CALCULUS AND IT'S APPLICATION

A PROJECT REPORT

Submitted by

SAYAN GOSWAMI

UID: 22013121051

DEPARTMENT OF MATHEMATICS

BANKURA CHRISTIAN COLLEGE

for

B.Sc. SEMESTER - VI EXAMINATION, 2024 – 25

in

MATHEMATICS (HONORS)

Of

BANKURA UNIVERSITY

CALCULUS AND IT'S APPLICATION

Name of the Student: SAYAN GOSWAMI

UID : 22013121051

Registration No. : 00419 of 2022-23

Semester : VI

Academic Session : 2024 – 25

Course Type : Honors [CBCS (NEW)]

Subject : MATHEMATICS

Course Title: Dissertation on any topic of Mathematics (Project Work)

Course Code : SH/MTH/604/DSE – 4

Course ID : 62127

Name of the Examination: B.Sc. Semester - VI Examination, 2024 – 25

Name of the Supervisor: Dr. Subhasis Bandyopadhyay

DEPARTMENT OF MATHEMATICS

BANKURA CHRISTIAN COLLEGE

(Estd: 1903)

Re-accredited with Grade ‘A’ (4th Cycle) by NAAC in 2025

College with Potential for Excellence (2nd Phase)

PO+Dist: Bankura, West Bengal, INDIA, PIN: 722101

Phone No. 03242-250924

Website: www.bankurachristiancollege.in

email: principal@bankurachristiancollege.in

<u>PROJECT COMPLETION CERTIFICATE</u>

<u>To Whom It May Concern</u>

This is to certify that SAYAN GOSWAMI (UID: 22013121051, Registration No: 00419 of 2022-23) of Department of Mathematics, Bankura Christian College, Bankura, has successfully carried out this project work entitled “CALCULUS AND IT’S APPLICATION” under my supervision and guidance.

This project has been undertaken as a part of the undergraduate CBCS (New) curriculum of Mathematics (Honors), Semester : VI, Paper: DSE – 4, Course Title: Dissertation of any topic in Mathematics (Project Work), Course ID: 62127 and for the partial fulfilment of the degree of Bachelor of Science (Honors) in Mathematics of Bankura University under CBCS (New) Curriculum in 2024 – 25.

Signature of the Supervisor

Name :

Designation:

Signature of the HOD

Department of Mathematics

Bankura Christian College

DECLARATION

I hereby declare that my project, titled 'CALCULUS AND IT'S APPLICATION', has been submitted by me to Bankura University for the DSE-4 paper in Semester VI, under the guidance of my professor of Mathematics at Bankura Christian College, Dr. Subhasis Bandyopadhyay. I also declare that the project has not been submitted here by any other student.

Name: SAYAN GOSWAMI

UID Number: 22013121051

Semester: VI

ACKNOWLEDGEMENT

First of all, I am immensely indebted to my institution, Bankura Christian College under Bankura University, and it is my humble pleasure to acknowledge my deep sense of gratitude to our Principal Sir, Dr. Fatik Baran Mandal, for his valuable suggestions and encouragement that made this project successful. I am grateful to the Head of the Mathematics Department, Dr. Utpal Kumar Samanta, for always lending his helping hand in every situation. A heartfelt thanks to my guide in this project, Dr. Subhasis Bandyopadhyay, whose guidance and valuable support have been instrumental in the completion of this project work.

Table of Contents

1. Introduction

Calculus is a fundamental branch of mathematics that focuses on change and accumulation. Developed in the 17th century by Newton and Leibniz, it is now indispensable in science, engineering, economics, and more. It provides a systematic method for analyzing dynamic systems, modeling real-world phenomena, and solving complex problems.

The study of calculus is traditionally divided into two main branches: **differential calculus** and **integral calculus**.

- **Differential calculus** focuses on the concept of the derivative, which represents the rate of change of a quantity. It is used to determine the slope of a curve, analyze motion, optimize functions, and understand how variables change with respect to one another.
- **Integral calculus**, on the other hand, deals with accumulation and area under curves. It involves the concept of the integral, which can be interpreted as the total accumulation of a quantity over an interval, such as distance travelled over time, area under a curve, or volume enclosed by surfaces.

Together, differential and integral calculus form a complete framework for understanding both local behaviour (through derivatives) and total accumulation (through integrals). The **Fundamental Theorem of Calculus** bridges these two branches by showing that differentiation and integration are inverse processes.

This project aims to explore the core ideas of calculus and illustrate its applications in various domains such as physics, biology, engineering, and economics. Through real-world examples, the usefulness and power of calculus in solving practical problems will be highlighted.

2. Aim and Objective of the Project

Aim:

The aim of this project is to explore the fundamental principles of calculus and demonstrate its applications across various disciplines, using both theoretical analysis and real-world case studies.

Objectives:

- To understand the origin and evolution of calculus.
- To study core concepts such as limits, derivatives, and integrals.
- To investigate how differential and integral calculus apply in fields like physics, economics, biology, and engineering.
- To analyze real-world problems and show how calculus helps in solving them.
- To introduce computational tools used to perform calculus-related operations.

3. Historical development of Calculus

Calculus has a rich and fascinating history, developing over centuries from ancient geometry to modern-day analysis. The term "calculus" itself comes from the Latin word for "small pebble," which was used in counting.

Ancient Foundations

- **Babylonians and Egyptians** used numerical methods for area calculations.
- **Greek mathematicians** like **Eudoxus** and **Archimedes** developed the method of exhaustion—an early form of integration.
- **Indian mathematicians** like **Aryabhata** and **Bhaskara II** introduced preliminary ideas of instantaneous rates of change and summation.
- **Chinese scholars**, notably **Liu Hui**, used early integration techniques to compute volumes and areas.

Birth of Calculus (17th Century)

- **Isaac Newton (1642–1727):** Developed “fluxions” to describe motion and change. He used calculus for his laws of motion and gravitation.
- **Gottfried Wilhelm Leibniz (1646–1716):** Introduced much of the notation still in use today (e.g., ∫, dy/dx). He emphasized the use of infinitesimals.
- Both men developed calculus independently, leading to a historic dispute but mutual recognition of their contributions today.

18th–19th Century: Formalization

- **Euler, Lagrange, and Laplace** expanded calculus into mechanics and astronomy.
- **Augustin-Louis Cauchy** introduced rigorous definitions of limits and continuity.
- **Karl Weierstrass** formalized the epsilon-delta definition of limits.
- **Riemann and Lebesgue** developed different integration theories.

20th Century to Present

- Calculus now includes **vector calculus**, **multivariable calculus**, and **computational calculus**.
- It underpins **numerical methods**, **AI**, **differential geometry**, **quantum mechanics**, and more.

4. Fundamentals of Calculus

4.1 Differential Calculus

- **Concepts:** Derivatives, limits, continuity.
- **Applications:** Rate of change, optimization, tangents.
- **Example:** Velocity as the derivative of displacement.

4.2 Integral Calculus

- **Concepts:** Definite and indefinite integrals, area under curves.
- **Applications:** Accumulated quantity, total area, solving differential equations.
- **Example:** Calculating area under a velocity-time graph to find distance.

4.3 Multivariable Calculus (Brief Overview)

- **Partial derivatives**
- **Multiple integrals**
- **Applications in thermodynamics, economics, and optimization**

5. Concepts of limits, derivatives, and integrals

5.1 Limits

A **limit** describes the value that a function approaches as the input (usually denoted by xxx) gets closer to a certain point. Limits are essential for defining both continuity and the derivative of a function.

Example:

$$\lim_{x \to a} f(x) = L$$

This means that as x approaches a, the function f(x) gets closer to the value L. If the limit exists, it gives a precise way to understand behaviour near a point, even if the function is not defined at that point.

5.2 Derivatives

The **derivative** measures the **rate of change** of a function with respect to one of its variables. It tells us how a quantity changes as its input changes and is a fundamental tool in analysing motion, growth, optimization, and more.

Mathematically, the derivative of f(x) at a point x is defined using a limit:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This represents the **slope of the tangent line** to the curve y = $f(x)$ at a given point. In real-life applications, derivatives are used to calculate velocity (rate of change of position), acceleration, and marginal cost/revenue.

5.3 Integrals

The **integral** represents the process of **accumulation**—finding the total effect of small, continuous changes. While derivatives break things down into instantaneous change, integrals build up the whole from many small parts.

There are two main types:

- **Indefinite Integral**: Represents a family of functions whose derivative is the given function.

$$\int f(x)\,dx$$

- **Definite Integral**: Represents the area under the curve of $f(x)$ between two limits a and b:

$$\int_a^b f(x)\,dx$$

Integrals are widely used in finding **area, volume, displacement**, and **total accumulation** in physics, engineering, economics, and biology.

.

6. Applications of Differential Calculus

In Physics:

- Motion equations, velocity, and acceleration.
- Example: v(t) = $\frac{ds}{dt}$, a(t) = $\frac{dv}{dt}$

In Economics:

- Marginal cost and marginal revenue.
- Optimization of profit: Set dP/dx = 0

In Biology:

- Modeling growth rates of populations.
- Example: Logistic growth using first-order derivatives.

In Engineering:

- Structural stress analysis.
- Optimization of design for minimal material usage.

6.1

In Physics: Motion Analysis Using Derivatives

Differential calculus is fundamental in physics, particularly in the study of **kinematics**, which deals with the motion of objects. The concepts of **position**, **velocity**, and **acceleration** are interrelated through derivatives:

- **Position (s)** represents the location of an object at a given time t.
- **Velocity (v)** is the **rate of change of position** with respect to time. It describes how fast and in what direction an object is moving. Mathematically,

$$V(t) = \frac{ds}{dt}$$

- **Acceleration (a)** is the **rate of change of velocity** with respect to time. It indicates how the velocity of an object is changing. It is given by:

$$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

Example:

If the position of an object is given by s(t) = $5t^2 + 3t$ then:

- Velocity:

$$v(t) = \frac{ds}{dt} = 10t + 3$$

- Acceleration:

$$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 10$$

This shows that the object moves with a linearly increasing velocity and constant acceleration.

7. Applications of Integral Calculus

In Physics:
- Work done: $W = \int F(x)\, dx$
- Area under velocity-time curve for displacement.

In Economics:
- Accumulated cost and revenue.
- Consumer and producer surplus.

In Biology:
- Total population over time.
- Area under infection-rate curve during epidemics.

In Engineering:
- Calculating volumes of materials.
- Heat and fluid flow models.

7.1

In Engineering: Volumes, Heat, and Flow Models

Integral calculus plays a vital role in engineering, especially in the design and analysis of systems involving geometry, materials, energy, and fluids.

- **Volume of Materials**:
  Engineers use integrals to calculate the volume of complex shapes and objects by revolving a curve around an axis (solid of revolution). For example, the volume of a solid generated by rotating a function y = f(x) about the x-axis from x=a to x=b is:

  $$V = \pi \int_a^b [f(x)]^2 \, dx$$

  This is useful in material cost estimation and design of components like pipes, tanks, and beams.

- **Heat and Fluid Flow Models**:
  In thermodynamics and fluid mechanics, integrals are used to calculate quantities such as:
  - **Total heat transfer** over time or space.
  - **Cumulative flow of fluids** through surfaces or along channels.

    For example, if q(t) is the rate of heat transfer over time, the total heat transferred from time t1 to t2 is:

    $$Q = \int_{t1}^{t2} q(t) \, dt$$

    Similarly, if F(x) represents the flow rate of a fluid along a pipe, the total volume that has passed through a section is:

    $$V = \int_a^b F(x) \, dx$$

8. Real-World problems that can be solved using Calculus

Case Study 1: Cost Minimization in Manufacturing

Case Study 2: Population Modeling

- Model: $\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$
- Application: Predict population dynamics using differential equations.

Case Study 3: Area Under Curve in Medical Research

- Example: Analyzing drug concentration over time using definite integrals.

Case Study 4: Traffic Flow Optimization

- **Problem:** Find the optimal number of vehicles that can pass through a signal to minimize wait time.
- **Approach:** Use differential equations to model vehicle density $\rho(x,t)$ and flow Rate $q(x,t)=\rho v$
-

Case Study 5: Economics – Maximum Profit

- **Given:** Profit function $P(x)=R(x)-C(x)$
- **Objective:** Find x such that $dP/dx=0$
- **Conclusion:** Maximum profit occurs when marginal revenue equals marginal cost.

Case Study 6: Epidemic Spread (SIR Model)

- **Model:**

$$dS/dt = -\beta SI \ , \ dI/dt = \beta SI - \gamma I, \ dR/dt = \gamma I$$

- **Application:** Used to predict the spread of diseases like COVID-19.

Case Study 7: Rocket Trajectory

- Calculus helps derive height, velocity, and acceleration using Newton's laws.
- Integral calculus is used to compute distance and fuel efficiency.

8.1 CASE STUDY 1 : Cost Minimization in Manufacturing

In manufacturing, one of the key objectives is to **minimize production cost** while maintaining output and efficiency. This can be achieved using the tools of **differential calculus**, particularly by analysing the **cost function** and finding its minimum point.

Let's take an example,

Let the **cost function** be given by:

$$C(x) = ax^2 + bx + c$$

where:

- x is the number of units produced,
- a,b,c are constants,
- a>0 ensures the graph opens upwards (a parabola), meaning a minimum exists.

Solution Using Calculus:

To find the **minimum cost**, we take the derivative of the cost function with respect to x

$$C'(x) = \frac{d}{dx}(ax^2 + bx + c) = 2ax + b$$

We set the first derivative equal to zero to find critical points:

$$2ax + b = 0 \quad \Rightarrow \quad x = -\frac{b}{2a}$$

This value of x gives the number of units that should be produced to minimize the cost. Since a>0, the function is convex, confirming that this point is a **minimum**.

8.2 CASE STUDY 2 : Population Modeling

One important application of differential calculus in biology is **modeling population growth**. Populations rarely grow indefinitely; instead, their growth is affected by limited resources, space, and competition. A widely used model to describe such behavior is the **logistic growth model.**

Mathematical Model:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

where:

- P(t) is the population at time t,
- $\frac{dP}{dt}$ is the **rate of change** of population with respect to time,
- r is the **intrinsic growth rate,**
- K is the **carrying capacity** of the environment (maximum sustainable population).

Explanation:

- When P is small compared to, the term $\left(1 - \frac{P}{K}\right) \approx 1$, so the population grows almost exponentially.
- As P approaches K, the term $\left(1 - \frac{P}{K}\right)$ becomes smaller, slowing the growth rate.
- When $P = K$, growth stops ($\frac{dP}{dt} = 0$), indicating that the population has reached its environmental limit.

Let's take an example,

Suppose a population of fish in a lake is initially 100, and the lake can support a maximum of 1000 fish. Assume the growth rate r=0.4

Then the logistic model becomes:

$$\frac{dP}{dt} = 0.4P\left(1 - \frac{P}{1000}\right)$$

We can use this equation to predict the fish population at different times.

Key Observations:

- When P is small, the population grows quickly (almost exponentially).
- As P increases and approaches 1000, growth slows down.
- When P = 1000, the growth stops:

$$\frac{dP}{dt} = 0$$

This shows how calculus—through differential equations—can help **model and predict biological systems** realistically over time.

9. Conclusion and Future Scope

Calculus is not just theoretical but highly applicable in practical life. This project highlights how calculus provides a framework to model, analyze, and predict dynamic systems across science and industry. From optimizing profit to understanding disease spread, its importance is unmatched. Through this project, we explored the core concepts of **differential and integral calculus** and their applications in fields such as **physics, economics, biology, and engineering**.

- **Differential calculus** helps in studying rates of change, motion, growth, optimization, and more.
- **Integral calculus** is essential for measuring quantities such as area, volume, total population, and accumulated cost.

By applying **calculus**, we are able to create **mathematical models** that accurately describe and predict **real-world** behavior. These tools not only enhance **theoretical** understanding but also improve **practical decision-making** in science and industry.

Future Scope:

The future of calculus is both broad and impactful. As technology and data science advance, calculus continues to find new applications in areas such as:

- **Artificial Intelligence and Machine Learning** (e.g., optimization algorithms)
- **Medical sciences** (e.g., tumour growth modeling, drug dosage design)
- **Climate modeling and environmental science**
- **Robotics and automated systems**
- **Financial engineering and risk analysis**

10. References

- James Stewart, *Calculus: Early Transcendentals*, 8th Edition, Cengage Learning, 2015.

- George B. Thomas, Maurice D. Weir, Joel R. Hass, *Thomas' Calculus*, 14th Edition, Pearson, 2017.

- Sheldon M. Ross, *Introduction to Mathematical Modeling*, 3rd Edition, Academic Press, 2003.

- William E. Boyce and Richard C. DiPrima, *Elementary Differential Equations and Boundary Value Problems*, 10th Edition, Wiley, 2012.

- Khan Academy, *Calculus Topics*, Available at: https://www.khanacademy.org/math/calculus-1

- Wolfram MathWorld, *Calculus*, Available at: https://mathworld.wolfram.com/Calculus.html

- World Health Organization, *Mathematical Modeling of Infectious Diseases*, Available at: https://www.who.int