

Module 5

1. Find $\text{curl}(\vec{F})$, where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$.
2. Find the directional derivative of $f(x, y, z) = x^2yz + 4xz^2$ at the point $p(1, -2, -1)$ in the direction of the vector $2\hat{i} - \hat{j} - \hat{k}$.
3. Show that $\text{curl}(\vec{\nabla}f) = 0$, where $f(x, y, z) = x^2y + 2xy + z^2$.
4. Show that $\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find the scalar function ϕ , such that $\vec{A} = \vec{\nabla}\phi$.
5. Find the angle between the surfaces $x^3 + y^3 + z^3 - 3xyz = 5$ and $x^2y + y^2z + z^2x - 5xyz = 8$ at the point $(1, 0, 1)$.
6. If $r = |\vec{r}|$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, then prove that $\vec{\nabla}(r^n) = n r^{n-2} \vec{r}$.
7. Find the equation of tangent plane and normal line to the surface $xyz = 4$ at the point $(1, 2, 2)$.
8. Show that $\vec{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Also find scalar function ϕ such that $\vec{A} = \vec{\nabla}\phi$.
9. Verify Green's theorem for $\vec{F} = (xy + y^2)\hat{i} + x^2\hat{j}$ where the curve C is bounded by $y = x$ and $y = x^2$.
10. Find a unit normal to the surface $x^2 - y^2 + z = 2$ at $(1, -1, 2)$.
11. In what direction from the point $(1, 1, -1)$ is the directional derivative of $f = x^2 - 2y^2 + 4z^2$ a maximum? What is the magnitude of this directional derivative?
12. Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$.
13. Find the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at $(2, 1, 3)$ in the direction $\vec{i} - 2\vec{k}$.
14. Find the angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point $(1, -2, 1)$.
15. If $u = x + y + z$, $v = x^2 + y^2 + z^2$ and $w = xy + yz + zx$, prove that $\text{grad } u$, $\text{grad } v$, and $\text{grad } w$ are coplanar.
16. Show that $\text{curl}(\text{grad}(f)) = 0$ where $f = x^2y + 2xy + z^2$.
17. If $r = |\vec{r}|$ where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ prove that
 - (i) $\vec{\nabla}\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$
 - (ii) $\vec{\nabla}(r^n) = nr^{n-2}\vec{r}$
18. State Green's theorem. Verify Green's theorem in the plane for $\oint_C [(xy + y^2)dx + x^2dy]$ where C is bounded by $y = x$ and $y = x^2$.
19. Find $\text{div } \vec{F}$ and $\text{curl } \vec{F}$ where $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$.

20. Find the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at the point $(2, 1, 3)$ in the direction of the vector $\hat{i} - 2\hat{k}$.
21. Find the angle between the surfaces $xy^2z = 3x + z^2$ and $3x^2 - y^2 + 2z = 1$ at the point $(1, -2, 1)$.
22. Find the equation of the tangent plane and the normal line to the surface $2x^2 + y^2 + 2z = 3$ at the point $(2, 1, -3)$.
23. Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$ around the rectangle bounded by $x = \pm a, y = 0, y = b$.