In[8]:=
$$p1[r\_] := \frac{27\,(r + r^2)}{8\,(1 + r + r^2)^{5/2}};$$

$$p2[r\_] := \frac{81\,\sqrt{3}\,(r + r^2)^2}{4\,\pi\,(1 + r + r^2)^4};$$

$$psp[r\_, \nu\_] := \left(\frac{\text{Gamma}[2\,\nu + 2]\,\text{Gamma}[\nu + 2]^2}{(\nu + 1)^2\,\text{Gamma}[\nu + 1]^4}\right)\left(\frac{r^\nu}{(1 + r)^{2\,\nu+2}}\right);$$

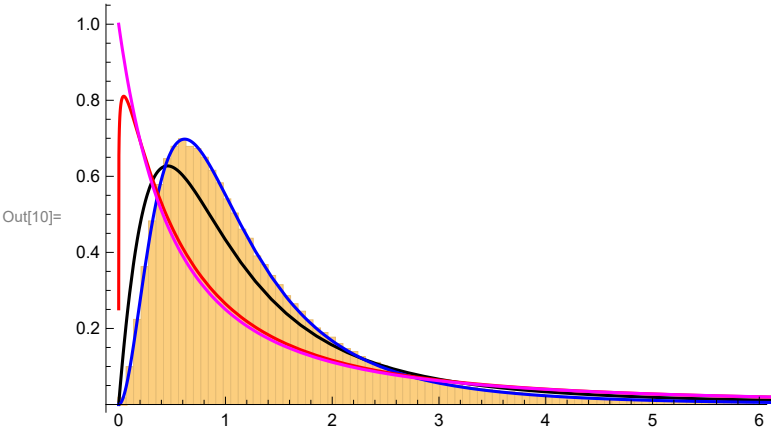
Singular value distribution (eigenvalues of $\sqrt{HH^+}$) for the GinUE

In[6]:=
```
n = 100; singev = {}; Sprd = {}; sprall = {};

Monitor[For[i = 1, i < 5001, i++, {
   (*ev=RandomReal[{-1,1},n]+i RandomReal[{-1,1},n];*)
   H0 = RandomVariate[NormalDistribution[0, 1/Sqrt[2 n]], {n, n}];
   H1 = RandomVariate[NormalDistribution[0, 1/Sqrt[2 n]], {n, n}] +
     i RandomVariate[NormalDistribution[0, 1/Sqrt[2 n]], {n, n}];
   ev = Sqrt[Eigenvalues[H0.ConjugateTranspose[H0]]];
   (*singev=Join[singev,ev];*)
   sev = Sort[ev];
   spr = Table[(sev[[j + 2]] - sev[[j + 1]]) / (sev[[j + 1]] - sev[[j]]), {j, 1, n - 2}];
   sprall = Join[sprall, spr];
   rtil = Min[spr, 1 / spr];
   Sprd = Append[Sprd, rtil];
  }], i]
```

In[10]:=
```
Show[Histogram[Flatten[sprall], {0, 6, 0.07}, "PDF"],
 Plot[{p1[r], p2[r], psp[r, 0.1], 1 / (1 + r)^2}, {r, 0, 7},
  PlotRange → All, PlotStyle → {Black, Blue, Red, Magenta}]]
```

Out[10]=



In[ ]:=
```
rtil
```

Out[ ]=
```
0.177616
```

In[ ]:=
```
singev
```

Out[ ]=
```
{30.222, 15.8192, 12.9628, 10.4016, 8.21801, 7.79928, 7.27352,
 -6.55373, 5.41825, -4.58046, ··· 99 980 ···, -4.54312, 3.94917, 3.61926,
 -3.57692, -2.40906, 2.1213, -1.83934, -1.43103, 1.02566, -0.618838}
```

large output | show less | show more | show all | set size limit...

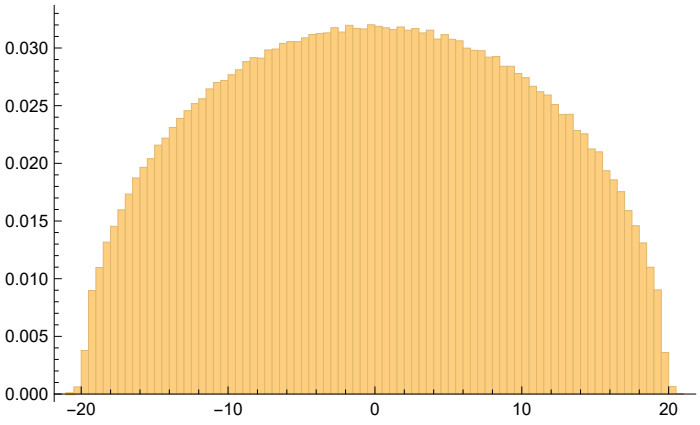
Normal Spacing Ratios of GUE

In[ ]:=
```
n = 100; singev = {}; Sprd = {}; Sprtil = {};
```

In[ ]:=
```
Monitor[For[i = 1, i < 5001, i++, {
   GUE = RandomVariate[GaussianUnitaryMatrixDistribution[n]] ;
   ev = Eigenvalues[GUE];
   singev = Join[singev, ev];
   sev = Sort[ev];
   rr = Table[(sev[[j + 2]] - sev[[j + 1]])/(sev[[j + 1]] - sev[[j]]), {j, 1, n - 2}];
   Sprd = Join[Sprd, rr];
   Sprtil = Join[Sprtil, Table[Min[rr[[j]], 1/rr[[j]]], {j, 1, n - 2}]];
  }], i]
```

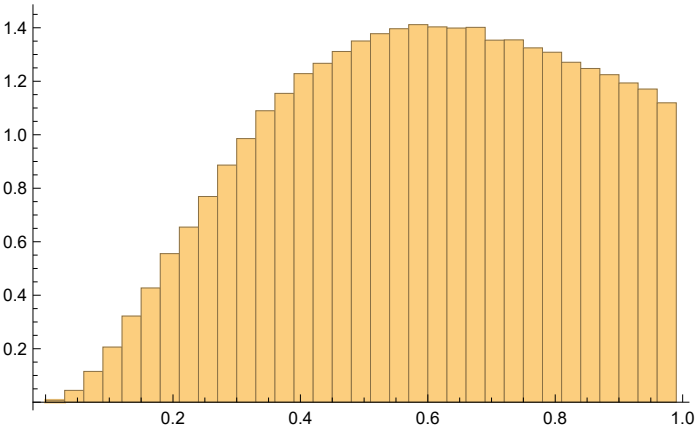
In[ ]:=
```
Show[Histogram[singev, 60, "PDF", PlotRange → All]]
```

Out[ ]=



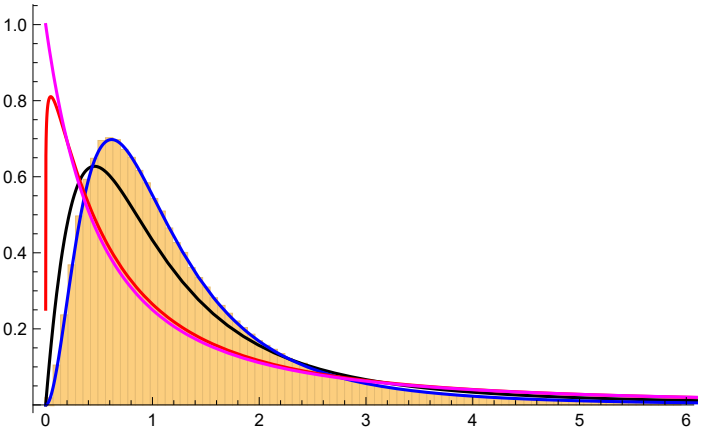
In[ ]:=
```
Histogram[Flatten[Sprtil], {0, 1, 0.03}, "PDF"]
```

Out[ ]=



In[ ]:=
```
Show[Histogram[Flatten[Sprd], {0, 6, 0.07}, "PDF"],
 Plot[{p1[r], p2[r], psp[r, 0.1], 1 / (1 + r)^2}, {r, 0, 7},
  PlotRange → All, PlotStyle → {Black, Blue, Red, Magenta}]]
```

Out[ ]=



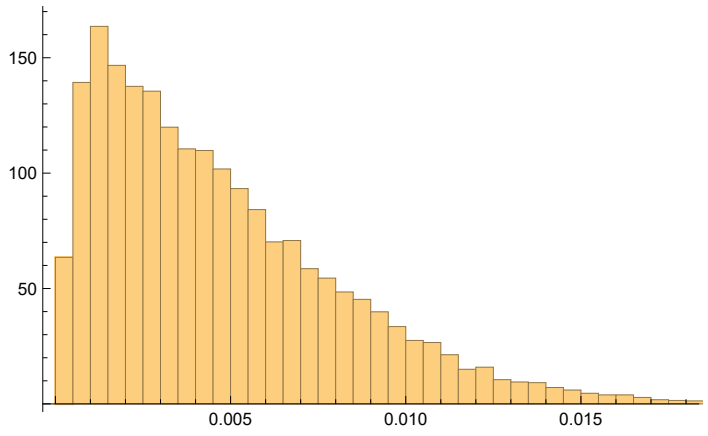
(*GinUE– GinOE crossover using the Pandey Mehta Hamiltonian*)

In[144]:=
```
n = 500; singev = {}; Sprd = {}; λ = 0.1; Sprtil = {}; minev = {};
```

In[145]:=
```
Monitor[For[i = 1, i < 201, i++, {
  H1 = RandomVariate[NormalDistribution[0, 1/Sqrt[2 n]], {n, n}]; (*GinOE*)
  H2 = RandomVariate[NormalDistribution[0, 1/Sqrt[2 n]], {n, n}] +
    i RandomVariate[NormalDistribution[0, 1/Sqrt[2 n]], {n, n}];
  (*GinUE*)
  H = 1/Sqrt[1 + λ^2] H1 + λ/Sqrt[1 + λ^2] H2;
  ev = Sqrt[Eigenvalues[H.ConjugateTranspose[H]]];
  (*minev= Append[minev,Min[ev]];*)(*Smallest of the singular values*)
  singev = Join[singev, ev];
  sev = Sort[ev];
  rr = Table[(sev[[j + 2]] - sev[[j + 1]])/(sev[[j + 1]] - sev[[j]]), {j, 1, n - 2}];
  (*Adjacent level spacing ratios of the singular values*)
  Sprd = Join[Sprd, rr]
  (*Sprtil=Join[Sprtil,Table[Min[rr[[j]],1/rr[[j]]],{j,1,n-2}]];*)
  (*Adjacent level spacing ratios (of the other kind) of the singular values*)
 }],
 i]
```

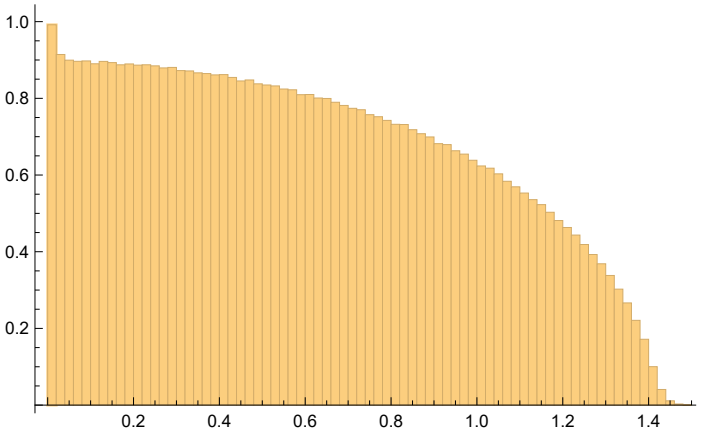
In[14]:=
```
Histogram[Chop[minev], 60, "PDF"]
```

Out[14]=



In[83]:=
```
Histogram[Chop[singev], 60, "PDF"]
```

Out[83]=



In[44]:=
```
Graphics[{Blue, AbsoluteThickness[2], Line[{{0, 1}, {1, 1}}]}]
```

Out[44]=

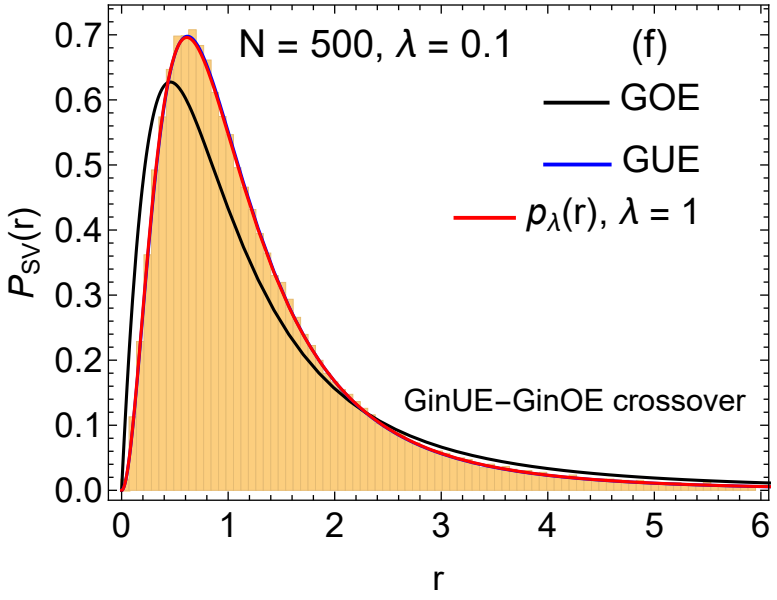
Graphics

In[22]:=
```
Show[Histogram[Flatten[Sprtil], {0, 1, 0.02}, "PDF"], Plot[prt[λ], {λ, 0, 1}]]
```

Out[22]=
```
$Aborted
```

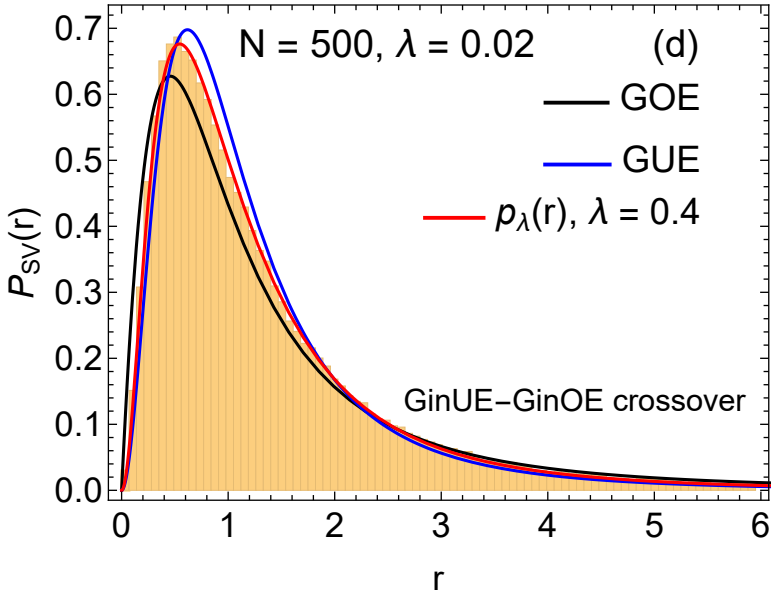
In[149]:=
```
N500crossλ01 = Show[Histogram[Chop[Sprd], {0, 6, 0.07}, "PDF"],
  Plot[{p1[r], p2[r], pλ[1, r]}, {r, 0, 7}, PlotRange → All,
   PlotStyle → {Black, Blue, Red}], Frame → {{True, True}, {True, True}},
  FrameStyle → Thickness[0.002], AspectRatio → 3 / 4, LabelStyle → {Black, 18},
  ImageSize → 400, FrameLabel → {"r", "PSV(r)"},
  Epilog → {{Inset[Style["N = 500, λ = 0.1            (f)", 20, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.2, .97}]], {Left, Top}],
     Inset[Style["GinUE-GinOE crossover", 16, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.45, .17}]], {Left, Bottom}]}, {Inset[
     Style["——— GOE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.94}]], {Right, Top}]},
   {Inset[Style["——— GUE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.82}]],
     {Right, Top}]}, {Inset[Style[" ——— pλ(r), λ = 1", 19],
     Offset[{-1, -1}, Scaled[{0.9, 0.7}]], {Right, Top}]}}]
```

Out[149]=



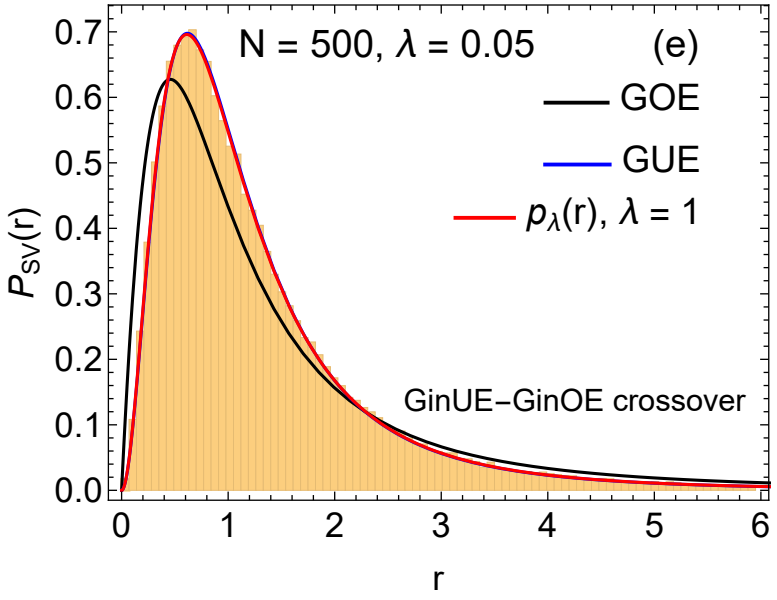
In[143]:=
```
N500cross$\lambda$002 = Show[Histogram[Chop[Sprd], {0, 6, 0.07}, "PDF"],
  Plot[{p1[r], p2[r], p$\lambda$[0.4, r]}, {r, 0, 7}, PlotRange → All,
   PlotStyle → {Black, Blue, Red}], Frame → {{True, True}, {True, True}},
  FrameStyle → Thickness[0.002], AspectRatio → 3 / 4, LabelStyle → {Black, 18},
  ImageSize → 400, FrameLabel → {"r", "P$_{SV}$(r)"},
  Epilog → {{Inset[Style["N = 500, $\lambda$ = 0.02          (d)", 20, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.2, .97}]], {Left, Top}],
     Inset[Style["GinUE-GinOE crossover", 16, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.45, .17}]], {Left, Bottom}]}, {Inset[
     Style["——— GOE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.94}]], {Right, Top}]},
   {Inset[Style["——— GUE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.82}]],
     {Right, Top}]}, {Inset[Style[" ——— p$_\lambda$(r), $\lambda$ = 0.4", 19],
     Offset[{-1, -1}, Scaled[{0.9, 0.7}]], {Right, Top}]}}]
```

Out[143]=



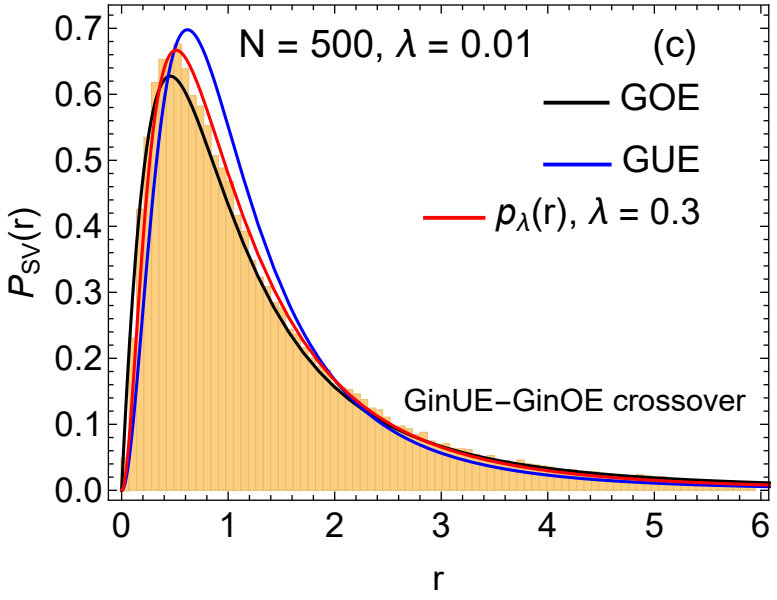
In[134]:=
```
N500crossλ005 = Show[Histogram[Chop[Sprd], {0, 6, 0.07}, "PDF"],
  Plot[{p1[r], p2[r], pλ[1, r]}, {r, 0, 7}, PlotRange → All,
   PlotStyle → {Black, Blue, Red}], Frame → {{True, True}, {True, True}},
  FrameStyle → Thickness[0.002], AspectRatio → 3 / 4, LabelStyle → {Black, 18},
  ImageSize → 400, FrameLabel → {"r", "P_SV(r)"},
  Epilog → {{Inset[Style["N = 500, λ = 0.05            (e)", 20, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.2, .97}]], {Left, Top}],
     Inset[Style["GinUE-GinOE crossover", 16, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.45, .17}]], {Left, Bottom}]}, {Inset[
     Style["——— GOE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.94}]], {Right, Top}]},
   {Inset[Style["——— GUE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.82}]],
     {Right, Top}]}, {Inset[Style[" ——— pλ(r), λ = 1", 19],
     Offset[{-1, -1}, Scaled[{0.9, 0.7}]], {Right, Top}]}}]
```

Out[134]=



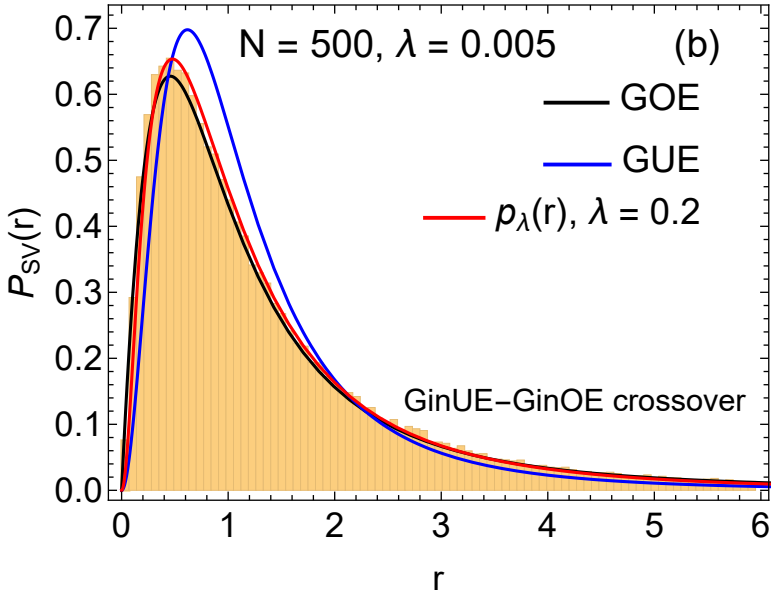
In[127]:=
```
N500crossλ001 = Show[Histogram[Chop[Sprd], {0, 6, 0.07}, "PDF"],
  Plot[{p1[r], p2[r], pλ[0.3, r]}, {r, 0, 7}, PlotRange → All,
   PlotStyle → {Black, Blue, Red}], Frame → {{True, True}, {True, True}},
  FrameStyle → Thickness[0.002], AspectRatio → 3 / 4, LabelStyle → {Black, 18},
  ImageSize → 400, FrameLabel → {"r", "P_SV(r)"},
  Epilog → {{Inset[Style["N = 500, λ = 0.01            (c)", 20, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.2, .97}]], {Left, Top}],
     Inset[Style["GinUE-GinOE crossover", 16, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.45, .17}]], {Left, Bottom}]}, {Inset[
     Style["——— GOE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.94}]], {Right, Top}]},
   {Inset[Style["——— GUE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.82}]],
     {Right, Top}]}, {Inset[Style[" ——— pλ(r), λ = 0.3", 19],
     Offset[{-1, -1}, Scaled[{0.9, 0.7}]], {Right, Top}]}}]
```

Out[127]=



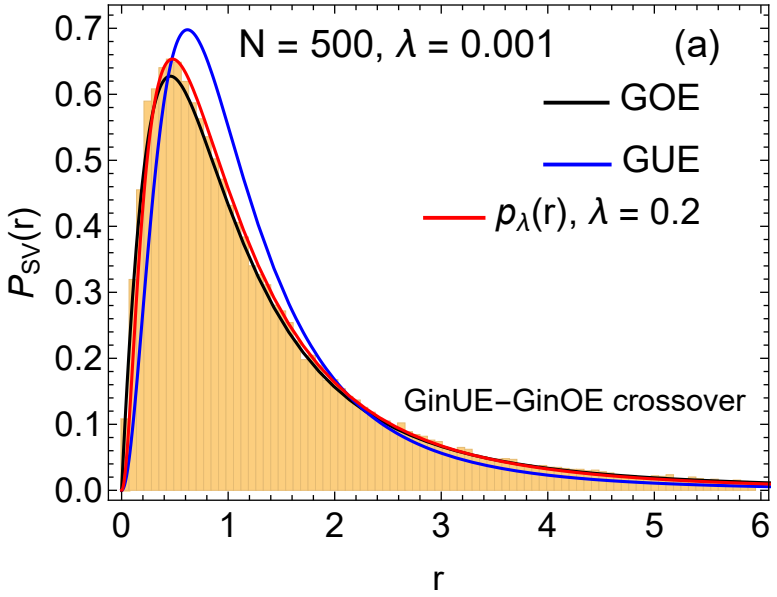
In[119]:=
```
N500crossλ0005 = Show[Histogram[Chop[Sprd], {0, 6, 0.07}, "PDF"],
  Plot[{p1[r], p2[r], pλ[0.2, r]}, {r, 0, 7}, PlotRange → All,
   PlotStyle → {Black, Blue, Red}], Frame → {{True, True}, {True, True}},
  FrameStyle → Thickness[0.002], AspectRatio → 3 / 4, LabelStyle → {Black, 18},
  ImageSize → 400, FrameLabel → {"r", "P_SV(r)"},
  Epilog → {{Inset[Style["N = 500, λ = 0.005          (b)", 20, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.2, .97}]], {Left, Top}],
     Inset[Style["GinUE-GinOE crossover", 16, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.45, .17}]], {Left, Bottom}]}, {Inset[
     Style["——— GOE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.94}]], {Right, Top}]},
    {Inset[Style["——— GUE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.82}]],
     {Right, Top}]}, {Inset[Style[" ——— pλ(r), λ = 0.2", 19],
     Offset[{-1, -1}, Scaled[{0.9, 0.7}]], {Right, Top}]}}]
```

Out[119]=



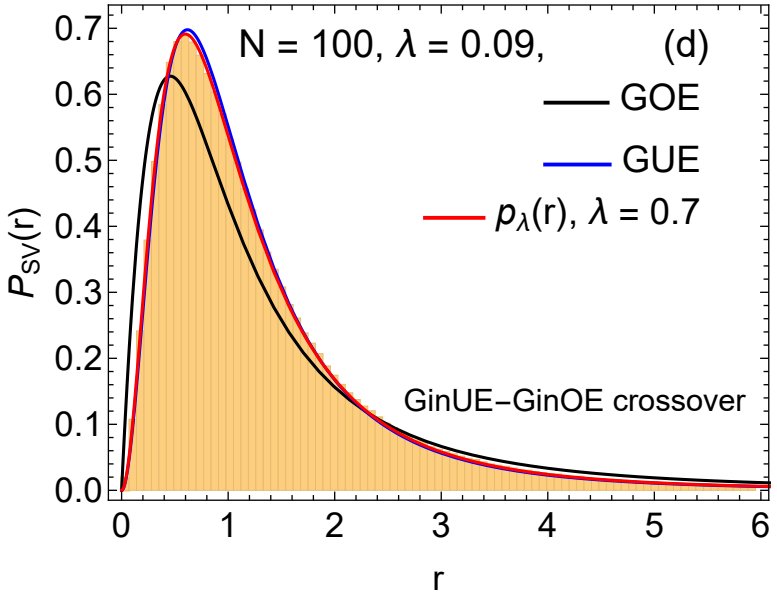
In[115]:=
```
N500crossλ0001 = Show[Histogram[Chop[Sprd], {0, 6, 0.07}, "PDF"],
  Plot[{p1[r], p2[r], pλ[0.2, r]}, {r, 0, 7}, PlotRange → All,
   PlotStyle → {Black, Blue, Red}], Frame → {{True, True}, {True, True}},
  FrameStyle → Thickness[0.002], AspectRatio → 3 / 4, LabelStyle → {Black, 18},
  ImageSize → 400, FrameLabel → {"r", "P_SV(r)"},
  Epilog → {{Inset[Style["N = 500, λ = 0.001          (a)", 20, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.2, .97}]], {Left, Top}],
     Inset[Style["GinUE-GinOE crossover", 16, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.45, .17}]], {Left, Bottom}]}, {Inset[
     Style["——— GOE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.94}]], {Right, Top}]},
    {Inset[Style["——— GUE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.82}]],
     {Right, Top}]}, {Inset[Style[" ——— pλ(r), λ = 0.2", 19],
     Offset[{-1, -1}, Scaled[{0.9, 0.7}]], {Right, Top}]}}]
```

Out[115]=



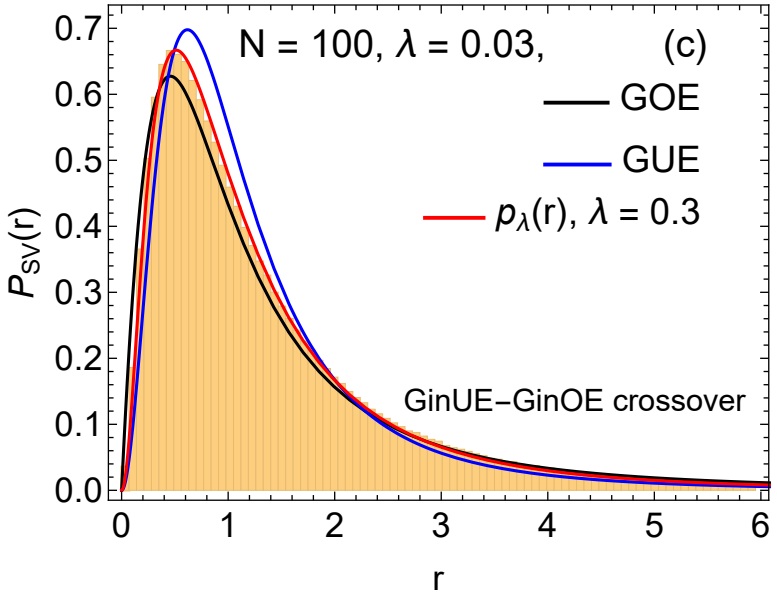
In[104]:=
```
N100crossλ009 = Show[Histogram[Chop[Sprd], {0, 6, 0.07}, "PDF"],
  Plot[{p1[r], p2[r], pλ[0.7, r]}, {r, 0, 7}, PlotRange → All,
   PlotStyle → {Black, Blue, Red}], Frame → {{True, True}, {True, True}},
  FrameStyle → Thickness[0.002], AspectRatio → 3 / 4, LabelStyle → {Black, 18},
  ImageSize → 400, FrameLabel → {"r", "P_SV(r)"},
  Epilog → {{Inset[Style["N = 100, λ = 0.09,          (d)", 20, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.2, .97}]], {Left, Top}],
     Inset[Style["GinUE-GinOE crossover", 16, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.45, .17}]], {Left, Bottom}]}, {Inset[
     Style["——— GOE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.94}]], {Right, Top}]},
    {Inset[Style["——— GUE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.82}]],
     {Right, Top}]}, {Inset[Style[" ——— p_λ(r), λ = 0.7", 19],
     Offset[{-1, -1}, Scaled[{0.9, 0.7}]], {Right, Top}]}}]
```

Out[104]=



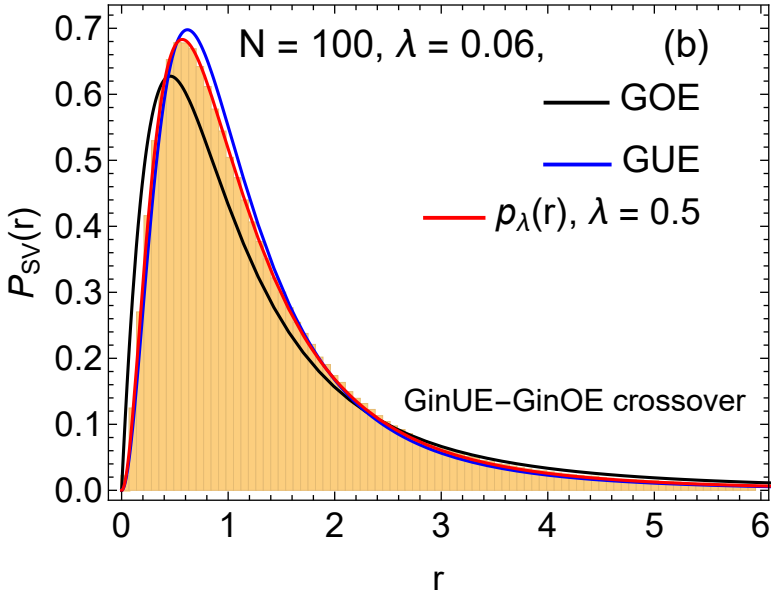
In[96]:=
```
N100crossλ003 = Show[Histogram[Chop[Sprd], {0, 6, 0.07}, "PDF"],
  Plot[{p1[r], p2[r], pλ[0.3, r]}, {r, 0, 7}, PlotRange → All,
   PlotStyle → {Black, Blue, Red}], Frame → {{True, True}, {True, True}},
  FrameStyle → Thickness[0.002], AspectRatio → 3 / 4, LabelStyle → {Black, 18},
  ImageSize → 400, FrameLabel → {"r", "P_SV(r)"},
  Epilog → {{Inset[Style["N = 100, λ = 0.03,          (c)", 20, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.2, .97}]], {Left, Top}],
     Inset[Style["GinUE-GinOE crossover", 16, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.45, .17}]], {Left, Bottom}]}, {Inset[
     Style["—— GOE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.94}]], {Right, Top}]},
   {Inset[Style["—— GUE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.82}]],
     {Right, Top}]}, {Inset[Style[" —— pλ(r), λ = 0.3", 19],
     Offset[{-1, -1}, Scaled[{0.9, 0.7}]], {Right, Top}]}}]
```

Out[96]=



In[90]:=
```
N100crossλ006 = Show[Histogram[Chop[Sprd], {0, 6, 0.07}, "PDF"],
  Plot[{p1[r], p2[r], pλ[0.5, r]}, {r, 0, 7}, PlotRange → All,
   PlotStyle → {Black, Blue, Red}], Frame → {{True, True}, {True, True}},
  FrameStyle → Thickness[0.002], AspectRatio → 3 / 4, LabelStyle → {Black, 18},
  ImageSize → 400, FrameLabel → {"r", "P_SV(r)"},
  Epilog → {{Inset[Style["N = 100, λ = 0.06,          (b)", 20, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.2, .97}]], {Left, Top}],
     Inset[Style["GinUE-GinOE crossover", 16, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.45, .17}]], {Left, Bottom}]}, {Inset[
     Style["——— GOE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.94}]], {Right, Top}]},
    {Inset[Style["——— GUE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.82}]],
     {Right, Top}]}, {Inset[Style[" ——— pλ(r), λ = 0.5", 19],
     Offset[{-1, -1}, Scaled[{0.9, 0.7}]], {Right, Top}]}}]
```

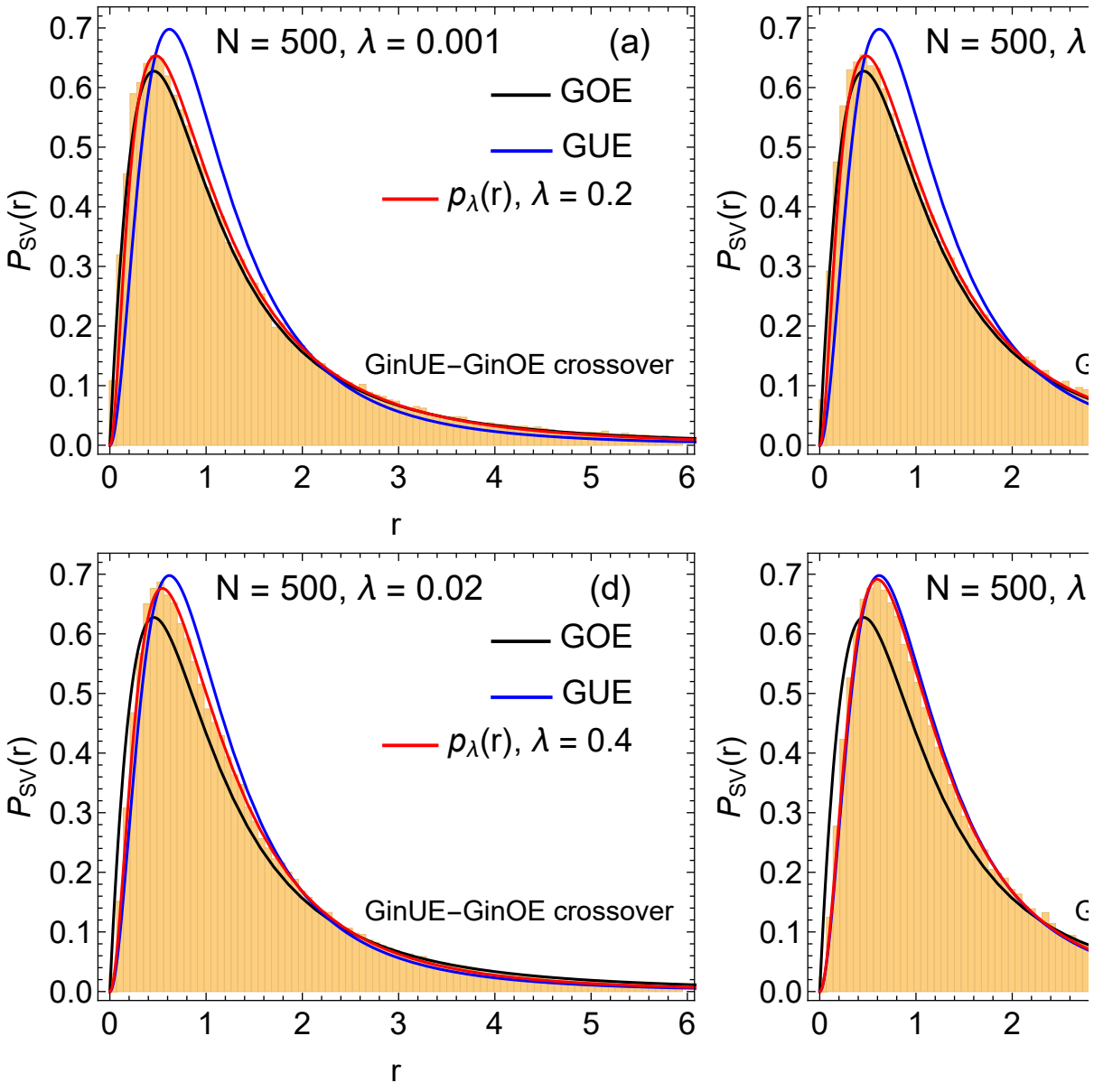
Out[90]=



In[80]:=
```
N100crossλ0 = Show[Histogram[Chop[Sprd], {0, 6, 0.07}, "PDF"],
  Plot[{p1[r], p2[r], pλ[0.2, r]}, {r, 0, 7}, PlotRange → All,
   PlotStyle → {Black, Blue, Red}], Frame → {{True, True}, {True, True}},
  FrameStyle → Thickness[0.002], AspectRatio → 3 / 4, LabelStyle → {Black, 18},
  ImageSize → 400, FrameLabel → {"r", "P_SV(r)"},
  Epilog → {{Inset[Style["N = 100, λ = 0,            (a)", 20, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.2, .97}]], {Left, Top}],
     Inset[Style["GinUE-GinOE crossover", 16, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.45, .17}]], {Left, Bottom}]}, {Inset[
     Style["——— GOE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.94}]], {Right, Top}]},
   {Inset[Style["——— GUE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.82}]],
     {Right, Top}]}, {Inset[Style[" ——— p_λ(r), λ = 0.2 ", 19],
     Offset[{-1, -1}, Scaled[{0.92, 0.7}]], {Right, Top}]}}]
```

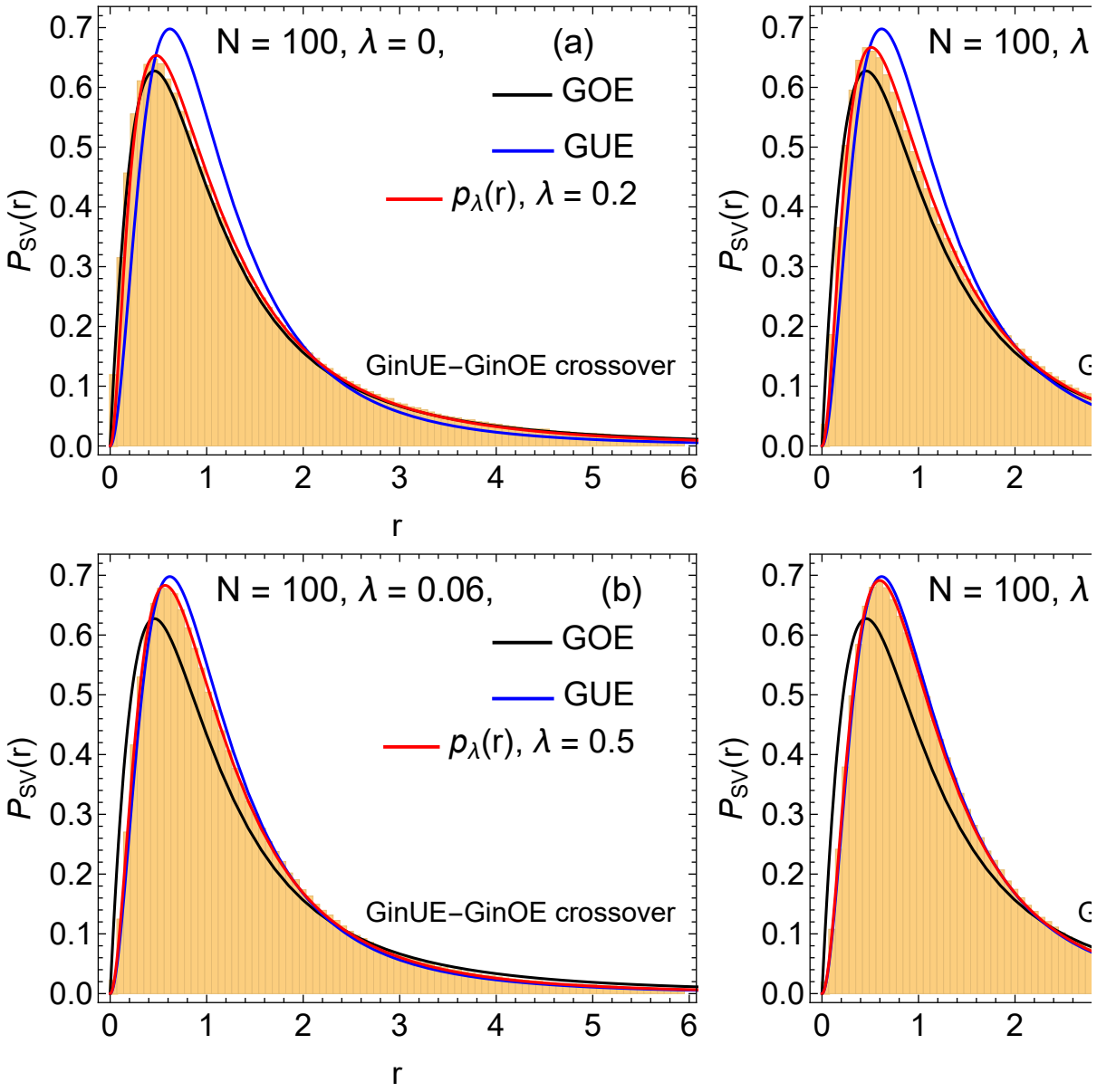
In[152]:=
```
FullcrossN500 = Grid[{{N500crossλ0001, N500crossλ0005, N500crossλ001},
    {N500crossλ002, N500crossλ003, N500crossλ01}}]
```

Out[152]=



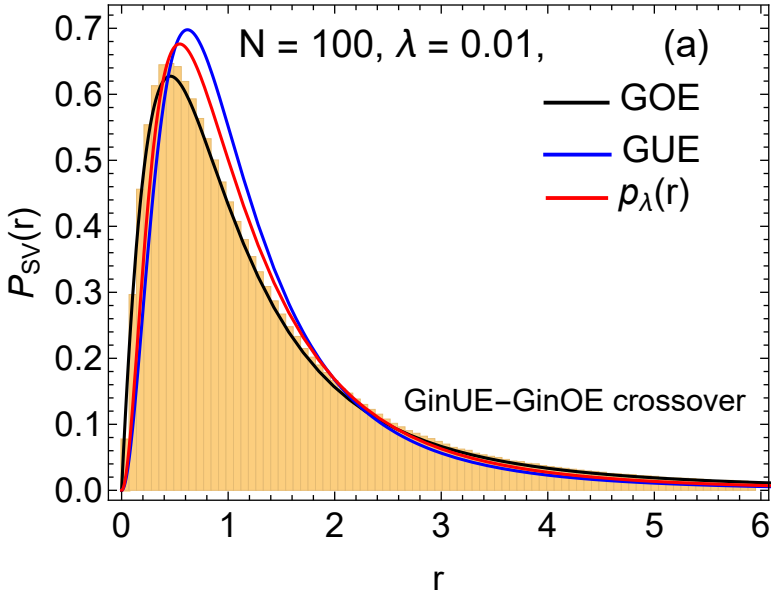
In[105]:=
```
FullcrossN100 = Grid[{{N100crossλ0, N100crossλ003}, {N100crossλ006, N100crossλ009}}]
```

Out[105]=



In[63]:=
```
N100crossλ001 = Show[Histogram[Chop[Sprd], {0, 6, 0.07}, "PDF"],
  Plot[{p1[r], p2[r], pλ[0.4, r]}, {r, 0, 7}, PlotRange → All,
   PlotStyle → {Black, Blue, Red}], Frame → {{True, True}, {True, True}},
  FrameStyle → Thickness[0.002], AspectRatio → 3 / 4, LabelStyle → {Black, 18},
  ImageSize → 400, FrameLabel → {"r", "P_SV(r)"},
  Epilog → {{Inset[Style["N = 100, λ = 0.01,          (a)", 20, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.2, .97}]], {Left, Top}],
     Inset[Style["GinUE-GinOE crossover", 16, FontColor → Black],
      Offset[{-1, -1}, Scaled[{.45, .17}]], {Left, Bottom}]}, {Inset[
     Style["—— GOE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.94}]], {Right, Top}]},
   {Inset[Style["—— GUE", 19], Offset[{-1, -1}, Scaled[{0.9, 0.84}]],
     {Right, Top}]}, {Inset[Style["—— p_λ(r) ", 19],
     Offset[{-1, -1}, Scaled[{0.9, 0.74}]], {Right, Top}]}}]
```

Out[63]=



In[65]:=
```
prt[0.2]
```

Out[65]= $Aborted

In[64]:=
```
Plot[prt[λ], {λ, 0, 5}]
```

Out[64]= $Aborted

In[23]:=
```
prt[λ_] := Integrate[2 pλ[λ, r] HeavisideTheta[1 - r], {r, 0, ∞}, Assumptions → λ > 0]
```

In[24]:=
$$g[\eta\_, \xi\_] := \frac{\xi\left(5\eta^2 + 3\xi^2\right)}{\eta^4\left(\eta^2 + \xi^2\right)^2} + \frac{3}{\eta^5}\,\text{ArcTan}\left[\frac{\xi}{\eta}\right];$$

$$a[r\_] := \sqrt{\frac{1}{6} * \left(1 + r + r^2\right)};$$

$$b[\lambda\_] := \frac{1}{2\sqrt{2}\,\lambda};$$

$$b\alpha[\alpha\_] := \sqrt{\frac{1-\alpha^2}{8\alpha^2}};$$

In[25]:=
$$p\lambda[\lambda\_, r\_] := \frac{r\,(1+r)}{16\sqrt{6}\,\pi}\left(1+\lambda^2\right)^{3/2}\left(g[a[r], b[\lambda]] + g[a[r], b[\lambda]\,r] - g[a[r], b[\lambda]\,(r+1)]\right);$$