Introduction to Neural and Cognitive Modelling

Assignment 4

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$Solution\ to\ Part\ 1$

$m = r(S) + n$

$\implies m = 10 + \frac{50}{1 + exp\left(-\frac{S - 0.25}{0.15}\right)} \ + \ n$

$where\ p(n) \ = \ \frac{1}{\sqrt{2\pi}(5)}.\,exp\left(\ -\ \frac{n^2}{2(5)^2}\right)$

$i.e\ p(n) \approx G(\mu = 0,\ \sigma = 5)$

$We\ have:$

$\frac{50}{1 + exp\left(-\frac{S - 0.25}{0.15}\right)} \ = \ m \ - \ n \ - \ 10$

$\implies 1 \ + \ exp\left(\ -\ \frac{S - 0.25}{0.15}\right) \ = \ \frac{50}{m - n - 10}$

$\implies exp\left(\ -\ \frac{S - 0.25}{0.15}\right) \ = \ \frac{50}{m - n - 10} \ - \ 1$

$\implies -\frac{S - 0.25}{0.15} = ln\left(\frac{50}{m - n - 10} \ - \ 1\right)$

$\implies \ S \ = \ \frac{25}{100} \ - \ \frac{15}{100} ln\left(\frac{50}{m - n - 10} \ - \ 1\right)$

$if\ we\ set\ m \ = \ 58,\ we\ get:$

$S \ = \ \frac{25}{100} \ - \ \frac{15}{100}.\,ln\left(\frac{50}{48 - n} \ - \ 1\right)$

$Now,\ S \in \ [0,\ 1]$

$To\ compute\ n\ from\ S:$

$n \ = \ m \ - \ 10 \ - \ \frac{50}{1 + exp\left(-\frac{S - 0.25}{0.15}\right)}$

$and\ with\ m\ set\ to\ 58,$

$n \ = \ 48 \ - \ \frac{50}{1 + exp\left(-\frac{S - 0.25}{0.15}\right)}$

$We\ have\ a\ Gaussian\ model\ for\ p(n).\ Every\ S\ gives\ us\ a\ unique\ n.\ Thus,\ we\ can\ estimate\ L(S; m)\ as:$

$$\frac{1}{\sqrt{2\pi}(5)}.\,exp\left(\frac{-1}{2(5)^2}\left(m \ - \ 10 \ - \ \frac{50}{1 + exp\left(-\frac{S - 0.25}{0.15}\right)}\right)^2\right)$$

$-\ We\ use\ the\ above\ relation\ to\ obtain\ L(S; 58)$

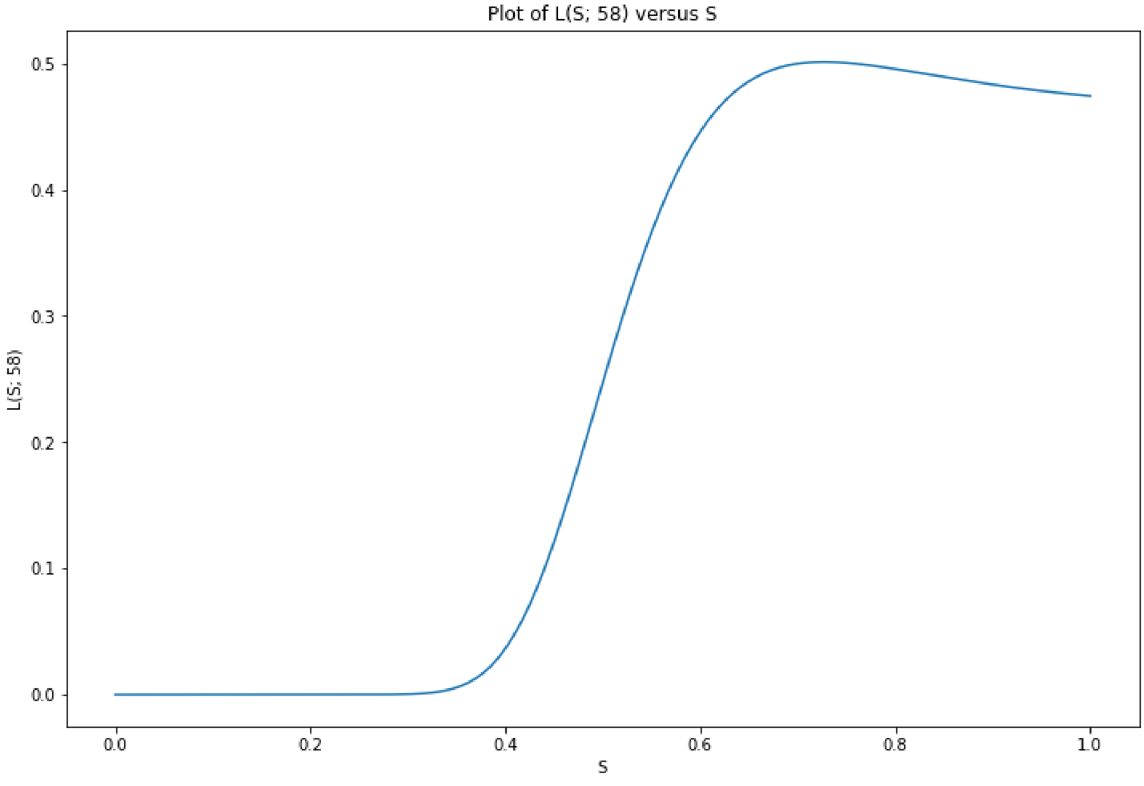
In [1]:
```
import numpy as np
import matplotlib.pyplot as plt

size = 10000
# Number of values used for plotting and estimations

def L(s, m = 58):
    # Estimate the Likelihood L(S; m)
    calc = np.power((m - 10 - 50 / (1 + np.exp(- (s - 0.25) / 0.15))), 2) * (-1 / 50)
    return (1 / 5 * np.sqrt(2 * np.pi)) * np.exp(calc)

S_values = [(i / size) for i in range(size)]
L_values = [L(s) for s in S_values]

fig = plt.figure(figsize=(12,8))
plt.xlabel('S')
plt.ylabel('L(S; 58)')
plt.title("Plot of L(S; 58) versus S")
plt.plot(S_values, L_values)
plt.show()
```



$Solution\ to\ Part\ 2$

$For\ ease\ of\ representation,\ let\ us\ define\ a\ few\ terms:$

- $A \ = \ \frac{-1}{2(5)^2}\left(m \ - \ 10 \ - \ \frac{50}{1 + exp\left(-\frac{S \ - \ 0.25}{0.15}\right)}\right)^2$
- $B \ = \ \left(m \ - \ 10 \ - \ \frac{50}{1 + exp\left(-\frac{S \ - \ 0.25}{0.15}\right)}\right)$
- $C \ = \ 1 \ + \ exp(- \ \frac{S \ - \ 0.25}{0.15})$

$To\ estimate\ maximum\ likelihood,\ we\ differentiate\ p(S)\ w.r.t\ \ p:$

$$\therefore \ p'(s) \ = \ \frac{1}{\sqrt{2\pi}(5)}.exp(A).\left(\frac{-1}{2(5)^2}\right).2.B.\frac{+50}{C^2}.\left(exp\left(- \ \frac{S - 0.25}{0.15}\right)\right).\frac{-S}{0.15}$$

$Now,\ we\ equate\ p'(s)\ to\ 0\ to\ obtain\ critical\ points:$

$p'(s) \ = \ 0$

$\implies \ B \ = \ 0\ or\ S \ = \ 0$

$Note\ that: \ B \ = \ 0 \implies \ A \ = \ 0$

$And: \ exp(A), \ \frac{50}{C^2}, \ exp\left(- \ \frac{S - 0.25}{0.15}\right) \ \neq \ 0$

$Now,\ S = 0\ is\ a\ boundary\ point\ and\ L(0;\ 58) \approx 0$

$Hence,$

$B = 0$

$$\implies \ \left(m \ - \ 10 \ - \ \frac{50}{1 + exp\left(-\frac{S^* \ - \ 0.25}{0.15}\right)}\right) = 0$$

$$\implies \ \frac{50}{1 + exp\left(-\frac{S^* \ - \ 0.25}{0.15}\right)} = m - 10$$

$$\implies \ 1 \ + \ exp(- \ \frac{S^* \ - \ 0.25}{0.15}) \ = \ \frac{50}{m - 10}$$

$$\implies \ exp(- \ \frac{S^* \ - \ 0.25}{0.15}) \ = \ \frac{50}{m - 10} - 1$$

$$\implies \ -\frac{S^* \ - \ 0.25}{0.15} = ln\left(\frac{50}{m - 10} - 1\right)$$

$$\implies \ S^* \ = \ -\frac{15}{100}.ln\left(\frac{50}{m - 10} - 1\right) + \frac{25}{100}$$

In [2]:
```python
def get_S_optima(m):
    return -0.15 * np.log(50/48 - 1) + 0.25

def get_maximum_likelihood(m):
    return L(get_S_optima(m))

best_S = get_S_optima(58)
best_ML = get_maximum_likelihood(58)

print("S* = " + str(best_S))
print("L* = " + str(best_ML))
```

```
S* = 0.7267080745521916
L* = 0.5013256549262001
```

$Setting\, m \;=\; 58:$

$S^{*} = -\frac{15}{100}.\, ln\left(\frac{50}{48} - 1\right) + \frac{25}{100} = 0.72670807455$

$-\, is\ the\ required\ S\ that\ maximizes\ L(S;\ 58)$

$We\ verify\ using:$

- $Estimating\ from\ the\ plot\ of\ L\ versus\ S$
- $Iteratively\ checking\ computed\ L\ values\ to\ get\ an\ estimate\ of\ which\ S\ maximizes\ L$

In [3]:
```
# Iteratively Checking L values to find which S maximizes L

def get_best_S_approx():
    position = np.argmax(np.asarray(L_values))
    return (position / size)

S_best_approx = get_best_S_approx()

print("An Approximation to S* using computed values: S*_approx = " + str(S_best_approx))
```

```
An Approximation to S* using computed values: S*_approx = 0.7267
```

$-\, Clearly\ everything\ checks\ out\ and \rightarrow$

$S^{*} \;=\; 0.727,\ and\ L^{*} \;=\; 0.501\ are\ optimum\ values\ for\ S\ and\ L\ respectively.$

$Thank\ You$