Project 3: Ordinary Differential Equations and the Solar System

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Abstract

- Develop code for simulating solar system
- Object orient for modularity
- Multiple solvers for ODE
  - Euler method
  - Velocity Verlet metho
- Test as it is created to isolate bugs before they grow

1 Introduction

- For ease of debugging, a simulation of a simple binary system is created with just the Sun and the Earth

The only force in the problem is gravity. Newton's law of gravitation is given by a force F_G

$$F_G = \frac{GM_\odot M_{\text{Earth}}}{r^2},$$

where $M_\odot$ is the mass of the Sun and M_{Earth} is the mass of the Earth. The gravitational constant is G and r is the distance between the Earth and the Sun. Eventually the other planets will be added, but for now just this system will be used to test the layout of the algorithms.

2 Description

This section will provide details about the general equations describing the sytem, and the general approach of the problem.

2.1 Block tied to wall

Newton's equation of motion for system:

$$m\frac{d^2x}{dt^2} = -kx,$$

aka:

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x = -\omega_0^2 x,$$

with the angular frequency $\omega_0^2 = k/m$.

see line 1116 of ode.tex @Author: mhjensen for more equation

2.2 Euler Method

- Euler
- Enhanced Euler
  - Better numerical stability

2.3 Verlet Method

- Verlet
- Velocity Verlet
  - Method with velocity is studied here

3 Implementation

This section will transform the equations described in the previous section into usable code, and also describe difficulties encountered, and implemented solutions.

For 2D simplification, get 4 coupled DEs:

$$\frac{dv_x}{dt} = -\frac{GM_\odot}{r^3}x,$$

$$\frac{dx}{dt} = v_x,$$

$$\frac{dv_y}{dt} = -\frac{GM_\odot}{r^3}y,$$

$$\frac{dy}{dt} = v_y$$

This would be the same for 1 or 3 dimensions. But,

$$GM_{\odot} = v^2 r,$$

and assuming Earth's velocity is: $v = 2\pi r/\mathrm{yr} = 2\pi \mathrm{AU}/\mathrm{yr}$
Thus can equate the following for the velocity in some direction:

$$\frac{dv_x}{dt} = -\frac{GM_{\odot}}{r^3}x = 4\pi^2\frac{(\mathrm{AU})^3}{\mathrm{yr}^2}$$

3.1 Euler

insert discretized
insert code

3.1.1 Enhanced Euler

- better numerical stability as show in results

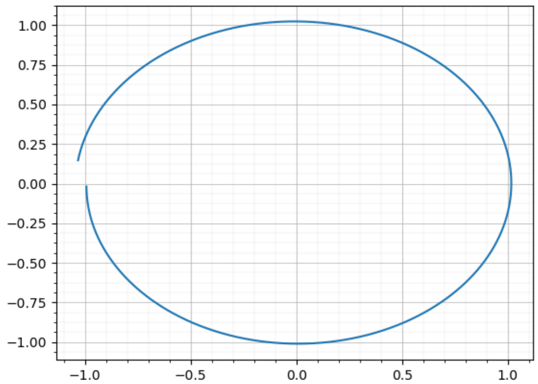
3.2 Velocity Verlet

- As numerically stable as enhanced Euler

4 Results

Results are presented here =.75

4.1 Euler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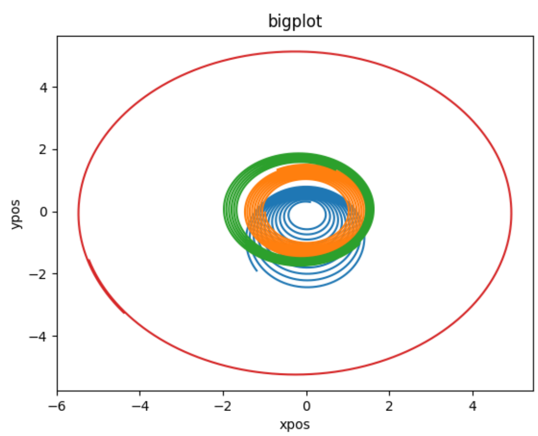
1.0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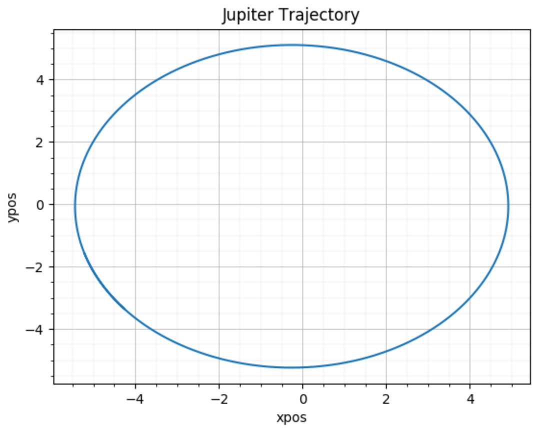
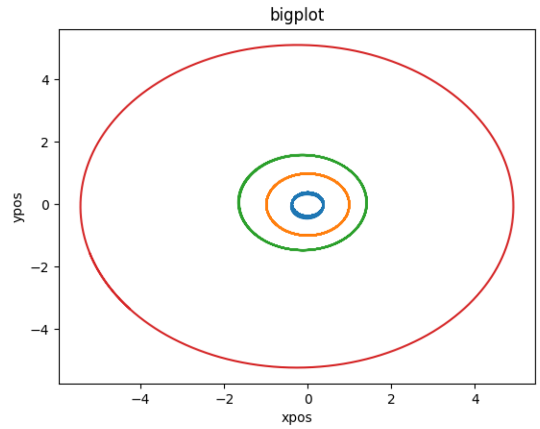
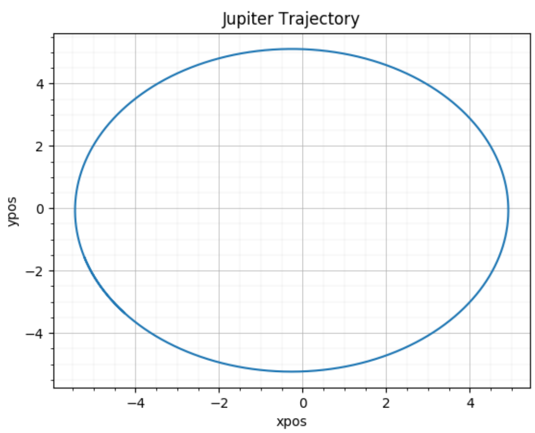
−4
−2
0
2
4
xpos

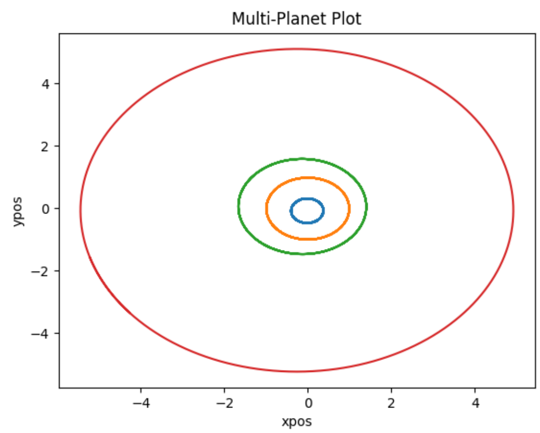
4.2 Enhanced Euler





4.3 Velocity Verlet





4.4 Accuracy

4.5 Gauss General:

4.6 LU Decomposition:

4.7 Speed of Execution