ASSIGNMENT - 2

① Obtain the dual of the following LPP

$Z_{max} = 3x_1 + 4x_2$

Subject to $2x_1 + 6x_2 \leq 16$ w_1

$5x_1 + 2x_2 \geq 20$ w_2

$x_1, x_2 \geq 0$

Sol^n We have $-5x_1 - 2x_2 \leq -20$.

So, $Z_{min} = 16w_1 - 20w_2$

subject to

$2w_1 - 5w_2 \geq 3$

$6w_1 - 2w_2 \geq 4$ $w_1, w_2 \geq 0$

② Solve the following LPP using dual simplex method

$Z_{min} = 3x_1 + x_2$

subject to $x_1 + x_2 \geq 1$

$2x_1 + 3x_2 \geq 2$

$x_1, x_2 \geq 0$

Sol^n $Z_{max} = -3x_1 - x_2$ subject to $-x_1 - x_2 \leq -1$

$-2x_1 - 3x_2 \leq -2$

converting to equations

$-x_1 - x_2 + U_1 = -1$

$-2x_1 - 3x_2 + U_2 = -2$

$Z_{max} = -3x_1 - x_2 + 0U_2 + 0U_1$

Basis	CB	x_1	x_2 ↓ EV	U_1	U_2	B	
U_1	0	-1	-1	1	0	-1	
LV ← U_2	0	-2	$\boxed{-3}$	0	1	-2	← PR
C_j		-3	-1	0	0		

Z_j		0	0	0	0	0
$C_j - Z_j$		-3	-1	0	0	0
PR		-2	-3	0	1	
Ratio		3/2	1/3	0	0	
			↑ PC			
					↓ EV	
LV ← u_1	0	-1/3	0	1	$\boxed{-1/3}$	-1/3 ← PR
x_2	-1	2/3	1	0	-1/3	2/3
C_j		-3	-1	0	0	
Z_j		-2/3	-1	0	1/3	
$C_j - Z_j$		-7/3	0	0	-1/3	
PR		-1/3	0	1	-1/3	
Ratio		7	0	0	1	
					↑ PC	
u_2	0	1	0	-3	1	1
x_2	-1	1	1	-1	0	1
C_j		-3	-1	0	0	
Z_j		-1	-1	1	0	
$C_j - Z_j$		-2	0	-1	0	

$x_1 = 0$ and $x_2 = 1$ $\quad Z_{max} = -3(0) - 1 = -1$

(3) Obtain the dual of the following primal problem

minimize $Z = 3x_1 - 2x_2 + x_3$

subject to $2x_1 - 3x_2 + x_3 \leq 5$ $\quad w_1$

$4x_1 - 2x_2 \geq 9$

$-8x_1 + 4x_2 + 3x_3 = 8$

$x_1, x_2 \geq 0$, x_3 is unrestricted.

Sol^n Here min so, $\quad -2x_1 + 3x_2 - x_3 \geq -5 \quad w_1$

$4x_1 - 2x_2 \geq 9 \quad w_2$

$$-8x_1 + 4x_2 + 3x_3 \geq 8 \quad w_3$$
$$8x_1 - 4x_2 - 3x_3 \geq -8 \quad w_4$$

So, $Z_{max} = -5w_1 + 9w_2 + 8w_3 - 8w_4$
subject to
$$-2w_1 + 4w_2 - 8w_3 + 8w_4 \leq 3$$
$$3w_1 - 2w_2 + 4w_3 - 4w_4 \leq -2$$
$$-w_1 + 3w_3 - 3w_4 \leq 1$$

(4) Use dual simplex method to maximize
$Z = -3x_1 - 2x_2$
subject to $x_1 + x_2 \geq 1$
$x_1 + x_2 \leq 7$
$x_1 + 2x_2 \geq 10$
$x_2 \leq 3$
$x_1, x_2 \geq 0$

Sol^n Constraints $-x_1 - x_2 \leq -1$
$-x_1 - 2x_2 \leq -10$

So, $Z_{max} = -3x_1 - 2x_2 + 0U_1 + 0U_2 + 0U_3 + 0U_4$
$-x_1 - x_2 + U_1 = -1$
$x_1 + x_2 + U_2 = 7$
$-x_1 - 2x_2 + U_3 = -10$
$x_2 + U_4 = 3$

Basis	C_B	x_1	x_2 ↓EV	U_1	U_2	U_3	U_4	B	
U_1	0	-1	-1	1	0	0	0	-1	
U_2	0	1	1	0	1	0	0	7	
LV ← U_3	0	-1	$\boxed{-2}$	0	0	1	0	-10	← PR
U_4	0	0	1	0	0	0	1	3	
C_j		-3	-2	0	0	0	0		

				0	0	0	0	
		0	0	0	0	0	0	
z_j		-3	-2	0	0	0	0	
$c_j - z_j$		-1	-2	0	0	1	0	
PR		3	1	0	0	0	0	
Ratio			↑ PC EV					

		$-1/2$	0	1	0	$-1/2$	0	4
U_1	0	$1/2$	0	0	1	$1/2$	0	2
U_2	0	$1/2$	1	0	0	$-1/2$	0	5
x_2	-2	$\boxed{-1/2}$	0	0	0	$1/2$	1	-2 ← PR
LV U_4	0	-3	-2	0	0	0	0	
c_j		-1	-2	0	0	2	0	
z_j		-2	0	0	0	-1	0	
$c_j - z_j$		$-1/2$	0	0	0	$1/2$	1	
PR		4	0	0	0	-2	0	
Ratio		↑ PC						

U_1	0	0	0	1	0	-1	-1	6
U_2	0	0	0	0	1	1	1	0
x_2	-2	0	1	0	0	0	1	3
x_1	-3	1	0	0	0	-1	-2	4
c_j		-3	-2	0	0	0	0	
z_j		-3	-2	0	0	3	4	
$c_j - z_j$		0	0	0	0	-3	-4	

$x_1 = 4$ $x_2 = 3$ So $Z_{max} = -3(4) - 2(3)$
$= -12 - 6$
$= -18$

(5) Solve the following LPP using dual simplex method.

$Z_{min} = 2x_1 + x_2$

Subject to $3x_1 + x_2 \geq 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \geq 3$

$x_1, x_2 \geq 0$

Sol^n $Z_{max} = -2x_1 - x_2$ subject to $-3x_1 - x_2 \leq -3$

$-4x_1 - 3x_2 \leq -6$

$-x_1 - 2x_2 \leq -3$

$Z_{max} = -2x_1 - x_2 + 0U_1 + 0U_2 + 0U_3$

$-3x_1 - x_2 + U_1 = -3$

$-4x_1 - 3x_2 + U_2 = -6$

$-x_1 - 2x_2 + U_3 = -3$

↓ EV

Basis	C_B	x_1	x_2	U_1	U_2	U_3	B	
U_1	0	-3	-1	1	0	0	-3	
← U_2 LV	0	-4	$\boxed{-3}$	0	1	0	-6	← PR
U_3	0	-1	-2	0	0	1	-3	
C_j		-2	-1	0	0	0		
Z_j		0	0	0	0	0		
$C_j - Z_j$		-2	-1	0	0	0		
PR		-4	-3	0	1	0		
Ratio		1/2	1/3	0	0	0		

↑ PC

↓ EV

← U_1 LV	0	$\boxed{-5/3}$	0	1	-1/3	0	-1	← PR
x_2	-1	4/3	1	0	-1/3	0	2	
U_3	0	5/3	0	0	-2/3	1	1	
C_j		-2	-1	0	0	0		
Z_j		-4/3	-1	0	1/3	0		
$C_j - Z_j$		-2/3	0	0	-1/3	0		
PR		-5/3	0	1	-1/3	0		

Scanned by CamScanner

Ratio		2/5	-	0	1	-	
		↑ PC					
x_1	-2	1	0	-3/5	1/5	0	3/5
x_2	-1	0	1	4/5	-3/5	0	6/5
U_3	0	0	0	1	-1	1	0
c_j		-2	-1	0	0	0	
z_j		-2	-1	2/5	1/5	0	
$c_j - z_j$		0	0	-2/5	-1/5	0	

$x_1 = 3/5 \quad x_2 = 6/5 \quad Z_{min} = 2\left(\frac{3}{5}\right) + \left(\frac{6}{5}\right)$

$= 12/5$

(6)

2	3	11	7	6
1	0	6	1	1
5	8	15	9	10
7	5	3	2	

Obtain the initial basic feasible solution to the transportation table shown above, the element of the matrix indicate cost. Using North west corner rule and least cost method.

Sol^n North-West Corner Rule.

	D1	D2	D3	D4	↓B
O1	[6] 2	3	11	7	6/0
O2	[1] 1	0	6	1	1
O3	5	[5] 8	[3] 15	[2] 9	10/5/2/0
D →	7/1/0	5/0	3/0	2/0	

Origin	Destination	No. of units	Cost
O1	D1	6	12

02	D1	1	1
03	D2	5	40
03	D3	3	45
03	D4	2	18
			116/-

least cost method.

	D1	D2	D3	D4	S↓
01	[6] 2	3	11	7	6/0
02	1	[1] 0	6	1	1/0
03	[1] 5	[4] 8	[3] 15	[2] 9	10/9/5/3/0
D→	7/1/0	5/4/0	3/0	2/0	

Origin	Destination	No of units	Cost
01	D1	6	12
02	D2	0	0
03	D1	1	5
03	D2	4	32
03	D3	3	45
03	D4	2	18
			112/-

(7) Obtain the initial basic feasible solution to the transportation table shown above, the element of the matrix indicate cost. Using row minima and penalty method.

Sol^n Row Minima

[6] 2	3	11	7	6/0
1	[1] 0	6	1	1/0
[1] 5	[4] 8	[3] 15	[2] 9	10/9/5/3/0
7/1/0	5/4/0	3/0	2/0	

Scanned by CamScanner

$$\text{Total Cost} = 6\times2 + 1\times0 + 5\times1 + 4\times8 + 3\times15 + 9\times2$$
$$= 12+5+32+45+18$$
$$= 112$$

Penaulty Method

[1] 2	[5] 3	11	7	6/1/0	1	1	5 ←
1	0	6	[1] 1	1/0	1		
[6] 5	8	[3] 15	[1] 9	10/4/1/0	3	3	4
7/6/0	5/0	3/0	2/1/0				
1	2	5	6				
3	5	4	↑				
3	↑	4					

$$\text{Total Cost} = 2\times1 + 5\times3 + 1\times1 + 6\times5 + 3\times15 + 1\times9$$
$$= 2+15+1+30+45+9$$
$$= 102$$

(8) Solve the following transportation problem in which col entries represent the unit costs (in lakhs of rupees) of transportation with usual notations.

2	7	4	5
3	3	1	8
5	4	7	7
1	6	2	14
7	9	18	

Sol^n

[5] 2	7	4	5/0	2 ←		
3	3	[8] 1	8/0	2 ←		
5	[3] 4	7	7/0	1	1	1
[2] 1	[2] 6	[10] 2	14/4/2/0	1	1	5 ←
7/2/0	9/2/0	18/10/0				
1	1	1				
2	1	1				
4	2	5 ↑				
4	2					

$$Cost = 5\times2 + 8\times1 + 7\times4 + 2\times1 + 6\times2 + 10\times2$$
$$= 10 + 8 + 28 + 2 + 12 + 20$$
$$= 80$$

Optimality check

$m+n-1 = 4+3-1 = 6 =$ no. of allocations

	$V_1=1$	$V_2=6$	$V_3=2$	
$U_1=1$	[5] 2	7	4	5
$U_2=-1$	3	3	[8] 1	8
$U_3=-2$	5	[3] 4	7	7
$U_4=0$	[2] 1	[2] 6	[10] 2	14
	7	9	18	

let $U_4=0$ $\quad U_4+V_1=1$

$V_1=1$

$U_4+V_2=6 \quad V_2=6$

$U_4+V_3=2 \quad V_3=2$

$V_2=6 \quad U_3+V_2=4 \quad U_3=-2$

$V_1=1 \quad U_1+V_1=2 \quad U_1=1$

$V_3=2 \quad U_2+V_3=1 \quad U_2=-1$

$C_{ij}-(u_i+v_j)$

$C_{12} = 7-(6+1) = 0$ $C_{22} = 3-(6-1) = -2$ $C_{31} = 5-(-2+1) = 6$

$C_{13} = 4-(2+1) = 1$ $C_{21} = 3-(-1+1) = 3$ $C_{33} = 7-(-2+2) = 7$

[5] 2	7	4	5
3	[x] 3	[8-x] 1	8
5	[7] 4	7	7
[2] 1	[2-x] 6	[10+x] 2	14
7	9	18	

let $x-2=0$ $x-8=0$

$x=8$ $x=2$

↑ min

Put $x=2$. $V_1=1$ $V_2=6$ $V_3=2$

$U_1=1$	[5] 2	7	4	5
$U_2=-3$	3	[2] 3	[6] 1	8
$U_3=-2$	5	[7] 4	7	7
$U_4=0$	[2] 1	6	[12] 2	14
	7	9	18	

$C_{ij}-(U_i+V_j)$

$C_{12} = 7-(1+6) = 0$ $C_{21} = 3-(-3+1) = 5$ $C_{33} = 7-(-2+2) = 7$

$C_{13} = 4-(2+1) = 1$ $C_{31} = 5-(1-2) = 6$ $C_{42} = 6-(6+0) = 0$

Since $C_{ij}-(U_i+V_j) \geq 0$

The solution is optimum

Optimum cost $= 5\times2 + 3\times2 + 6\times1 + 7\times4 + 2\times1 + 12\times2$

$= 10+6+6+28+2+24$

$= 22+54 = 76$

(9)

2	3	11	7	6
1	0	6	1	1
5	8	15	9	10
7	6	3	2	

Obtain the initial basic feasible solution to the transportation table shown above using vogel's approximation method.

[1] 2	[5] 3	11	7	6/1/0	1	1	5 ←
1	0	6	[1] 1	1/0	1		
[6] 5	8	[3] 15	[1] 9	10/4/3/0	3	3	4
7/6/0	5/0	3/0	2/1/0				
1	3	5	6				
3	5	4	↑ 2				
3	↑	4	2				

Cost $= 2\times1 + 5\times3 + 1\times1 + 6\times5 + 3\times15 + 1\times9$

$= 2 + 15 + 1 + 30 + 45 + 9$

$= 18 + 75 + 9 = 102$

(10) A company is spending Rs 1000 everyday on transportation of its units from three plants to four distribution centres. The supply and demand units with unit cost of transportation are given as

Distribution Centre

Plant	D1	D2	D3	D4	Capacity
P1	19	30	50	12	7 ↓
P2	70	30	40	60	10
P3	40	10	60	20	18
demand	5 →	8	7	15	

Sol^n

[5] 19	30	50	[2] 12	7/2/0	7	18	38	38
70	30	[7] 40	[3] 60	10/3/0	10	10	20	20
40	[8] 10	60	[10] 20	18/10/0	10	10	40 ←	
5/0	8/0	7/0	15/5/3/0					
21	20	10	8					
↑	20	10	8					
	↑	10	8					
		10	48 ↑					

	V1=27	V2=10	V3=0	V4=20	
U1=-8	[5] 19	20	50	[2] 12	7
U2=40	70	30	[7] 40	[3] 60	10
U3=0	40	[8] 10	60	[10] 20	18
	5	8	7	15	

Let U3 = 0   U3 + V1 = 40   V1 = 40

U3 + V4 = 20   V4 = 20

V4 = 20   U2 + V4 = 60   U2 = 40

U2 = 40   U2 + V3 = 40   V3 = 0

V4 = 20   U1 + V4 = 12   U1 = -8

U1 = -8   U1 + V1 = 19   V1 = 27

$C_{ij} - (U_i + V_j)$

$C_{12} = 30 - (-8 + 10) = 28$ $C_{21} = 70 - (40 + 27) = 3$

$C_{13} = 50 - (-8 + 0) = 58$ $C_{22} = 30 - (40 + 10) = -20$

$C_{31} = 40 - (0 + 27) = 13$ $C_{33} = 60 - (0 + 0) = 60$

[5] 19	30	50	[2] 12	7
70	[x] 30	[7] 40	[3 - x] 60	10
40	[8 - x] 10	60	[10 + x] 20	18
5	8	7	15	

$3 - x = 0$ $8 - x = 0$

$\underline{x = 3}$

	V1=27	V2=10	V3=20	V4=20	
U1=-8	[5] 19	30	50	[2] 12	7
U2=20	70	[3] 30	[7] 40	60	10
U3=0	40	[5] 10	60	[13] 20	18
	5	8	7	15	

let $U_3=0$ $U_3+V_2=10$ $V_2=10$

$U_3+V_4=20$ $V_4=20$

$V_2=10$ $U_2+V_2=30$ $U_2=20$

$U_2=20$ $U_2+V_3=40$ $V_3=20$

$V_4=20$ $U_1+V_4=12$ $U_1=-8$

$C_{ij}-(U_i+V_j)$

$C_{12}=30-(-8+10)=28$ $C_{21}=70-(20+27)=23$

$C_{13}=50-(-8+20)=38$ $C_{24}=60-(20+20)=20$

$C_{31}=40-(0+27)=13$ $C_{33}=60-(0+20)=40$

Since $C_{ij}-(U_i+V_j)\geq 0$ for all values. Sol^n is optimum

optimum cost $= 5\times19+2\times12+3\times30+7\times40+5\times10+13\times20$

$= 95+24+90+280+50+260$

$= 799$

(ii) A company has 3 car manufacturing factories located in cities C_1, C_2, C_3 which can supply cars to 4 show rooms located in towns T_1, T_2, T_3, T_4. Each plant can supply 6, 1 and 10 truckload of cars daily respectively. The daily requirements of showrooms are 7, 5, 3 and 2 truck loads respectively. The transportation cost per truckload of cars (in thousands of rupees) from each factory to each show room are as follows. Find optimum distribution schedule & cost

2	3	4	7
1	0	6	1
5	8	15	9

[1] 2	[5] 3	11	7	6/1/0	1	1	5 ←
1	0	6	[1] 1	1/0	1		
[6] 5	8	[3] 15	[1] 9	10/4/3/0	3	3	4
7/6/0	5/0	3/0	2/1/0				
1	3	5	6				
3	5	4	2				
3	7	4	2				

Cost = 2×1 + 5×3 + 1×1 + 6×5 + 3×15 + 1×9

= 2+15+1+30+45+9

= 102

	$V_1=5$	$V_2=6$	$V_3=15$	$V_4=9$	
$U_1=-3$	[1] 2	[5] 3	11	7	6
$U_2=-8$	1	0	6	[1] 1	1
$U_3=0$	[6] 5	8	[3] 15	[1] 9	10
	7	5	3	2	

Let $U_3=0$ $\quad V_1+U_3=5 \quad V_1=5$

$V_3+U_3=15 \quad V_3=15$

$V_4+U_3=9 \quad V_4=9$

$V_4=9 \quad V_4+U_2=1 \quad U_2=-8$

$V_1=5 \quad V_1+U_1=2 \quad U_1=-3$

$U_3=-3 \quad V_2+U_3=3 \quad V_2=6$

$C_{13}=11-(-3+15)=-1 \qquad C_{21}=1-(-8+5)=4$

$C_{14}=7-(-3+9)=1 \qquad C_{22}=0-(-8+6)=2$

$C_{23}=6-(-8+15)=-1 \qquad C_{32}=8-(0+6)=2$

[1] 2	[5] 3	[x] 11	7	6
1	0	6	[1] 1	1
[6+x] 5	8	[3-x] 15	[1] 9	10
7	5	3	2	

$1-x=0 \quad 3-x=0$

$x=1$

	$V_1=7$	$V_2=5$	$V_3=3$	$V_4=2$	
$U_1=8$	2	[5] 3	[1] 11	7	6
$U_2=-1$	1	0	6	[1] 1	1
$U_3=0$	[7] 5	8	[2] 15	[1] 9	10
	7	5	3	2	

Scanned by CamScanner