

salvage value → money received after selling a used product.

Date 04/09/2019



## Comparison of Alternatives:

- ① Present worth method,
- ② Future worth method,
- ③ Annual worth method,
- ④ Rate of return method,
- ⑤ Cost Benefit Analysis.

Present Worth method is most popular in business.  
In case of comp. project.

$$NPW(i\%) = PW(B) - PW(C).$$

If  $NPW > 0$ , project will be selected.

If  $NPW < 0$ , project will be rejected.

If  $NPW = 0$ , project may or may not be selected.

In case of mutually exclusive project.

Methods:

- ① Revenue Based method,
- ② Cost Based method.

Revenue Based method,

It refers to that method where all types of benefit i.e. profit, revenue, income or earning and salvage value will be assigned with +ve sign and all type of cost i.e. payment or investment will be assigned with -ve sign.

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## Cost Based Method.

It refers to that method where all types of benefit will be assigned with +ve sign and cost will be assigned with -ve sign.

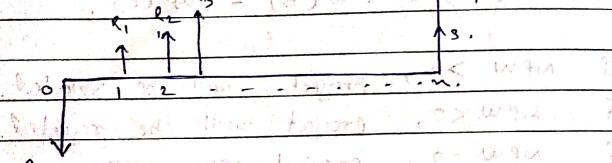
## Comparison of Alternatives.

- ① Present worth method

- ② Revenue based method.

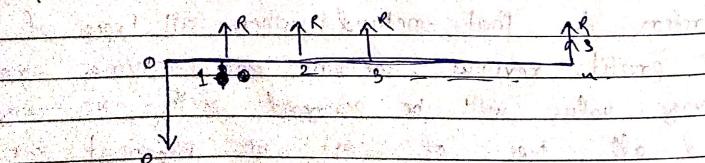
Present worth method. ① Different series.

$$NPW(i\%) = -P + R_1 \left[ \frac{1}{(1+i)^1} \right] + R_2 \left[ \frac{1}{(1+i)^2} \right] + R_3 \left[ \frac{1}{(1+i)^3} \right]$$



$$NPW(i\%) = -P + R_1 \left[ \frac{1}{(1+i)^1} \right] + R_2 \left[ \frac{1}{(1+i)^2} \right] + R_3 \left[ \frac{1}{(1+i)^3} \right] + \dots + R_m \left[ \frac{1}{(1+i)^m} \right] + S \left[ \frac{1}{(1+i)^n} \right].$$

② Equal Payment series.



$$NPW(i\%) = -P + R \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] + S \left[ \frac{1}{(1+i)^n} \right].$$

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Cost Based Method:	
* Different terms:	
* Equal payment terms:	

Diagram illustrating cash flows over time (Year 0 to Year 6). Arrows indicate inflows (down) and outflows (up).

NPW (i%) =  $P + C_1 \left[ \frac{1}{(1+i)^1} \right] + C_2 \left[ \frac{1}{(1+i)^2} \right] + \dots + C_m \left[ \frac{1}{(1+i)^m} \right] - S \left[ \frac{1}{(1+i)^n} \right]$ .

NPW (i%) =  $P + C \left[ \frac{1 - (1+i)^{-m}}{1 - (1+i)^{-1}} \right] - S \left[ \frac{1}{(1+i)^n} \right]$ .

Note:  $C = C_1 + C_2 + \dots + C_m$

Q) From the following table find out the project is financially feasible or not on the basis of present worth method if  $i = 20\%$ , compounded annually.

From the following table find out project will be selected or rejected if  $i = 18\%$ , compounded annually.

End of Year	Cash Flows
0	-65,000
1	-20,000
2	-30,000
3	30,000
4	5,00,000
5	5,00,000
6	"

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Q)  $NPW(i\%) = -P + R_1 \left[ \frac{1}{(1+i)^1} \right] + R_2 \left[ \frac{1 - \frac{1}{(1+i)^2}}{1 - \frac{1}{(1+i)}} \right] + \dots$

So,

$NPW(i\%) = -65,000 + 20,000 \left[ \frac{1}{(1+0.18)} \right] + 22,000 \left[ \frac{1}{(1+0.18)^2} \right]$

$+ 30,000 \left[ \frac{1}{(1+0.18)^3} \right] + 36,000 \left[ \frac{1}{(1+0.18)^4} \right]$

$\therefore NPW(18\%) = -65,000 + 16949.152 + 15800.052$

$+ 18258.926 + 18568.399$

$\therefore 4,576.53$

selected because  $NPW(18\%) > 0$ .

Q) From the following table find out the project is financially feasible or not on the basis of present worth method if  $i = 20\%$ , compounded annually.

End of year cash flows.

0	-40,00,000
1	5,00,000
2	5,00,000
3	5,00,000
4	5,00,000
5	"
6	"

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7	Initial investment	"
8	Annual income	"
9	Annual income	"
10	Annual income	"
11	Annual income	"
12	Annual income	"
13	Annual income	"
14	Annual income	"
15	Annual income	"

Solve:

$$NPW(17\%) = -P + R \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$= -40,00,000 + 5,00,000 \left[ \frac{(1+0.2)^{15} - 1}{0.2(1+0.2)^{15}} \right]$$

$$= -166,226.3 \cdot 6.79 = -1,14,000,000.87$$

It is not feasible because,

$$NPW(20\%) < 0.$$

- (d) From the following table with which technology will be selected from Present worth method if  $i = 16\%$  compounded annually.

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Technology	Initial outlay	Annual income	Life in yrs.
1	10,00,000	5,00,000	15 yrs
2	18,00,000	7,00,000	15 yrs
3	16,00,000	6,00,000	15 yrs

\* revenue based method

Solve: Technology 1:

$$NPW(16\%) = -10,00,000 + 5,00,000 \left[ \frac{(1+0.16)^{15} - 1}{0.16(1+0.16)^{15}} \right]$$

$$\Rightarrow 13,87,728.061.$$

Technology 2:

$$NPW(16\%)$$

$$= -18,00,000 + 7,00,000 \left[ \frac{(1+0.16)^{15} - 1}{0.16(1+0.16)^{15}} \right]$$

$$\Rightarrow 21,02,819.314$$

Technology 3:

$$NPW(16\%)$$

$$= -16,00,000 + 6,00,000 \left[ \frac{(1+0.16)^{15} - 1}{0.16(1+0.16)^{15}} \right]$$

$$\Rightarrow 12,43,273.198.$$

So, Tech 2 will be selected because it has greater NPW

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From the following table of firm which machine will be selected by Present worth method if  $i = 19\%$ . compounded annually.

Machine	Initial cost	Service life in yrs.	Annual op. & maintenance cost	salvage value.
A	6,00,000	20 yrs.	35,000	15,000
B	7,00,000	20 yrs.	40,000	12,000

Sol:

Machine A:

$$NPW(19\%) = 6,00,000 + 35,000 \left[ \frac{(1+0.19)^{20} - 1}{0.19(1+0.19)^{20}} \right] - 15,000 \times \left[ \frac{1}{(1+0.19)^{20}} \right]$$

$$\Rightarrow NPW(19\%) = 7,96,322.664$$

$$NPW(19\%) = 7,00,000 + 40,000 \left[ \frac{(1+0.19)^{20} - 1}{0.19(1+0.19)^{20}} \right] - 12,000 \times \left[ \frac{1}{(1+0.19)^{20}} \right]$$

$$NPW(19\%) = 9,24,591.339$$

Machine A will be selected because cost on machine A is less.

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Future Worth Method.

\* In case of one project:

$$NFW(i\%) = Fw(B) - Fw(c).$$

If  $NFW > 0$ , project will be selected

If  $NFW < 0$ , project will be rejected

If  $NFW = 0$ , project may or may not be selected.

\* In case of mutually exclusive projects:

Revenue Based Method:

Different semi-annuals:

$$NFW(i\%) = -P(1+i)^m + R_1(1+i)^{m-1} + R_2(1+i)^{m-2} + \dots + R_m + S$$

Equal payment semi-annuals:

$$NFW(i\%) = -P(1+i)^m + R \left[ \frac{(1+i)^m - 1}{i} \right] + S$$

Cost Based Method:

Different semi-annuals:

$$NFW(i\%) = P(1+i)^m + C_1(1+i)^{m-1} + C_2(1+i)^{m-2} + \dots + C_m - S$$

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\* Equal payment series.

$$NFW(i\%) = P(1+i)^n + C \left[ \frac{(1+i)^n - 1}{i} \right] - S$$

- (Q) From the following table find out which alternative will be selected on the basis of future worth method, if  $i = 15\%$  compounded annually.

Particulars	Alternative A	Alternative B
Initial cost	4,00,000	6,00,000
Uniform annual benefit	64,000	96,000
useful life (yrs)	15 yrs	15 yrs.

Sol: Alternative A:

$$NFW(15\%) = -4,00,000 (1+0.15)^{15} + 64,000 \left[ \frac{(1+0.15)^{15} - 1}{0.15} \right]$$

$$\Rightarrow NFW(15\%) = 85009.573.$$

Alternative B:

$$NFW(B\%) = -6,00,000 (1+0.15)^{15} + 96,000 \left[ \frac{(1+0.15)^{15} - 1}{0.15} \right]$$

$$\approx NFW(15\%) = 127514.36.$$

Alternative B will be selected because  $NFW(15\%)$  for B > A.  
(Annual benefit is more).

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- (Q) From the following table find out which machine will be selected on the basis of future worth method if  $i = 11\%$  compounded annually.

Particulars machine 1 machine 2 machine 3

Particulars	Machine 1	Machine 2	Machine 3
Initial investment	80,00,000	170,00,000	90,00,000
Annual op. & f	8,00,000	9,00,000	8,50,000
Annual insurance	0.00,000	0.00,000	0.00,000
Salvage value	5,00,000	4,00,000	3,00,000

Sol: Machine 1:

$$NFW(11\%) = 80,00,000 (1.11)^{17} + 8,00,000 \left[ \frac{(1.11)^{17} - 1}{0.11} \right] - 5,00,000 \\ = 82361415.91$$

Machine 2:

$$NFW(11\%) = 170,00,000 (1.11)^{17} + 9,00,000 \left[ \frac{(1.11)^{17} - 1}{0.11} \right] - 4,00,000 \\ = 8,09,16,409.48.$$

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- Q) From the following table find out which machine will be selected on the basis of future worth method if  $i = 11\%$  compounded annually.

Particulars	Machine 1	Machine 2	Machine 3
Initial investment	80,00,000	70,00,000	90,00,000
Life (yrs)	17 yrs	17 yrs	17 yrs
Annual op. & maintenance cost	8,00,000	9,00,000	8,50,000
Salvage value	5,00,000	4,00,000	3,00,000

Sol: Machine 1:

$$\text{NFW}(11\%) = 80,00,000 (1.11)^{-17} + 8,00,000 \left[ \frac{(1.11)^{17} - 1}{0.11} \right] - 5,00,000 \\ = 8,236,415.91$$

Machine 2:

$$\text{NFW}(11\%) = 70,00,000 (1.11)^{-17} + 9,00,000 \left[ \frac{(1.11)^{17} - 1}{0.11} \right] - 4,00,000 \\ = 8,09,16,409.48.$$

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Machine 3:

$$\text{NFW}(11\%) = 90,00,000 (1.11)^{-17} + 8,50,000 \left[ \frac{(1.11)^{17} - 1}{0.11} \right] - 7,00,000$$

$$= 9,01,81,550.76.$$

Machine 3 out!

Machine 2 will be selected because cost will be less.

- Q) From the following table find out which alternative will be selected on the basis of future worth method if  $i = 12\%$  compounded annually?

Particulars	Alternative 1	Alternative 2
First cost	15,00,000	20,00,000
Annual property taxes	70,000	90,000
Annual income	60,00,50,000	70,00,000
Life (yrs)	15 yrs	15 yrs
Net annual income	4,30,000	6,10,000

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SOL

$$NFW(12\%) = -15,00,000 (1.12)^{15} + 4,30,000 \left[ \frac{(1.12)^{15} - 1}{0.12} \right]$$

$$NFW(12\%) = 78,19,928.665$$

$$NFW(12\%) = -20,00,000 (1.12)^{15} + 6,10,000 \left[ \frac{(1.12)^{15} - 1}{0.12} \right]$$

$$NFW(12\%) = 1,17,93,4984.42$$

Alternative 2 will be selected.

- (ii) From the following table find out which machine will be selected on the basis of FWM if  $i=16\%$ , compounded annually.

Particulars	Machine A	Machine B
Initial cost	5,00,000	3,00,000
Life (yrs)	4 yrs	7 yrs.
Salvage value	1,00,000	2,00,000
Annual maintenance cost	30,000	0

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Machine A: Present Value Method

$$NFW(16\%) = 5,00,000 (1.16)^4 + 30,000 \left[ \frac{(1.16)^4 - 1}{0.16} \right] - 1,00,000$$

$$\Rightarrow NFW(16\%) = 16,55,526.067.$$

Machine B:

$$NFW(16\%) = 3,00,000 (1.16)^7 - 2,00,000.$$

$$\Rightarrow 19,78,353.814$$

Machine A will be selected.

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## Annual Worth Method / Annual Equivalent Method

- \* In case of same project:

$$NAW(i\%) = AW(B) - AW(c).$$

If  $NAW > 0$ , project will be selected.

If  $NAW < 0$ , project will be rejected.

If  $NAW = 0$ , project will be may or may not be selected

- \* In case of mutually exclusive projects.

Different series.

- \* Equal payment series: (Revenue based method)

$$NAW(i\%) = -P \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right] + R \rightarrow A \left[ \frac{i}{(1+i)^n - 1} \right]$$

- \* Cost based method.

$$NAW(i\%) = P \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right] + c \rightarrow A \left[ \frac{i}{(1+i)^n - 1} \right] - s \left[ \frac{i}{(1+i)^n - 1} \right]$$

- \* Different series,

$$NAW(i\%) = NPW \left[ \frac{i(1+i)^n}{(1+i)^n - 1} \right]$$

same for both revenue and cost based method.

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Q) From the following table find out which technology will be selected on the basis of annual equivalent method if  $i = 18\%$  compounded annually.

Particulars	Technology A	Technology B
Initial cost	5,00,000	3,00,000
End of Year	cash inflows year end	cash inflows year end.
1	10,000	15,000
2	20,000	30,000
3	30,000	0
4	45,000	0.

Soln: Technology A:

$$NAW(i\%) = \left\{ -50,00,000 + 10,000 \left[ \frac{1}{(1+18)} \right] + 20,000 \left[ \frac{1}{(1+18)^2} \right] + 30,000 \left[ \frac{1}{(1+18)^3} \right] + 45,000 \left[ \frac{1}{(1+18)^4} \right] \right\} \cdot \left[ \frac{0.18(1+18)^4}{(1+18)^4 - 1} \right]$$

$$\Rightarrow -4,35,692 \cdot 30 \cdot \left[ \frac{0.18(1+18)^4}{(1+18)^4 - 1} \right]$$

$$\approx -1,61,963 \cdot 676 \cdot$$

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Technology B:

$$NAW(18\%) = \left\{ -5,00,000 + 15,000 \left[ \frac{1}{1.18^1} \right] + 30,000 \left[ \frac{1}{1.18^2} \right] \right\}$$

$$\frac{0.18 \times (1.18)^2}{(1.18)^2 - 1}$$

$$\Rightarrow -4,25,220.18$$

Technology A will be selected

$$\therefore NAW_A(18\%) > NAW_B(18\%)$$

Q. From the following which machine will be selected on the basis of AEM, if  $i = 20\%$ . compounded annually.

Machine	Down payment	8 Yearly equal instalment	No. of instalments
1	5,00,000	2,00,000	15
2	4,00,000	3,00,000	15
3	6,00,000	1,50,000	15

Machine 1:

$$NAW(20\%) = 5,00,000 \left[ \frac{0.20 (1.20)^{15}}{(1.20)^{15} - 1} \right] + 2,00,000$$

$$\Rightarrow NAW(20\%) = 3,06,941.059$$

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Machine 2:

$$NAW(20\%) = 4,00,000 \left[ \frac{0.20 (1.20)^{15}}{(1.20)^{15} - 1} \right] + 3,00,000$$

$$\Rightarrow NAW(20\%) = 3,85,552.847$$

Machine 3:

$$NAW(20\%) = 6,00,000 \left[ \frac{0.20 (1.20)^{15}}{(1.20)^{15} - 1} \right] + 1,50,000$$

$$\Rightarrow NAW(20\%) = 2,78,329.271$$

Machine B will be selected, because cost is minimum.

(i) From the following table find out which alternative will be selected on the basis of AEM if  $i = 25\%$ . compounded annually. and life of both the alternatives is 5 yrs

Particulars	Alternative A	Alternative B
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Investment	1,50,000	1,75,000
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Annual equal return	10,000, 60,000	70,000
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Salvage value	15,000	35,000
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Alternative A:

$$NAW(25\%) = -1,50,000 \left[ \frac{(1.25)^5 \cdot (0.25)}{(1.25)^5 - 1} \right] + 60,000$$

$$= 18,000 \left[ \frac{0.25}{(1.25)^5 - 1} \right]$$

$$NAW(25\%) = 60,50,69.$$

Alternative B:

$$NAW(25\%) = -125,000 \left[ \frac{(1.25)^5 \cdot (0.25)}{(1.25)^5 - 1} \right] + 70,000.$$

$$= -35,000 \left[ \frac{0.25}{(1.25)^5 - 1} \right]$$

$$NAW(25\%) = 9,191,456$$

As return from alternative B is more so B will be selected.

- (Q) From the following table find out which machine will be selected on the basis of annual equivalent method if  $i = 25\%$ . compounded annually.

Particulars	Machine A	Machine B
Initial cost	3,00,000	6,00,000

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10 yrs

12 yrs

Life in yrs

Salvage value

2,00,000

5,00,000

Annual maintenance

30,000

0

Machine A:

$$NAW(22\%) = 3,00,000 \left[ \frac{(1.22)^{10} \cdot (0.22)}{(1.22)^{10} - 1} \right] + 30,000$$

$$= 2,00,000 \left[ \frac{0.22}{(1.22)^{10} - 1} \right]$$

$$\Rightarrow NAW_A(22\%) = 99,489.498.$$

Machine B:

$$NAW(22\%) = 6,00,000 \left[ \frac{(1.22)^{12} \cdot (0.22)}{(1.22)^{12} - 1} \right] - 3,00,000 \left[ \frac{0.22}{(1.22)^{12} - 1} \right]$$

$$\Rightarrow NAW_B(22\%) = 1,42,468.494.$$

$NAW_A(22\%) > NAW_B(22\%)$  machine A is selected because.

$$NAW_A(22\%) > NAW_B(22\%)$$

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## Rate of Return Method :-

### Types of Return.

(1) Minimum acceptable rate of return / Minimum attractive rate of return. (MARR).

(2) Net Present Value (NPV)

(3) Internal Rate of Return (IRR)

### Minimum Acceptable Rate of Return

It refers to the lower limit of project acceptability beyond which if rate of return falls project will be rejected.

### Net Present Value; (NPV).

It refers to the addition of present values of all the future stream of benefits during the life span of a project.

### Internal Rate of Return

It refers to the rate of return which equals the present value of benefit.

If  $NPV \neq 0$ .

$$IRR = \left( \text{Interest rate of the last positive result} \right) - \frac{\text{the difference between the two interest rates}}{\text{true value}}$$

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$$IRR = \frac{\text{Interest rate of the last result} + \frac{\text{Difference b/w the two interest rates}}{\text{true value - salvage value}}}{\text{true value - (up value)}}$$

\* Selection or rejection of the project:

If  $IRR > MARR$ , project will be selected.

If  $IRR < MARR$ , project will be rejected.

If  $IRR = MARR$ , project may or may not be selected.

(Q) From the following table find for what the investor should go with the new buys. or not on the basis of RRM.

0	Initial investment	-1,00,000
1	Annual cash flows	30,000
2	Annual cash flows	30,000
3	Annual cash flows	30,000
4	Annual cash flows	30,000
5	Annual cash flows	30,000

$$NPV = -P + R \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right] + 0.$$

$$\Rightarrow -100,000 + 30,000 \left[ \frac{(1+0.1)^5 - 1}{0.1(1+0.1)^5} \right] = 13723.60.$$

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$$\Rightarrow NPW(10\%) = -1,00,000 + 30,000 \left[ \frac{(1+0.12)^5 - 1}{0.12(1+0.12)^5} \right]$$

$$\Rightarrow NPW(10\%) = 8143.286.$$

$$NPW(15\%) = 564.65.$$

$$NPW(17\%) = -1771.19.$$

$$\therefore IRR = 15\% + 2\% \times \frac{564.65 - 0}{564.65 + 1771.19} = 15.48\%$$

$$\therefore IRR = 15.48\%$$

The investment may go with the new business because  $IRR > MARR$ .

- A company is trying to diversify its bus. in a new product line. the life of the project is 10 yrs with no salvage value. at the end of its life, the initial outlay of the project is 20,00,000 Rs. annual cash inflow is 3,50,000. Rs. Find the rate of return for the new business.

Taking MARR as 10%.

$$NPW(10\%) = -P + R \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right].$$

$$-20,00,000 + 3,50,000 \left[ \frac{(1+1)^{10} - 1}{0.1(1+1)^{10}} \right].$$

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$$NPW(10\%) = 150598.487. Rs. = 150598.487.$$

$$NPW(11\%) = 61231.82039. Rs. = 61231.82039.$$

$$NPW(12\%) = -22421.94. Rs. = -22421.94.$$

$$\therefore IRR = 11\% + 1\% \times \frac{61231.82039 - 0}{61231.82039 + 22421.94}$$

$$= 11.738\%.$$

$$= 11.7\%$$

- Q) From the following table find out rate of return for all the alternatives and find out which alternative will be selected on the basis of RRM if MARR is 12%.

Particulars	Alternative A1	Alternative A2	Alternative A3
Investment	Rs. 1,50,000	Rs. 10,000	Rs. 55,000
Annual net income	Rs. 570	Rs. 58,200	Rs. 9,000

So/yr	5 yrs	5 yrs	5 yrs
Alternative A1			

So/yr	5 yrs	5 yrs	5 yrs
Alternative A1			
NPW(10%)	-150598.487	61231.82039	-22421.94

$$\therefore NPW(10\%) = 14269.651.$$

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$$\begin{aligned} IRR_A &= 15.8\% \\ IRR_{A2} &= 12\% \\ IRR_{A3} &= 11\% \end{aligned}$$

$$NPW(13\%) = 10280 + 228$$

$$NPW(14\%) = 6448 + 499$$

$$NPW(15\%) = 2457 - 707$$

$$NPW(16\%) = -790.438$$

$$\begin{aligned} IRR &= 15\% + 1\% \times \frac{12457 - 707}{2457 - 707 + 790.438} \\ &= 15\% + 1\% \times \frac{11750}{2457.438} \\ IRR &= 15.77\% \end{aligned}$$

Alternative A2: ~~NPW(12%) = 58,260~~  
~~IRR = 12.00%~~

$$NPW(12\%) = -210,000 + 58,260 \left[ \frac{0.12 \cdot (1.12)^5 - 1}{0.12 \times (1.12)^5} \right]$$

Alternative A3: ~~NPW(12%) = 6270.44~~  
~~IRR = 14.261%~~

$$NPW(12\%) = -210,000 + 6270.44 \left[ \frac{0.12 \cdot (1.12)^5 - 1}{0.12 \times (1.12)^5} \right]$$

Alternative A3: ~~NPW(12%) = 6270.44~~  
~~IRR = 14.261%~~

$$NPW(12\%) = -210,000 + 6270.44 \left[ \frac{0.12 \cdot (1.12)^5 - 1}{0.12 \times (1.12)^5} \right]$$

Alternative A3: ~~NPW(12%) = 6270.44~~  
~~IRR = 14.261%~~

$$NPW(12\%) = -210,000 + 6270.44 \left[ \frac{0.12 \cdot (1.12)^5 - 1}{0.12 \times (1.12)^5} \right]$$

Alternative A3: ~~NPW(12%) = 6270.44~~  
~~IRR = 14.261%~~

$$NPW(12\%) = -210,000 + 6270.44 \left[ \frac{0.12 \cdot (1.12)^5 - 1}{0.12 \times (1.12)^5} \right]$$

Alternative A3: ~~NPW(12%) = 6270.44~~  
~~IRR = 14.261%~~

$$NPW(12\%) = -210,000 + 6270.44 \left[ \frac{0.12 \cdot (1.12)^5 - 1}{0.12 \times (1.12)^5} \right]$$

Alternative A3: ~~NPW(12%) = 6270.44~~  
~~IRR = 14.261%~~

$$NPW(12\%) = -210,000 + 6270.44 \left[ \frac{0.12 \cdot (1.12)^5 - 1}{0.12 \times (1.12)^5} \right]$$

Alternative A3: ~~NPW(12%) = 6270.44~~  
~~IRR = 14.261%~~

$$NPW(12\%) = -210,000 + 6270.44 \left[ \frac{0.12 \cdot (1.12)^5 - 1}{0.12 \times (1.12)^5} \right]$$

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$$NPW(11\%) = 16.894$$

$$\therefore IRR = 11\% + 1\% \times \frac{16.894}{16.894 + 6270.44}$$

$$\Rightarrow IRR = 11.00\%$$

Alternative A2 will be selected because it has more ~~return~~ <sup>rate of return</sup> i.e. 15.77%.

Benefit-Cost Analysis:

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## Benefit Cost Analysis

\* Selection or rejection of the project:

Benefit cost ratio:

$$\frac{B}{C} = \frac{\text{Present value of Benefit}}{\text{Present value of cost}} = \frac{PW(B)}{PW(C)}$$

If  $\frac{B}{C} > 1$ , project will be selected.

If  $\frac{B}{C} < 1$ , project will be rejected.

If  $\frac{B}{C} = 1$  then project may or may not be selected.

(Q) From the following table find out which project will be selected on the basis of B/C ratio.

Project	Present worth of Benefit	Present worth of cost
1	60,00,000	40,00,000
2	30,00,000	20,00,000
3	90,00,000	35,00,000

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$$\left(\frac{B}{C}\right)_1 = \left[ \frac{PW(B)}{PW(C)} \right] = 1.5$$

$$\left(\frac{B}{C}\right)_2 = \left[ \frac{PW(B)}{PW(C)} \right] = 4$$

$$\left(\frac{B}{C}\right)_3 = \left[ \frac{PW(B)}{PW(C)} \right] = 2.5$$

Project 2 will be selected ( $\because B/C$  ratio is more).

Q) In a particular locality of a state, the vehicle users take around about road to reach certain places because of presence of a river, this results in excessive travel time and increased fuel cost. So the state government is planning to reconstruct a bridge across the river. The estimated initial investment for constructing the bridge is 40,00,000. The estimated life of the bridge is 15 yrs. The annual maintenance and op. cost is 1,50,000. The value of fuel savings due to the construction of the bridge is 6,00,000/- in first year and it increases by 50,000/- every year thereafter till the end of the life of the bridge. Check whether the project is justified based on B/C ratio by assuming an interest rate of 12% compounded annually.

$$\text{Initial investment} = 40,00,000$$

$$\text{Life} = 15 \text{ yrs}, R = 1,50,000$$

$$\text{Benefit} = 6,00,000$$

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Sol:

$$\text{B/P}^W(\%) = -40,00,000 + 1,50,000 \left[ \frac{(1+0.12)^{15} - 1}{0.12(1+0.12)^{15}} \right]$$

$$\Rightarrow \text{Cost} = -41,80,000 \text{ rs. } 5021629.673$$

★

$$A_1 = 6,00,000 + 50,000$$

$$C = 50,000.$$

$$A(B) = -6,00,000 + 50,000 \left[ \frac{1 - (1+0.12)^{15}}{0.12(1+0.12)^{15} - 1} \right]$$

$$\Rightarrow A(B) = -849015.169$$

★

$$\text{Pw}(B) = A(B) \left[ \frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

$$\Rightarrow -849015.169 \left[ \frac{(1+0.12)^{15} - 1}{0.12 \times (1+0.12)^{15}} \right]$$

B

$$= 5782527.266 \text{ rs. } 5782527.266$$

C

$$= 5021629.673$$

∴ Project is justified because  $B/C > 1$

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state government is planning a hydro electric project in addition of to the production of electric power, this project will provide flood control, irrigation and reforestation benefits, the estimated benefits and cost are given as follows

initial cost  $\rightarrow$  8 crore.

Annual power sales  $\rightarrow$  60,00,000 rs.

Annual flood control savings  $\rightarrow$  30,00,000.

Annual irrigation benefits  $\rightarrow$  50,00,000.

Annual reforestation benefits  $\rightarrow$  20,00,000.

Annual operating and maintenance cost is 130,00,000.

Life of project is 50 yrs.

$i = 15\%$ .

Check whether the government should implement the project or not on the basis of benefit cost analysis.

Sol:  $\text{Pw}(C) = 8,00,00,000 + (30,00,000 + 50,00,000) \times$

$$\left[ \frac{(1+0.15)^{50} - 1}{0.15(1+0.15)^{50}} \right]$$

$\Rightarrow \text{Pw}(C) = +9965088 \text{ rs. } 99981543.98.$

$\text{Pw}(B) = A = 1,60,00,000.$

$\text{Pw}(B) = 1,60,00,000$

$$\left[ \frac{(1+0.15)^{50} - 1}{0.15(1+0.15)^{50}} \right]$$

$$= 106568234.6.$$

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$$\frac{PW(B)}{PW(C)} = \frac{106568234.6}{99981543.98}$$

$$= 1.06$$

$\therefore$  Project must be implemented

$$\because PW(B) / PW(C) > 1$$

- (Q) Two mutually exclusive projects are being considered for investment, project A, requires an initial outlay of 30,00,000 with net receipts estimated as 9,00,000 Rs per year for the next 5 years. The initial outlay for the project A<sub>2</sub> is 60,00,000 and net receipts have been estimated as 15,00,000 Rs per year for the next 7 yrs. Using B/C ratio, find out which project will be selected if  $i = 10\%$  compounded annually.

Sol:

Project A<sub>1</sub>:

$$PW(C) = 30,00,000$$

$$A = 9,00,000$$

$$PW(B) = 9,00,000 \times \left[ \frac{(1+1)^5 - 1}{(1+1)^5 \times 0.1} \right]$$

$$\therefore PW(B) = 3411708.092$$

$$B/C = \frac{3411708.092}{30,00,000} = 1.137$$

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Project A<sub>2</sub>:

$$PW(C) = 60,00,000$$

$$PW(B) = 15,00,000 \left[ \frac{(1+1)^7 - 1}{(1+1)^7 \times 0.1} \right]$$

$$\therefore PW(B) = 7302628.227$$

$$B/C = 7302628.227$$

$$= 60,00,000$$

Project A<sub>2</sub> will be selected

$$(B/C)_{A_1} < (B/C)_{A_2}$$

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### Pay-Back Period Method :

\* In case of equal return.

$$P = \frac{I}{C}$$

I → Initial investment.  
C → Yearly equal cash inflows.  
Pay back years.

(Q) Suppose an investor has invested ₹ 50,00,000 in a project for which he is getting equal return of ₹ 1,00,000 at the end of every year. Find out how many yrs it will take to the investors to get back his money, which he has initially invested with the help of pay-back period method.

Sol:  $P = \frac{I}{C}$

$$\Rightarrow P = \frac{50,00,000}{1,00,000} = 50$$

≈ 25 years.

\* In case of different amount of return.

Remaining amount  
next cash inflow × 12.

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One investor has invested ₹ 10,00,000 in a project, the investor wants to purchase a machine for which there are two machines available in the market find out which machine will be selected on the basis of pay-back period method.

	Cash inflow from	
	Machine - 1	Machine - 2
1	1,00,000	2,00,000
2	3,00,000	4,00,000
3	5,00,000	6,00,000
4	7,00,000	8,00,000
5	8,00,000	7,00,000

\* For machine - 1.

$$\text{Number of months} = \frac{1,00,000}{7,00,000} \times 12 = 1.714$$

≈ 2 month

Total time = 3 years 2 month.

\* For machine - 2.

$$\text{Number of months} = \frac{2,00,000}{4,00,000} \times 12 = 6$$

≈ 6 months.

Total time = 2 yrs 6 months

Machine 2 will be selected because it requires less time.

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A government is planning three projects for which estimated benefits and costs are given as follows.

Particulars	Project A			Project B			Project C		
	A	B	C	A	B	C	A	B	C
Initial cost	15,000,000	25,000,000	30,000,000						
Annual operating and maintenance costs per year	20,000,000	25,000,000	35,00,000						
Annual saving	1,20,00,000	1,20,00,000	1,80,00,000						
Annual benefit	35,00,000	45,00,000	60,00,000						
Repayment	10,00,000	20,00,000	35,00,000						

Find out which project will be selected on the basis of B/C ratios if  $i = 9\%$  compounded annually.

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Project A:

$$PW(A) = -15,00,00,000 + \frac{15,00,00,000}{(1.09)^5} - \frac{1}{0.09(1.09)^5}$$

$$\Rightarrow 171923365.8 \text{ rupees}$$

$$PW(B) = -25,00,00,000 + \frac{25,00,00,000}{(1.09)^5} - \frac{1}{0.09(1.09)^5}$$

$$\Rightarrow PW(B) = 186348609.3$$

$$B/C = \frac{186348609.3}{171923365.8} = 1.08$$

Project B:

$$PW(C) = -30,00,00,000 + \frac{30,00,00,000}{(1.09)^5} - \frac{1}{0.09(1.09)^5}$$

$$\Rightarrow 277404207.3$$

$$PW(B) = -20,00,00,000 + \frac{20,00,00,000}{(1.09)^5} - \frac{1}{0.09(1.09)^5}$$

$$\Rightarrow 241157023.8$$

$$\Rightarrow B/C = \frac{241157023.8}{277404207.3} = 0.869$$

$$277404207.3$$

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Project C:

$$PW(C) = 40,00,00,000 + 35,00,000 \left[ \frac{(1.09)^{10} - 1}{0.09(1.09)^{10}} \right]$$

$$\pi \cdot PW(C) = 43,836,589.02$$

$$PW(B) = 3,25,00,000 \left[ \frac{(1.09)^{10} - 1}{0.09(1.09)^{10}} \right]$$

$$\pi \cdot PW(B) = 3,56,25,4094.3 \quad \therefore B/C = 0.812,$$

so, Project A is selected

- (d) From the following table, find out a company should select which alternative on the basis of rate of return method if MARR = 12%.

Alternative	Initial investment	Yearly
1	5,00,000	8,00,000
2	8,00,000	2,70,000
3	10,00,000	1,70,000

m=5

so? Alternative 1:

$$NPW(12\%) = -5,00,000 + 1,70,000 \left[ \frac{(1.12)^5 - 1}{0.12(1.12)^5} \right]$$

$$\pi \cdot NPW(12\%) = 112,811.95$$

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$$NPW(15\%) = -9,79,29,314$$

$$NPW(14\%) = 8,36,23,704$$

$$NPW(13\%) = 69,806.36$$

$$NPW(12\%) = 316,19,04$$

$$NPW(20\%) = 84,04,063$$

$$NPW(21\%) = -25,82,662$$

$$IRR = 20\% + 1\% \times \frac{84,04,063}{84,04,063 + 25,82,662}$$

$$= 20.76\%$$

Alternative 2:

$$NPW(12\%) = -8,00,000 + 2,70,000 \left[ \frac{(1.12)^5 - 1}{0.12(1.12)^5} \right]$$

$$NPW(13\%) = 173,289.57$$

$$NPW(14\%) = 149,652.44$$

$$NPW(15\%) = 74,652.297$$

$$NPW(16\%) = -99,84,229$$

$$IRR = 20\% + \left( \frac{-99,84,229}{74,652.297 + 99,84,229} \right) = 20.4278\%$$

∴ Alternative 1 will be selected because IRR > MARR

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- (b) From the following table find out which route will be selected on the basis of annual worth method if  $i = 15\%$  compounded annually.

Particulars	route around the lake	route under the lake
length	15 km	15 km
first cost	$\text{Rs } 7,50,000$	$\text{Rs } 7,50,000$
useful life	$15 \text{ years} + 0.5 = 15.5 \text{ years}$	$15 \text{ years}$
maintenance cost	$6,000 \text{ km}^{-1} \text{ yr}^{-1}$	$12,000 \text{ km}^{-1} \text{ yr}^{-1}$
salvage value	$90,000 \text{ km}^{-1}$	$1,50,000 \text{ km}^{-1}$
yearly power loss	$15,000 \text{ km}^{-1}$	$15,000 \text{ km}^{-1}$
solt:	Route around the lake	Route under the lake
$P = 1,50,000$		
$C = 6,000 \text{ km}^{-1} \text{ yr}^{-1}$		
$S = 90,000 \text{ km}^{-1}$		
$\text{Total} = 90,000 \times 15.5 = \text{Rs } 1,35,00,000$		
Yearly power loss = $15,000 \text{ km}^{-1}$		
$= 15,000 \times 15 = 2,25,000$		

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$$\therefore \text{NAW (15\%)} = 1,50,000 \left[ \frac{0.15 \times (1.15)^{15}}{(1.15)^{15} - 1} \right] + (90,000 + 2,25,000)$$

$$= \text{Rs } 1,35,000 \left[ \frac{0.15}{1.15^{15} - 1} \right]$$

$$\therefore \text{NAW (15\%)} = 31,2,99.536$$

### \* Route under the lake.

$$P = 7,50,000$$

$$C_{\text{Total}} = 12,000 \times 15$$

$$= 6,0,000$$

$$S = 1,50,000 \times 5$$

$$= 7,50,000$$

$$\text{Loss} = 75,000$$

$$\therefore \text{NAW (15\%)} = 7,50,000 \left[ \frac{0.15 \times 1.15^{15}}{1.15^{15} - 1} \right] + (60,000 + 75,000)$$

$$= 7,50,000 \left[ \frac{0.15}{1.15^{15} - 1} \right]$$

$$\text{NAW (15\%)} = 247,600$$

Route under the lake will be selected because the cost is less.

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## Production

It refers to the process of physical transformation of inputs into output.

## Production Function:

It refers to the functional relationship between inputs and output.

$$Q = f(N, L, K)$$

$$Q = f(m_1, m_2, m_3, \dots, m_n)$$

If there are n numbers of inputs.

## Factors of Production:

Fixed factor.

Variable factors.

Fixed factor:

It refers to those factors which cannot be changed or remain fixed during the process of production.

Eg: Plant size, big machines and land.

## Variable factors:

It refers to those factors which can be changed during the process of production.

Eg: Labour and raw materials.

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## Production time period:

Short period of production. (Short run)

Long period of production (long run)

## Short period:

It refers to that period of production where fixed factor cannot be changed but variable factor can be changed.

## Long period:

It refers to that period of production where nothing is called fixed, all the factors are variable factors.

## \* Three Concepts of Production:

- Total Product (TP)
- Marginal Product (MP)
- Average Product (AP)

### Total Product:

It refers to the total amount of output produced with a fixed amount of variable factors.

### Marginal Product:

It refers to the net addition to the total product by employing one more unit of input.

$$MP_m = TP_m - TP_{m-1}$$

If Labour is input

$$MP_L = \frac{d(TP)}{dL} = dQ/dL$$

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- If capital ( $k$ ) is input

$$MP_k = \frac{d(TP)}{dk} = \frac{dQ}{dk}$$

### Average Product:

It refers to the total amount of output produced per unit of a variable factor.

- If  $L$  is the input

$$AP_L = \frac{TP}{L} = \frac{Q}{L}$$

- If  $k$  is the input

$$AP_k = \frac{TP}{k} = \frac{Q}{k}$$

### Theories of Production Function: (1)

- Law of Variable Production (stage of short run)

- Law of Return to Scale (stage of long run).

### Law of Variable Production

It is a short run prod<sup>n</sup> function which discusses the relationship b/w output and one variable input.

It refers to the prod<sup>n</sup> function where as equal increments of one input

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are added, the input of other productive services being constant, the resulting increment of product beyond a certain point will decrease i.e. marginal product will diminish (given by  $c$  slightly).

### Assumptions of the theory:

- The state of technology remains constant.
- There must be some inputs whose quantities can be kept as fixed.

Unit of Labour used	Total Product	Marginal Product	Average Product
---------------------	---------------	------------------	-----------------

1	80	80	80
2	170	90	85
3	290	100	90
4	368	98	92
5	430	62	86
6	503	73	83.83
7	580	0	71.43
8	655	-8	61.9
9	720	-15	53.33
10	780	-50	43.

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Units of labour      TP      MP      AP      stage of operation,

because TP increases at an increasing rate

1	80	80	80	Stage I
2	170	90	85	Stage II (stage of increasing returns)
3	240	100	90	Stage II (stage of diminishing returns)
4	308	98	92	Stage II (stage of diminishing returns)
5	360	62	86	Stage III (negative)
at a diminishing rate	6	480	50	Diminishing returns
7	503	23	71.8	negative
rate	8	503	0	negative
9	490	-13	54.4	negative
10.	485	-5	48.5	negative

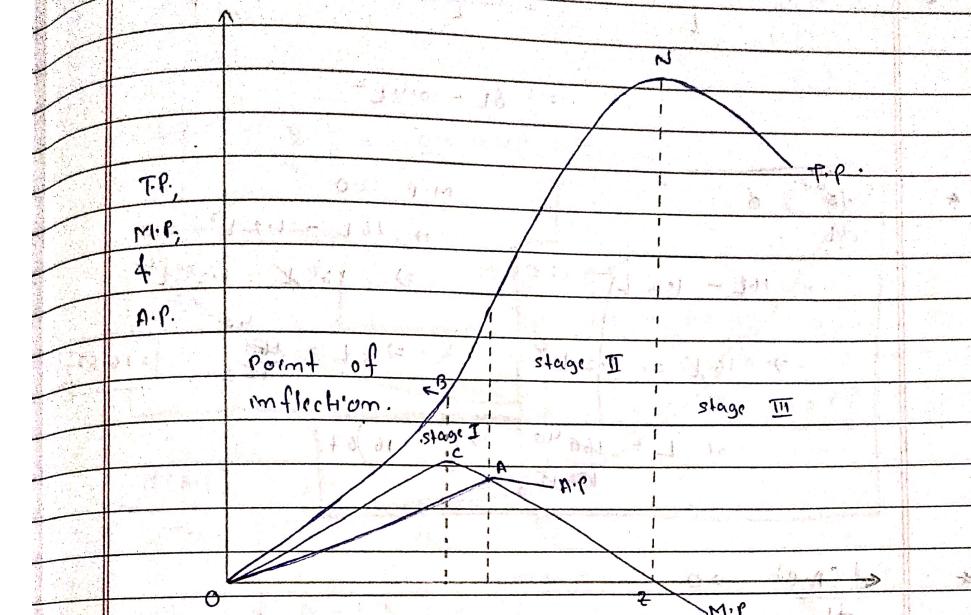
Producer will want to produce in Stage II because total capacity is utilised.

- Q) From the following table find out Marginal product and AP & also find stages of operation

Units of labour used (L)	TP	MP	AP	
1	30	30	30	
2	80	50	40	Stage I
3	140	60	46.67	Stage II
4	190	50	47.5	Stage II
5	190	0	38	Stage III
6	180	-10	30	Stage III

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### Diagrammatic Representation :-



Quantity of variable factor →

- Q) From the following short run production function find out

① Marginal product function.

② Average product function.

③ Value of L at which output will be maximum.

④ Value of L at which average product is maximum.

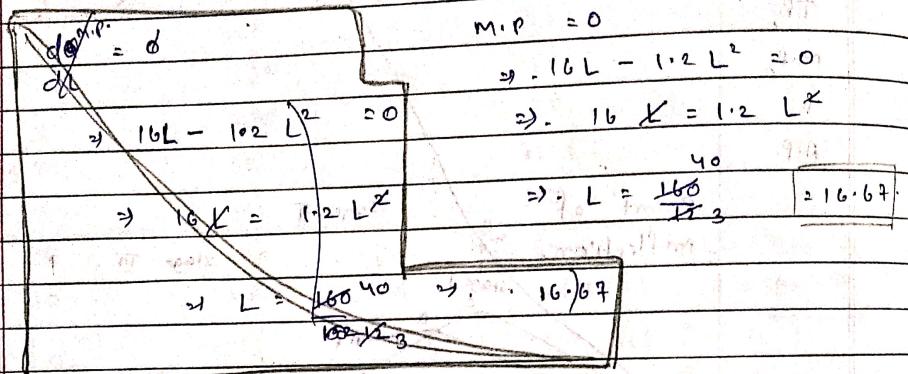
$$Q = 8L^2 - 0.4L^3$$

$$\text{M.P.} = \frac{dQ}{dL} = 16L - 1.2L^2$$

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$$\star A \cdot P = \frac{Q}{L} = \frac{8L^2 - 0.4L^3}{L}$$

$$\Rightarrow 8L - 0.4L^2$$



$$\frac{d(A \cdot P)}{dL} = 0$$

$$\therefore 8 - 0.8L = 0.$$

$$\therefore 8 = 0.8L \Rightarrow L = \frac{8}{0.8} = 10$$

(i) From the following short run production function find out :-

M.P. function.

A.P. function.

Find  $m$  at which output will be maximum

Find  $m$  at which  $A.P.$  is max.

$$Q = 6m^2 - 0.8m^3$$

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$$\frac{dQ}{dm} = 12m - 1.5m^2 = M.P.$$

$$\frac{dQ}{dm} = 12m - 1.5m^2 = 0$$

$$A \cdot P = \frac{Q}{m} = 6m - 0.8m^2$$

$$\frac{d(A \cdot P)}{dm} = 0$$

$$\frac{d(A \cdot P)}{dm} = 12 - 3m = 0$$

$$12 - 3m = 0 \Rightarrow m = 4$$

$$\frac{d(A \cdot P)}{dm} = 0 \Rightarrow 12 - 3m = 0 \Rightarrow m = 4$$

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### Law of Returns to Scale:

It is a long-run homogeneous production function which shows the relationship between output and all the variable inputs.

### Types of returns to scale:

① Increasing returns to scale.

② Constant returns to scale.

③ Decreasing returns to scale.

### \* Increasing returns to scale:

when the rate of change in output is more than the rate of change in input or doubling of input results in more than doubling of output it is called increasing returns to scale.

Eg:

C	K	Q
20	30	200
40	60	600

### Constant returns to scale.

when the rate of change in output is equal to the rate of change in input or doubling of input results in doubling of output it is called constant returns to scale.

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C	K	Q
20	30	200
40	60	400

### \* Decreasing returns to scale.

when the rate of change in output is less than the rate of change in input or doubling of input results in less than doubling of output, it is called decreasing returns to scale.

Eg:

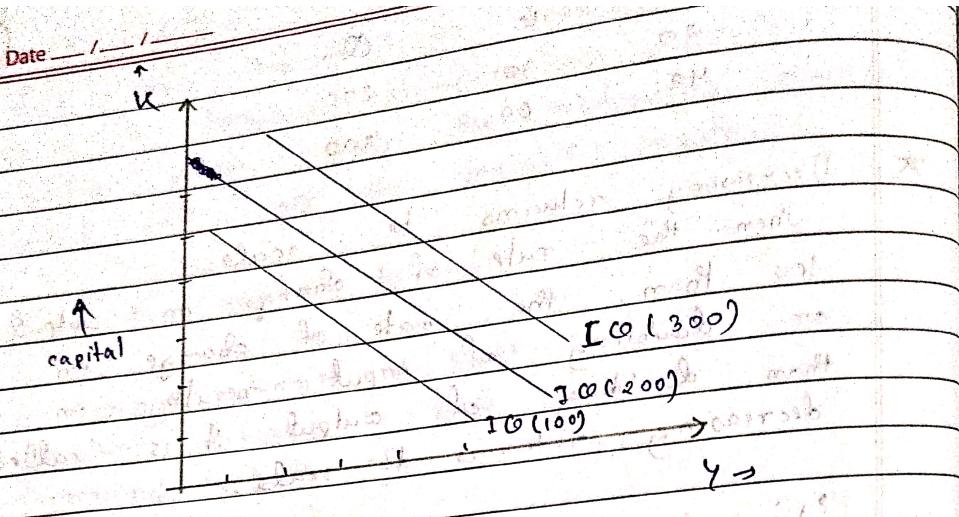
C	K	Q
20	30	200
40	60	300

### \* I see quant

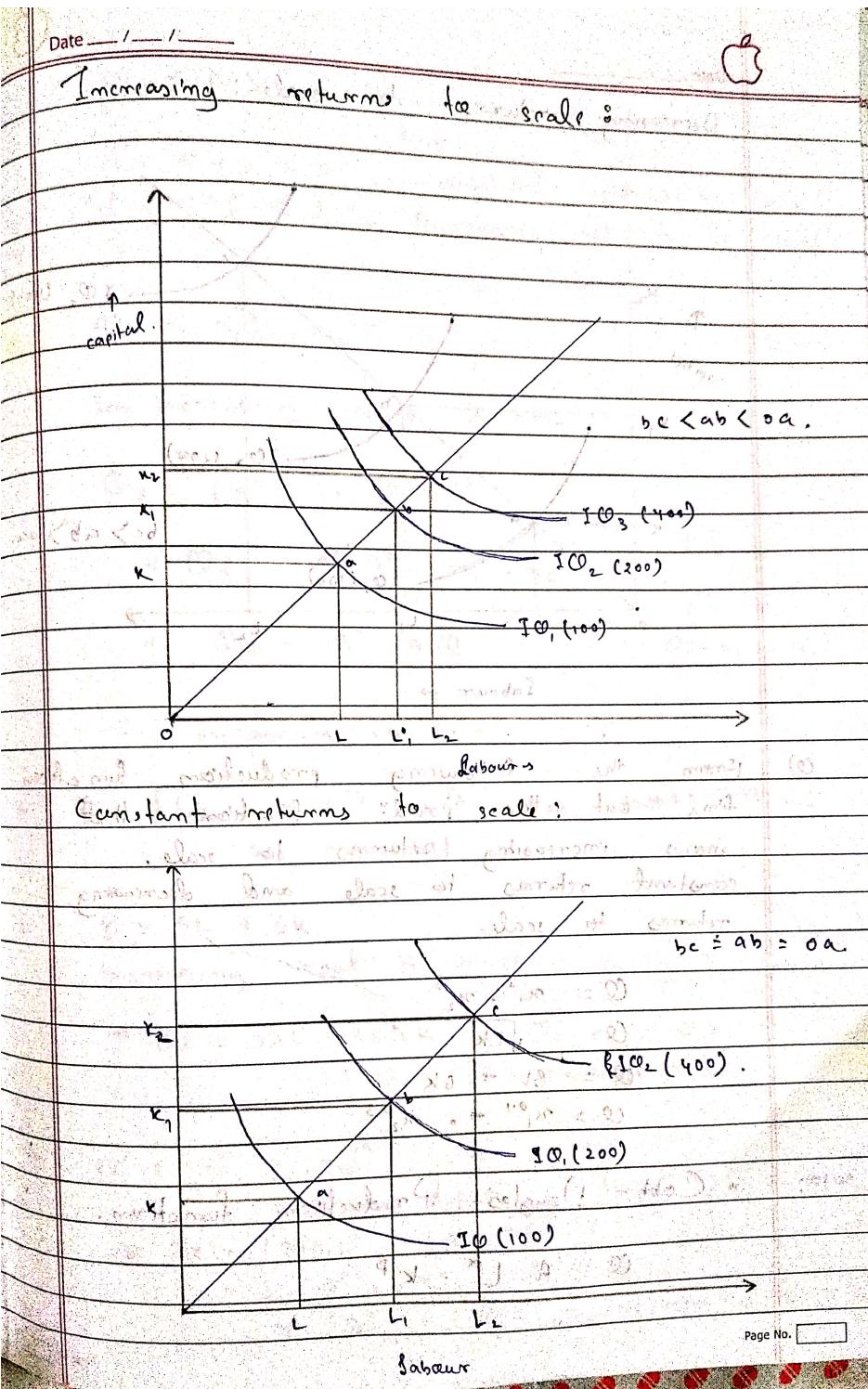
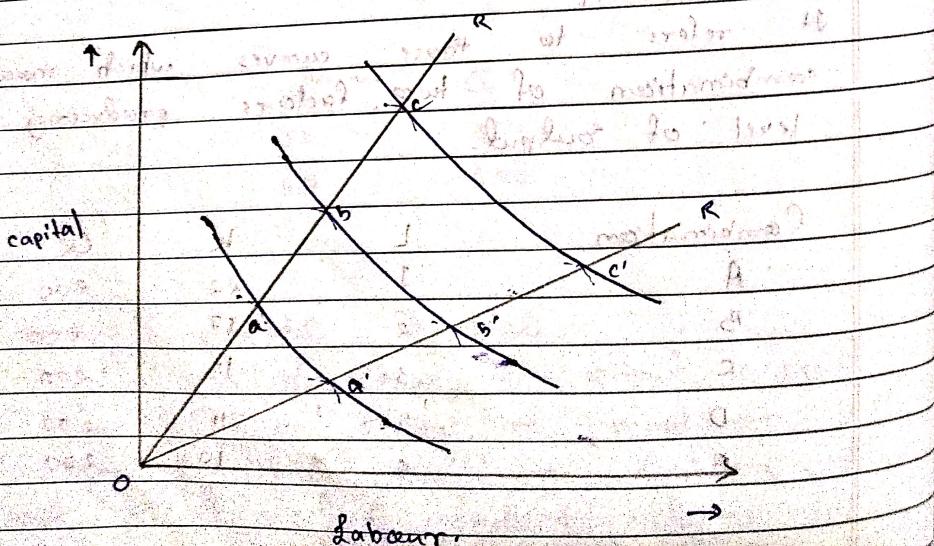
It refers to those curves which shows various combinations of two factors producing same level of output.

### Combinations

	L	K'	Q'
A	1	22	200
B	2	17	200
C	3	13	200
D	4	11	200
E	5	10	200

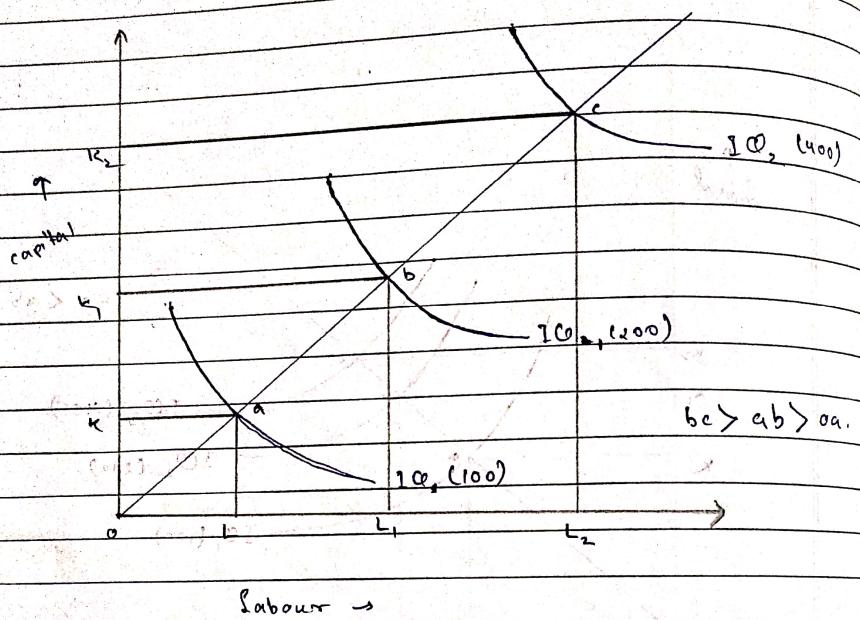


Isocline :  
It refers to the locus of points on various isoquants.



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Decreasing returns to scale:



Labour →

- (a) From the following production functions find out the production functions that shows increasing returns to scale, constant returns to scale and decreasing returns to scale.

$$Q = x_1^2 \cdot x_2^3$$

$$Q = \sqrt{LK}$$

$$Q = BL + GK$$

$$Q = x_1^{0.1} \cdot x_2^{0.3}$$

Soln: \* Cobb-Douglas Production function.

$$Q = A \cdot L^\alpha \cdot K^\beta$$

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If  $\alpha + \beta > 1$ , Increasing returns to scale.

If  $\alpha + \beta = 1$ , constant returns to scale.

If  $\alpha + \beta < 1$ , Decreasing returns to scale.

$$Q = x_1^2 \cdot x_2^3$$

Increasing inputs & time

$$Q_* = (\lambda x_1)^2 \cdot (\lambda x_2)^3$$

$$\therefore Q_* = \lambda^2 \cdot \lambda^3 \cdot x_1^2 \cdot x_2^3$$

$$\therefore Q_* = \lambda^5 \cdot Q \quad \left\{ \because Q = x_1^2 \cdot x_2^3 \right.$$

∴ Increasing m.t.s.  $\Rightarrow$  Increasing

$$Q = \sqrt{LK} = L^{1/2} \cdot K^{1/2} \Rightarrow \text{that } \alpha + \beta = 1$$

so, constant m.s.

$$Q = BL + GK$$

Increasing input  $\&$  times

$$\therefore Q_* = 3\lambda L + 3G\lambda K \Rightarrow Q_* = 3(Q)$$

∴ Increasing m.s.  $\Rightarrow$  Increasing

$$Q = x_1^{0.1} \cdot x_2^{0.3}$$

a increasing input  $\&$  times.

$$\therefore Q_* = (\lambda \cdot x_1)^{0.1} \cdot (\lambda x_2)^{0.3}$$

$$\therefore 2^{0.4} \times (x_1^{0.1} \cdot x_2^{0.3})$$

$$\therefore 2^{0.4} \times (Q)$$

Decreasing m.s.

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## \* Properties of Cobb-Douglas Production function :-

1. The ratios of marginal product and average product gives its exponents.  
 $(MP_L/AP_L = \alpha)$  &  $(MP_K/AP_K = \beta)$ .

2. Elasticity of output is equal to the respective exponents.

Proof:

$$Q = A \cdot L^\alpha \cdot K^\beta$$

$$\frac{dQ}{dL} = A \cdot \alpha \cdot L^{\alpha-1} \cdot K^\beta$$

$$\Rightarrow \frac{dQ}{dL} = \alpha \cdot \frac{A L^\alpha \cdot K^\beta}{L}$$

$$\Rightarrow \frac{dQ}{dL} = \alpha \cdot Q$$

$$\Rightarrow \frac{dQ}{dL} = \alpha \cdot AP_L$$

$$\Rightarrow MP_L = \alpha \cdot AP_L$$

$$\Rightarrow \frac{MP_L}{AP_L} = \alpha$$

$$Q = A L^\alpha K^\beta$$

$$\frac{dQ}{dK} = \beta A L^\alpha K^{\beta-1}$$

$$\Rightarrow MP_K = \beta \cdot \frac{Q}{K} \quad \left\{ Q = A L^\alpha K^\beta \right.$$

$$\Rightarrow MP_K = \beta \cdot AP_K$$

$$\frac{MP_K}{AP_K} = \beta$$

Result:

Elasticity of Output

= Proportionate change in output  
Proportionate change in input.

$$E_L = \frac{\frac{dQ}{dL} \times L}{Q}$$

$$= \frac{MP_L \times 1}{AP_L} = \frac{MP_L}{AP_L}$$

$$\boxed{E_L = \alpha}$$

$$E_K = \frac{\frac{dQ}{dK} \times K}{Q}$$

$$= MP_K \times \frac{1}{AP_K}$$

$$\Rightarrow \frac{MP_K}{AP_K} = \beta$$

$$\therefore E_K = \beta$$

- (Q) From the following product fun<sup>ctn</sup>. find out  
→ m.p. function (short run prod<sup>ctn</sup> fun<sup>ctn</sup> if  
the fixed quantity of capital is 10000 units.

$$O = L^{0.75} K^{0.25}$$

$$\alpha = 0.75 \quad \beta = 0.25$$

Marginal production function:

$$* MP_L = \frac{dO}{dL} = 0.75 L^{-0.25} K^{0.25}$$

$$* MP_K = \frac{dO}{dK} = 0.25 \cdot L^{0.75} K^{-0.75}$$

$$\begin{aligned} * O &= L^{0.75} K^{0.25} \\ &= L^{0.75} (10000)^{0.25} \\ &\propto 10 L^{0.75} \end{aligned}$$

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From the following m.p. fun<sup>ctn</sup> and A.p. fun<sup>ctn</sup> find out elasticity of output w.r.t input.

$$O = 1.50 L^{0.75} K^{0.25}$$

$$\star M.P_L = 1.50 \times 0.75 \times L^{0.75-0.25} K^{0.25} \\ = 1.125 L^{-0.25} K^{0.25}$$

$$M.P_K = 0.375 L^{0.75} K^{-0.75}$$

$$\star A.P_L = \frac{M.P_L}{\alpha} = \frac{1.125 L^{-0.25} K^{0.25}}{0.75} \Rightarrow A.P_L$$

$$\star A.P_K = \frac{M.P_K}{\beta} = \frac{0.375 L^{0.75} K^{-0.75}}{0.25} \\ = 1.5 L^{0.75} K^{-0.75}$$

$$A.P_K = \frac{B M.P_K}{\beta} = \frac{0.375 L^{0.75} K^{-0.75}}{0.25} \\ = 1.5 L^{0.75} K^{-0.75}$$

$$\star E_L = \alpha = 0.75 : \frac{dO/dL \times L/C}{AP_L} = \frac{MP_L}{AP_L}$$

$$E_K = \beta = 0.25 : \frac{dO/dK \times K/C}{AP_K} = \frac{MP_K}{AP_K}$$

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## Short run production function with two variable input.

### Producer's Equilibrium

- Output maximisation.
- Cost minimisation.

- ① Iso-quant / Iso-product curves.
- ② Marginal Rate of Technical Substitution.

### Iso-quant / Iso-product curves.

It refers to those curves which shows various combinations of two factors producing same level of output.

### Marginal Rate of Technical Substitution

It refers to the rate at which number of units of one factor substituted to have one more unit of another factor.

### MRTS<sub>LK</sub>

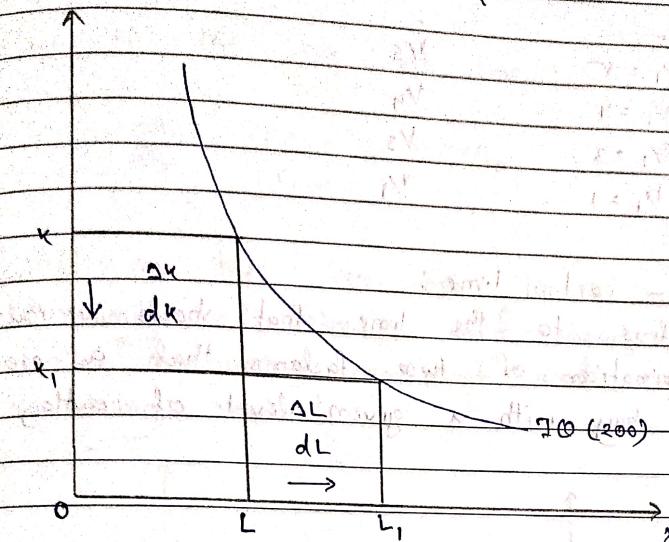
It refers to the rate at which number of units of capital substituted to have one more unit of labour.

### MRTS<sub>KL</sub>

It refers to the rate at which the number of units of labour substituted to have one more unit of labour (capital).

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Slope of the isoquant.



$$MRTS_{LK} = \frac{dK}{dL} = \frac{MP_L}{MP_K}$$

$$MRTS_{KL} = \frac{dL}{dK} = \frac{MP_K}{MP_L}$$

- (i) From the following table find out MRTS<sub>LK</sub> and MRTS<sub>KL</sub>.

Combinations.

	L	K	Q	MRTS <sub>KK</sub>
A	1	25	200	25
B	2	20	200	10
C	3	16	200	16/3
D	4	13	200	13/4
E	5	12	200	12/5

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MRTSLK

$$S_1 = 5$$

$$V_1 = 4$$

$$S_1 = 3$$

$$V_1 = 1$$

MRTSKL

$$V_5$$

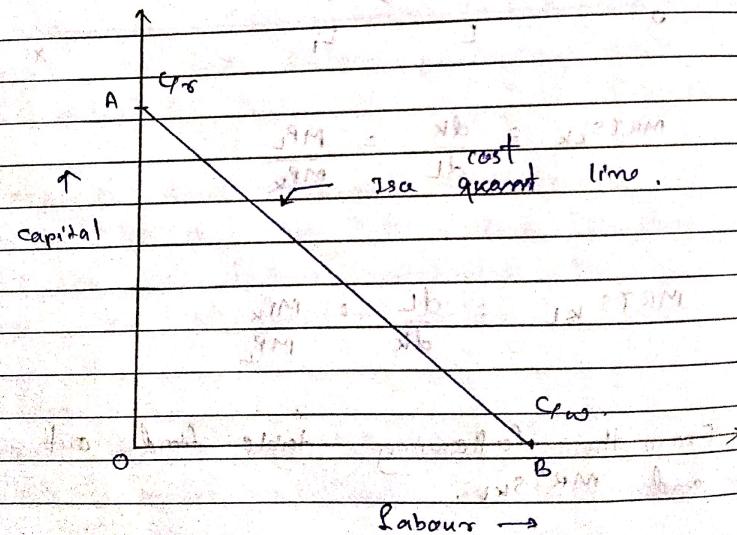
$$V_4$$

$$V_3$$

$$V_1$$

Iso-cost line:

It refers to the line that shows various combinations of two factors that a producer can buy with a given level of outlay.



Equation of the iso-cost line:

$$C = WL + rK$$

L = No. of units of labour used,

w = wages of the labour.

K = No. of units of capital used

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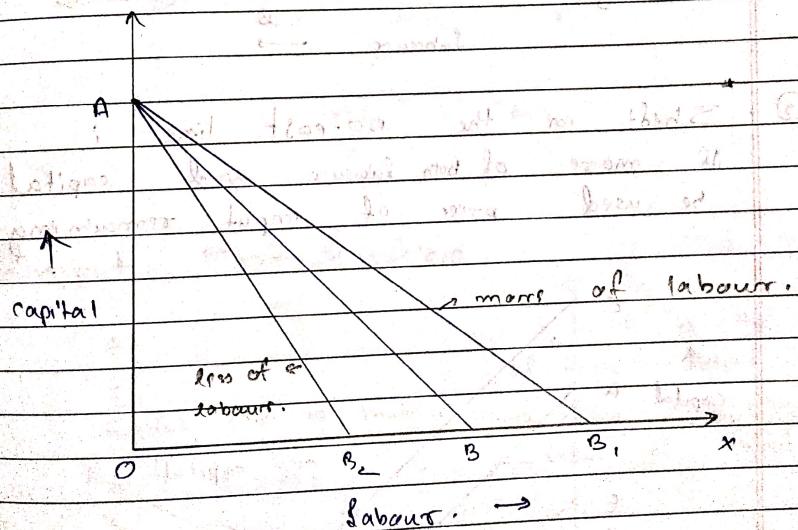
$r$  = price of capital.

slope of the iso-cost line.

$$\frac{OA}{OB} = \frac{C/r}{C/w} = w/r$$

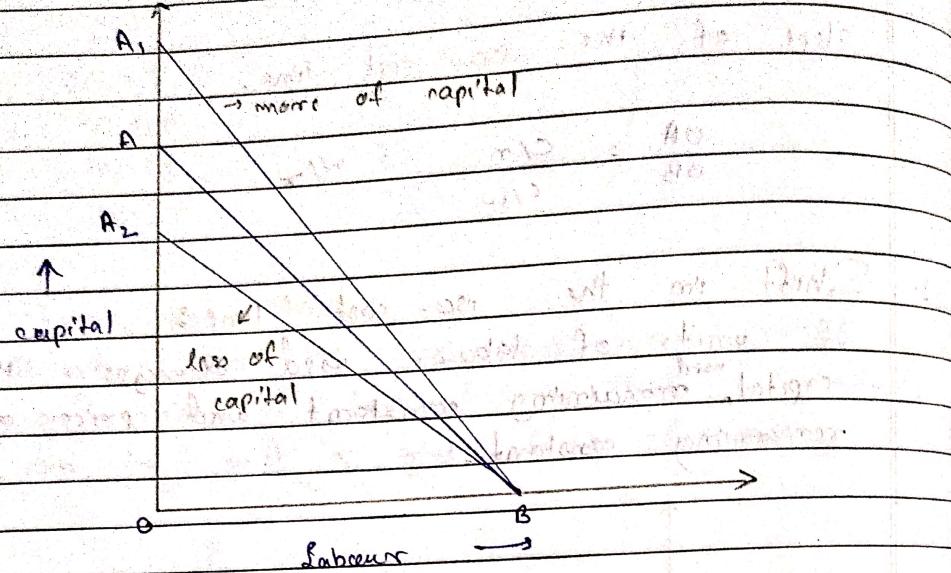
Shift in the iso-cost line:

If units of labour used changes that of capital remaining constant and price of input remaining constant.

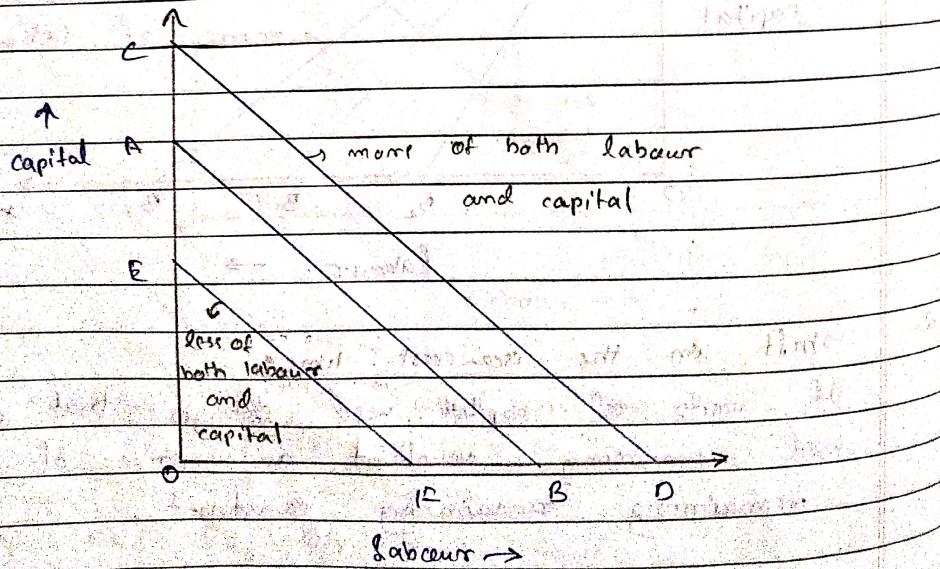


Shift in the iso-cost line:

If units of capital used changes that of labour used remaining constant and price of input remaining constant.



③ Shift in the isocost line : If more of both labour and capital will be used power of input remaining constant



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Conditions for the producer to be in equilibrium.

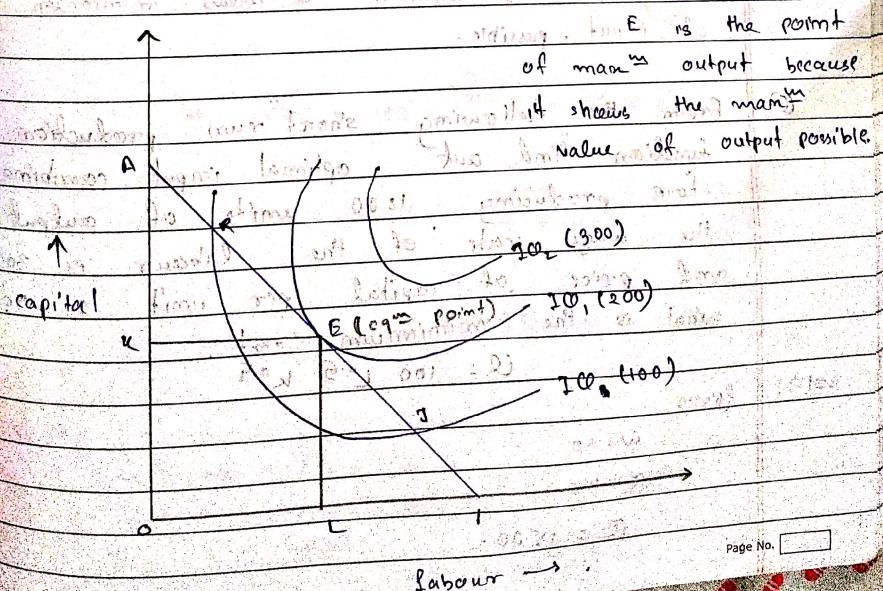
- ① The slope of the isoquant should be equal to the slope of iso-cost line.
- ② The isoquant should be convex at equilibrium point.

Condition 1 :

$$MRTS_{LK} = \frac{dk}{dL} = \frac{MP_L}{MP_K} = \frac{w}{r}$$

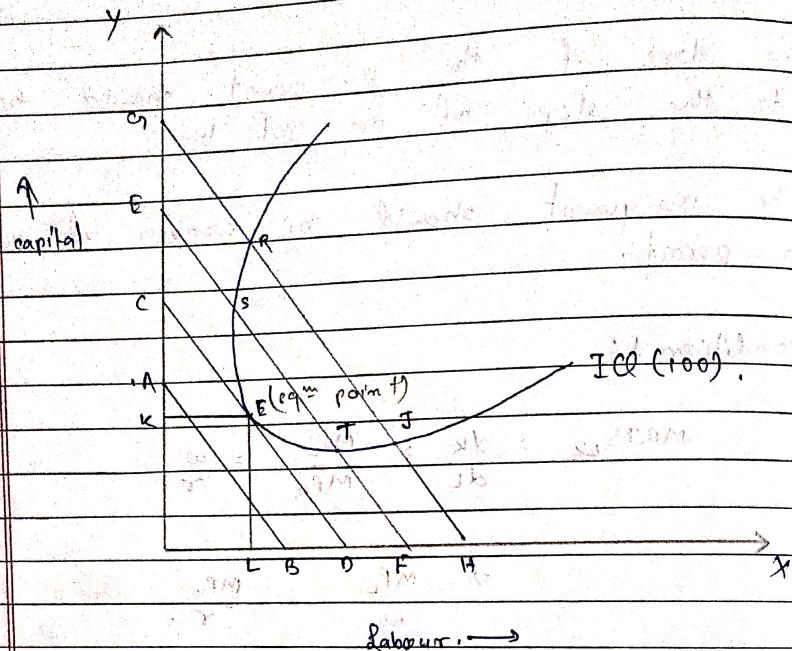
$$2) \frac{MP_L}{w} = \frac{MP_K}{r}$$

Output Maximization.



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## Cost minimisation:



- (\*) E is the point of minimum cost because it lies on C0 which is the shows minimum cost of input, possible.

(Q) From the following short run production function find out optimal input combination for producing 1500 units of output if the wage rate of the labour is 30Rs and price of capital per unit is 40Rs what is the minimum cost.

$$CL = 100 L^{0.5} K^{0.5}$$

SOL:

$$w = 30$$

$$r = 40$$

$$CL = 1500$$

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MP

$$MP_L = \frac{dCL}{dL} = 50 L^{-0.5} K^{0.5}$$

$$MP_K = \frac{dCL}{dK} = 50 L^{0.5} K^{-0.5}$$

$$\text{now, } \frac{MP_L}{MP_K} = \frac{w}{r}$$

$$\therefore \frac{50 L^{-0.5} K^{0.5}}{50 L^{0.5} K^{-0.5}} = \frac{3}{4}$$

$$\therefore L^1 \cdot k^1 = \frac{3}{4}$$

$$\therefore \frac{k}{L} = \frac{3}{4}$$

$$\text{now, so, } 1500 = 100 \times L^{0.5} \times \left(\frac{3L}{4}\right)^{0.5}$$

$$\therefore 1500 = 100 \times L^{0.5} \times 0.866 \times L^{0.5}$$

$$\therefore L = \frac{1500}{100 \times 0.866} = 17.32 \approx 17$$

$$k = 17.32 \times \frac{3}{4} = 12.99 \approx 13$$

so,

$$C = wL + rk$$

$$\therefore C = (30 \times 17.32) + (40 \times 13)$$

$$\therefore C = 1030$$

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For output man =

From the following prod^n fun^n find out  
the quantity of labour and capital that  
the company should use in order to  
maximise output and also find man.  
output

$$W = 30.$$

$$T =$$

$$C = 1800.$$

- \* Fixed cost can never be zero.
- \* Variable cost can be zero.

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## Cost

It refers to the amount of money incurred  
on a thing.

### Cost of Production.

It refers to the expenditures incurred on factors  
of production to produce the output.

### Types of cost:

- ① Fixed cost.
- ② Variable cost.
- ③ Marginal cost.
- ④ Sunk cost.

### Fixed cost:

It refers to those cost which remain fixed  
or remain independent of the level of output  
which doesn't change with level of output.

### Variable cost:

It refers to those cost which changes in direct  
proportion to the level of output.

Fixed cost:  $C_f \rightarrow$  cost of machine installation.

### Output

Output	Cost of machine
100	$10,000$
200	$10,000$
300	$10,000$
400	$10,000$

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Variable cost:

output

100

200

300

400

500

600

cost of raw material.

10,000
20,000
30,000
40,000
50,000
60,000

Marginal cost:

It refers to the net addition to the total cost from producing one extra unit of output.

Output

20

40

60

80

100

Total cost.

200

400

600

800

1000

$$MC = \frac{d(TC)}{dQ}$$

Sunk cost:

It refers to those costs which can't be recovered.



$$\text{Total cost} = TFC + TVC$$

TFC → Total fixed cost

TVC → Total variable cost.

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y

cost

o

TFC

TVC

output → x

① Average Fixed cost (AFC).

It refers to the total fixed cost divided by units of output produced.

AFC =  $\frac{TFC}{Q}$

cost

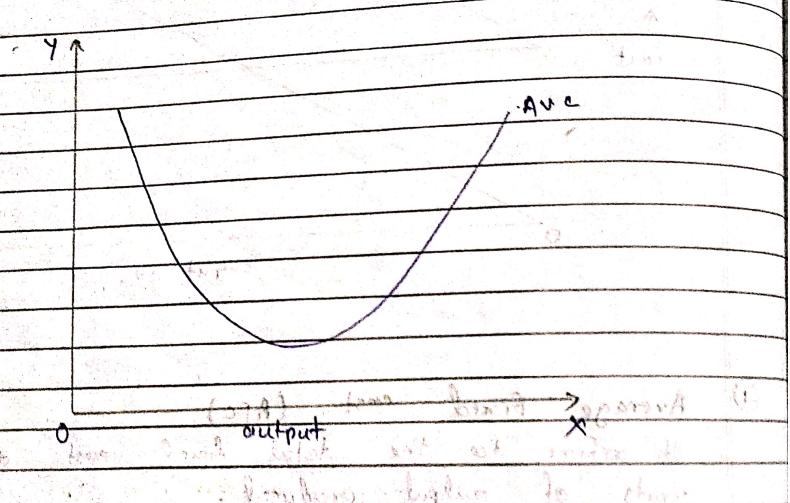
o

output → x

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Average Variable cost.  
It refers to the total variable cost divided by the units of output produced.

$$AVC = \frac{TVC}{Q}$$



Average Total cost.

It refers to total cost of units divided by the units of output produced.

$$AC (ATC) = \frac{TC}{Q}$$

\*  $AC = AFC + AVC$

Proof:

$$AVC = \frac{TVC}{Q}$$

$$AVC = \frac{TVC}{Q}$$

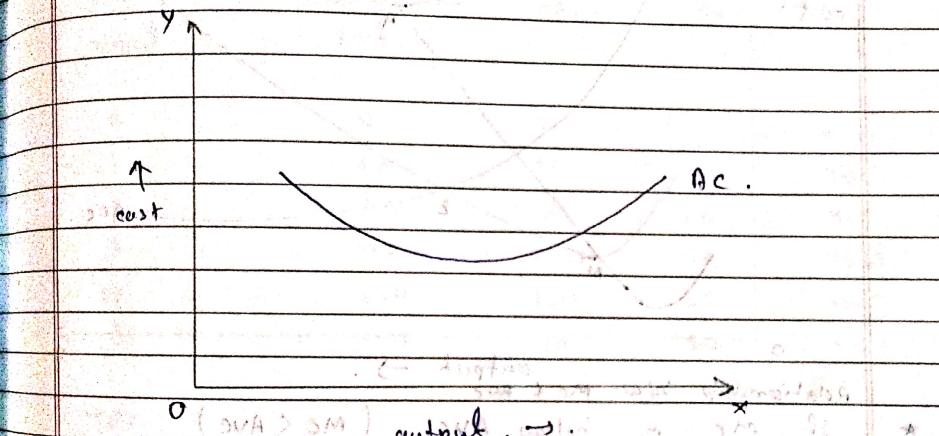
$$AVC + AVC = \frac{TVC}{Q} + \frac{TVC}{Q}$$

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∴ refer 2.  $AVC + AFC = \frac{TC}{Q}$

where  $\frac{TC}{Q} = AC (ATC)$

∴  $AFC + AVC = AC$ .



Proof

$$MC_m = TVC_m - TVC_{m-1}$$

and also  $AVC_m - AVC_{m-1}$

Proof:

$$MC_m = TC_m - TC_{m-1}$$

$$\Rightarrow MC_m = TFC_m + TVC_m - TFC_{m-1} - TVC_{m-1}$$

( $\because TFC_m$  is remaining constant)

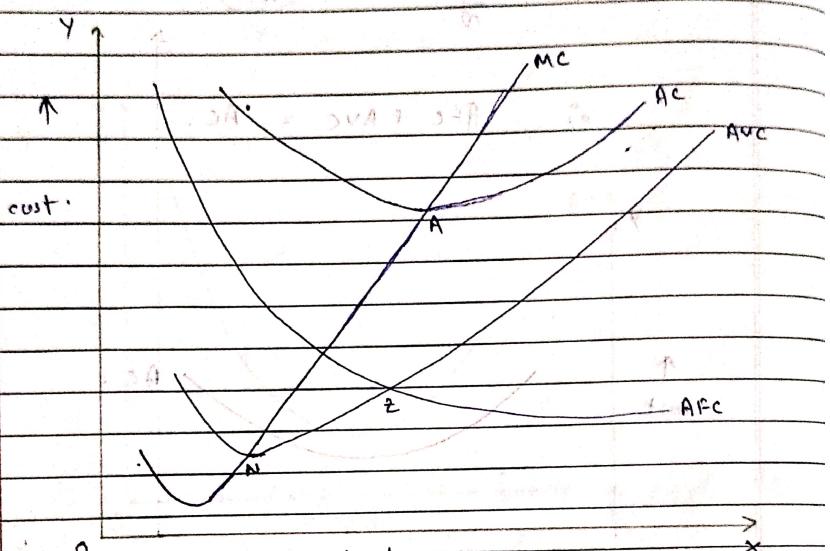
∴  $MC_m = TVC_m - TVC_{m-1}$

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Sc:

$$MC = \frac{d(TVC)}{dQ}$$



Relationship b/w MC & AVC  
output  $\rightarrow$ .

\* If MC is below AVC. ( $MC < AVC$ )

AVC decreases.

\* If MC is above AVC. ( $MC > AVC$ )

AVC is increasing.

\* If  $MC = AVC$  ( $MC = AVC$ )

AVC is constant.

Relationship b/w MC & AC.

\* If  $MC < AC$ , AC is decreasing.

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\* If  $MC > AC$ , AC is increasing.

\* If  $AC = MC$ , AC is constant or minimum.

\* Relationship b/w  $AVC$ ,  $AFC$ ,  $AC$

(i) From the following table find out  $AVC$ ,  $AFC$ ,  $AC$ , if the fixed cost  $AVC$  is 50.

units of Total variable AFC.  $AVC$   $AC$   $AFC$   
output  $\rightarrow$

0	0	50	50	50
1	20	50	20	70
2	40	25	20	45
3	60	19.0	20	46.67
4	80	12.0	20	42.5
5	100	10	20	40

TFC  $\rightarrow$  TC  $\rightarrow$  MC

50  $\rightarrow$  50  $\rightarrow$  -

50  $\rightarrow$  70  $\rightarrow$  20

50  $\rightarrow$  90  $\rightarrow$  20

50  $\rightarrow$  110  $\rightarrow$  20

50  $\rightarrow$  130  $\rightarrow$  20

50  $\rightarrow$  150  $\rightarrow$  20

50  $\rightarrow$  200  $\rightarrow$  20

(ii) From the following cost function find out MC function, AVC function, AC function and also find out at what level of output  $AVC$  &  $MC$  is minimum. If fixed cost is 200.

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$$\begin{aligned} TVC &= 300Q - 12Q^2 + 0.25Q^3 \\ TC &= 300Q - 12Q^2 + 0.25Q^3 + 8200 \end{aligned}$$

Sol:  $\star MC = \frac{d(TC)}{dQ}$

$$\Rightarrow MC = 300 - 24Q + 0.75Q^2$$

$\star AVC = \frac{TVC}{Q}$

$$\Rightarrow AVC = 300 - 12Q + 0.25Q^2$$

$$\star AC = \frac{TC}{Q}$$

$$AC = 300 - 12Q + 0.25Q^2 + 8200/Q$$

$$AC = 300 - 12Q + 0.25Q^2 + 8200/Q$$

$$\star \frac{d(MC)}{dQ} = \star d(MC) = 0.$$

$$\Rightarrow -24 + 0.15Q = 0$$

$$\Rightarrow Q = 24/0.15 = 240/48 = 16.$$

$$\Rightarrow 0Q = 16,$$

$$\star \frac{d(AVC)}{dQ} = 0.$$

$$\Rightarrow -12 + 0.5Q = 0$$

$$\Rightarrow Q = 24.$$

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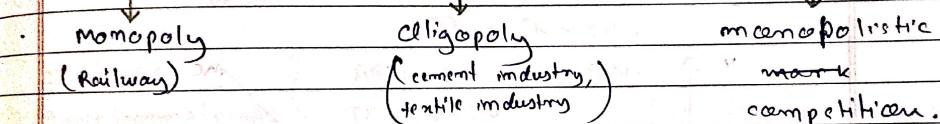
## Market

It refers to the place where non-homogeneous goods are purchased or sold but there is a contact b/w buyers and sellers who agreed at a common price.

Types of market:

→ Perfect competition / perfectly competitive market.

→ Imperfect competition (monopolistic competition)



Perfectly competitive market (not possible in real world)

It refers to that type of market where:

① There are large number of buyers and sellers.

② There is free entry and free exit.

★ Uniform Price.

③ Buyers and sellers are producing and selling homogeneous products.

④ Buyers and sellers have knowledge about the actual price of the product prevailing in the market.

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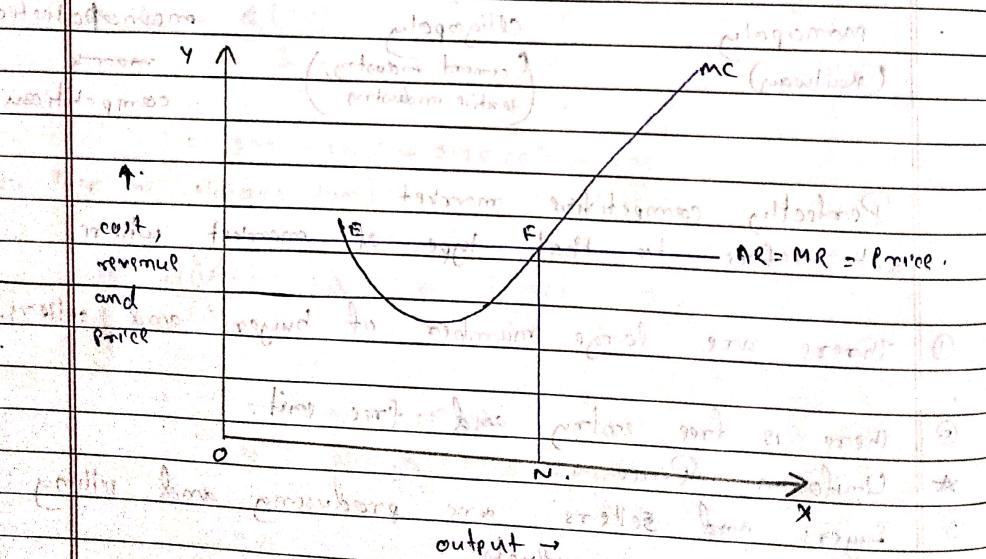


Equilibrium of the firms under perfectly competitive market

- (1) Short run equilibrium of the firms.
- (2) Long-run eqm of the firm.

Condition for the firms short run eqm in the short run under perfectly competitive market.

- (1)  $MR = MC$
- (2) MC is must be increasing at equilibrium point.

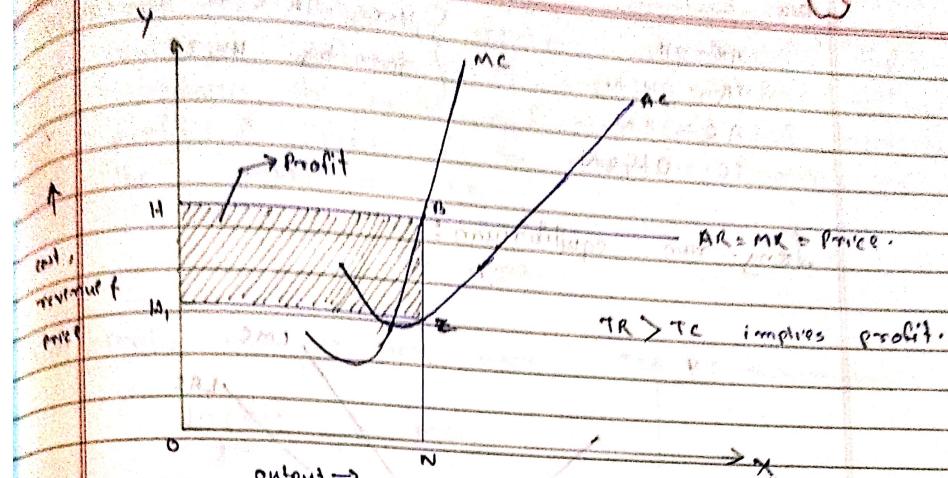


Point F shows the eqm point because it satisfies both the condition, i.e.  $MR = MC$  &  $MC$  is increasing.

①  $TR > TC$ ,  $\Rightarrow \text{Profit}$ .



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if  $AR > AC$

super normal profit

if  $AR = AC$

normal profit.

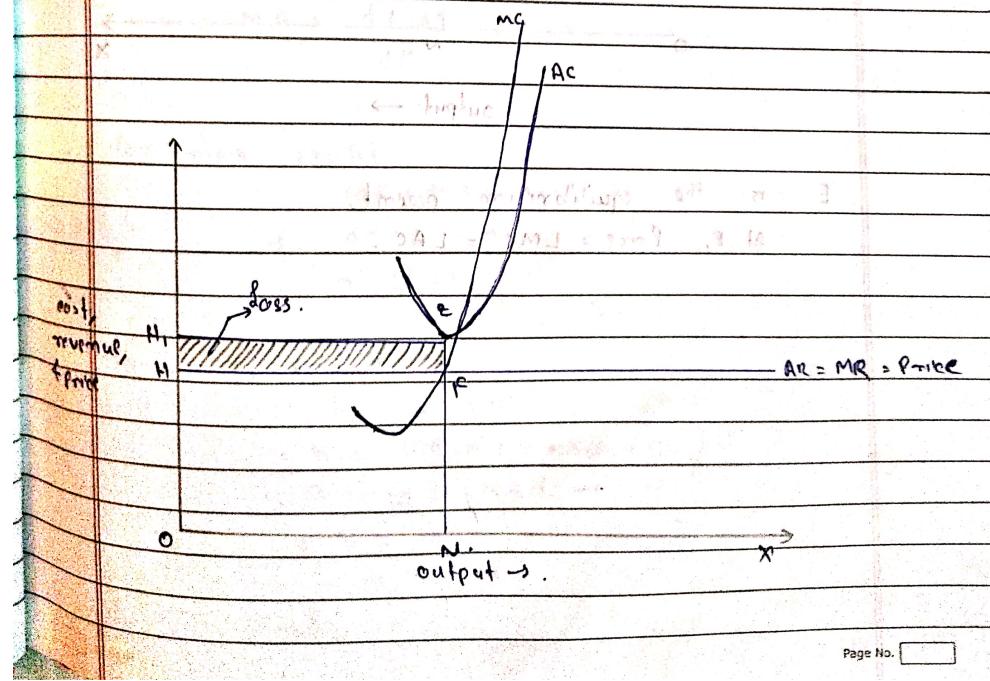
$\Pi = TR - TC$ .

$AR = OH$  or  $BN$ .

$TR = OHBN$

$TC = OHZN$ .

(i.e.  $ARBN = OHZN$ )



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$$AR = OH$$

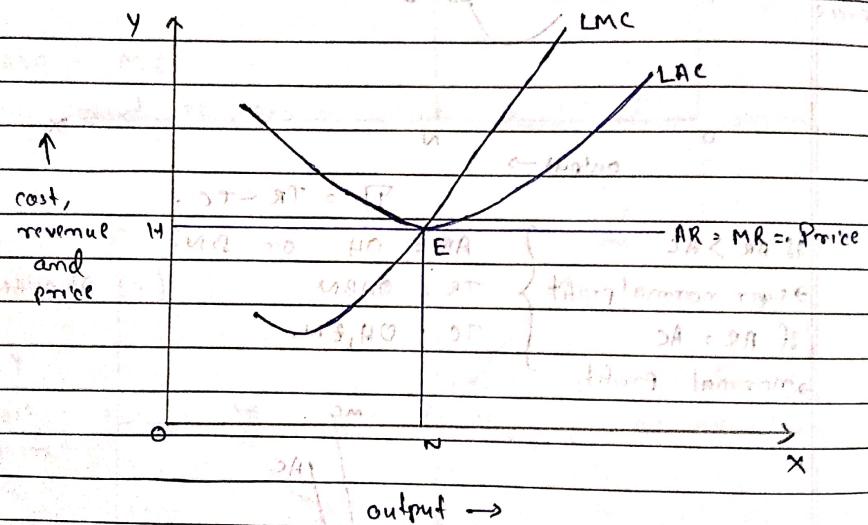
$$TR = OH \cdot FN.$$

$$AC = ZN.$$

$$TC = OH, 2N.$$

Hence,  $TR < TC$  so, loss.  
given by  $HH, ZF$

long-run equilibrium!



E is the equilibrium point,  
At E, Price = LMC = LAC.

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Q) From the following short-run cost function find out at what level of output the firm will maximise its profit and also find out maximum profit if price of the product prevailing in the market is 10 Rs.

$$TC = 4 + 8Q + Q^2$$

Ans: Profit will be max. when  $TC$  is minimum.

$$\frac{d(TC)}{dQ} = MC$$

$$\therefore MC = 8 + 2Q.$$

$$\therefore MR - TR = 10 \times Q$$

$$\therefore MR = \frac{d(TR)}{dQ} = 10 \times Q$$

For max. profit,

$$MR = MC$$

$$\Rightarrow 8 + 2Q = 10 \times Q$$

$$\Rightarrow Q = 1. \quad \text{or} \quad Q = 0.5$$

For max. profit:

$$\Pi = TR - TC = (10 \times Q) - (4 + 8Q + Q^2)$$

$$\Rightarrow \Pi = 10Q - (4 + 8Q + Q^2)$$

$$\Rightarrow \Pi = 10 - 13.$$

$$\Rightarrow \Pi = -3.$$

Profit = -3.

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(Q)

From the following cost function and revenue function find out max. profit of the firm under perfectly competitive market in the short run.

$$TR = 12Q$$

$$TC = 2000 + 4Q + 0.02Q^2$$

soln:  $\text{MR} = d(TR) = 12 \quad \text{and} \quad d(Q) = 12$

$$\frac{dC}{dQ} = MC = \frac{d(TC)}{dQ} = 4 + 0.04Q$$

At equilibrium,

$$MR = MC$$

$$\Rightarrow 12 = 4 + 0.04Q$$

$$\Rightarrow Q = \frac{12 - 4}{0.04} = \frac{8}{0.04} = 200$$

$$\Rightarrow Q = \frac{8}{0.04} = 200$$

now,

$$\Pi = TR - TC$$

$$\Rightarrow 12Q - (2000 + 4Q + 0.02Q^2)$$

$$\Rightarrow (12 \times 200) - (2000 + 800 + 800)$$

$$\Rightarrow 2400 - 3600$$

$$\Rightarrow \Pi = -1200$$

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## Shutting Down Point.

conditions for the short shut down of a firm in perfectly competitive market in short run.

$$\text{Price} = \text{AVC} = \text{MC}$$

(Q) From the following total variable cost fun<sup>n</sup> in the short run under perfectly competitive market, find out the price at which the firm should shut down its production.

$$TVC_Q = 150Q - 20Q^2 + Q^3$$

soln:  $\text{AVC} = \frac{TVC}{Q} = 150 - 20Q + Q^2$

$$MC \Rightarrow$$

\* Minimum AVC

$$\frac{d(\text{AVC})}{dQ} = 0$$

$$\Rightarrow -20 + 2Q = 0$$

$$\Rightarrow Q = 10$$

∴ Price at which

$$\therefore \text{AVC} = 150 - 20Q + Q^2$$

$$= 150 - (20 \times 10) + 100$$

$$= 150 - 200 + 100$$

$$\Rightarrow 50$$

∴ Price at which firm's shut down is 50.

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### Monopoly:

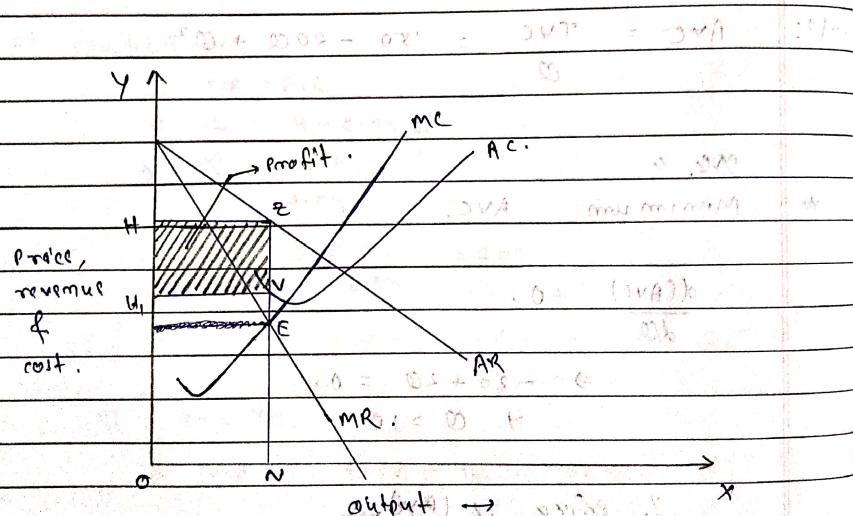
It refers to that type of market where there is a single seller and large number of buyers and there is no restriction for the new firms enter into the industry and firms are producing selling the goods which have no close substitute.

Condition for the monopolist to be in equilibrium

if the short run, long run, firm

medium term of market fails

- ①  $MR = MC$ .
- ② MC must cut MR at firm below.



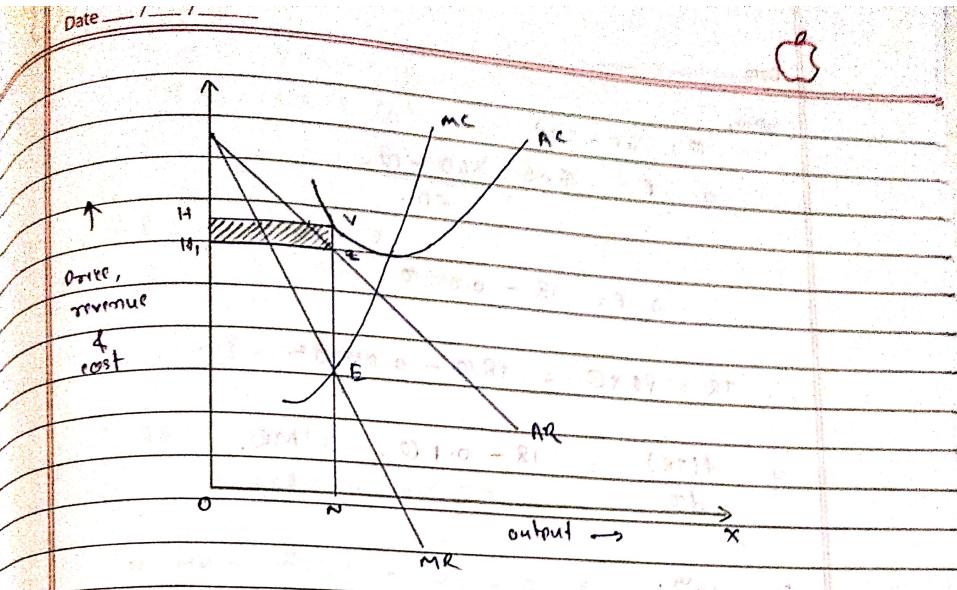
$$AR = OH$$

$$\therefore TR = OH \cdot VN = (OH \times ON)$$

$$AC = OH$$

$$\therefore TC = OH \cdot VN = (OH \times ON)$$

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$$AR = OH$$

$$TR = OH \cdot VN = (OH \times ON)$$

$$AC = OH$$

$$TC = OH \cdot VN = (OH \times ON)$$

- ③ From the following short run cost function and demand function find out max. profit of the monopolist - in the short run.

$$TC = 60 + 0.05Q^2 \quad \left\{ \begin{array}{l} P \rightarrow \text{Price} \\ Q \rightarrow \text{Quantity} \end{array} \right.$$

$$Q = 360 - 20P \quad \left\{ \begin{array}{l} Q \rightarrow \text{Demand} \end{array} \right.$$

$$MC = \frac{d(TC)}{dQ}$$

$$\therefore MC = 6 + 0.1Q$$

$$\text{Now, } TR = P \times Q = 360P - 20P^2$$

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now,

$$0 > 360 - 20P$$

$$\Rightarrow P = \frac{360 - Q}{20}$$

$$\Rightarrow P = 18 - 0.05Q$$

$$\therefore TR = P \times Q = 18Q - 0.05Q^2$$

$$\Rightarrow \frac{d(TR)}{dQ} = 18 - 0.1Q = MR$$

For eq<sup>m</sup>: MR = MC

$$MR = MC$$

$$\Rightarrow 18 - 0.1Q = 6 + 0.1Q \quad (MC = 6)$$

$$\Rightarrow 0.2Q = 12$$

$$\Rightarrow Q = 120, \quad MC = 6$$

$$\text{Total cost} = (MC \times Q) = 6 \times 120 = 720$$

Now, for max profit, profit must be zero

$$\Pi = TR - TC \quad \text{and} \quad \Pi = (18Q - 0.05Q^2) - (6Q + 0.05Q^2)$$

$$\therefore \Pi = 12Q - 0.1Q^2$$

$$\therefore \Pi = 360 - 9Q - 0.05Q^2 - 6Q - 0.05Q^2$$

(c) From the following cost and demand fun<sup>m</sup>  
find the eq<sup>m</sup> price and quantity for  
the monopolist and also find out  
max profit.

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$$Q = 400 - 20P$$

$$TC = 5Q + C_2$$

$$P = \frac{400 - Q}{20}$$

$$\Rightarrow P = 20 - 0.05Q$$

$$\therefore TR = P \times Q$$

$$= 20Q - 0.05Q^2$$

$$\Rightarrow MR = \frac{d(TR)}{dQ} = 20 - 0.1Q$$

$$\star \quad MC = \frac{d(TC)}{dQ} = 5 + \frac{Q}{25}$$

For eq<sup>m</sup>, we have to solve

i.e., MR = MC. i.e., 20 - 0.1Q = 5 + Q/25

$$\Rightarrow 20 - 0.1Q = 5 + \frac{Q}{25}$$

$$\Rightarrow 15 = 0.1Q + 0.04Q$$

$$\Rightarrow 15 = 0.14Q$$

$$\Rightarrow Q = 107.14$$

$$\therefore \text{Max } \Pi = TR - TC - \frac{d(TC)}{dQ} \cdot Q$$

$$= (20Q - 0.05Q^2 - 5Q - \frac{Q^2}{25})$$

$$\therefore \Pi = 803.57$$

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## Depreciation.

It refers to the loss in the value of any asset due to its constant use.

Methods:

- (1) Straight line method.
- (2) Declining Balance method.
- (3) Sum-of-the-years-digit-method.
- (4) Sinking fund method.

Straight line method:

It refers to that method of depreciation where a fixed rate of depreciation is charged directly on the initial value of asset on various years.

$$(1) \text{ Depreciation amount per year } (D) = \frac{I - S}{N}.$$

I → initial value of the asset.

S → Salvage value / reduced value of asset.

N → number of years / life of the asset.

(2) Rate of depreciation (d)

$$= \frac{D}{I} \times 100.$$

$$(3) \text{ Book value } (B_t) = I - (t \times D) \quad \{ \text{or } B_{t-1} - D_t \}$$

t → the number of years the asset has been used.

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An equipment has been purchased at 80,000 rs. If estimated salvage value of the equipment at the end of the life of the equipment 10<sup>th</sup> year left of life of equipment is 1000. Find out depreciation amount and book value of the asset after 5<sup>th</sup> year, with the help of straight line method.

$$I = 80,000 \text{ rs.}$$

$$S = 1000 \text{ rs.}$$

$$N = 5 \text{ yrs.}$$

$$D = \frac{I - S}{N}.$$

$$\therefore D = \frac{80,000 - 1000}{5} = 15800. 7900 \text{ rs.}$$

10

$$* \text{ Book value} = I - (t \times D)$$

$$= 80,000 - (5 \times 7900).$$

$$\therefore 40500. \text{ rs.}$$

(4) Rate of depreciation

If initial value of a machine is 50,000 rs. with estimated salvage value of 5000 rs. at the end of its service life of 8 years find out depreciation amount and book value of various / every year.

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End of the year  $B_t$ .

0	50,000	50,000
1	5625	44375
2	5625	38750
3	5625	33125
4	5625	27500
5	5625	21875
6	5625	16250
7	5625	1
8.	5625	

Given,

$$I = 50,000 \cdot 0.03 = 1500 - 0.000375 \cdot 0$$

$$S = 5000 \cdot 0.03 = 1500$$

N.F.

$$(1 + I) - S = \text{value next year}$$

$$0 = 50,000 - 5000 \cdot 0.03 = 45,000$$

$$50,000 \cdot (0.87 \times 0.03) = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^2 = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^3 = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^4 = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^5 = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^6 = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^7 = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^8 = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^9 = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^{10} = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^{11} = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^{12} = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^{13} = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^{14} = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^{15} = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^{16} = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^{17} = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^{18} = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^{19} = 1,890$$

$$50,000 \cdot (0.87 \times 0.03)^{20} = 1,890$$

### Declining Balance method.

It refers to that method of depreciation where a fixed rate of depreciation is charged on the declining balance for various years. This is also known as the straight line method.

$$\textcircled{1} \quad D_t = k \times B_{t-1}$$

$k$  = fixed percentage.

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$$B_t = B_{t-1} - D_t$$

- \( \textcircled{2} \) If a machine has been purchased at 90,000 rs. find out its depreciation amount and book value for every year till the end of the  $t^{\text{th}}$  year of the life of the machine if the fixed rate of depreciation is 30% using declining balance method.

End of the year.

$D_t$ .

$B_t$

0	90,000
1	63,000
2	44,100
3	30,870
4	21,609
5	15,126.3
6	10,588.41
7	7,411.887

### \* Sum of the year's digit method of depreciation:

$$\textcircled{1} \quad \text{Sum of the year } \frac{N(N+1)}{2}$$

$$\textcircled{2} \quad D_t = \text{Rate } (I-S)$$

$$\text{Rate} = \frac{\text{Rank}}{\text{sum of the year}}$$

$$\textcircled{3} \quad B_t = B_{t-1} - D_t$$

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- (Q) If an asset has been purchased at ₹1,00,000 with estimated salvage value of ₹40,000 at the end of its service life of 10 yrs. Find out depreciation amount and book value at the end of various years with the help of sum of the year digit method.
- Soln:  $I = 2,00,000 \text{ rs.}$  sum of the years =  $10 \times 11 / 2 = 55$   
 $S = 40,000.$   $\text{Rate } D_t = \frac{I}{55} (I-s).$   
 $N = 10.$

End of the year.	$D_t$	$B_t$
1	10	29090.90
2	9.55	26181.81
3	8.33	23272.72
4	7.22	20363.63
5	6.25	17454.55
6	5.45	14545.45
7	4.76	11636.36
8	4.17	8727.27
9	3.65	5818.18
10.	1	2909.09

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### Sinking Fund Method.

$$\begin{aligned} ① A &= (I-s) \left[ \frac{i}{(1+i)^m - 1} \right] \\ ② D_t &= A \cdot (1+i)^{t-1} \\ ③ B_t &= B_{t-1} - D_t \end{aligned}$$

- (Q) If  $I = 100,000 \text{ rs.}, S = 20,000, m = 4 \text{ yrs.}$  Find out depreciation amount and book value for various years if  $i = 12\% \text{ compounded annually}$  with the help of sinking fund method.

Soln:  $I = 1,00,000.$   $i = 0.12$   
 $S = 20,000.$   $m = 4$  yrs.  $t = 4$  years  
 $A = ?$   $\therefore A = I/(I-s) \left[ \frac{i}{(1+i)^m - 1} \right]$

$$A = 100,000 / (100,000 - 20,000) \times \left[ \frac{0.12}{(1.12)^4 - 1} \right]$$

$$A = 16738.75 \text{ rs.}$$

$$D_t = A \cdot (1+i)^{t-1}$$

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End of year

	A	B	C
0	16738.754	1,00,000.	
1	16738.754	83201.246.	
2	16938.754	64513.842	
3	16938.754	43516.749	
4	16938.754	20000.	

### Break - Even Analysis

It refers to that analysis which discusses the behaviour of total revenue and total cost as the level of output changes.

#### Break - Even point (BEP)

It refers to that point where  $TR = TC$ .

It is also called point of no profit.

$$TR = TC$$

so,  $\Pi = 0$

$$\{ \begin{aligned} TR &= P \times N \\ \Pi &= 0 \end{aligned}$$

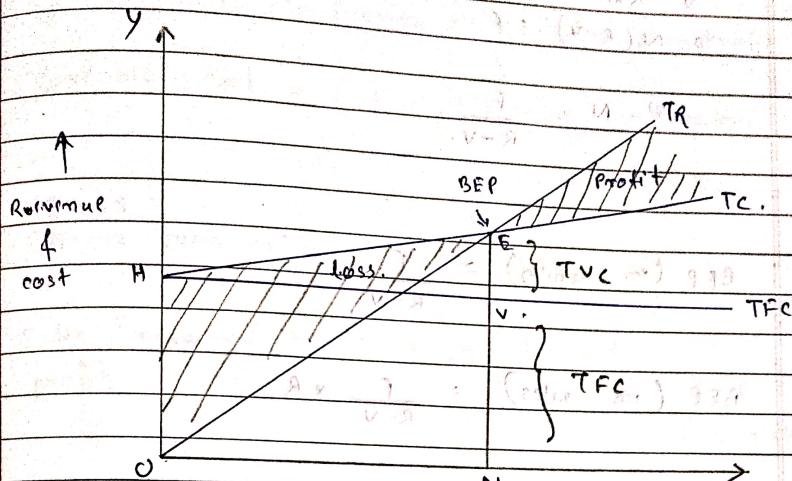
#### Methods of calculating BEPA

① Graphical method.

② Algebraic method.

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Graphical method:



Algebraic method:

R → selling price per unit.

N → number of units of output sold.

V → variable cost per unit.

F → Total Fixed cost.

At BEP,

$$TR = TC$$

$$\bullet TR = P \times N$$

$$\bullet TR = R \times N$$

$$\bullet TC = TFC + TN$$

$$\bullet TC = F + VN$$

From ①

$$\therefore R \times N = F + VN$$

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$$\begin{aligned} \textcircled{1} \quad & RN - VN = F \\ \Rightarrow & N(R-V) = F \end{aligned}$$

$$\Rightarrow N = \frac{F}{R-V}.$$

$$\textcircled{1} \quad \text{BEP (in units)} = \frac{F}{R-V}.$$

$$\textcircled{2} \quad \text{BEP (in sales)} = \frac{F}{R-V} \times R$$

$$\textcircled{3} \quad \text{Contribution to per unit (c)} = R-V.$$

$$\textcircled{4} \quad \text{Profit} = \text{Total contribution} - F.$$

$$\textcircled{5} \quad \text{Total contribution} = F + P.$$

$$\textcircled{6} \quad \text{Units required to have a targeted profit} = \frac{F+P - \text{targeted profit}}{R-V}.$$

$$\textcircled{7} \quad \text{Sales required to have a targeted profit}$$

$$= \frac{F+P}{R-V} \times R$$

$$\textcircled{8} \quad \text{Margin of safety} = \text{Actual sales} - \text{BES}.$$

$$\textcircled{9} \quad \text{P/V ratio} = \frac{\text{Contribution per unit}}{\text{Selling price per unit}}$$

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$$\textcircled{10} \quad \text{Fixed cost} = (\text{sales} \times \text{P/V ratio}) - \text{Profit}.$$

$$\textcircled{11} \quad \text{Variable cost} = \left(1 - \frac{P}{V} \text{ ratio}\right) \times \text{sales}.$$

\textcircled{12} BES.

$$(\text{Break even sales}) = \frac{F}{\text{P/V ratio}}$$

$$\textcircled{13} \quad \text{Sales required to have a targeted profit}$$

$$= \frac{F+P}{\text{P/V ratio}}$$

$$\textcircled{14} \quad \text{Margin of safety} = \frac{\text{Profit}}{\text{P/V ratio}}.$$

$$\textcircled{15} \quad \text{P/V ratio} = \frac{\text{change in profit} \times 100}{\text{change in sales}}$$

\textcircled{16} From the following information calculate.

BEP, P/V ratio.

Selling price / unit = 80 rs

variable cost / unit = 20 rs

Total Fixed cost = 20,000 rs

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SOL:

$$\text{Given, } R = 30 \text{ Rs.}, V = 20 \text{ Rs.}$$

$$P = 20,000.$$

$$\star \text{ BEP} = \frac{F}{R-V} = \frac{20,000}{(30-20)} = 20,000 \text{ Rs.}$$

$$\star \text{ BEP (in units)} = \frac{F}{R-V} = \frac{20,000}{10} = 2000 \text{ units.}$$

$$\star \text{ P/V ratio} = \frac{R-V}{R} \times 100 = \frac{10}{30} \times 100 \% = 33.33\%.$$

(Q) P/V ratio, fixed cost, variable cost in 2010, BEP (in sales), sales required to earn a profit of 20,000 Rs.

Year	Sales	Cost
2010	1,20,000	1,10,000
2011	1,40,000	1,27,000

$$\text{Given, } R = 30 \text{ Rs.}, V = 20 \text{ Rs.}$$

$$\text{Profit} = 20,000 \text{ Rs.}$$

$$13,000.$$

$$\star \text{ P/V ratio} = \frac{\text{change in profit}}{\text{change in sales}} \times 100.$$

$$\Rightarrow \text{P/V ratio} = \frac{20,000}{10} \times 100 = 200\%.$$

$$\Rightarrow \text{P/V ratio} = 20\%.$$

$$\star \text{ Fixed cost} = (\text{sales} \times \text{P/V ratio}) - \text{Profit}$$

$$= (1,20,000 \times 0.2) - 9,000.$$

$$\star \text{ Variable cost} = (1 - \text{P/V ratio}) \text{ sales.}$$

$$\Rightarrow (1 - 0.2) \times 1,20,000.$$

$$\star \text{ BEP (Sales)} = \frac{F}{\text{P/V ratio}} = \frac{15,000}{0.2}$$

$$\star \text{ Sales to have a profit of 20,000} = 15,000 + 20,000$$

$$\Rightarrow 1,75,000 \text{ Rs.}$$

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- ① Topic name, & roll no., name, submitted to.
- ② Certificate.
- ③ Acknowledgement.
- ④ Indian.
- ⑤ Introduction.
- ⑥ Topic. (photos). (10 pages → write up + diagram)
- ⑦ Conclusion.

(Q) From the following information find out  
i) P/V ratio: ii) BEP (units) iii) Profit when the output  
is 50,000 units.

$$F = 1,20,000 \text{ Rs} = V(= 5 \times 10,000) R(= 7) =$$

$$\text{Soln: i) P/V ratio} = \frac{R-V}{R} \times 100 = \frac{4}{7} \times 100 = 57.143\%.$$

$$\text{ii) BEP (units)} = \frac{F}{R-V} = \frac{1,20,000}{4} = 30,000 \text{ units}$$

$$\text{iii) BEP (sales)} = \text{BEP (units)} \times \frac{R}{V} = 30,000 \times 7 = 2,10,000 \text{ Rs}$$

iv) Given, units required to have a target profit = 50,000

$$\Rightarrow 50,000 = \frac{F + P}{R-V}$$

$$\Rightarrow P = 50,000 \times (4) - 1,20,000$$

$$\Rightarrow P = 80,000 \text{ Rs.}$$

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## Inflation

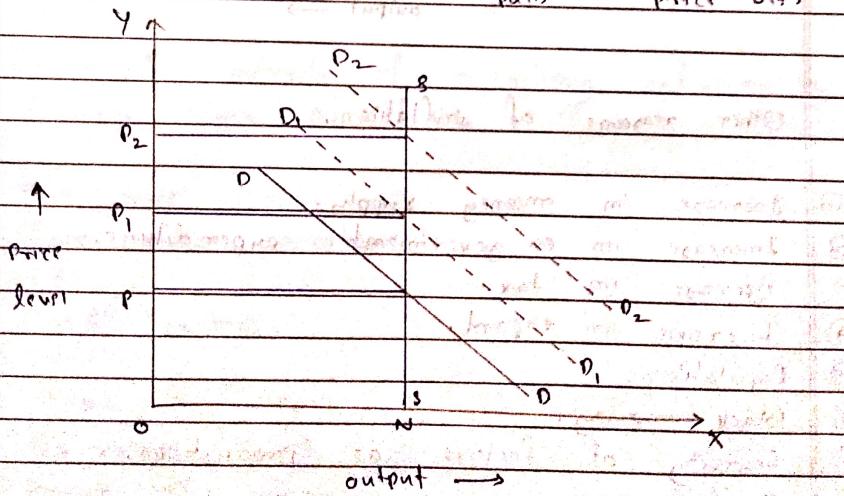
It refers to a substantial and continuous rise in price level which reduces purchasing power of money.

Causes of inflation:

- ① Demand - Pull inflation.
- ② Cost - Push inflation.

Demand - Pull inflation.

It refers to that type of inflation where aggregate demand for goods and services of an economy is more than aggregate supply of goods and services resulting in shortage of goods and services which pulls the price off.

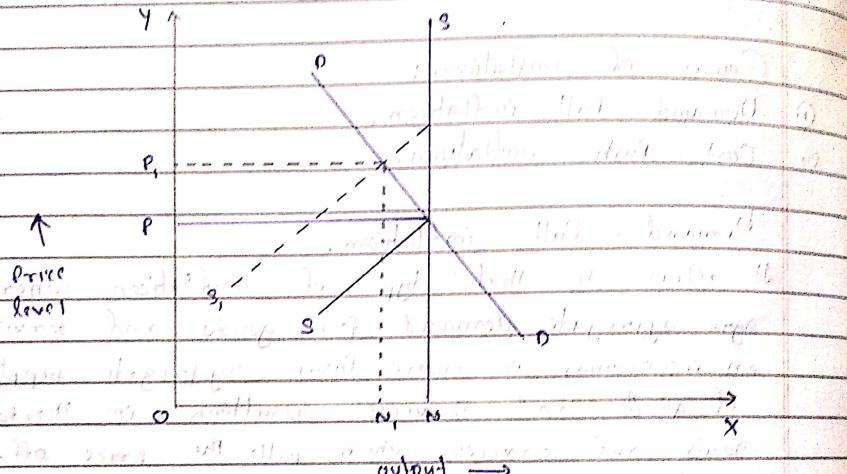


Cost - Push inflation.

It refers to that type of inflation which results from increase in the cost of production because of which instead of increasing the price,

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producers reduces the quantity supplied to the market. i.e. Inflation is due to the increase in demand, because there hasn't been any change in supply.



Other reasons of inflation.

- ① Increase in money supply.
- ② Increase in govt. expenditure.
- ③ Decrease in tax.
- ④ Increase in export.
- ⑤ Population.
- ⑥ Black money.
- ⑦ Scarcity of factors of production.
- ⑧ Trade union.

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Factors of controlling inflation.

- ① Monetary Policy.
- ② Fiscal Policy.

Monetary policy.

- ① Increase in bank rate.
- ② sale and purchase of government securities.
- ③ Increase in cash reserve ratio (C.R.R.).
- ④ Credit control.

Fiscal Policy:

- ① Increase in tax.
- ② Decrease in government expenditure.
- ③ Increase in public borrowing.

Quantitative methods of controlling inflation by Reserve Bank of India

- ① Bank rate.
- ② Open market operation.
- ③ CRR.
- ④ Credit control.

Q) From the following information find out P/V ratio, Fixed cost, BEP, variable cost, Margin of safety.

Particulars	2006	2007
Sales	1,50,00,000	2,00,00,000
Profit	30,00,000	50,00,000

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sol 9:

\* P/V ratio =  $\frac{\text{change in profit} \times 100}{\text{change in sales}}$

$$= \frac{20,00,000 \times 100}{50,00,000}$$

$\therefore$  P/V ratio  $= \frac{20,00,000 \times 100}{50,00,000} = 40\%$   
 (i.e.,  $\frac{2}{5} \times 100 = 40\%$ )

\* Fixed cost = (sales  $\times$  P/V ratio) - Profit.

$$= (1,50,00,000 \times 0.4) - 30,00,000$$

$$= 80,00,000 \text{ rupees}$$

\* B.E.S =  $\frac{\text{Profit}}{\text{P/V ratio}} = \frac{30,00,000}{0.4} = 75,00,000$

\* Margin of Safety =  $\frac{\text{Profit}}{\text{P/V ratio}}$

$\therefore$  Margin of Safety =  $30,00,000 \text{ at } 0.4 = 75,00,000$

∴ Total sales =  $75,00,000 \text{ at } 0.4 = 187,50,000$

\* Variable cost  $MOS_{2007} = 187,50,000 \text{ at } 0.4 = 75,00,000$

$\therefore$  Total sales =  $187,50,000 \text{ at } 0.4 = 468,75,000$

$\therefore$  Total sales =  $468,75,000 \text{ at } 0.4 = 1,25,00,000$

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Ques.

\*  $V_{2006} = (1 - \text{P/V ratio}) \times \text{sales}$

$$= (1 - 0.4) \times 1,50,00,000$$

$$= 90,00,000 \text{ rupees}$$

\*  $V_{2007} = (1 - 0.4) \times 2,00,00,000$

$$= 1,20,00,000 \text{ rupees}$$