Introduction to Neural and Cognitive Modelling

Assignment 4

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$Solution\ to\ Part\ 1$

$$m = r(S) + n$$

$$\implies m = 10 + \frac{50}{1 + exp\left(-\frac{S - 0.25}{0.15}\right)} \ + \ n$$

$$where\ p(n) \ = \ \frac{1}{\sqrt{2\pi}(5)}.\, exp\left(- \frac{n^2}{2(5)^2}\right)$$

$$i.e\ p(n) \approx G(\mu = 0,\ \sigma = 5)$$

$We\ have:$

$$\frac{50}{1 + exp\left(-\frac{S - 0.25}{0.15}\right)} \ = \ m \ - \ n \ - \ 10$$

$$\implies 1 \ + \ exp\left(- \frac{S - 0.25}{0.15}\right) \ = \ \frac{50}{m - n - 10}$$

$$\implies exp\left(- \frac{S - 0.25}{0.15}\right) \ = \ \frac{50}{m - n - 10} \ - \ 1$$

$$\implies -\frac{S - 0.25}{0.15} = ln\left(\frac{50}{m - n - 10} \ - \ 1\right)$$

$$\implies S \ = \ \frac{25}{100} \ - \ \frac{15}{100} ln\left(\frac{50}{m - n - 10} \ - \ 1\right)$$

$if\ we\ set\ m \ = \ 58,\ we\ get:$

$$S \ = \ \frac{25}{100} \ - \ \frac{15}{100}.\, ln\left(\frac{50}{48 - n} \ - \ 1\right)$$

$Now,\ S \in \ [0,\ 1]$

$To\ compute\ n\ from\ S:$

$$n \ = \ m \ - \ 10 \ - \ \frac{50}{1 + exp\left(-\frac{S - 0.25}{0.15}\right)}$$

$and\ with\ m\ set\ to\ 58,$

$$n \ = \ 48 \ - \ \frac{50}{1 + exp\left(-\frac{S - 0.25}{0.15}\right)}$$

$We\ have\ a\ Gaussian\ model\ for\ p(n).\ Every\ S\ gives\ us\ a\ unique\ n.\ Thus,\ we\ can\ estimate\ L(S; m)\ as:$

$$\frac{1}{\sqrt{2\pi}(5)}.\, exp\left(\frac{-1}{2(5)^2} \left(m \ - \ 10 \ - \ \frac{50}{1 + exp\left(-\frac{S - 0.25}{0.15}\right)} \right)^2 \right)$$

$-\ We\ use\ the\ above\ relation\ to\ obtain\ L(S; 58)$

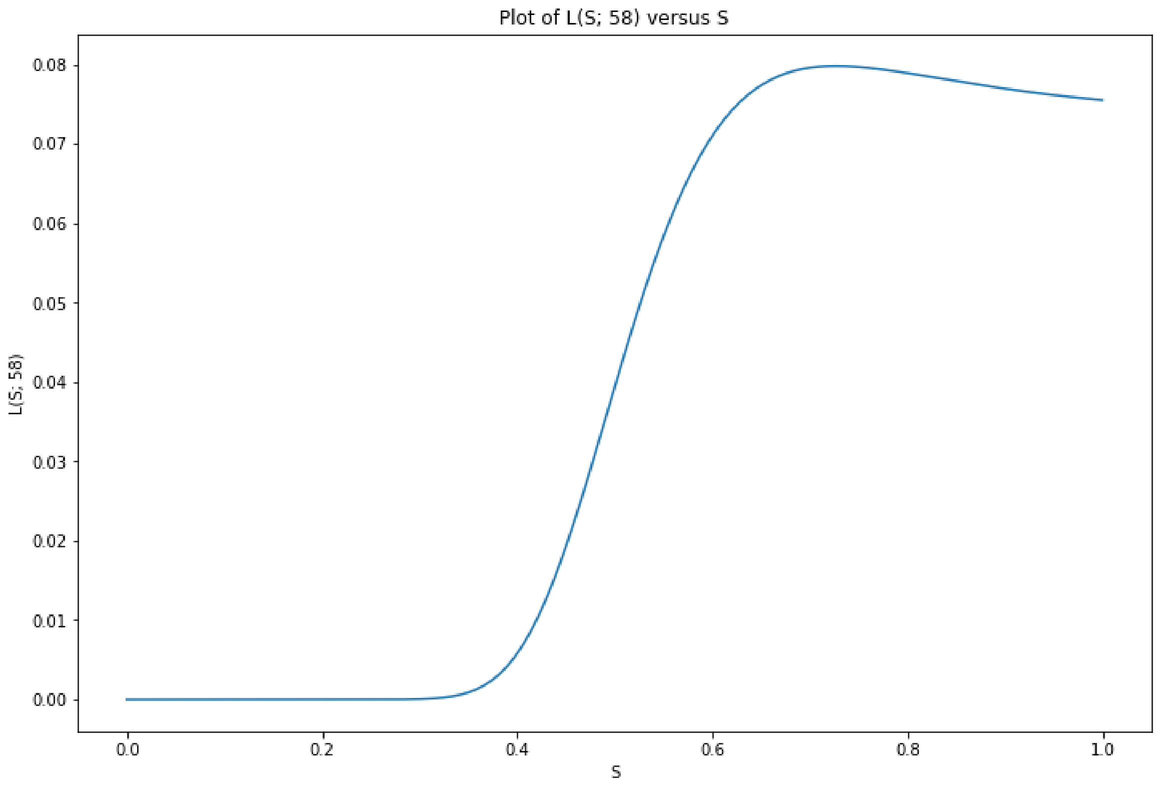
In [1]:
```python
import numpy as np
import matplotlib.pyplot as plt

size = 10000
# Number of values used for plotting and estimations

def L(s, m):
    # Estimate the Likelihood L(S; m)
    calc = np.power((m - 10 - 50 / (1 + np.exp(- (s - 0.25) / 0.15))), 2) * (-1 / 50)
    return (1 / (5 * np.sqrt(2 * np.pi))) * np.exp(calc)

S_values = [(i / size) for i in range(size)]
L_values = [L(s, 58) for s in S_values]

fig = plt.figure(figsize=(12,8))
plt.xlabel('S')
plt.ylabel('L(S; 58)')
plt.title("Plot of L(S; 58) versus S")
plt.plot(S_values, L_values)
plt.show()
```



Solution to Part 2

$For\ ease\ of\ representation,\ let\ us\ define\ a\ few\ terms:$

- $A = \frac{-1}{2(5)^2}\left(m - 10 - \frac{50}{1 + exp\left(-\frac{S - 0.25}{0.15}\right)}\right)^2$
- $B = \left(m - 10 - \frac{50}{1 + exp\left(-\frac{S - 0.25}{0.15}\right)}\right)$
- $C = 1 + exp(-\frac{S - 0.25}{0.15})$

$To\ estimate\ maximum\ likelihood,\ we\ differentiate\ p(S)\ w.r.t\ \ p:$

$$\therefore\ p'(s) = \frac{1}{\sqrt{2\pi}(5)}.exp(A).\left(\frac{-1}{2(5)^2}\right).2.B.\frac{+50}{C^2}.\left(exp\left(-\frac{S-0.25}{0.15}\right)\right).\frac{-S}{0.15}$$

$Now,\ we\ equate\ p'(s)\ to\ 0\ to\ obtain\ critical\ points:$

$p'(s) = 0$

$\implies B = 0\ or\ S = 0$

$Note\ that:\ B = 0 \implies A = 0$

$And:\ exp(A),\ \frac{50}{C^2},\ exp\left(-\frac{S-0.25}{0.15}\right) \neq 0$

$Now,\ S = 0\ is\ a\ boundary\ point\ and\ L(0;\ 58) \approx 0$

$Hence,$

$B = 0$

$$\implies \left(m - 10 - \frac{50}{1 + exp\left(-\frac{S^* - 0.25}{0.15}\right)}\right) = 0$$

$$\implies \frac{50}{1 + exp\left(-\frac{S^* - 0.25}{0.15}\right)} = m - 10$$

$$\implies 1 + exp(-\frac{S^* - 0.25}{0.15}) = \frac{50}{m-10}$$

$$\implies exp(-\frac{S^* - 0.25}{0.15}) = \frac{50}{m-10} - 1$$

$$\implies -\frac{S^* - 0.25}{0.15} = ln\left(\frac{50}{m-10} - 1\right)$$

$$\implies S^* = -\frac{15}{100}.ln\left(\frac{50}{m-10} - 1\right) + \frac{25}{100}$$

In [2]:
```
def get_S_optima(m):
    return -0.15 * np.log(50/(m - 10) - 1) + 0.25

def get_maximum_likelihood(m):
    return L(get_S_optima(m), m)

best_S = get_S_optima(58)
best_ML = get_maximum_likelihood(58)

print("S* = " + str(best_S))
print("L* = " + str(best_ML))
```

```
S* = 0.7267080745521916
L* = 0.07978845608028655
```

$Setting\, m \;=\; 58:$

$S^* = -\frac{15}{100}.\, ln\left(\frac{50}{48} - 1\right) + \frac{25}{100} = 0.72670807455$

$-\, is\; the\; required\; S\; that\; maximizes\; L(S;\; 58)$

$We\; verify\; using:$

- $Estimating\; from\; the\; plot\; of\; L\; versus\; S$
- $Iteratively\; checking\; computed\; L\; values\; to\; get\; an\; estimate\; of\; which\; S\; maximizes\; L$

In [3]:
```
# Iteratively Checking L values to find which S maximizes L

def get_best_S_approx():
    position = np.argmax(np.asarray(L_values))
    return (position / size)

S_best_approx = get_best_S_approx()

print("An Approximation to S* using computed values: S*_approx = " + str(S_best_approx))
```

```
An Approximation to S* using computed values: S*_approx = 0.7267
```

$-\, Clearly,\; everything\; checks\; out\; and \rightarrow$

$S^* \;=\; 0.7267,\; L^* \;=\; 0.0798\; are\; optimum\; values\; for\; S\; and\; L\; respectively.$

$Thank\;\; You$