$$Q_x = 67,000$$

$$E = -15 \times \frac{200}{67,000}$$

$$E = -1 - 0.0441$$

$$E = 0.044$$

II) Income elasticity of demand: It refers to the degree of responsiveness of Q.D. for a commodity in response to a change in the income of the consumer.

$$e_Y = \frac{\text{Proportionate Change in Q.D.}}{\text{Proportionate change in Income}}$$

$$= \frac{\% \text{ change in Q.D.}}{\% \text{ change in Income}}$$

$$e_Y = \frac{\Delta Q}{\Delta Y} \times \frac{Y}{Q}$$

$\downarrow$   
income elasticity of demand

$$e_Y = \frac{dQ}{dY} \times \frac{Y}{Q}$$

$Y$ : Original Income of consumer

$Q$ : Original Q.D. of a commodity

$\Delta Y$ : change in Income

$\Delta Q$ : change in Q.D.

Q.) A consumer's purchase of a commodity increases from 20 to 23 units due to a rise in his income from 25,000 INR to 30,000 INR per month. Find income elasticity of demand.

$$e_Y = \frac{\Delta Q}{\Delta Y} \times \frac{Y}{Q}$$

$$= \frac{3}{5000} \times \frac{25,000}{20}$$

$$= 0.75$$

Q.) From the following Demand function, find Income elasticity of demand if income of consumer is 80,000 INR per month.

$$\text{exp: } Q = 80,000 + 0.6m$$

all in regards to original unit of time and not changing per

$$e_Y = \frac{dQ}{dY} \times \frac{Y}{Q}$$

$$Q = 80,000 + 0.6 \times 1,10,000$$

$$e_Y = 0.6 \times \frac{15,000}{80,000} = \frac{50,000}{1,10,000} = 0.273$$

Q.) From the following Demand function, find price elasticity of demand and income elasticity of demand if price of the commodity is 30 INR per unit and income of consumer is 20,000 INR per month.

$$Q = 30,000 - 15P + 0.7Y$$

P → Price of commodity

$$Q = 30,000 - 15(30) + 0.7(20,000)$$

$$= 343,550.$$

$$e_P = \frac{dQ}{dP} \times \frac{P}{Q}$$

$$= -15 \times \frac{30}{343,550}$$

$$= 1 - 0.010 \\ = 0.990$$

$$e_Y = \frac{dQ}{dY} \times \frac{Y}{Q}$$

$$= 0.7 \times \frac{20,000}{343,550}$$

$$= 0.321$$

for substitute and complementary goods.  
 Cross elasticity of demand: When two goods are so closely related that Q.D. for good X depends on the price of good Y, then cross elasticity of demand is defined as the degree of responsiveness for Q.D. for good 'X' in response to a change in the price of good 'Y'.

It is +ve in case of substitute goods and -ve in complementary goods

$$e_{\text{cross}} = \frac{\text{Proportional change in Q.D. of } X}{\text{Proportional change in Price of } Y}$$

$$e_{\text{cross}} = \frac{\Delta Q_x}{\Delta P_y} \times \frac{P_y}{Q_x}$$

$$e_{\text{cross}} = \frac{dQ_x}{dP_y} \times \frac{P_y}{Q_x}$$

If Q.D. for coffee increases from 10,000 units to 15,000 units, find out cross elasticity of demand b/w Tea and coffee, if price of Tea increases from 700 to 900 INR per 250 grams pack.

$$\Rightarrow e_{\text{cross}} = \frac{15,000 - 10,000}{900 - 700} \times \frac{700}{10,000}$$

$$\Rightarrow \frac{5000}{200}$$

$$\Rightarrow 25 \times \frac{700}{10,000}$$

$$= 1.75$$

Q.1 Form the following Demand fn: find out cross-elasticity of demand b/w Tea and coffee. If price of Tea is 2000 INR per 500 grams pack

$$Q = 60,000 + 0.3 P_t$$

$$= 60,000 + 0.3 (2000)$$

$$= 60,600$$

$$e_{\text{cross}} = 0.3 \times \frac{2000}{60,600}$$

$$= 0.009$$

Q.2 Form the following table, find out

i) Cross elasticity of demand b/w 'X' and 'Y', if price of X increases from 200 to 400 per unit.

ii) Good 'X' and 'Y' are what type of goods on the basis of i).

iii) Income elasticity of Demand for good Y if income of the consumer will increase from 40,000 to 55,000 INR per month.

iv) Good 'Y' is what type of good on the basis of above question

Price of Good X	Q.D for Good Y	Income
200	80	20,000
300	70	40,000
400	65	50,000
500	40	55,000

→ i)  $e_c = \frac{\Delta Q_{xy}}{\Delta P_x} \times \frac{P_x}{Q_{xy}}$

$$= \frac{65-80}{400-200} \times \frac{200}{80}$$

$$= -\frac{15}{200} \times \frac{200}{80}$$

$$= -0.1875$$

ii) ~~Complementary goods~~ ~~inferior goods~~ ~~Supplementary~~

$$e_y = \frac{\Delta Q_y}{\Delta P_y} \times \frac{P_y}{Q_y}$$

$$\Rightarrow \frac{40-50}{55,000-40,000} \times \frac{40,000}{50}$$

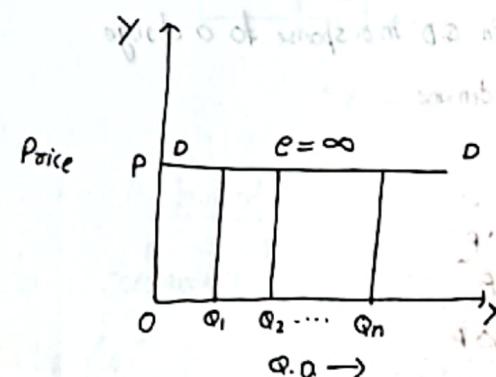
$$= -1.142$$

### → Degrees of elasticity of Demand

- i) Perfectly elastic Demand.
- ii) Relatively " / More elastic demand
- iii) Unitary "
- iv) Relatively inelastic / Less elastic demand
- v) Perfectly Inelastic Demand

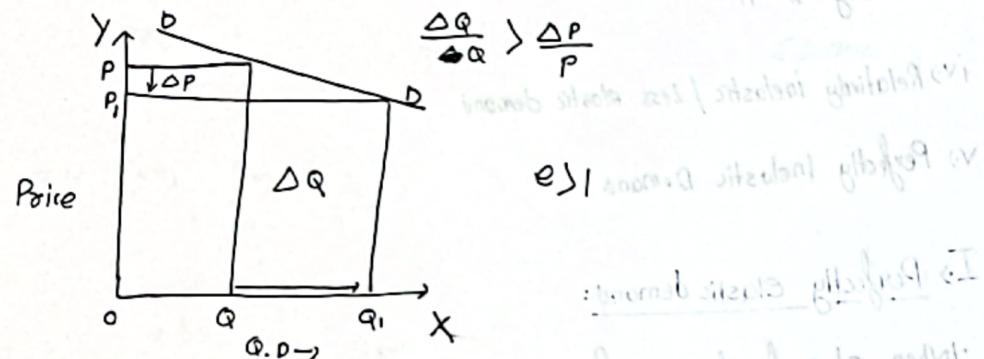
### I) Perfectly elastic demand:

When at a fix price, infinite quantities of commodity are demanded and with a small increase in the price, Q.D falls to zero. It is called perfectly elastic demand.



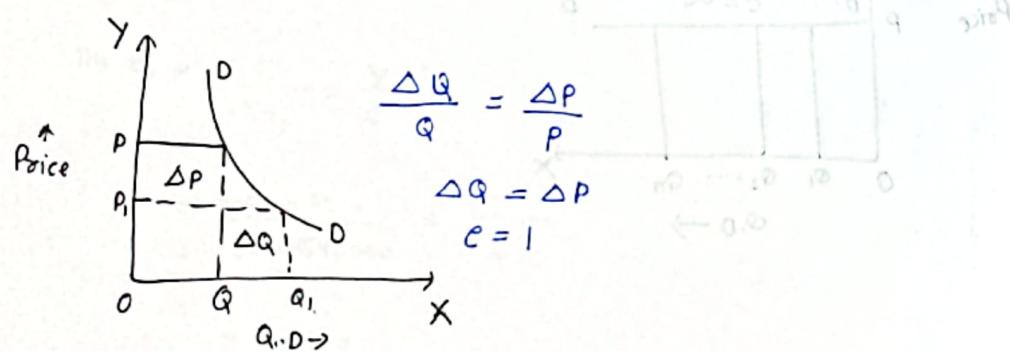
## II, Relatively Elastic Demand:

→ When there is more than proportional change in Q.D for a commodity in response to a change in its price. It is called R.E. Demand. Ex → Durable and Luxurious goods.

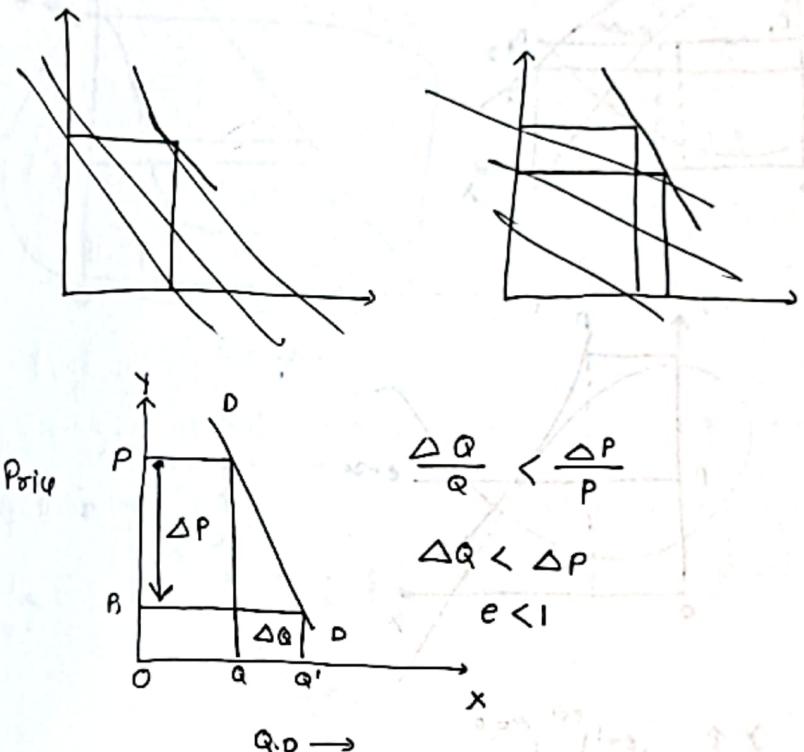


## III, Unitary Elastic Demand:

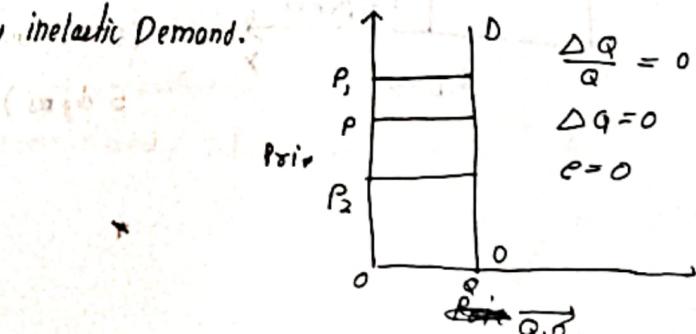
When there is proportional change in Q.D in response to a change in its price, it is called unitary elastic demand.

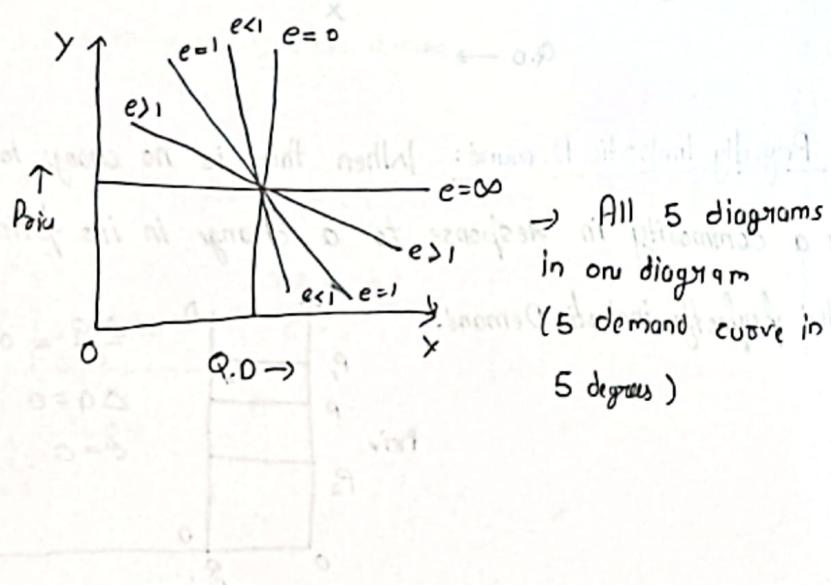
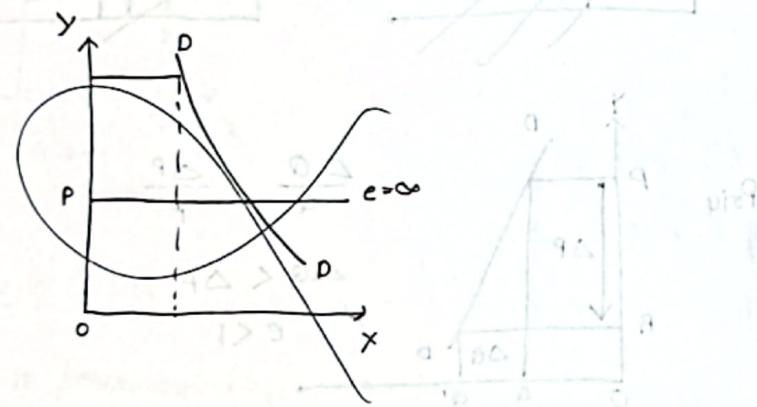
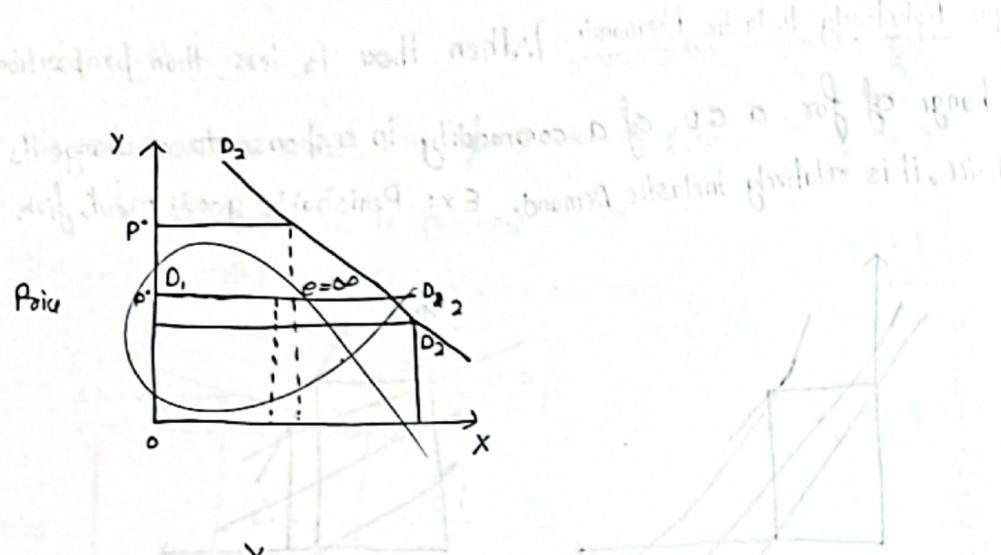


**IV) Relatively Inelastic Demand:** When there is less than proportional change of for a Q.D. of a commodity in response to a change in its price, it is relatively inelastic Demand. Ex: Perishable good: meat, fish.

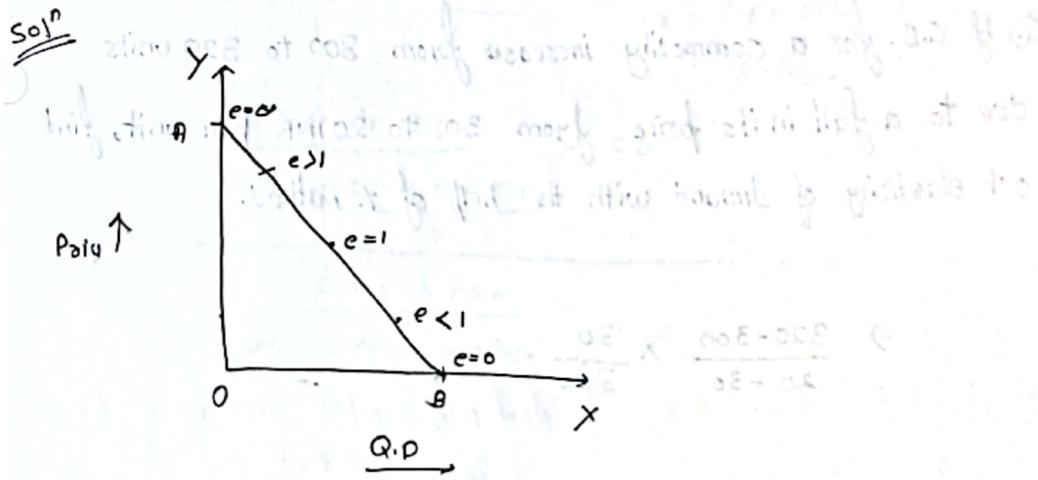
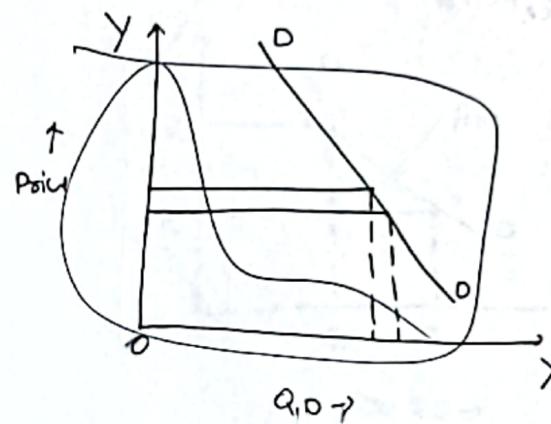


**V) Perfectly Inelastic Demand:** When there is no change in Q.D. for a commodity in response to a change in its price, it is called perfectly inelastic Demand.





Show all the 5 degrees of elasticity on a Downward Sloping straight line demand curve.



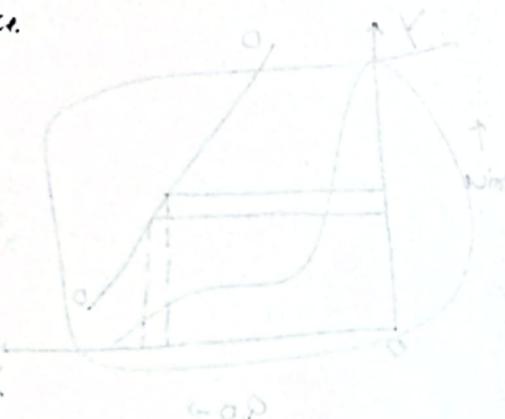
### Methods of measuring elasticity of demand.

- i) Percentage method
- ii) Arc method / mid-point method
- iii) Total expenditure method / Total outlay method.

I) Percentage Method: It refers to that method where elasticity of demand can be measured on the basis of % change in Q.D. in response to % change in its price.

$$E(P_p) = \frac{\% \text{ change in Q.D.}}{\% \text{ change in Price}}$$

$$\Rightarrow \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$$



Q) If Q.D. for a commodity increases from 300 to 320 units due to a fall in its price from 30 to 20 INR per unit, find out elasticity of demand with the help of % method.

$$\Rightarrow \frac{320 - 300}{20 - 30} \times \frac{30}{300}$$

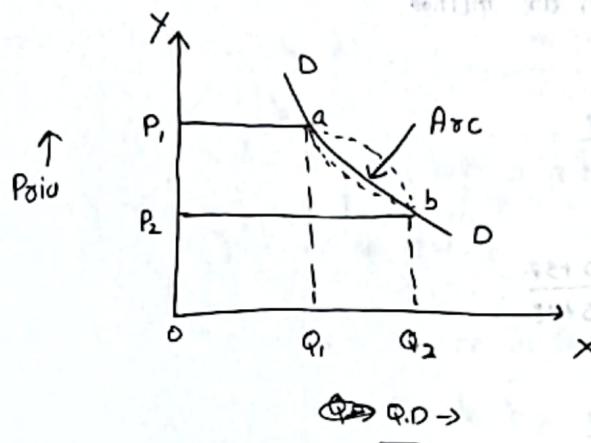
$$= \frac{20}{10} \times \frac{30}{300}$$

$$\Rightarrow -0.2$$

$$\Rightarrow 1 - 0.2$$

$$= 0.2$$

ii) Arc Method: It refers to that method where elasticity of demand can be measured b/w two separate points.



$$e_{arc} = \frac{\text{Change in Q.D.}}{\frac{\text{Original Q.D.} + \text{New Q.D.}}{2}} \times 100$$

$$= \frac{\text{Change in Price}}{\frac{\text{Original Price} + \text{New Price}}{2}} \times 100$$

$$= \frac{\frac{\Delta Q}{Q_1 + Q_2} \times 100}{\frac{\Delta P}{P_1 + P_2}}$$

$$= \frac{\frac{\Delta P}{P_1 + P_2} \times 100}{\frac{\Delta Q}{Q_1 + Q_2}}$$

also for price elasticity

$$\Rightarrow \frac{\Delta Q}{\Delta P} \times \frac{P_1 + P_2}{Q_1 + Q_2}$$

for price

$$\frac{\Delta Q}{\Delta Y} \times \frac{Y_1 + Y_2}{Q_1 + Q_2}$$

for income elasticity

$$P = \text{Price}$$

$$Y = \text{Income}$$

Q. If Q.O. of a commodity decreases from 55 to 48 units due to a rise in its price from 30 to 37 INR per unit, find elasticity of demand, with arc method

$$e_{arc} = \frac{\Delta Q}{\Delta P} \times \frac{P_1 + P_2}{Q_1 + Q_2}$$

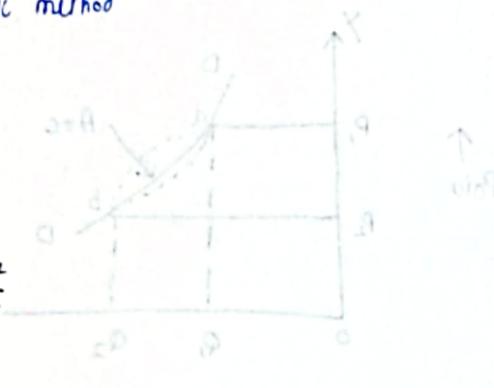
$$= \frac{48 - 55}{37 - 30} \times \frac{30 + 37}{55 + 48}$$

$$\Rightarrow \frac{-7}{7} \times \frac{67}{102}$$

$$= -1 - 0.65$$

$$1 - 0.65$$

$$\Rightarrow -0.65$$



→ 3 possibilities:

i) If ~~TE~~ Total expenditure increases with a fall in the price. or decreases with a ~~fall~~ rise in the price ~~fall~~, then price elasticity will be  $e > 1$ .

ii) If T.E. remains constant with a rise or fall in the price then price elasticity will be  $e = 1$ .

iii) If T.E. increases with a rise in the price or decreases with a fall in the price, the price elasticity will be  $e < 1$ .

Price <del>&amp;</del> Q.O	T.E. <del>=</del>	$e_p = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$
5.0      30	150	<del>e &gt; 1</del> $e > 1$
4.75     40	190	<del>e &gt; 1</del> $e > 1$
4.50     50	225	<del>e &gt; 1</del> $e > 1$
4.25     60	255	<del>e &gt; 1</del> $e > 1$
4.00     75	300	<del>e &gt; 1</del> $e > 1$
3.75     80	300	<del>e &gt; 1</del> $e > 1$
3.50     83	290.5	<del>e &lt; 1</del> $e < 1$
3.25     87	282.75	<del>e &lt; 1</del> $e < 1$

Q. If ~~the~~ Q.O. for a commodity declines from 100 to 80 units due to a rise in its price from 8 to 10 INR per unit. Find out elasticity of demand with Total expenditure method.

$$P \text{ Price} \quad Q \text{ O.D} \quad TE \text{ Total Expenditure} \quad e_p$$

$$\begin{array}{l} 8 \text{ } 100 \text{ } - 800 \\ 10 \text{ } 80 \text{ } - 800 \end{array} \quad \begin{array}{l} e_p = 1 \\ e_p = 1 \end{array}$$

## Determinants / Factors affecting elasticity of demand.

- i.) Availability of Substitutes  $e=0$   
is No Substitute
- ii.) Number of " :  $e \geq 1$
- iii.) Nature of the good.  $e \geq 1$   
Luxury:  $e \geq 1$   
Perishable:  $e < 1$   
Necessary:  $e = 0$
- iv.) Alternative Uses (luxurious Goods)  
Number of uses:  $e \geq 1$   
only one use:  $e = 0$
- v.) Postponement of Consumption Can be postponed now:  $e \geq 1$   
Short period:  $e = 0$   
Can't be postponed now:  $e = 0$
- vi.) Time Period Can't be postponed now:  $e = 0$   
short period:  $e = 0$   
long period:  $e \geq 1$
- vii.) Habit:  $e = 0$

## Demand Forecasting

$y$  is dependent

$$y = a + bx$$

Years	(y) Sales	X	XY	$\Sigma Y = Na + b \Sigma x$
1990	10	-2		$\Sigma XY = a \Sigma x + b \Sigma x^2$
1991	15	-1		
1992	25	0		
1993	36	1		
1994	42	2		
		$\Sigma x =$		

Q.) From the following table, forecast sales for the year 2021 and 22

Year	Sales (\$1000)	X	XY	$\Sigma X^2$	$\Sigma Y = 5a + b \Sigma x$
2015	20				$28 = 5a + b \cdot 10$
2016	35	-2	-40	-4	$a = 43.2$
2017	42	-1	-35	-1	
2018	56	0	0	0	$\Sigma XY = a \Sigma x + b \Sigma x^2$
2019	63	1	56	1	$107 = 43.2(0) + b \cdot 10$
N=5	$\Sigma Y = 216$	$\Sigma x = 0$	$\Sigma = 107$	$\Sigma x^2 = 10$	$b = 10.7$

$$y = 43.2 + 10.7x$$

for 2021,  ~~$x=4$~~

$$Y_{2021} = 43.2 + 0.7(4)$$

$$= 86$$

$$Y_{2022} = 43.2 + 0.7(5)$$

$$= 96.7$$

## Supply

Stock: Refers to actual amount of production whereas supply refers to the actual amount offered for sale in the market.

Supply Schedule: Refers to the tabular representation of different quantities of a commodity supplied to the market at different prices.

Price of X (in INR)	Q.S of X (in units)
5	10
10	17
15	19

## Types of Supply Schedule:

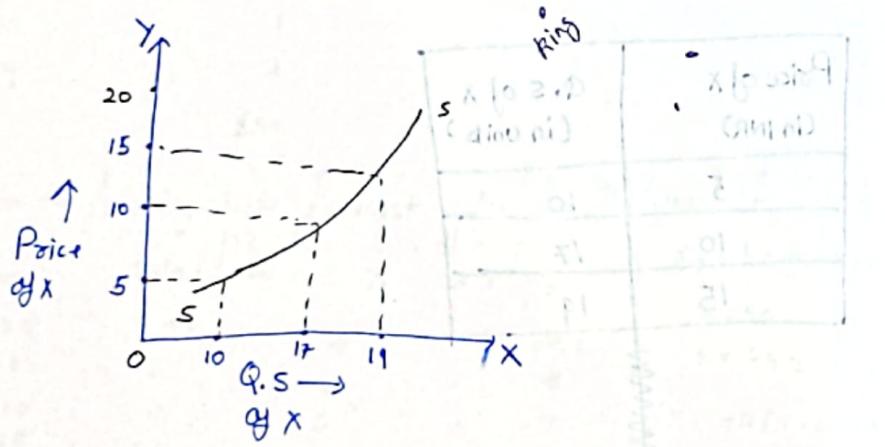
i) Individual Supply Schedule.

ii) Market Supply Schedule.

Individual Supply Schedule: Refers to the tabular representation of different quantities of a commodity supplied to the market by an individual producer at different prices (Draw schedule)

Market Supply Schedule: Refers to the tabular representation of different quantities of a commodity supplied to the market by all the producers at different prices (Draw schedule)

## Supply Curve



Law of Supply: It is defined as other factors remaining constant, quantity supply of a commodity increases with a rise in price and decreases with a fall in the price.

## Assumptions of the law:

- The technique of production remain constant.
- Prices of Input remain constant.
- Prices of other products remain constant
- No. of producers in the market remain constant.
- Prices of other ~~fixed~~ producer does not change.

(Prepare Supply schedule, then Supply curve, limitation/critism of law.)

## Limitation:

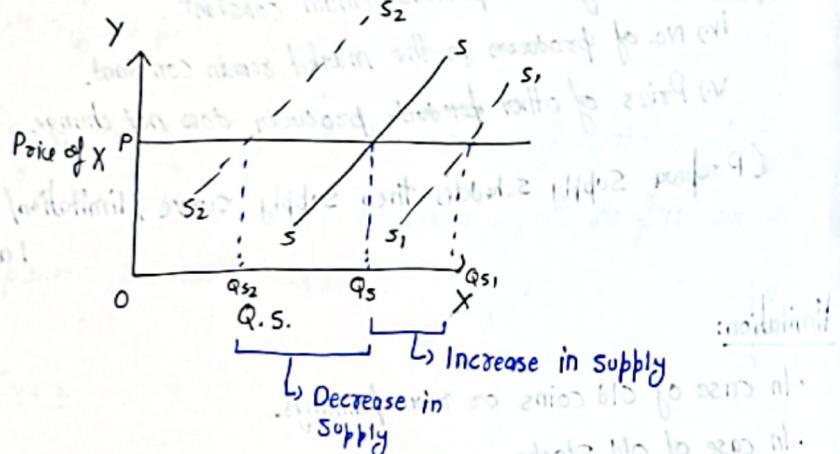
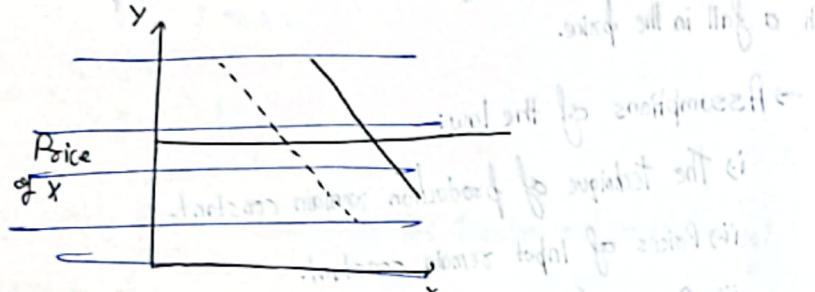
- In case of old coins or rare paintings.
- In case of old stocks.
- If labour price increases.
- When a business person closes his/her business.

Change in supply: When supply of a commodity changes due to change in the other factors, price remaining constant, it is called change in supply.

- Types:
- Increase in supply
  - Decrease in supply.

### Change in supply curve:

When there is a change in demand, it leads to shift in demand curve.

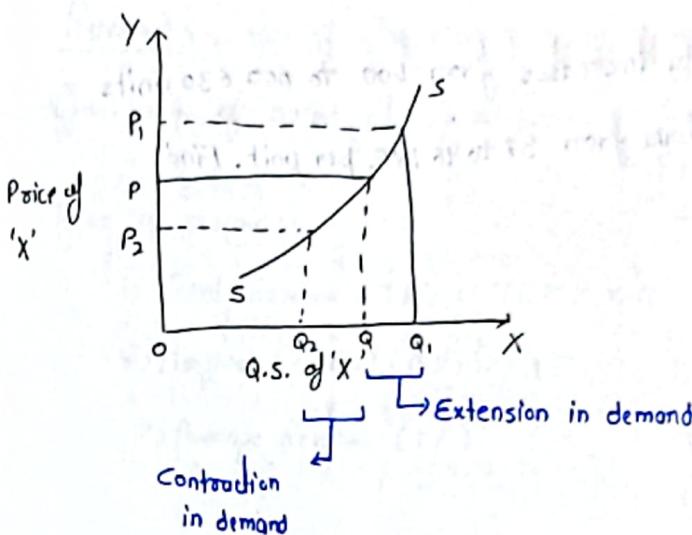


### Change in Quantity Supply:

When supply of a commodity changes due to a change in its price, other factors remaining constant, it is called change in Q.S.

**Types:** i) Extension in supply.

ii) Contraction in supply.



**Elasticity of Supply:** Refers to the degree of responsiveness of Q.S. of a commodity in response to a change in its price.

$$e_s = \frac{\% \text{ change in Q.S.}}{\% \text{ change in Price}}$$

$$= \frac{\Delta Q_s}{\Delta P} \times \frac{P}{Q_s}$$

$$= \frac{dQ_s}{dP} \times \frac{P}{Q_s}$$

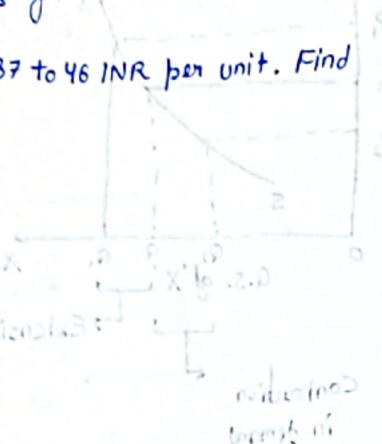
Q.) If Q.S. of a commodity increases from 500 to 630 units due to a rise in its price from 37 to 46 INR per unit. Find elasticity of supply.

$$e_s = \frac{\Delta Q.S.}{\Delta P} \times \frac{P}{Q.S.}$$

$$= \frac{130}{9} \times \frac{37}{500}$$

$$= \frac{4810}{4500}$$

$$\Rightarrow 1.068$$



→ Degree of elasticity of supply:

i) Perfectly elastic Supply ( $e_s=\infty$ )

ii) Relatively " " ( $e_s>1$ )

iii) Unitary " " ( $e_s=1$ )

iv) Relatively inelastic " " ( $e_s<1$ )

v) Perfectly " " ( $e_s=0$ )

→ Perfectly elastic supply: When price is fixed infinitely

→ Relatively elastic supply:

Revenue: Refers to the income earned by a producer by selling different units of output at different prices.

[P: Selling price per unit]

[Q: No. of units of output sold]

Types of revenue:

i) Total Revenue (TR) :  $TR = P \times Q$

ii) Marginal Revenue (MR)

iii) Average Revenue (AR)

Marginal Revenue: It refers to the net addition to the total revenue by selling one extra unit of Output:

$$MR_n = TR_n - TR_{n-1}$$

$$MR = \frac{d(TR)}{dQ}$$

Total Revenue: Total income earned by a producer by selling different unit of output at different prices.  $TR = P \times Q$

Average Revenue: It refers to the total income <sup>earned</sup> per unit of output sold.

$$AR = \frac{TR}{Q} \quad -(I)$$

$$TR = AR \times Q$$

$$TR = P \times Q \quad -(II)$$

$$P = AR$$

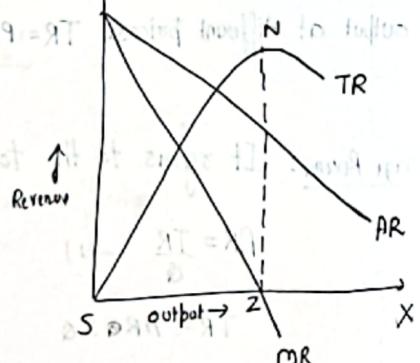
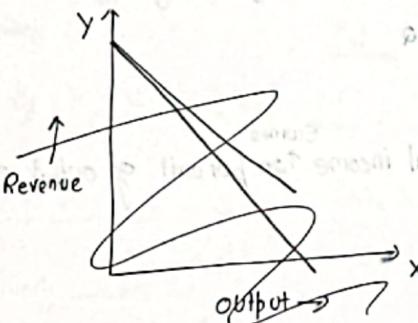
Q.) From the following table find total revenue and marginal revenue.

Unit of Output Sold	A.R.	M.R.
1	16	16
2	15	14
3	14	12
4	13	10
5	12	8
6	11	6
7	10	4
8	9	2
9	8	0
10	7	-2

$TR = 16 + 30 + 42 + 52 + 60 + 66 + 70 + 72 + 72 + 70 = 420$

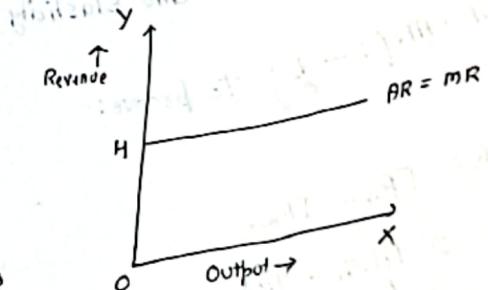
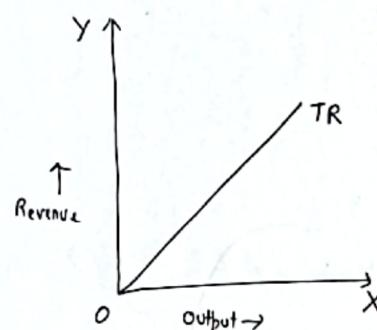
$MR = \frac{d(TR)}{dQ} = TR_n - TR_{n-1}$

TR is max when MR is zero



Q.) From the following table find total revenue and Marginal Revenue.

Unit of Output Sold	AR	TR	MR
1	30	30	30
2	30	60	30
3	30	90	30
4	30	120	30
5	30	150	30
6	30	180	30
7	30	210	30
		$\Sigma$	



Q.) From the following demand function find i) TR and MR ii) Price and Quantity when  $MR=0$  iii) Price and quantity when  $TR=\max$

$$Q = 40,000 - 15P$$

$$\text{i) } Q = 40,000 - 15P$$

$$\text{TR} = 15P = \frac{40,000}{15} - \frac{Q}{15} \Rightarrow 2666 - \frac{Q}{15} \Rightarrow TR = 26660 - \frac{Q^2}{15}$$

$$MR = 2666 - 0.13Q$$

Value of elasticity of demand when price starts falling will be

$$MR=0$$

$$\frac{2666}{13} = Q$$

$$Q = \frac{2050}{13} = 20507.69$$

$$Q = 20508$$

$$\Rightarrow Q = 40,000 - 15P$$

$$\frac{20508 - 40,000}{-15} = P$$

$$\Rightarrow P = 1298.8$$

→ Relationship b/w Revenue and Elasticity

$$MR = AR \left(1 - \frac{1}{e}\right) \text{ To prove:}$$

$$MR = TR_n - TR_{n-1}$$

$$\Rightarrow ARQ_1 - ARQ_2$$

$$\Rightarrow AR(Q_1 - Q_2)$$

$$\Rightarrow \left(e = \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}\right)$$

$$\Rightarrow AR \left( \frac{\Delta Q}{\Delta P} \times \frac{P}{e_1} \right) - AR \left( \frac{\Delta Q}{\Delta P} \times \frac{P}{e_2} \right)$$

$$\Rightarrow AR \left( \frac{\Delta Q}{\Delta P} \times \frac{1}{e_1} \right) - AR \left( \frac{\Delta Q}{\Delta P} \times \frac{1}{e_2} \right)$$

$$MR = \frac{d(TR)}{dQ}$$

$$= \frac{d(P \times Q)}{dQ}$$

$$\Rightarrow \frac{\partial P}{\partial Q} \cdot Q + \frac{\partial Q}{\partial Q} \cdot P$$

$$\Rightarrow P + \frac{\partial P}{\partial Q} Q$$

$$MR = P \left(1 + \frac{\partial P}{\partial Q} \cdot \frac{Q}{P}\right)$$

$$= P \left(1 + \frac{1}{\frac{\partial Q}{\partial P} \frac{P}{Q}}\right)$$

$$= P \left[1 + \frac{1}{\frac{1}{e}}\right] \quad [P = AR]$$

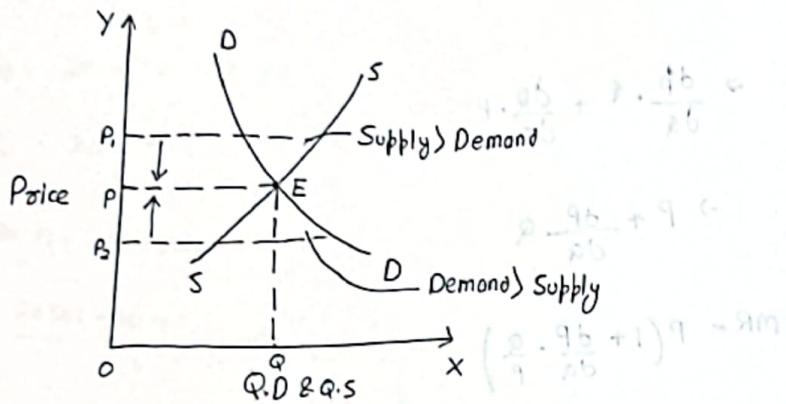
$$MR = AR \left(1 - \frac{1}{e}\right) \quad \underline{\text{Proved}}$$

Q.) Find MR If price of the commodity is 3INR per unit and coefficient of elasticity is 5

$$MR = AR \left(1 - \frac{1}{e}\right) \quad [AR = P]$$

$$= 3 \left(1 - \frac{1}{5}\right) \Rightarrow 3 - \frac{3}{5} \Rightarrow \frac{15-3}{5} \Rightarrow \underline{\underline{2.4}}$$

Equilibrium b/w demand and Supply :- (Y: Independent  
X: Dependent).



Q) From the following Demand and Supply function. find Equilibrium price and quantity.

$$Q_d = 800 - 15b$$

$$Q_s = 500 + 5b$$

Find new equilibrium price and quantity if Supply remaining constant

Demand decreases to  $Q_d = 700 - 10P$

$$\rightarrow 800 - 15b = 700 - 10P$$

$$300 = 5b$$

$$b = 15$$

$$Q \Rightarrow 800 - 15(15)$$

$$Q \Rightarrow 575$$

$$Q_d \Rightarrow 575 = 700 - 10P$$

$$+125 = 700$$

$$12.5 = b$$

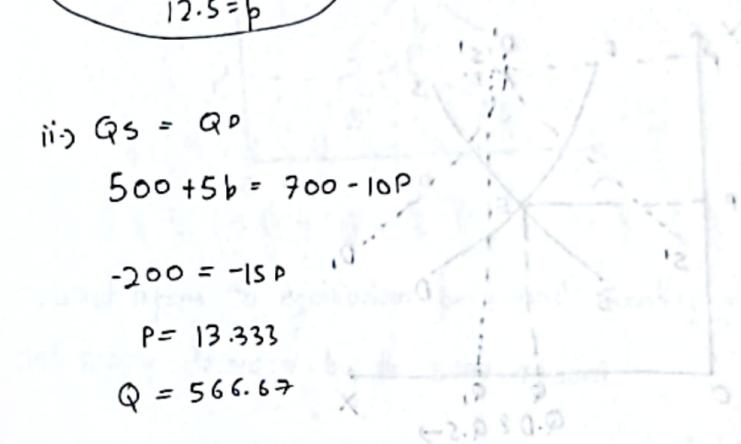
$$\text{ii) } Q_s = Q_d$$

$$500 + 5b = 700 - 10P$$

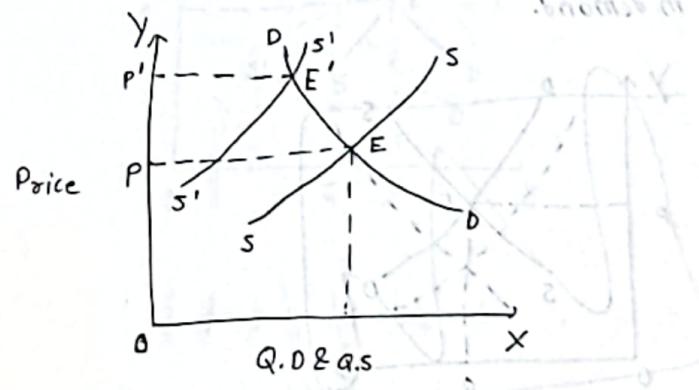
$$-200 = -15P$$

$$P = 13.333$$

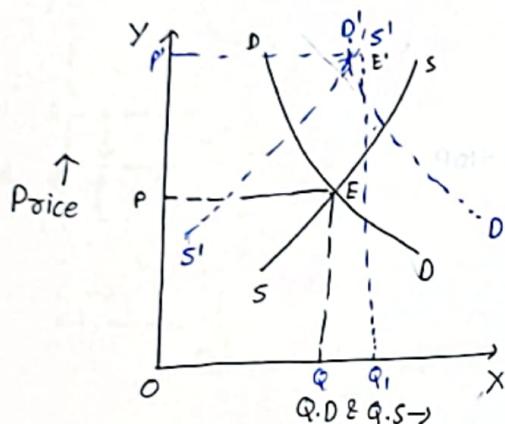
$$Q = 566.67$$



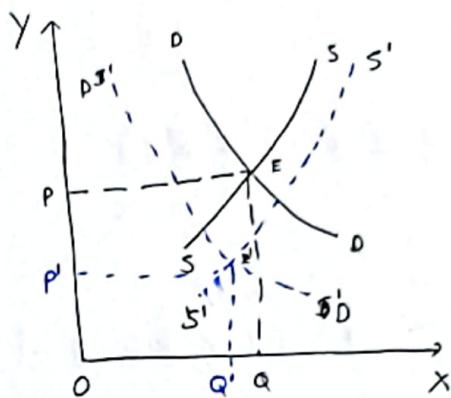
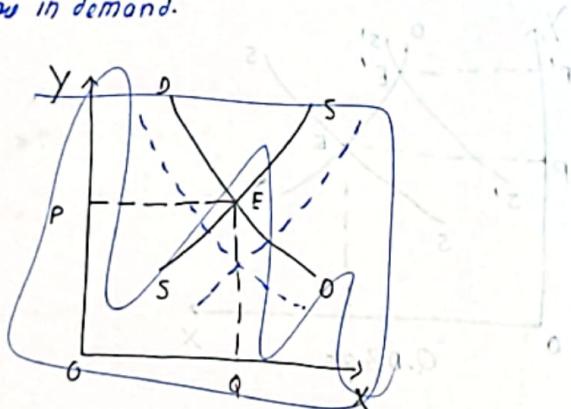
What happens to equilibrium price and quantity, if demand remaining constant, supply decreases.



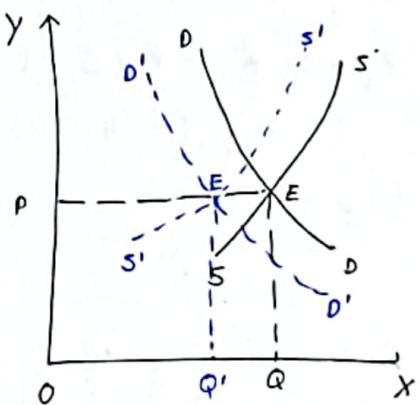
Q) What happens to equilibrium price and quantity if increase in demand is more than decrease in supply.



(b) Suppose there is an increase in demand and a fall in supply of fruit.  
What happens to equilibrium price and quantity if increase in supply is less than decrease in demand.



Q) What happens to equilibrium price and quantity if both demand and supply decreases by the same amount.



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Q1) Explain the degrees of elasticity of demand with the help of their respective diagrams.

Q2) Distinguish b/w % method and Arc method of measuring elasticity of Demand.

Q3) From the following demand function, find price elasticity of demand and income elasticity of demand if price of the commodity is 50INR per unit, and income of the consumer is 30,000 per month

$$Q = 50,000 - 5P + 0.7Y$$

Q4) From the following demand function find  
i.) TR function and MR function

ii.) Price and Quantity for TR to be maximum

$$Q = 80,000 - 25P$$

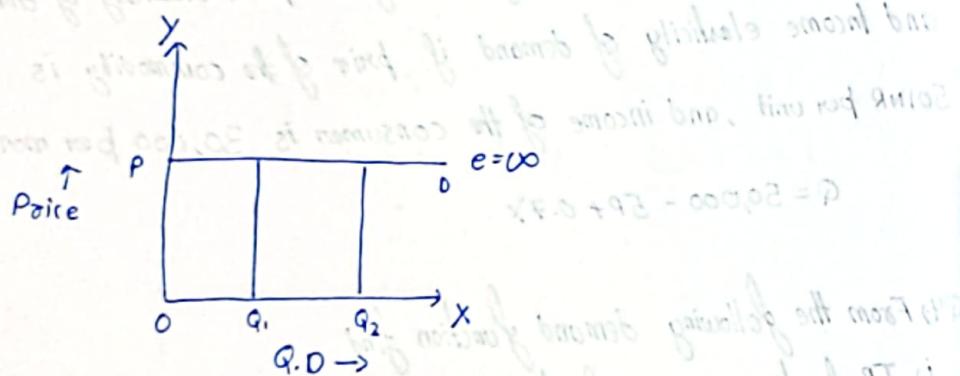
1. There are 5 degrees of elasticity of demand.

### i) Perfectly Elastic Demand:

In perfectly elastic demand, the Q.D. of a commodity is

infinite with a fixed price, but if there is a change in price

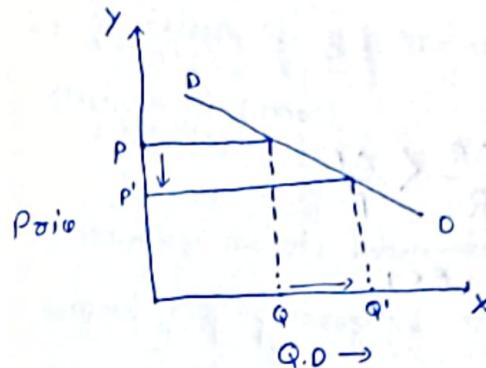
of a commodity, Q.D. falls to zero



### ii) Relatively Elastic Demand:

In Relatively elastic demand, the Q.D. of a commodity is more proportionate with respect to price of a commodity

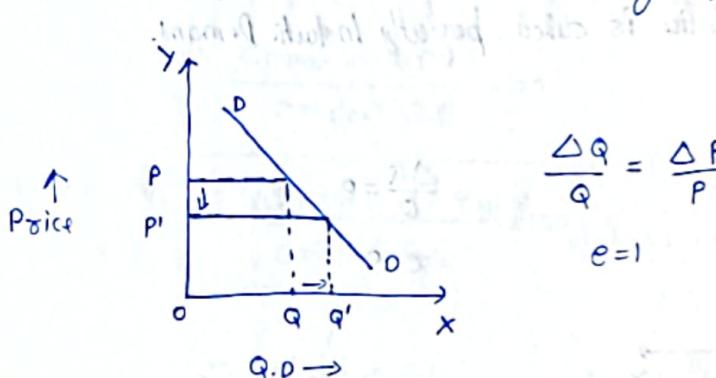
i.e., when there is more than proportional change in Q.D. for a commodity in response to change in price. Ex: Luxurious, Durables goods



$$\frac{\Delta Q}{Q} > \frac{\Delta P}{P}$$
$$e > 1$$

### iii) Unitary Elastic Demand:

When there is a proportional change in Q.D. for a commodity for a change in price is called Unitary Elastic Demand.

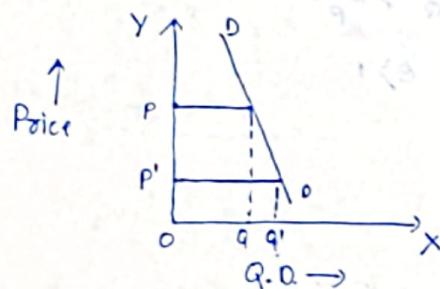


$$\frac{\Delta Q}{Q} = \frac{\Delta P}{P}$$
$$e = 1$$

### iv) Relatively Inelastic Demand:

When there is less than proportional change in Q.D. for a commodity for a change in price is called Relatively Inelastic Demand. Ex: Fish, meat

### ~~Perfectly Inelastic Demand:~~



$$\frac{\Delta Q}{Q} < \frac{\Delta P}{P}$$

$$e < 1$$

2.) % method and Arc methods are ways to compute the elasticity of Demand.

i.) Percentage method: ~~better than~~ method when elasticity of demand can be measured on the basis of percentage change in Q.D. of a commodity in response to percentage change in its price

$$E(e_r) = \frac{\% \text{ change in Q.D.}}{\% \text{ change in Price}}$$

$$= \frac{\text{Change in Q.D.}}{\text{Original Q.D.}} \times 100$$

$$\frac{\text{Change in Price}}{\text{Original Price}} \times 100$$

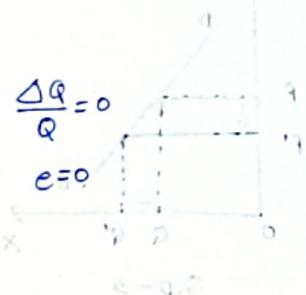
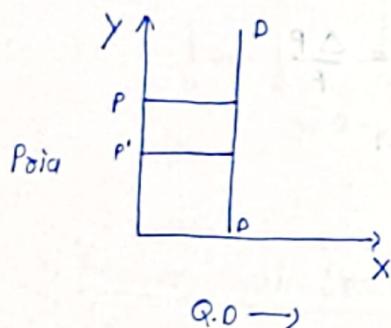
$$\Rightarrow \frac{\text{Change in Q.D.}}{\text{Change in Price}} \times \frac{\text{Original Price}}{\text{Original Q.D.}}$$

$$\Rightarrow \frac{\Delta Q}{\Delta P} \times \frac{P}{Q}$$

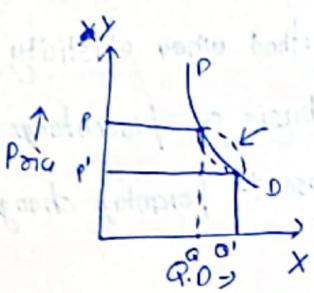
[ $\because Q \rightarrow \text{Quantity Demand}$   
 $P \rightarrow \text{Price}$ ]

### Perfectly Inelastic Demand:

When there is no change in Q.D. for a commodity with respect to change in its price is called perfectly Inelastic Demand.



ii) Arc Method: It is the method where elasticity of demand can be measured b/w two specific points.



$$\text{E}_\text{arc} = \frac{\text{Change in Q.D.}}{\text{Original Q.D.} + \text{New Q.D.}} \times 100$$

$$\frac{\text{Change in } \Delta \text{ Price}}{\text{Original Price} + \text{New Price}} \times 100$$

$$\Rightarrow \frac{\text{Change in Q.D.}}{\text{Original Q.D.} + \text{New Q.D.}} \times \frac{\Delta Q}{\Delta P} \times \frac{P_1 + P_2}{Q_1 + Q_2}$$

$$\left[ \begin{array}{l} \therefore Q_1 = \text{Quantity Demand of 1} \\ Q_2 = " " \text{ of 2} \\ P_1 = \text{Price of commodity 1} \\ P_2 = " " 2 \end{array} \right]$$

3.) Given

$$Q = 50,000 - 5P + 0.7Y \quad \dots(1)$$

Price of commodity = 50 INR

Income of consumer = 30,000 per month

Now, putting the values in (1)

$$Q = 50,000 - 5(50) + 0.7(30,000)$$

$$\underline{Q = 70,750}$$

i.) Price elasticity of demand

We know

$$\begin{aligned} e_p &= \frac{dQ}{dP} \times \frac{P}{Q} \\ &= -5 \times \frac{50}{70,750} \end{aligned}$$

$$= -0.0031$$

$$= \underline{0.003}$$

ii.) Income elasticity of demand

We know

$$\begin{aligned} e_y &= \frac{dQ}{dY} \times \frac{Y}{Q} \Rightarrow 0.7 \times \frac{30,000}{70,750} \\ &= \underline{0.296} \end{aligned}$$

## Interest Rates:

- i.) Simple Interest Rate
- ii.) Compound " "
- iii.) Nominal " "
- iv.) Effective " "

Simple Interest Rate: Refers to that type of interest rates where principal amount remains same for various years.

$$I = P \cdot i \cdot N$$

$i \rightarrow$  Interest Rate

$N \rightarrow$  No. of Years

$P \rightarrow$  Principal Amount

$I \rightarrow$  Simple Interest Rate

$$F = P + I$$

$$= P + (P \cdot i \cdot N)$$

$$F = P(1 + i \cdot N)$$

$F \rightarrow$  Future Value

$P \rightarrow$  Principal

Q.) Find future value of 4 lakh INR at 6% interest rate, after 6 years, with the help of Simple Interest Rate.

$$F = 4,00,000 (1 + 0.06 \times 6)$$

$$= 4,00,000 (1 + 0.36)$$

$$= 4,00,000 \underline{\underline{+}}$$

$$\Rightarrow 5,44,000/-$$

Compound Interest Rate: Refers to that type of interest rate where the principal amount keeps on changing every year.

$$F_n = P(1+i)^n$$

$N \Rightarrow$  Number of years

$P \rightarrow$  Principal,  $i \rightarrow$  Interest rate

Q.) If a person deposits 7 lakh INR in a bank at 9.5% interest rate compounded annually, find out the maturity amount of his account after 10 years.

$$F_n = 7,00,000 (1 + 0.095)^{10}$$

$$= 17,34,759.339/-$$

Nominal Interest Rate: Refers to that type of interest rate where interest rate is calculated several times in a year, i.e., quarterly, monthly, half-yearly/semi-annually and each day to find out compound interest rate.

i) Quarterly:  $F_4 = P \left(1 + \frac{i}{4}\right)^{4N}$

ii) Monthly:  $F_{12} = P \left(1 + \frac{i}{12}\right)^{12N}$

iii) Half-yearly / Semi-Annually:  $F_2 = P \left(1 + \frac{i}{2}\right)^{2N}$

iv) Each Day:  $F_{365} = P \left(1 + \frac{i}{365}\right)^{365N}$  [not including leap year]

Q. Find future value of 5 lakh INR at 4.5% interest rate after 7 years if the compounding is monthly.

$$F_{12} = 5,00,000 \left(1 + \frac{0.045}{12}\right)^{12(7)}$$

$$= 684,726.12/-$$

Effective Interest Rate: Refers to the ratio of interest charged for 1 year to the principal amount. ( $N=1$ )

$$i_{\text{eff}} = \frac{F-P}{P} \quad (i_{\text{eff}} \rightarrow \text{Interest Rate})$$

Q. Find effective interest rate of 8,00,000 INR at 9% interest rate if the compounding is quarterly.

$$\begin{aligned} \text{to } F_4 &= P \left(1 + \frac{i}{4}\right)^{4N} \\ \cancel{P} &= 8,00,000 \left(1 + \frac{0.09}{4}\right)^4 \\ &= 8,74,466.65/- \end{aligned}$$

$$\begin{aligned} i_{\text{eff}} &= \frac{8,74,466.65 - 8,00,000}{8,00,000} \quad \left(i_{\text{eff}} = \frac{F-P}{P}\right) \\ &= 0.093 \\ &= \underline{\underline{9.3\%}} \end{aligned}$$

Q. If Principal amount is not given:

$$i_{eff} = \left(1 + \frac{\alpha}{m}\right)^m - 1$$

$\alpha \rightarrow$  Nominal Interest Rate

$m \rightarrow$  No. of times interest rate is calculated in a year.

~~No.~~ <sup>or</sup> No. of compounding

Q. If a credit amount charges 19% interest rate, find out effective interest rate if the compounding is half-yearly.

$$i_{eff} = \left(1 + \frac{0.19}{2}\right)^2 - 1$$

$$= 0.199$$

$$= 19.9\%$$

### Utility-

Refers to the want satisfying capacity of the commodity.

It can be +ve or -ve.

- When utility has -ve impact on health and society, it is -ve utility. Ex- Smoking
- When utility has +ve impact on health and society, it is +ve utility.

### Types of Utility:

i)  $\rightarrow$  (TU)

Total Utility: Refers to the total satisfaction that a consumer can get by consuming various units of a commodity.

ii)  $\rightarrow$  (MU)

Marginal Utility: Refers to the net addition to the total utility by consuming 1 extra unit of a commodity.

$$\text{Marginal Utility (MU)} = \text{TU}_n - \text{TU}_{n-1}$$

X  $\sim$  commodity 'x'

$$MU_x = \frac{d(TU)}{dx}$$

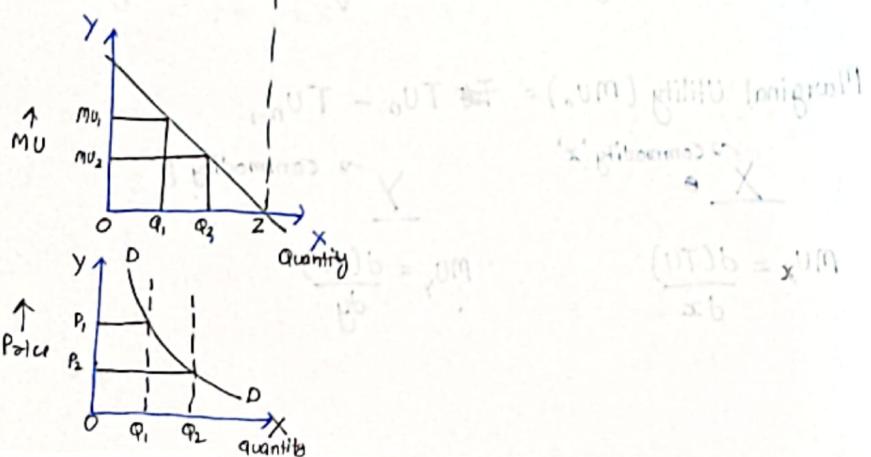
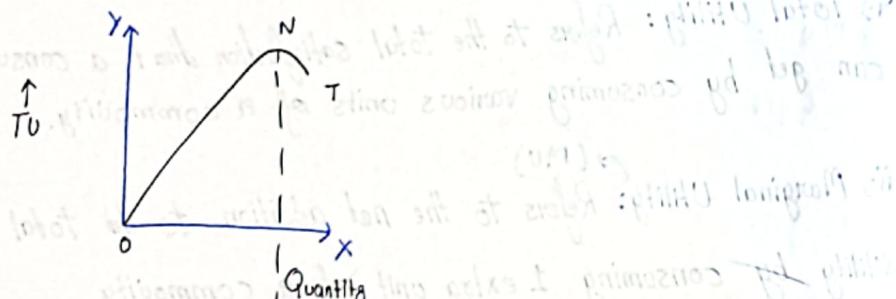
Y  $\sim$  commodity 'y'

$$MU_y = \frac{d(TU)}{dy}$$

Q.) From the following table find out marginal utility

Units of commodity consumed	Total Utility	Marginal Utility
1	10	-
2	18	8
3	24	6
4	28	4
5	30	2
6	30	0
7	26	-4

→ Total utility is Max when marginal utility is Zero.



### Theory of consumer behaviour.

1.) Indifference Curve (IC)

2.) Marginal rate of Substitution (MRS)

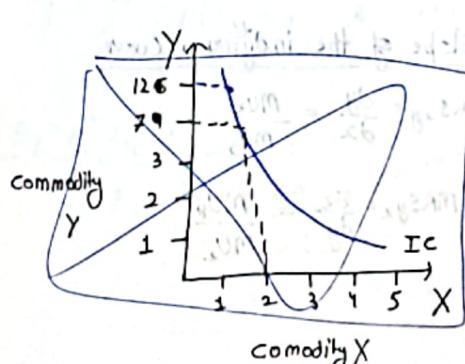
3.) Budget line

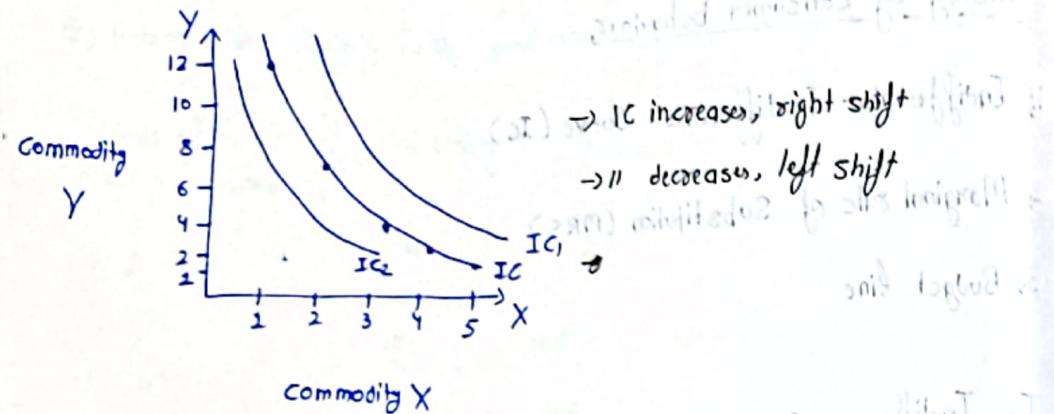
I.) Indifference Curve:

It refers to those curves which shows various combination of two commodities that give equal level of satisfaction to the consumer.

Combination

- A  $1$
- B  $2$
- C  $3$
- D  $4$
- E  $5$



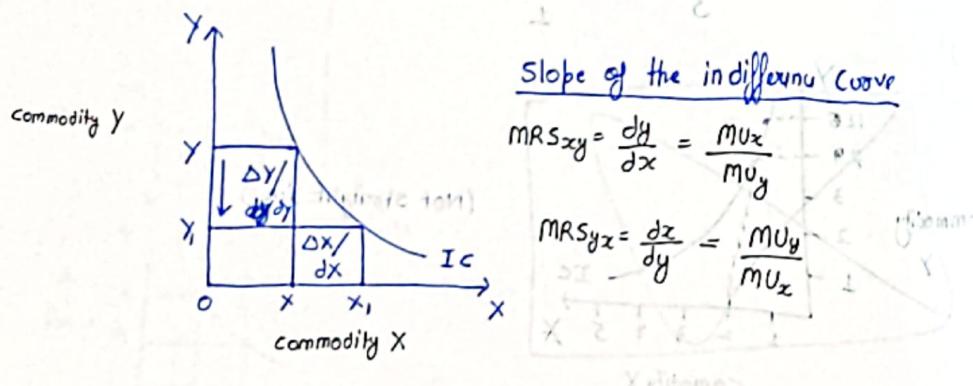


## II) Marginal Rate of Substitution (MRS):

H refers to the rate at which number of units of 1 commodity substituted to have 1 more unit of another commodity.

i) MRS<sub>xy</sub>: Refers to the rate at which the number of unit of commodity Y substituted to have one more unit of commodity X.

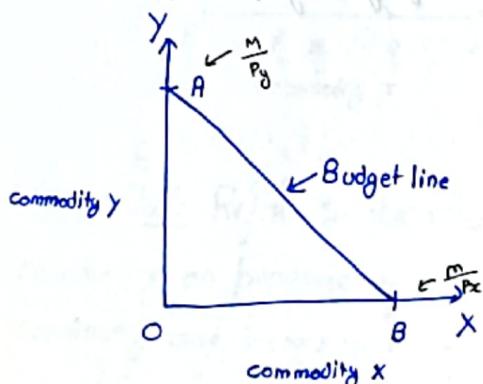
ii) MRS<sub>yx</sub>: Refers to the rate at which the number of unit of commodity X substituted to have one more unit of commodity Y.



Q) From the following table find out  $MRS_{xy}$ ,  $MRS_{yx}$ .

Combination	X	Y	$MRS_{xy}$	$MRS_{yx}$
A	1	12		
B	2	7	-5	-0.2
C	3	3	-4	-0.25
D	4	2	-1	-1
E	5	1	-1	-1

Budget Line: Refers to the line that shows various combination of two commodities that a consumer can purchase with a given level of income.



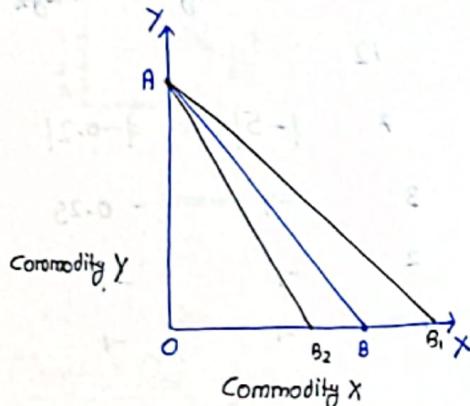
Slope of the budget line:

$$\frac{OA}{OB} = \frac{m}{P_y} \Rightarrow \frac{P_x}{P_y} = \frac{m}{P_x}$$

Eqn of the budget line:

$$M = P_x q_x + P_y q_y$$

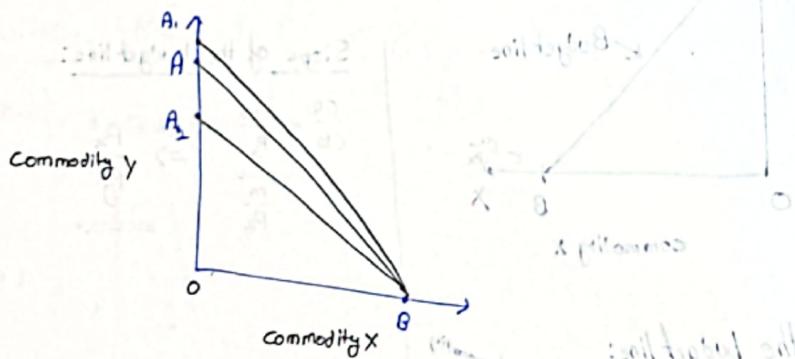
Shift in the budget line if "purchase" of commodity  $x$  changes,  $y$  and income remaining constant.



$B_1 \rightarrow$  Decrease in price of  $X$ , increase in Demand.

$B_2 \rightarrow$  Increase " " " " decrease in Demand.

Shift in the budget line if purchase of commodity  $y$  changes,  $x$  and income remaining constant.

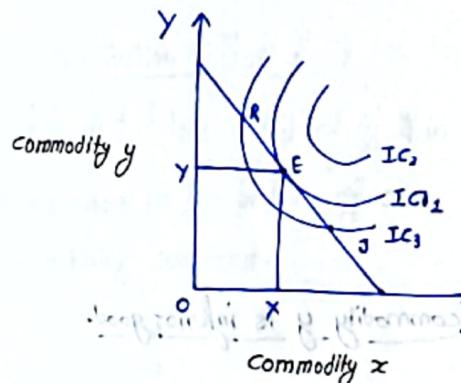


Condition for a consumer to be in equilibrium.

- i) Slope of the indifference curve is equal to slope of the budget line.
- ii) The indifference curve should be convex at equilibrium point.

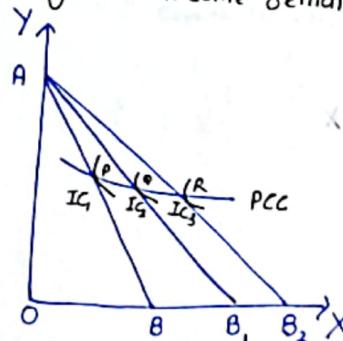
$$MRS_{xy} = \frac{dy}{dx} = \frac{MU_x}{MU_y} = \frac{P_x}{P_y}$$

$\Rightarrow \frac{MU_x}{P_x} = \frac{MU_y}{P_y} \rightarrow$  Optimum condition bundle for  $x$  and  $y$ .



- maximum level of satisfaction
- E → lying on the maximum possible indifference curve, IC2
- OX, OY → Optimal combination of commodity  $x$ ,  $y$  for consumer to be in Equilibrium

Price Effect: Refers to the effect of change in the price of one commodity on purchase of commodities, price of another commodity and income remaining constant.

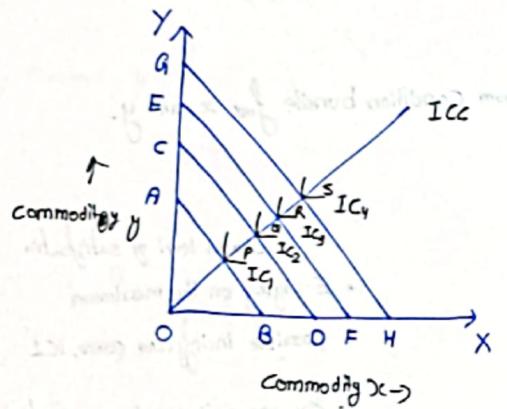


Joining P, Q, R → Price Consumption Curve.

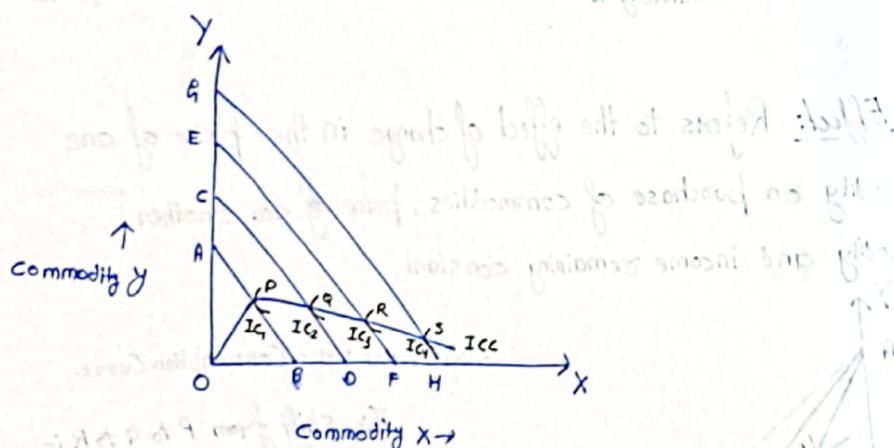
This shift from P to Q to R is Price Effect

### Income Effect:

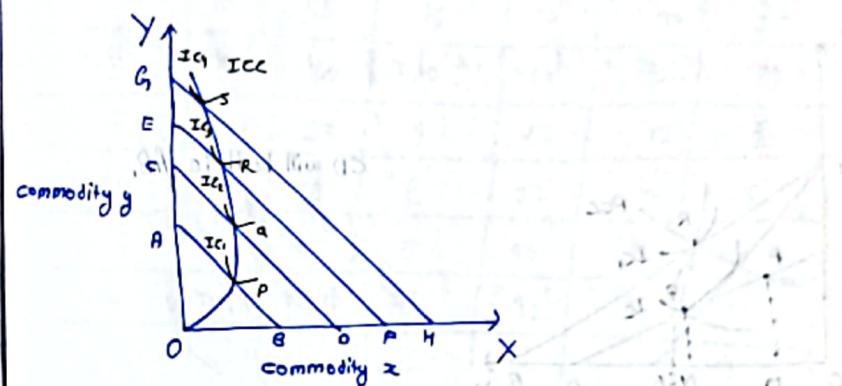
Refers to the effect of change of income of the consumer on purchase of commodities, price of both commodities remaining constant.



### Income Consumption Curve if commodity y is inferior good.

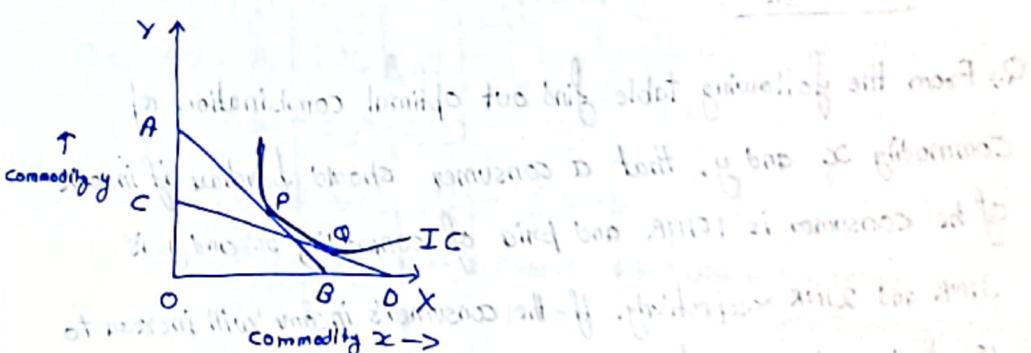


### Income Consumption Curve if commodity x is an inferior good.

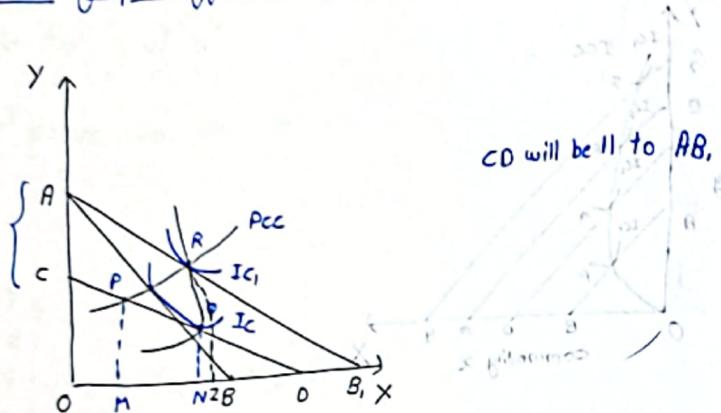


### Substitution Effect:

Refers to the effect of fall in the price of one commodity and increase in the price of other commodity, income of the consumer remaining constant.



Relationship among price effect, income effect, Substitution effect.



$$MZ = PE$$

$$NZ = IE$$

$$MN = SE$$

$$MZ = MPN + NZ$$

$$PE = SE + IE$$

Q. From the following table find out optimal combination of Commodity x and y, that a consumer should purchase if income of the consumer is 10 INR and price of commodity x and y is 3 INR and 2 INR respectively. If the consumer's income will increase to 18, find out the no. of unit of commodity x and y, consumer should purchase to maximize utility.

Quantity (in units)	commodity x			commodity y		
	T.U.	M.U.	MU.Pen Rs	T.U.	M.U.	MU.Pen Rs
1	45	45	15	40	40	20
2	75	30	10	60	20	10
3	102	27	9	72	12	6
4	120	18	6	82	10	5
5	135	15	5	90	8	4
6	144.99	9.99	3.33	92	2	1

$$\frac{MU_x}{P_x} = 15 \rightarrow \text{Marginal Utility of X is 15 times its price in units of } \frac{\text{Marginal Utility}}{\text{Price}}$$

$$MU_x = 15 \times 3 \Rightarrow 45$$

$$" \\ MU = T_n - T_{n-1}$$

$$\text{Now, A/Q, income of the consumer} = 10 \text{ INR}$$

$$\text{Budget line} = P_x \times Q_x + P_y Q_y$$

$$10 = 3(2) + 2(2)$$

at

$$18 = 3(4) + 2(3) \rightarrow \text{Consumer satisfaction will be maximum by purchasing 4 units of x and 3 units of y giving } 6$$

Q) If a consumer has 22 INR to spend on both a and b, whose prices are 2 INR each, find out

i) How many units of a and b should be purchased for maximum utility of the consumer.

ii) If income of the consumer increases to 28 INR, find optimal consumption bundle of good a and b for maximum satisfaction of consumer.

Q) What is the marginal utility per INR spent on 4<sup>th</sup> unit of good 'a' and 6 unit of good 'b'.

Quantity for good 'a'	T.U.	M.U.	M.U. per INR	Quantity for good 'b'	T.U.	M.U.	M.U. per INR
1	10	10	5	1	16	16	8
2	19	9	4.5	2	30	14	7
3	27	8	4	3	42	12	6
4	34	7	3.5	4	52	10	5
5	40	6	3	5	60	8	4
6	45	5	2.5	6	66	6	3
7	49	4	2	7	70	4	2

$$\underline{22} = \underline{2(7)} + \underline{2(4)} \quad \checkmark$$

$$\underline{22} = \underline{2(6)} + \underline{2(5)} \quad \checkmark$$

$$\cancel{22 = 2(7) + 2(4)}$$

$$\cancel{22 = 2(7)}$$

$$\underline{22} = \underline{2(5)} + \underline{2(6)} \quad \checkmark$$

$$28 = 2(7) + 2(1)$$

Q

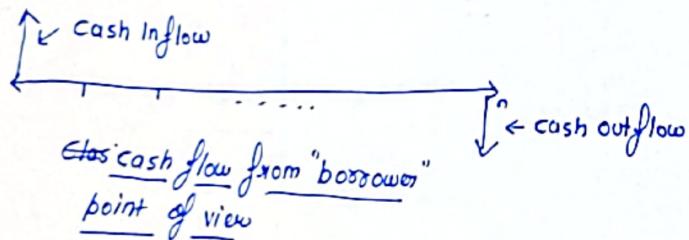
## Time value of Money.

Refers to the value of money at a particular time period.

Value of money in present is greater than value of money in future.

Cash flow diagram: Refers to graphical representation of cash inflow and outflow among a timeline.

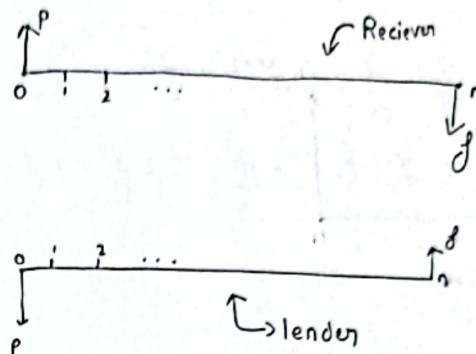
Timeline: Horizontal Scale that shows us time period.



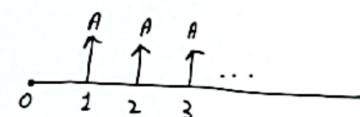
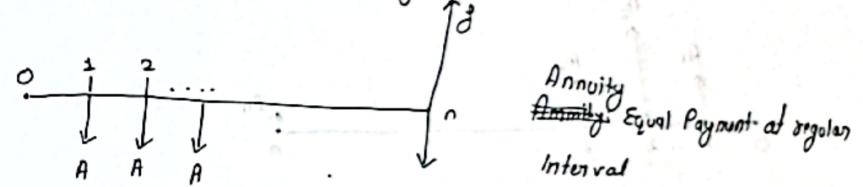
## Types of cashflow Diagram:

- Single Payment Cashflow
- Uniform payment series
- Linear Gradient Series
- Geometric Gradient Series.
- Irregular Payment series.

### I) Single Payment Cashflow:



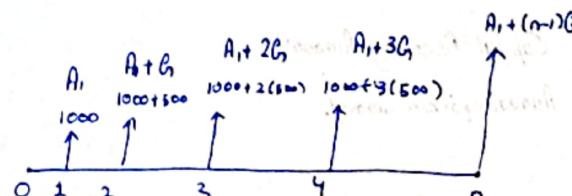
### II) Uniform Payment Series: (Starts from 1)



### III) Linear Gradient Series: Refers to the series of cashflow, increasing or decreasing by a fixed amount.

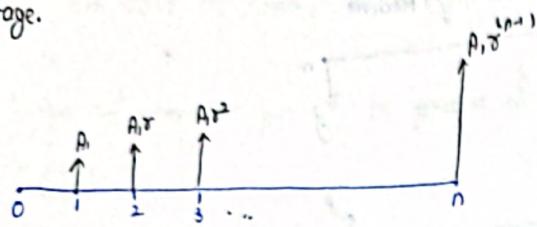
$$A_1 = 1000$$

$$G = 500$$



#### IV) Geometric Gradient Series

Refers to series of cashflow increasing or decreasing by a fixed percentage.



#### V) Irregular Payment Series:



#### • Types of compound Amount Factors:

i) Single Payment Compound Amount

ii) Single Payment Present Worth Amount

iii) Equal " Series Compound Amount

iv) " " " Sinking Fund

v) " " " Present worth amount.

VI) Equal-Payment Series Capital Recovery Amount.

VII) Linear gradient series Annual equivalent amount.

#### I) Single Payment Compound Amount

↳ Going to calculate future value.

Here the objective is to find future value of a present sum after  $n^{th}$  year, compounded at an interest rate ' $i$ '.

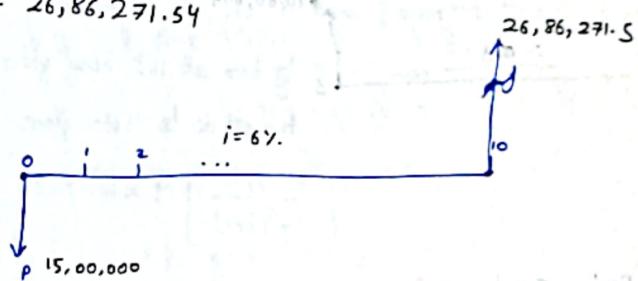
$$F = P(1+i)^n \rightarrow \text{Single Payment Compound Amount.}$$

Q.) A person deposits 15 lakh INR in a bank for 10 years. Find the maturity amount of his account if interest rate is 6% compounded annually.

$$F = P(1+i)^n$$

$$= 15,00,000 (1 + 0.06)^{10}$$

$$= 26,86,271.54$$



#### II) Single Payment Present Worth Amount.

Here the objective is to find the present value of a future sum after  $n^{th}$  year, compounded at an interest rate ' $i$ '.

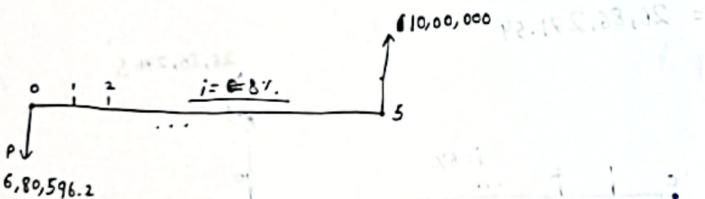
$$P = \frac{F}{(1+i)^n} \rightarrow \underline{\text{Single payment present worth amount.}}$$

Q.) A person needs 10 lakh INR after 5 years. Find how much money the person has to deposit now to get 10 lakh after 5 years if interest rate is 8% compounded Anually.

$$P = \frac{F}{(1+i)^n}$$

$$P = \frac{10,00,000}{(1+0.08)^5}$$

$$= 6,80,596.20 \text{ INR}$$



#### Equal payment series compound Amount.

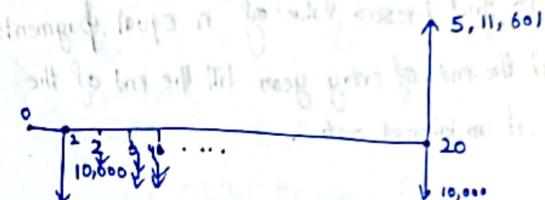
Here the objective is to find future value of n equal payments that is to be made at the end of every year, till the end of  $n^{\text{th}}$  year, Compounded at an interest rate i;

$$F = A \left[ \frac{(1+i)^n - 1}{i} \right]$$

Q.) A person invest an equal amount of 10,000INR at the end of every year for 20 years, find the maturity amount of his account if interest rate is 9% compounded Anually.

$$\Rightarrow F = 10000 \left( \frac{(1+0.09)^{20} - 1}{0.09} \right)$$

$$= 511601.1964$$



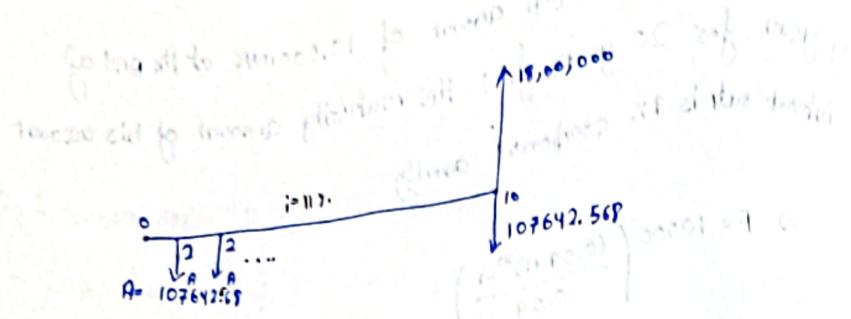
#### Equal payment series sinking fund.

Here the objective is to find n equal payments that is to be collected at the end of every year till the end of the  $n^{\text{th}}$  year to realise a future sum after  $n^{\text{th}}$  year compounded at an interest rate i;

$$\text{using } \cancel{F = A} \rightarrow A = \frac{i}{(1+i)^n - 1}$$

Q.) A person needs 18,00,000 INR after 10 years, find out how much equal amount of money the person has to deposit at the end of every year for 10 years if interest rate is 11% compounded anually.

$$A = 18,00,000 \left[ \frac{0.11}{(1+0.11)^{10} - 1} \right] \Rightarrow A = 107642.5688$$



### Equal-Payment Series present worth amount.

Here the objective is to find present value of  $n$  equal payments that is to be made at the end of every year till the end of the  $n^{\text{th}}$  year compounded at an interest rate  $i$ .

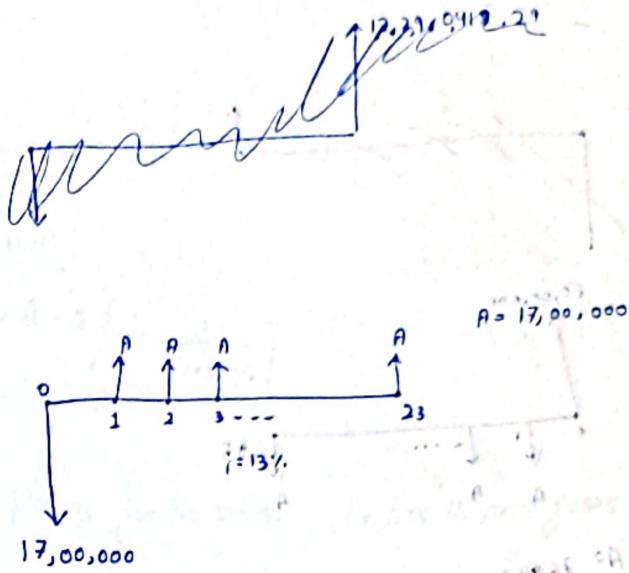
$$P = A \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

Q) A company wants to set up a reserve which will help it to have an annual equivalent amount of 17 lakh INR for next 23 years

towards its employee welfare majors. The reserve is assumed to grow at the rate of 13% compounded annually. Find single payment that must be made as the reserve amount now.

$$P = 17,00,000 \left[ \frac{(1.13+1)^{23} - 1}{0.13} \right]$$

$$P = 12,29,041.829$$



### Equal payment series Capital Recovery Amount.

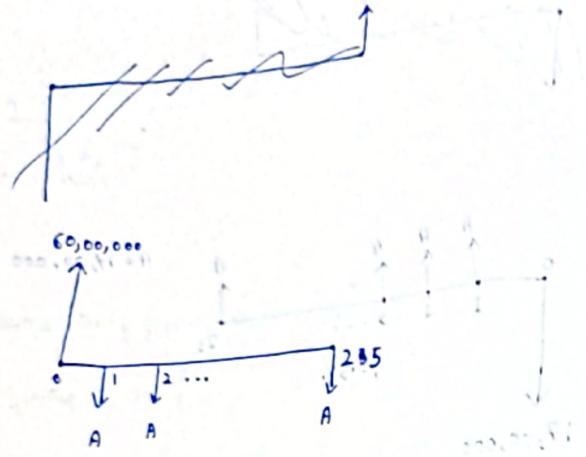
Here the objective is to find present value of  $n$  equal payments that is to be recovered at the end of every year till the end of  $n^{\text{th}}$  year for a loan i.e. Sanctioned now, compounded at an interest rate  $i$ .

$$A = P \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

Q) A company has taken a loan of 60,00,000 INR find out the instalment amount that the company has to pay at 12% interest rate compounded annually if the no. of instalment is 25.

$$A = 60,00,000 \left[ \frac{0.12 (1+0.12)^{25}}{(1+0.12)^{25} - 1} \right]$$

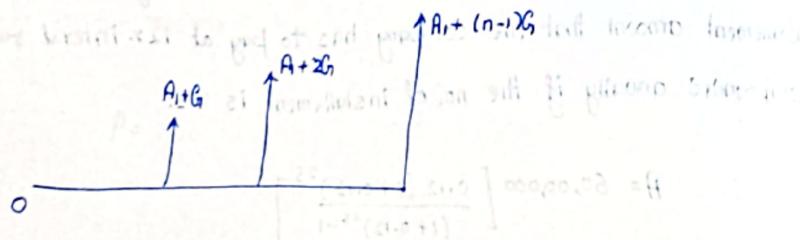
$$= 7,64,999.8189$$



$$A = 76,04,999.8181$$

### Linear gradient series Annual equivalent amount

Here the objective is to find annual equivalent amount, series of equal amounts, starting in the first year ( $A_1$ ), with a fixed amount increasing or decreasing ( $G$ ) at the end of every year, till the end of the  $n^{\text{th}}$  year, following at the first year compounded at an interest rate  $i$ .



### Increasing series:

$$A = A_1 + G \left[ \frac{1}{i} - \frac{n}{(1+i)^{n-1}} \right]$$

### Decreasing Series:

$$A = A_1 - G \left[ \frac{1}{i} - \frac{n}{(1+i)^{n-1}} \right]$$

Q) A person is planning for his retired life, he has 15 more years of service, he would like to deposit 10% of his salary, which is 5000 INR, starting at the end of first year, with an annual increase of ~~1000~~ 1000 thereafter for the next 14 years. If the interest rate is 10% compounded annually find out the total amount of the above series after 15 years.

Future Value

$$A_1 = 5000$$

$$G = 1,000$$

for  $n = 14$  yrs

Total amount = 15 yrs

$$A = 5000 \left[ \frac{1}{0.1} - \frac{15}{(1+0.1)^{15}-1} \right]$$

$$A = 4994.7210 \Rightarrow \cancel{10,278.93} \quad \underline{\underline{10,278.93}}$$

$$F = A \left[ \frac{(1+i)^n - 1}{i} \right]$$

$$= 4994.7210 \left[ \frac{(1+0.1)^{15} - 1}{0.1} \right] = \underline{\underline{3,26,587.226}}$$

$$\cancel{F = 10,278.93}$$

$$\cancel{F = 10,278.93}$$

## Comparison of Alternatives

i) Present Worth method

ii) Future Worth Method

iii) Annual Worth Method

iv) Rate of Return method

v) Cost-Benefit Analysis

vi) Pay-back period method.

• Present worth Method

I) In case of One Project

$$NPW(i\%) = PW(B) - PW(C)$$

NPW → Net Present worth

B → Benefit

C → Cost

If  $NPW(i\%) > 0$ , Project will be Selected

If  $NPW(i\%) < 0$ , " " " Rejected

If  $NPW(i\%) = 0$ , " may or may not be selected

II, In case of mutually exclusive projects. (more than 1 Project)

i) Revenue Based method.

ii) Cost Based Method.

• Revenue Based Method:

Refers to that method where all types of benefit where all types of benefits that is profit, income or earning and ~~salvage~~ Salvage value will be assigned with a +ve sign and all types of the cost that is spending, expenditure, payment and investment will be assigned ~~with~~ with -ve sign

• Cost Based Method:

Refers to that method where all types of benefit will be assigned with -ve sign and all types of cost will be assigned with +ve sign

→ Revenue Based Method:

$$\begin{aligned}
 &\rightarrow E \rightarrow convert F to P \\
 &= -P + R_1 \left[ \frac{1}{(1+i)^1} \right] + R_2 \left[ \frac{1}{(1+i)^2} \right] \dots + R_n \left[ \frac{1}{(1+i)^n} \right] \\
 &\quad + S \left[ \sum_{i=1}^n \frac{1}{(1+i)^i} \right]
 \end{aligned}$$

$\uparrow S$   
 $\uparrow R_i$   
 $\uparrow$   
 $P = \text{Initial Investment}$

Equal payment Series:

$$\Rightarrow -P + R \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] + S \left[ \frac{1}{(1+i)^n} \right]$$

Cost Based Method:

a) Different Series

to get the value based on cash inflow

$$\text{Net利} = P + C_1 \left[ \frac{1}{(1+i)^1} \right] + C_2 \left[ \frac{1}{(1+i)^2} \right] + \dots + C_n \left[ \frac{1}{(1+i)^n} \right] - S \left[ \frac{1}{(1+i)^n} \right]$$

P = Initial Amount

b) Equal payment Series

~~$$\text{Net利} = P + C \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] - S \left[ \frac{1}{(1+i)^n} \right]$$~~

From the following table find out, project will be selected

or not, on the basis of present worth method, if  $i=14\%$  compounded annually,

Year End

Cash flow

0	-80,000	→ construction
1	25,000	→ construction
2	37,000	...
3	40,000	...
4	48,000	...

every year cashflow → ~~Revenue~~ ~~Board met.~~

just to show cash outflow

$$= -P + R_1 \left[ \frac{1}{(1+i)^1} \right] + \dots + S \left[ \frac{1}{(1+i)^n} \right] \quad S=0$$

$$= -80,000 + 25,000 \left[ \frac{1}{(1+14)^1} \right] + \dots + 48,000 \left[ \frac{1}{(1+14)^4} \right]$$

$$\text{NPW}(14\%) = \underline{\underline{+85818 - 877}} \quad 25818.837$$

NPW(14%) =

$$\text{NPW}(14\%) = -80,000 + \dots + 25818.837$$

$$\text{NPW}(14\%) \geq 0 \quad \underline{\underline{\text{Selected}}} \quad \text{selected}$$

$$\text{NPW}(14\%) = 25818.837$$

Q) From the following table find out project is financially feasible or not on the basis of present worth method if  $i = 12\%$ .

Year End	Cashflow
0	-90,000
1	40,000
2	40,000
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	2040,000

$$\Rightarrow -P + R \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] + S \left[ \frac{1}{(1+i)^n} \right] \quad S=0$$

$$\Rightarrow -90,000 + 40,000 \left[ \frac{(1+0.12)^{20} - 1}{0.12(1+0.12)^{20}} \right]$$

$$= 2,08,777.745 \quad \text{as } NPW(12\%) \geq 0 \\ \hookrightarrow \text{feasible}$$

Q) From the following table find out which technology will be selected on the basis of present worth method if  $i = 16\%$  compounded annually.

Technology	Initial Outlay	Annual Income	Life in years
1	10,00,000	5,00,000	15
2	18,00,000	7,00,000	15
3	16,00,000	6,00,000	15

Technology 1:

$$-P + R \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] + S \left[ \frac{1}{(1+i)^n} \right] \quad S=0$$

$$-10,00,000 + 5,00,000 \left[ \frac{(1+0.16)^{15} - 1}{0.16(1+0.16)^{15}} \right] + 0 \left[ \frac{1}{(1+0.16)^{15}} \right] \quad S=0$$

$$= 17,877,28.081$$

Technology 2:

$$= 21,02,819.314$$

$$\underline{\text{Technology 3:}} \quad 17,45,273.698$$

Technology 2 will be selected.

Q) From the following table find out which machine will be selected on the basis of present worth if  $i=13\%$ . compounded annually.

Machine	Initial Cost	Service life (in years)	Annual Operation and maintenance	Sale value
A	6,00,000	20	35,000	15,000
B	7,00,000	20	40,000	12,000

$$P + C \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] - S \left[ \frac{1}{(1+i)^n} \right]$$

$$A \Rightarrow 796322.6635 \rightarrow \text{cost low} \Rightarrow \text{Machine A will be selected}$$

$$B \Rightarrow 924591.3388 \rightarrow \text{cost high}$$

→ Future Worth Method:

In case of 1 project.

$$NFW(i\%) = FW(B) - FW(c)$$

If  $NFW(i\%) > 0$ , Project will be selected

If  $NFW(i\%) < 0$ , " " " rejected.

If  $NFW(i\%) = 0$ , " may or may not be selected.

In case of mutually exclusive projects

a) Revenue Based Method

i) Different Series

$$NFW(i\%) = -P(1+i)^n + R_1(1+i)^{n-1} + R_2(1+i)^{n-2} + \dots + R_n + S^F$$

ii) Equal-Payment Series

$$NFW(i\%) = -P(1+i)^n + R \left[ \frac{(1+i)^n - 1}{i} \right] + S$$

b) Cost Based Method

i) Different Series

$$NFW(i\%) = P(1+i)^n + C_1(1+i)^{n-1} + C_2(1+i)^{n-2} + \dots + C_n - S$$

ii) Equal Payment Series

$$NFW(i\%) = P(1+i)^n + C \left[ \frac{(1+i)^n - 1}{i} \right] - S$$

Q) From the following table find out which alternative will be selected on the basis of future worth if  $i=13\%$ . compounded annually.

Particulars	Alternative A	Alternative B
-------------	---------------	---------------

Initial Cost	4,00,000	6,00,000
--------------	----------	----------

Uniform annual benefit	64,000	96,000
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Life in years	15 <span style="float: right;">for A</span>	15 <span style="float: right;">for B</span>
	85009.57333	127514.36

Q) From the following table find out which machine will be selected on the basis of future worth method if  $i = 11\%$ .

Particulars	Machine 1	Machine 2	Machine 3
Initial Investment	80,00,000	70,00,000	90,00,000
life in years	17	17	17
Annual operation and maintenance	8,00,000	9,00,000	8,50,000
Salvage	5,00,000	4,00,000	7,00,000

$$\text{NFW}(i) = P(1+i)^n + C \left[ \frac{(1+i)^n - 1}{i} \right] - S$$

$$= -P \left[ \frac{1}{(1+i)^n} \right] + C \left[ \frac{1}{(1+i)^n} \right] - S$$

$$\text{NFW}(11\%) = -1.082261415.91$$

$$\text{NFW}(11\%) = -11.080916407.48$$

$$\text{NFW}(11\%) = -11.0890121550.76$$

(Q) Which alternative will be selected on the basis of future worth method if  $i = 12\%$  compounded annually.

Particulars	Alternative 1	Alternative 2
First cost	15,00,000	20,00,000
Annual Property tax	70,000	90,000
Annual Income	5,00,000	7,00,000
life in years	15	15
Net annual income	4,30,000	6,10,000

$$\text{NFW}(i) = -P(1+i)^n + R \left[ \frac{(1+i)^n - 1}{i} \right] + S$$

$$1: 7819929.665$$

$$2: 11793499.042$$

Annual Worth Method / Annual Equivalent Method:

In case of one project:

$$NAW(i\%) = Aw(B) - Aw(C)$$

• if  $NAW(i\%) > 0$ , Project will be selected

" "  $< 0$ , " " not be "

" "  $= 0$ , Project may or may not be selected.

In case of mutually exclusive projects/more than 1

a) Equal-Payment series:

i) Revenue Based Method:

$$NAW(i\%) = -P \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right] + R + S \left[ \frac{1}{(1+i)^n - 1} \right]$$

ii) Cost Based Method:

$$NAW(i\%) = P \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right] + C - S \left[ \frac{1}{(1+i)^n - 1} \right]$$

b) Different Series:

$$NAW(i\%) = NPW \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

Q) From the following Table, find out which technology will be selected on the basis of present worth method if  $i = 18\%$ .

Particulars	Tech A	Tech B
• First Cost	5,00,000	7,00,000
• End of Year	Cashflow Year End	Cashflow Year End
1	10,000	15,000
2	20,000	30,000
3	30,000	0
4	45,000	0

→ Different Series:

$$NAW(i\%) = NPW \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

Tech A

$$NPW(i\%) = -P + R_1 \left[ \frac{1}{(1+i)^1} \right] + R_2 \left[ \frac{1}{(1+i)^2} \right] + R_3 \left[ \frac{1}{(1+i)^3} \right] + R_4 \left[ \frac{1}{(1+i)^4} \right]$$

$$= -435692.309 \Rightarrow -4,35,692.309 \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right] \\ = -1,61,963.69$$

Tech B

$$NPW(i\%) = -665742.6027$$

$$\downarrow \\ -2,42,482.22 - 4,25,020.1835$$

Q1) From the following table find out which machine selected on the basis of annual equivalent method if  $i = 20\%$ . compounded annually

Machine	Down Payment	Yearly equal installment
1	5,00,000	2,00,000
2	4,00,000	3,00,000
3	6,00,000	1,50,000

Name: Vaibhav Sharma, Roll: 22053123

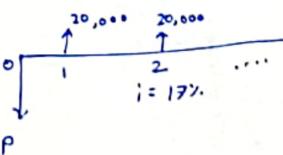
Q2) A person needs 7,00,000 INR after 3 years. Find out how much money the person has to deposit now to get 7,00,000 INR after 3 years if the interest rate is 8% compounded annually.

Q3) A person invest an equal amount of 25,000 INR at the end of every year, starting from the end of next year. Find maturity amount of his account after 12 years, if the interest rate is 7.5% compounded annually.

Q4) A company has taken a loan of 20,00,000 INR. Find out the installment amount the company has to pay if interest rate is 12% compounded annually and no. of installments is 10.

Q5) A person plans to invest an equal amount of 30,000 INR starting in the first year with an annual decrease of 500 INR for next 10 years. Find out total amount of the above sum after 11 years if the interest rate is 10% compounded annually.

Q6) Solve the following



$$P = \frac{20,000}{0.17} \left[ \frac{1 - (1 + 0.17)^{-25}}{0.17} \right] = 103,32$$

$$1) F = 7,00,000 \text{ INR}$$

$$n = 3$$

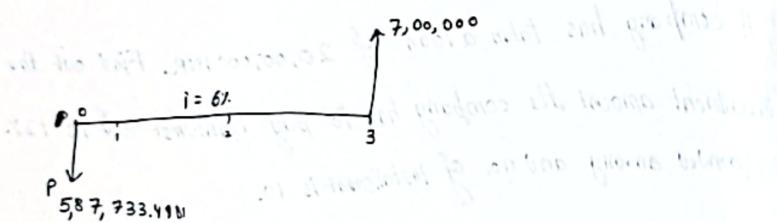
$$\text{Interest} = 6\%$$

$$P=?$$

$$P = \frac{F}{(1+i)^n}$$

$$= \frac{7,00,000}{(1+0.06)^3}$$

$$P = 587733.4981 \text{ INR}$$



$$2) A = 25,000 \text{ INR}$$

$$n = 12 \text{ years}$$

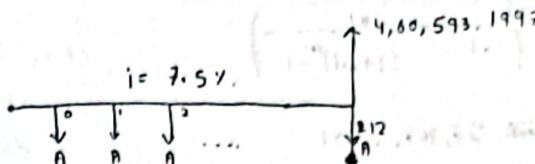
$$i = 7.5\%$$

$$F=?$$

$$F = A \left[ \frac{(1+i)^n - 1}{i} \right]$$

$$= 25,000 \left[ \frac{(0.075+1)^{12} - 1}{0.075} \right]$$

$$F = 460593.1997 \text{ INR}$$



$$A = 25,000$$

$$3, P = 20,00,000 \text{ INR}$$

$$i = 12\%$$

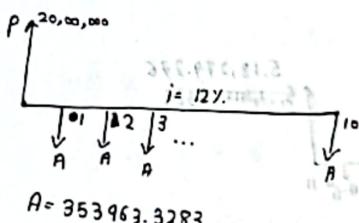
$$n = 10$$

$$A = ?$$

$$A = P \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

$$= 20,00,000 \left[ \frac{0.12(1+0.12)^{10}}{(1+0.12)^{10} - 1} \right]$$

$$A = 353963.9283 \text{ INR}$$



$$A = 353963.9283$$

$$4) A_1 = 35,000 \text{ INR}$$

$$G = 500$$

$$n = 10$$

$$i = 10\%$$

$$A = A_1 - G \left[ \frac{1}{i} - \frac{1}{(1+i)^n - 1} \right]$$

$$A = 30,000 - 500 \left( \frac{1}{.1} - \frac{1.1^n}{(1+1)^n - 1} \right)$$

$$A = 28,137.269 \text{ INR } 27,967.97281$$

Now, P/A

Total amount after 11 years.

$$\Rightarrow A = 28,137.269 \text{ INR } 27,967.97281$$

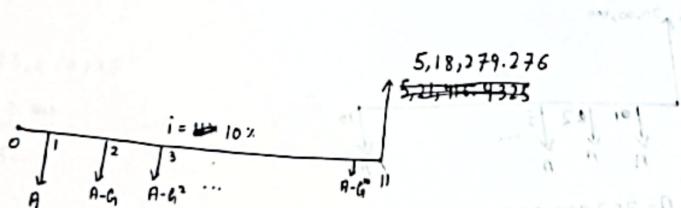
$$i = 10\%, n = 11$$

$$F = ?$$

$$F = A \left[ \frac{(1+i)^n - 1}{i} \right]$$

$$= A \left[ \frac{(1.1 + 1)^{11} - 1}{.1} \right] \text{ INR } 518,279.276$$

$$F = 518,279.276 \text{ INR}$$



5. ~~P?~~? From the following diagram, we can deduce that:

$$A = 20,000$$

$$i = 17\%$$

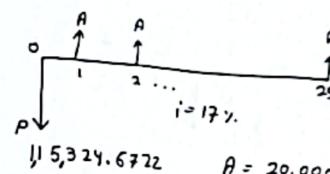
$$n = 25$$

$$P = ?$$

$$P = P \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$= 20,000 \left[ \frac{(1.17 + 1)^{25} - 1}{.17 (1 + .17)^{25}} \right]$$

$$P = 115,324.6722$$



## Rate of Return Method

### • Types of return:

i) ~~Minimum Acceptable Rate of Return~~

ii) Minimum Attractive Rate of Return /

Minimum Acceptable Rate of Return (MARR)

iii) Net Present Value (NPV)

iv) Internal Rate of Return (IRR)

### I. Minimum Attractive Rate of Return

Refers to the lower limit of project acceptability beyond which if rate of return falls, project will be rejected.

### II. Net present Value (Same as Present Worth)

Refers to the sum of all the ~~present~~ present values from a future stream of benefit during the lifespan of the project.

### III. Internal Rate of Return (IRR)

Refers to that Rate of Return which equates <sup>worth</sup> present value of benefit to <sup>worth</sup> present value of cost.

$$\therefore PW(B) = 0 \quad PW(C)$$

$$NPW = 0$$

$$ex: i = 12\%$$

$$P = 10,00,000$$

$$R = 80,000$$

$$\text{Suppose, } NPW(12\%) = 0$$

$$12\% = IRR$$

if  $NPW \neq 0$

$IRR = \text{Interest Rate of last +ve result} + \text{Diff. b/w two}$

i) If 1<sup>st</sup> try = +ve,

Take interest rate more

if 1<sup>st</sup> try = -ve,

Take interest rate less

$$\text{ex: MARR} = 10\%$$

$$P = 5,00,000$$

$$NPW(10\%) = 6000$$

$$\underline{NPW(12\%) = 2000}$$

$$\underline{NPW(15\%) = -1000}$$

$$IRR = 12\% + 3\% \left[ \frac{2000 - 0}{2000 - (-1000)} \right] \times 100\%$$

$$= 12\% + 3\% \left[ \frac{2000 - 0}{2000 + 1000} \right] \times 100\%$$

$$\left[ \frac{1}{(1+i+1)} \right]^2 + \left[ \frac{100,000}{(1+i+1)^3} \right] = 0 - 12\%$$

$$\left[ \frac{1}{(1+i+1)} \right]^3 + \frac{100,000}{(1+i+1)^4} = 0 - 12\%$$

## Selection or Rejection of Project

i) if  $IRR > MARR$  : Project Selected

ii) if  $MARR > IRR$  : Project Rejected

iii) if  $MARR = IRR$  : Project may or may not be selected

Q) From the following information, find out the investor should go with the new business or not with the help of Rate of Return,

if  $MARR = 10\%$ .

Year	Cashflow
0	-1,00,000
1	30,000
2	30,000
3	30,000
4	30,000
5	30,000

$$\Rightarrow -P + R \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] + S \left[ \frac{1}{(1+i)^n} \right]$$

$$\Rightarrow -1,00,000 + 30,000 \left[ \frac{(1+1)^5 - 1}{(1+1)\cdot 1} \right] = \underline{\underline{12,000}} \\ 13,723.6030$$

$$at i = 10\% \Rightarrow -ve$$

$$NPW(10\%) = 13,723.6030$$

$$(11\%) = 10,876.91$$

$$(12\%) = 8143.28$$

$$(13\%) = 2992.42$$

$$(14\%) = 564.6526$$

$$(15\%) = -1771.191$$

Now,

$$IRR = 15\% + 1\% \cdot \left[ \frac{564.656 - 0}{564.656 - (-1771.191)} \right]$$

$$= 15\% + 1\% \cdot \left[ \frac{564.659 - 0}{564.656 + 1771.191} \right]$$

$$= 15.24\%$$

as  $IRR > MARR$ , Yes

Q.) A company is trying to diversify its business in a new product line, life of the project is 10 years with no salvage value at the end of its life. Initial outlay of project is 20,00,000 INR with annual equal return of ~~3,00,000~~ 3,50,000 INR. Find Rate of Return for new business.

Since MARR is not given, take any  $i$ , for which it is final

for  $i = 10\%$ .

$$= -P + R \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$\text{for } i = 11\% = 61,231.203$$

$$i = 12\% = -22,421.940$$

$$\Rightarrow 11\% + 1\% \left[ \frac{61,231.203}{61,231 + 22,421.940} \right] = 12\%$$

$$i = \cancel{0.117} \quad 0.117$$

$$= 11.7\%$$

a) From the following table find out rate of return for all alternatives and find out which alternative will be selected on the basis of ROR method, if MARR = 12%.

Particulars	Alt. A <sub>1</sub>	Alt. A <sub>2</sub>	Alt. A <sub>3</sub>
Investment	1,50,000	2,10,000	2,55,000
annual net income	45,570	58,260	69,000
life in years	5	5	5

MARR = 12%

$$= -P + R \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$\geq -1,50,000 + 45,570 \left[ \frac{(0.12+1)^5 - 1}{(0.12)(0.12+1)^5} \right]$$

$$= 14,269.651$$

$$at = 10\% \Rightarrow 6445.499$$

$$15\% = 2757.707$$

$$16\% = -790.438$$

$$IRR = 15\% + \frac{1}{15\% - 14,269.651} \cdot \frac{14,269.651}{14,269.651 + 790.438}$$

$$= 0.529$$

$$\geq 15\% + 1\% \left[ \frac{2757.707}{2757.707 + 790.438} \right]$$

$$= 0.157$$

AH2  $12\%$

$$-2,10,000 + 58260 \left[ \frac{(1+0.12)^5 - 1}{0.12(1+0.12)^5} \right]$$

$$= 14.261$$

$$13\% = -5086.106$$

$$\text{IRR} = 12\% + 1\% \left[ \frac{14.261}{14.261 + 5086.106} \right]$$

$$= 0.12$$

AH3  $12\%$

$$\text{IRR} = 11\%$$

$A_1$  will be selected as ROR is more

## Benefit cost Analysis

• Benefit Cost Ratio ( $B/C$ )

$$\frac{B}{C} = \frac{\text{PW}(B)}{\text{PW}(C)} = \frac{\text{Present value Benefit}}{\text{Present value of cost}}$$

### Selection or Rejection of the project

If  $\frac{B}{C} > 1$ : Selected

If  $\frac{B}{C} < 1$ : Rejected

If  $\frac{B}{C} = 1$ : May or may not be selected.

a) From the following table find out which project will be selected on the basis of benefit cost analysis.

Project	PW(B)	PW(C)
1	60,00,000	40,00,000
2	80,00,000	20,00,000
3	90,00,000	35,00,000

$$\left(\frac{B}{C}\right)_1 = \frac{\text{PW}(B)}{\text{PW}(C)} = 1.5$$

$$1 = 1.5$$

$$2 = 24$$

$$3 = 2.57 | 42857143$$

2 will be selected.

Q) In a particular locality of space state, the vehicle users take a round about route to reach certain places because of presence of a river. It results in excessive travel time and increased fuel cost. So, the state govt. is planning to construct a bridge across the river. The estimated initial investment for constructing the bridge is 40,00,000. Estimated life of bridge is 15 years, annual maintenance and operation cost is 1,50,000 INR. The value of fuel savings due to construction of bridge is 60,00,000 INR in 1<sup>st</sup> year and increases by 50,000 every year thereafter till the end of the life of the bridge; check whether the project is justified based on B/C ratio by assuming an interest rate of 12% compounded annually.

$$A_1 = 6,00,000$$

$$G = 50,000$$

$$A = A_1 + G \left[ \frac{1}{i} + \frac{n}{(1+i)^n - 1} \right]$$

$$\text{Pw}(C) = P + C \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

if initial and cost given

$$\text{Pw}(B) = \frac{\text{Pw}(C)}{1+i} \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$A_1 = 6,00,000 + 50,000 \left[ \frac{1}{0.12} + \frac{15}{(1+0.12)^{15}} \right]$$

$$= 849,015.168 \text{ INR}$$

$$\text{Pw}(B) = 8,49,015.168 \left[ \frac{(1+0.12)^{15} - 1}{(0.12)(0.12+0.12)^{15}} \right]$$

$$= 5,782,527.266$$

~~$$\text{Pw}(C) = 40,00,000 + 150,000 \left[ \frac{(1+0.12)^{15} - 1}{0.12(1+0.12)^{15}} \right]$$~~

$$= 5,021,629.673$$

$$\Rightarrow \frac{\text{Pw}(B)}{\text{Pw}(C)} = \frac{5,782,527.266}{5,021,629.673}$$

$$= 1.15$$

Project justified as ~~Pw(C)~~ B/C is > 1

Q) The state govt. is planning a hydro electric project in addition to the production of electric power the project will provide flood control, irrigation and recreation. Estimated benefit and cost given as follow.

$$\text{Initial cost} = 8,00,00,000 \text{ (c)}$$

$$\text{Annual Power sales} = 60,00,000 \text{ (B)}$$

$$\text{Annual flood control saving} = 30,00,000 \text{ (B)}$$

$$\text{Annual Irrigation Benefit} = 50,00,000 \text{ (B)}$$

$$\text{Annual Recreation Benefit} = 20,00,000 \text{ (B)}$$

$$\text{Annual operating and maintenance cost} = 30,00,000 \text{ (C)}$$

$$\text{Life of project} = 50 \text{ years}$$

$$i = 15\%$$

~~if~~

total annual Benefit = , then convert to P

$$\text{Total annual benefit} = 16,000,000$$

$$P(W(C)) = 8,00,00,000 + 30,00,000 \left[ \frac{(1+0.15)^{50} - 1}{0.15(1+0.15)^{50}} \right]$$

$$= \cancel{8,00,00,000} \quad 99,981,543.983$$

$$P(W(B)) = 16,00,00,000 \left[ \frac{(1+0.15)^{50} - 1}{0.15(1+0.15)^{50}} \right] \\ = 106,568,234.576$$

$$B/C = \frac{106,568,234.576}{99,981,543.983} = 1.06$$

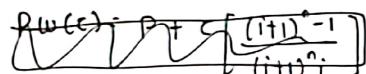
= > 1 project will be selected

Q.) Two mutually exclusive projects are being considered for investment. Project A1 requires an initial outlay of 30,00,000 INR with net receipts of 9,00,000 INR per year, for the next 5 years. The initial outlay for project A2 is 60,00,000 INR with net receipts of 15,00,000 INR per year for next 7 years. Using B/c ratio find out which project will be selected if  $i=10\%$  compounded annually

$$\text{A1} \\ \text{Pw}(c) = \frac{I}{C}$$

Cost = 30,00,000 INR (already in present)

Benefit = 9,00,000 INR  $\rightarrow$  convert to (P)



$$\text{Pw}(B) = 9,00,000 \left[ \frac{(1+1)^5 - 1}{(1+1)^5 \cdot 1} \right] \\ = 3411708.092$$

B/C =

$B/C = 1.137$

A2

Cost = 60,00,000 INR

$$\text{Pw}(B) = 15,00,000 \left[ \frac{(1+1)^7 - 1}{(1+1)^7 \cdot 1} \right] \\ = 7302628.226$$

$B/C = 1.2$

### Pay-Back Period Method

#### Equal Return

Gets equal amount of return every year.

$$P = \frac{I}{C}$$

I  $\rightarrow$  Initial Investment

C  $\rightarrow$  Cash Inflow (Yearly equal)

P  $\rightarrow$  Pay back year

Q.) An investor has invested 50,00,000 INR. Find out how many years it'll take for investor to get back his money with the help of Equal Return method, if the estimated return is 4,00,000 INR every year.

$$P = \frac{I}{C} = \frac{50,00,000}{4,00,000} = 12.5 \text{ years} = 13 \text{ years}$$

#### In case of different amount of return

The remaining amount  $\times 12$

Next Cash Inflow

Investment = 5,00,000

ex: 1  $\rightarrow$  1,00,000  $\rightarrow$  4,00,000 (5,00,000 remain)

2  $\rightarrow$  2,50,000  $\rightarrow$  350,000 (1,50,000 remain)

3  $\rightarrow$  4,20,000  $\rightarrow$  7,70,000

4  $\rightarrow$  5,00,000

$$\Rightarrow \frac{1,50,000}{4,20,0000} \times 12 \Rightarrow 4.28 \Rightarrow 4.3 \Rightarrow 4 \text{ years } 4 \text{ months}$$

Q.) One investor has invested 10,00,000 INR on a project. The investor wants to purchase a machine for which there are two machines available in the market. Find out which machine will be selected with the help of payback period method.

End of Year	Cashflow from Machine 1	Cashflow from Required machine 2
1	1,00,000	2,00,000
2	1,00,000	4,00,000
3	5,00,000	4,50,000
4	7,00,000	5,60,000
5	8,00,000	7,00,000
Total		

### Machine 1

$$\frac{1,00,000}{7,00,000} \times 12 = 1.71 \Rightarrow 3 \text{ years } 2 \text{ months}$$

### Machine 2

$$\frac{4,00,000}{4,50,000} \times 12 = 10.66 \Rightarrow 2 \text{ years } 11 \text{ months}$$

As machine 2 requires less time than machine 1, machine 2 will be selected.

### Depreciation

Refers to the loss of value in any asset due to its constant use.

#### Types of method

i) Straight line method.

ii) Declining balance method.

iii) Sum-of-the-year digit method.

iv) Sinking fund method.

#### Straight line Method

Refers to that method of depreciation, where a fixed rate of depreciation is charged directly on the initial value of the asset for various years.

##### Depreciation amount per year (D)

$$D = \frac{I-S}{N}$$

I = Initial value of the asset / Purchase price

S = Salvage value / Reduced value of asset

N = life of asset in Years.

##### Rate of Depreciation (d):

$$d = \frac{D}{I} \times 100$$

iii) Book Value ( $B_t$ ),

$$B_t = I - (t \times D)$$

$t$  = No. of years the asset has been used.

or

$$B_t = B_{t-1} - D_t$$

Declining Balance Method:

It refers to that method of depreciation where a fixed rate of depreciation will be charged on the declining balance every year.

$$D_t = K \times B_{t-1}$$

$K \rightarrow$  A fixed rate of depreciation

$$B_t = B_{t-1} - D_t$$

Q: If a machine has been purchased at 90,000 INR with estimated service life of 10 years, find out depreciation amount and Book Value for various years if rate of depreciation is 30%, with the help of Declining-Balance Method

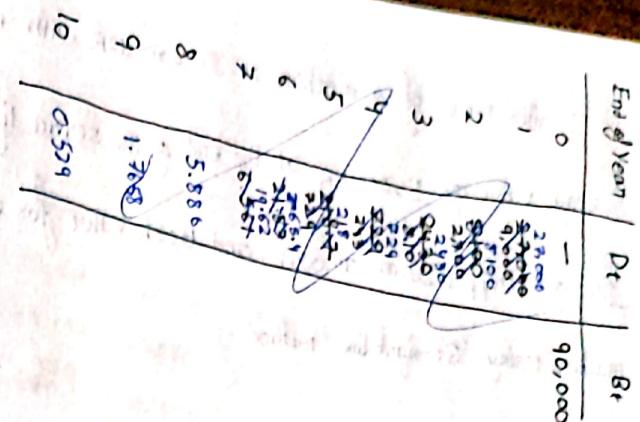
$$I = 90,000$$

$$N = 10$$

$$K = 30\%$$

$$D = \frac{I-S}{N} = \frac{90,000}{10} = 9,000$$

(NexPay)



Q) If an equipment has been purchased at 80,000 INR with estimated salvage value of 8,000 INR at the end of its service life of 10 years, find out rate of Depreciation and Book value of its after 5 years, with the help of straight line method.

$$I = 80,000$$

$$S = 8,000$$

$$N = 10$$

$$D = \frac{I-S}{N} = \frac{80,000 - 8,000}{10} \Rightarrow 7,200$$

$$d = \frac{D}{I} \times 100$$

$$= \frac{7,200}{80,000} \times 100 = 9\%$$

$$B_t = I - (t \times D)$$

$$= 80,000 - (5 \times 7,200)$$

$$= 44,000$$

Q. If initial value of a machine is 50,000 INR with estimated Salvage value of 5,000 INR at the end of its service life of 8 years find out Depreciation amount and Book value for every year of machine, using Straight line method

End of Year	D	B <sub>t</sub>
0	—	50,000
1	5625	44375
2	5625	38750
3	5625	33125
4	5625	27500
5	5625	21875
6	5625	16250
7	5625	10625
8	5625	5000

$$D = \frac{50,000 - 5,000}{8} = 5625$$

BS Q1.2

End of Year	D <sub>t</sub>	B <sub>t</sub>
0	—	90,000
1	27,000	63,000
2	18,900	44,100
3	13,230	30,870
4	9,261	21,609
5	6,482	15,0126.3
6	4,537.887	10,588.41
7	3,176.523	7,941.887
8	2,222	5,188.321
9	1,556.496	6,3631.824
10	1,089.54	25,42177

→ Sum of the year digit Method

$$\text{i) Sum of the year} = \frac{N(N+1)}{2}$$

$$\text{ii) } D_t = \frac{\text{Rate}}{\text{Sum of the year}} \cdot D$$

$$\text{Rate} = \frac{\text{Rank}}{\text{Sum of the Year}}$$

$$\text{iii) } B_t = B_{t-1} - D_t$$

if total year = 5, highest rank = 5  
Since for 1<sup>st</sup> year, Value = highest,  
rank = 5, for 2<sup>nd</sup>, Rank = 4

Q2 If an equipment has been purchased with 1 lakh INR with estimated salvage of 20,000 at the end of its service life of 8 years, find out depreciation amount and Book-value, using

Sum of the years' digits =

$$I = 1,00,000 \text{ INR}$$

$$S = 20,000$$

$$N = 8$$

$$D_t = \text{Rate} (I - S)$$

$$\text{Rate} = \frac{\text{Rank}}{\text{Sum of years}}$$

$$\Rightarrow \text{Sum of the years} = \frac{8(8+1)}{2} = 36 \quad B_t = B_{t-1} - D_t$$

End of Year	Rank	D <sub>t</sub>	B <sub>t</sub>
0	-	-	1,00,000
1	8	17777.778	82,222.222
2	7	15555.556	66,666.668
3	6	13333.333	53,333.333
4	5	11111.111	42,222.224
5	4	8888.888	33,333.336
6	3	6666.667	26,666.667
7	2	4444.444	22,222.226
8	1	2222.222	20,000.004

### Sinking Fund Method

$$\text{i) } A = (I - S) \left[ \frac{1}{(1+i)^{n-1}} \right]$$

$$\text{ii) } D_t = A (1+i)^{t-1}$$

$$\text{iii) } B_t = B_{t-1} - D_t$$

Q3 If an asset has been purchased at 90,000 INR with estimated salvage value of 5,000 INR at the end of its service life of 10 years. Find out depreciation amount and book value of asset every year with the help of sinking fund, when i = 12%.

$$\Rightarrow \text{End of Year} \quad D_t \quad B_t$$

$$A = 15,721.3$$

End of Year	A	D <sub>t</sub>	B <sub>t</sub>
0	4843.6539	4324.61	90,000
1	"	4324.61	85,673.38
2	"	50421.81	79,731.459
3	"	60751.71	73,655.375
4	"	6804.984	66,850.59
5	"	7621.513	59,229.007
6	"	8536.173	50,692.834
7	"	9560.513	41,132.32
8	"	10703.776	30,132.545
9	"	11992.708	18,434.837
10	"	13431.83	5000.003

### -Production

Refers to the process of physical transformation of input  $\times$  output.

### Production Function:

It refers to the functional relationship b/w input and output.

Let  $Q$  = Output

~~N~~ = Land  $\rightarrow$  Capital  $\rightarrow$  Labour  $\rightarrow$  Services

L = Labour

K = Capital

$$Q = f(N, L, K) \quad , \text{ if } N \text{ inputs then} \\ Q = f(x_1, x_2, x_3, \dots, x_n)$$

i) Law of variable production / Short-run theory of production with one variable input.

ii) Law of Returns to scale / long-run production theory

iii) Short-run production function with two variable input.

### Factors of production

i) Fixed Factor.

ii) Variable Factor. (No. of labour etc.)

### Fixed Factor

Factors which remains fixed. ex: Land, Building, Machine etc

• It refers to those factors which can't be changed during the process of production. Ex: Big machine, Plant size, land, Big Building.

### Variable Factor

• It refers to those factors which can be changed during the process of production. Ex: Labour, Raw materials.

### Production Time Period

i) Short Period / Short Run

ii) Long Period / Long Run (Nothing is fixed)

### Short Period

It refers to that time period of production, where fixed factors can't be changed but variable factors can be changed.

Long Period: Refers to that time period of production, where nothing is called fixed. All factors are variable factors.

### 3 concepts of Production

i) Total Product (TP)

ii) Marginal Product (MP)

iii) Average Product (AP)

#### - Total Product (TP):

• It refers to the total amount of production with a given amount of variable factors.

#### - Marginal Product (MP):

• It refers to the net addition to the total product by implying one extra unit of input.

$$MP_n = TP_n - TP_{n-1}$$

$$\text{ex: } MP_L = \frac{d(TP)}{dL} = \frac{dQ}{dL}$$

$L \rightarrow$  Labour

$K \rightarrow$  Capital

$$\text{ex: } MP_K = \frac{d(TP)}{dK} = \frac{dQ}{dK}$$

$$MP_x = \frac{d(TP)}{dx}$$

$x \rightarrow$  input

#### - Average Product (AP):

• It refers to the total production per unit of input used.

$$AP_L = \frac{TP}{L} = \frac{Q}{L}$$

$L \rightarrow$  Labour

$$AP_K = \frac{TP}{K} = \frac{Q}{K}$$

### Law of Variable Production (G. Stigler)

It is defined as "equal increments of one input are added, the i/p of other productive services being held constant, the resulting increment of product beyond a certain point decrease, i.e., marginal product will diminish."

#### - Assumption of the Law

i) Rate of technology remains constant

ii) There must be <sup>some</sup> inputs whose quantities can be fixed

Units of Labour used (L)	Total Product (Q)	MP	AP	Stage of operation
under utilization of Fixed Factors	80	-	80	Stage-I
	170	90	85	Increasing return
	270	100	90	
full utilization of Fixed Factors (optimum stage of production)	368	98	92	Stage-II
	430	62	86	Diminishing return
	480	50	80	
	504	24	72	
	504	0	63	
	490	-14	54.44	Stage-III
	485	-5	48.5	Negative return

Stage I: Increasing return: In this stage, as MP increases, TP also increases at a increasing rate

Stage II: Diminishing return: In this Stage, as MP increases, TP decreases at a diminishing rate

Shift

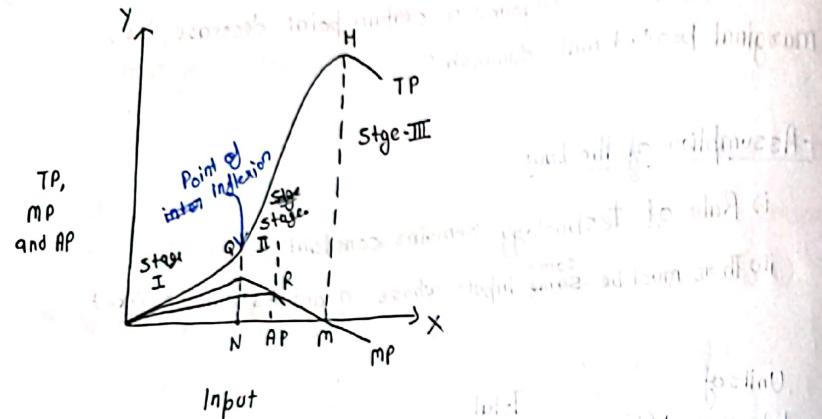
Z

X

C

Q. What is the optimal stage of production? (Last Page)

All factors are fixed and "factors in fixed" are considered  
productivity and efficiency. Total product of labour is increasing at decreasing rates of return.  
Second derivative of total product of labour is negative.



Q. From the following table find out MP, AP and also stage of production

Units of Labour	TP	MP	AP	Stage
1	30	-	30	Stage I
2	80	50	40	Stage I
3	140	60	46.666	Stage I
4	190	50	47.5	Stage II
5	190	0	38	Stage III
6	180	-10	30	Stage III

Q. From the following short run production fun.

- i) MP Fun.
- ii) AP Fun.

iii) The value of L at which the o/p will be max.

iv) At what level of L, AP will be max.

Short run production function, derived from the above production function

$$Q = 8L^2 - 0.4L^3$$

$$i) MP_L = \frac{dQ}{dL} = 16L - 1.2L^2$$

$$ii) AP_L = \frac{dQ}{L} = 8L - 0.4L^2$$

$$iii) MP_L = 0$$

$$16L - 1.2L^2 = 0$$

$$L = 16.67$$

$$L \approx 17$$

$$iv) MP = AP$$

$$16 - 16L - 1.2L^2 = 8L - 0.4L^2$$

$$L = 10$$

$$\frac{d(AP_L)}{dL} = 0 \Rightarrow 8 - 0.8L = 0$$

$$L = 10$$

$$\frac{d(MP_L)}{dL} = 0$$

Q. From the following short run production fun.

i) MP Fun.

ii) AP Fun.

iii) At what level of X, output will be max, and at what level of X, AP and MP will be max

iv) Output will be max when MP = 0

$$Q = 6x^2 - 0.5x^3$$

$$i) MP: \frac{dQ}{dx} = 12x - 1.5x^2$$

$$ii) AP: \frac{dQ}{x} = 6x - 0.5x^2 \text{ and AP is max when } x = 12$$

$$MP = AP \quad \frac{dQ}{x} = 0$$

$$\Rightarrow 12x - 1.5x^2 = 6x - 0.5x^2$$

$$\Rightarrow 6x = 0.5x^2, x = 12$$

$$MP \text{ max when } \frac{\partial}{\partial x} \frac{dMP}{dx} = 0$$

$$12 - 12x = 0$$

$$12 = 12x$$

$x = 1$ , at this level of  $x$ , MP is max.

### Law of Returns to Scale (Ex of short run production)

- It refers to the long run homogeneous production func, which discusses the relation b/w output and all the variable input.

$$Q = f(L, K)$$

: If we change production by  $1\tau$  times,

$$Q^* = f(LK, KK)$$

### Types of Returns to Scale

i) Increasing Returns to Scale

ii) Constant Returns to Scale

iii) Decreasing Returns to Scale

### Increasing Returns to Scale:

When the rate of change in output is more than the rate of change in input or doubling of input results in more than doubling of output, it is called increasing returns to scale.

### Constant Returns to Scale:

When the rate of change in output is equal to the rate of change in input or doubling of inputs result in doubling of output, it is called constant returns to scale.

### Decreasing Returns to Scale:

When the rate of change in output is less than the rate of change in input or doubling of input results in less than doubling of output, it is called decreasing returns to scale.

### Isoquant / Iso-Product Curve:

It refers to those curves, which shows various combination of two factors, producing equal level of output.

Combination	L	K	Q
A	1	22	200
B	2	17	200

If  $L$  increases

if  $K$  remains

constant

to same output

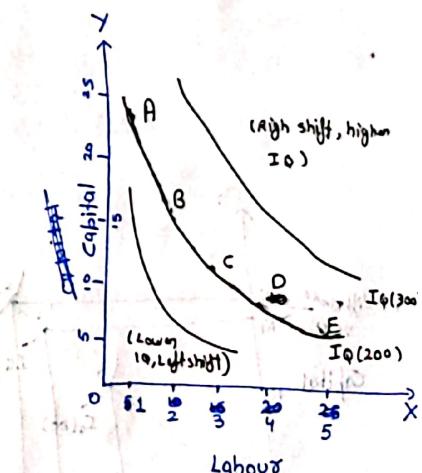
units of output

reduces

costs

and

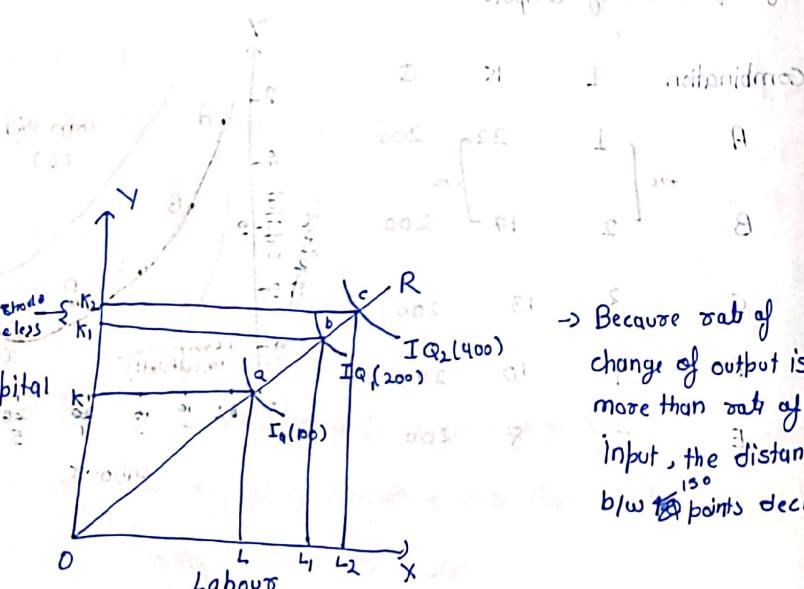
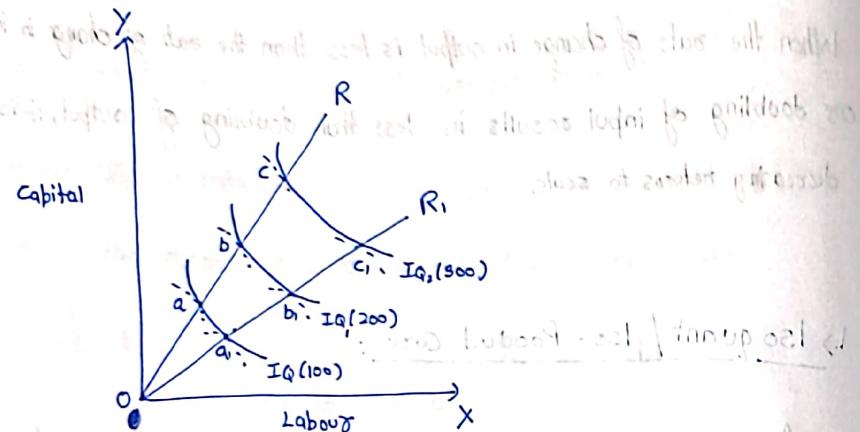
revenue



## II) Isocline: Iso-cline

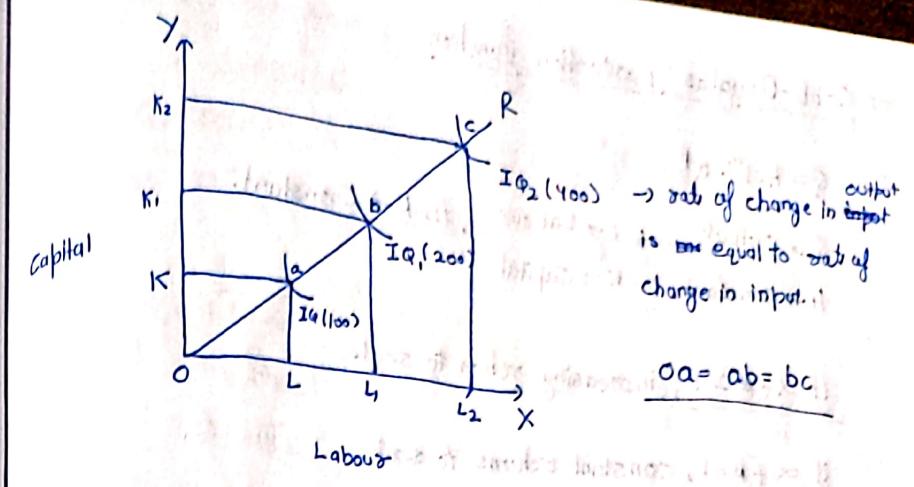
Refers to the line that shows us locus of various points on

different iso-quants.

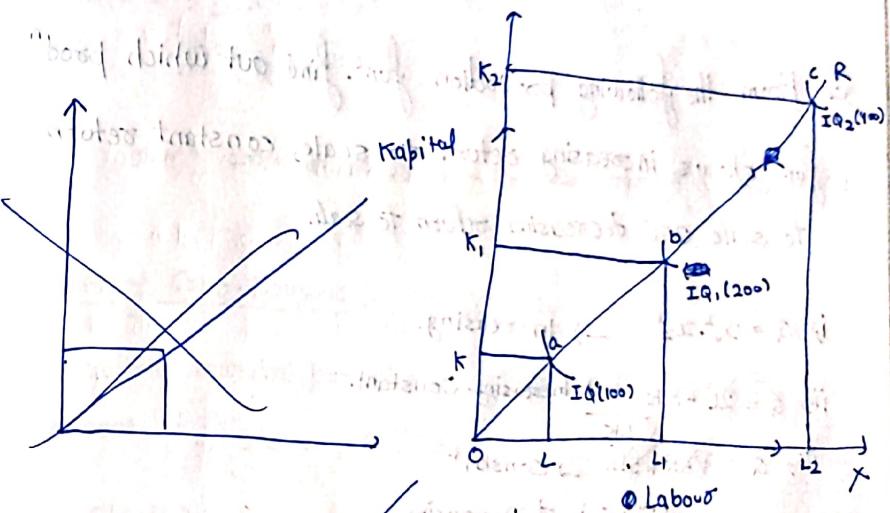


Because rate of change of output is more than rate of input, the distance b/w points declines.

↳ Increasing Returns to Scale



↳ Constants Returns to Scale.



↳ Decreasing Returns to Scale

rate of change in output is less than rate of change of input.

$oa < ab < bc$

↳ More distant from origin positive slope of isoquants.

## Cobb-Douglas production function.

$$Q = A \cdot L^\alpha \cdot K^\beta$$

$Q \rightarrow$  Output,  $L \rightarrow$  Labour,  $A, B \rightarrow$  Constant.

$A \rightarrow$  Constant,  $K \rightarrow$  Capital

If  $\alpha + \beta > 1$ , increasing return to scale

If  $\alpha + \beta = 1$ , constant returns to scale.

If  $\alpha + \beta < 1$ , decreasing returns to scale.

Q) From the following production func. find out which prod' fun' shows increasing return to scale, constant return to scale and decreasing return to scale.

i)  $Q = x_1^2 \cdot x_2^3 \rightarrow$  Increasing.

ii)  $Q = 2L + 3K \rightarrow$  Increasing Constant.

iii)  $Q = \sqrt{LK} \rightarrow$  Constant

iv)  $Q = 0.5 L^{0.2} \cdot K^{0.1} \rightarrow$  Decreasing

v)  $Q = x_1^2 \cdot x_2^3$ , let change input by  $\lambda$

$$Q_1 = (x_1 \lambda)^2 \cdot (x_2 \lambda)^3$$

$$= x_1^2 \cdot \lambda^2 \cdot x_2^9 \cdot \lambda^3$$

$$\Rightarrow \lambda^5 (x_1^2 \cdot x_2^3) \rightarrow Q_1 = \lambda^5 Q (Q = x_1^2 \cdot x_2^3)$$

as rate of output is more than rate of change in input., increasing.

$$\text{I. } Q_1 = 2AL + 3AK$$

$$= 2L + 3K$$

$$Q \neq Q(\lambda)$$

as rate of change in input is rate of change in output.

$$\text{II. } Q_1 = \sqrt{LK}$$

$$= \sqrt{\lambda^2 KL}$$

$$\Rightarrow \lambda \sqrt{KL}$$

$Q_1 = \lambda Q \rightarrow$  rate of change in input is rate of change in output

$$\text{IV. } Q_1 = 0.5 L^{0.2} \lambda^{0.2} \cdot K^{0.1} \lambda^{0.1}$$

$$= \lambda^{0.3} (Q)$$

rate of change in output is less than rate of change in input

## Properties of Cobb-Douglas production func.

i) The ratio of marginal product and average product gives its exponent. :  $\frac{MP_L}{AP_L} = \alpha$ ,  $\frac{MP_K}{AP_K} = \beta$

ii) The Elasticity of output is equal to the respective exponents.

$$\text{I. } Q = A \cdot L^\alpha \cdot K^\beta$$

$$\frac{dQ}{dL} = \alpha \cdot A L^{\alpha-1} \cdot K^\beta$$

$$= \alpha \cdot \frac{A L^\alpha \cdot K^\beta}{L}$$

$$= \alpha \cdot \frac{Q}{L}$$

$$MP = \alpha \cdot AP_L$$

$$\frac{MP_L}{AP_L} = \alpha \quad \text{prooved}$$

$$Q = A \cdot L^\alpha \cdot K^\beta$$

$$\frac{dQ}{dK} = \beta \cdot A \cdot L^\alpha \cdot K^{\beta-1}$$

$$\frac{dQ}{dK} = \beta \cdot \frac{A \cdot L^\alpha \cdot K^\beta}{K}$$

$$\frac{dQ}{dK} = \beta \cdot \frac{Q}{K}$$

$$MP_K = \beta \cdot AP_K$$

$$\frac{MP_K}{AP_K} = \beta$$

Prooved.

II

$$E_L = \frac{\text{Proportionate change in Outputs}}{\text{Proportionate change in Input.}}$$

$$= \frac{\frac{dQ}{Q} \times 100}{\frac{dL}{L} \times 100}$$

$$= \frac{\frac{dQ}{dL} \times \frac{L}{Q}}{\frac{dL}{L} \times 100} \times 100$$

$$= \frac{dQ}{dL} \times \frac{L}{Q}$$

$$E_L = MP_L \times \frac{1}{AP_L} \Rightarrow \frac{MP_L}{AP_L} = \alpha$$

$$\Rightarrow E_L = \alpha \quad (\text{Prooved})$$

$E_K = \frac{\text{Proportionate change in Output}}{\text{Proportionate change in Input}}$

$$= \frac{\frac{dQ}{Q} \times 100}{\frac{dK}{K} \times 100}$$

$$= \frac{\frac{dQ}{dK} \times K}{Q}$$

$$= MP_K \times \frac{1}{AP_K}$$

$$= \frac{MP_K}{AP_K}$$

$$E_K = \beta \quad \underline{\text{Prooved.}}$$

Q) From the following, find out marginal production func and short run production if fixed quantity of capital is 10000 units.

$$Q = L^{0.75} K^{0.25}$$

$\Rightarrow$

$$\frac{dQ}{dK} = MP_K = 0.25 L^{0.75} K^{-0.75} \quad \mid \frac{dQ}{dL} = 0.75 L^{-0.25} K^{0.25}$$

Short run production f^n:

$$Q = L^{0.75} (10000)^{0.25}$$

$$Q = 10 L^{0.75}$$

Q.) From the following, find out MP, AP and also find out elasticity of output w.r.t inputs.

$$Q = 1.50 L^{0.75} K^{0.25}$$

$$\Rightarrow MP_K = \frac{dQ}{dK} = 0.375(L^{0.75})(K^{-0.75})$$

$$MP_L = \frac{dQ}{dL} = 1.125(L^{-0.25})(K^{0.25})$$

$$AP_L = \frac{Q}{L} = 1.50(L^{-0.25})(K^{0.25})$$

$$AP_K = \frac{Q}{K} = 1.50(L^{0.75})(K^{-0.75})$$

→ Elasticity of Output:

~~Labor~~ = 0.75 (i.e. if one unit of labour is added)

↳ Capital = 0.25

w.r.t Labour

$$E_L = \frac{dQ}{dL} \times \frac{1}{AP_L}$$

$$= \frac{MP_L}{AP_L} = 0.75$$

$$E_K = \frac{MP_K}{AP_K} = 0.25$$

### Producer's Equilibrium

i) Iso quant / Iso Product Curve.

ii) Marginal Rate of Technical substitution (MRTS).

iii) Iso-Cost line.

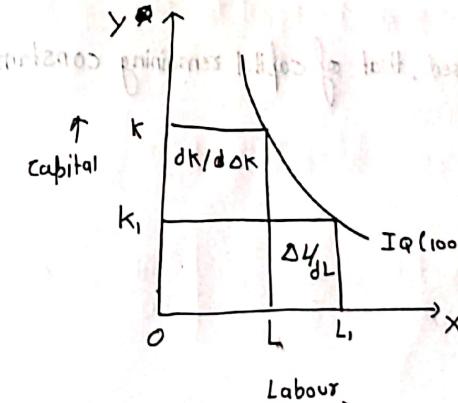
### MRTS:

The rate at which number of unit of 1 factor substituted to have one more unit of another factor.

i) MRTS<sub>LK</sub>: Refers to the rate at which the number of unit of Capital Substituted to have one more unit of labour.

ii) MRTS<sub>KL</sub>: Refers to the rate at which the number of unit substituted to have one more unit of capital.

### Slope of the iso-quant.

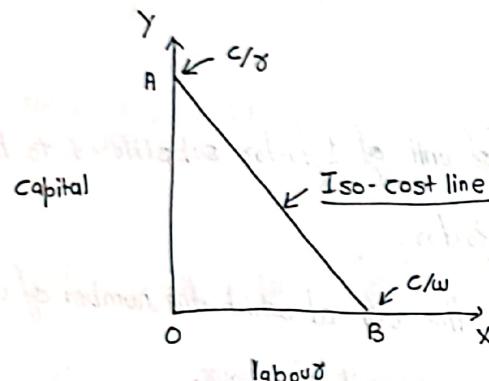


$$\text{Slope} \\ MRTS_{LK} = \frac{dK}{dL} = \frac{MP_L}{MP_K}$$

$$MRTS_{KL} = \frac{dL}{dK} = \frac{MP_K}{MP_L}$$

### III> Iso-Cost line:

It refers to the line, that shows us various combination of two factors, that a producer can buy with a given level of expenditure.



Slope of the iso-cost line:

$$\frac{OA}{OB} = \frac{C/\rho}{C/w} = \frac{w}{\rho}$$

### Eq'n of the iso-cost line

$$C = wL + \rho K$$

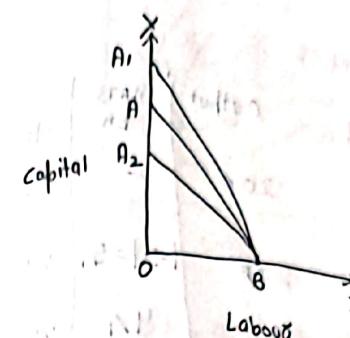
C → cost of Production

w → wage rate of the labour

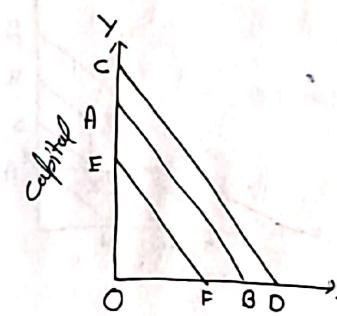
~~ρ~~ → Price of capital per unit

K → No. of unit of capital used

- Shift in iso-cost line, if units of capital used changes, that of labour remaining constant.



- Shift in the iso-cost line, if number of units of labour used and capital used changes.



Q) From the following find out  $MRTS_{LK}$ ,  $MRTS_{K_L}$

Combination	labour	Capital	Output	$MRTS_{LK}$	$K_L$
A	1	22	200	-	-
B	2	17	200	-5 = 5/1	1/5
C	3	13	200	4/1	1/4
D	4	11	200	2/1	1/2
E	5	10	200	1/1	1/1

$$MRTS_{LK} = \frac{\Delta \theta K}{\Delta \theta L} = \frac{MP_L}{MP_K}$$

$$MRTS_{K_L} = \frac{\Delta \theta L}{\Delta \theta K} = \frac{MP_K}{MP_L}$$

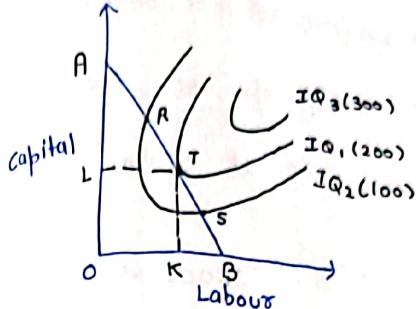
→ Condition for a producer to be in equilibrium.

i) Slope of the iso-quant should be equal to slope of the iso-cost line.

ii) The isoquant should be convex at equilibrium point.

$$MRTS_{LK} = \frac{\partial K}{\partial L} = \frac{MP_L}{MP_K} = \frac{w}{\gamma} \Rightarrow \frac{MP_L}{w} = \frac{MP_K}{\gamma}$$

### Profit Maximization

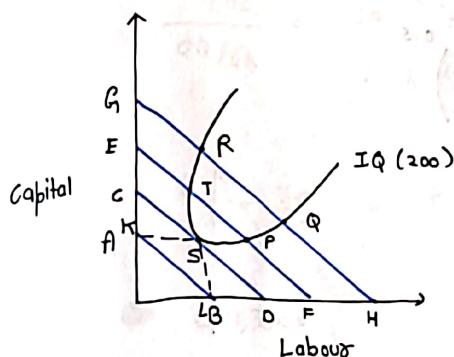


out of point R, T, S

As point T is lying on maximum IQ point (200), output ↑, profit will be maximum

$O^L$ ,  $O^K$  are optimal input combination.

### Cost Minimization



lying on the lowest point S gives minimum cost.

Q) From the following short run production fun'. find out optimal input combination, for producing 1500 unit of output if the wage-rate of the labour is 30INR and price of capital per unit is 40INR.

What is the minimum cost.

$$\text{cond for Optimal Input Combination: } \frac{MP_L}{MP_K} = \frac{w}{\gamma}$$

$w=30, \gamma=40$

$$Q = 100 \cdot L^{0.5} \cdot K^{0.5}$$

$$MP_L = \frac{dQ}{dL} = 50L^{-0.5} \times 0.5$$

$$MP_K = \frac{dQ}{dK} = 50 \cdot 0.5 L^{0.5}$$

$$\Rightarrow \frac{50L^{0.5}K^{0.5}}{50K^{0.5}L^{0.5}} = \frac{30}{40}$$

$$\text{or profit} = \frac{100}{L} \cdot \frac{30}{40} = \frac{30}{40}$$

Profit maximization & L minimum

$$\Rightarrow \frac{K}{L} = \frac{3}{4}$$

$$\Rightarrow K = \frac{3}{4}L$$

~~$$Q = 100 \cdot L^{0.5} \cdot K^{0.5}$$~~



$$1500 = 100 \cdot L^{0.5} \cdot \left(\frac{3}{4}L\right)^{0.5}$$

$$L = 17.320$$

$$L = 17$$

$$K = \frac{3}{4}L$$

$$= 12.75$$

$$K = 13$$



→ Eqn of isoquants

$$C = WL + \sigma K$$

$$C = 1030$$

$$\frac{10}{L} + \frac{40}{K} = 1030$$

zero minimum cost of output

$$\frac{10}{L} = \frac{40}{K}$$

$$10L = 40K$$

$$10L = 40K \Rightarrow L = 4K$$

a) From the following production fun' find Quantity of labour and capital, that the company should use in order to maximise output and also findout if cost of production is 1,300 INR wage rate is 30 and price of capital = 40.

$$Q = 100 \cdot L^{0.5} \cdot K^{0.5}$$

$$\rightarrow \text{Optimal input combination} \quad \frac{MP_L}{MP_K} = \frac{\omega}{\sigma}$$

$$\frac{dQ/dL}{dQ/dK} = \frac{50L^{-0.5}K^{0.5}}{50K^{-0.5}L^{0.5}} = \frac{K}{L}$$

$$\frac{K}{L} = \frac{30}{40} \Rightarrow K = \frac{3}{4}L$$

$$C = 1300$$

$$1300 = \omega L + \sigma K$$

$$1300 = 30L + 40 \cdot \frac{3}{4}L$$

$$1300 = 30L + 30L$$

$$1300 = 60L$$

$$L = 21.6$$

$$L = 22$$

$$K = \frac{3}{4}L$$

$$K = 16.5 \Rightarrow 17$$

$$Q = 100 \cdot L^{0.5} \cdot K^{0.5}$$

$$Q = 1933.90$$

$$Q = 1934$$

$$AC(A\bar{C}T\bar{C}) = \frac{T\bar{C}}{Q}$$

$$TC = TFC + TVC$$

$$AFC = \frac{TFC}{Q}$$

$$AVC = \frac{TVC}{Q}$$

$$AC = AFC + AVC$$

~~$$MC = TC - TC_{n-1}$$~~

$$MC_n = TFC + TVC_n - TFC - TVC_{n-1}$$

$$\boxed{MC_n = TVC_n - TVC_{n-1}}$$

### Break-Even Analysis:

It is that analysis which analyzes the behaviour of total revenue and total cost as the level of output changes

$$TR = P \times Q$$

$$TC = TFC + TVC$$

$$TVC = AVC \times Q$$

### Break-even Point

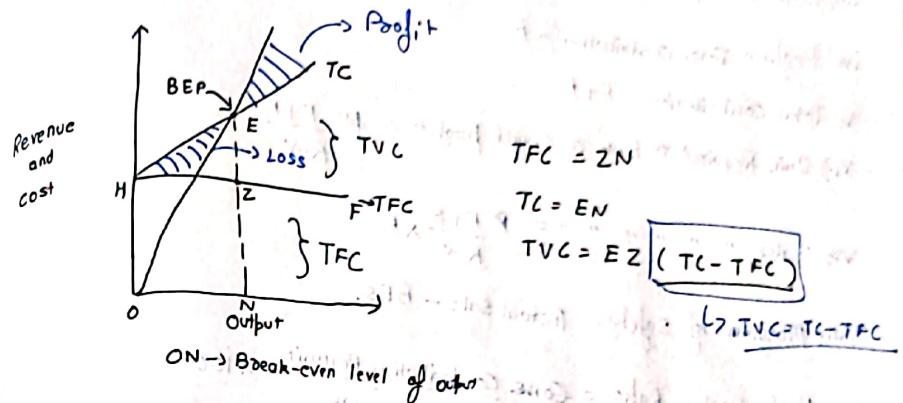
Refers to the point where  $TR = TC$

If  $\pi$  is profit / loss, formula of profit is  $= TR - TC$

$$\begin{cases} TR = TC \\ \pi = 0 \end{cases}$$

(if +ve, profit  
-ve, loss)

- Methods of break-even analysis.
- i) Graphical Method.
- ii) Algebraic Method.



Selling price per unit =  $R$

Variable cost per unit =  $V$

Total Fixed Cost =  $F$

No. of units of output =  $N$

At, BEP,  $TR = TC$  (1)

$$TR = P \times Q$$

$$TR = RN$$

~~$$TC = F + VN$$~~

$$= F + V.N$$

$$RN = F + V.N$$

$$F = RN - VN$$

$$F = N(R-V)$$

$$\boxed{N = \frac{F}{R-V}} \rightarrow \text{Break-Even level of output.}$$

$$\text{i) BEP in (units)} \Rightarrow N = \frac{F}{R-V}$$

$$\text{ii) BES / BEP (in Sales)} = \frac{F}{R-V} \times R$$

$$\text{iii) Contribution per unit (C)} = R-V$$

$$\text{iv) Profit} = \text{Total Contribution} - F$$

$$\text{v) Total Contribution} = F+P$$

$$\text{vi) Units Required to have a target profit} = \frac{F+P}{R-V}$$

$$\text{vii) Sales} = \frac{F+P}{R-V} \times R$$

$$\text{viii) Margin of Safety} = \text{Actual Sales} - \text{BES}$$

$$\text{ix) Profit/Value Ratio} = \frac{\text{Contribution per unit}}{\text{Selling price per unit}} \times 100$$

$$= \frac{R-V}{R} \times 100$$

$$\text{x) Break-even-Sales} = \frac{F}{R-V} \times R$$

$$= \frac{F}{P/V \text{ ratio}} \quad (\text{if } R \text{ and } V \text{ not given})$$

$$\text{xi) Sales Required to have a target profit} = \frac{F+P}{P/V \text{ ratio}}$$

$$\text{xii) Margin of Safety} = \frac{\text{Profit}}{\text{P/V ratio}}$$

$$\text{xiii) Contribution per unit} = R-V = \frac{P}{R}$$

$$\text{xiv) P/V ratio} = \frac{\text{Change in Profit}}{\text{Change in Sales}} \times 100$$

### Formula for Break-Even Analysis

xv) Fixed Cost = (Sales  $\times$  P/V ratio) - Profit

xvi) Variable Cost =  $(1 - P/V \text{ ratio}) \times \text{Sales}$

a) From the following, calculate BEP and P/V ratio

$$\text{Selling price per unit} = 30 \text{ INR (R)}$$

$$\text{Variable Cost per unit} = 20 \text{ INR (V)}$$

$$\text{Total Fixed Cost} = 20,000 \text{ INR (F)}$$

$$\text{BEP} = \frac{F}{R-V} = \frac{20,000}{30-20}$$

$$= 2,000 \text{ units}$$

$$\text{BEP (Sales)} = \frac{F}{R-V} \times R$$

$$= 2,000 \times 30$$

$$= 60,000 \text{ INR/-}$$

$$\text{P/V ratio} = \frac{R-V}{R} \times 100$$

$$= \frac{10}{30} \times 100 = 33.33\%$$

b) From the following, P/V ratio, Fixed Cost, Variable cost in 2010, BEP and Sales required to have a target profit of 20,000 INR

Years	Sales	Cost	Profit
2010	1,20,000	72,000	48,000
2011	1,40,000	84,000	56,000

$$P/V \text{ ratio} = \frac{\text{change in profit}}{\text{change in sales}} \times 100$$

$$\begin{aligned} &= \frac{(1,40,000 - 1,23,000) - (1,20,000 - 1,11,000)}{1,40,000 - 1,20,000} \times 100 \\ &= \frac{17,000 - 9,000}{20,000} \times 100 \\ &= \underline{20\%} \end{aligned}$$

$$\text{Profit} = TR - TC$$

$$\begin{aligned} \text{Fixed Cost} &= (\text{Sales} \times P/V \text{ ratio}) - \text{Profit} \\ &= (1,40,000 \times .20) - 13,000 \rightarrow \text{can take for 2010 and} \\ &= 15,000 \end{aligned}$$

$$\text{Variable Cost (2010)} = 96,000 \rightarrow (1 - 0.2) \cancel{96,000} \rightarrow 1,20,000$$

$$\text{Sales required to have target} = 1,75,000 \left| \left( \frac{F+P}{P-N} \right) = \frac{15,000 + 20,000}{.2} \right.$$

$$\text{Break-even Sales} = \frac{F}{P-N} = \frac{15,000}{0.2} \rightarrow 75,000 \text{ units}$$

Q) From the following info. to find P/V ratio, BEP, Profit when output is 50,000 INR.

$$\text{Fixed Cost} = 1,20,000$$

$$\text{Variable Cost per unit} = 3 \text{ INR}$$

$$\text{Selling Price per unit} = 7 \text{ INR}$$

$$\frac{R}{P} \times 100$$

$$\begin{aligned} &= \frac{7-3}{7} \times 100 \\ &= 57 \text{ or } 143\% \end{aligned}$$

$$\text{BEP} = \frac{F}{R-N} = \frac{1,20,000}{7-3} = 30,000$$

$$\text{Units for target profit} = 50,000 = \frac{F+P}{R-N}$$

$$50,000 = \frac{1,20,000 + P}{7-3}$$

$$P = 80,000$$



## Inflation

Whenever there is a rapid continuous and substantial rise in which reduces value of money, it is called inflation.

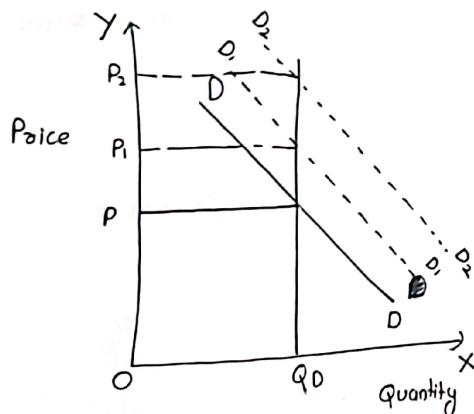
### Types of Inflation:

I) Demand-pull inflation.

II) Cost-push inflation.

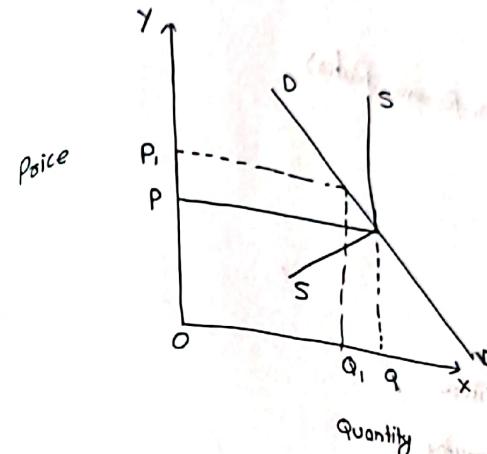
### I, Demand-Pull Inflation:

When aggregate demand for a commodity is more than supply of goods and services of an economy, it results in gap in the a gap of an economy, which pulls the price up.



### II, Cost-Push Inflation

When cost of production increases instead of increasing the price the supply of goods and services to the market, which push the price up.



(Explain in exam)

### Causes of Inflation

- Increase in money supply
- Decrease in tax
- Increase in govt' expenditure
- Increase in export
- Increase in population
- Black money
- Increase in
- Shortage in factor of production
- Trade union
- Hoarding
- War and emergency

Demand-side inflation

→ people get more money, more they buy.  
The more demand, increase in inflation.

{ Natural calamities  
Supply-side → inflation. }

Supply-side inflation

{ Explanation is recorded in the phone. }

## Factors of Control of Inflation

i) Monetary Policy

ii) Fiscal Policy

i) a) Increase in bank Rate

b) Increase in CRR (Cash-Reserve-Ratio)

c) Credit Control

ii) Sale of Govt. Security

(max. of money)

iii) a) Increase in tax

b) Decrease in Govt. expenditure

c) Increase in Public Borrowing

## Quantitative Methods of Controlling Inflation by Reserve-Bank of India

i) Bank Rate

ii) Open Market operation.

iii) CRR

iv) Credit Control

{  
a) notes  
b) coins  
c) bank deposits  
d) advances}

Bank Rate

Open Market operation

Credit Control

Bank Deposits

Bank Advances

Notes and Coins

Bank Reserves

Bank Capital

Bank Profit

Bank Assets

Bank Liabilities

Bank Reserves

Bank Capital

Bank Profit

Bank Assets

Bank Liabilities

Bank Reserves

Bank Capital

Bank Profit

Bank Assets

Bank Liabilities

Bank Reserves

Bank Capital

Bank Profit

Bank Assets

Bank Liabilities

Bank Reserves

Bank Capital