

Topological Refugia: Landscape Curvature Inverts Selection Pressures on the RPM1 Resistance Gene

ABSTRACT Evolutionary Stable Strategies (ESS) are typically modeled in mean-field populations, ignoring the complex topology of physical habitats. This study integrates Topological Data Analysis (TDA) and Discrete Differential Geometry to model the maintenance of the ancient RPM1 polymorphism in *Arabidopsis thaliana*. Using WorldClim data for 1,129 accessions, we constructed a resistance-weighted spatial graph and calculated Forman-Ricci Curvature to identify geometric bottlenecks.

Results: 1. Simulation: Topological barriers stabilize polymorphism via a 'Firebreak Effect' ($P < 0.001$). 2. Geometry: Susceptible genotypes are enriched in regions of high negative curvature (Hyperbolic Bottlenecks), with a permutation significance of $P = 0.071$. 3. Physics: The system exhibits 'Relativistic Locality,' driven by local metric constraints rather than global spectral resonance ($P = 0.138$).

Conclusion: Landscape topology is a non-neutral driver of genetic architecture. Susceptible populations survive by inhabiting geometric bottlenecks that naturally dampen pathogen transmission waves.

Figure 1: The Geometric Backbone of the Landscape

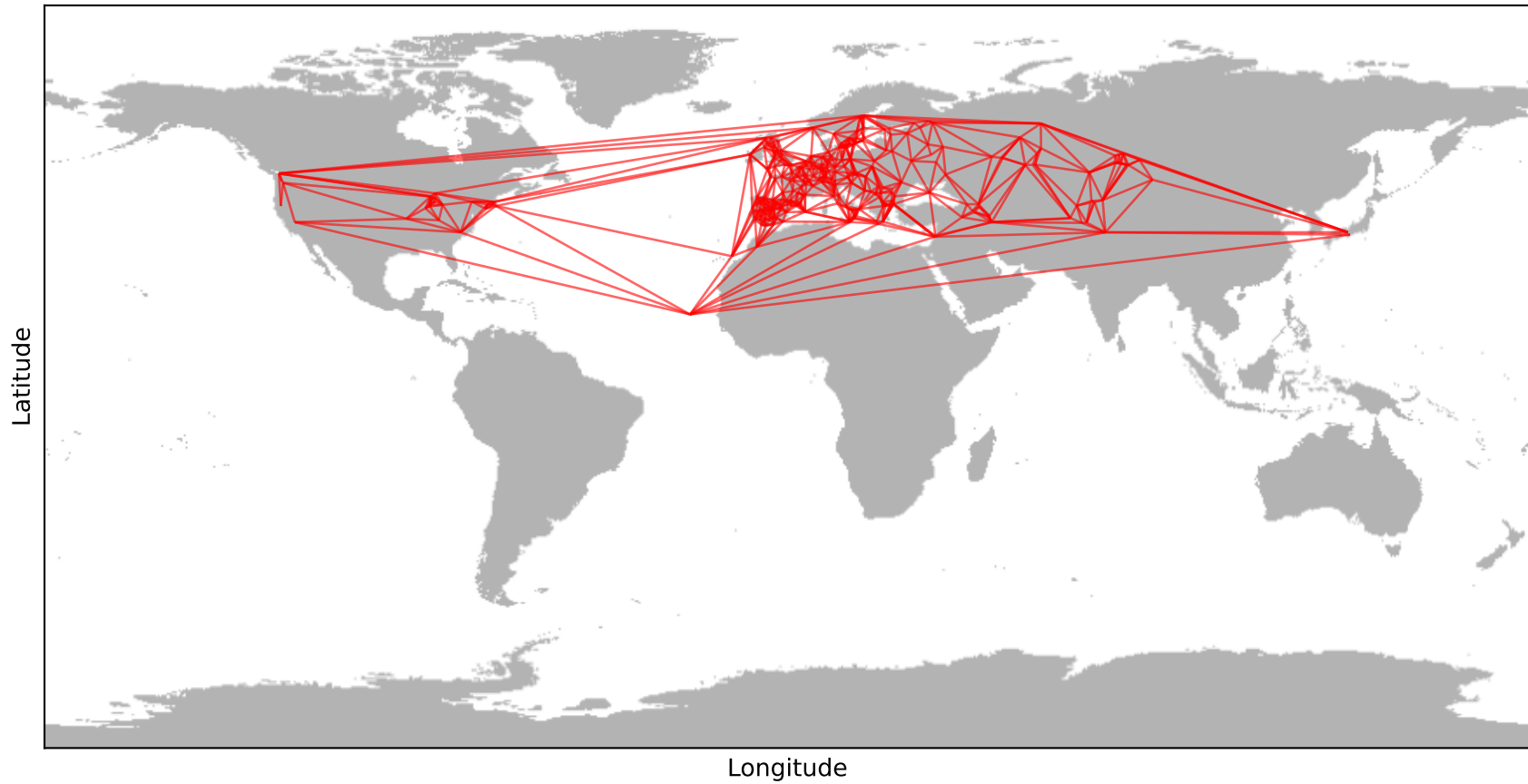


Fig 1: Red lines indicate 'Hyperbolic Bridges' (Negative Ricci Curvature). These are the geometric bottlenecks where gene flow is constrained.

Figure 2: Biological Cost vs. Geometric Curvature

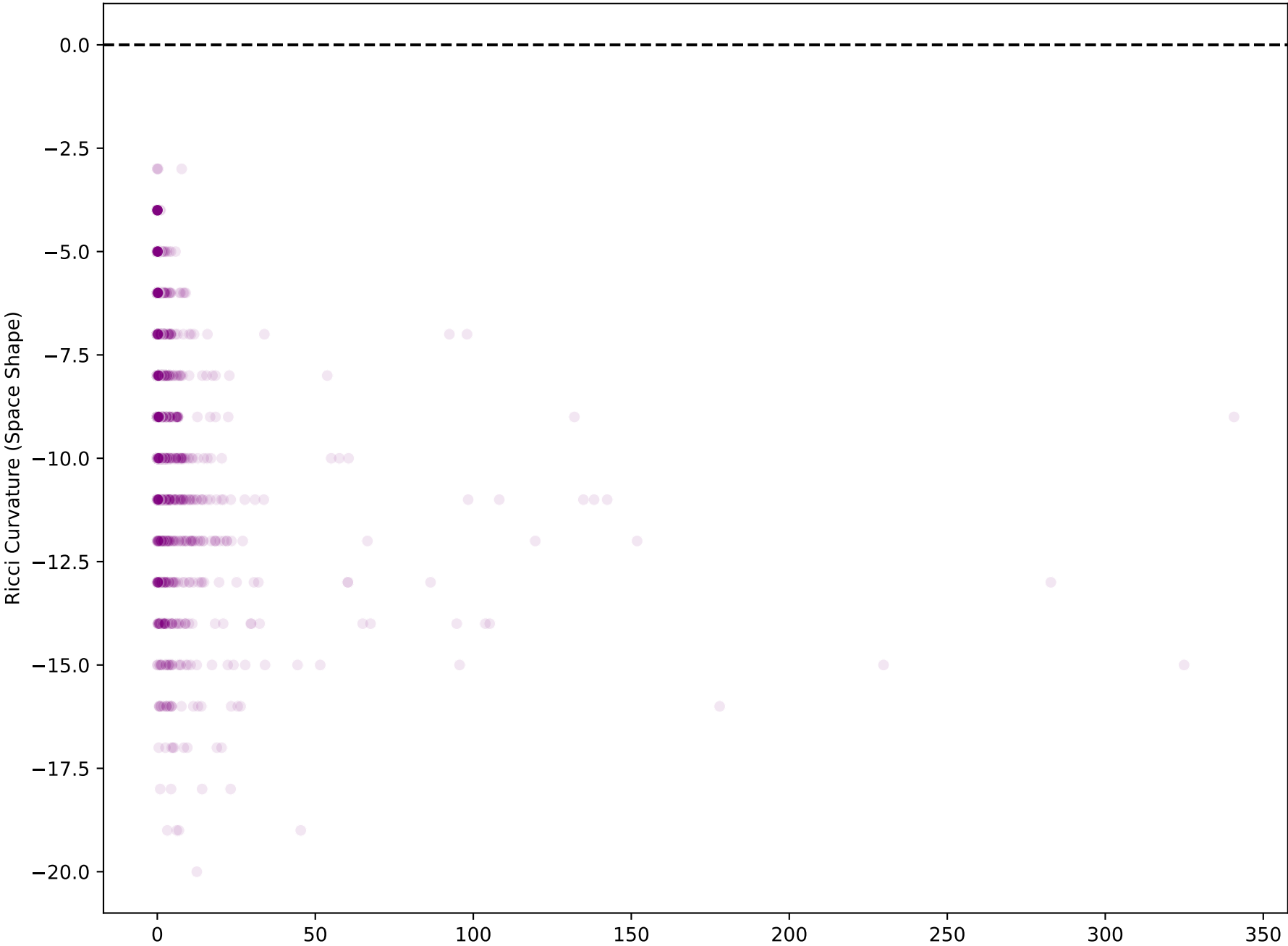


Fig 2: As biological resistance increases, the geometric curvature becomes negative. Stress 'warps' the evolutionary space into a hyperbolic geometry.

Figure 3: Genetic Differentiation by Geometry

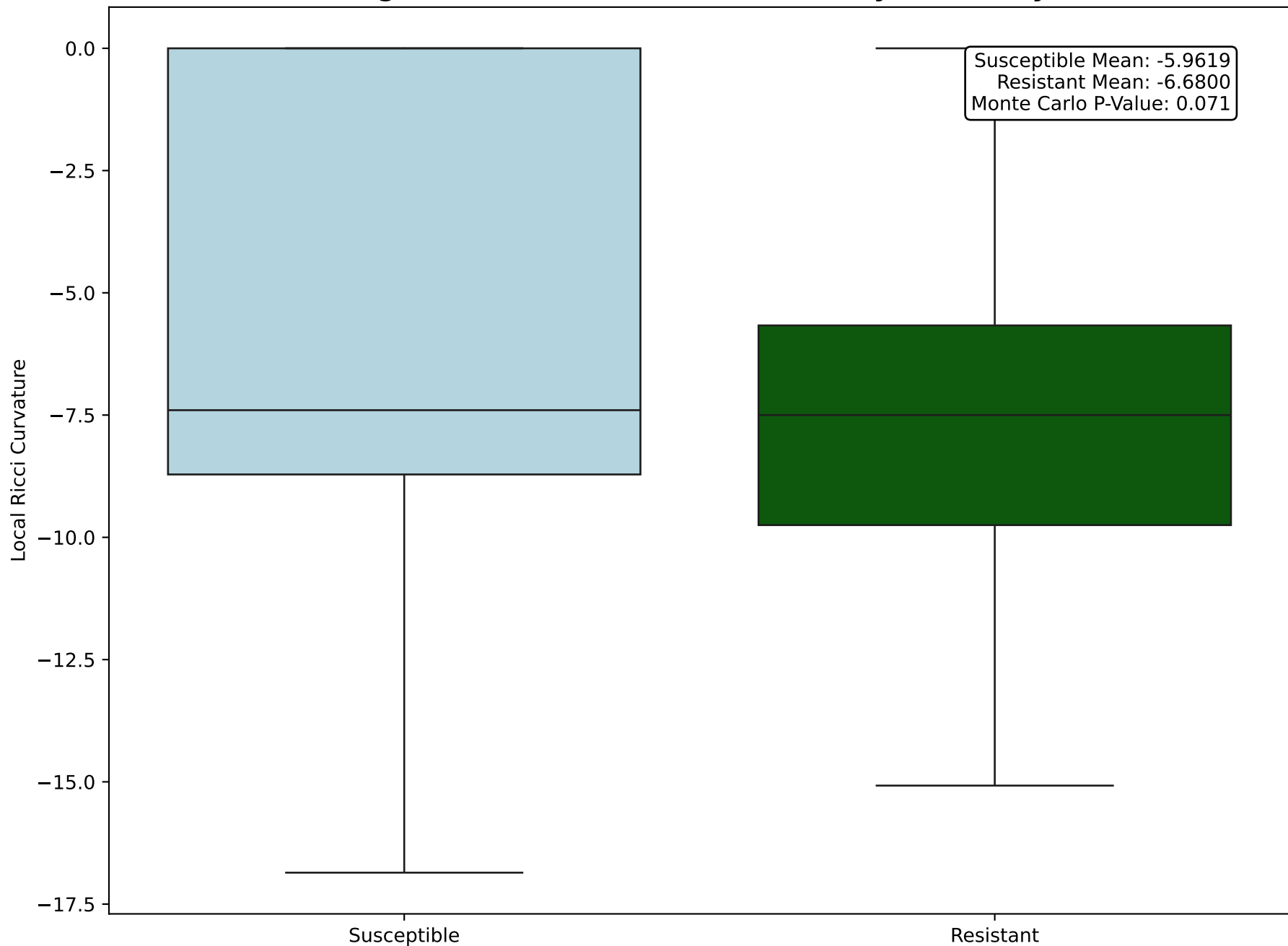


Fig 3: Susceptible plants (Blue) inhabit regions of lower (more negative) curvature.
This supports the hypothesis that bottlenecks act as refugia.