

### Module 3

1. Test the convergence of the series  $\frac{1^2 \cdot 2^2}{1!} + \frac{2^2 \cdot 3^2}{2!} + \frac{3^2 \cdot 4^2}{3!} + \dots \infty$

2. Define conditional convergent and absolute convergent. Discuss the convergence of the series

$\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n^2 + 1}$ . Is it absolutely convergent?

3. State Leibnitz theorem and apply it to examine the convergence of  $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$ .

4. State D'Alembert's Ratio test for infinite series of positive terms. Discuss the convergence of the series  $\sum_{n=1}^{\infty} n^4 e^{-n^2}$

5. Examine the convergence of the infinite series  $\left(\frac{1}{3}\right)^2 + \left(\frac{1 \cdot 2}{3 \cdot 5}\right)^2 + \left(\frac{1 \cdot 2 \cdot 3}{3 \cdot 5 \cdot 7}\right)^2 + \dots \infty$

6. Test the convergence of the infinite series  $\frac{6}{1 \cdot 3 \cdot 5} + \frac{8}{3 \cdot 5 \cdot 7} + \frac{10}{5 \cdot 7 \cdot 9} + \dots \infty$

7. Test the conditional convergence of the alternating series  $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots \infty$

8. Examine the convergence of the infinite series  $1 + \frac{1}{2} \cdot \frac{1}{3} + \frac{1 \cdot 3}{2 \cdot 4} \cdot \frac{1}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} \cdot \frac{1}{7} + \dots \infty$

9. Examine the convergence of the infinite series

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{n^p}$$

10. Examine the convergence and divergence of the series  $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \dots \infty$ .

11. State Leibnitz's test for convergence of an alternating series. Prove that the series

$x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \infty$  is absolutely convergent when  $|x| < 1$  and conditionally convergent when  $x=1$

12. Test the convergence of the series  $1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} \dots$  to  $\infty$ , ( $x > 0$ )

13. Discuss the convergence of the series  $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^2 + 1}$

14. Test the convergence of the series  $4x + \frac{4 \cdot 7}{1 \cdot 2} x^2 + \frac{4 \cdot 7 \cdot 10}{1 \cdot 2 \cdot 3} x^3 + \dots$  to  $\infty$ , ( $x > 0$ )

15. State Leibnitz's test for the convergence of an alternating series. Show that the series  $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{(n+1)} \cdot \frac{x^n}{n} + \dots \infty$  is absolutely convergent if  $|x| < 1$  and is conditional convergent when  $x=1$ .

16. For what values of  $x$  the following series is convergent?

$$\frac{x}{1.3} + \frac{x^2}{3.5} + \frac{x^3}{5.7} + \dots$$

17. Test the series for convergence of the series

i)  $\frac{1^p}{2^q} + \frac{2^p}{3^q} + \frac{3^p}{4^q} + \dots$ .

ii)  $1 + \frac{2^2}{3^2}x + \frac{2^2 \cdot 4^2}{3^2 \cdot 5^2}x^2 + \frac{2^2 \cdot 4^2 \cdot 6^2}{3^2 \cdot 5^2 \cdot 7^2}x^3 + \dots (x \neq 1)$ .

iii)  $\left(\frac{1}{3}\right)^2 + \left(\frac{1.2}{3.5}\right)^2 + \left(\frac{1.2.3}{3.5.7}\right)^2 + \dots \infty$ .

iv)  $\sin\left(\frac{1}{1^{3/2}}\right) + \sin\left(\frac{1}{2^{3/2}}\right) + \sin\left(\frac{1}{3^{3/2}}\right) + \sin\left(\frac{1}{4^{3/2}}\right) + \dots \infty$ .

v)  $\left(\frac{2^2}{1^2} - \frac{2}{1}\right)^{-1} + \left(\frac{3^3}{2^3} - \frac{3}{2}\right)^{-2} + \left(\frac{4^4}{3^4} - \frac{4}{3}\right)^{-3} + \dots \infty$ .

vi)  $\frac{\sqrt{1}}{a \cdot 1^{3/2} + b} + \frac{\sqrt{2}}{a \cdot 2^{3/2} + b} + \frac{\sqrt{3}}{a \cdot 3^{3/2} + b} + \dots$  where  $a > 0$ .

vii)  $\frac{1}{1^p} - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \dots \infty$ .