Problems_4_and_5

April 5, 2021

```
In [1]: import numpy as np
        import scipy as sc
        from scipy.stats import f
        from scipy.stats import t
        import matplotlib.pyplot as plt
```

1 *Problem* 4

```
In [2]: data_males = [[15, 17, 24, 14],
                      [17, 15, 32, 26],
                      [15, 14, 29, 23],
                      [13, 12, 10, 16],
                      [20, 17, 26, 28],
                      [15, 21, 26, 21],
                      [15, 13, 26, 22],
                      [13, 5, 22, 22],
                      [14, 7, 30, 17],
                      [17, 15, 30, 27],
                      [17, 17, 26, 20],
                      [17, 20, 28, 24],
                      [15, 15, 29, 24],
                      [18, 19, 32, 28],
                      [18, 18, 31, 27],
                      [15, 14, 26, 21],
                      [18, 17, 33, 26],
                      [10, 14, 19, 17],
                      [18, 21, 30, 29],
                      [18, 21, 34, 26],
                      [13, 17, 30, 24],
                      [16, 16, 16, 16],
                      [11, 15, 25, 23],
                      [16, 13, 26, 16],
                      [16, 13, 23, 21],
                      [18, 18, 34, 24],
                      [16, 15, 28, 27],
                      [15, 16, 29, 24],
                      [18, 19, 32, 23],
              [18, 16, 33, 23],
              [17, 20, 21, 21],
              [19, 19, 30, 28]]

data_females = [[13, 14, 12, 21],
                [14, 12, 14, 26],
                [12, 19, 21, 21],
                [12, 13, 10, 16],
                [11, 20, 16, 16],
                [12, 9, 14, 18],
                [10, 13, 18, 24],
                [10, 8, 13, 23],
                [12, 20, 19, 23],
                [11, 10, 11, 27],
                [12, 18, 25, 25],
                [14, 18, 13, 26],
                [14, 10, 25, 28],
                [13, 16, 23, 28],
                [16, 21, 26, 26],
                [14, 17, 14, 14],
                [16, 16, 15, 23],
                [13, 16, 23, 24],
                [2, 6, 16, 21],
                [14, 16, 22, 26],
                [14, 17, 22, 28],
                [16, 13, 16, 14],
                [15, 14, 20, 26],
                [12, 10, 12, 9],
                [14, 17, 24, 23],
                [13, 15, 18, 20],
                [11, 16, 18, 28],
                [7, 7, 19, 18],
                [12, 15, 7, 28],
                [6, 5, 6, 13]]

data_females = np.asarray(data_females)
data_males = np.asarray(data_males)
# As there are two classes
k = 2
p = data_females.shape[1]

count_females = np.shape(data_females)[0]
count_males = np.shape(data_females)[0]
total_count = count_females + count_males

mean_females = np.mean(data_females, axis = 0)
mean_males = np.mean(data_males, axis = 0)
```

```
        covariance_females = np.cov(data_females.T)
        covariance_males = np.cov(data_males.T)

        pooled_covariance = ((count_females - 1) * covariance_females +
                             (count_males - 1) * covariance_males) / (total_count - k)
```

1.1 (*a*)

```
In [3]: disc_coeff_vector = np.matmul(np.linalg.inv(pooled_covariance),
                                      (mean_males - mean_females))

        print('Required Discriminant Function Coefficient Vector:')
        print(np.ndarray.tolist(np.round(disc_coeff_vector, 4)))
```

```
Required Discriminant Function Coefficient Vector:
[0.5621, -0.2294, 0.4412, -0.2863]
```

1.2 (*b*)

```
In [4]: std_coefficients = np.multiply(np.sqrt(np.diag(pooled_covariance)),
                                       disc_coeff_vector)
        print('Standardised Coefficients:')
        print(np.ndarray.tolist(np.round(std_coefficients, 4)))
        print()

        print('Absolute Values of Standardised Coefficients:')
        print(np.ndarray.tolist(np.abs(np.round(std_coefficients, 4))))
```

```
Standardised Coefficients:
[1.4958, -0.9133, 2.3669, -1.3436]

Absolute Values of Standardised Coefficients:
[1.4958, 0.9133, 2.3669, 1.3436]
```

1.3 (*c*)

```
In [5]: # Computing Student's Two Sample t-test statistics
        t_test_stats = []

        for variable in range(p):
          numerator = mean_males[variable] - mean_females[variable]
          pooled_sd = np.sqrt((covariance_males[variable, variable] +
                               covariance_females[variable, variable]) / 2)
          denominator = pooled_sd * np.sqrt(2 / count_males)
          t_test_stats.append(numerator / denominator)

        t_test_stats = np.asarray(t_test_stats)
        print('Two Sample t-test Statistics for the ' + str(p) + ' variables:')
        print(np.ndarray.tolist(np.round(t_test_stats, 4)))
```

```
Two Sample t-test Statistics for the 4 variables:
[5.5334, 1.822, 7.3063, 0.5364]
```

1.4 (*d*)

```
In [6]: print('Ranking variables in order of their contribution to separating of the groups.')
        print('  Based on:')
        print('  Two Sample t-test Statistics = '
              + str(1 + np.argsort(-1 * np.abs(t_test_stats))))
        print('  Standardized Coefficient = '
              + str(1 + np.argsort(-1 * np.abs(std_coefficients))))
```

```
Ranking variables in order of their contribution to separating of the groups.
  Based on:
  Two Sample t-test Statistics = [3 1 2 4]
  Standardized Coefficient = [3 1 4 2]
```

The same rankings are obtained for the two significant variables based on the Two Sample t-test Statistics and standardized coefficient values.

1.5 (*e*)

```
In [7]: def partial_hotelling_two_sample_T_square(neglect = -1):
          # neglect can take values 0, 1, ..., (p - 1)
          # if neglect = -1, consider complete two sample T square
          global pooled_covariance
          global mean_females, mean_males
          global count_females, count_males
          global p

          indices = np.asarray([index for index in range(p) if index != neglect])
          cov = []

          for row in range(p):
            if row == neglect:
              continue
            cov.append([])
            for col in range(p):
              if col == neglect:
                continue
              cov[-1].append(pooled_covariance[row, col])

          cov = np.asmatrix(cov)
          mean_1 = mean_males[indices]
          mean_2 = mean_females[indices]
          difference = mean_1 - mean_2

          factor = (mean_males * mean_females) / (mean_males + mean_females)
          T_square = np.matmul(np.matmul(difference.T, np.linalg.inv(cov)), difference)

          return np.ravel(T_square)[0]

        complete_T_square = partial_hotelling_two_sample_T_square()
        nu = count_females + count_males - k
        factor = nu - p + 1
        alpha = 0.05

        critical_partial_F = f.isf(alpha, 1, factor)
        print('Critical Partial F value = ' + str(np.round(critical_partial_F, 4)))
        print()

        partial_F_s = []
        for drop in range(p):
          T_square_partial = partial_hotelling_two_sample_T_square(drop)
          F = factor * (complete_T_square - T_square_partial) / (nu + T_square_partial)
          partial_F_s.append(F)
          print('p = ' + str(drop + 1) + ': ')
          print('  Computed Partial F value = '  + str(np.round(F, 4)))
          print('  Standardized Coefficient = '  + str(np.round(std_coefficients[drop], 4)))
          print()

        partial_F_s = np.asarray(partial_F_s)

        print('Ranking variables in order of their contribution to separating the groups.')
        print('  Based on:')
        print('  Computed Partial F value = '
              + str(1 + np.argsort(-1 * np.abs(partial_F_s))))
        print('  Standardized Coefficient = '
              + str(1 + np.argsort(-1 * np.abs(std_coefficients))))
```

```
Critical Partial F value = 4.0162

p = 1:
  Computed Partial F value = 1.189
  Standardized Coefficient = 1.4958

p = 2:
  Computed Partial F value = 0.4386
  Standardized Coefficient = -0.9133

p = 3:
  Computed Partial F value = 3.2927
  Standardized Coefficient = 2.3669

p = 4:
  Computed Partial F value = 1.0822
  Standardized Coefficient = -1.3436

Ranking variables in order of their contribution to separating the groups.
  Based on:
  Computed Partial F value = [3 1 4 2]
  Standardized Coefficient = [3 1 4 2]
```

The same individual variable rankings are obtained based on Partial F values and the standardized coefficient values.

2 *Problem* 5

```
In [8]: method_1 = [[5.4, 6.0, 6.3, 6.7],
                     [5.2, 6.2, 6.0, 5.8],
                     [6.1, 5.9, 6.0, 7.0],
                     [4.8, 5.0, 4.9, 5.0],
                     [5.0, 5.7, 5.0, 6.5],
                     [5.7, 6.1, 6.0, 6.6],
                     [6.0, 6.0, 5.8, 6.0],
                     [4.0, 5.0, 4.0, 5.0],
                     [5.7, 5.4, 4.9, 5.0],
                     [5.6, 5.2, 5.4, 5.8],
                     [5.8, 6.1, 5.2, 6.4],
                     [5.3, 5.9, 5.8, 6.0]]

        method_2 = [[5.0, 5.3, 5.3, 6.5],
                     [4.8, 4.9, 4.2, 5.6],
                     [3.9, 4.0, 4.4, 5.0],
                     [4.0, 5.1, 4.8, 5.8],
                     [5.6, 5.4, 5.1, 6.2],
                     [6.0, 5.5, 5.7, 6.0],
                     [5.2, 4.8, 5.4, 6.0],
                     [5.3, 5.1, 5.8, 6.4],
                     [5.9, 6.1, 5.7, 6.0],
                     [6.1, 6.0, 6.1, 6.2],
                     [6.2, 5.7, 5.9, 6.0],
                     [5.1, 4.9, 5.3, 4.8]]

        method_3 = [[4.8, 5.0, 6.5, 7.0],
                     [5.4, 5.0, 6.0, 6.4],
                     [4.9, 5.1, 5.9, 6.5],
                     [5.7, 5.2, 6.4, 6.4],
                [4.2, 4.6, 5.3, 6.3],
                [6.0, 5.3, 5.8, 5.4],
                [5.1, 5.2, 6.2, 6.5],
                [4.8, 4.6, 5.7, 5.7],
                [5.3, 5.4, 6.8, 6.6],
                [4.6, 4.4, 5.7, 5.6],
                [4.5, 4.0, 5.0, 5.9],
                [4.4, 4.2, 5.6, 5.5]]

method_1 = np.asarray(method_1)
method_2 = np.asarray(method_2)
method_3 = np.asarray(method_3)
methods = [method_1, method_2, method_3]

all_data = np.vstack(methods)

# As there are three methods
k = 3
n = methods[0].shape[0]
p = methods[0].shape[1]
N = all_data.shape[0]

total_mean = np.mean(all_data, 0)
means = []

for method in methods:
  means.append(np.mean(method, 0))
```

```
In [9]: # Computing between Sum of Squares
        H = np.zeros((p, p))
        for mean in means:
          unit = np.asmatrix(mean - total_mean)
          H += np.matmul(unit.T, unit)

        H = n * H
```

```
In [10]: # Computing within Sum of Squares
         E = np.zeros((p, p))

         for i in range(k):
           for j in range(n):
             unit = np.asmatrix(methods[i][j] - means[i])
             E += np.matmul(unit.T, unit)

         consider = np.matmul(np.linalg.inv(E), H)
```

2.1 (*a*)

```
In [11]: # Computing eigenvectors of inverse(E).H

         eigen_values, eigen_vectors = np.linalg.eig(consider)
         positions = np.argsort(-1 * eigen_values)

         print('Eigen Vectors in Decreasing order of importance:')
         print()

         for vector_index, index in enumerate(positions):
           print('Eigen Vector ' + str(vector_index + 1) + ' = ', end = '')
           print(np.ndarray.tolist(np.round(eigen_vectors[:, index], 4)))
```

```
Eigen Vectors in Decreasing order of importance:

Eigen Vector 1 = [-0.013, 0.8274, -0.5496, -0.1149]
Eigen Vector 2 = [0.6057, -0.5857, -0.5088, 0.1767]
Eigen Vector 3 = [-0.2557, 0.0156, -0.4918, 0.8322]
Eigen Vector 4 = [0.8065, -0.3107, -0.0554, 0.4999]
```

2.2 (*b*)

```
In [12]: # Tests of significance

         sorted = eigen_values[positions]

         def test_statistic_V_m(m = 0):
           global N, p, k
           global sorted

           factor = N - 1 - (p + k) / 2
           sum = 0
           for index in range(m, p):
             sum += np.log(1 + sorted[index])

           return factor * sum

         # Computing relative importance of each eigen vector based on V_m test statistic
         print('Contribution to the separation based on V_m test statistic of:')
         for index in range(p):
           print('  Eigen Vector ' + str(index + 1) + ' = ', end = '')
           print(np.abs(np.round(test_statistic_V_m(index), 4)))
```

```
Contribution to the separation based on V_m test statistic of:
  Eigen Vector 1 = 45.4099
  Eigen Vector 2 = 3.9405
  Eigen Vector 3 = 0.0
  Eigen Vector 4 = 0.0
```

```
In [13]: # Computing relative importance of each eigen vector
         # based on weighted average of eigen values

         sum_eigen_values = np.sum(eigen_values)

         print('Contribution to the separation based on eigen value spectrum of:')
         for vector_index, index in enumerate(positions):
           print('  Eigen Vector ' + str(vector_index + 1) + ' = ', end = '')
           print(np.abs(np.round(eigen_values[index] / sum_eigen_values, 4)))
```

```
Contribution to the separation based on eigen value spectrum of:
  Eigen Vector 1 = 0.9535
  Eigen Vector 2 = 0.0465
  Eigen Vector 3 = 0.0
  Eigen Vector 4 = 0.0
```

- *The two procedures agree on the number of important discriminant functions.*
- *The relative importance of the functions as discerned based on the two methods are also similar.*

2.3 (*c*)

```
In [14]: principal_eigen_vector = eigen_vectors[:, positions[0]]
         print('Discriminant Function Coefficient Vector:')
         print(np.ndarray.tolist(np.round(principal_eigen_vector, 4)))
         print()

         S_pl = E / (N - k)
         std_coefficients = np.multiply(np.sqrt(np.diag(S_pl)), principal_eigen_vector)
         print('Standardised Coefficients:')
         print(np.ndarray.tolist(np.round(std_coefficients, 4)))
         print()

         print('Absolute Values of Standardised Coefficients:')
         print(np.ndarray.tolist(np.abs(np.round(std_coefficients, 4))))
```

```
Discriminant Function Coefficient Vector:
[-0.013, 0.8274, -0.5496, -0.1149]

Standardised Coefficients:
[-0.0083, 0.41, -0.326, -0.0668]

Absolute Values of Standardised Coefficients:
[0.0083, 0.41, 0.326, 0.0668]
```

Based on the absolute values of the standardized coefficients, we conclude that y_2, and y_3 contribute significantly to the separation of groups but y_1 and y_4 do not.

In [15]:
```
def partial_Wilk_s_two_sample_T_square(neglect = -1):
  # neglect can take values 0, 1, ..., (p - 1)
  # if neglect = -1, consider complete two sample T square
  global H, E

  if neglect == -1:
    return np.linalg.det(E) / np.linalg.det(H + E)

  indices = np.asarray([index for index in range(p) if index != neglect])
  H_consider = []
  E_consider = []

  for row in range(p):
    if row == neglect:
      continue
    H_consider.append([])
    E_consider.append([])

    for col in range(p):
      if col == neglect:
        continue
      H_consider[-1].append(H[row, col])
      E_consider[-1].append(E[row, col])

  H_consider = np.asmatrix(H_consider)
  E_consider = np.asmatrix(E_consider)

  wilk_s = np.linalg.det(E_consider) / np.linalg.det(H_consider + E_consider)

  return wilk_s
```

2.4 (*d*)

In [16]:
```
nu_E = N - k
nu_H = k - 1
alpha = 0.05
factor = (nu_E - p + 1) / nu_H

critical_partial_F = f.isf(alpha, nu_H, nu_E - p + 1)
print('Critical Partial F value = ' + str(np.round(critical_partial_F, 4)))
print()

wilk_s_total = partial_Wilk_s_two_sample_T_square()

partial_F_s = []
```

```
        for drop in range(p):
          wilk_s = partial_Wilk_s_two_sample_T_square(drop) / wilk_s_total
          F = factor * (1 - wilk_s) / wilk_s
          partial_F_s.append(F)

          print('p = ' + str(drop + 1) + ': ')
          print('  Computed Partial F value = '  + str(np.round(F, 4)))
          print('  Standardized Coefficient = '  + str(np.round(std_coefficients[drop], 4)))
          print()

        partial_F_s = np.asarray(partial_F_s)

        print('Ranking variables in order of their contribution to separating the groups.')
        print('Based on:')
        print('  Computed Partial F value = '
              + str(1 + np.argsort(-1 * np.abs(partial_F_s))))
        print('  Standardized Coefficient = '
              + str(1 + np.argsort(-1 * np.abs(std_coefficients))))
```

```
Critical Partial F value = 3.3158

p = 1:
  Computed Partial F value = -1.0668
  Standardized Coefficient = -0.0083

p = 2:
  Computed Partial F value = -8.3064
  Standardized Coefficient = 0.41

p = 3:
  Computed Partial F value = -6.1863
  Standardized Coefficient = -0.326

p = 4:
  Computed Partial F value = -0.483
  Standardized Coefficient = -0.0668

Ranking variables in order of their contribution to separating the groups.
Based on:
  Computed Partial F value = [2 3 1 4]
  Standardized Coefficient = [2 3 4 1]
```

Partial F values rank the two worthy variables i.e. y_2*, and* y_3 *in the same order as standardized coefficitents.*

2.5 (e)

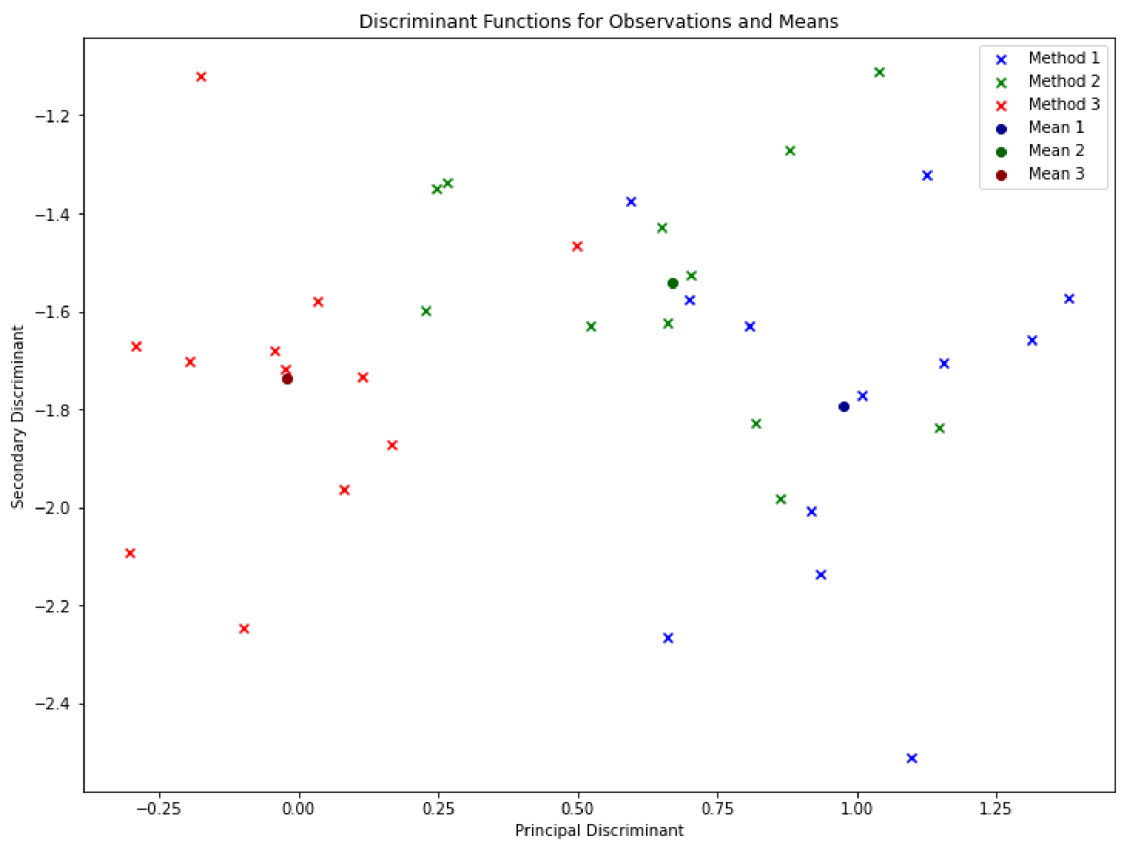
```
In [17]: # Constructing the Plots
         x = [[], [], [], []]
```

```
y = [[], [], [], []]

for index in range(N):
  unit = all_data[index]
  x[int(index / n)].append(np.dot(eigen_vectors[:, positions[0]], unit))
  y[int(index / n)].append(np.dot(eigen_vectors[:, positions[1]], unit))

for index in range(k):
  unit = means[index]
  x[k].append(np.dot(eigen_vectors[:, positions[0]], unit))
  y[k].append(np.dot(eigen_vectors[:, positions[1]], unit))

fig = plt.figure(figsize = (12, 9))
p = fig.add_subplot('111')
p.set_title('Discriminant Functions for Observations and Means')
p.set_xlabel('Principal Discriminant')
p.set_ylabel('Secondary Discriminant')
p.scatter(x[0], y[0], color = 'blue', label = 'Method 1', marker = 'x')
p.scatter(x[1], y[1], color = 'green', label = 'Method 2', marker = 'x')
p.scatter(x[2], y[2], color = 'red', label = 'Method 3', marker = 'x')
p.scatter(x[3][0], y[3][0], color = 'darkblue', label = 'Mean 1')
p.scatter(x[3][1], y[3][1], color = 'darkgreen', label = 'Mean 2')
p.scatter(x[3][2], y[3][2], color = 'darkred', label = 'Mean 3')
p.legend()
plt.show()
```



```
In [18]: # ^_^ Thank You
```