Project 1

Michael Saybolt
Bjon Charlery

May 2, 2017

Abstract

This project explored solving differential equations, specifically linear second order equations, by using a couple of different methods which include LU Decomposition, Gauss Forward and Backward Substitution : General algorithm

1 Introduction

The objective of this project was to solve a differential equation in the form of:

$$-u^{"}(x) = f(x), x\epsilon(0,1), u(0) = u(1) = 0.$$

that can be written into the linear equation ($Ax = b$). Since A is a tridiagonal matrix there is an actual or analytical solution that can be made for this particular matrix. The following sections will describe the origins for the algorithms used and their respective results.

2 Description

This section will detail the methods used to solve the linear equation system. It is solved using two methods:

- LU decomposition
- Tridiagonal Solver - Gauss Elimination

2.1 LU Decomposition Solver

This method consists of creating a Lower-Triangular matrix L and a Upper-Triangular matrix U for factorizing the linear system ($A = LU$).

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 & 0 \\ l_{31} & l_{32} & 1 & 0 & 0 \\ l_{41} & l_{42} & l_{43} & 1 & 0 \\ l_{51} & l_{52} & l_{53} & l_{54} & 1 \end{bmatrix} and \begin{bmatrix} 1 & l_{12} & l_{13} & l_{14} & l_{15} \\ 0 & 1 & l_{23} & l_{24} & l_{25} \\ 0 & 0 & 1 & l_{34} & l_{35} \\ 0 & 0 & 0 & 1 & l_{45} \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Once (L) and (U) are found they can be used to solve the following equation: ($Av = LUv = h^2f$) by allowing ($Ly = h^2f$) and ($Uv = y$) with backwards substitution, in this case that would be required two times.

2.2 Tridiagonal Solver: General Gauss Elimination

This method consists of using Gauss elimination due to the matrix (A) only having values along the three diagonals of the matrix.

$$\begin{bmatrix} b_1 & c_1 & 0 & 0 & ... & ... & 0 \\ a_1 & b_2 & c_2 & 0 & ... & ... & 0 \\ 0 & a_2 & b_3 & c_3 & 0 & ... & 0 \\ \vdots & ... & \ddots & \ddots & \ddots & ... & \vdots \\ \vdots & ... & ... & \ddots & \ddots & \ddots & 0 \\ 0 & ... & ... & 0 & a_{\text{n-1}} & b_{\text{n-1}} & c_{\text{n-1}} \\ 0 & ... & ... & ... & 0 & a_{\text{n}} & b_{\text{n}} \end{bmatrix}$$

The diagonal of the matrix will be changed using the following formula while the "Upper Diagonal" will not be changed:

$$d_{\text{i}} = b_{\text{i}} - a_{\text{i}}c_{\text{i}}/d_{\text{i-1}}$$

The "Lower Diagonal" will be changed so that all elements below the diagonal are zeros. The vector (f) changes with the following formula:

$$w_{\text{i}} = f_{\text{i}} - a_{\text{i}}w_{\text{i-1}}/d_{\text{i-1}}$$

With backwards substitution then can be performed after following:

$$v_{\text{i}} = w_{\text{i-1}} - c_{\text{i-1}}v_{\text{i}}/d_{\text{i-1}}$$

with

$$d_1 = b_1, w_1 = f_1, and v_{\text{n}} = h^2 w_{\text{n}}/d_{\text{n}}$$