Module 1

1. Find the rank and nullity of the following matrices

i) $\begin{bmatrix} -1 & 2 & -1 & 0 \\ 2 & 4 & 4 & 2 \\ 0 & 0 & 1 & 5 \\ 1 & 6 & 3 & 2 \end{bmatrix}$ ii) $\begin{bmatrix} 0 & 0 & 5 & -3 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{bmatrix}$ iii) $\begin{bmatrix} 0 & 0 & 2 & 2 & 0 \\ 1 & 3 & 2 & 4 & 1 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{bmatrix}$

iv) $\begin{bmatrix} 2 & 4 & 8 & 12 & 8 \\ 1 & 2 & 4 & 6 & 4 \\ 2 & 2 & 2 & 2 & 2 \\ -1 & 0 & 2 & 4 & 2 \end{bmatrix}$ v) $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ vi) $\begin{bmatrix} 1 & 3 & 2 & 4 & 1 \\ 0 & 0 & 2 & 2 & 0 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{bmatrix}$

2. Find the values of μ for which the rank of the following matrix is 2

$$\begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 5 & 3 & \mu \\ 1 & 1 & 6 & \mu + 1 \end{pmatrix}$$

3. Check the consistency of the following system of equations

$$x + y + z = 1$$
$$2x + y + 2z = 2$$
$$3x + 2y + 3z = 5$$

4. Examine whether the following system is consistent

$$x + 2y = 5$$
$$2x + 5y = 11$$
$$3x + 7y = 17$$

5. Check the consistency of the following system of equations and then solve if possible

$$x - y + 2z = 6,$$
$$3x + y + z = 4,$$
$$x + 2y - z = -3$$

6. Check the consistency of the following system of equations and solve if possible

$$x + y + z = 1$$
$$2x + y + 2z = 1$$
$$x + 2y + 3z = 0$$

7. Solve the following homogeneous system

$$x + 2y - z = 0$$
$$2x + y - 2z = 0$$

8. For what values of k the following equations

$$x + y + z = 1$$
$$2x + y + 4z = k$$
$$4x + y + 10z = k^2$$

have solutions and solve them completely in each case.

9. Investigate for what values of λ and μ the equations
   x + y + z = 6,
   x + 2y + 4z = 10,
   2x + 3y + λz = μ
   have (1) no solution, (2) a unique solution, and (3) infinitely many solutions.

10. Determine the values of a and b so that the system of equations
$$2x + 3y + 4z = 9$$
$$x - 2y + az = 5$$
$$3x + 4y + 7z = b$$
have i) unique solution, ii) many solutions, and iii) no solution

11. Determine the conditions under which the system of equations
$$x + y + z = 1$$
$$x + 2y - z = b$$
$$5x + 7y + az = b^2$$
admits a) Unique solution b) No solution c) Many solution.

12. Find the eigenvalues and eigen vector of the following matrices

i) $\begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$ ii) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ iii) $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -2 & 4 \\ 3 & -3 & 6 \end{bmatrix}$ iv) $\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

v) $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ vi) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ vii) $\begin{pmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{pmatrix}$

13. Find eigen values of A^{-1} and A^5 where $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$

14. If $A = \begin{bmatrix} 0 & 0 & 1 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$, using Cayley-Hamilton theorem find A^{-1}

15. If $A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$, then using Cayley- Hamilton theorem find A^{-1} and A^8.

16. State Cayley-Hamilton theorem. Verify the theorem for the matrix $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Hence find A^{-1} and A^{50}.

17. Verify that the matrix $A = \begin{pmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{pmatrix}$ satisfies its own characteristic equation.

18. Find the inverse of the following matrices by finding the characteristic equation
$$\begin{pmatrix} 1 & 2 & 1 \\ -1 & 0 & 3 \\ 2 & -1 & 1 \end{pmatrix}$$

20. Show that the matrix $A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$ is diagonalizable. Find the matrix P such that $P^{-1}AP$ is a diagonal matrix.

19. If possible diagonalise the matrix $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$. Find the matrix which diagonalise this. Verify your result.

20. If possible diagonalise the matrix $\begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$. Find the matrix which diagonalise this. Verify your result.