

A4 — Analysis of Variance

1. Test the hypothesis $H_0: \mu_1 = \mu_2 = \dots = \mu_5$ at the 0.05 level of significance for the data given on mean absorption of moisture in concrete by 5 different concrete aggregates. Six observations are made in each sample, and the 5 times 6 equal to 30 observations are given in the table below.

551	595	639	417	563
457	580	615	449	631
450	508	511	517	522
731	583	573	438	613
499	633	648	415	656
632	517	677	555	679

2. An experiment was carried out over a long period on apple trees of different rootstocks. The data for eight trees from each of the six rootstocks are given below. The variables are:

- y1: trunk girth at 4 years (mm x 100)
- y2: extension growth at 4 years (m)
- y3: trunk girth at 15 years (mm x 100)
- y4: weight of tree above ground level at 15 years (lb x 1000)

1.11 2.569 3.58 0.760 (1)

1.19 2.928 3.75 0.821

1.09 2.865 3.93 0.928

1.25 3.844 3.94 1.009

1.11 3.027 3.60 0.766

1.08 2.336 3.51 0.726

1.11 3.211 3.98 1.209

1.16 3.037 3.62 0.750

1.05 2.074 4.09 1.036 (2)

1.17 2.885 4.06 1.094

1.11 3.378 4.87 1.635

1.25 3.906 4.98 1.517

1.17 2.782 4.38 1.197

1.15 3.018 4.65 1.244

1.17 3.383 4.69 1.495

1.19 3.447 4.40 1.026

1.07 2.505 3.76 0.912 (3)

0.99 2.315 4.44 1.398

1.06 2.667 4.38 1.197

1.02 2.390 4.67 1.613

1.15 3.021 4.48 1.476

1.20 3.085 4.78 1.571

1.20 3.308 4.57 1.506

1.17 3.231 4.56 1.458

1.22 2.838 3.89 0.944 (4)

1.03 2.351 4.05 1.241

1.14 3.001 4.05 1.023

1.01 2.439 3.92 1.067

0.99 2.199 3.27 0.693

1.11 3.318 3.95 1.085

1.20 3.601 4.27 1.242

1.08 3.291 3.85 1.017

0.91 1.532 4.04 1.084 (5)

1.15 2.552 4.16 1.151

1.14 3.083 4.79 1.381

1.05 2.330 4.42 1.242

0.99 2.079 3.47 0.673

1.22 3.366 4.41 1.137

1.05 2.416 4.64 1.455

1.13 3.100 4.57 1.325

1.11 2.813 3.76 0.800 (6)

0.75 0.840 3.14 0.606

1.05 2.199 3.75 0.790

1.02 2.132 3.99 0.853

1.05 1.949 3.34 0.610

1.07 2.251 3.21 0.562

1.13 3.064 3.63 0.707

1.11 2.469 3.95 0.952

a) Test $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6$

b) Test the following two orthogonal contrasts

2	-1	-1	-1	-1	2
1	0	0	0	0	-1

c) Test the 4 individual variables using the 0.05 level of significance.

3. An experiment was performed to determine whether two forms of iron (Fe^{2+} and Fe^{3+}) are retained differently. The investigators divided 108 mice randomly into 6 groups of 18 each; 3 groups were given Fe^{2+} in three different concentrations, 10.2, 1.2, and 0.3 millimolar, and 3 groups were given Fe^{3+} at the same three concentrations.

The mice were given the iron orally; the iron was radioactively labeled so that a counter could be used to measure the initial amount given. At a later time, another count was taken for each mouse, and the percentage of iron retained was calculated. The data for the two forms of iron are listed in the table below.

Fe ³⁺			Fe ²⁺		
10.2	1.2	.3	10.2	1.2	.3
.71	2.20	2.25	2.20	4.04	2.71
1.66	2.93	3.93	2.69	4.16	5.43
2.01	3.08	5.08	3.54	4.42	6.38
2.16	3.49	5.82	3.75	4.93	6.38
2.42	4.11	5.84	3.83	5.49	8.32
2.42	4.95	6.89	4.08	5.77	9.04
2.56	5.16	8.50	4.27	5.86	9.56
2.60	5.54	8.56	4.53	6.28	10.01
3.31	5.68	9.44	5.32	6.97	10.08
3.64	6.25	10.52	6.18	7.06	10.62
3.74	7.25	13.46	6.22	7.78	13.80
3.74	7.90	13.57	6.33	9.23	15.99
4.39	8.85	14.76	6.97	9.34	17.90
4.50	11.96	16.41	6.97	9.91	18.25
5.07	15.54	16.96	7.52	13.46	19.32
5.26	15.89	17.56	8.36	18.4	19.87
8.15	18.3	22.82	11.65	23.89	21.60
8.24	18.59	29.13	12.45	26.39	22.25

We will use a logarithmic transformation of the data to make it nearly normal. Perform two way ANOVA on the transformed data.

4. In an experiment, a two-way classification of measurements is made on bar steel. The experiment involved a 2×4 design with 4 replications, for a total of 32 observation vectors. The factors were rotational velocity (A1-fast, A2-slow) and four types of lubricants (B1, B2, B3, B4). The experimental units were 32 homogeneous pieces of bar steel. Two variables were measured on each piece of bar steel:

x_1 : ultimate torque

x_2 : ultimate strain

	A1		A2	
B1	7.80	90.4	7.12	85.1
	7.10	88.9	7.06	89.0
	7.89	85.9	7.45	75.9
	7.82	88.8	7.45	77.9
B2	9.00	82.50	8.19	66.0
	8.43	92.40	8.25	74.5
	7.65	82.40	7.45	83.1
	7.70	87.40	7.45	86.4
B3	7.28	79.60	7.15	81.2
	8.96	95.10	7.15	72.0
	7.75	90.20	7.70	79.9
	7.80	88.00	7.45	71.9
B4	7.60	94.1	7.06	81.2
	7.00	86.6	7.04	79.9
	7.82	85.9	7.52	86.4
	7.80	88.8	7.70	76.4

Test for:

- (i) The significance of the effect of velocity
- (ii) The significance of lubricants
- (iii) The interaction effect