CAP 5610
Assignment #2 Solution

February 4, 2025

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1 Types of Attributes [10 points]

Solution for Q1:

Ans: The classification of the attributes is as follows:

(a) Rating of an Amazon product by a person on a scale of 1 to 5: **Ordinal**.

Ratings on a scale have a meaningful order or ranking, but the differences between each rating values are not necessarily equal. For example, the difference between a rating of 1 and 2 may not be the same as the difference between a rating of 4 and 5, but we can sort these ratings from 5 as the best all the way to the 1 as the worst rating. Furthermore, for ordinal attributes like rating, we cannot perform arithmetic operations such as addition or multiplication as they are not meaningful. Hence, rating has the distinctness and order properties of ordinal attributes.

(b) The Internet Speed: **Ratio**.

The Internet speed has a meaningful difference and a true zero, meaning that ratios also make sense. For instance, 100 Mbps is twice as fast as 50 Mbps, and 0 Mbps means no internet speed. Furthermore, we can perform arithmetic operations such as addition and multiplication on internet speed. Hence, internet speed has the distinctness, order, and all arithmetic properties of ratio attributes.

(c) Number of customers in a store: **Ratio**.

The number of customers has a meaningful difference and a true zero, meaning that ratios also make sense. To give an example, 100 customers is twice as many as 50 customers, and 0 customers means no customers. Furthermore, we can perform arithmetic operations such as addition and multiplication on the number of customers. Hence, the number of customers has the distinctness, order, and all arithmetic properties of ratio attributes.

(d) UCF Student ID: **Nominal**.

The UCF student ID is a unique identifier or label for each student, but it does not have any meaningful order or ranking. To give an instance, a UCF student ID of 1234567 is not better or worse than a student ID of 4567890. Furthermore, we cannot perform arithmetic operations such as addition or multiplication on UCF student IDs as they are not meaningful. Hence, UCF student ID has the distinctness property of nominal attributes.

(e) Letter grade (A, B, C, D): **Ordinal**.

Letter grades have a meaningful order or ranking, but the differences between each grade are not necessarily equal. To illustrate, the difference between a grade of A (4.0) and B (3.25) may not be the same as the difference between a grade of C (2.50) and D (2.0), but we can sort these letters from the best grade to achieve to worst grade to achieve. Furthermore, we cannot perform arithmetic operations such as addition or multiplication on letter grades as they are not meaningful. Hence, letter grades have the distinctness and order properties of ordinal attributes.

2 Distance/Similarity Measures [20 points]

1. [10 points] Solution for Q2 Part 1:

Ans: To group the boxes based on their shapes (length-width ratio), the best proximity measure to use is a **correlation similarity measure** instead of the Euclidean distance. This is because correlation measures the strength and direction of the linear relationship between two variables, which in this case are the length and width of the boxes. By using correlation, we can capture the shape of the boxes regardless of their size or position.

In other words, correlation measures the similarity in the pattern of the length and width values, which is what we are interested in when grouping the boxes based on their shapes.

Since we are interested in shape similarity, the key idea is to check if the length-width ratio of two boxes is consistent.

For the given 4 boxes, we can calculate the correlation between their length and width values as

$$ratio_{TL} = \frac{2}{1} = 2.0$$
$$ratio_{TR} = \frac{1}{1} = 1.0$$
$$ratio_{BL} = \frac{6}{3} = 2.0$$
$$ratio_{BR} = \frac{3}{3} = 1.0$$

By looking at the figure and ratios calculated, we can see that the boxes have different sizes and positions, but their shapes are similar. To give an instance, we can see that the top-left box (2,1) has the same ratio as bottom-left box (6,3), making them similar in shape. Similarly, the top-right box (1,1) has the same ratio as bottom-right box (3,3), making them similar in shape.

Euclidean distance would not reflect that similarity because it would consider the absolute difference between the length and width values.

2. [10 points] Solution for Q2 Part 2:

Ans: To group the boxes based on their size, the best proximity measure to use is **Euclidean distance** instead of correlation similarity measure. This is because Euclidean distance measures the absolute difference in both length and width values, which is what we are interested in when grouping the boxes based on their overall dimensions.

In other words, Euclidean distance can be used to group by any size property, which is what we are interested in when grouping the boxes based on their size.

To give an example of grouping by size, we can calculate the Euclidean distance between two boxes (L_1, W_1) and (L_2, W_2), based on their length and width values:

$$d = \sqrt{(L_1 - L_2)^2 + (W_1 - W_2)^2}$$

We can apply this formula to the boxes in the figure to group them based on their size. For instance, the Euclidean distance between the top-left box (2,1) and other boxes is:

$$d_{TL,BL} = \sqrt{(2-6)^2 + (1-3)^2} = \sqrt{20} \approx 4.47$$
$$d_{TL,TR} = \sqrt{(2-1)^2 + (1-1)^2} = \sqrt{1} = 1$$
$$d_{TL,BR} = \sqrt{(2-3)^2 + (1-3)^2} = \sqrt{5} \approx 2.24$$

The lower the Euclidean distance, the more similar the boxes are in size. For this reason, we can say that the top-left box (2,1) is more similar in size to the top-right box (1,1) than to the bottom-left box (6,3).

Correlation similarity measure would not reflect that size similarity because it would consider the pattern of the length and width values.

3 Coding Question [20 points]

Solution for Q3:

Ans:

Please check the source code and outputs included in the appendix named as

CAP_5610_Assignment_2_Solution_Arman_Sayan.ipynb

for the solution.

4 Coding Question [25 points]

Solution for Q4:

Ans:

Please check the source code and outputs included in the appendix named as

CAP_5610_Assignment_2_Solution_Arman_Sayan.ipynb

for the solution.

5 Coding Question [25 points]

Solution for Q5:

Ans:

Please check the source code and outputs included in the appendix named as

CAP_5610_Assignment_2_Solution_Arman_Sayan.ipynb

for the solution.

A Appendix

CAP_5610_Assignment_2_Solution_Arman_Sayan

February 4, 2025

CAP 5610 Assignment #2: Data and Linear Regression

This source code is written by Arman Sayan.

Last Edit: February 4, 2024

1 Q3 - Coding Question

Please write a Python code to calculate Cosine similarity, and Euclidean distance using NumPy. The input can be two randomly generated vectors or fixed vectors written by yourself.

1.1 Answer:

The formula for the cosine similarity is

$$CS = \frac{A \cdot B}{||A|| \cdot ||B||}$$

where $A \cdot B$ is the dot product of vectors A and B, $||A||$ is the magnitude of vector A, and $||B||$ is the magnitude of vector B.

[1]:
```
# Import necessary libraries
import numpy as np
import random as rnd
import math
```

[2]:
```
# Define a function to calculate Cosine similarity
# using Numpy
def CosineSimilarity_v1(A, B):
  # Calculate norm/magnitude of each vector
  norm_vecA = np.linalg.norm(A)
  norm_vecB = np.linalg.norm(B)

  if norm_vecA == 0 or norm_vecB == 0:
    return None  # to indicate undefined similarity

  # Calculate dot product
  dot_product = np.dot(A, B)

  # Combine results
```

```python
    return dot_product / (norm_vecA * norm_vecB)
```

[3]:
```python
# Define a function to calculate Cosine similarity
# without Numpy
def CosineSimilarity_v2(A, B):
    # Calculate norm/magnitude of each vector
    norm_vecA = math.sqrt(sum(a_i ** 2 for a_i in A))
    norm_vecB = math.sqrt(sum(b_i ** 2 for b_i in B))

    if norm_vecA == 0 or norm_vecB == 0:
        return None  # to indicate undefined similarity

    # Calculate dot product
    dot_product = sum(a_i * b_i for a_i, b_i in zip(A, B))

    # Combine results
    return dot_product / (norm_vecA * norm_vecB)
```

The formula for the Eucledian distance is

$$|A - B| = \sqrt{\sum_{i=1}^{n} (a_i - b_i)^2}$$

where n is the length of the vector.

[4]:
```python
# Define a function to calculate Euclidean distance
# using Numpy
def EuclideanDistance_v1(A, B):
    return np.linalg.norm(A - B)
```

[5]:
```python
# Define a function to calculate Euclidean distance
# without Numpy
def EuclideanDistance_v2(A, B):
    distance = 0
    for i in range(len(A)):
        distance += (A[i] - B[i]) ** 2
    return distance ** 0.5
```

[6]:
```python
# Learn the vector size from the user
inputCollected = False
vectorSize = 0
while not inputCollected:
    try:
        vectorSize = int(input("Enter the vector size: "))
        if vectorSize <= 0:
            raise ValueError
        maxValue = int(input("Enter the maximum value of the vector elements: "))
        if maxValue <= 0:
```

```
            raise ValueError
        inputCollected = True
    except ValueError:
        print("Invalid input. Please enter a valid integer.")

# Generate two random vectors based on the vector size
vector1 = np.random.randint(low=-maxValue, high=maxValue, size=vectorSize)
vector2 = np.random.randint(low=-maxValue, high=maxValue, size=vectorSize)

# Print the results
print("Vector 1:", vector1)
print("Vector 2:", vector2)
print("Cosine Similarity with Numpy:", CosineSimilarity_v1(vector1, vector2))
print("Cosine Similarity without Numpy:", CosineSimilarity_v2(vector1, vector2))
print("Euclidean Distance with Numpy:", EuclideanDistance_v1(vector1, vector2))
print("Euclidean Distance without Numpy:", EuclideanDistance_v2(vector1,␣
 ↪vector2))
```

```
Enter the vector size: 5
Enter the maximum value of the vector elements: 2
Vector 1: [ 0 -2 -1  1  0]
Vector 2: [ 0  0  0 -1  0]
Cosine Similarity with Numpy: -0.4082482904638631
Cosine Similarity without Numpy: -0.4082482904638631
Euclidean Distance with Numpy: 3.0
Euclidean Distance without Numpy: 3.0
```

2 Q4 - Coding Question

Please implement a Linear Regression to find the best linear model for the provided linear data. Please plot the result using "matplotlib.pyplot".

2.1 Answer:

[7]:
```
# Import necessary libraries
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

[8]:
```
# Load HW2_linear_data.csv and preprocess data
data = pd.read_csv("HW2_linear_data.csv")
X = data.iloc[:, 0].values  # treat the first column as the input feature
y = data.iloc[:, 1].values  # treat the second column as the target variable

# Reshape X and y for computations
X = X.reshape(-1, 1)
y = y.reshape(-1, 1)
```

[9]:
```
# Initialize necessary parameters that will be used
m = 0  # Slope coefficient
c = 0  # Intercept value
learning_rate = 0.0001  # Fixed learning rate
epochs = 1000  # Number of iterations
N = len(y)  # Number of data points
```

For a dataset with N samples, the MSE is:

$$MSE = \frac{1}{N}\sum_{i=1}^{N}(y_i - (\hat{m}x_i + \hat{c}))^2$$

where y_i is the actual target value, $\hat{m}x_i + \hat{c}$ is the predicted value.

$\hat{m}$ the slope, and $\hat{c}$ the intercept are the parameters we want to optimize.

Then, we need to take the derivative of MSE with respect to each of these parameters:

$$\frac{\partial MSE}{\partial m} = -\frac{2}{N}\sum_{i=1}^{N} x_i(y_i - (\hat{m}x_i + \hat{c}))$$

$$\frac{\partial MSE}{\partial c} = -\frac{2}{N}\sum_{i=1}^{N} (y_i - (\hat{m}x_i + \hat{c}))$$

[10]:
```
# Perform Gradient Descent algorithm
for epoch in range(epochs):
  y_pred = m * X + c  # Compute predictions
  error = y_pred - y  # Compute error

  # Compute gradients
  DwrtM = (2 / N) * np.sum(error * X)  # Derivative wrt m
  DwrtC = (2 / N) * np.sum(error)  # Derivative wrt c

  # Update parameters
  m -= learning_rate * DwrtM
  c -= learning_rate * DwrtC

  # Print loss in every 50 iterations
  if epoch % 50 == 0:
    mse = np.mean(error ** 2)
    print(f"For epoch {epoch}, MSE: {mse:.6f}")
```

```
For epoch 0, MSE: 5611.166154
For epoch 50, MSE: 111.060791
For epoch 100, MSE: 111.058150
For epoch 150, MSE: 111.055510
For epoch 200, MSE: 111.052873
For epoch 250, MSE: 111.050237
For epoch 300, MSE: 111.047604
For epoch 350, MSE: 111.044972
```

```
For epoch 400, MSE: 111.042342
For epoch 450, MSE: 111.039715
For epoch 500, MSE: 111.037089
For epoch 550, MSE: 111.034465
For epoch 600, MSE: 111.031843
For epoch 650, MSE: 111.029223
For epoch 700, MSE: 111.026605
For epoch 750, MSE: 111.023989
For epoch 800, MSE: 111.021374
For epoch 850, MSE: 111.018762
For epoch 900, MSE: 111.016152
For epoch 950, MSE: 111.013543
```

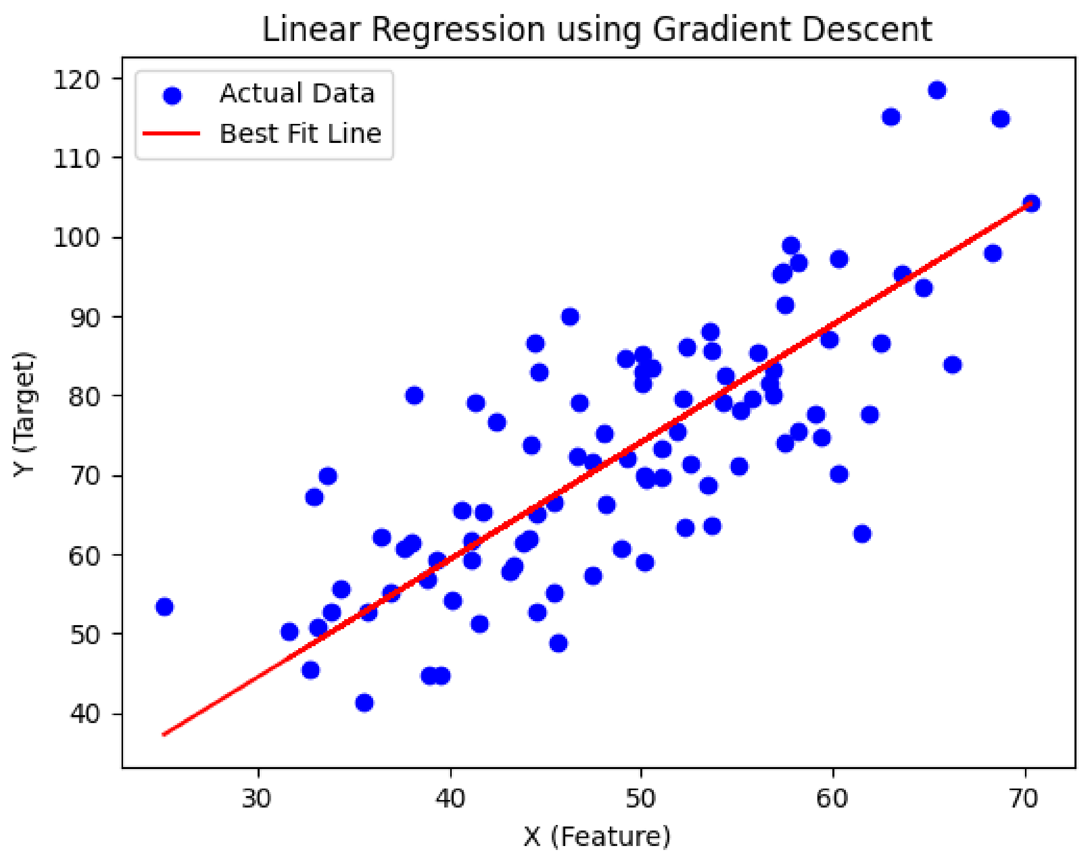
[11]:

```python
# Final linear model
print(f"Final model: Y = {m:.3f}X + {c:.3f}")
```

```
Final model: Y = 1.480X + 0.101
```

[12]:

```python
# Plot the data
plt.scatter(X, y, color="blue", label="Actual Data")
plt.plot(X, m * X + c, color="red", label="Best Fit Line")
plt.xlabel("X (Feature)")
plt.ylabel("Y (Target)")
plt.title("Linear Regression using Gradient Descent")
plt.legend()
plt.show()
```



3 Q5 - Coding Question

Please implement a non-linear regression to find the best cubic function model for the provided non-linear data. Please plot the result, too.

3.1 Answer:

[13]:
```
# Import necessary libraries
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

[14]:
```
# Load HW2_linear_data.csv and preprocess data
data = pd.read_csv("HW2_nonlinear_data.csv")
X = data.iloc[:, 0].values  # treat the first column as the input feature
y = data.iloc[:, 1].values  # treat the second column as the target variable

# Expand X to include cubic terms
X_poly = np.vstack([X**3, X**2, X, np.ones_like(X)]).T
```

[15]:
```
# Initialize necessary parameters and hyperparameters that will be used
a, b, c, d = np.random.randn(4)  # Random initialization
params = np.array([a, b, c, d])
learning_rate = 0.000001  # Fixed learning rate
epochs = 10000  # Number of iterations
N = len(X)  # Number of data points
```

[16]:
```
# Perform Gradient Descent algorithm
for epoch in range(epochs):
  y_pred = X_poly @ params  # Compute predictions
  errors = y_pred - y  # Compute error

  # Compute gradients
  gradients = (2 / N) * (X_poly.T @ errors)

  # Update parameters
  params -= learning_rate * gradients

  # Print loss in every 500 iterations
  if epoch % 500 == 0:
    mse = np.mean(errors ** 2)
    print(f"For epoch {epoch}, MSE: {mse:.6f}")
```

```
For epoch 0, MSE: 19202881.150963
For epoch 500, MSE: 619216.402912
For epoch 1000, MSE: 592605.881596
For epoch 1500, MSE: 591483.788087
For epoch 2000, MSE: 591151.065418
For epoch 2500, MSE: 590843.210586
For epoch 3000, MSE: 590536.558239
For epoch 3500, MSE: 590230.373919
For epoch 4000, MSE: 589924.632958
For epoch 4500, MSE: 589619.332675
For epoch 5000, MSE: 589314.471084
For epoch 5500, MSE: 589010.046238
For epoch 6000, MSE: 588706.056208
For epoch 6500, MSE: 588402.499078
For epoch 7000, MSE: 588099.372951
For epoch 7500, MSE: 587796.675946
For epoch 8000, MSE: 587494.406197
For epoch 8500, MSE: 587192.561853
For epoch 9000, MSE: 586891.141080
For epoch 9500, MSE: 586590.142058
```

[17]:
```
# Extract learned parameters
a, b, c, d = params
print(f"Learned coefficients: a={a:.6f}, b={b:.6f}, c={c:.6f}, d={d:.6f}")
```

```python
# Final non-linear model
print(f"Final model: Y = {a:.3f}X^3 + {b:.3f}X^2 + {c:.3f}X + {d:.3f}")
```

```
Learned coefficients: a=10.675649, b=20.947880, c=-3.191184, d=8.925191
Final model: Y = 10.676X^3 + 20.948X^2 + -3.191X + 8.925
```

[18]:
```python
# Generate predictions for visualization
X_sorted = np.sort(X)
Y_fitted = a * X_sorted**3 + b * X_sorted**2 + c * X_sorted + d

# Plot results
plt.scatter(X, y, color='blue', label='Original Data')
plt.plot(X_sorted, Y_fitted, color='red', linewidth=2, label='Fitted Cubic␣
↪Model')
plt.xlabel("X (Feature)")
plt.ylabel("Y (Target)")
plt.title("Cubic Regression using Gradient Descent")
plt.legend()
plt.show()
```