

# A Study on Automatic Driving Technique of Tractor-semitrailer

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**Abstract**—In order to elevate the driving stability of tractor-semitrailer for automated vehicle control, the issue of automated steering and independent braking control of a tractor-semitrailer vehicle system is delivered. The input/output linearization and Lyapunov minimax approach were used and combined in the design of the control. The design was programmed and simulated in Matlab. The results show that in spite of the presence uncertainty, the proposed design achieves lane following and prevents the jack-knifing phenomenon of tractor-semitrailer system.

**Keywords**—vehicle engineering; steering and braking control; style; tractor-semi trailer; driving stability

## I. INTRODUCTION

With the advantages of large loadage, convenient and fast, tractor-semitrailer has been the main truck styles of highway transportation. Automatic driving technique of tractor-semitrailer will reduce labor intensity remarkably and reduce traffic accident induced by fatigue driving. Furthermore, it decrease the cost of transportation enterprises.

Articulated vehicle control is more complicated than light passenger due to the relative motions between the tractor and the trailer. A control design employs the input/output linearization and adaptive backstepping methodology has been proposed in [1]. The control rule for the braking torque includes the use of parameters which are regulated by an on-line adaptation rule. This is mainly for the presence of an uncertain parameter appears in the tire dynamics and it is presumed constant. In another design the Lyapunov minimax approach was used. The uncertainty which is addressed in the setting may be time-varying. The possible bound of the uncertainty is needed for the control design. The resulting system performance can be prescribed and guaranteed. In this paper, A new design was proposed. The input/output linearization methodology is used; The Lyapunov minimax approach is applied for the control law which regulates the braking torque. In addition, the tire dynamics uncertainty can be extended to be time-varying, the design may also help to prevent the unboundedness control issue appeared earlier.

## II. CONSTRUCTION OF THE MODEL

The model of tractor-semitrailer was described in Fig 1. It is assumed that the longitudinal velocity is constant, the lateral and yaw motions are small. The equations of the model can be described as.

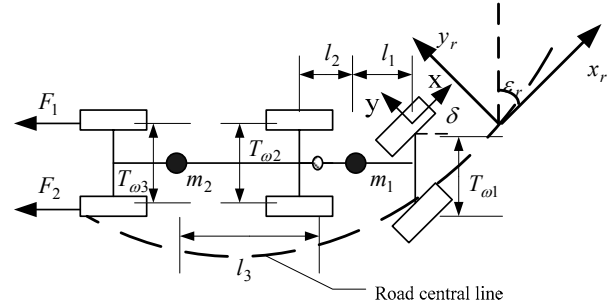


Figure 1 the model of tractor-semitrailer

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} = F_g(q, \dot{q}) \quad (1)$$

Where

$$M(q) = \begin{bmatrix} m_1 + m_2 & -m_2(d_1 + d_3 \cos \epsilon_f) & -m_2 d_3 \cos \epsilon_f \\ -m_2(d_1 + d_3 \cos \epsilon_f) & I_{z1} + I_{z2} + m_2(d_1^2 + d_3^2) & I_{z2} + m_2 d_3 \\ -m_2 d_3 \cos \epsilon_f & I_{z2} + m_2 d_3 & I_{z2} + m_2 d_3 \end{bmatrix}$$

$$C(q, \dot{q}) = \begin{bmatrix} 0 & 0 & m_2 d_3 (\dot{\epsilon} + \dot{\epsilon}_f) \sin \epsilon_f \\ m_2 d_3 \dot{\epsilon}_f \sin \epsilon_f & -m_2 d_1 d_3 \dot{\epsilon}_f \sin \epsilon_f & m_2 d_3 \dot{\epsilon} \cos \epsilon_f \\ -m_2 d_3 \dot{\epsilon} \sin \epsilon_f & m_2 d_1 d_3 \dot{\epsilon} \sin \epsilon_f & -m_2 d_1 d_3 (\dot{\epsilon} + \dot{\epsilon}_f) \sin \epsilon_f \\ & -m_2 d_3 \dot{\epsilon} \cos \epsilon_f & 0 \end{bmatrix}$$

$q$  is the generalized coordinate;  $F_g$  is the generalized force vector;  $\epsilon_f$  is semitrailer yaw angle relative to tractor sprung mass coordinate;  $d_1, d_2, d_3, d_4$  are relative position from tractor or semitrailer C.G. to fifth wheel.

Define  $\dot{q}_r = [\dot{y}_r \ \dot{\epsilon}_r \ \dot{\epsilon}_f]^T$  and  $\dot{q}_e = [-\dot{x}\epsilon_r \ \dot{\epsilon}_d \ 0]^T$ , then

$$M(q_r)(\ddot{q}_r + \ddot{q}_e) + C(q_r, \dot{q}_r + \dot{q}_e)(\dot{q}_r + \dot{q}_e) = F_g(q_r, \dot{q}_r, \dot{q}_e) \quad (2)$$

$F_g$  is the steering and braking control and it is expressed as

$$F_g = [F_y, M_{\sigma}, M_{\delta}]^T = G(q_r)[\delta, F_1, F_2]^T - K(q_r, \dot{q}_r, \dot{\epsilon}_d) \quad (3)$$

$$\text{Where } G(q_r) = \begin{bmatrix} 2C_{\sigma f} \cos \epsilon_r - \sin \epsilon_f \cos \epsilon_r & -\sin \epsilon_f \cos \epsilon_r \\ 2C_{\sigma f} l_1 & \frac{T_{w3}}{2} + d_1 \sin \epsilon_f & -\frac{T_{w3}}{2} + d_1 \sin \epsilon_f \\ 0 & \frac{T_{w3}}{2} & -\frac{T_{w3}}{2} \end{bmatrix}$$

$$\underline{K}(q_r, \dot{q}_r, \dot{\varepsilon}_d) = \begin{bmatrix} 2(C_{af}\zeta_f + C_{ar}\zeta_r) + C_{at}\zeta_t \cos \varepsilon_r \\ 2(C_{af}l_1\zeta_f - C_{ar}l_2\zeta_r - C_{at}(l_3 + d_1 \cos \varepsilon_f))\zeta_t \\ -2C_{at}l_3\zeta_t \end{bmatrix}$$

$\delta$  is the steering angle of tractor front wheel;  $F_1, F_2$  are the longitudinal forces of the trailer wheels;  $\varepsilon_r$  is tractor yaw angle relative to the road centerline coordinate;  $\dot{\varepsilon}_d$  is desired yaw rate of the vehicle at curved section;  $\zeta_f$  is lateral slip angle at tractor front wheel;  $C_{af}, C_{ar}, C_{at}$  are cornering stiffness of tractor front wheel, rear wheel and semitrailer rear wheel.  $\zeta_f, \zeta_r, \zeta_t$  are the lateral slip angle at tractor front wheel, tractor rear wheel and semitrailer rear wheel.

Let  $T = F_1 - F_2$ , then  $F_1 + F_2 = \text{sgn}(T)T$ . so, (3) becomes

$$[F_y, M_{\varphi}, M_{\varphi}]^T = H(q_r, \text{sgn}(T))U - \underline{K}(q_r, \dot{q}_r, \dot{\varepsilon}_d) \quad (4)$$

Where

$$H(q_r, \text{sgn}(T)) = \begin{bmatrix} 2C_{af} \cos(\varepsilon_r) & -\sin(\varepsilon_f) \cos(\varepsilon_r) \text{sgn}(T) \\ 2C_{af}l_1 & \frac{T_{w3}}{2} + d_1 \sin(\varepsilon_f) \text{sgn}(T) \\ 0 & \frac{T_{w3}}{2} \end{bmatrix}, U = \begin{bmatrix} \delta \\ T \end{bmatrix}.$$

Define

$\underline{C}(q_r, \dot{q}_r, \dot{\varepsilon}_d, \ddot{\varepsilon}_d) = M(q_r)\ddot{q}_e + C(q_r, \dot{q}_r + \dot{q}_e)(\dot{q}_r + \dot{q}_e)$  the equation of motion (2) then becomes:

$M(q_r)\ddot{q}_e + \underline{C}(q_r, \dot{q}_r, \dot{\varepsilon}_d, \ddot{\varepsilon}_d) + \underline{K}(q_r, \dot{q}_r, \dot{\varepsilon}_d) = H(q_r, \text{sgn}(T))U$   
 $F_1$  and  $F_2$  are generated by the wheel and tire dynamics, which are represented by

$$I_{\omega}\dot{\omega}_i = -B\omega_i + F_i r - \tau_i \quad (5)$$

$F_i$  can be approximated as

$$F_i = C_{lt}\lambda_i \quad (6)$$

Where  $\lambda_i = \frac{V_{\omega} - \omega_i r}{V_{\omega i}}$ ;  $C_{lt}$  is longitudinal stiffness of

semitrailer rear wheel. Then we know that actually we exert  $\tau_i$  to control the tractor-semitrailer vehicle.

### III. CONTROLLER DESIGN

To render the tractor-semitrailer vehicle practically stable a robust controller is proposed. The control strategy includes two parts. 1. Input/output linearization methodology was used to simplify the error dynamics of the vehicle. 2. A control rule for the braking torque based on the Lyapunov minimax approach is put forward.

$$\text{Define } e_1 = y_r + c\varepsilon_r, e_2 = \varepsilon_f - \varepsilon_f d \quad (7)$$

Where  $c$  is a weighting factor and  $c > 0$   $\varepsilon_{fd}$  is determined by the tractor and trailer wheel base and the radius of the curved section.

$$\text{Let } M(q_r) = \begin{bmatrix} M^{-1}(1) + cM^{-1}(2) \\ M^{-1}(3) \end{bmatrix}$$

Then the error dynamics can be described as:  $[\ddot{e}_1, \ddot{e}_2]^T =$

$$-\underline{M}(q_r)(\underline{C}(q_r, \dot{q}_r, \dot{\varepsilon}_d, \ddot{\varepsilon}_d) + \underline{K}(q_r, \dot{q}_r, \dot{\varepsilon}_d) + \underline{M}(q_r, \text{sgn}(T))U) \quad (8)$$

$$\text{Define: } J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} = M(q_r)H(q_r, \text{sgn}(T))$$

$$U = J^{-1}M(q_r)(C(q_r, \dot{q}_r, \dot{\varepsilon}_d, \ddot{\varepsilon}_d) + K(q_r, \dot{q}_r, \dot{\varepsilon}_d) - J^{-1}(K_D \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} + K_p \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}))$$

So the resulting system is:

$$[\ddot{e}_1, \ddot{e}_2]^T = K_D [\dot{e}_1, \dot{e}_2]^T + K_p [e_1, e_2]^T \quad (9)$$

If  $K_D$  and  $K_p$  are chosen appropriately, both  $e_1$  and  $e_2$  will approach 0 when  $t \rightarrow \infty$ . If  $y_r(t)\varepsilon_r(t) \geq 0, t \in [t_0, \infty)$  come into existence, the lane following control is achieved and the trailer's unstable yaw motion is prevented.

Since the braking force  $F_i$  is not the real control being applied, we have to consider the wheel and tire dynamics. In order to know how  $F_i$  is generated by  $\tau_i$ , we have to ponder the wheel and tire dynamics. Combining (5) and (6), and choose

$$K_D = \begin{bmatrix} K_{D1} & 0 \\ 0 & K_{D2} \end{bmatrix}, K_p = \begin{bmatrix} K_{p1} & 0 \\ 0 & K_{p2} \end{bmatrix} \quad (10)$$

Define

$$\eta = T - T_d(q, \dot{q}, \dot{\varepsilon}_d, \ddot{\varepsilon}_d) \quad (11)$$

And the system becomes:

$$\begin{aligned} \ddot{e}_1 &= -K_{D1}\dot{e}_1 - K_{p1}e_1 - J_{12}\eta \\ \ddot{e}_2 &= -K_{D2}\dot{e}_2 - K_{p2}e_2 - J_{22}\eta \end{aligned} \quad (12)$$

$$\dot{\eta} = C(\frac{\omega_i r}{V_{\omega i}^2} \dot{V}_{\omega i} - \frac{r}{I_{\omega} V_{\omega i}} (-B\omega_i + C_{lt}\lambda_i r - \tau_i)) - \dot{T}_d$$

The tire longitudinal stiffness  $C_{lt}$  depends on the tire pressure, tire baldness, and road conditions. They are always varying. It is, therefore, practical to treat  $C_{lt}$  as an uncertain time-varying parameter  $C_{lt}(t)$ . It is further decomposed as:

$$C_{lt}(t) = C_{lt}^0 + \Delta C_{lt}(t) \quad (13)$$

Where  $C_{lt}^0$  is the nominal value of  $C_{lt}$ , and

$$\Delta C_{lt}^l \leq \Delta C_{lt}(t) \leq \Delta C_{lt}^u \quad (t \geq 0)$$

Because  $C_{lt}$  is strictly positive, so  $C_{lt}^0 + \Delta C_{lt}^l > 0$ . Let

$z^T = [e_1, \dot{e}_1, e_2, \dot{e}_2]^T$ . System (12) becomes:

$$\dot{z} = A_1 z + B_1 \eta$$

$$\dot{\eta} = -a\eta + (B_2^0 + \Delta B_2(\Delta C_{lt}))\tau_i + \Delta f(\eta, \Delta C_{lt}, q, \dot{q}, \dot{\varepsilon}_d) \quad (14)$$

Where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -K_{p1} & -K_{D1} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -K_{p2} & -K_{D2} \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ J_{12} \\ 0 \\ J_{22} \end{bmatrix}, B_2^0 = \frac{C_{lt}^0 r}{I_{\omega} V_{\omega i}}, \Delta B_2(\Delta C_{lt}) = \frac{\Delta C_{lt} r}{I_{\omega} V_{\omega i}}$$

$$\Delta f(\eta, \Delta C_{lt}, q, \dot{q}, \dot{\varepsilon}_d) = a\eta + (C_{lt}^0 + \Delta C_{lt})(\frac{\omega_i r}{V_{\omega i}^2} \dot{V}_{\omega i} - \frac{r}{I_{\omega} V_{\omega i}} (-B\omega_i + C_{lt}\lambda_i r - \tau_i)) - \dot{T}_d$$

So the following the matching condition is satisfied:

$$\Delta B_2(\Delta C_{lt}) = B_2^0 E(\Delta C_{lt})$$

$$\Delta f(\eta, \Delta C_{lt}, q, \dot{q}, \dot{\epsilon}_d) = B_2^0 N(\eta, \Delta C_{lt}, q, \dot{q}, \dot{\epsilon}_d) \quad (15)$$

$$\text{Where } E(\Delta C_{lt}) = \frac{\Delta C_{lt}}{C_{lt}^0}, N(\eta, \Delta C_{lt}, q, \dot{q}, \dot{\epsilon}_d) = \frac{a I_{\omega} V_{\omega}}{C_{lt}^0 r} \eta - \frac{I_{\omega} V_{\omega}}{C_{lt}^0 r} +$$

$$\frac{C_{lt}^0 + \Delta C_{lt}}{C_{lt}^0} \left( \frac{I_{\omega} \omega_i}{V_{\omega}} \dot{V}_{\omega} - (-B \omega_i + C_{lt} \lambda_i r - \tau_i) \right); V_{\omega} \text{ is forward}$$

velocity of the semitrailer wheel;  $\omega$  is angular velocity of the semitrailer wheel;  $\lambda$  is slip ratio of the semitrailer wheel;  $r$  is radius of semitrailer rear wheel;  $I_{\omega}$  is moment of inertia of semitrailer rear wheel;  $B$  is bearing friction coefficient of semitrailer rear wheel.

Since  $C_{lt}^0 + \Delta C_{lt}^l > 0$ , there exists a known scalar,  $\beta, \rho$  so that  $\min_{\Delta C_{lt}} E(\Delta C_{lt}) \geq \beta > -1$

$$\max_{\Delta C_{lt}} |N(\eta, \Delta C_{lt}, q, \dot{q}, \dot{\epsilon}_d)| \leq \rho(\eta, q, \dot{q}, \dot{\epsilon}_d)$$

$$\text{Let } \bar{\rho}(\eta, q, \dot{q}, \dot{\epsilon}_d) \geq \frac{\rho(\eta, q, \dot{q}, \dot{\epsilon}_d)}{1+\beta}, \mu(\eta, q, \dot{q}, \dot{\epsilon}_d) = B_2^0 \bar{\rho}(\eta, q, \dot{q}, \dot{\epsilon}_d)$$

$$\text{The control becomes: } \tau_i(t) = p(\eta(t), q(t), \dot{q}(t), \dot{\epsilon}_d(t)) \quad (16)$$

$$\text{Where } P(\eta, q, \dot{q}, \dot{\epsilon}_d) = \begin{cases} -\frac{\mu}{|\mu|} \bar{\rho} & |\mu| \geq \xi \\ -\frac{\mu}{\xi} \bar{\rho} & |\mu| < \xi \end{cases} \quad \xi > 0$$

The control given by (16) renders system (14) globally practically stable. The size of the uniform ultimate boundedness region can be made arbitrarily small by a suitable choice of  $\xi$ . In addition, the control magnitude is no greater than  $\bar{\rho}$  and is hence finite for any  $\eta, q, \dot{q}, \dot{\epsilon}_d$ .

#### IV. THE SIMULATION OF SYSTEM

In this section the system was simulated. To show the performance due to the use of the control method, following parameters were set: lateral displacement,  $y_r = 10\text{cm}$ ; tractor yaw angle  $\epsilon_r = 3\text{deg}$ ; longitudinal velocity is  $27\text{ m/s}$ ; The nominal value of the longitudinal stiffness of semitrailer rear wheel is 3500; This vehicle enters a curved section with radius 450 m from  $t = 0$  to  $t = 20$  second. Fig. 2 shows the lateral displacement of the vehicle  $y_r$ . The tractor yaw angle  $\epsilon_r$  is shown in Fig. 3. The trailer yaw motion  $\epsilon_f$  is shown in Fig. 4. The corresponding steering input is shown in Fig. 5. Fig. 6 and Fig. 7 are the corresponding braking force at the trailer's left and right wheel respectively. Both the lateral displacement and tractor yaw angle reach a region close to 0 after around 20 seconds. The trailer yaw angle remains to be close to 0 during the maneuvering.

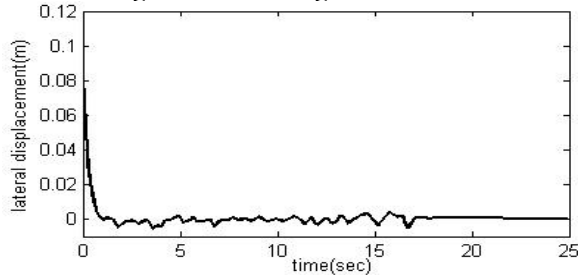


Figure 2 lateral displacement of the vehicle

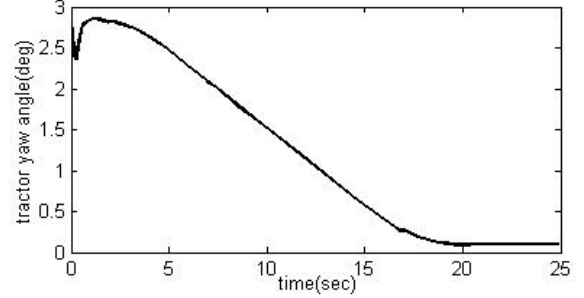


Figure 3 tractor yaw angle

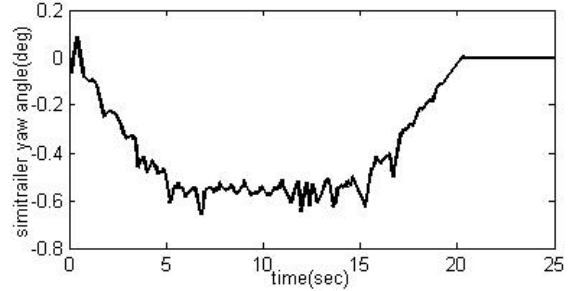


Figure 4 trailer yaw motion angle

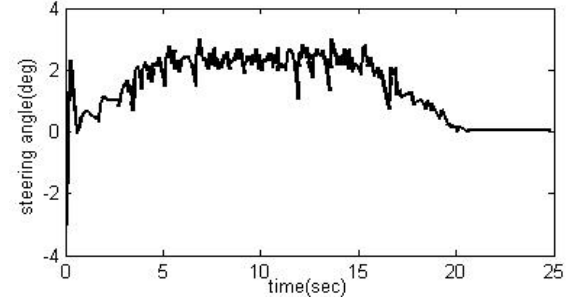


Figure 5 steering input of vehicle

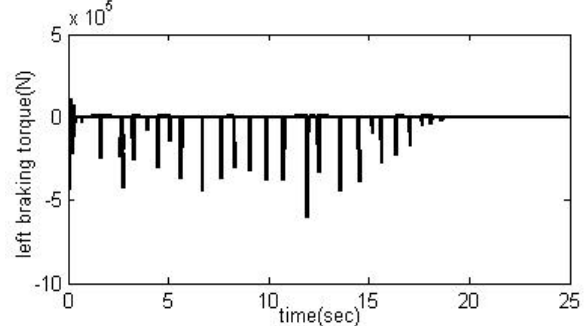


Figure 6 braking force at the trailer's left wheel

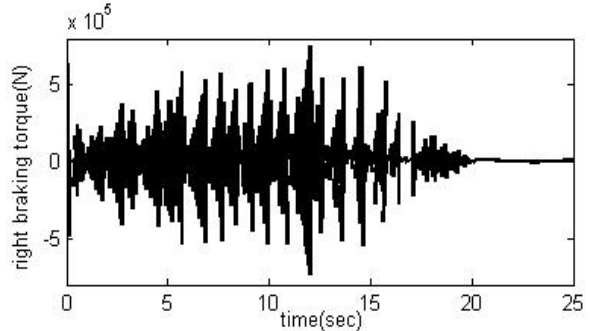


Figure 7 braking force at the trailer's right wheel

## V. CONCLUSION

In this paper, We a robust control design for the steering and independent braking control of tractor-semitrailer vehicles was proposed. The steering control is presented by using input/output linearization methodology. The magnitude of the braking torque is no greater than a bounded function which is continuous in the system state variables. Moreover, the uncertain parameter  $C_{lt}$  under consideration is allowed to be time-varying. And we concluded that the control design renders the vehicle/wheel dynamics globally practically stable. It is educed that the design is able to prevent the occurrence of jack-knifing as well as to achieve lane following maneuvering.

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