

# Biswas\_Sayan\_HW3

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```
[1]: import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn import preprocessing
from sklearn import metrics
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LogisticRegression
from sklearn.metrics import confusion_matrix
from sklearn.metrics import classification_report
from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
from sklearn.naive_bayes import GaussianNB
from sklearn.tree import DecisionTreeClassifier
from sklearn.metrics import roc_curve, auc
import warnings
```

```
[2]: # f = open("spambase.names", 'r')
# count = 0
# column_names = []
# for line in f:
#     count = count+1
#     if count >=34:
#         temp = line.split(":")
#         column_names.append(temp[0])
# print(len(column_names))
```

## 1 Problem 1

### 1.1 1a

```
[3]: column_names = ['word_freq_make', 'word_freq_address', 'word_freq_all',
    → 'word_freq_3d', 'word_freq_our', 'word_freq_over', 'word_freq_remove',
    'word_freq_internet', 'word_freq_order', 'word_freq_mail', 'word_freq_receive',
    → 'word_freq_will', 'word_freq_people',
    'word_freq_report', 'word_freq_addresses', 'word_freq_free',
    → 'word_freq_business', 'word_freq_email', 'word_freq_you',
```

```

'word_freq_credit', 'word_freq_your', 'word_freq_font', 'word_freq_000',
→ 'word_freq_money', 'word_freq_hp', 'word_freq_hpl',
'word_freq_george', 'word_freq_650', 'word_freq_lab', 'word_freq_labs',
→ 'word_freq_telnet', 'word_freq_857', 'word_freq_data',
'word_freq_415', 'word_freq_85', 'word_freq_technology', 'word_freq_1999',
→ 'word_freq_parts', 'word_freq_pm', 'word_freq_direct',
'word_freq_cs', 'word_freq_meeting', 'word_freq_original', 'word_freq_project',
→ 'word_freq_re', 'word_freq_edu', 'word_freq_table',
'word_freq_conference', 'char_freq_;', 'char_freq_', 'char_freq_',
→ 'char_freq_!', 'char_freq_$', 'char_freq_#', 'capital_run_length_average',
'capital_run_length_longest', 'capital_run_length_total', 'class']
org_data = pd.read_csv("/Users/snehaagarwal/Desktop/SML/HW3/spambase/spambase.
→data", names = column_names)
#org_data.iloc[:,0:-1]
#org_data

```

```

[4]: X = org_data.iloc[:,0:-1]
y = org_data[['class']]

```

```

[5]: # splitting the data
X_train, X_test, y_train, y_test = train_test_split(X, y, test_size = 0.25,
→stratify=y, random_state = 57)

```

```

[6]: # preprocessing
ss_scaler = preprocessing.StandardScaler()
X_train = pd.DataFrame(ss_scaler.fit_transform(X_train), columns=X_train.columns)
X_test = pd.DataFrame(ss_scaler.transform(X_test), columns = X_test.columns)
y_train = np.ravel(y_train)

```

```

[7]: # check of the proportion of the class after splitting

# y_train.sum(axis = 0, skipna = True)[0]/y_train.shape[0]
# y_test.sum(axis = 0, skipna = True)[0]/y_test.shape[0]

```

```

[8]: warnings.filterwarnings("ignore", category=FutureWarning)
log_reg = LogisticRegression()
log_reg.fit(X_train, y_train)
y_pred = log_reg.predict(X_test)

```

```

[9]: # 1. Confusion matrix
conf_mat = confusion_matrix(y_test, y_pred)
print(conf_mat)

```

```

[[667  30]
 [ 42 412]]

```

```
[10]: # 2. True Positives, False Positives, True Negatives, False Negatives
true_negatives = conf_mat[0][0]
false_negatives = conf_mat[1][0]
true_positives = conf_mat[1][1]
false_positives = conf_mat[0][1]
print("True Positives:", true_positives, ", False Positives: ", false_positives,
      ↪", True Negatives: ", true_negatives,
      ↪", False Negatives: ", false_negatives)
```

True Positives: 412 , False Positives: 30 , True Negatives: 667 , False Negatives: 42

```
[11]: # 3. Accuracy, Error
accuracy = (true_positives + true_negatives)/y_test.shape[0]
error = 1-accuracy
print("Accuracy: ", accuracy, ", Error: ", error)
#print('Accuracy of logistic regression classifier on test set: {:.2f}'.
      ↪format(log_reg.score(X_test, y_test)))
```

Accuracy: 0.9374456993918332 , Error: 0.0625543006081668

```
[12]: #print("Accuracy on testing set:",metrics.accuracy_score(y_test, y_pred))
print("Precision on testing set:",metrics.precision_score(y_test, y_pred))
print("Recall on testing set:",metrics.recall_score(y_test, y_pred))
print("F1-score on testing set:",metrics.f1_score(y_test,y_pred))
```

Precision on testing set: 0.9321266968325792  
 Recall on testing set: 0.9074889867841409  
 F1-score on testing set: 0.9196428571428571

## 1.2 1b

```
[13]: coeff = pd.DataFrame(log_reg.coef_.T, X_train.columns, columns=['Coefficients'])  
      print("Intercept:", log_reg.intercept_)  
      coeff
```

Intercept: [-2.05031456]

```
[13]:
```

	Coefficients
word_freq_make	-0.128952
word_freq_address	-0.229919
word_freq_all	0.021797
word_freq_3d	0.864687
word_freq_our	0.375941
word_freq_over	0.162546
word_freq_remove	1.072679
word_freq_internet	0.227923
word_freq_order	0.184366
word_freq_mail	0.026171
word_freq_receive	0.052486
word_freq_will	-0.147935
word_freq_people	-0.066259
word_freq_report	0.027453
word_freq_addresses	0.327778
word_freq_free	0.851626
word_freq_business	0.313429
word_freq_email	0.107110
word_freq_you	0.147704
word_freq_credit	0.297572
word_freq_your	0.249094
word_freq_font	0.204894
word_freq_000	1.528238
word_freq_money	0.333764
word_freq_hp	-2.060978
word_freq_hpl	-0.840894
word_freq_george	-3.708191
word_freq_650	0.222779
word_freq_lab	-0.768961
word_freq_labs	-0.182240
word_freq_telnet	-0.225475
word_freq_857	0.337067
word_freq_data	-0.317132
word_freq_415	-1.268668
word_freq_85	-0.700761
word_freq_technology	0.249106
word_freq_1999	-0.025615
word_freq_parts	-0.134884
word_freq_pm	-0.380025

word_freq_direct	-0.341114
word_freq_cs	-1.563493
word_freq_meeting	-1.479615
word_freq_original	-0.178957
word_freq_project	-0.753836
word_freq_re	-0.588157
word_freq_edu	-0.822307
word_freq_table	-0.176247
word_freq_conference	-0.998389
char_freq_;	-0.288732
char_freq_(	0.030094
char_freq_[	-0.277017
char_freq_!	0.656580
char_freq_\$	1.091596
char_freq_#	0.612885
capital_run_length_average	1.377071
capital_run_length_longest	1.160721
capital_run_length_total	0.344884

```
[14]: coeff["new_coeff"] = abs(coeff["Coefficients"])
coeff = coeff.sort_values(by = 'new_coeff',ascending=False)
coeff = coeff.drop('new_coeff',axis =1)
print("The features that contribute mostly to the prediction are given below:")
coeff.head(n=10)
```

The features that contribute mostly to the prediction are given below:

```
[14]:
```

	Coefficients
word_freq_george	-3.708191
word_freq_hp	-2.060978
word_freq_cs	-1.563493
word_freq_000	1.528238
word_freq_meeting	-1.479615
capital_run_length_average	1.377071
word_freq_415	-1.268668
capital_run_length_longest	1.160721
char_freq_\$	1.091596
word_freq_remove	1.072679

```
[15]: #coeff = pd.DataFrame(log_reg.coef_.T,X_train.columns,columns=['Coefficients'])
coeff = pd.DataFrame(log_reg.coef_,columns = X_train.columns)
pos_corr = []
neg_corr = []
for i in coeff:
    if coeff[i][0] > 0:
        pos_corr.append(i)
    else:
        neg_corr.append(i)
print("Features positively correlated with spam class: ", pos_corr)
print("\n")
print("Features negatively correlated with spam class: ", neg_corr)
```

Features positively correlated with spam class: ['word\_freq\_all', 'word\_freq\_3d', 'word\_freq\_our', 'word\_freq\_over', 'word\_freq\_remove', 'word\_freq\_internet', 'word\_freq\_order', 'word\_freq\_mail', 'word\_freq\_receive', 'word\_freq\_report', 'word\_freq\_addresses', 'word\_freq\_free', 'word\_freq\_business', 'word\_freq\_email', 'word\_freq\_you', 'word\_freq\_credit', 'word\_freq\_your', 'word\_freq\_font', 'word\_freq\_000', 'word\_freq\_money', 'word\_freq\_650', 'word\_freq\_857', 'word\_freq\_technology', 'char\_freq(', 'char\_freq\_!', 'char\_freq\_\$', 'char\_freq\_#', 'capital\_run\_length\_average', 'capital\_run\_length\_longest', 'capital\_run\_length\_total']

Features negatively correlated with spam class: ['word\_freq\_make', 'word\_freq\_address', 'word\_freq\_will', 'word\_freq\_people', 'word\_freq\_hp', 'word\_freq\_hpl', 'word\_freq\_george', 'word\_freq\_lab', 'word\_freq\_labs', 'word\_freq\_telnet', 'word\_freq\_data', 'word\_freq\_415', 'word\_freq\_85',

```
'word_freq_1999', 'word_freq_parts', 'word_freq_pm', 'word_freq_direct',  
'word_freq_cs', 'word_freq_meeting', 'word_freq_original', 'word_freq_project',  
'word_freq_re', 'word_freq_edu', 'word_freq_table', 'word_freq_conference',  
'char_freq_;', 'char_freq_[']
```

```
[16]: # log_reg.predict_proba(X_test)
```

```
[17]: # log_reg.predict(X_test)
```

### 1.3 1c

```
[18]: y_prob = log_reg.predict_proba(X_test)[: ,1]

decision_threshold = [0.25,0.5,0.75,0.9]

for i in decision_threshold:
    y_pred_dt = []
    for j in y_prob:
        if j>=i:
            y_pred_dt.append(1)
        else:
            y_pred_dt.append(0)
    print("Accuracy when decision threshold is ",i," : ",metrics.
    →accuracy_score(y_test, y_pred_dt))
    print("Precision when decision threshold is ",i," : ",metrics.
    →precision_score(y_test, y_pred_dt))
    print("Recall when decision threshold is ",i," : ",metrics.
    →recall_score(y_test, y_pred_dt))
    #print(confusion_matrix(y_test, y_pred_dt))
    print("\n")
```

```
Accuracy when decision threshold is  0.25 :  0.9148566463944396
Precision when decision threshold is  0.25 :  0.8503937007874016
Recall when decision threshold is  0.25 :  0.9515418502202643
```

```
Accuracy when decision threshold is  0.5 :  0.9374456993918332
Precision when decision threshold is  0.5 :  0.9321266968325792
Recall when decision threshold is  0.5 :  0.9074889867841409
```

```
Accuracy when decision threshold is  0.75 :  0.895742832319722
Precision when decision threshold is  0.75 :  0.9441489361702128
Recall when decision threshold is  0.75 :  0.7819383259911894
```

```
Accuracy when decision threshold is  0.9 :  0.8479582971329279
Precision when decision threshold is  0.9 :  0.9603960396039604
Recall when decision threshold is  0.9 :  0.6409691629955947
```

For decision threshold = 0.5, the combination of the values of the accuracy, precision and recall are best, hence the decision threshold = 0.5 is the best choice. Accuracy of the model increases as the threshold increases till threshold=0.5, post which the accuracy decreases. Precision increases as



the decision threshold increases as the count of false positives decreases. Recall decreases as the threshold increases as the count of false negatives increases.

## 1.4 1d

```
[19]: X = org_data.iloc[:,0:-1]
      y = org_data[['class']]

      # splitting the data
      X_train, X_test, y_train, y_test = train_test_split(X, y, test_size = 0.25,
      →stratify=y, random_state = 57)

      # preprocessing
      ss_scaler = preprocessing.StandardScaler()
      X_train = pd.DataFrame(ss_scaler.fit_transform(X_train), columns=X_train.columns)
      X_test = pd.DataFrame(ss_scaler.transform(X_test), columns = X_test.columns)

[20]: # Compute Cost function is implemented to check if the cost is decreasing with
      →iterations
      def cross_entropy_obj(X,y,theta):
          N = y.shape[0]
          z = np.dot(X, theta)
          h = sigmoid(z)
          # print("h",h)
          # print("first half", -(np.transpose(y).dot(np.log(h))))
          # print("second half", -(np.transpose(1-y).dot(np.log(1-h))))
          cost = (1/N)*(-(np.transpose(y).dot(np.log(h)))-(np.transpose(1-y).dot(np.
          →log(1-h))))
          return cost

      # Sigmoid function
      def sigmoid(z):
          return 1 / (1 + np.exp(-z))

      # Method to compute the theta values using gradient descent
      def gradient_descent_log_reg(X, y, alpha, num_iters):
          iters = num_iters
          if 'x0' not in X.columns:
              x0 = np.ones((X.shape[0], 1), dtype=int)
              X.insert(0, "x0", x0, True)
          # d is the number of features
          d = X_train.shape[1]
          # Initializing theta with zeros
          theta = np.zeros((d,1))
          X=X.values
          y=y.values
          diff_cost = 0
          N = y.shape[0]
          theta_old = theta
```

```

#while diff_cost > 0.000001 or num_iters > 0

while num_iters > 0:
    z = np.dot(X, theta_old)
    h = sigmoid(z)
    gradient = np.transpose(X).dot(h-y)
    theta_new = theta_old - alpha * (1/N) * gradient
    #print("theta_new", theta_new)
    # checking for convergence
    delta_theta = theta_new - theta_old
    delta = np.sqrt(np.transpose(delta_theta).dot(delta_theta))
    #print(delta)
    if delta < 0.00001 and iters != num_iters :
        print("Gradient descent converged at iteration", iters-num_iters)
        break
    num_iters = num_iters - 1
    old_cost = cross_entropy_obj(X,y,theta_old)
    theta_old = theta_new
    new_cost = cross_entropy_obj(X,y,theta_new)
    diff_cost = old_cost - new_cost
    #print("old_cost", old_cost)
    #print("new cost", new_cost)
    #print("diffcost:", diff_cost)
return theta_new

def predict_prob(X,theta):
    if 'x0' not in X.columns:
        x0 = np.ones((X.shape[0], 1), dtype=int)
        X.insert(0, "x0", x0, True)
    return(sigmoid(np.dot(X,theta)))

# predict the y values using theta and X
def predict(y, threshold):
    y_pred_gd=[]
    for i in y:
        if i>=threshold:
            y_pred_gd.append(1)
        else:
            y_pred_gd.append(0)
    return y_pred_gd

```

```

[21]: # theta = gradient_descent_log_reg(X_train, y_train, 0.3, 10)
      # print(cross_entropy_obj(X_train,y_train,theta)[0][0])

```

```
[22]: lr_rates = [0.01, 0.05, 0.4]
iter_var = [10, 50, 100]

for i in lr_rates:
    for j in iter_var:
        theta = gradient_descent_log_reg(X_train, y_train, i, j)
        print("Cross Entropy for alpha =", i, "and iterations = ", j, "is: ",
→cross_entropy_obj(X_train, y_train, theta)[0][0])
        if j == 100:
            pred_prob = predict_prob(X_test, theta)
            y_pred_val = predict(pred_prob, 0.5)
            print("Accuracy when decision threshold is", i, "and iter = 100:
→", metrics.accuracy_score(y_test, y_pred_val))
            print("F1_score when decision threshold is", i, "and iter = 100:
→", metrics.f1_score(y_test, y_pred_val))
            print("\n")
```

```
Cross Entropy for alpha = 0.01 and iterations = 10 is: 0.6495252510572939
Cross Entropy for alpha = 0.01 and iterations = 50 is: 0.5365732045629628
Cross Entropy for alpha = 0.01 and iterations = 100 is: 0.46158848382339607
Accuracy when decision threshold is 0.01 and iter = 100: 0.8992180712423979
F1_score when decision threshold is 0.01 and iter = 100: 0.8719646799116998
```

```
Cross Entropy for alpha = 0.05 and iterations = 10 is: 0.5348795325629325
Cross Entropy for alpha = 0.05 and iterations = 50 is: 0.36599881153810176
Cross Entropy for alpha = 0.05 and iterations = 100 is: 0.3126833535736034
Accuracy when decision threshold is 0.05 and iter = 100: 0.9026933101650738
F1_score when decision threshold is 0.05 and iter = 100: 0.8747203579418344
```

```
Cross Entropy for alpha = 0.4 and iterations = 10 is: 0.3232409114693962
Cross Entropy for alpha = 0.4 and iterations = 50 is: 0.25107070946974663
Cross Entropy for alpha = 0.4 and iterations = 100 is: 0.23462056796440348
Accuracy when decision threshold is 0.4 and iter = 100: 0.9131190269331017
F1_score when decision threshold is 0.4 and iter = 100: 0.8876404494382022
```

Compared to the metrics given by the package, the accuracy and F1 score obtained is less using my implementation of the logistic regression using gradient descent which might be due to the fact that the gradient descent is still not converged with 100 iterations and the alpha values selected.

## 2 Problem 2

### 2.1 2a

```
[23]: # removing bias again
```

```
# X_train = X_train.iloc[:,1:]
# X_test = X_test.iloc[:,1:]
if X_train.columns[0] == "x0":
    X_train = X_train.iloc[:,1:]
if X_test.columns[0]=="x0":
    X_test = X_test.iloc[:,1:]
y_train = np.ravel(y_train)
```

```
[24]: from sklearn.neighbors import KNeighborsClassifier
```

```
for i in range(1,17,2):
    classifier = KNeighborsClassifier(n_neighbors=i)
    classifier.fit(X_train, y_train)
    #predictions
    y_pred_train_knn = classifier.predict(X_train)
    y_pred_test_knn = classifier.predict(X_test)
    print("Accuracy in training data using KNN for k=",i," : ",metrics.
→accuracy_score(y_train, y_pred_train_knn))
    print("Error in training data using KNN for k=",i," : ",1-metrics.
→accuracy_score(y_train, y_pred_train_knn))
    print("Accuracy in testing data using KNN for k=",i," : ",metrics.
→accuracy_score(y_test, y_pred_test_knn))
    print("Error in testing data using KNN for k=",i," : ",1-metrics.
→accuracy_score(y_test, y_pred_test_knn))
    print("\n")
```

```
Accuracy in training data using KNN for k= 1 : 0.9994202898550725
Error in training data using KNN for k= 1 : 0.0005797101449275255
Accuracy in testing data using KNN for k= 1 : 0.9183318853171155
Error in testing data using KNN for k= 1 : 0.08166811468288449
```

```
Accuracy in training data using KNN for k= 3 : 0.9504347826086956
Error in training data using KNN for k= 3 : 0.04956521739130437
Accuracy in testing data using KNN for k= 3 : 0.9252823631624674
Error in testing data using KNN for k= 3 : 0.07471763683753263
```

```
Accuracy in training data using KNN for k= 5 : 0.9307246376811594
Error in training data using KNN for k= 5 : 0.06927536231884057
Accuracy in testing data using KNN for k= 5 : 0.9235447437011295
Error in testing data using KNN for k= 5 : 0.07645525629887051
```

Accuracy in training data using KNN for k= 7 : 0.9260869565217391  
Error in training data using KNN for k= 7 : 0.07391304347826089  
Accuracy in testing data using KNN for k= 7 : 0.9087749782797567  
Error in testing data using KNN for k= 7 : 0.09122502172024327

Accuracy in training data using KNN for k= 9 : 0.9234782608695652  
Error in training data using KNN for k= 9 : 0.07652173913043481  
Accuracy in testing data using KNN for k= 9 : 0.9096437880104257  
Error in testing data using KNN for k= 9 : 0.09035621198957433

Accuracy in training data using KNN for k= 11 : 0.9214492753623188  
Error in training data using KNN for k= 11 : 0.0785507246376812  
Accuracy in testing data using KNN for k= 11 : 0.9087749782797567  
Error in testing data using KNN for k= 11 : 0.09122502172024327

Accuracy in training data using KNN for k= 13 : 0.9162318840579711  
Error in training data using KNN for k= 13 : 0.08376811594202893  
Accuracy in testing data using KNN for k= 13 : 0.9131190269331017  
Error in testing data using KNN for k= 13 : 0.08688097306689835

Accuracy in training data using KNN for k= 15 : 0.9098550724637681  
Error in training data using KNN for k= 15 : 0.09014492753623193  
Accuracy in testing data using KNN for k= 15 : 0.9009556907037359  
Error in testing data using KNN for k= 15 : 0.09904430929626407

k=3, gives the highest accuracy in testing.

## 2.2 2b

```
[25]: from sklearn.discriminant_analysis import LinearDiscriminantAnalysis
      from sklearn.naive_bayes import GaussianNB
      from sklearn.tree import DecisionTreeClassifier
      from sklearn.metrics import roc_curve, auc

      classifiers = ["Logistic Regression", "LDA", "kNN", "Naive Bayes", "Decision_
        ↳Tree"]

      def log_reg(X,y):
          log_reg = LogisticRegression()
          log_reg.fit(X,y)
          return log_reg

      def knn(X,y):
          classifier = KNeighborsClassifier(n_neighbors=3)
          classifier.fit(X,y)
          return classifier

      def LDA(X,y):
          lda = LinearDiscriminantAnalysis()
          lda.fit(X,y)
          return lda

      def NB(X,y):
          nb = GaussianNB()
          nb.fit(X,y)
          return nb

      def DTClassifier(X,y):
          clf = DecisionTreeClassifier()
          clf.fit(X,y)
          return clf

      def predict(classifier,X_train,y_train,X_test,y_test):
          if classifier == "Logistic Regression":
              model = log_reg(X_train,y_train)
          elif classifier == "kNN":
              model = knn(X_train, y_train)
          elif classifier == "LDA" :
              model = LDA(X_train, y_train)
          elif classifier == "Naive Bayes":
              model = NB(X_train, y_train)
          elif classifier == "Decision Tree":
```

```

        model = DTClassifier(X_train, y_train)

        y_pred_train = model.predict(X_train)
        y_pred_test = model.predict(X_test)
        print("Accuracy in training set using",classifier,":",metrics.
→accuracy_score(y_train, y_pred_train))
        print("Error in training set using",classifier,":",1 - metrics.
→accuracy_score(y_train, y_pred_train))
        print("Accuracy in testing set using",classifier,":",metrics.
→accuracy_score(y_test, y_pred_test))
        print("Error in testing set using",classifier,":",1 - metrics.
→accuracy_score(y_test, y_pred_test))
        print("Precision in testing set using",classifier,":",metrics.
→precision_score(y_test, y_pred_test))
        print("Recall in testing set using",classifier,":",metrics.
→recall_score(y_test, y_pred_test))
        print("\n")

for i in classifiers:
    predict(i,X_train,y_train,X_test,y_test)

```

Accuracy in training set using Logistic Regression : 0.9318840579710145  
 Error in training set using Logistic Regression : 0.06811594202898552  
 Accuracy in testing set using Logistic Regression : 0.9374456993918332  
 Error in testing set using Logistic Regression : 0.0625543006081668  
 Precision in testing set using Logistic Regression : 0.9321266968325792  
 Recall in testing set using Logistic Regression : 0.9074889867841409

Accuracy in training set using LDA : 0.896231884057971  
 Error in training set using LDA : 0.10376811594202895  
 Accuracy in testing set using LDA : 0.894005212858384  
 Error in testing set using LDA : 0.10599478714161603  
 Precision in testing set using LDA : 0.9088669950738916  
 Recall in testing set using LDA : 0.8127753303964758

Accuracy in training set using kNN : 0.9504347826086956  
 Error in training set using kNN : 0.04956521739130437  
 Accuracy in testing set using kNN : 0.9252823631624674  
 Error in testing set using kNN : 0.07471763683753263  
 Precision in testing set using kNN : 0.9088888888888889  
 Recall in testing set using kNN : 0.9008810572687225

Accuracy in training set using Naive Bayes : 0.8136231884057971  
 Error in training set using Naive Bayes : 0.18637681159420294



Accuracy in testing set using Naive Bayes : 0.8166811468288445  
Error in testing set using Naive Bayes : 0.18331885317115548  
Precision in testing set using Naive Bayes : 0.6925515055467512  
Recall in testing set using Naive Bayes : 0.9625550660792952

Accuracy in training set using Decision Tree : 0.9994202898550725  
Error in training set using Decision Tree : 0.0005797101449275255  
Accuracy in testing set using Decision Tree : 0.9026933101650738  
Error in testing set using Decision Tree : 0.09730668983492619  
Precision in testing set using Decision Tree : 0.8622881355932204  
Recall in testing set using Decision Tree : 0.8964757709251101

Logistic regression is performing best with accuracy = 0.937, precision = 0.93 and recall = 0.907. Naive Bayes is performing worst among the models selected accuracy = 0.816, precision = 0.69 and recall = 0.96. Naive Bayes predicts the actual class correctly far better than any other classifiers but the accuracy and precision is lower than any other classifier probably due to the fact that Naive Bayes assumption of independence among features. The accuracy for LDA is 0.89, accuracy for kNN = 0.925 and accuracy for decision trees = 0.910.

## 2.3 2c

```
[26]: import matplotlib.pyplot as plt

def ROC(classifiers,X_train,y_train,X_test,y_test):
    for classifier in classifiers:
        if classifier == "Logistic Regression":
            model = log_reg(X_train,y_train)
        elif classifier == "kNN":
            model = knn(X_train, y_train)
        elif classifier == "LDA" :
            model = LDA(X_train, y_train)
        elif classifier == "Naive Bayes":
            model = NB(X_train, y_train)
        elif classifier == "Decision Tree":
            model = DTClassifier(X_train, y_train)

        #y_pred_test = model.predict(X_test)
        y_pred_test = model.predict_proba(X_test)[:,-1]
        #print(y_pred_test)
        fpr, tpr, thresholds = roc_curve(y_test,y_pred_test)
        #print(fpr,tpr,thresholds)
        #    plot_ROC(fpr,tpr,classifier)

        # def plot_ROC(false_positive_rate,true_positive_rate,classifier): (area =
        →%0.2f) % auc(fpr,tpr)
            lw=2
            plt.plot(fpr,tpr, lw=lw,
                    label='ROC curve for %s (area = %0.2f)' %
        →(classifier, auc(fpr,tpr)) )
            plt.plot([0, 1], [0, 1], lw=lw, color='navy', linestyle='--')
            plt.xlim([0.0, 1.0])
            plt.ylim([0.0, 1.05])
            plt.xlabel('False Positive Rate')
            plt.ylabel('True Positive Rate')
            title = 'Receiver operating characteristic'
            plt.title(title)
            plt.legend(loc="lower right")
            #plt.show()
            print("AUC for",classifier,":",auc(fpr,tpr))
        plt.show()

    #for i in classifiers:
    ROC(classifiers,X_train,y_train,X_test,y_test)
```

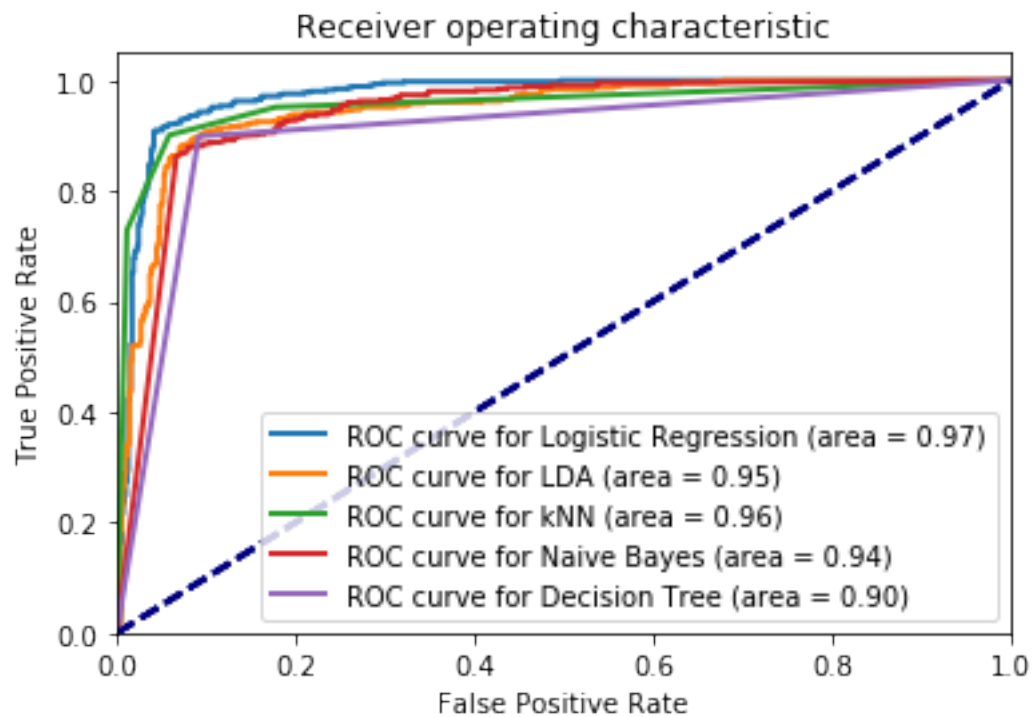
AUC for Logistic Regression : 0.9712913746136683

AUC for LDA : 0.9456402202011135

AUC for kNN : 0.9552408370676087

AUC for Naive Bayes : 0.9378377438866381

AUC for Decision Tree : 0.9034281597026905



### 3 Problem 3

#### 3.1 3a

```
[27]: X = org_data.iloc[:,0:-1]
      y = org_data[['class']]
      # splitting the data
      X_train, X_test, y_train, y_test = train_test_split(X, y, test_size = 100,
      ↪train_size = 100, stratify=y, random_state = 57)

[28]: # preprocessing
      ss_scaler = preprocessing.StandardScaler()
      X_train = pd.DataFrame(ss_scaler.fit_transform(X_train),columns=X_train.columns)
      X_test = pd.DataFrame(ss_scaler.transform(X_test),columns = X_test.columns)

[29]: #a
      def euclidean_distance(X,Y):
          temp = np.transpose(X-Y).dot(X-Y)
          distance = np.sqrt(temp)
          return distance
```

#### 3.2 3b

```
[30]: #b

      def test_kNN(k,train_feature,train_class,test_point):
          distance_list = []
          nearest_points = []
          pred_class_dict = {}

          for i, sample in enumerate(train_feature.values):
              distance_list.append([euclidean_distance(sample,test_point.values)])
              distance_list[i].append(i)

          k_nearest_distances = sorted(distance_list,key=key_func)[0:k]

          for i in k_nearest_distances:
              nearest_points.append(i[1])

          for point in nearest_points:
              class_val = train_class.iloc[point][0]
              if class_val not in pred_class_dict:
                  pred_class_dict[class_val] = 1
              else:
                  pred_class_dict[class_val] += 1
          #print(pred_class_dict)
```

```
pred_class = max(pred_class_dict, key=pred_class_dict.get)
return(pred_class)

def key_func(s):
    return s[0]

print(test_kNN(5, X_train, y_train, X_test.iloc[0]))
```

1

### 3.3 3c

```
[31]: # c
      # Using my own implementation
      import time

      n_row = X_test.shape[0]
      for k in range(1,17,2):
          y_pred_kNN = []
          start_time = time.time()
          for row in range(n_row):
              y_pred_kNN.append(test_kNN(k, X_train, y_train, X_test.iloc[row]))
          print("Running time of kNN testing averaged over all the points in the_
→testing set for k=%d is %s seconds "
              % (k,((time.time() - start_time)/n_row)))
          #print(y_pred_kNN)
          print("Accuracy in testing data using KNN for k=",k," : ",metrics.
→accuracy_score(y_test,y_pred_kNN))
          print("Error in testing data using KNN for k=",k," : ",1-metrics.
→accuracy_score(y_test, y_pred_kNN))
          print("\n")
```

Running time of kNN testing averaged over all the points in the testing set for k=1 is 0.0014136815071105957 seconds

Accuracy in testing data using KNN for k= 1 : 0.77

Error in testing data using KNN for k= 1 : 0.22999999999999998

Running time of kNN testing averaged over all the points in the testing set for k=3 is 0.0016277718544006347 seconds

Accuracy in testing data using KNN for k= 3 : 0.88

Error in testing data using KNN for k= 3 : 0.12

Running time of kNN testing averaged over all the points in the testing set for k=5 is 0.002272059917449951 seconds

Accuracy in testing data using KNN for k= 5 : 0.87

Error in testing data using KNN for k= 5 : 0.13

Running time of kNN testing averaged over all the points in the testing set for k=7 is 0.002150590419769287 seconds

Accuracy in testing data using KNN for k= 7 : 0.89

Error in testing data using KNN for k= 7 : 0.10999999999999999

Running time of kNN testing averaged over all the points in the testing set for

k=9 is 0.003649299144744873 seconds  
Accuracy in testing data using KNN for k= 9 : 0.86  
Error in testing data using KNN for k= 9 : 0.14

Running time of kNN testing averaged over all the points in the testing set for  
k=11 is 0.0034282684326171877 seconds  
Accuracy in testing data using KNN for k= 11 : 0.82  
Error in testing data using KNN for k= 11 : 0.18000000000000005

Running time of kNN testing averaged over all the points in the testing set for  
k=13 is 0.002761719226837158 seconds  
Accuracy in testing data using KNN for k= 13 : 0.86  
Error in testing data using KNN for k= 13 : 0.14

Running time of kNN testing averaged over all the points in the testing set for  
k=15 is 0.003134438991546631 seconds  
Accuracy in testing data using KNN for k= 15 : 0.9  
Error in testing data using KNN for k= 15 : 0.09999999999999998

### 3.4 3d

```
[32]: # d
      #Using existing package
      y_train = np.ravel(y_train)
      for k in range(1,17,2):
          classifier = KNeighborsClassifier(n_neighbors=k)
          classifier.fit(X_train, y_train)
          #predictions
          #y_pred_train_knn = classifier.predict(X_train)
          y_pred_test_knn = classifier.predict(X_test)
          #print("Accuracy in training data using KNN for k=",i," : ",metrics.
          →accuracy_score(y_train, y_pred_train_knn))
          #print("Error in training data using KNN for k=",i," : ",1-metrics.
          →accuracy_score(y_train, y_pred_train_knn))
          print("Accuracy in testing data using KNN for k=",k," : ",metrics.
          →accuracy_score(y_test, y_pred_test_knn))
          print("Error in testing data using KNN for k=",k," : ",1-metrics.
          →accuracy_score(y_test, y_pred_test_knn))
          print("\n")
```

Accuracy in testing data using KNN for k= 1 : 0.77

Error in testing data using KNN for k= 1 : 0.22999999999999998

Accuracy in testing data using KNN for k= 3 : 0.88

Error in testing data using KNN for k= 3 : 0.12

Accuracy in testing data using KNN for k= 5 : 0.87

Error in testing data using KNN for k= 5 : 0.13

Accuracy in testing data using KNN for k= 7 : 0.89

Error in testing data using KNN for k= 7 : 0.10999999999999999

Accuracy in testing data using KNN for k= 9 : 0.86

Error in testing data using KNN for k= 9 : 0.14

Accuracy in testing data using KNN for k= 11 : 0.82

Error in testing data using KNN for k= 11 : 0.18000000000000005

Accuracy in testing data using KNN for k= 13 : 0.86

Error in testing data using KNN for k= 13 : 0.14



Accuracy in testing data using KNN for k= 15 : 0.9  
Error in testing data using KNN for k= 15 : 0.09999999999999998

The result obtained using my own implementation of kNN is similar to those obtained using the package for different values of k. The accuracy increase till k=7, post which the accuracy decreases on increasing k.

### **3.5 3e**

The running time of kNN testing averaged over all the points in the testing set is printed along with problem 3c.

## 4 Problem 4

### 4.1 4a,b

```
[33]: data = org_data.copy()
```

```
[34]: def k_fold_CV(data,k_fold,classifier):
    N = data.shape[0]
    fold_size = int(N/k_fold)
    start = 0
    end = fold_size
    error = 0
    for k in range(k_fold):
#         print("start",start)
#         print("end",end)
        test = data[start:end]
        if start != 0:
            train = data[0:start]
            train = train.append(data[end:N],ignore_index = True)
        else:
            train = data[end:N]
#         print("test shape",test.shape)
#         print("train shape",train.shape)
#         print("\n")
        X_train, X_test, y_train, y_test = preprocessing_data(train,test)
#         print("y_train shape",y_train.shape)
#         print("y_test shape", y_test.shape)
        if classifier == "LDA":
            model = LDA(X_train,y_train)
        elif classifier == "Logistic Regression":
            model = Log_Reg(X_train, y_train)
        y_pred = model.predict(X_test)
#         print("Error in testing set using",model,"for fold",k,":",1 - metrics.
→accuracy_score(y_test, y_pred))
        error += 1 - metrics.accuracy_score(y_test, y_pred)

        start = start + fold_size
        if end + fold_size > N:
            end = N
        else:
            end = end + fold_size
    print("Average validation error using",classifier,"for number of folds_
→=",k_fold,":", error/k_fold)
    return (error/k_fold)

def LDA(X,y):
```

```

lda = LinearDiscriminantAnalysis()
lda.fit(X,y)
return lda

def Log_Reg(X,y):
    log_reg = LogisticRegression()
    log_reg.fit(X,y)
    return log_reg

def preprocessing_data(train,test):
    X_train = train.iloc[:,0:-1]
    y_train = train["class"]
    X_test = test.iloc[:,0:-1]
    y_test = test["class"]
    ss_scaler = preprocessing.StandardScaler()
    X_train = pd.DataFrame(ss_scaler.fit_transform(X_train),columns=X_train.
→columns)
    X_test = pd.DataFrame(ss_scaler.transform(X_test),columns = X_test.columns)
    return X_train, X_test, y_train, y_test

#k_fold_CV(data,5,"LDA")
classifiers = ["Logistic Regression", "LDA"]
for model in classifiers:
    for k in [5,10]:
        k_fold_CV(data,k,model)

```

Average validation error using Logistic Regression for number of folds = 5 :  
 0.1408695652173913  
 Average validation error using Logistic Regression for number of folds = 10 :  
 0.10934782608695652  
 Average validation error using LDA for number of folds = 5 : 0.18391304347826087  
 Average validation error using LDA for number of folds = 10 :  
 0.14804347826086955

## 4.2 4c

From the above run, we can say that the Logistic Regression works better than LDA as the average validation error for k=5 using logistic regression is 0.140 and using LDA is 0.183. And the trend is similar for k = 10, however the average validation error decreases as the value of k increases from 5 to 10 for both the classifiers.

### Problem 5

a)  $P(Y=1 | X_1=1, X_2=0, X_3=1)$  using Naive Bayes rule with the Naive Bayes assumption (stating that the conditional probabilities of feature given the label are independent).

$$P(Y=1 | X_1=1, X_2=0, X_3=1) = \frac{P(X_1=1, X_2=0, X_3=1 | Y=1) P(Y=1)}{P(Y=1) P(X_1=1, X_2=0, X_3=1 | Y=1) + P(Y=0) P(X_1=1, X_2=0, X_3=1 | Y=0)}$$

Using the Naive Bayes assumption, we can write,

$$= \frac{P(X_1=1 | Y=1) P(X_2=0 | Y=1) P(X_3=1 | Y=1) P(Y=1)}{P(Y=1) P(X_1=1 | Y=1) P(X_2=0 | Y=1) P(X_3=1 | Y=1) + P(Y=0) P(X_1=1 | Y=0) P(X_2=0 | Y=0) P(X_3=1 | Y=0)}$$

from the table we get,

$$P(X_1=1 | Y=1) = 2/4 \text{ (stored)}$$

$$P(X_1=0 | Y=1) = 1 - P(X_1=1 | Y=1) = 2/4$$

$$P(X_2=1 | Y=1) = 3/4 \text{ (stored)}$$

$$P(X_2=0 | Y=1) = 1 - P(X_2=1 | Y=1) = 1/4$$

$$P(X_3=1 | Y=1) = 2/4 \text{ (stored)}$$

$$P(X_3=0 | Y=1) = 1 - P(X_3=1 | Y=1) = 2/4$$

$$P(Y=1) = 4/7 \text{ (stored)}$$

$$P(Y=0) = 1 - P(Y=1) = 3/7$$

$$P(X_1=1 | Y=0) = 2/3 \text{ (stored)}$$

$$P(X_1=0 | Y=0) = 1 - P(X_1=1 | Y=0) = 1/3$$

$$P(X_2=1 | Y=0) = 1/3 \text{ (stored)}$$

$$P(X_2=0 | Y=0) = 1 - P(X_2=1 | Y=0) = 2/3$$

$$P(X_3=1 | Y=0) = 2/3 \text{ (stored)}$$

$$P(X_3=0 | Y=0) = 1 - P(X_3=1 | Y=0) = 1/3$$

$$= \frac{2/4 \times 1/4 \times 2/4 \times \frac{4}{7}}{4/7 \times \frac{2}{4} \times 1/4 \times 2/4 + 3/7 \times 2/3 \times 2/3 \times 2/3}$$

$$= 0.219$$

$$\therefore P(Y=1 | X_1=1, X_2=0, X_3=1) = 0.219$$

$$\text{For, } P(Y=1 | X_1=1, X_2=1, X_3=1)$$

$$= \frac{P(X_1=1|Y=1) P(X_2=1|Y=1) P(X_3=1|Y=1) P(Y=1)}{P(Y=1) P(X_1=1|Y=1) P(X_2=1|Y=1) P(X_3=1|Y=1) + P(Y=0) P(X_1=1|Y=0) P(X_2=1|Y=0) P(X_3=1|Y=0)}$$

$$= \frac{2/4 \times 3/4 \times 2/4 \times \frac{4}{7}}{4/7 \times 2/4 \times 3/4 \times 2/4 + 3/7 \times 2/3 \times 1/3 \times 2/3}$$

$$= 0.627$$

\* The parameters that are estimated ~~that~~ and stored by the Naive Bayes classifier can be generalized as ~~(n-1)k~~ for ~~each~~  $(n-1)k$  values for each feature, when  $n$  = number of values an input feature can take, and  $k$  = number of values of the class. Along with it we need to store the priors, which would be  $(k-1)$ . Here, in this case, each feature takes 2 values  $\{1, 0\}$ ,  $n=2$ ,  $d=3$  (input features)  $k = \{1, 0\} = 2$ . Therefore,  $(n-1)k \times d + k - 1 = (2-1) \times 2 \times 3 + 2 - 1 = 6 + 1 = 7$ .

In this case, 7 values can be stored, and the remaining 7 can be estimated using them.



Problem 5.

b)

$P(Y=1 | X_1=1, X_2=0, X_3=1)$  using Bayes Rule, without the Naive Bayes assumption.

$$P(Y=1 | X_1=1, X_2=0, X_3=1) = \frac{P(X_1=1, X_2=0, X_3=1 | Y=1) P(Y=1)}{\sum_{k=0,1} P(Y=k) P(X_1=1, X_2=0, X_3=1 | Y=k)}$$

$$= \frac{P(X_1=1 \cap X_2=0 \cap X_3=1 \cap Y=1) / P(Y=1) \times P(Y=1)}{\sum_{k=0,1} P(Y=k) P(X_1=1, X_2=0, X_3=1 | Y=k)}$$

$$= \frac{0}{\sum_{k=0,1} P(Y=k) P(X_1=1, X_2=0, X_3=1 | Y=k)}$$

$$= \frac{0}{0} \left[ \because P(X_1=1 \cap X_2=0 \cap X_3=1 \cap Y=1) = 0 \right]$$

~~undefined~~

\*  $P(Y=1 | X_1=1, X_2=1, X_3=1)$  using Bayes Rule, without the Naive Bayes Assumption.

$$P(Y=1 | X_1=1, X_2=1, X_3=1) = \frac{P(X_1=1, X_2=1, X_3=1 | Y=1) P(Y=1)}{\sum_{k=0,1} P(Y=k) P(X_1=1, X_2=1, X_3=1 | Y=k)}$$

$$= \frac{P(X_1=1 \cap X_2=1 \cap X_3=1 \cap Y=1) / P(Y=1) \times P(Y=1)}{\sum_{k=0,1} P(Y=k) P(X_1=1, X_2=1, X_3=1 | Y=k)}$$

$$= \frac{0}{\sum_{k=0,1} P(Y=k) P(X_1=1, X_2=1, X_3=1 | Y=k)}$$

$$= \frac{0}{0} \left[ \because P(X_1=1 \cap X_2=1 \cap X_3=1 \cap Y=1) = 0 \right]$$

= undefined.

~~the number~~

The number of parameters that needs to be stored in this case will be: combinations of  $x_1, x_2, x_3 = 2^n$  <sup>no</sup> ~~(features)~~ where  $(n = \text{number of features})$  and we can store only  $2^n - 1$  combinations of the features, multiplied by the number of values the class can take, i.e. 2 in this case,  $(2^n - 1) \times 2$ .

- Also the prior probability needs to be stored, hence the no. of parameters would be

$$(2^n - 1) \times 2 + 1 = (2^3 - 1) \times 2 + 1 = 15 \text{ parameters.}$$

$\{n = \text{no. of features}\}$

## 5 Problem 6

### 5.1 6a

```
[35]: def sigmoid(z):  
        return 1 / (1 + np.exp(-z))  
  
        theta = np.matrix('-6;0.05;1')  
  
        X = np.matrix('1;40;3.5')  
  
        temp = np.transpose(theta).dot(X)  
  
        prob = sigmoid(temp)  
  
        print("Recieve A in Class with probability:", prob.A1[0])
```

Recieve A in Class with probability: 0.3775406687981454



## Problem 6

b)  $\theta_0 = -6$ ,  $\theta_1 = +0.05$ ,  $\theta_2 = 1$ ,  
 $x_0 = 1$ ,  $x_1 = ?$   $x_2 = 3.5$ .  
 $h\theta(x) = 1/2$ .

$$\therefore h\theta(x) = g(\theta^T x) = \frac{1}{1 + e^{-(\theta^T x)}} = \frac{1}{2}$$

$$\Rightarrow 2 = 1 + e^{-(\theta^T x)}$$

$$\Rightarrow 1 = e^{-(\theta^T x)}$$

⊙ Taking log on both sides,

$$\Rightarrow -(\theta^T x) = \log(1)$$

$$\Rightarrow \theta^T x = 0$$

$$\Rightarrow -6 + 0.05x_1 + 3.5 = 0$$

$$\Rightarrow 0.05x_1 = 2.5$$

$$\Rightarrow x_1 = \frac{2.5}{0.05} = 50$$

$\therefore$  The student in part (a) need to study 50 hrs, to have a 50% chance of getting an A in the class.