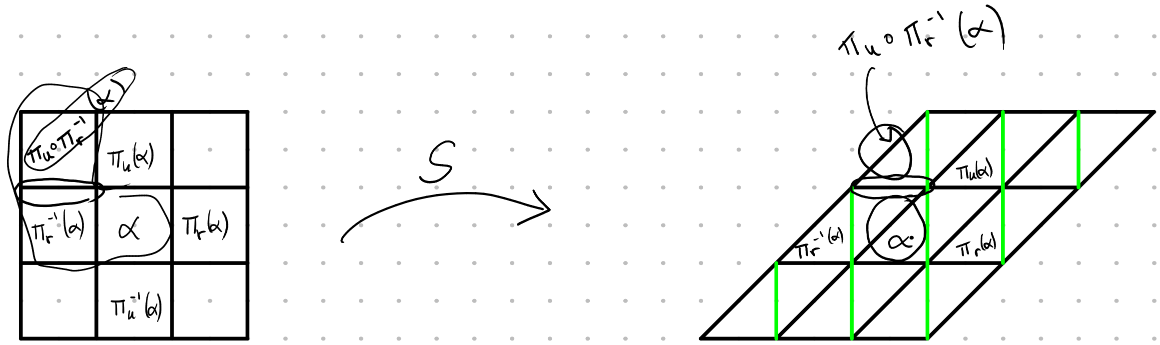


We adopt the convention that the bottom left of a sheared square inherits the label of the original square.

Clearly, the right edge of new α is glued to the new $\pi_r(\alpha)$.

What about the upper edge?

Say the new perms a $\tilde{\pi}$

$\tilde{\pi}_r(\alpha) = \pi_r(\alpha)$

$$\tilde{\pi}_u(\alpha) = \pi_u \circ \pi_r^{-1}(\alpha)$$

Algorithm for factoring elements of $SL_2(\mathbb{Z})$.

Let $\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2(\mathbb{Z})$

Recall

$$R = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad S^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

We want to multiply a sequence of R, S, & S^{-1} to the left of $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ until we get to $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Consider what each of the matrices do to $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

$$R \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} -c & -d \\ a & b \end{pmatrix}$$

$$S \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c+a, & d+b \end{pmatrix}$$

$$S^{-1} \cdot \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c-a & d-b \end{pmatrix}$$

Consider the first column $\begin{pmatrix} a \\ c \end{pmatrix}$. Can we bring it to the form $\begin{pmatrix} a' \\ 0 \end{pmatrix}$?

Special case ; IF $c = na$

IF $a = nc$

In general: $\boxed{c = na + r}$ $0 \leq r < a$

$$\begin{pmatrix} a \\ r \end{pmatrix} \xrightarrow{\text{mult by } R} \begin{pmatrix} -r \\ a \end{pmatrix} \xrightarrow{\text{division again}}$$

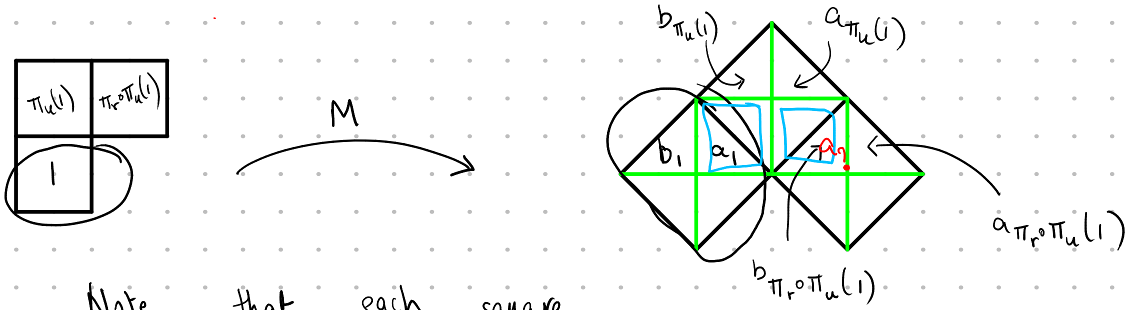
Now that we've brought the first column to the form $\begin{pmatrix} a' \\ 0 \end{pmatrix}$, the matrix looks like $\underbrace{\begin{pmatrix} a' & b' \\ 0 & d' \end{pmatrix}}_{\in SL_2(\mathbb{Z})}$

What can a' & d' be?

both 1 or -1.

$$\underbrace{\begin{pmatrix}1 & b'\\ 0 & 1\end{pmatrix}}_{(S^{-1})^{b'}} \quad \text{or} \quad \underbrace{\begin{pmatrix}-1 & b'\\ 0 & -1\end{pmatrix}}_{(S)^{b}} \longrightarrow \begin{pmatrix}-1 & 0\\ 0 & -1\end{pmatrix} \xrightarrow{R^2} \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

Understanding the action of $M = \underbrace{\begin{pmatrix}1 & 1\\ -1 & 1\end{pmatrix}}$ on n-origamis.



Note that each square becomes two squares. If the original square is labelled i, then we will label the resulting two squares as a_i & b_i.

Clearly, one permutation is easy to determine.

$$\boxed{\tilde{\pi}_r(b_i) = a_i}$$

What about $\tilde{\pi}_r(a_i)$?

From the picture, we can tell

$$\tilde{\pi}_r(a_i) = b_{\pi_r \circ \pi_u(i)}$$

The task of determining $\tilde{\pi}_u$ is an exercise.