15/07/2022

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Modules (CS-3031)

1. Introduction to Neuro Fassy and Soft Computing.

2. Fossy Set Theory.

3. Fossy rules, Fossy Reasoning and Fossy Inference System.

4. Optimisation.

5. Artificial Neural Network

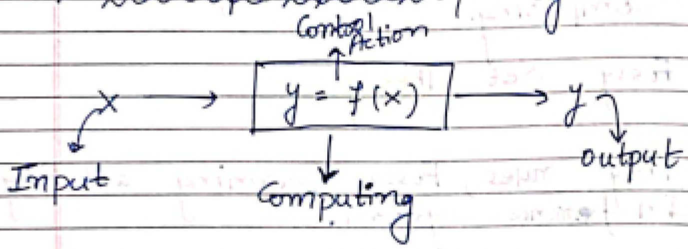
6. Neuro Fassy Models.

* Ref book → Neural Networks, Fuzzy Logic and Genetic Algorithms. {by Rajasekaran}

* Soft Computing is the synonym of Computational Intelligence.

→ Lotfi A Zadek (1994)
↳ the scientist who invented Computational Intelligence / Soft Computing.

⇒ Concept of Computing



f → mapping function / Algorithm

⇒ The Characteristics of Computing

[*] Precise Solution.

[*] Control Action should be unambiguous and accurate.

[*] Mathematical Model

⇒ Hard Computing

↳ LA Zade (1996)

[*] Characteristics

1. Precise Result is guaranteed.
2. Control Action is unambiguous.
3. Control Action is formally defined with Mathematical Model, or algorithm.

→ Examples → Solving numerical problems, searching and sorting techniques, Solving Computing Geometry Problems. {eg. finding closest pair of points given a set of points.

⇒ Soft Computing

→ Characteristics

1. Imprecision
2. Ambiguous data / Noisy data / Inaccurate data

③ It does not require any mathematical model.

④ Low solution cost.

⑤ Uncertanity

⑥ Adaptive / Dynamic

** Def

Soft Computing is a set of methodology that aim to exploit the tolerance for impreecision and uncertanity to acheive tractability, robustness and low solution cost.

* Role Model for soft computing is the human mind.

ANN {Artificial Neural Networks}	Fuzzy Logic	GA {Genetic Algorithms}
↳ Knowledge Base	↳ Uncertanity	↳ Past Experience
↳ eg. Student-Teacher Interaction	eg:- Doctor-Patient Interaction	eg:- Who will win the IPL
eg:- Hand Written Character Recognition	eg. Weather forecasting	eg. Invest money in MF
eg. Face Recognition	↳ It can be low, medium or high.	
eg. Fingers Point Detection		

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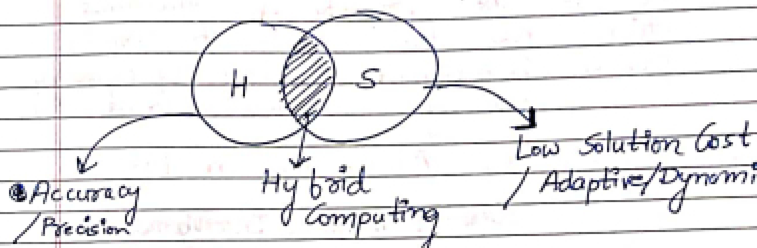
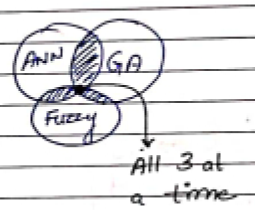
Difference between Hard Computing and Soft Computing

Hard Computing	Soft Computing
(1) It deals with Accurate & precise Solutions	(1) It deals with imprecise solutions.
(2) It deals with Accurate data or unambiguity data.	(2) It deals with noisy or ambiguity data.
(3) It deals with Certainity.	(3) It deals with uncertainity.
(4) It requires mathematical model.	(4) It does not require mathematical model.
(5) Costly.	(5) Low solution costs
(6) Sequential Computation.	(6) Parallel Computation.
(7) Binary logic {0, 1 True/False Low/High}	(7) Crisp logic / Fuzzy logic {0, 0.5, 0.3 High/Med/Low

⇒ Hybrid Computing



(*) Hybrid Soft Computing



→ ANN - GA

→ GA - Fuzzy

→ ANN - Fuzzy

→ ANN - GA - Fuzzy.

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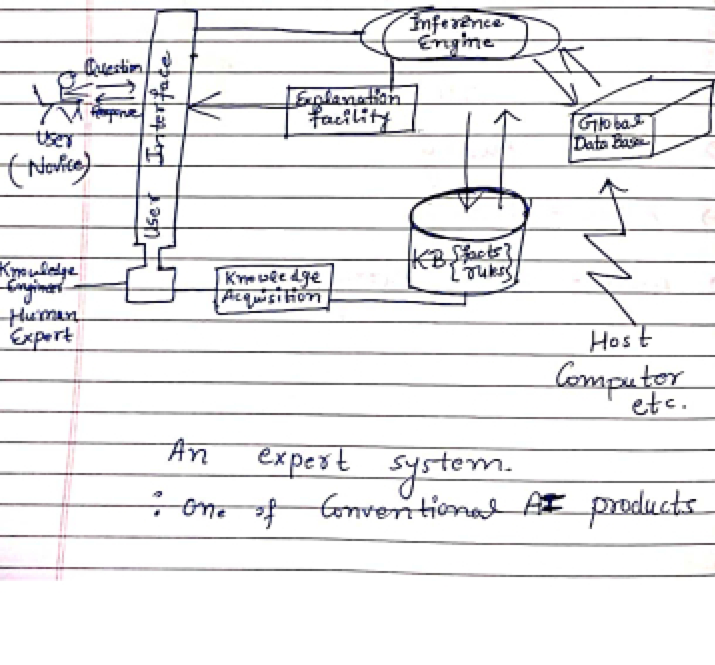
Intro to Neuro-Fuzzy & Soft Computing characteristics

Artifical Intelligence

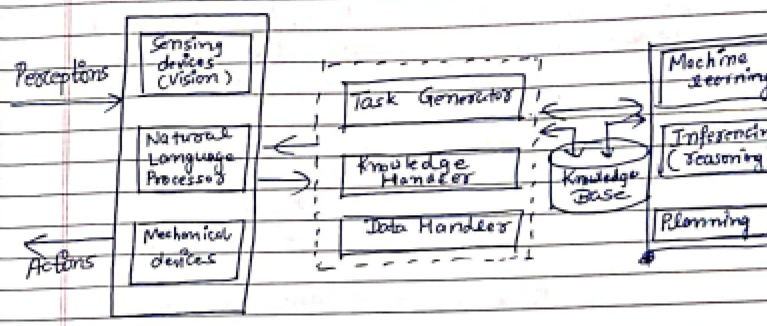
→ It is the property of machines which has the ability to mimic human intelligent behaviour by expressing it in language or symbolic rules

→ Coined by McCarthy in 1956.
Father of Artificial Intelligence.



An expert system.
: one of Conventional AI products

* Intelligent System



→ A subset of AI that focuses on getting machines to make decisions by feeding them data. → Machine Learning

→ A subset of Machine Learning that uses the concept of Neural Networks to solve complex problems.

Frank Rosenblatt

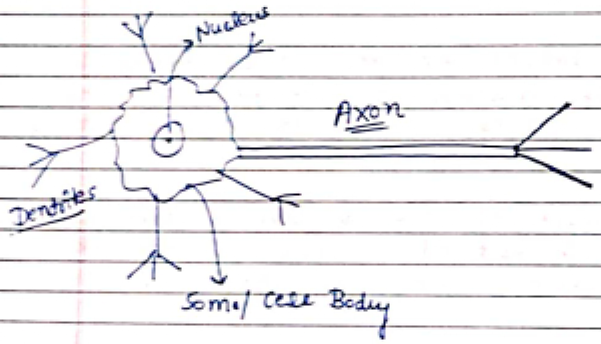
⇒ Artificial Neural Network

It is an information processing model that resembles the characteristics of biological nervous system such as Brain.

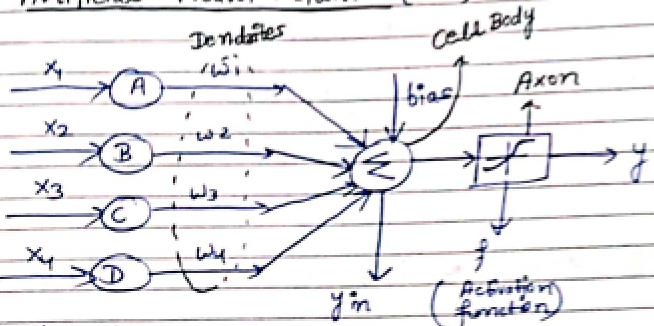
[*] Biological Nervous System

Brain → Cell → Neurons
(10^{11} neurons)

1 neuron → 2 Lakh neurons



[*] Artificial Neural Networks (ANN)



x_1, x_2, x_3, x_4
→ Input Signals

A, B, C, D
→ Input Nodes

$$y_{in} = w_1 x_1 + w_2 x_2 + w_3 x_3 + \dots + b$$
$$= \sum_{i=1}^{n} w_i x_i + b$$

$$\boxed{y = f(y_{in})}$$

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(Long Ques)

Characteristics of Neuro Fuzzy and Soft Computing

1. Human Expertise

2. Biologicallay Inspired Computing Models

3. New Optimisation Techniques

4. Numerical Computation

5. New Application domains

6. Model Free learning

7. Intensive Computation.

8. Fault Tolerance

9. Goal driven Characteristics

10. Real World Applications

Lotfi A Zadeh (1965) (University of California)

Fuzzy Set Theory

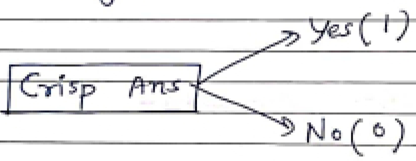
Introduction to Fuzzy Logic

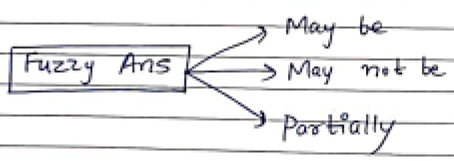
Fuzzy logic is a mathematical language to express something.

Earlier Methods

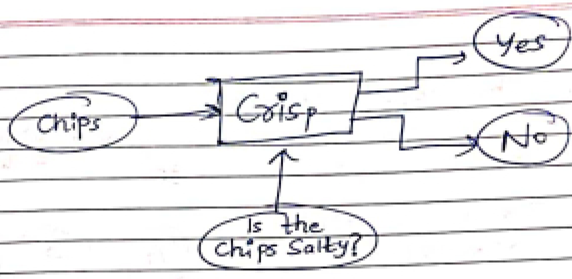
1. Boolean Algebra (0/1)
2. Relational Algebra
3. Predictional Algebra eg, $A = \{x \mid x \geq 6\}$

Combining all 3 → then it becomes fuzzy logic

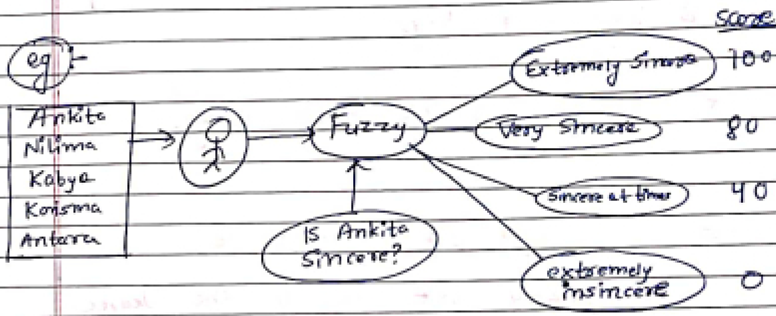




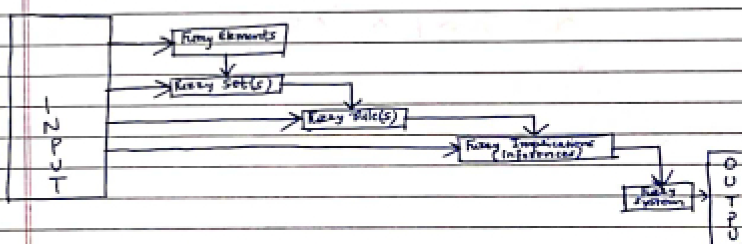
eg:-



eg:-



⇒ Concept of Fuzzy System

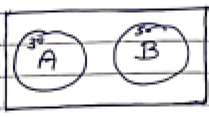


=> X = The total students in CI class

X = Universe of discourse

$\overline{\overline{80}}$

A = Total no. of girl students



B = Total no. of boys students

[*] It should be finite, always.

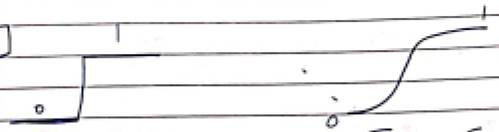
=> Concept of Fuzzy Set

A fuzzy set express the degree to which an element belongs to a set, which membership function varies between 0 & 1.

$$A = \{ x \mid x > 6 \}$$

Crisp Set / Classical Set

[*]



Crisp Set A   Fuzzy Set A

A = Set of tall people.

=> A fuzzy set A in X is expressed as a set of ordered pairs.

$$A = \{ (x, \underbrace{\mu_A(x)}_{\text{Membership function}}) \mid x \in X \}$$

eg) A = All Good Students

$$A = \{ (x, \mu_A(x)) \mid x \in X \}$$

$\mu_A(x)$ is a measurement of goodness of the student X.

→ A = {(Ankita, 0.8), (Kabya, 0.7), (Antara, 0.1), (Ankit, 0.9)

classmate
Date ____
Page ____

⇒ Fuzzy Set vs Crisp Set

Fuzzy Set	Crisp Set
① $F = (x, \mu_A(x)) \mid x \in X)$ & $\mu_A(x)$ is the degree of x.	① $A = \{ x \mid x \in X \}$
② It is a collection of ordered pairs.	② It is a collection of elements.
③ Inclusion of an element $x \in X$ into F is fuzzy that is, if present, with a degree of membership.	③ Inclusion of an element $x \in X$ into A is crisp, i.e, has strict boundary, Yes or No.

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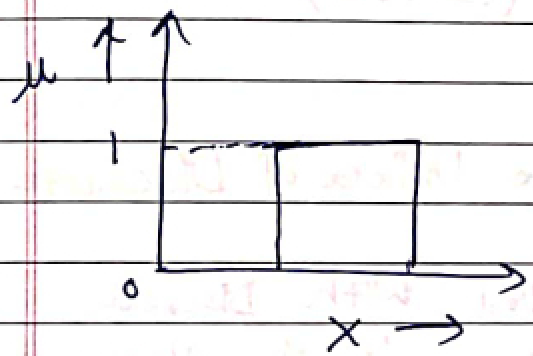
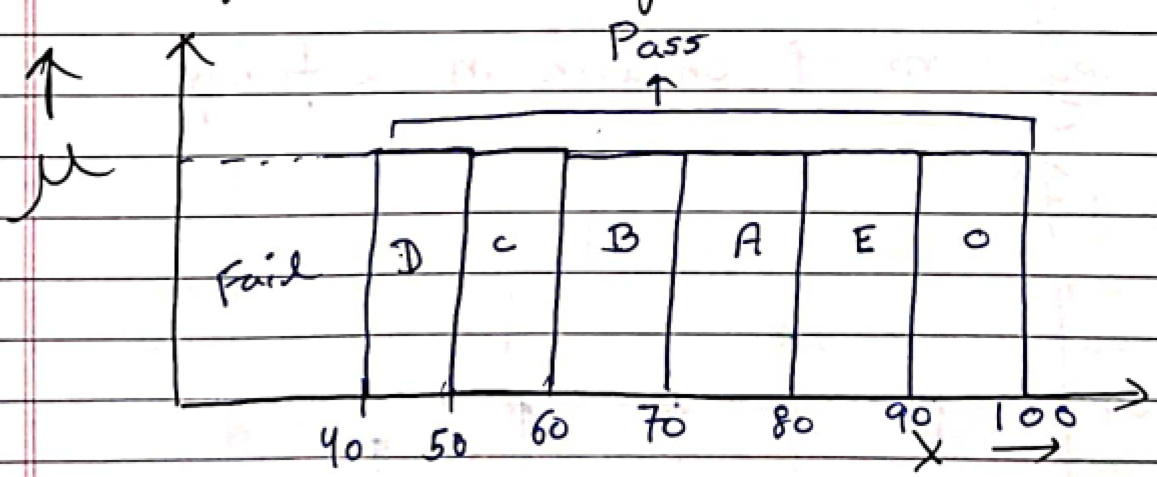
$X = \{\text{Kabita}, \text{Anu}, \text{Rani}\}$ → Crisp set
→ All good students

$F = \{(\text{Kabita}, 0.8), (\text{Anu}, 0.9), (\text{Rani}, 1)\}$
→ Fuzzy set

Crisp



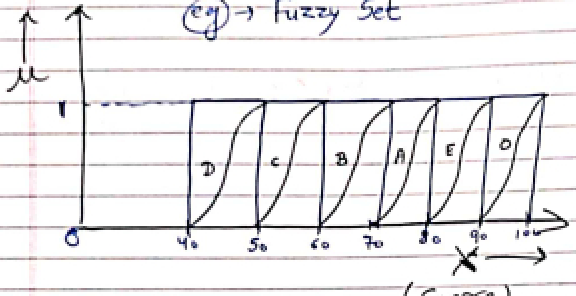
eg — Grading of Student



Crisp Set.

Fuzzy Membership Function

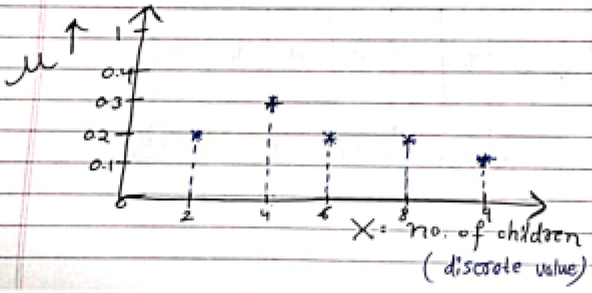
eg → Fuzzy Set



⇒ **Fuzzy Set with Discrete Universe of Discourse**

Fuzzy set can be called with Discrete Universe of Discourse if the element or the membership function or both having discrete values.

eg :- no. of children in a family



$A = \{(0, 0.1), (1, 0.30), (2, 0.78) \ldots \ldots (10, 0.1)\}$

⇒ Fuzzy set A = "sensible no. of children".

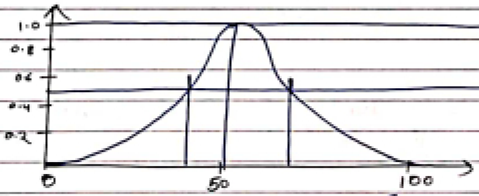
$X = \{0, 1, 2, 3, 4, 5, 6\}$ (discrete and ordered universe)

$A = \{(0, 0.1), (1, 0.3), (2, 0.7), (3, 1), (4, 0.6), (5, 0.2), (6, 0.1)\}$

⇒ $X = \{$Bangalore, Hyderabad, Delhi$\}$

$A = \{(B, 0.8), (H, 0.9), (D, 1)\}$

⇒ **Fuzzy Set with Continuous Universe of Discourse**



$\mu_B(x) = \frac{1}{1+\left(\frac{x-50}{10}\right)^2}$

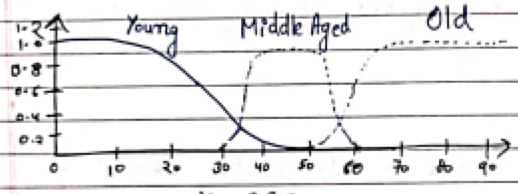
B = "Middle Aged" $B = \{(x, \mu_B(x))\}$

→ Three Linguistic Variables in Age :-

① Midde Aged

② Young

③ Old.



$x = age.$

Representation

$$A = \sum_{x_i \in X} \frac{\mu_A(x_i)}{x_i} \rightarrow \text{discrete}$$

$$= \int_{x \in X} \frac{\mu_A(x)}{x} \rightarrow \text{Continuous}$$

· eg. $X = \{0, 1, 2, 3, 4, 5\}$

no. of children

$$A = \left\{ \frac{0.1}{0} + \frac{0.2}{1} + \frac{0.3}{2} + \frac{0.4}{3} \right\}$$

Membership function

elements

eg. $B = \left\{ \frac{0.1}{\text{Bhubaneswar}} + \frac{0.2}{\text{Delhi}} + \frac{0.3}{\text{Patna}} \right\}$

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Basic Definitions & Terminology

- → Support
- → Core
- → Cross Over Points
- → Boundary
- → Fuzzy Singleton
- → Height & Cardinality
- → Normality
- → Alpha Cut & Strong Alpha Cut
- → Symmetry
- → Open & Closed.
- → Convexity

A = {b,c,d}
Support(A) = {b,c,d}

Support

The support of a fuzzy set A is the set of all points $x \in X$ such that $\mu_A(x) > 0$.

$Support(A) = \{ x \mid \mu_A(x) > 0 \}$

Crossover Points

The crossover point of a fuzzy set A is the set of all the points $x \in X$ such that $\mu_A(x) = 0.5$.

$Crossover(A) = \{ x \mid \mu_A(x) = 0.5 \}$

Fuzzy Singleton

A fuzzy set which consists of one single point whose membership function $(\mu_A(x)) = 1$.

Core

The Core of a fuzzy set A is the set of all the points $x \in X$ such that $\mu_A(x) = 1$.

$Core(A) = \{ x \mid \mu_A(x) = 1 \}$

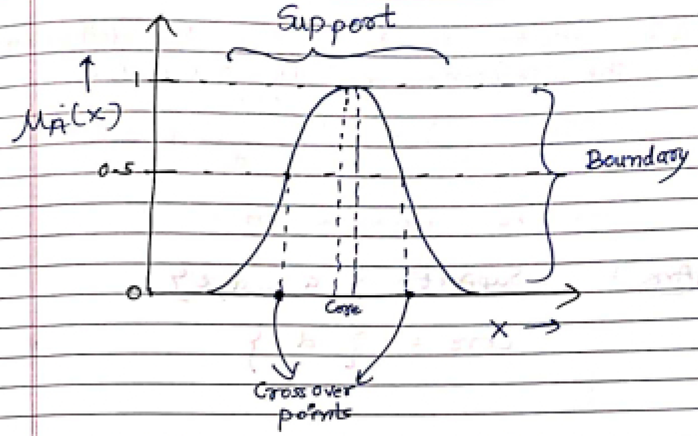
$\mu_A(x) = \{0, 0.1, 0.2, 0.3\}$

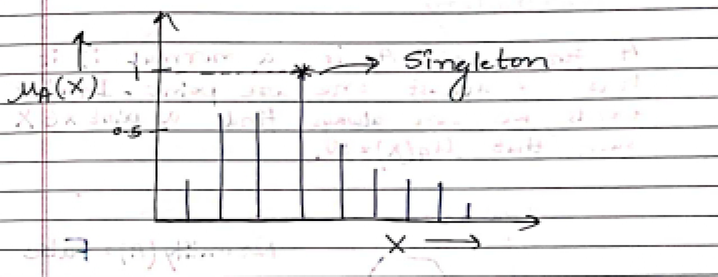
$\therefore Core(A) =$ Null.

Boundary

The boundary of a fuzzy set A is the set of all points $x \in X$ such that its $\mu_A(x) > 0$ & $\mu_A(x) < 1$.

$Boundary(x) = \{ x \mid 0 < \mu_A(x) < 1 \}$





Q:- Consider the fuzzy set, M defined on the reference set $U = \{a, b, c, d, e\}$

$$M = \frac{0.375}{a} + \frac{0.5}{c} + \frac{1}{d} + \frac{0.875}{e}$$

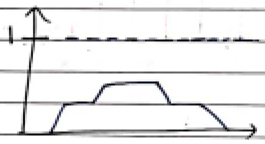
Find the support & core?

Ans:- Support = $\{a, c, d, e\}$

Core = $\{d\}$

=> Normality

A fuzzy set A is a normal if it has atleast one core point. In other words, we can always find a point $x \in X$ such that $\mu_A(x) = 1$.



Normality(A) = False

$A = \{a, b, c, d\}$

$\mu_A(x) = \{0.1, 0.2, 0.3, 0.4\}$

$\therefore$ Normality(A) = False

=> Height.

The height of a fuzzy set A is defined as the maximum membership value attained by it's elements.

From previous example.

Height(A) = 0.4

(*) $\boxed{\text{Height}(A) = \max_{x \in X} (\mu_A(x))}$

=> Cardinality

The Cardinality of a fuzzy set A is defined as the sum of all the membership values of set A.

$$\text{Cardinality}(A) = |A| = \sum_{x \in X} \mu_A(x)$$

$$\sum \mu_A(x) = 0.1 + 0.2 + 0.3 + 0.4$$

$\mu \neq 1$

$\therefore$ Cordinality (A) = 1

⇒ Alpha Cut & Strong Alpha Cut

The alpha cut of a fuzzy set A is defined as:-

$$A_\alpha = \{x \mid \mu_A(x) \geq \alpha\}$$

Strong α-cut is defined similarly

$$A_\alpha' = \{x \mid \mu_A(x) > \alpha\}$$

eg :- $A_\alpha = \{c, d\}$

$A_\alpha' = \{d\}$

** Support$(A) = A_0'$ and Core$(A) = A_1$.

Q1. Obtain 0.3-cut & strong 0.3-cut for following fuzzy set:-

$$A = \frac{0.2}{-3} + \frac{0.5}{-2} + \frac{0.2}{-1} + \frac{1}{0} + \frac{0.7}{1} + \frac{0.2}{2} + \frac{0.3}{3} + \frac{0.1}{4}$$

∴ 0.3-cut = $\{-2, 0, 1, 3\}$

Strong 0.3 cut = $\{-2, 0, 1\}$

Q2. $\{(x_1, 0.3), (x_2, 0.5), (x_3, 0.2), (x_4, 0.1)\}$

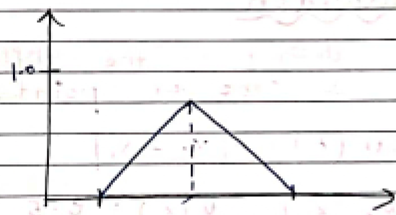
0.2-cut & strong 0.2-cut?

∴ 0.2 cut = $\{x_1, x_2, x_3\}$

Strong 0.2 cut = $\{x_1, x_2\}$

⇒ Symmetry

A fuzzy set A is symmetric if it's membership function around a certain point $x = c$, such that $\mu_A(x+c) = \mu_A(x-c)$ $\forall x \in X$.



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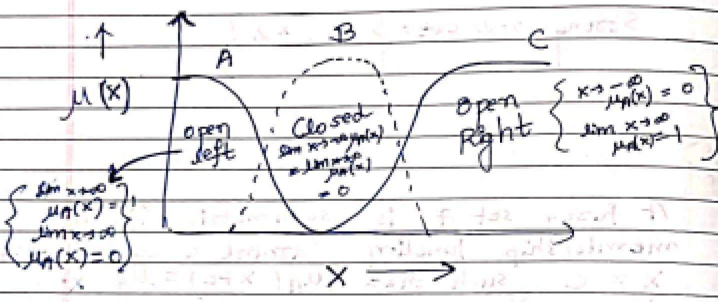
→ Open Left

→ Open Right

→ Closed

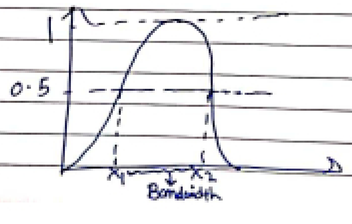


(*) Bandwidth

It is defined as the difference between 2 crossover points.

width (A) = $|x_2 - x_1|$

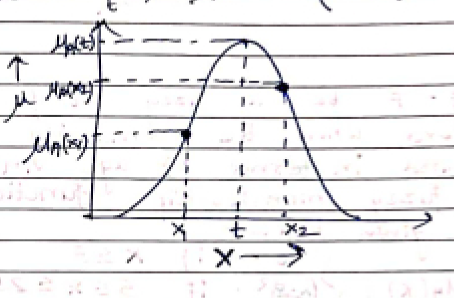
Where $\mu_A(x_1) = \mu_A(x_2) = 0.5$



⇒ Convexity

A fuzzy set A is convex iff for any $x_1, x_2 \in X$ and any $\lambda \in [0,1]$, if it satisfies the relation:-

$$\mu_A(\underbrace{\lambda x_1 + (1-\lambda)x_2}_{t}) \geq \min(\mu_A(x_1), \mu_A(x_2))$$

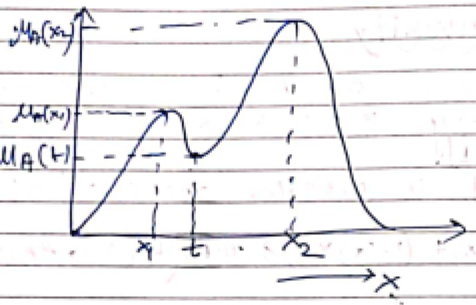


$\therefore \mu_A(t) \geq \min(\mu_A(x_1), \mu_A(x_2))$

$\mu_A(x_2) \geq \mu_A(x_1) \quad \lambda = 0$

$\mu_A(x_1) \geq \mu_A(x_1) \quad \lambda = 1$

$\Rightarrow$



Q:- Let F be a fuzzy set of matured persons where the maturity is measured in terms of age in years. The fuzzy membership function followed is given below:-

$$\mu_A(x) = \begin{cases} 0 & ; \text{ if } x \le 5 \\ \left(\frac{x-5}{20}\right)^2 & ; \text{ if } 5 \le x \le 25 \\ 1 & ; \text{ if } x \ge 25 \end{cases}$$

The universe consists of the individuals Sunny, Moon, Pikoo, Gina, Osho, Chong, Paul, Lalu, Lila, Poly whose ages are: 15, 20, 10, 27, 32, 12, 18, 24, 3 & 8 years respectively.

Find
1. Normality
2. Height
3. Support
4. Core
5. Cardinality

Ans:-

$\{$ Sunny, Moon, Pikoo, Gina, Osho, Chong, Paul, Lalu, Lila, Poly $\}$
15 20 10 27 32 12 18 24 3 8

$\mu_A(x) = \{0.25, 0.5625, 0.0625, 1, 1, 0.1225, 0.4225, 0.9025, 0, 0.0$

Support(A) = $\{$ except Lila All the elements $\}$

Core(A) = $\{$ Gina Osho $\}$

Height(A) = 1

Cardinality(A) = 4.345

Normality(A) =

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Chapter-6

Set Theoretic Operation

→ Fuzzy set operations

① Union

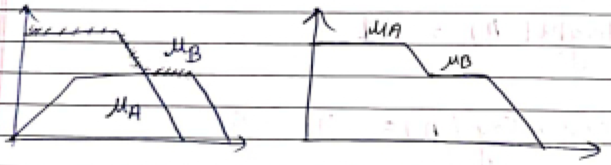
$$\mu_{A\cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}$$

Example

$A = \{(x_1, 0.5), (x_2, 0.1), (x_3, 0.4)\}$

$B = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5)\}$

$C = A\cup B = \{(x_1, 0.5), (x_2, 0.3), (x_3, 0.5)\}$



② Intersection

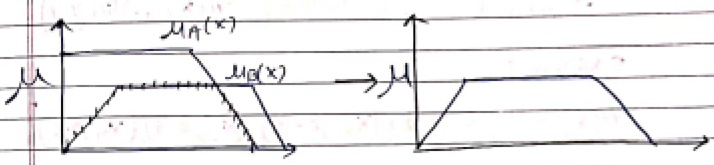
$$\mu_{A\cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}$$

Example

$A = \{(x_1, 0.5), (x_2, 0.1), (x_3, 0.4)\}$

$B = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5)\}$

$C = A\cap B = \{(x_1, 0.2), (x_2, 0.1), (x_3, 0.4)\}$



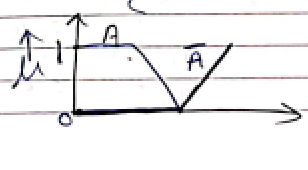
③ Complement

$$\mu_{\bar{A}}(x) = \{1 - \mu_A(x)\}$$

Example→

$A = \{(x_1, 0.5), (x_2, 0.1), (x_3, 0.4)\}$

$C = A^c = \{(x_1, 0.5), (x_2, 0.9), (x_3, 0.6)\}$



④ Subset / Containment

$A \subseteq B \Rightarrow \mu_A(x) \leq \mu_B(x)$

⑤ Cartesian Product & Cartesian co-product

$\mu_{A \times B}(x, y) = \min(\mu_A(x), \mu_B(y))$

→ Cartesian Product

$\mu_{A+B}(x, y) = \max(\mu_A(x), \mu_B(y))$

example

$A(x) = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5), (x_4, 0.6)\}$

$B(y) = \{(y_1, 0.8), (y_2, 0.6), (y_3, 0.3)\}$

$A \times B = \min(\mu_A(x), \mu_B(y)) =$

Cartesian Product

	y_1	y_2	y_3
x_1	0.2	0.2	0.2
x_2	0.3	0.3	0.3
x_3	0.5	0.5	0.3
x_4	0.6	0.6	0.3

$A + B = \max(\mu_A(x), \mu_B(y)) =$

Cartesian Co product.

	y_1	y_2	y_3
x_1	0.8	0.6	0.3
x_2	0.8	0.6	0.3
x_3	0.8	0.6	0.5
x_4	0.8	0.6	0.6

⑥ Algebraic Product or Vector Product (A·B)

$\mu_{A \cdot B}(x) = \mu_A(x) \cdot \mu_B(x)$

From prev. example

$\mu_{A \cdot B}(x) = \{(x_1, 0.16), (x_2, 0.18), (x_3, 0.15)\}$

⑦ Scalar Product ($a \times A$)

$\mu_{aA}(x) = \alpha \cdot \mu_A(x)$.

⑧ Equality ($A = B$)

$\mu_A(x) = \mu_B(x)$

⑨ Power of fuzzy set A^α

$\mu_{A^\alpha}(x) = \{\mu_A(x)\}^\alpha$

$\alpha < 1 \rightarrow$ dilation

$\alpha > 1 \rightarrow$ concentration.

⇒ Fuzzy set operations

① Sum $(A+B)$

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x)\cdot\mu_B(x)$$

② Difference $(A - B = A \cap B^c)$

$$\mu_{A-B}(x) = \mu_{A\cap B^c}(x)$$

eg $A: \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5)\}$

$\bar{B} = \{(x_1, 0.2), (x_2, 0.4), (x_3, 0.7)\}$

$\therefore A - B = A \cap B^c = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5)\}$

③ Disjunctive Sum

$$A \oplus B = ((A^c \cap B) \cup (A \cap B^c))$$

④ Bounded Sum

$$|A(x) \oplus B(x)|$$

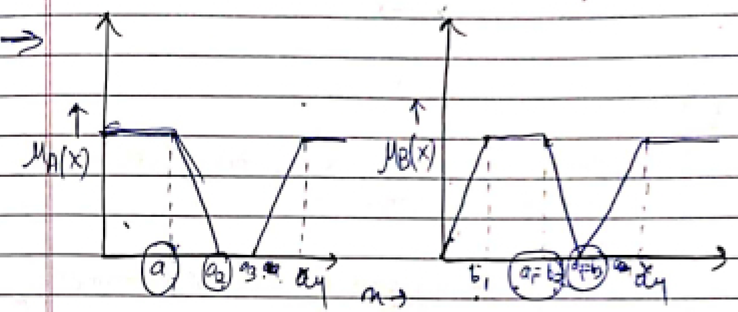
$$\mu_{|A(x) \oplus B(x)|} = \min\{1, \mu_A(x) + \mu_B(x)\}$$

⑤ Bounded Difference

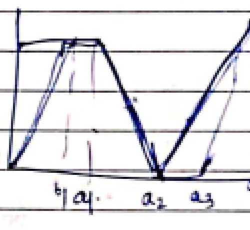
$$|A(x) \ominus B(x)|$$

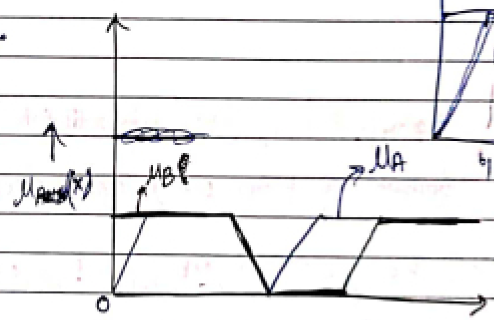
$$\mu_{|A(x) \ominus B(x)|} = \max\{0, \mu_A(x) + \mu_B(x) - 1\}$$

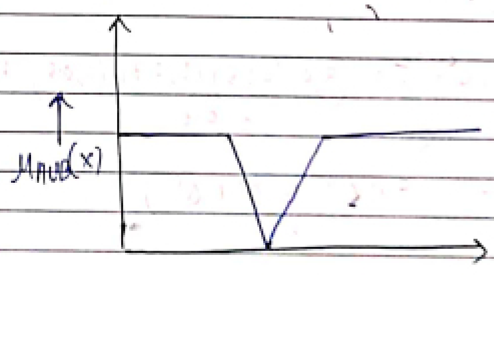
→

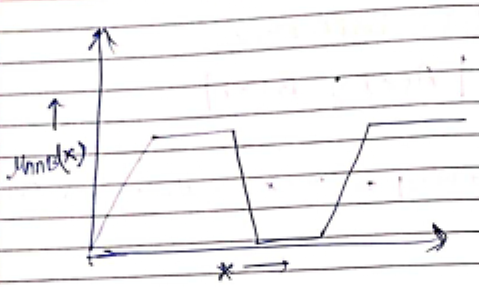


Ans :-









Q :-

$P = \{(\text{Jasmine}, 0.3), (\text{Rose}, 0.9), (\text{Lotus}, 1.0), (\text{Daffodil}, 0.7), (\text{Sunflower}, 0.5), (\text{Hibiscus}, 0.4), (\text{Chrysanth}, 0.6)\}$

$Q = \{(\text{Jasmine}, 1.0), (\text{Rose}, 1.0), (\text{Lotus}, 0.5), (\text{Daffodil}, 0.2), (\text{Sunflower}, 0.2), (\text{Hibiscus}, 0.1), (\text{Chrysanth}, 0.4)\}$

Ans :-

1) $\mu_{P+Q}(x) = \{(\text{Jasmine}, 1), (\text{Rose}, 1), (\text{Lotus}, 1), (\text{Daffodil}, 0.76), (\text{Sunflower}, 0.8), (\text{Hibiscus}, 0.46), (\text{Chrysanth}, 0.76)\}$

2) $Q^c = \{(J, 0), (R, 0), (L, 0.5), (D, 0.8), (S, 0.8), (H, 0.9), (C, 0.6)\}$

$P - Q = P \cap Q^c = \{(J, 0), (R, 0), (L, 0.5), (D, 0.7), (S, 0.5), (H, 0.4), (C, 0.6)\}$

3) $P \oplus Q = ((P^c \cap Q) \cup (P \cap Q^c))$

$P^c = \{(J, 0.7), (R, 0.1), (L, 0), (D, 0.3), (S, 0.5), (H, 0.6), (C, 0.4)\}$

$Q^c = \{(J, 0), (R, 0), (L, 0.5), (D, 0.8), (S, 0.8), (H, 0.9), (C, 0.6)\}$

$P^c \cap Q = \{(J, 0.7), (R, 0.1), (L, 0), (D, 0.2), (S, 0.2), (H, 0.1), (C, 0.4)\}$

$P \cap Q^c = \{(J, 0), (R, 0), (L, 0.5), (D, 0.7), (S, 0.5), (H, 0.4), (C, 0.6)\}$

$(P^c \cap Q) \cup (P \cap Q^c)$

$= \{(J, 0.7), (R, 0.1), (L, 0.5), (D, 0.7), (S, 0.5), (H, 0.4), (C, 0.6)\}$

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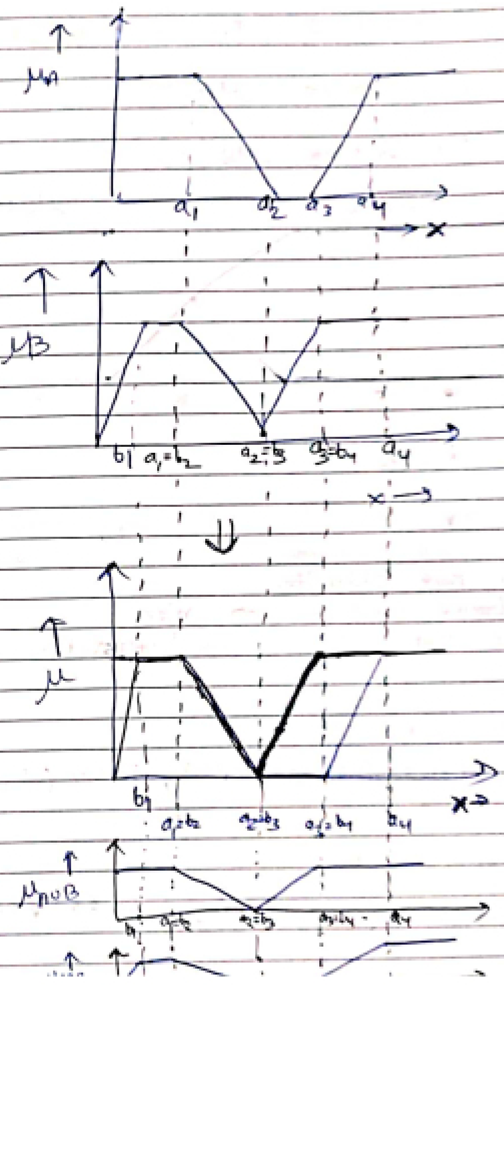
(4) $|P \oplus Q| = \mu_{|P(x) \oplus Q(x)|} = \min\left(1, \mu_p(x) + \mu_q(x)\right)$

$= \{(J, 1), (R, 1), (L, 1), (D, 0.9), (S, 0.7), (H, 0.5), (C, 1)\}$

(5) $|P(x) \odot Q(x)| = \mu_{|P(x) \odot Q(x)|}$

$= \max\{0, \mu_p(x) + \mu_q(x) - 1\}$

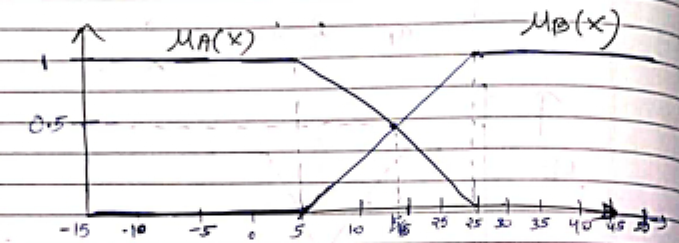
$= \{(J, 0.3), (R, 0.9), (L, 0.5), (D, 0), (S, 0), (H, 0), (C, 0)\}$



Q:- Two membership functions $\mu_A(x)$ & $\mu_B(x)$

A = Cold Climate $\mu_A(x)$

B = Hot Climate $\mu_B(x)$



X → range of discourse

What are the fuzzy sets representing

1. Not cold climate
2. Not hot climate
3. Extreme Climate → union
4. Pleasant climate → intersection

Ans:-

1. Not cold climate

$= \{(-15,0), (-10,0), (-5,0), (0,0), (5,0), (15,0.5), (25,1)\}$

2. Not hot climate

$= \{(-15,1), (-10,1), (-5,1), (0,1), (5,1), (15,0.5), (25,0)\}$

Q:-

$$\mu_A(x) = \begin{cases} 0 & \text{age} \le 30 \\ \frac{x-30}{40} & 30 \le \text{age} \le 70 \\ 1 & \text{age} \ge 70 \end{cases}$$

$$\mu_B(x) = \begin{cases} 0 & \text{age} \le 10 \\ \frac{x-10}{15} & 10 \le \text{age} \le 25 \\ 1 & 25 \le \text{age} \le 50 \\ \frac{80-x}{30} & 50 \le \text{age} \le 80 \\ 0 & \text{age} \ge 80 \end{cases}$$

Ans:- (Senior Person)

$\mu_A(x) = \{(GP, 1), (GM, 0.825), (D, 0.275), (M, 0.2), (A, 0.55)\}$

(Active)

$\mu_B(x) = \{(GP, 0.267), (GM, 0.567), (D, 1), (M, 1), (D, 0.333), (S, 0.2), (A, 0.90)\}$

1. Senior & Active {Intersection : min}

$= \{(GP, 0.267), (GM, 0.567), (D, 0.275), (M, 0.2), (A, 0.55)\}$

(2) Senior or Active {max}

$= \{(GP, 1), (GM, 0.825), (D, 1), (M, 1), (D, 0.33), (S, 0.2), (A, 0.933)\}$

(3) Not Senior

$= \{(GP, 0), (GM, 0.175), (D, 0.725), (M, 0.8), (A, 0.43)\}$

(4) Not Active

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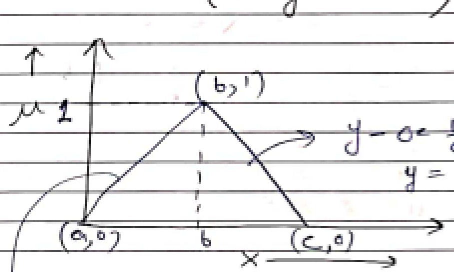
Chapter-7

→ Fuzzy Membership Functions

(1) Triangular Membership function
(Triangular MF)



$\left(\frac{1-0}{b-a}\right)$

$y - 0 = \frac{1-0}{c-b}(x - c)$

$y = \frac{-1}{c-b}(x - c)$

$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$

$y - 0 = \left(\frac{1-0}{b-a}\right)(x - a)$

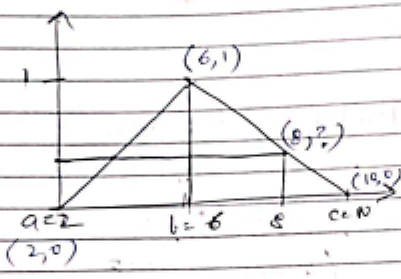
$y = \frac{1}{b-a}(x - a)$

$\therefore\ y = \frac{1}{b-a}(x - a)$

$$\mu(x, a, b, c) = \begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & a \le x \le b \\ \frac{c-x}{c-b} & b \le x \le c \\ 0 & x \ge c \end{cases}$$

(*) $\text{triangle}(x; a, b, c) = \max\left(\min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right)$

(a)



$6 \le x \le 10$

$\therefore \frac{10-8}{10-6} \left\{ \frac{c-x}{c-b} \text{ if } b \le x \le c \right\}$

$= \frac{2}{4} = \frac{1}{2}$

$\mu = 0.5$

$\frac{x-a}{b-a} = \frac{8-2}{6-2} = \frac{6}{4}$

$= 1.5 = \mu$

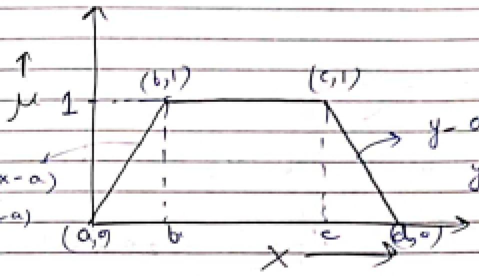
$\{ 2 \le x \le 6 \}$

$\therefore \max \left(\min \left(0.5, 1.5 \right), 0 \right)$

$= \max (0.5, 0)$

$= 0.5$

(2) Trapezoidal Function



$y - 0 = \frac{1-0}{a-b}(x-a)$

$y = \frac{1}{a-b}(x-a)$

$y - 0 = \frac{1-0}{d-c}(x-d)$

$y = \frac{1}{d-c}(x-d)$

$y = \frac{x-d}{d-c}$

$$\mu(x; a, b, c, d) = \begin{cases} 0 & x \le a \\ \frac{x-a}{b-a} & a \le x \le b \\ 1 & b \le x \le c \\ \frac{d-x}{d-c} & c \le x \le d \\ 0 & x \ge d \end{cases}$$

$\therefore \max \left\{ \min \left(\frac{x-a}{b-a}, 1, \frac{d-x}{d-c} \right), 0 \right\}$

① :- triangle $(x; 20, 60, 80)$

→ for trapezoidal
$x = 3.5$

$a = 2,\ b = 4,\ c = 8,\ d = 10$

$\frac{x-a}{b-a} = \frac{3.5-2}{4-2} \quad 2 \le x \le 4$

$= \frac{1.5}{2} \quad \frac{3}{4} = 0.75$

0

$1 \qquad 4 \le x \le 8$

$\frac{10-3.5}{10-8} \qquad 8 \le x \le 10$

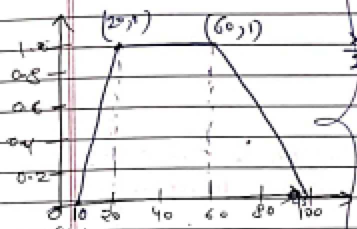
$= \frac{6.5}{2} = 3.25$

$\therefore \max\{\min(0.75, 1, 3.25), 0\}$

$\therefore \max\{0.75, 0\}$

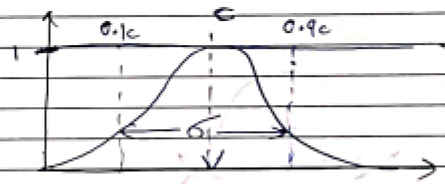
$= 0.75$ Ans

② :- trapezoid $(x; 10, 20, 60, 95)$;



$$\begin{cases} 0 & x \le 10 \\ \frac{x-10}{20-10} & 10 \le x \le 20 \\ 1 & 20 \le x \le 60 \\ \frac{95-x}{95-60} & 60 \le x \le 95 \\ 0 & x \ge 95 \end{cases}$$

③ Gaussian Membership function



gaussian $(x; c, \sigma) = e^{-\frac{1}{2}\left(\frac{x-c}{\sigma}\right)^2}$

$x = 9,\ m = 10,\ \sigma = 3$

$\therefore \mu = e^{-\frac{1}{2}\left(\frac{9-10}{3}\right)^2}$

$= e^{-\frac{1}{2}(1/9)} = e^{-1/18} = 0.9459$

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$x = 20$, $\sigma = 8$

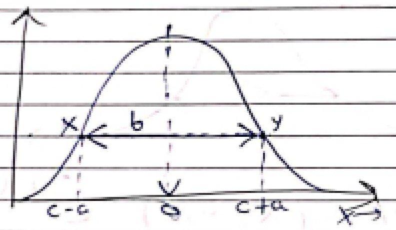
$$e^{-1/2\left(\frac{x-c}{\sigma}\right)^2}$$

$$= e^{-1/2\left(\frac{20-c}{8}\right)^2}$$

$\Rightarrow$ Generalised Bell Membership function

{Cauchy Membership function}

$$bell(x; a, b, c) = \frac{1}{1+\left|\frac{x-c}{a}\right|^{2b}}$$



Slope at $x = b/2a$

Slope at $y = -b/2a$

$c - a = 10$
$c + a = 40$
$a = 25$

$a = 2$, $b = 3$, $c = 10$

$x = 8$

$$\therefore \mu_{bell} = \frac{1}{1+\left|\frac{8-10}{2}\right|^6} = \frac{1}{1+1^6}$$

$$= \frac{1}{2} = 0.5$$

$\mu_{bell} = 0.5$ at $x = 8$

Q :-

$$\mu_{gaussian(x,50,20)} = e^{-1/2\left(\frac{x-c}{\sigma}\right)^2}$$

$$= e^{-1/2\left(\frac{x-50}{20}\right)^2}$$

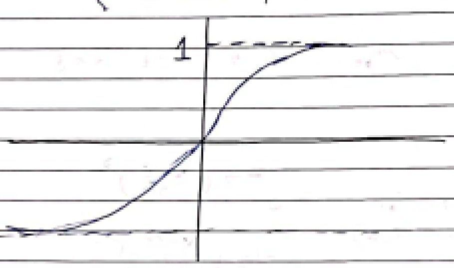
$$\mu_{bell(x,20,4,50)} = \frac{1}{1+\left|\frac{x-50}{20}\right|^8}$$

⇒ Sigmoid Membership Function

$$sig(x; a, c) = \frac{1}{1 + e^{-a(x-c)}}$$

where a controls the slope at the crossover point $x = c$.

{ ~~crossover point where~~ $\mu = 0.5$ }



⊛ Concentration

$$A^k = [\mu_A(x)]^k \quad ; k > 1$$

⊛ Dilation

$$A^k = [\mu_A(x)]^k \quad ; k < 1$$

⇒ Membership Function of Two Dimensions

If A is a fuzzy set in X, then it's cylindrical extension in $X \times Y$ is a fuzzy set $C(A)$ defined by:-

$$\boxed{C(A) = \int_{X \times Y} \frac{\mu_A(x)}{(x, y)}} \rightarrow 2D$$

$$1D \rightarrow \int_X \frac{\mu_A(x)}{x}$$

⊛ Projections of Fuzzy Sets

The projection of R in X :-

$$R_X = \int_X [\max \mu_R(x, y)] / x$$

$$R_Y = \int_Y [\max \mu_R(x, y)] / y$$

R =

	y_1	y_2	y_3
x_1	0.1	0.2	0.3
x_2	0.4	0.5	0.6
x_3	0.5	0.6	0.7

$$R_X = \frac{0.3}{x_1} + \frac{0.6}{x_2} + \frac{0.7}{x_3}$$

$$R_y = \frac{0.5}{y_1} + \frac{0.6}{y_2} + \frac{0.7}{y_3}$$

Differentiation of MF's (HW)

⇒ Composite & Non-Composite MF's

If the membership function of 2 dimensions can be expressed as an analytical expression of 2 membership functions of 1 dimension, then it is composite, otherwise non-composite.

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Chapter-8
Crisp & Fuzzy Relation

→ Crisp Relation

$A \times B = \{(a,b) \mid a \in A \;\&\; b \in B\}$

A & B are two crisp sets
$A \times B \rightarrow$ cartesian product

* $A \times B \neq B \times A$
* $|A \times B| = |B \times A|$

eg:- $A = \{1, 2, 3, 4\}$

$B = \{3, 5, 7\}$

$A \times B = \{(1,3), (1,5), (1,7), (2,3), (2,5), (2,7), (3,3), (3,5), (3,7), (4,3), (4,5), (4,7)\}$

eg. $R = \{(a,b) \mid b = a+1\}$

$\therefore R = \{(2,3), (4,5)\}$

$$R = \begin{array}{c|ccc} & 3 & 5 & 7 \\ \hline 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 \end{array}$$

→ Operations on Crisp Relations

$R(x,y)$ & $S(x,y)$ → Relations
$x \in A,\ y \in B$

Union

$$R(x,y) \cup S(x,y) = \max\big(R(x,y),\ S(x,y)\big)$$

Intersection

$$R(x,y) \cap S(x,y) = \min\big(R(x,y),\ S(x,y)\big)$$

Complement

$$\overline{R}(x,y) = 1 - R(x,y)$$

Q.

$$R = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad S = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Find
1. $R \cup S$
2. $R \cap S$
3. $\overline{R}$

Ans :-

1. $$R \cup S = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. $$R \cap S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3. $$\overline{R} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

⇒ Composition of two Crisp Relations

$$R \circ S = \{(x,z) \mid (x,y) \in R \ \&\ (y,z) \in S \ \&\ \forall y \in Y\}$$

$$T(x,z) = \max\{\min\{R(x,y),\ S(y,z) \ \&\ \forall y \in Y\}\}$$

Q. $X = \{1, 3, 5\}$

$Y = \{1, 3, 5\}$

$R = \{(x, y) \mid y = x + 2\}$;

$S = \{(x, y) \mid x < y\}$

Ans. $X \times Y = \{(1,1), (1,3), (1,5), (3,1), (3,3), (3,5), (5,1), (5,3), (5,5)\}$

$R = \{(x,y) \mid y = x+2\}$ $S = \{(x,y) \mid x < y\}$

$R = \{(1,3), (3,5)\}$ $S = \{(1,3), (1,5), (3,5)\}$

$$R = \begin{array}{c|ccc} & 1 & 3 & 5 \\ \hline 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 \end{array} \qquad S = \begin{array}{c|ccc} & 1 & 3 & 5 \\ \hline 1 & 0 & 1 & 1 \\ 3 & 0 & 0 & 1 \\ 5 & 0 & 0 & 0 \end{array}$$

$T = R \circ S = \{(0,0), (1,0), (0,0)\}$ → $0, 0, 0$ {min} → 0 {max}

$\{(0,1), (1,0), (0,0)\}$ → $0, 1, 0$ → 1 {max}

$\{(0,1), (1,0), (0,0)\}$ → 0

$$T = R \circ S = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\{(0,0), (0,0), (1,0)\}$ → $0, 0, 0$ → 0 $\quad$ $\{(0,1), (0,0), (1,0)\}$ → $0, 0, 0$ → 0 $\quad$ $\{(0,1), (0,1), (1,0)\}$ → $0, 0, 0$ → 0

$\{(0,0), (0,0), (0,0)\}$ → $0, 0, 0$ → 0 $\quad$ $\{(0,1), (0,0), (0,0)\}$ → $0, 0, 0$ → 0 $\quad$ $\{(0,1), (0,1), (0,0)\}$ → $0, 0, 0$ → 0

Fuzzy Relation

$$\mu_R(x, y) = \mu_{A \times B}(x, y) = \min\{\mu_A(x), \mu_B(y)\}$$

eg. $A = \{(a_1, 0.2), (a_2, 0.7), (a_3, 0.4)\}$

$B = \{(b_1, 0.5), (b_2, 0.6)\}$

$$R = A \times B = \begin{array}{c|cc} & b_1 & b_2 \\ \hline a_1 & 0.2 & 0.2 \\ a_2 & 0.5 & 0.6 \\ a_3 & 0.4 & 0.4 \end{array}$$

⊛ Union

$$\mu_{R\cup S}(a,b) = \max\left(\mu_R(a,b), \mu_S(a,b)\right)$$

⊛ Intersection

$$\mu_{R\cap S}(a,b) = \min\left\{\mu_R(a,b), \mu_S(a,b)\right\}$$

⊛ Complement

$$\mu_{\bar{R}}(a,b) = 1 - \mu_R(a,b)$$

⊛ Composition

$T = R \circ S$

$$\mu_{R\circ S} = \max\left\{\min\left(\mu_R(x,y), \mu_S(y,z)\right)\right\}$$

$X = \{x_1, x_2, x_3\}$ $\quad Y = \{y_1, y_2\}$ $\quad Z = \{z_1, z_2, z_3\}$

$$R = \begin{array}{c|cc} & y_1 & y_2 \\ \hline x_1 & 0.5 & 0.1 \\ x_2 & 0.2 & 0.9 \\ x_3 & 0.8 & 0.6 \end{array}_{3\times 2} \qquad S = \begin{array}{c|ccc} & x_1 & x_2 & x_3 \\ \hline y_1 & 0.6 & 0.4 & 0.7 \\ y_2 & 0.5 & 0.8 & 0.9 \end{array}_{2\times 3}$$

$$R \circ S = \begin{array}{c|ccc} & z_1 & z_2 & z_3 \\ \hline x_1 & 0.5 & 0.4 & 0.1 \\ x_2 & 0.5 & 0.8 & 0.9 \\ x_3 & 0.6 & 0.6 & 0.6 \end{array}_{3\times 3}$$

0.5 0.1 → 0.5 | 0.4 0.1 → 0.4
0.2 0.5 → 0.5 | 0.2 0.8 → 0.8
0.8 0.5 → 0.6 | 0.4 0.6 → 0.6

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Q

$$R_{P\times D} = \begin{array}{c|cccc} & D_1 & D_2 & D_3 & D_4 \\ \hline P_1 & 0.6 & 0.6 & 0.9 & 0.8 \\ P_2 & 0.1 & 0.2 & 0.9 & 0.8 \\ P_3 & 0.9 & 0.3 & 0.4 & 0.8 \\ P_4 & 0.9 & 0.8 & 0.4 & 0.2 \end{array}_{4\times 4} \qquad S_{D\times S} = \begin{array}{c|cccc} & S_1 & S_2 & S_3 & S_4 \\ \hline D_1 & 0.1 & 0.2 & 0.7 & 0.9 \\ D_2 & 1 & 1 & 0.4 & 0.6 \\ D_3 & 0 & 0 & 0.5 & 0.9 \\ D_4 & 0.9 & 1 & 0.8 & 0.2 \end{array}_{4\times 4}$$

$$R\circ S_{P\times S} = \begin{array}{c|cccc} & S_1 & S_2 & S_3 & S_4 \\ \hline P_1 & 0.8 & 0.8 & 0.8 & 0.9 \\ P_2 & 0.8 & 0.8 & 0.8 & 0.9 \\ P_3 & 0.8 & 0.8 & 0.8 & 0.9 \\ P_4 & 0.8 & 0.8 & 0.7 & 0.9 \end{array}$$

0.1, 0.6, 0, 0.8 → 0.8
0.2, 0.6, 0, 0.8 → 0.8
0.6, 0.4, 0.5, 0.8 → 0.8
0.6, 0.6, 0.9, 0.2 → 0.9
0.1, 0.2, 0, 0.8 → 0.8
0.1, 0.2, 0, 0.8 → 0.8
0.1, 0.2, 0.5, 0.8 → 0.8
0.1, 0.2, 0.9, 0.2 → 0.9
0.1, 0.3, 0, 0.8 → 0.8
0.2, 0.3, 0, 0.8 → 0.8
0.7, 0.3, 0.4, 0.8 → 0.9
0.7, 0.4, 0.4, 0.2
0.9, 0.6, 0.4, 0 → 0.9

HW:

$$R = \begin{array}{c|cccc} & \alpha & \beta & \gamma & \delta \\ \hline 1 & 0.1 & 0.3 & 0.5 & 0.7 \\ 2 & 0.4 & 0.2 & 0.8 & 0.9 \\ 3 & 0.6 & 0.8 & 0.3 & 0.2 \end{array}_{3\times 4} \qquad S = \begin{array}{c|cc} & a & b \\ \hline \alpha & 0.9 & 0.1 \\ \beta & 0.2 & 0.3 \\ \gamma & 0.5 & 0.6 \\ \delta & 0.7 & 0.2 \end{array}_{4\times 2}$$

Ans:

$$R\circ S = \begin{array}{c|cc} & a & b \\ \hline 1 & 0.7 & 0.5 \\ 2 & 0.7 & 0.6 \\ 3 & 0.6 & 0.3 \end{array}_{3\times 2}$$

0.1, 0.2, 0.5, 0.7 → 0.7
0.1, 0.3, 0.5, 0.2
0.4, 0.2, 0.5, 0.7
0.1, 0.2, 0.6, 0.2
0.6, 0.2, 0.3, 0.2
0.1, 0.3, 0.3, 0.2

⇒ Max Product Composition

$$T = R \circ S$$

$$\mu_{R\circ S}(x,z) = \max\{(\mu_R(x,y)\,\mu_S(y,z))\}$$

Q.

$$R = \begin{array}{c|cccc} & \alpha & \beta & \gamma & \delta \\ \hline 1 & 0.1 & 0.3 & 0.5 & 0.7 \\ 2 & 0.4 & 0.2 & 0.8 & 0.9 \\ 3 & 0.6 & 0.8 & 0.3 & 0.2 \end{array}_{3\times4} \qquad S = \begin{array}{c|cc} & a & b \\ \hline \alpha & 0.9 & 0.1 \\ \beta & 0.2 & 0.3 \\ \gamma & 0.5 & 0.6 \\ \delta & 0.7 & 0.2 \end{array}_{4\times2}$$

Ans

max(0.1×0.9, 0.3×0.2, 0.5×0.5, 0.7×0.7)
max(0.09, 0.06, 0.25, 0.49) → 0.49

max(0.1×0.1, 0.3×0.3, 0.5×0.6, 0.7×0.2)
max(0.01, 0.09, 0.3, 0.14) → 0.3

max(0.36, 0.04, 0.4, 0.63) → 0.63

max(0.04, 0.06, 0.48, 0.18) → 0.48

max(0.54, 0.16, 0.15, 0.14) → 0.54

max(0.06, 0.24, 0.18, 0.04) → 0.24

$$A\,T = \begin{array}{c|cc} & a & b \\ \hline 1 & 0.49 & 0.30 \\ 2 & 0.63 & 0.48 \\ 3 & 0.54 & 0.24 \end{array}_{3\times2}$$

(Prev Yr Question)

Q. (a) $\sim A = \left\{ \frac{0}{5} + \frac{0.1}{30} + \frac{0.3}{50} + \frac{0.8}{100} + \frac{1}{300} \right\}$

$\sim B = \left\{ \frac{0.7}{2} + \frac{0.8}{4} + \frac{0.2}{8} + \frac{0.1}{10} + \frac{0.7}{1.2} \right\}$

$\sim A = \{(5,0), (30,0.1), (50,0.3), (100,0.8), (300,1)\}$

$\sim B = \{(2,0.7), (4,0.8), (8,0.2), (10,0.1), (1.2,0.7)\}$

$\sim(\sim A) = \{(5,1), (30,0.9), (50,0.7), (100,0.2), (300,0)\}$

$\sim(\sim B) = \{(2,0.3), (4,0.2), (8,0.8), (10,0.9), (1.2,0.3)\}$

$$R = A \times B = \begin{array}{c|ccccc} & 2 & 4 & 5 & 10 & 1\cdot2 \\ \hline 5 & 0 & 0 & 0 & 0 & 0 \\ 30 & 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 50 & 0.3 & 0.3 & 0.2 & 0.1 & 0.3 \\ 100 & 0.7 & 0.8 & 0.2 & 0.1 & 0.7 \\ 300 & 0.7 & 0.8 & 0.2 & 0.1 & 0.7 \end{array}_{5\times5}$$

(b) $C = \{(5,1),(30,0.8),(50,0.1),(100,0.2),(300,0)\}$

(a) Max-Min Composition

0 5×1

0.1, 0.1, 0.1, 0.1, 0

0.3, 0.3, 0.1, 0.1, 0

0.7, 0.8, 0.1, 0.1, 0

0.7, 0.8, 0.1, 0.1, 0

$$\begin{bmatrix} 0 \\ 0.1 \\ 0.3 \\ 0.8 \\ 0.8 \end{bmatrix}_{5\times1}$$

(b) Max Product Composition

0.1, 0.08, 0.01, 0.02, 0

0.3, 0.24, 0.02, 0.02, 0

0.7, 0.64, 0.02, 0.02, 0

0.7, 0.64, 0.02, 0.02, 0

$$\begin{bmatrix} 0 \\ 0.1 \\ 0.3 \\ 0.7 \\ 0.7 \end{bmatrix}_{5\times1}$$

$\Rightarrow$ 2D Membership function

$$\mu_R(x,y) = \begin{cases} \dfrac{(y-x)}{4} & \text{if } y > x \\ 0 & \text{if } y \leq x \end{cases}$$

Suppose $X = \{3,4,5\}$ & $Y = \{3,4,5,6,7\}$

$$R = \begin{array}{c|ccccc} & 3 & 4 & 5 & 6 & 7 \\ \hline 3 & 0 & 0.25 & 0.5 & 0.75 & 1 \\ 4 & 0 & 0 & 0.25 & 0.5 & 0.75 \\ 5 & 0 & 0 & 0 & 0.25 & 0.5 \end{array}$$

Q:- $$\mu_R(x,y) = \begin{cases} \dfrac{(y-x)}{x+y+3} & \text{if } x < y \\ 0 & \text{if } x \geq y \end{cases}$$

$x = \{2,3,4\}$ $\quad y = \{2,3,4,5,6\}$

Ans:-

$$R = \begin{array}{c|ccccc} & 2 & 3 & 4 & 5 & 6 \\ \hline 2 & 0 & 0.125 & 0.22 & 0.3 & 0.36 \\ 3 & 0 & 0 & 0.1 & 0.18 & 0.25 \\ 4 & 0 & 0 & 0 & 0.083 & 0.153 \end{array}$$

$\frac{1}{12}$ $\frac{2}{13}$ $\frac{1}{8}$ $\frac{2}{9}$ $\frac{3}{10}$ $\frac{4}{11}$ $\frac{1}{10}$ $\frac{2}{11}$ $\frac{2}{12}$

19/08/2022

upto chap-8 { fuzzy rel^n & fuzzy proposition }

⇒ Fuzzy Extension Principle

Procedure for extending any expression from crisp domain to fuzzy domain.

$$A = \mu_A(x_1)/x_1 + \mu_A(x_2)/x_2 + \cdots + \mu_A(x_n)/x_n$$

$$B = f(A) = \mu_A(x_1)/y_1 + \mu_A(x_2)/y_2 + \cdots + \mu_A(x_n)/y_n$$

Where $y_i = f(x_i)$, $i = 1, 2, \ldots n$

Q. $A = \frac{0.1}{-2} + \frac{0.4}{-1} + \frac{0.8}{0} + \frac{0.9}{1} + \frac{0.3}{2}$

$$f(x) = x^2 - 3$$

Ans
Upon applying the extension principle, we have

$$B = \frac{0.1}{1} + \frac{0.4}{-2} + \frac{0.8}{-3} + \frac{0.9}{-2} + \frac{0.3}{1}$$

$$= \frac{0.8}{-3} + (0.4 \vee 0.9)/-2 + (0.1 \vee 0.3)/1$$

$$= \frac{0.8}{3} + \frac{0.9}{-2} + \frac{0.3}{1}$$ { where $\vee$ represents max }

Q. $A = \frac{0.2}{-3} + \frac{0.4}{-2} + \frac{0.6}{-1} + \frac{1}{0} + \frac{0.7}{1} + \frac{0.3}{2} + \frac{0.1}{3}$

$$y = f(x) = x^2 + 2x$$

Ans $B = \frac{0.2}{3} + \frac{0.4}{0} + \frac{0.6}{-1} + \frac{1}{0} + \frac{0.7}{3} + \frac{0.3}{8} + \frac{0.1}{15}$

$$B = \frac{0.6}{-1} + \frac{0.4}{0} + \frac{0.7}{3} + \frac{0.3}{8} + \frac{0.1}{15}$$

$9+2(-3)$
$9-6 = 3$
$4+2(-2)$
$4-4$
$1-2$
$1+2(1)$
$4+4$
$9+2(3)$

⇒ Fuzzy Propositions

$A \Rightarrow B$ $\quad (\bar{A} \cup B)$

$A = B$ $\quad (\bar{A} \cup B) \cap (A \cup \bar{B})$

a	b	$\wedge$	$\vee$	$\sim a$	$\longrightarrow$	=
0	0	0	0	1	1	1
0	1/2	0	1/2	1	1	1/2
0	1	0	1	1	1	0
1/2	0	0	1/2	1/2	1/2	1/2
1/2	1/2	1/2	1/2	1/2	1/2	1
1/2	1	1/2	1	1/2	1	1/2
1	0	0	1	0	0	0
1	1/2	1/2	1	0	1/2	1/2
1	1	1	1	0	1	1

AND($\wedge$)
OR($\vee$)
NOT($\sim$)
IMPLICATION ($\Rightarrow$)
EQUAL(=)

NOT ($\sim P$)	:	$1 - T(P)$
OR ($P \vee Q$)	:	$\max[T(P), T(Q)]$
AND ($P \wedge Q$)	:	$\min[T(P), T(Q)]$
IMPLICATION ($P \to Q$)	:	$\max[(1 - T(P)), T(Q)]$
EQUALITY ($P = Q$)	:	$1 - [T(P) - T(Q)]$

Notation

P : x is intelligent

P : v is F

Where v is an element that takes values V from some universal set V & F is a fuzzy set on V that represents a fuzzy predicate.

$\mu_F(V)$ → membership function of v.

P : v is F

$T(P) = \mu_F(V)$ for a $v \in V$

23/08/2022

Chapter-9

Fuzzy Implications (If-then Rule)

If x is A then y is B

x is A → Antecedent / Premise

y is B → Consequence / Conclusion.

$\boxed{R : A \to B}$ → fuzzy implication

eg:

$P = \{1, 2, 3, 4\}$

$T = \{10, 15, 20, 25, 30, 35, 40, 45, 50\}$

let the linguistic variable High Temperature & low pressure are given as

Given

$T_{high} = \{(20, 0.2), (25, 0.4), (30, 0.6), (35, 0.6), (40, 0.7), (45, 0.8), (50, 0.8)\}$

$P_{low} = \{(1, 0.8), (2, 0.8), (3, 0.6), (4, 0.4)\}$

$R : T_{high} \rightarrow P_{low}$

	1	2	3	4
20	0.2	0.2	0.2	0.2
25	0.4	0.4	0.4	0.4
30	0.6	0.6	0.6	0.4
35	0.6	0.6	0.6	0.4
40	0.7	0.7	0.6	0.4
45	0.8	0.8	0.6	0.4
50	0.8	0.8	0.6	0.4

⇒ Interpretation of Fuzzy Rules

* Interpretation as A coupled with B (using T-norm Operator)

Intersection → T-norm operator

Union → T-Conorm operator / S-norm operator

T-norm operator: $\mu_{A\cap B}(x) = T(\mu_A(x), \mu_B(x)) = \mu_A(x) * \mu_B(x)$

• T-norm Operator

$R : A \rightarrow B = A \times B = \int_{X \times Y} \mu_A(x) * \mu_B(y) \,|\, (x, y)$

where $*$ is called a T-norm operator.

⇒ The most frequently used T-norm operators are :-

(1) Minimum — ** (Mamdani Rule)

$\mu_A(x) = a \,;\, \mu_B(x) = b$

$T_{min}(a, b) = min(a, b) = a \wedge b$

$R_{min} = \int_{X \times Y} \mu_A(x) \wedge \mu_B(y) \,|\, (x, y)$

(2) Algebraic Product (** Larsen Rule)

$T_{ap}(a, b) = ab$

$R_{ap} = \int_{X \times Y} \mu_A(x) \cdot \mu_B(y) \,|\, (x, y)$

(3) Bounded Product

$T_{bp} = 0 \vee (a + b - 1)$ | $R_{bp} = \int_{X \times Y} 0 \vee (\mu_A(x) + \mu_B(x) - 1) \,|\, (x, y)$ $= \int_{X \times Y} \mu_A(x) \odot \mu_B(y) \,|\, (x, y)$

(4) Drastic Product $\int_{X \times Y} \mu_A(x) \cdot \mu_B(y) \,|\, (x,y)$

$$T_{dp} = \begin{cases} a & \text{if } b = 1 \\ b & \text{if } a = 1 \\ 0 & \text{if } a, b < 1 \end{cases}$$

$$R_{dp} = \begin{cases} \int_{X \times Y} \mu_A(x) & \text{if } b = 1 \\ \int_{X \times Y} \mu_B(x) & \text{if } a = 1 \\ 0 & \text{if } a, b < 1 \end{cases}$$

- S-norm Operator

(1) Maximum

$T_{max}(a,b) = a \vee b$ $\quad R_{max} = \int_{X \times Y} \mu_A(x) \vee \mu_B(y) \,|\, (x,y)$

(2) Algebraic Sum

$T_{as} = a + b - ab$ $\quad R_{as} = \int_{X \times Y} (\mu_A(x) + \mu_B(y) - \mu_A(x)\mu_B(y)) \,|\, (x,y)$

(3) Bounded Sum

$T_{bs} = 1 \wedge (a + b)$ $\quad R_{bs} = \int_{X \times Y} 1 \wedge (\mu_A(x) + \mu_B(y)) \,|\, (x,y)$

(4) Drastic Sum

$$S(a,b) = \begin{cases} a & \text{if } b = 0 \\ b & \text{if } a = 0 \\ 1 & \text{if } a, b > 0 \end{cases}$$

24/08/2022

Interpretation of A entails B

(1) Material Implication

$R : A \to B = \bar{A} \cup B$

(~~Zadeh's Arithmetic Rule~~)

(2) Propositional Calculus

$R : A \to B = \boxed{\bar{A} \cup (A \cap B)} = \boxed{(A \times B) \cup (\bar{A} \times Y)}$

** (Zadeh's Max-Min Rule)

(3) Extended Propositional Calculus

$R : A \to B = (\bar{A} \cap \bar{B}) \cup B$

Q:

$\text{Risky-Job} = \frac{0.3}{Job1} + \frac{0.8}{Job2} + \frac{0.7}{Job3} + \frac{0.9}{Job4}$

$\text{High Compensation} = \frac{0.2}{c_1} + \frac{0.4}{c_2} + \frac{0.6}{c_3} + \frac{0.4}{c_4}$

Using Zadeh's interpretation the truth value of Rule R is expressed by the relation.

$R = (\text{Risky-Job} \times \text{High Compensation})$

$\quad \cup \; (\overline{\text{Risky-Job}} \times \text{Compensation})$

$\therefore A = (\text{Job}1, 0.3), (\text{Job}2, 0.8), (\text{Job}3, 0.7), (\text{Job}4, 0.9)$

$B = (c_1, 0.2), (c_2, 0.4), (c_3, 0.6), (c_4, 0.8)$

$$A \times B = \begin{array}{c} \\ \text{Job}1 \\ \text{Job}2 \\ \text{Job}3 \\ \text{Job}4 \end{array} \begin{array}{c} \begin{array}{cccc} c_1 & c_2 & c_3 & c_4 \end{array} \\ \begin{bmatrix} 0.2 & 0.3 & 0.3 & 0.3 \\ 0.2 & 0.4 & 0.6 & 0.8 \\ 0.2 & 0.4 & 0.6 & 0.7 \\ 0.2 & 0.4 & 0.6 & 0.8 \end{bmatrix} \end{array}$$

$\bar{A} = (\text{Job}1, 0.7), (\text{Job}2, 0.2), (\text{Job}3, 0.3), (\text{Job}4, 0.1)$

$y = \{(y_1, 1), (y_2, 1), (y_3, 1), (y_4, 1)\}$

$$\bar{A} \times y = \begin{array}{c} \\ \text{Job}1 \\ \text{Job}2 \\ \text{Job}3 \\ \text{Job}4 \end{array} \begin{array}{c} \begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \end{array} \\ \begin{bmatrix} 0.7 & 0.7 & 0.7 & 0.7 \\ 0.2 & 0.2 & 0.2 & 0.2 \\ 0.3 & 0.3 & 0.3 & 0.3 \\ 0.1 & 0.1 & 0.1 & 0.1 \end{bmatrix} \end{array}$$

Now, $(A \times B) \cup (\bar{A} \times y) = \begin{bmatrix} 0.7 & 0.7 & 0.7 & 0.7 \\ 0.2 & 0.4 & 0.6 & 0.8 \\ 0.3 & 0.4 & 0.6 & 0.7 \\ 0.2 & 0.4 & 0.6 & 0.8 \end{bmatrix}$

$\Rightarrow$ Fuzzy If Then Else Rule.

IF x is A THEN y is B ELSE y is C.

$R = (A \times B) \cup (\bar{A} \times C)$

26/08/2020

$A = \left\{ \frac{0.1}{100}, \frac{0.3}{500}, \frac{0.7}{1000}, \frac{1.0}{5000} \right\}$ (Long Distance)

$B = \left\{ \frac{0.1}{30}, \frac{0.3}{50}, \frac{0.5}{70}, \frac{0.7}{90}, \frac{0.9}{120} \right\}$ (High Speed)

$C = \text{Moderate speed} = \left\{ \frac{0.3}{30}, \frac{0.8}{50}, \frac{0.6}{70}, \frac{0.4}{90}, \frac{0.1}{120} \right\}$

Ans. $$A \times B = \begin{array}{c} \\ 100 \\ 500 \\ 1000 \\ 5000 \end{array} \begin{array}{c} \begin{array}{ccccc} 30 & 50 & 70 & 90 & 120 \end{array} \\ \begin{bmatrix} 0.1 & 0.1 & 0.1 & 0.1 & 0.1 \\ 0.1 & 0.3 & 0.3 & 0.3 & 0.3 \\ 0.1 & 0.3 & 0.5 & 0.7 & 0.7 \\ 0.1 & 0.3 & 0.5 & 0.7 & 0.9 \end{bmatrix} \end{array}_{4 \times 5}$$

$\bar{A} = \left\{ \frac{0.9}{100}, \frac{0.7}{500}, \frac{0.3}{1000}, \frac{0}{5000} \right\}$

$$(\bar{A} \times C) = \begin{array}{c} \\ 100 \\ 500 \\ 1000 \\ 5000 \end{array} \begin{array}{c} \begin{array}{ccccc} 30 & 50 & 70 & 90 & 120 \end{array} \\ \begin{bmatrix} 0.3 & 0.8 & 0.6 & 0.4 & 0.1 \\ 0.3 & 0.7 & 0.6 & 0.4 & 0.1 \\ 0.3 & 0.3 & 0.3 & 0.3 & 0.1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}_{4 \times 5}$$

$\mu > 0$ ($\mu = 1$) 20 35
35 - 20 = 15

$(A \times B) \cup (\bar{A} \times C)$

$$= \begin{bmatrix} 0.3 & 0.8 & 0.6 & 0.4 & 0.1 \\ 0.3 & 0.7 & 0.6 & 0.4 & 0.3 \\ 0.3 & 0.3 & 0.5 & 0.7 & 0.7 \\ 0.1 & 0.3 & 0.5 & 0.7 & 0.9 \end{bmatrix}_{4 \times 5}$$

30/08/2022

Chap-10

Fuzzy Inference
(Fuzzy Reasoning)

1. Modus Ponens (MP)
2. Modus Tollens (MT)

1. Modus Ponens (MP)

fact x is A

rule if x is A then y is B

Inference → y is B.
(Conclusion)

2. GMP → Generalised Modus Ponens.

fact → x is A'

rule → if x is A then y is B.

Inference → y is B'

② Modus Tollens (MT)

Fact → y is B

Rule → If x is A, then y is B

Inference → x is A

⊛ GMT

Fact → y is B'

Rule → If x is A then y is B

Inference → x is A'

GMP → $B' = A' \circ R(x, y)$

GMT → $A' = B' \circ R(x, y)$

$$R(x,y) = (A \times B) \cup (\bar{A} \times Y)$$

Q. $A = \{(x_1, 0.5), (x_2, 1), (x_3, 0.6)\}$

$B = \{(y_1, 1) \ (y_2, 0.4)\}$

$A' = \{(x_1, 0.6), (x_2, 0.9), (x_3, 0.7)\}$

$B' = ?$ using GMP.

$\boxed{\text{Ans}}$:-

$$\therefore R(x,y) = (A \times B) \cup (\bar{A} \times Y) = \begin{array}{c|cc} & y_1 & y_2 \\ x_1 & 0.5 & 0.4 \\ x_2 & 1 & 0.4 \\ x_3 & 0.6 & 0.4 \end{array}_{3\times 2} = (A \times B)$$

Now, $B' = A' \circ R(x, y)$ {By GMP rule}

$$= \begin{bmatrix} 0.6 & 0.9 & 0.7 \end{bmatrix}_{1\times 3} \begin{bmatrix} 0.5 & 0.4 \\ 1 & 0.4 \\ 0.6 & 0.4 \end{bmatrix}_{3\times 2}$$

0.5, 0.9, 0.6 → 0.9

0.4, 0.4, 0.4 → 0.4

$$B' = \begin{bmatrix} 0.9 & 0.4 \end{bmatrix}_{1\times 2}$$

$$(\bar{A} \times Y) = \begin{array}{c|cc} & y_1 & y_2 \\ x_1 & 0.5 & 0.5 \\ x_2 & 0 & 0 \\ x_3 & 0.4 & 0.4 \end{array}_{3\times 2}$$

$Y = \{(y_1, 1), (y_2, 1)\}$

$$\therefore R(x,y) = \begin{bmatrix} 0.5 & 0.4 \\ 1 & 0.4 \\ 0.6 & 0.4 \end{bmatrix}_{3\times 2} \begin{bmatrix} 0.5 & 0.5 \\ 0 & 0 \\ 0.4 & 0.4 \end{bmatrix}_{3\times 2} = \begin{bmatrix} 0.5 & 0.5 \\ 1 & 0.4 \\ 0.6 & 0.4 \end{bmatrix}_{3\times 2}$$

$B' = A' \circ R(x, y)$

$$= \begin{bmatrix} 0.6 & 0.9 & 0.7 \end{bmatrix}_{1\times 3} \begin{bmatrix} 0.5 & 0.5 \\ 1 & 0.4 \\ 0.6 & 0.4 \end{bmatrix}_{3\times 2}$$

$$B' = \begin{bmatrix} 0.9 & 0.5 \end{bmatrix}_{1\times 2}$$

Now,

$A' = ?$

Given $B' = \{(y_1, 0.9), (y_2, 0.7)\}$

$A' = B' \circ R(x, y)$

$$\begin{bmatrix} 0.9 & 0.7 \end{bmatrix}_{1\times 2}$$

$$\begin{bmatrix} 0.9 & 0.7 & 0 \end{bmatrix}_{1\times 3} \begin{bmatrix} 0.5 & 0.5 \\ 1 & 0.4 \\ 0.6 & 0.4 \end{bmatrix}_{3\times 2}$$

$$= \begin{bmatrix} 0.5 & 0.5 \\ 1 & 0.4 \\ 0.6 & 0.4 \end{bmatrix}_{3\times 2} \begin{bmatrix} 0.9 \\ 0.7 \end{bmatrix}_{2\times 1} \left\{ \text{we will take transpose of } A' \right\}$$

0.5, 0.7, 0.7

3×2

0.5, 0.4, 0, 0.5

1×2

$$\begin{bmatrix} 0.7 & 0.5 \end{bmatrix}$$

3×1

$$= \begin{bmatrix} 0.5 \\ 0.9 \\ 0.6 \end{bmatrix}_{3\times 1} = A'$$

$\therefore A' = \{(x_1, 0.5), (x_2, 0.9), (x_3, 0.6)\}$

Q:- $H = \{(70,1), (80,1), (90,0.3)\}$

$VH = \{(80,0.6), (90,0.9), (100,1)\}$

$S = \{(30,0.8), (40,1.0), (50,0.6)\}$

$QS = \{(10,1), (20,0.8), (30,0.5)\}$

$\boxed{\text{Ans}}$:-

$R = (H \times S) \cup (\bar{H} \times y)$

$$(H \times S) = \begin{array}{c} \\ 70 \\ 80 \\ 90 \end{array} \begin{array}{c} \begin{array}{ccc} 30 & 40 & 50 \end{array} \\ \begin{bmatrix} 0.8 & 1 & 0.6 \\ 0.8 & 1 & 0.6 \\ 0.3 & 0.3 & 0.3 \end{bmatrix}_{3\times 3} \end{array}$$

$H' = \{(70,0), (80,0), (90,0.7)\}$

$y = \{(y_1,1), (y_2,1), (y_3,1)\}$

$$(\bar{H} \times y) = \begin{array}{c} \\ 70 \\ 80 \\ 90 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.7 & 0.7 & 0.7 \end{bmatrix}_{3\times 3} \end{array}$$

Now $(H \times S) \cup (\bar{H} \times Y) = \begin{bmatrix} 0.8 & 1 & 0.6 \\ 0.8 & 1 & 0.6 \\ 0.7 & 0.7 & 0.7 \end{bmatrix}_{3\times 3} = R$

$QS = VH \circ R(x,y)$

$$= \begin{bmatrix} 0.6 & 0.9 & 1 \end{bmatrix} \circ \begin{bmatrix} 0.8 & 1 & 0.6 \\ 0.8 & 1 & 0.6 \\ 0.7 & 0.7 & 0.7 \end{bmatrix}_{3\times 3}$$

$$QS = \begin{bmatrix} 0.8 & 0.9 & 0.7 \end{bmatrix}_{1\times 3}$$

2/09/2022

(*) 3 types of Infering (fact / input)

1. Single Rule with Single Antecedent
2. Single Rule with Multiple Antecedent.
3. Multiple Rule with Multiple Antecedent.

① Single Rule with Single Antecedent

fact → x is A'

rule → if x is A, then y is B

o/p → x is B'

$B' \longrightarrow A' \circ R(x, y)$

$R(x, y) \rightarrow \int_{x \times y} \mu_A(x) \wedge \mu_B(y) \Big|_{(x, y)}$

{ By Mamdani Rule }

$\therefore \; B' = A' \cdot R(x, y)$

$= \bigvee_{\text{(random)}} \left[\mu_{A'}(x) \wedge \mu_A(x) \right] \wedge \mu_B(y)$

$= w \wedge \mu_B(y)$

② Single Rule with Multiple Antecedent

fact → x is A' and y is B'

rule → if x is A and y is B then z is C. $\{ A \times B \rightarrow C \}$

o/p → z is c'

By ~~Zadeh Max Min Rule~~

$R(x, y) = (A \times B) \cup (\bar{A} \times C)$ { Replace y with C }

By Mamdani Rule { Take all the minimum of μ }

$R(x, y) = \int \mu_A(x) \wedge \mu_B(y) \wedge \mu_C(z) \Big|_{(x, y, z)}$

$\therefore \; \boxed{C' = (A' \times B') \circ (A \times B \rightarrow C)}$

{ Find out by Mamdani Rule }

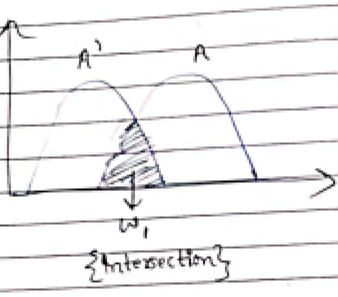
$\therefore \; \mu_{C'}(x) = \bigvee \left[(\mu_{A'}(x) \wedge \mu_{B'}(y)) \wedge (\mu_A(x) \wedge \mu_B(y) \wedge \mu_C(z)) \right]$

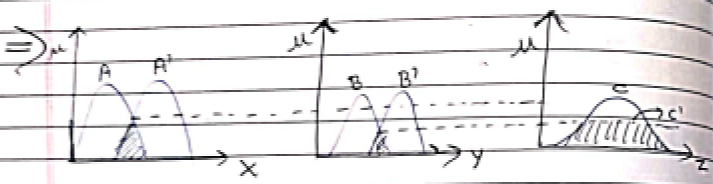
$= \overbrace{\bigvee \left[\mu_{A'}(x) \wedge \mu_A(x) \right]}^{w_1} \wedge \overbrace{\bigvee \left[\mu_{B'}(y) \wedge \mu_B(y) \right]}^{w_2} \wedge \mu_C(z)$

$= (w_1 \wedge w_2) \wedge \mu_C(z)$

Firing strength → Compatibility b/w A & A' or B & B' { common values }

(*)





③ Multiple Rule with Multiple Antecedents

premise 1 (fact) : x is A' and y is B'

premise 2 (rule 1) : if x is A_1 and y is B_1 then z is C_1

premise 3 (rule 2) : if x is A_2 and y is B_2 then z is C_2.

Conclusion : x is C'.

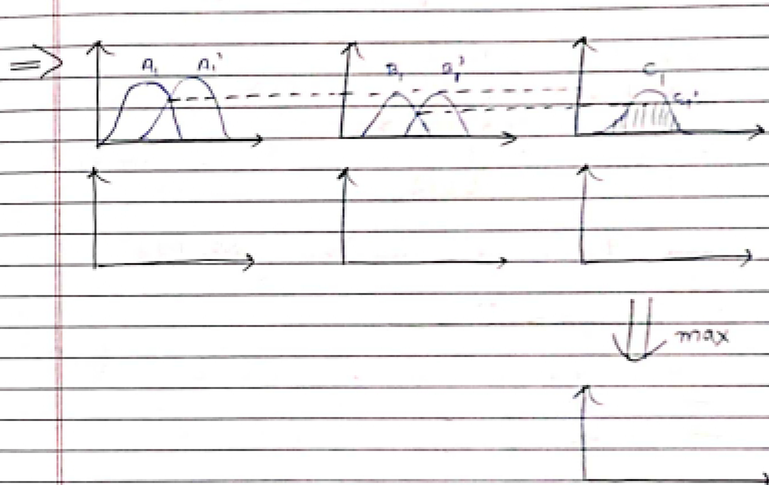
$$C' = (A' \times B') \circ (R_1 \cup R_2)$$

$$= [(A' \times B') \circ R_1] \cup [(A' \times B') \circ R_2]$$

$$= C_1' \cup C_2'$$

$$R_1 = (A_1 \times B_1) \rightarrow C_1$$

$$R_2 = (A_2 \times B_2) \rightarrow C_2$$



6/09/2022

⇒ Defuzzification

It converts fuzzy value to crisp value.

→ Defuzzification Methods (** Imp)

(WLCM)

1. Lambda Cut Method
2. Maxima Methods
   - → Height Method
   - → First of Maxima (FoM) / Smallest of Maxima (SOM)
   - → Last of Maxima (LoM) / Largest of Maxima (LoM)
   - → Mean of maxima (MoM)
3. Centroid Methods (GAS)
   - → Center of Gravity method (CoG)
   - → Center of Sum method (CoS)
   - → Center of Area method (CoA)
4. Weighted Average Method.

Upto Chap-4 From book

(1) Lambda Cut Method {α-cut method}

$A = \{(x_1, 0.1), (x_2, 0.3), (x_3, 0.4), (x_4, 0.9)\}$

$A_{0.3} = \{(x_2, 1), (x_3, 1), (x_4, 1)\}$

(*) $A_\lambda = \{x \mid \mu_A(x) \geq \lambda\}$

HW

$\mu(x)$	x_1	x_2	x_3	x_4	x_5
P	0.1	0.2	0.7	0.5	0.4
Q	0.9	0.6	0.3	0.2	0.8

(a) $P_{0.2}$, $Q_{0.3}$

$P_{0.2} = \{(x_2, 0.2), (x_3, 0.7), (x_4, 0.5), (x_5, 0.4)\}$

$Q_{0.3} = \{(x_1, 0.9), (x_2, 0.6), (x_3, 0.3), (x_5, 0.8)\}$

(b) $(P \cup Q)_{0.6}$

0.9 0.6 0.7 0.5 0.8

$(P \cup Q)_{0.6} = \{(x_1, 0.9), (x_2, 0.6), (x_3, 0.7), (x_5, 0.8)\}$

(c) $(P \cap \bar{P})_{0.8}$ 0.9 0.8 0.3 0.5 0.6

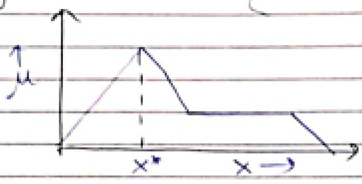
$(P \cap \bar{P})_{0.8} = \{(x_1, 0.9), (x_2, 0.8)\}$

(d) $(P \cap S)_{0.4}$

$= \{x_5, 0.4\}$

② Maxima Method

⊛ Height Method {Maximum Membership Method}

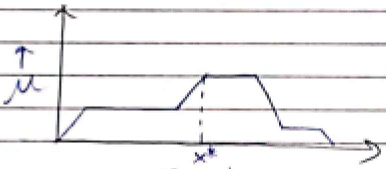


$$\mu_c(x^*) \geq \mu_c(x) \quad \forall \; x \in X$$

→ Here x^* is the height of the output fuzzy set c.

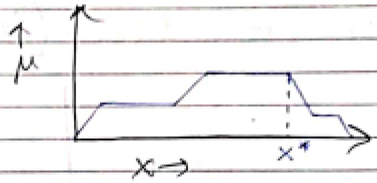
→ This method is applicable when height is unique.

⊛ First of Maxima (FoM)



$$x^* = \min\{x \mid C(x) = \max c\{\omega\}\}$$

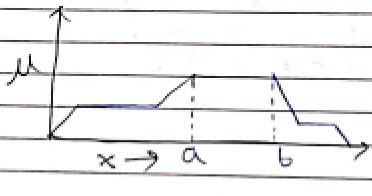
⊛ Last of Maxima (LoM)



$$x^* = \max\{x \mid C(x) = \max c\{\omega\}\}$$

⊛ Mean of Maxima (MoM)

$$x^* = \frac{\sum_{x_i \in M} (x_i)}{|M|}$$



$$x^* = \frac{a+b}{2}$$

(3) Centroid Methods

(*) Centre of Gravity Method (CoG)

$$\boxed{x^* = \frac{\int x \cdot \mu_c(x)\, dx}{\int \mu_c(x)\, dx}}$$ {Continuous}

$$\boxed{x^* = \frac{\sum_{i=1}^{n} x_i \cdot \mu(x_i)}{\sum_{i=1}^{n} \mu(x_i)}}$$ {discrete}

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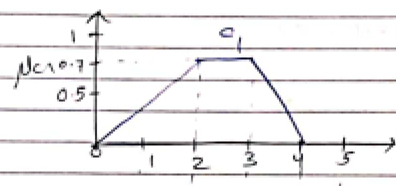
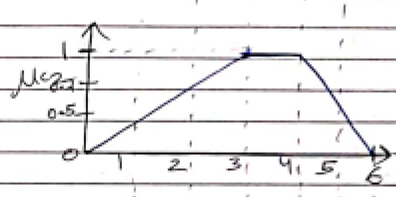
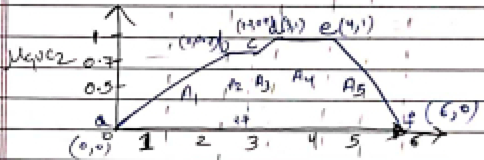
1. Center of Gravity Method / Centroid of Area Method
2. Center of Sum method.
3. Center of Area Method.

→







from $(0,0) \rightarrow (2, 0.7)$

$y - 0 = \frac{0.7}{2}(x - 0)$

$\boxed{y = \frac{0.7x}{2} = 0.35x}$

from $(2, 0.7), (2.7, 0.7)$

$\boxed{y = 0.7}$

from $(2.7, 0.7) \rightarrow (3, 1)$

$y - 0.7 = \frac{1 - 0.7}{3 - 2.7}(x - 2.7)$

$y - 0.7 = \frac{0.3}{0.3}(x - 2.7)$

$y = x - 2.7 + 0.7$

$\boxed{y = x - 2}$

from $(3, 1) \rightarrow (4, 1)$

$\boxed{y = 1}$

from $(4, 1) \rightarrow (6, 0)$

$y - 1 = \frac{0 - 1}{6 - 4}(x - 4)$

$y - 1 = \frac{-1}{2}(x - 4)$ → $2y - 2 = -x + 4$

$\boxed{y = \frac{1}{2}(6 - x)}$

$$\therefore \quad x^* = \frac{\int x \cdot \mu(x)\, dx}{\int \mu(x)\, dx}$$

$\int x \mu(x)\, dx$

$$= \int_0^2 0.35x^2 dx + \int_2^{2.7} 0.7x^2 dx + \int_{2.7}^{3} (x^2 - 2x)\, dx + \int_3^4 x\, dx$$

$$+ \int_4^6 (-0.5x^2 + 3x)\, dx$$

$N = 10.98$

$$D = \int_0^2 0.35x\, dx + \int_2^{2.7} 0.7x\, dx + \int_{2.7}^{3} (x-2)\, dx + \int_3^4 dx + \int_4^6 (-0.5x+3)\, dx$$

$D = 3.445$

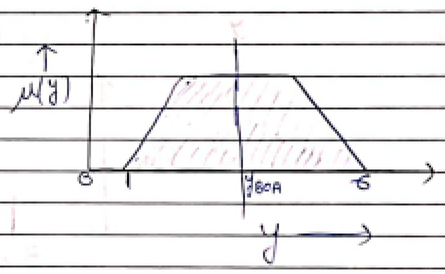
$$\therefore \quad x^* = \frac{N}{D} = \frac{10.98}{3.445} = 3.187$$

(*) Centre of Sum Method (COS)

$$\boxed{x^* = \frac{\sum_{i=1}^{n} x_i\, A_{c_i}}{\sum_{i=1}^{n} A_{c_i}}}$$

$\boxed{9/09/2022}$

Fuzzy Inference Systems



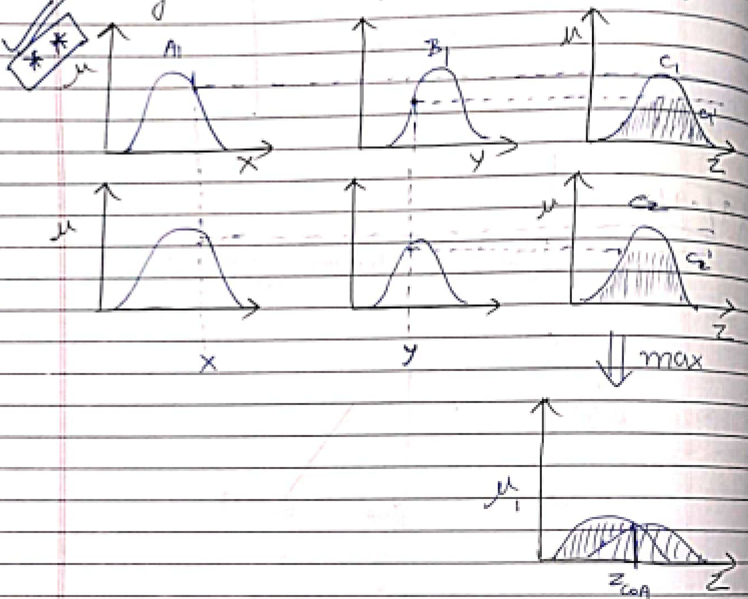
Bisector of area BOA satisfies

$$\int_{\alpha}^{z_{BOA}} \mu(z)\, dz = \int_{z_{BOA}}^{\beta} \mu_A(z)\, dz$$

Mamdani Fuzzy Model

(using min-max operator) (2 input 2 rules)

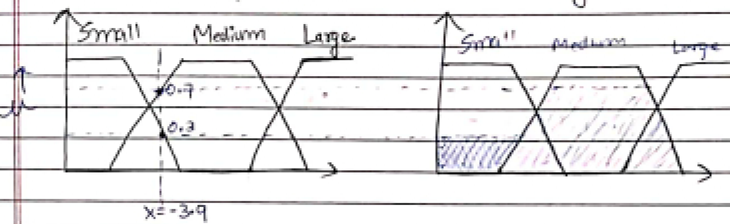
**



We can also do by max product rule.

eg 1 :- Design a fuzzy Mamdani Model using single i/p single o/p system (SISO) by taking 3 rules:

If x is small, then y is small.
If x is medium, then y is medium
If x is large, then y is large.



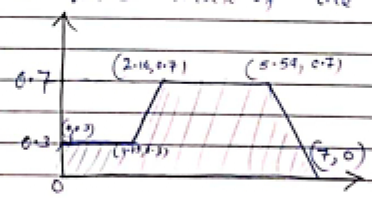
What will be the output y for input x=-3.9 ?

(Ans) For input x = -3.9

$\mu(x = -3.9)|_{small} = 0.3$

$\mu(x = -3.9)|_{medium} = 0.7$

Now, we take max of the 2 areas obtained.



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⟹ Sugeno Model or TSK Model (Example)

i/p → x, y

o/p → x

Rule 1: If x is low or y is high then output $z = 10 + 0.2x + 0.3y \Rightarrow z_1$

Rule 2: If x is low or y is medium then output $z = x + 2y \Rightarrow z_2$

Rule 3: If x is medium or y is low then output $z = 2x + y \Rightarrow z_3$

Rule 4: If x is high or y is high then output $z = 0.1x + 0.03y \Rightarrow z_4$.

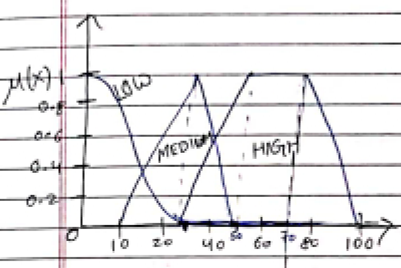
The membership function of inputs A & B with the universe of discourse X & Y respectively are given as follows:-

$\mu_{A_{low}}(x) = gaussm(x, 0, 20)$ $\mu_{B_{low}} = \cancel{gauss}\ trapmf(y, 0, 0, 30, 45)$

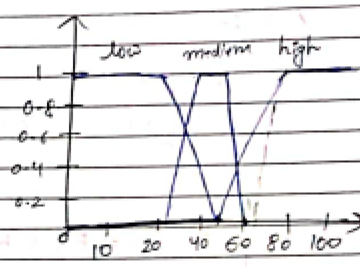
$\mu_{A_{medium}}(x) = trim(x, 10, 30, 50)$ $\mu_{B_{medium}} = trapmf(y, 20, 40, 50, 60)$

$\mu_{A_{high}}(x) = trap(x, 30, 50, 70, 100)$ $\mu_{B_{high}} = trapmf(y, 45, 60, 100, 100)$

b/w (0, 45) μ = 1
= (0, 20) μ = 0



X = 70



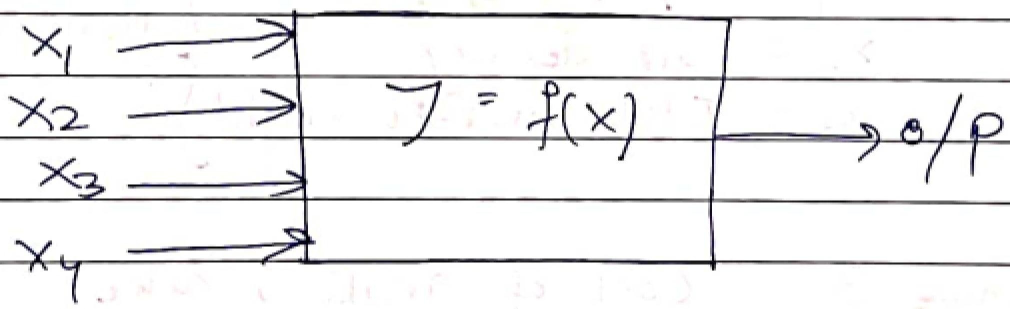
y = 60

Optimisation / Mathematical Programming

It is the selection of a best element (with regard to some criteria) from some set of alternations.

Optimization Problem:-

An optimization problem consists of maximising or minimising a real function by systematically choosing input values from within an allowed set and computing the value of the function.



$x_1, x_2, x_3, x_4 \rightarrow$ input are called control / decision Variable.

y is the function which is called Objective / Cost function.

$$y = f(x_1, x_2, x_3, \dots)$$

Maximize $f \rightarrow$ utility / objective function
Minimize $f \rightarrow$ cost function.

When there is a single input/ depend on single variable

$\rightarrow$ Single objective

When there are multiple input

$\rightarrow$ Multi objective

Example 1

Aircraft Endurance {optimization Problem without constraint}

x_1 = fuel supply
x_2 = air density
x_3 = Total aircraft weight
} parameters

Example 3 Cost of making cake

x_1 = Amount of ingredient 1
x_2 = Amount of ingredient 2

Constraints

$x_1 > 0$ $\quad x_2 \geq 0.5$
$x_1 \leq 0.75$ $\quad x_2 \leq 2.5$

Inequality $>, <, =$
Equality Constraint

There are 2 types of optimization problems:-

$\rightarrow$ Linear & Non-Linear

$2x+3y=10;\ x \geq 0;\ y \geq 0 \rightarrow$ Linear

$f(x,y) = 2x^2 - 3y$

$y \leq \sqrt{x}+2$
$y \geq x^2 - 4x$
} non linear

$\rightarrow$ Types of Problem

Single $\rightarrow$ When we need to focus on the search of one solution.

Multimodel $\rightarrow$ More than one optimal solution

Multi-Objective $\rightarrow$ Problem has set of solutions and we need to find a diverse set.

Multi-Objective $\rightarrow$ Having more than three objectives $\rightarrow$ Many objective

Dynamic Problem: Where the conditions for a problem change over time & there is need to crack the optima.

Notation for Single objective Problem (SOP)

Minimize / Maximize $f(x)$ subject to:-

$$g_j(x) \geq 0 \quad j = 1, 2, \ldots J$$

$$h_k(x) = 0 \quad k = 1, 2, \ldots K$$

$$x_i^{(L)} \leq x_i \leq x_i^{(U)} \quad i = 1, 2, \ldots n$$

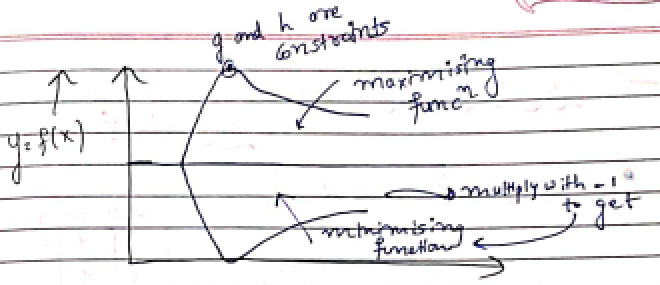
Many objective optimisation Problem

Minimise/Maximise $f_m(x)$ subject to $m = 1, 2, \ldots M$.

$$g_j(x) \geq 0 \; ; \; j = 1, 2, \ldots J$$

$$h_k(x) = 0 \; ; \; k = 1, 2, \ldots K$$

$$x_i^{(L)} \leq x_i \leq x_i^{(U)} \quad i = 1, 2, 3, \ldots n$$



(*) Various Methods of derivative based optimization

- → Least Square Estimator (LSE)
- → Recursive LSE
- → Maximum likelihood
- → Descent methods
- → Gradient Based Method.
- → Steepest descent method (or Gradient Method)

⇒ Basic principles of Derivative Based optimization

- Gradient Descent Method

$$J(m,c) = \sum_{i=1}^{n} \left[y_i - (mx_i + c)\right]^2$$

Data set given

x	y
3	4
5	6

initial values $m = 0,\ c = 1$

Formula

step size

$$c_{new} = c_{old} - [\text{Learning Rate} \times \text{Slope}]$$

$$c_{new} = c_{old} - \alpha\left(\frac{dJ}{dc}\right)$$ $\alpha = 0.001$ (given)

$$m_{new} = m_{old} - \alpha\left(\frac{dJ}{dm}\right)$$

$$J(m,c) = \left[4 - (3m+c)\right]^2 + \left[6 - (5m+c)\right]^2$$

$$c_{new} = 1 - 0.001 \times \frac{dJ}{dc}$$

$$J(m,c) = (4 - 3m - c)^2 + (6 - 5m - c)^2$$

$$= 2[4 - 3m - c](-1) + 2[6 - 5m - c](-1)$$

$$= -2(4 - 3m - c) - 2(6 - 5m - c)$$

$$= -2(4 - 0 - 1) - 2(6 - 0 - 1)$$

$$= -2(3) - 2(5)$$

$$= -4$$

$$\therefore c_{new} = 1 - 0.001 \times (-4)$$

$$= 1 + 0.004$$

$$= 1.004$$

$$J(m,c) = -2m - 2m$$
$$= -16$$

⇒ Genetic Algorithm

Principles of GA based on two fundamentals:-

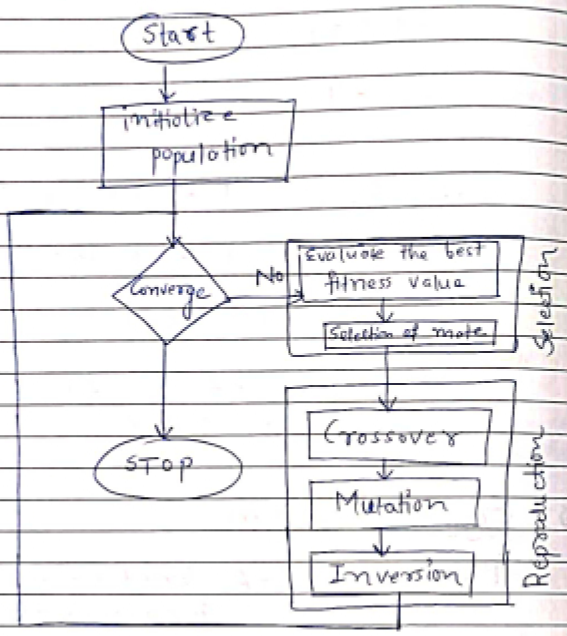
→ Genetics

→ Evaluation

Limitations

— Computationally Expensive

→ Framework of GA



(*) Four GA parameters / operators :-

1. Encoding : Convert decimal to binary
2. Selection : Select best fit value.
3. Crossover : Interchange characteristics
4. Mutation : To escape from ______ optima

Crossover : Exchange parts of chromosome with a crossover probability.

Select crossover points randomly.

One point Crossover

eg :-

| 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 |

| 0 | 1 | 1 | 1 | 0 | 1 | 1 | 0 | 1 | 1 | 1 | 0 |

Crossover point

| 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |

| 0 | 1 | 1 | 1 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |

2 point crossover

eg :

1 1 1 0 | 0 1 | 0 1 1 1
0 1 1 0 | 1 1 | 1 0 1 0

1 1 1 0 1 1 0 1 1 1
0 1 1 0 0 1 1 0 1 0

Uniform Mutation :-

0	1	0	1	1	1	1	1	1	1	1	0
0	1	0	1	1	1	0	1	1	1	1	0

Mutated bit

** Ex: Maximize the function $f(x) = x^2$ in the interval [0, 31].

N = no. of string = 4

String	Selection x		Initial population	$f(x) = x^2$	P_i	NP_i Max Expected Count (Probability count)	Actual Count
1	13	→	01101	169	0.14	0.56	1
2	24	→	11000	576	0.49	1.96	2
3	8	→	01000	64	0.05	0.20	0
4	19	→	10011	361	0.3	1.24	1
				1170	1	4	

$$P_i = \frac{f_i}{\sum_{i=1}^{n} f_i} = \frac{x^2}{1170}$$

13 → 0110|1 → 01100
24 → 1100|0 → 11001

24 → 11|000 → 11011
19 → 10|011 → 10000

Max = 1.96 = string 2 = 24

Crossover

String	Initial population	Crossover points	Population after crossover	x	x^2	P_i	NP_i
1	0110|1	4	01100	12	144	0.08	0.32
2	1100|0	4	11001	25	625	0.356	1.424
3	11|000	2	11011	27	729	0.415	1.660
4	10|011	2	10000	16	256	0.145	0.580
					1754		

Lower Expected Value - replace with max count

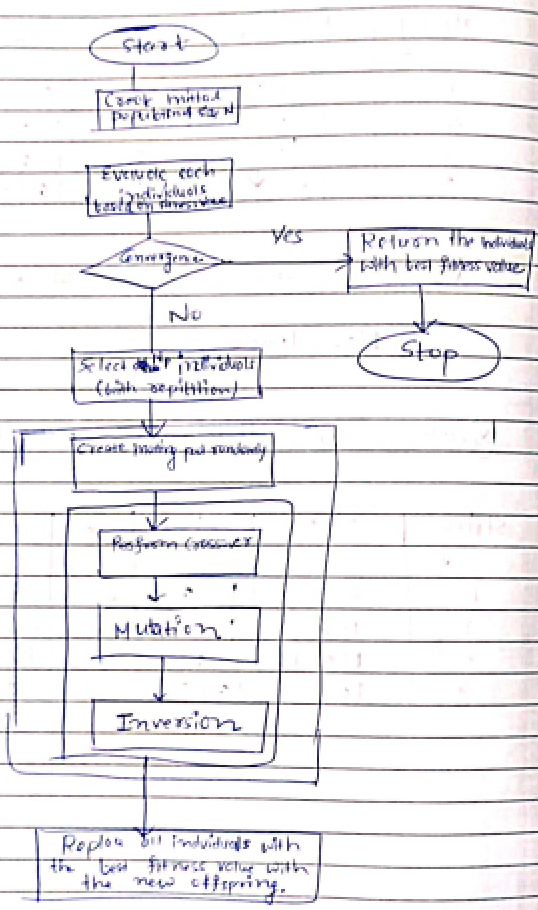
Max = 1.660

Mutation (Replace those also) (only lower one will have mutation)

String	Initial	Final	x	x^2	P_i	NP_i
1	01100	11100	28	784		
2	11001	11001	25	625		
3	11011	11011	27	729		
4	10000	10100	20	400		
				2538		

18/10/2022

Simple GA



→ 4 GA Operators

① Selection

The process of calculating the best fitness value among individuals.

② Cross Over

Exchange of Genetic material between two chromosomes.

- One point crossover
- Two point crossover

0 1 0 1 | 1 1 1 | 0 1 1
0 1 1 1 | 0 1 0 | 1 1 1 0

⇓

0 1 0 1 | 0 1 0 | 1 0 1 1
0 1 1 1 | 1 1 1 | 1 1 1 0

③ Mutation

Mutation is the process by altering the genetic material.

(4) Elitism

The process so that the individual with the best fitness value may not be lost in the process.

Q:- Explain the steps of Genetic Algorithm (GA). Carry out atleast 2 iterations of GA for the following optimization problem. Maximize $f(x) = 15x - x^2$ where $0 <= x <= 15$.

Ans:- N = no. of string = 4

String	Selection x	Initial population	$f(x) = 15x - x^2$	P_i	NP_i (Max Expected Count)	Actual Count
1	2 →	0010	26	0.156	0.624	1
2	5 →	0101	50	0.301	1.204	1
3	9 →	1001	54	0.325	1.300	1
4	12 →	1100	36	0.216	0.864	1
			166			

Max = 1.300 = String 3 = 9

Cross Over

String	Initial Population	Crossover Points	Population after Crossover	x	f(x)	P_i	NP_i
1	001'0	3	0011	3	36	0.22	[illegible]
2	0101	3	0100	4	44	0.27	1.09[illegible]
3	10'01	2	1101	13	26	0.16	0.6[illegible]
4	11'00	2	1000	8	56	0.34	[illegible]
					162		

Mutation

String	Initial	Final	x	f(x)	P_i	NP_i	Actual Count
1	0011	1011	11	44	0.268	1.072	1
2	0100	0100	4	44	0.268	1.072	1
3	1101	1101	13	26	0.158	0.632	1
4	1000	1010	10	50	0.304	1.216	1
				164			

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0101110101 0

Uniform Crossover

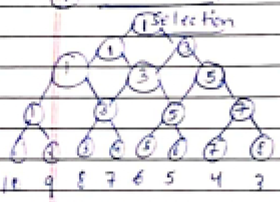
Mask =	0	1	0	1	0	0	1	0	0	1	1	
P1	0	1	0	1	1	1	1	1	0	1	1	
P2	0	1	1	1	0	1	0	1	1	1	0	

$\Downarrow$

O1	0	1	0	1	1	1	0	1	0	1	0	
O2	0	1	1	1	0	1	1	1	1	1	1	

Selection

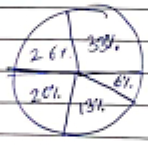
① Tournament Selection



More fitness value item will be selected

② Roullete wheel Selection

	$f(x)$	
1	5	$\frac{5}{15}$ = 33%
2	4	26%
3	3	20%
4	2	13%
5	1	6%
	15	



③ Rank Based Selection

Whose fitness will be less will be ranked more.

Rank

5 → 1

4 → 2

3 → 3

2 → 4

1 → 5

Q. $f(x) = 15x - x^2 \quad 0 \le x \le 15$

Ans.

String no.	Initial population		$f(x)$	P_i	NP_i	Expected count
1	1100	12	36	0.165	0.99	1
2	0100	4	44	0.202	1.206	1
3	0001	1	14	0.064	0.384	0
4	1110	14	14	0.064	0.384	0
5	0111	7	56	0.256	1.536	2
6	1001	9	54	0.244	1.464	1
			218	1		5

Cross over

X_6 [1|0|0|1] [0|1|0|0] × 2

X_1 [1|1|0|0] [0|1|1|1] × 5

X_2 [0|1|0|0] [0|1|1|1] × 5

String no	Initial Population	X	f(x)	Pi	NPi
6	1000	8	56	0.224	1.344
2	0101	5	50	0.2	1.2
1	1111	15	0	0	0
5	0100	4	44	0.176	1.056
2	0100	4	44	0.176	1.056
5	0111	7	56	0.224	1.334
			250		

Mutation

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Practice

Q1 :- $x = a\ b\ c\ d\ e\ f\ g\ h$

$f(x) = (a+b) - (c+d) + (e+f) - (g+h)$

$x1 = 6\ 5\ 4\ 1\ 3\ 5\ 3\ 2$

$x2 = 8\ 7\ 1\ 2\ 6\ 6\ 0\ 1$

$x3 = 2\ 3\ 9\ 2\ 1\ 2\ 8\ 5$

$x4 = 4\ 1\ 8\ 5\ 2\ 0\ 9\ 4$

(a) Evaluate fitness of each individual & arrange them in order with fittest first and the least fit last.

11-5+8-5
19-10
= 9
15-3+12-1
27-4
5-14 3-13
8-24
= -16
5-13 2-13
7-26

String no.	X	f(x)
1	65413532	9
2	87126601	23
3	23921285	-16
4	41852094	-19

Order

= x2, x1, x3, x4

(b) (i) Cross the fittest 2 individuals using 1 point-crossover at mid-point.

$x_2 = 8\ 7\ 1\ 2 \,|\, 6\ 6\ 0\ 1$ ⇒ $O_1 = 87123532$

$x_1 = 6\ 5\ 4\ 1 \,|\, 3\ 5\ 3\ 2$ ⇒ $O_2 = 65416601$

(ii) Cross the 2nd & 3rd fittest individuals using 2 point cross over at points b & f.

Ans

$x_1 = 6\ 5 \mid 4\ 1\ 3\ 5 \mid 3\ 2$

$x_3 = 2\ 3 \mid 9\ 2\ 1\ 2 \mid 8\ 5$

65921232

23413585

⇓

$O1 = 65 \mid 9\ 2\ 1\ 2 \mid 3\ 2$

$O2 = 23 \mid 4\ 1\ 3\ 5 \mid 8\ 5$

(iii) Cross the 1st & 3rd ~~[illegible]~~ fittest individuals using a uniform crossover.

Ans

$x_2 = 8\ 7\ 1\ 2\ 6\ 6\ 0\ 1$

$x3 = 2\ 3\ 9\ 2\ 1\ 2\ 8\ 5$

0 → P1

1 → P2

Mask = 0 1 0 1 0 0 1 0

$O1 = 8\ 3\ 1\ 2\ 6\ 6\ 8\ 1$

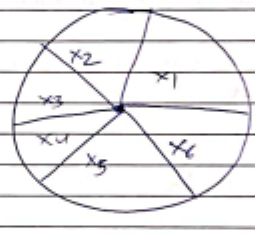
Q2 :- $N = 6$

$f(x) = 15x - x^2$

1100, 0100, 0001, 1110, 0111, 1001

(a)

String no	Initial Population	x	f(x)	Pi	
1	1100	12	36	0.165	16%
2	0100	4	44	0.201	20%
3	0001	1	14	0.064	6.4%
4	1110	14	14	0.064	6.4%
5	0111	7	56	0.256	25%
6	1001	9	54	0.247	24%
			218		



(b)

$X_6 = 1\ 0\ 0 \mid 1$

$X2 = 0\ 1\ 0 \mid 0$

$O1 = 1\ 0\ 0\ 0$

$O2 = 0\ 1\ 0\ 1$

$x_6 = 1\ 1\ 1 \mid 0$

$x_2 = 1\ 0\ 0 \mid 1$

$\boxed{O1 = 1111}$

$\boxed{O2 = 1000}$

Now after Roulette wheel, we will rank -

f(x)	
56	0111
54	1001
44	0100
36	1100
14	0001
14	1110

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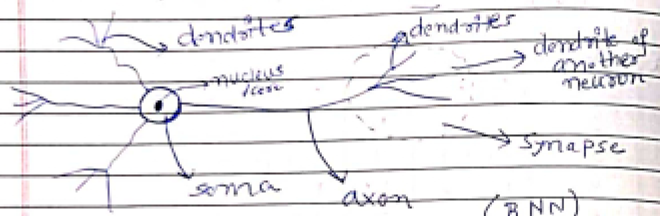
Module-6

Artificial Neural Network



(BNN)

Neuron: Basic Unit of Nervous System.

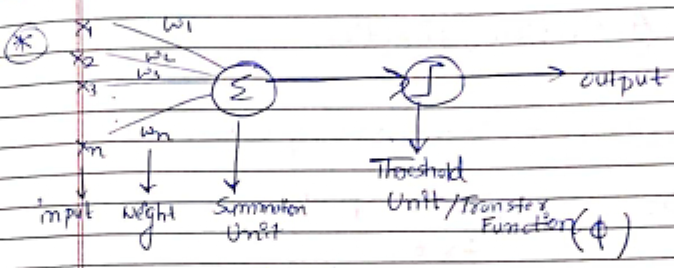
① Soma/ Cell Body receives information from other neurons through dendrites.

② Axon is an interconnecting link between input & output.

③ Synapse is the cell body of output which receives information from other neurons through dendrites.

(* *) Neurons ⇄ nodes
Dendrites ⟷ weight
Cell Body ⟷ Summation Unit
Axon ⟷ Connecting link b/w Summation unit & Threshold Unit

(*)



ANN

$$I = \sum_{i=1}^{n} w_i x_i$$

$$\phi(I) = \phi\left(\sum_{i=1}^{n} w_i x_i\right)$$

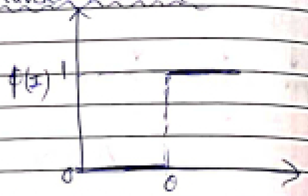
$$\phi(I) = \phi\left(\sum_{i=1}^{n} w_i x_i - \theta\right)$$

⟹ ϕ is called Transfer function or Activation function

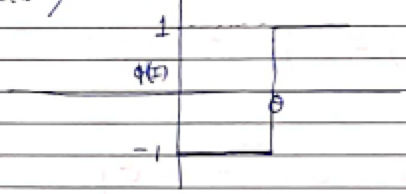
⇒ Activation Function

① Step Function / Heaviside function

$$\phi(I) = \begin{cases} 1 & \text{if } I > \theta \\ 0 & \text{if } I \leq \theta \end{cases}$$



② Signum Transfer Function / Signum Activation Function.
(Quantizer function)



$$\phi(I) = \begin{cases} +1 & , \text{if } I > \theta \\ -1 & , \text{if } I \leq \theta \end{cases}$$

③ Sigmoid Transfer Function

$$\phi(I) = \frac{1}{1 + e^{-\alpha I}}$$

α = slope
I = Input

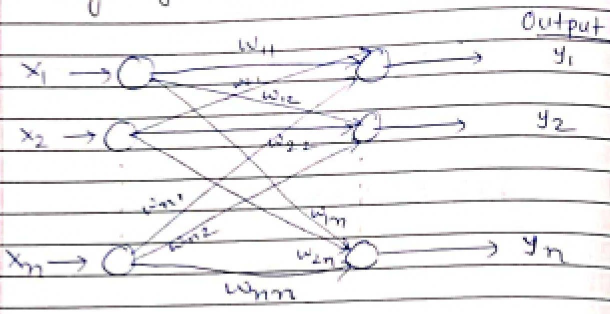
④ $\phi(I) = \tanh(I) = \dfrac{e^{\alpha I} - e^{-\alpha I}}{e^{\alpha I} + e^{-\alpha I}}$

(Hyperbolic Tangent Function)

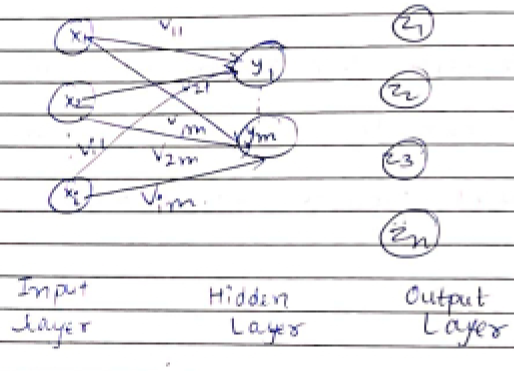
⇒ Neural Network Architectures

① Single layer feed forward architecture
② Multi layer feed forward architecture
③ Recurrent networks architecture.

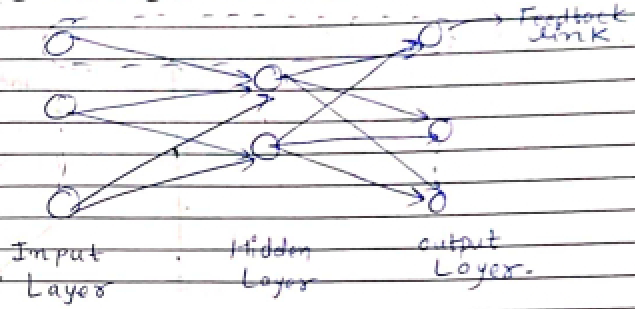
① Single Layer feed forword Architecture



② Multi Layer feed forword Architecture



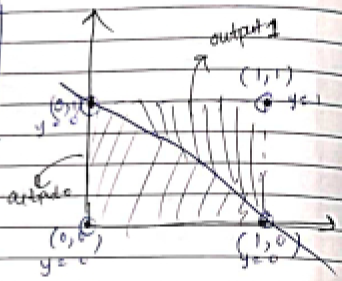
③ Recurrent Networks



2/11/2022

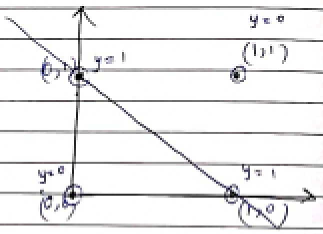
→ AND problem is linearly separable

x_1	x_2	O/P
0	0	0
0	1	0
1	0	0
1	1	1



→ XOR

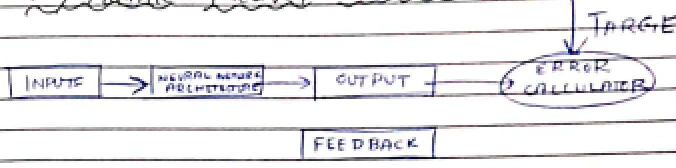
x_1	x_2	O/P
0	0	0
0	1	1
1	0	1
1	1	0



∴ Linearly not Separable

(*) For Linearly Separable Network, Single layer feed forward network will be applied & for linearly non-separable problem, multi layer feed forward network will be applied.

⇒ Dynamic Neural Network



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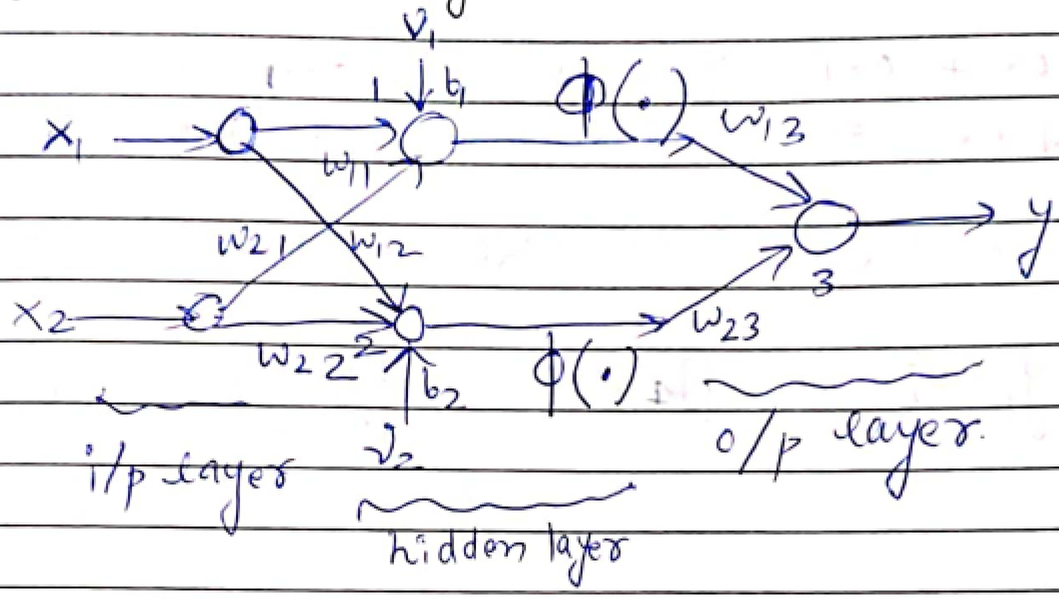
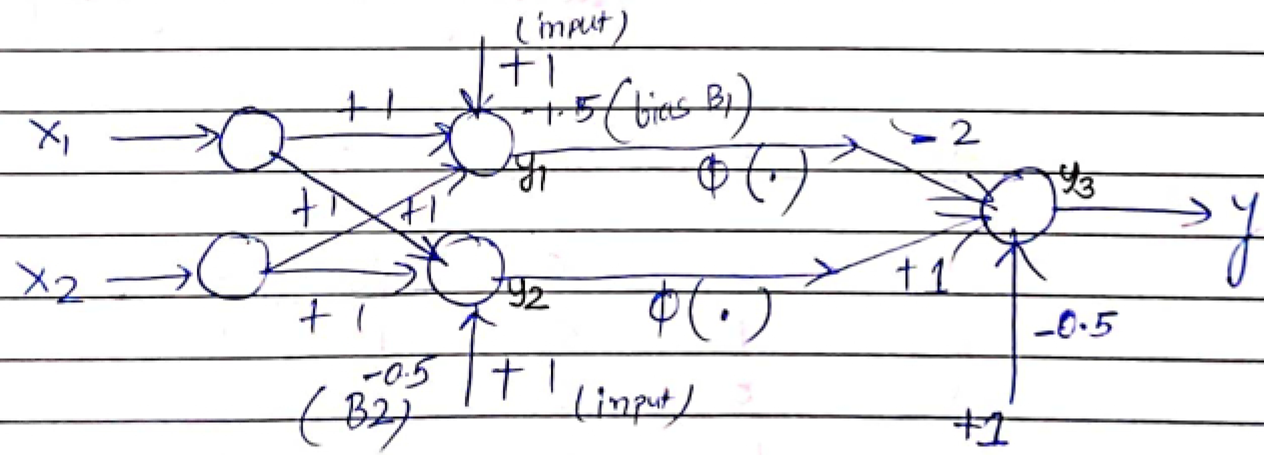
(IMP)

→ XOR Problem using multilayer feedforward (n/w)

XNOR can also come



x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0



$x_1 = 0, x_2 = 0$ | $x_1 = 0; x_2 = 1$ | $x_1 = 1, x_2 = 0$ | $x_1 = 1, x_2 = 1$

$y_1 = 0\times1 + 0\times1 + 1\times(-1.5)$ | $y_1 = 0\times1 + 1\times1 + 1\times(-1.5)$ | $y_1 = 1\times1 + 0\times1 + 1\times(-1.5)$ | $y_1 = 1\times1 + 1\times1 + 1\times(-1.5)$

$y_1 = -1.5 < 0$ | $y_1 = -0.5 < 0$ | $y_1 = -0.5 < 0$ | $y_1 = 0.5 > 0$

$\boxed{y_1 = 0}$ | $\boxed{y_2 = 0}$ | $\boxed{y_1 = 0}$ | $\boxed{y_1 = 1}$

{working as AND gate}

$y_2 = 1\times0 + 1\times0 + 1(-0.5)$ | $y_2 = 0\times1 + 1\times1 + 1(-0.5)$ | $y_2 = 1\times1 + 1\times0 + 1(-0.5)$ | $y_2 = 1\times1 + 1\times1 + 1(-0.5)$

$y_2 = -0.5 < 0$ | $y_2 = +0.5 > 0$ | $y_2 = 0.5 > 0$ | $y_2 = 1.5 > 0$

$\boxed{y_2 = 0}$ | $\boxed{y_2 = 1}$ | $\boxed{y_2 = 1}$ | $\boxed{y_2 = 1}$

{working as OR gate}

x_1	x_2	y_1	y_2	y_3
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

$y_1 = 0, y_2 = 0$: $y_3 = 0\times(-2) + 0\times1 + 1(-0.5)$, $y_3 = -0.5 < 0$, $\boxed{y_3 = 0}$

$y_1 = 0, y_2 = 1$: $y_3 = 0\times(-2) + (1\times1) + 1(-0.5)$, $y_3 = 0.5 > 0$, $\boxed{y_3 = 1}$

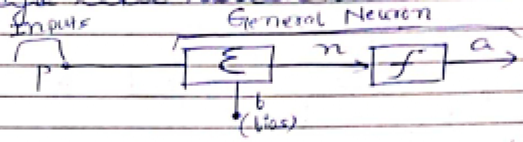
~~$y_3 = 1\times(-2) + 0\times1 + 1(-0.5)$~~, ~~$y_3 = -2.5 < 0$~~, $\boxed{y_3 = 0}$

$y_1 = 1, y_2 = 1$: $y_3 = 1\times(-2) + 1\times1 + 1(-0.5)$, $y_3 = -2 + 1 - 0.5 = -1.5$, $\boxed{y_3 = 0}$

2/11/22 {more notes are there 2 pages before this page}

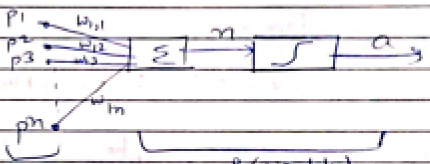
Activation Functions

→ Single Input Neuron Model



$a = f(wp + b)$

→ Multi Input Neuron Model



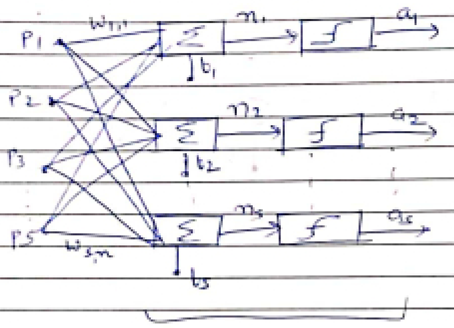
$a = f(wp + b)$

Weight Matrix $= W = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1n} \end{bmatrix}$ $\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_n \end{bmatrix}$

$n = w_{11} p_1 + w_{12} p_2 + \cdots + w_{1n} p_n + b$

$n = wp + b$ $\qquad a = f(wp + b)$

→ Multi Input Multi Neuron model



$a = f(wp+b)$

→ Activation Function

Name	Input/output Rel^n	Icon	MATLAB func^n
Hard limit	$a=0,\ n<0$ $a=1,\ n\geq 0$		hardlim
Symmetrical Hard limit	$a=-1\ \ n<0$ $a=+1\ \ n\geq 0$		hardlims
Linear	$a=n$		purelin
Saturating linear	$a=0\ \ n<0$ $a=n\ \ 0\leq n\leq 1$ $a=1\ \ n>1$		satlin
Symmetric Saturating Linear	$a=-1\ \ n<-1$ $a=n\ \ -1\leq n\leq 1$ $a=1\ \ n>1$		satlins
Log sigmoid	$a = \frac{1}{1+e^{-s}}$		~~tansig~~ logsig
Positive linear	$a = \frac{e^{n}-e^{-s}}{e^{n}+e^{-n}}$		tansig
Competitive	$a=1$ neuron with max n $a=0$ all other neurons		compet

Q1: The i/p to a single-i/p neuron is 2.0, weight = 2.3, bias = -3

Ans: Neuron output = $wp + b$

$= 2(2.3) - 3 = 1.6$

Q2: O/P of neuron of Q1 if it has the following transfer functions?

(i) Hard Limit (ii) Linear (iii) Log-Sigmoid.

$1.6 > 0$ $a = \text{purelin}(1.6)$ $a = \text{logsig}(1.6)$

O/P = 1 $a = 1.6$ $= \frac{1}{1+e^{-1.6}} = 0.8320$

(4?) [5]

Q3. Given 2 i/p neuron $b = 1.2$, $W = [3\ 2]$ & $p = [-5\ 6]^T$, calculate neuron o/p for the following transfer functions

(i) A symmetrical hard limit transfer function

(ii) A saturating linear transfer function

Q1. 2 i/p neuron
$b = 1.4$ $W = [3, 2]$ Input $p = [-4\ 5]^T$
Calculate neuron o/p for symmetrical hard limit transfer function

Q2. $W = [2, 4]$ $P = [-6, 5]^T$

$b = 1.3$

Ans

$$[2\ 4]\begin{bmatrix}-6\\5\end{bmatrix} + 1.3$$

$$= 9.3$$

$n > 1$

$\therefore \boxed{a = 1}$

$$[3\ 2]\begin{bmatrix}-4\\5\end{bmatrix} + b = [-12 + 10] + 1.4$$

$$= -2 + 1.4$$

$$= -0.6$$

$wp + b$

$$= [3\ 2]\begin{bmatrix}-5\\6\end{bmatrix} + 1.2$$

$$= -15 + 12 + 1.2$$

$$= -3 + 1.2$$

$n = -1.8$

(i) $n < 0$

$a = -1$

(ii) $n < 0$

$a = 0$

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Perceptron & Perceptron Learning Rules

→ Supervised (Babies) & Unsupervised Learning

- Supervised: There is target value
- Unsupervised: There is no target value

⇒ Perceptron Networks

⇒ Single layer Perceptron Networks

- In Step 1, initialise the weight and bias.

  $w_1 = 0$

  $w_2 = 0$

  Bias $= 0$

- Step 2 → Define the learning rate α and target value t

- Step 3 → Define the activation function

$$\underbrace{f(y_{in})}_{\text{output of the neuron}} = \begin{cases} 1 & \text{if } y_{in} > \theta \\ 0 & \text{if } \theta \le y_{in} \le \theta \\ -1 & \text{if } y_{in} \le \theta \end{cases}$$

- Step-4 → If $f(y_{in}) \neq t$ ($y \neq t$)

$$w_i(\text{new}) = w_i(\text{old}) + \alpha t x_i$$

$\Rightarrow \Delta w_i = \alpha t x_i$

$$b(\text{new}) = b(\text{old}) + \alpha t$$

Q1. Implement AND function using perceptron network

$\alpha = 1$

$w_1 = w_2 = 0$

$b = 0$

x_1	x_2	t	y_{in}	y	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	1	0	0	1	1	1	1	1	[illegible]
1	-1	-1	1	1	-1	1	-1	0	2	0
-1	1	-1	2	1	1	-1	-1	1	1	-
-1	-1	-1	-3	-1	0	0	0	1	1	-1

$\Delta w_1 = \alpha t x_1$ (weight change), $\Delta b = \alpha t$

$\{y = t\}$ $\{\Delta w = 0,\ \Delta b = 0\}$

$$y = \begin{cases} 1 & \text{if } y_{in} > 0 \\ 0 & \text{if } y_{in} = 0 \\ -1 & \text{if } y_{in} < 0 \end{cases}$$

$y_{in1} = w_1 x_1 + w_2 x_2 + b$

$= 0(1) + 0(1) + 0$

$= 0$

$y_{in2} = w_1 x_1 + w_2 x_2 + b$

$= (1)(1) + 1(-1) + 1$

$= 1$

$y_{in3} = 0(-1) + 2(1) + 0$

$= 2$

$y_{in4} = 1(-1) + 1(-1) - 1$

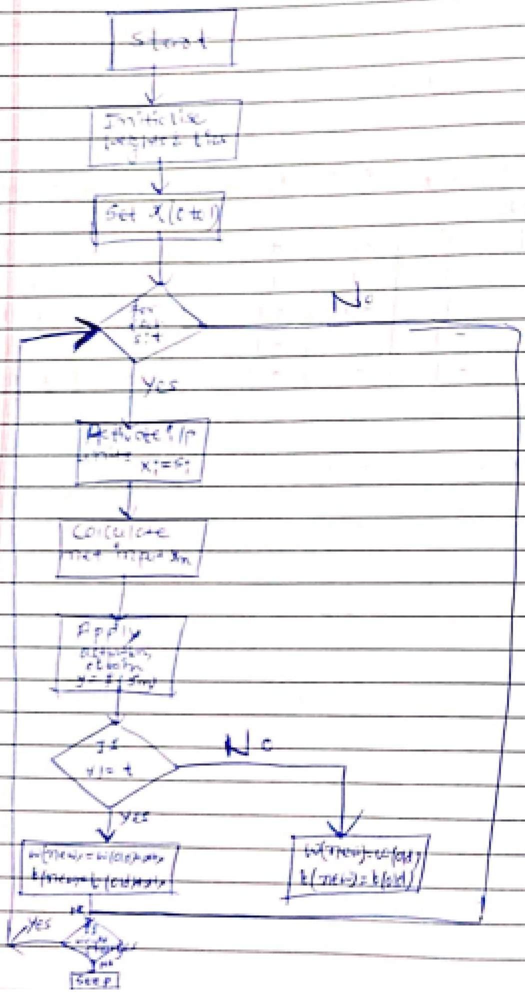
$= -1 - 1 - 1$

$= -3$

11/11/2022

(Practice perceptron question with binary i/p 0 & 1)

Perceptron network for Single Output



Unsupervised Learning

⟹ Mc Culloch - Pitts Neuron Model

Step-1 → $y_{in} = b + \sum_{i=1}^{n} x_i w_i$

$y_{in} = b + w_1 x_1 + w_2 x_2 = 0$

$\therefore x_2 = -\frac{w_1}{w_2} x_1 - \frac{b}{w_2}$

Net input received $> \theta$ (threshold)

$y_{in} > \theta$

$x_1 w_1 + x_2 w_2 > \theta$

$\therefore$ line eqn will be

$x_1 w_1 + x_2 w_2 = \theta$

$\therefore x_2 = \frac{-w_1}{w_2} x_1 + \frac{\theta}{w_2}$ (with $w_2 \neq 0$)

Q. Implement AND function using Single i/p neuron model or McPitts neuron model.

Ans :-

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

$w_1 = w_2 = 1$
bias = 0
(assume)

$y_{in} = b + w_1 x_1 + w_2 x_2$

$(0,0) \rightarrow y_{in} = 0\times1 + 0\times1 + 0 = 0$

$(0,1) \rightarrow y_{in} = 1\times0 + 1\times1 + 0 = 1$

$(1,0) \rightarrow y_{in} = 1\times1 + 1\times0 + 0 = 1$

$(1,1) \rightarrow y_{in} = 1\times1 + 1\times1 + 0 = 2$

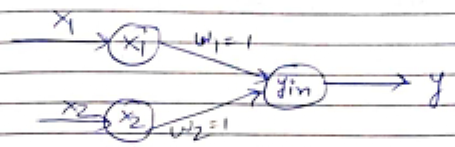
$$\therefore y = f(y_{in}) = \begin{cases} 1 & y_{in} \geq \theta \\ 0 & y_{in} < \theta \end{cases}$$

$\theta = nw - p$

$= 2\times1 - 0$

$\boxed{\theta = 2}$

AND
OR
ANDNOT
XOR



Q. Implement OR.

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

$w_1 = w_2 = 1$
bias = 0

$y_{in} = 0\times1 + 1\times0 = 0$

$y_{in} = 1\times0 + 1\times1 = 1$

$y_{in} = 1\times1 + 1\times0 = 1$

$y_{in} = 1\times1 + 1\times1 = 2$

~~But~~ $$y = f(y_{in}) = \begin{cases} 1 & y_{in} \geq 2 \\ 0 & y_{in} < 2 \end{cases}$$

But it is not satisfying here

$w_1 = 1$, $w_2 = -1$ we will take.

15/11/2022

Q. Solve XOR gate using MLP

Q: Why in XOR we use multi layer not single layer perceptron?

Ans: Because XOR is not linearly separable.

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

Calculate $w_{11}, w_{22}, w_{12}, w_{21}$.

Case-1

$$y = \overline{x_1}\, x_2 + x_1 \overline{x_2}$$
$$= z_2 + z_1$$
$$= z_2 \text{ or } z_1$$

$\therefore z_1 = x_1 \overline{x_2}$

x_1	x_2	z_1
0	0	0
0	1	0
1	0	1
1	1	0

Now $w_{11} = ?$ $w_{21} = ?$

$\theta = ?$

Case (i)

let $w_{11} = 1$, $w_{21} = 1$

$(0, 0)$ $z_{1in} = x_1 w_{11} + x_2 w_{21} = 0\times1 + 0\times1 = 0$

$(0, 1)$ $z_{1in} = 1$

$(1, 0)$ $z_{1in} = 1$

$(1, 1)$ $z_{1in} = 2$

$1, \theta \geq 1$
$0, \theta < 1$ → But it is not satisfying the truth table so we take $w_{11} = 1$, $w_{21} = -1$

Case (ii)

let $w_{11} = 1$; $w_{21} = -1$

$(0,0) \longrightarrow 0\times1 + 0\times-1 = 0 = z_{1in}$

$(0,1) \longrightarrow 0\times1 + 1\times-1 = -1 = z_{1in}$

$(1,0) \longrightarrow 1\times1 + 0\times-1 = 1 = z_{1in}$

$(1,1) \longrightarrow 1\times1 + 1\times-1 = 0 = z_{1in}$

Now $\theta \geq 1$, 1
$\theta < 1$, 0 } it is satisfying the truth table

$w_{11} = 1$

$w_{21} = -1$ $\Big\}$ z_1 input

$\theta = 1$

Now, for $z_2 = \overline{x_1}\, x_2$

x_1	x_2	z_2
0	0	0
0	1	1
1	0	0
1	1	0

Now

Case(i)

~~$w_{11} = 1$, $w_{21} = 1$~~

$w_{12} = +1 \quad w_{22} = +1$

$\therefore (0,0) \rightarrow z_{2in} = 0$

$(0,1) \rightarrow z_{2in} = 1$

$(1,0) \rightarrow z_{2in} = 1$

$(1,1) \rightarrow z_{in} = 2$

$\theta \geq 1,\ 1$
$\theta < 1,\ 0$ $\Big\}$ ∴ not satisfying with truth table

Case(ii) $\quad w_{12} = -1, \quad w_{22} = +1$

$(0,0) \rightarrow 0$

$(0,1) \rightarrow 1$

$(1,0) \rightarrow -1$

$(1,1) \rightarrow 0$

$\theta \geq 1,\ 1$
$\theta < 1,\ 0$ $\Big\}$ it is satisfying truth table

$\therefore \quad w_{12} = -1$

$w_{22} = 1$ $\Big\}$ z_2 input

$\theta \geq 1$

For output 'y'

$$y = z_1 v_1 + z_2 v_2$$

x_1	x_2	y	z_1	z_2
0	0	0	0	0
0	1	1	0	1
1	0	1	1	0
1	1	0	0	0

Case (i) $v_1 = 1, \quad v_2 = 1$

$\therefore \quad (0,0) \rightarrow 1\times 0 + 1\times 0 = 0$

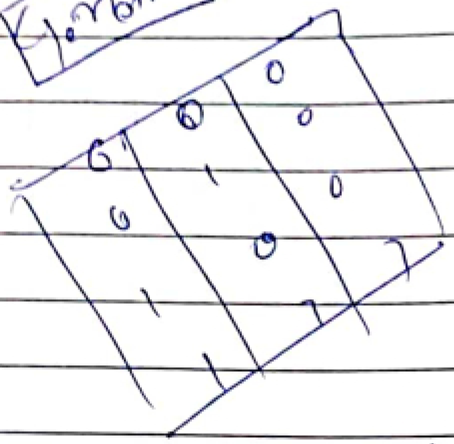
$(0,1) \rightarrow 1\times 0 + 1\times 1 = 1$

$(1,0) \rightarrow 1\times 1 + 1\times 0 = 1$

$(1,1) \rightarrow 1\times 0 + 1\times 0 = 0$

$$\therefore \boxed{\begin{array}{l} v_1 = 1 \\ v_2 = 1 \end{array}}$$

Symonic



3.5
1.5
1.75
3.5
4.25
1.50
2.75

$w = 3.5$

$b = 1.5$

$w_1 x_1 + w_2 x_2 + b$

$(1.5)(3.5) - 1.5$

$(3.5)(1.5) - 1.5$

1st Module

chap 1

1. Diff b/w soft computing & hard computing
2. soft computing constituents.

chap-2

1. Diff b/w expert system & intelligent system with diagram.
2. Characteristics of neuro fuzzy & soft computing (10 points)

classmate
Date
Page

Chap 3

1. Basic def. & terminologies of fuzzy set (with problems)
like core, support, etc...

2. Fuzzy set theory operations
(union, intersection, Cartesian product & co-product)

3. Diff types of membership functions.

4. Fuzzy Relation:
   - Max Min composition
   - Max Product composition
   - Extension principle
   - 2 D principle
     { eg - $\frac{x-y}{x+y+2}$ }

5. Operators of A coupled with B
   & A entails B
   - T-norm operator
   - S-norm operator

6. Linguistic Variables
   - Concentration
   - dilation
   - intensifier
   - contrast intensifier.

(7) Composite linguistic variables problems
{e.g. ~~old, young, very old, very young~~}

(8) Fuzzy reasoning
↳ Single with single antecedent
↳ Single with multiple "
↳ Multiple " " antecedent.

(9) Fuzzy defuzzification method
(theory) (no numericals)
α - cut
λ - cut (Problems)

(10) Mamdani
TSK
Sugeno } (Theory)

(11) Characteristics of derivative free optimization

(12) Flow chart of GA
& it's operators.

(13) ANN
↳ Comparison of ANN & BNN

(14) Linear seperable & Non-seperable

(15) AND, OR, XOR problems.

(16) Single layer netw. arch.
Multi " " "
Recurrent " " "

(17) Diff types of Activation funcns.
(long ques)
(name & it's operator)

(18) Activation function problems
(short Q.)

(19) Diff. learning methods (theory)
(supervised, unsupervised etc.)

(20) Adaline, Madaline, BPN & RBFN

(21) Merits & demerits of back propagation network

(22) ANFIS architecture.
(diagram & theory)