

Ratios of Spacings for GSE

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na = 50; nm = 2000;
eigall = {}; rath = {}; eigvalall = {};
AbsoluteTiming[Monitor[For[ty = 1, ty ≤ nm, ty++,
  {H8 = RandomVariate[GaussianSymplecticMatrixDistribution[na]];
   (*creating the GSE matrix*)
   eigval = Eigenvalues[H8]; (*find the eigenvalues*)
   eigvalall = Join[eigvalall, eigval]; (*All the eigenvalues from the ensemble*)
   seigval = Sort[eigval]; (*sorting to calculate the ratios of spacings*)
   p = Table[seigval[[i]], {i, 1, Length[seigval], 2}];
   (*making a list of alternate eigenvalues due to the degeneracy in GSE*)
   rath = Append[rath, Table[ $\frac{p[[j+2]] - p[[j+1]]}{p[[j+1]] - p[[j]]}$ , {j, 1, Length[p] - 2}]]
   (*then calculating the ratios of spacings*);}], ty]]

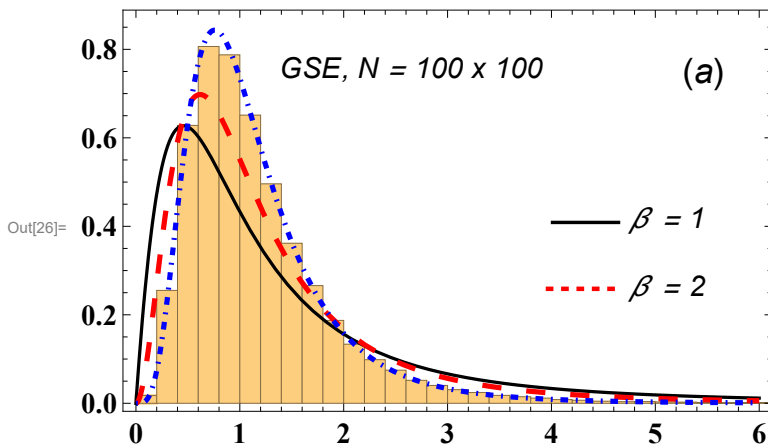
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Out[18]= {4.42474, Null}

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In[26]:= am10n7ps1 = Show[Histogram[Flatten[rath], 90, "PDF", PlotRange → {{0, 6}, All}],
  Plot[{pgoe[r], pgue[r], pSE[r]}, {r, 0, 6}, PlotRange → {{0, 6}, All},
    PlotStyle → {{Black, Thickness[0.005]}, {Red, Dashing[Large], Thickness[0.008]},
      {Blue, DotDashed, Thickness[0.008]}}], Frame → {{True, True}, {True, True}},
    Epilog → {{Inset[Style["GSE, N = 100 x 100", 15, FontSlant → Italic],
      Offset[{-1, -1}, Scaled[{0.65, 0.9}]], {Right, Top}]],
      {Inset[Style["(a)", 18, FontSlant → Italic], Offset[{-1, -1}, Scaled[{0.93, 0.9}]],
        {Right, Top}]], {Inset[Style["——  $\beta = 1$ ", 15, FontSlant → Italic],
        Offset[{-1, -1}, Scaled[{0.9, 0.65}]], {Right, Top}]]},
      {Inset[Style["- - -  $\beta = 2$ ", 15, FontSlant → Italic],
        Offset[{-1, -1}, Scaled[{0.9, 0.5}]], {Right, Top}]]}},
    LabelStyle → {15, Black, Bold, FontFamily → Times}]

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All analytical results for ratios of spacings of OE, UE, SP, SE and Wishart

$$p\beta 1N4[r_] := \frac{32}{9} r (1+r) \left(\frac{23+23r+2r^2}{(2+r)^7} + \frac{9*(42+91r+47r^2)}{(3+2r)^7} \right);$$

(*Wishart ensemble 4 x 4 *)

$$pSE[r_] := \frac{729 \sqrt{3} (r+r^2)^4}{4 \pi (1+r+r^2)^7};$$

$$pgoe[r_] := \frac{27 (r+r^2)}{8 (1+r+r^2)^{5/2}};$$

$$pgue[r_] := \frac{81 \sqrt{3} (r+r^2)^2}{4 \pi (1+r+r^2)^4};$$

$$psp[r_, v_] := \left(\frac{\text{Gamma}[2v+2] \text{Gamma}[v+2]^2}{(v+1)^2 \text{Gamma}[v+1]^4} \right) \left(\frac{r^v}{(1+r)^{2v+2}} \right);$$

All analytical results for eigenvector distributions of OE, UE, SP, SE and Wishart

$$R1[l_, R_] := \frac{\text{Gamma}[l/2] (1-R)^{\frac{1-3}{2}}}{\text{Gamma}\left[\frac{1-1}{2}\right] \sqrt{\pi R}};$$

$$R2[l_, R_] := (1-l) (1-R)^{1-2};$$

$$S2[l_, S_] := \left(\frac{1-l}{1} \right) e^S \left(1 - \frac{e^S}{1} \right)^{1-2};$$

$$S1[l_, S_] := \frac{\text{Gamma}[l/2] \sqrt{e^S}}{\sqrt{\pi} \text{Gamma}[(1-l)/2] \sqrt{1}} \left(1 - \frac{e^S}{1} \right)^{\frac{(1-3)}{2}};$$

na = 50; nm = 2000;

eigall = {}; rath = {}; eigvalall = {}; eigphall = {};

AbsoluteTiming[Monitor[For[ty = 1, ty ≤ nm, ty++,

{H8 = RandomVariate[CircularSymplecticMatrixDistribution[na]];

(*creating the CSE matrix, the matrix dimen is 2*na*)

eigval = Eigenvalues[H8]; (*find the eigenvalues*)

eigph = ArcTan[Re[eigval], Im[eigval]]; (*find the eigenphases*)

eigvalall = Join[eigvalall, eigval]; (*All the eigenvalues from the ensemble*)

eigphall = Join[eigphall, eigph];

seigval = Sort[eigph]; (*sorting to calculate the ratios of spacings*)

p = Table[seigval[[i]], {i, 1, Length[seigval], 2}];

(*making a list of alternate eigenvalues due to the degeneracy in CSE*)

rath = Append[rath, Table[$\frac{p[[j+2]] - p[[j+1]]}{p[[j+1]] - p[[j]]}$, {j, 1, Length[p] - 2}]]

(*then calculating the ratios of spacings*);}], ty]]

Out[33]= {23.2589, Null}

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In[34]:= am10n7ps1 = Show[Histogram[Flatten[rath], 90, "PDF", PlotRange → {{0, 6}, All}],
  Plot[{pgoe[r], pgue[r], pSE[r]}, {r, 0, 6}, PlotRange → {{0, 6}, All},
    PlotStyle → {{Black, Thickness[0.005]}, {Red, Dashing[Large], Thickness[0.008]},
      {Blue, DotDashed, Thickness[0.008]}}], Frame → {{True, True}, {True, True}},
    Epilog → {Inset[Style["CSE, N = 100 x 100", 15, FontSlant → Italic],
      Offset[{-1, -1}, Scaled[{0.65, 0.9}]], {Right, Top}],
      {Inset[Style["(a)", 18, FontSlant → Italic], Offset[{-1, -1}, Scaled[{0.93, 0.9}]],
        {Right, Top}], {Inset[Style["——  $\beta = 1$ ", 15, FontSlant → Italic],
        Offset[{-1, -1}, Scaled[{0.9, 0.65}]], {Right, Top}]}},
      {Inset[Style["- - -  $\beta = 2$ ", 15, FontSlant → Italic],
        Offset[{-1, -1}, Scaled[{0.9, 0.5}]], {Right, Top}]}},
    LabelStyle → {15, Black, Bold, FontFamily → Times}]

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