2020 IEEE Intelligent Vehicles Symposium (IV)
October 20-23, 2020. Las Vegas, USA

Formalization of Interstate Traffic Rules in Temporal Logic

Sebastian Maierhofer, Anna-Katharina Rettinger, Eva Charlotte Mayer, and Matthias Althoff

***Abstract*— To allow autonomous vehicles to safely participate in traffic and to avoid liability claims for car manufacturers, autonomous vehicles must obey traffic rules. However, current traffic rules are not formulated in a precise and mathematical way, so that they cannot be directly applied to autonomous vehicles. Additionally, several legal sources other than national traffic laws must be considered to infer detailed traffic rules. Thus, we formalize traffic rules for interstates based on the German Road Traffic Regulation, the Vienna Convention on Road Traffic, and legal decisions from courts. This makes it possible to automatically and unambiguously check whether traffic rules are being met by autonomous vehicles. Temporal logic is used to express the obtained rules mathematically. Our formalized traffic rules are evaluated for recorded data on more than 2,500 vehicles.**

I. Introduction

Compliance with traffic rules is mandatory for autonomous vehicles, and manufacturers do not want to be held responsible for accidents caused by a traffic rule violation. A general guideline for traffic rules is defined in the Vienna Convention on Road Traffic (VCoRT) [1]. Many countries have ratified the VCoRT and use it as a basis for their national traffic laws, e.g., the German Road Traffic Regulation (StVO).

Traffic rules are usually not precise and concrete because courts must be able to judge legal cases despite a lack of evidence. However, autonomous vehicles record surrounding traffic scenes and can precisely evaluate traffic rules. Thus, an unambiguous formalization of traffic laws makes sense for autonomous vehicles, as argued in [2]. Strict compliance with traffic rules will not necessarily affect the traffic flow. Due to their shorter reaction times, autonomous vehicles can obey traffic rules and still harmonize with traffic flow [3]. Both legal texts and judicial decisions that interpret traffic rules for specific situations must be considered to ensure traffic rule compliance. By combining all of these legal sources, traffic rules can be concretized. Additionally, these concretized traffic rules must be formalized in a machine-interpretable way, e.g., using logic. Traffic rules often specify a sequence of actions, making temporal logic a natural choice for such formalization. The formalization process can be separated into four steps:

1) **Extracting rules from legal sources**: All rules related to traffic can be extracted from legal sources. These sources may contain different sections of traffic law, judicial decisions, and consultancy by lawyers.
2) **Concretizing extracted rules**: The legal texts and judicial decisions are combined and concretized using natural language. As part of this concretization, the situation in which a rule is applicable should be expressed. It may also be the case that a single traffic rule pertains to different cases and each case must therefore be considered separately. For example, § 4(1) StVO addresses the distance between vehicles and also restricts when a vehicle can brake. The use of natural language allows lawyers to evaluate the concretization so that consistency between the formalized rules for autonomous vehicles and traffic rules for humans can be ensured.
3) **Extracting functions, predicates, and propositions**: We use predicates, functions, and atomic propositions to evaluate traffic situations, e.g., the allowed speed limit or the velocity of a preceding vehicle. These elements should be defined in a modular way, such that they can be easily reused.
4) **Creating temporal logic formulas**: The extracted predicates, functions, and propositions must be combined into a temporal logic formula that matches the concretized sentence from 2).

A. Related Work

One of the first approaches to codifying law is presented in [4], which focuses upon the formalization of the British Nationality Act. Selected traffic rules from the VCoRT are formalized in [2] and [5]–[9]. However, the VCoRT is not as concrete as national traffic rules, making it difficult to create formalized traffic rules. If not all traffic rules have to be obeyed, rulebooks allow the ordering and prioritization of rules based on their importance [10]. National overtaking traffic rules from Germany and Romania are formalized in [11] and [12], respectively. The authors of [13] present an approach to formalizing parts of Austrian law to detect inconsistencies in infrastructure elements of road networks, but they do not evaluate traffic laws related to the movement of vehicles. Elements like speed limits can also be evaluated online for their relevance to the current driving situation [14]. Apart from formalizing international or national law, one can also formalize the so-called Responsibility-Sensitive Safety (RSS) rules [15], [16] which describe a safe behavior of autonomous vehicles.

Monitoring algorithms based on temporal logics allow one to specify traffic rules and specifications that are defined over time. Different temporal logics that may be used include linear temporal logic (LTL) [6], [11] or signal temporal logic (STL) [16], [17]. The online evaluation of temporal logic

All authors are with the Department of Informatics, Technical University of Munich, 85748 Garching, Germany.
sebastian.maierhofer@tum.de, anna-katharina.rettinger@tum.de, eva.mayer@tum.de, althoff@tum.de

978-1-7281-6673-5/20/$31.00 ©2020 IEEE

specifications can be performed using runtime verification monitors [18], [19]. Control strategy synthesis makes monitors for the evaluation of rules superfluous [20], [21].

Another possible technique for determining compliance with traffic rules is to model traffic scenes as ontologies [22], [23] and to apply queries to the modeled ontologies. A probabilistic means of evaluating traffic rules is presented in [24] by converting rules and environmental information from first-order logic into a Bayesian network in which reasoning is performed.

Compliance with traffic rules can be 1) directly integrated into trajectory planning [9], [25], [26], 2) evaluated with monitors [11], [16], [17], or 3) considered in high-level planning [6], [22], [23], [27].

To summarize, selected rules from the VCoRT or national laws have been formalized, but there exists no set of formalized rules that draws upon a combination of several legal sources. Temporal logic is beneficial for the formalization of traffic rules because of its temporal properties. However, previous approaches based on temporal logics do not evaluate trajectories in terms of a set of formalized traffic rules for a specific use case like interstates, but rather use single rules, such as overtaking or maintaining a safe distance.

B. Contributions

This paper provides a set of formalized interstate traffic rules that can be used to evaluate trajectories. In summary, this work offers the following contributions:

1) We formalize traffic rules for autonomous vehicles based on the StVO, VCoRT, judicial decisions, and consultancy of lawyers.
2) Our evaluation of traffic rule compliance is independent of the trajectory planner used.
3) We mathematically formalize the environment of an autonomous vehicle.
4) Our defined propositions, predicates, and functions are modular and can easily be used for new traffic rules.
5) We evaluate our rules on more than 2,500 vehicles based on recorded real-world data.

The remainder of this paper is structured as follows. Section II introduces legal and mathematical foundations. Afterward, the formalization of traffic rules is presented in Sec. III, and the definition of auxiliary elements for the formalization of traffic rules is presented in Sec. IV. In Sec. V, the formalized traffic rules are evaluated for vehicle trajectories. Section VI concludes the paper.

II. Preliminaries

A. Legal Information

Please note that no official translated version of the StVO exists. The translation used for this paper is based on the *German Law Archive*[1]. The formalized rules in this work consider the ego vehicle to be the autonomous vehicle from whose perspective the rules are written. All other traffic participants are either controlled by humans or other autonomous vehicles.

We assume that the considered interstates have no intersections, driving directions that are structurally separated so that only lanes with the same driving direction are adjacent, and right-hand traffic. However, our formalization, can with minor adjustments, also be used for left-hand traffic. On German interstates, it is possible that no speed limit exists. In the following sections, we differentiate between main carriageways, access ramps, exit ramps, shoulder lanes as potential lane types, with solid, dashed, broad solid, and broad dashed lines as possible line markings. We assume that only a single broad line marking can exist for parallel lanes.

B. Metric Temporal Logic

We use metric temporal logic (MTL) [28] interpreted over finite traces of Boolean elements to monitor traffic rules since it allows us to specify an interval over which a property must be fulfilled. We use a fragment of MTL, which can specify properties for the future and the past. In the following, we introduce the operators used throughout this paper. Given a set $\mathcal{AP}$ of atomic propositions, where each atomic proposition $\sigma_\mathrm{i} \in \mathcal{AP}$ represents a Boolean statement, an MTL formula ϕ is defined as

$$\phi ::= \sigma_i \,|\, \neg\phi \,|\, \phi_1 \wedge \phi_2 \,|\, \phi_1 \vee \phi_2$$
$$\phi ::= \mathrm{G_I}(\phi) \,|\, \mathrm{F_I}(\phi) \,|\, \mathrm{P_I}(\phi) \,|\, \mathrm{O_I}(\phi)$$

where G, F, P, and O are temporal operators. The subscript I represents an interval $\mathbb{R}_{\geq 0}$ expressing time constraints relative to the current time. If the interval is not specified for an operator, we assume that the interval is specified until the end of the trace, which is always finite in this work. We also require the Boolean operators $\neg$, $\wedge$, and $\vee$. The implication $a \implies b$ is defined as $\neg a \vee b$. Unary connectives have precedence over binary connectives.

We describe the semantics of the presented MTL operators informally. For a detailed description of the semantics, we refer the reader to [29]. The future globally operator G specifies that ϕ holds within a time interval for all future states, and the future operator F specifies that ϕ holds within a time interval for some future state. The previously operator P expresses that ϕ holds within a time interval for the previous state, and the once operator O specifies that ϕ holds within a time interval for some previous state.

C. Road Network

We describe our road network in terms of lanelets [30], which are atomic, drivable road segments geometrically represented by a left and right bound, which are in turn represented by polylines. The set of all lanelets in a road network is denoted by $\mathcal{L} \cup \{\perp\}$, where $\perp$ is the bottom element in the case where no elements exist. Subsequently, we refer to a single lanelet as l, where $l \in \mathcal{L}$. A lanelet can be described by a type and an attribute, where the set of all types and attributes are denoted by $\mathcal{T} \cup \{\perp\}$ and $\mathcal{A} \cup \{\perp\}$, respectively. For interstates, we

[1] https://germanlawarchive.iuscomp.org/?p=1290

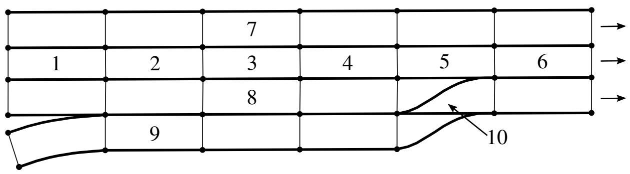

Fig. 1: Road network defined by lanelets. Lanelets 1-6 form a lane. Lanelets 2, 4, 7, and 8 are the predecessor, successor, left adjacent, and right adjacent lanelets of lanelet 3, respectively. Lanelets 1-8 are of type $main\_carriage\_way$, lanelet 9 is of type $access\_ramp$, and lanelet 10 is of type $access\_ramp$ and has the attribute $merge$.

have $\mathcal{T} = \{access\_ramp,\ exit\_ramp,\ main\_carriageway,\ shoulder\}$ and $\mathcal{A} = \{fork,\ merge\}$. The functions $\mathrm{type}(l)$ and $\mathrm{attr}(l)$ return the valid attribute and type for a lanelet (cf. Fig. 1), where $\mathrm{type}(\bot) = \mathrm{attr}(\bot) = \bot$. Likewise, the left and right bounds of a lanelet may have line markings. We denote the set of possible line markings by $\mathcal{LM} \cup \{\bot\}$, where $\mathcal{LM} = \{solid, dashed, broad\_solid, broad\_dashed\}$. The left and right line marking of a lanelet are provided by $\mathrm{mark_l}(l)$ and $\mathrm{mark_r}(l)$, where $\mathrm{mark_l}(\bot) = \mathrm{mark_r}(\bot) = \bot$. The function $\mathrm{speed\_limit}(ls)$, where $ls \subseteq \mathcal{L}$, returns a set of speed limits for a provided set of lanelets. The functions $\mathrm{l_{lb}}(l)$, $\mathrm{l_{rb}}(l)$, $\mathrm{l_{ini}}(l)$, and $\mathrm{l_{fin}}(l)$ return the left and right bounds, initial points, and final points of a lanelet, respectively. The operator $\mathrm{occ}(l)$ returns the spatial occupancy of a lanelet or of a lanelet bound. The previously defined functions return $\emptyset$ for $\bot$ as input parameter. The adjacent left, adjacent right, successor, and predecessor lanelets are defined as (cf. Fig. 1):

$$\mathrm{adj_l}(l) = \begin{cases} l' & \text{if } \ l' \in \mathcal{L} \wedge \mathrm{occ}(\mathrm{l_{lb}}(l)) \subseteq \mathrm{occ}(l') \\ & \quad \wedge\, \mathrm{l_{ini}}(l) \cap \mathrm{l_{ini}}(l') \neq \emptyset \\ & \quad \wedge\, \mathrm{l_{fin}}(l) \cap \mathrm{l_{fin}}(l') \neq \emptyset \\ \bot & \text{otherwise,} \end{cases}$$

$$\mathrm{adj_r}(l) = \begin{cases} l' & \text{if } \ l' \in \mathcal{L} \wedge \mathrm{occ}(\mathrm{l_{rb}}(l)) \subseteq \mathrm{occ}(l') \\ & \quad \wedge\, \mathrm{l_{ini}}(l) \cap \mathrm{l_{ini}}(l') \neq \emptyset \\ & \quad \wedge\, \mathrm{l_{fin}}(l) \cap \mathrm{l_{fin}}(l') \neq \emptyset \\ \bot & \text{otherwise,} \end{cases}$$

$$\mathrm{succ}(l) = \begin{cases} l' & \text{if } \ l' \in \mathcal{L} \wedge \mathrm{l_{fin}}(l) \setminus \mathrm{l_{ini}}(l') = \emptyset \\ & \quad \wedge\, \mathrm{type}(l) = \mathrm{type}(l') \wedge \mathrm{attr}(l) \neq merge \\ & \quad \wedge\, \mathrm{attr}(l') \neq fork \\ \bot & \text{otherwise,} \end{cases}$$

$$\mathrm{pre}(l) = \begin{cases} l' & \text{if } \ l' \in \mathcal{L} \wedge \mathrm{l_{ini}}(l) \setminus \mathrm{l_{fin}}(l') = \emptyset \\ & \quad \wedge\, \mathrm{type}(l) = \mathrm{type}(l') \wedge \mathrm{attr}(l) \neq fork \\ & \quad \wedge\, \mathrm{attr}(l') \neq merge \\ \bot & \text{otherwise.} \end{cases}$$

The lane originating from a lanelet is defined recursively by the lanelet's predecessor and successor lanelets:

$$\mathrm{lane}(l) = \begin{cases} \{l\} \cup \mathrm{lane}(\mathrm{pre}(l)) \cup \mathrm{lane}(\mathrm{succ}(l)) & \text{if } \ l \neq \bot \\ \emptyset & \text{if } \ l = \bot. \end{cases}$$

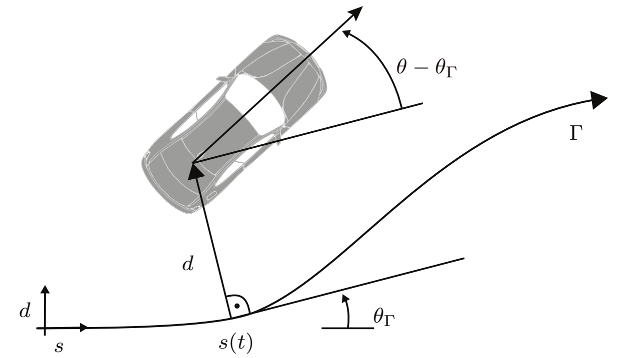

Fig. 2: A curvilinear coordinate system aligned with the reference path Γ of orientation θ_Γ. The vehicle is described by the longitudinal position s, the lateral deviation of the reference path d, and the orientation θ. We assume that Γ corresponds to the centerline of a lane occupied by the ego vehicle.

D. Vehicle Configuration

We use a curvilinear coordinate system [31], as shown in Fig. 2. The state x of a vehicle consists of the longitudinal state $x_{\text{lon}} \in \mathbb{R}^n$ and the lateral state $x_{\text{lat}} \in \mathbb{R}^n$. The longitudinal state $x_{\text{lon}} = \begin{bmatrix} s & v & a & j \end{bmatrix}^T$ consists of position s, velocity v, acceleration a, and jerk j. The lateral state $x_{\text{lat}} = \begin{bmatrix} d & \theta \end{bmatrix}^T$ consists of the lateral distance to the reference path Γ and orientation θ (cf. Fig. 2). The operator $\mathrm{proj}_\square(x)$ projects x to the elements specified by $\square$.

The input of the vehicle is denoted by $u(t)$. The underlying vehicle model can be arbitrary as long as a transformation to our state representation is possible. The spatial occupancy of a vehicle shape for a given state is denoted by $\mathcal{O}(x(t))$. The functions $\mathrm{front}(x(t))$, $\mathrm{rear}(x(t))$, $\mathrm{right}(x(t))$, and $\mathrm{left}(x(t))$ return the position of the front bumper, rear bumper, rightmost point, and leftmost point of a vehicle, respectively, and $v_{\text{type}}(x(t))$ returns the maximum allowed velocity for a vehicle type, e.g., a truck. The functions $\mathrm{ori_l}(l, s_l)$ and $\mathrm{width_l}(l, s_l)$, where s_l is the longitudinal position, return the orientation of the reference path θ_Γ and the width of a lanelet at s_l, respectively, where $\mathrm{ori_l}(\bot, s_l) = \mathrm{width_l}(\bot, s_l) = 0$. All variables associated with the ego vehicle are denoted by the subscript $\square_{\text{ego}}$; all variables associated with other vehicles are denoted by the subscript $\square_{\text{o}}$, and arbitrary vehicles, including the ego vehicle, are either denoted by the subscripts $\square_{\text{k}}$ or $\square_{\text{p}}$. Maximum and minimum values of state variables are denoted by $\square^{\max}$ and $\square^{\min}$, respectively. The set of existing vehicles at a specific time step without the ego vehicle, without the k$^{\text{th}}$ vehicle, or without the o$^{\text{th}}$ vehicle are denoted by $\mathcal{X}_{\neg\text{ego}}(\text{t})$, $\mathcal{X}_{\neg\text{k}}(\text{t})$, or $\mathcal{X}_{\neg\text{o}}(\text{t})$, respectively. For a given initial state x_0 and an input trajectory $u(\cdot)$, we introduce the solution of the model $\dot{x} = f(x, u)$ for the ego vehicle over time t as $\xi(t; x_0, u(\cdot))$. We denote a braking input by $u_{\text{br}}(\cdot)$. The time at which the ego vehicle is at a standstill, given x_0, $u(\cdot)$, and the system dynamics, is denoted by $t_s(u(\cdot))$.

III. Traffic Rule Formalization

We formalize selected rules from the StVO (e.g., establishing an emergency lane or keeping a safe distance) which are necessary for driving on German interstates and also provide

TABLE I: Overview of formalized traffic rules.

Rule	Law reference	MTL formula
R_G1	§ 4(1) StVO; § 13(5) VCoRT	$\mathrm{G}\big(\text{in\_same\_lane}(x_{\text{ego}}, x_{\text{o}}) \land \text{in\_front\_of}(x_{\text{ego}}, x_{\text{o}}) \land \neg \mathrm{O}_{[0,t_{\text{c}}]}\big(\text{cut\_in}(x_{\text{o}}, x_{\text{ego}}) \land \mathrm{P}(\neg\, \text{cut\_in}(x_{\text{o}}, x_{\text{ego}}))\big) \implies \text{keeps\_safe\_distance\_prec}(x_{\text{ego}}, x_{\text{o}})\big)$
R_G2	§ 4(1) StVO; § 17(1) VCoRT; [32] StVO § 4 Rn. 15-16	$\mathrm{G}\big(\neg\, \text{unnecessary\_braking}(x_{\text{ego}}, \mathcal{X}_{\neg\text{ego}})\big)$
R_G3	§ 3(1), § 3(3), § 18(1), § 18(5), § 18(6) StVO; traffic sign 274	$\mathrm{G}\big(\text{keeps\_lane\_speed\_limit}(x_{\text{ego}}) \land \text{keeps\_fov\_speed\_limit}(x_{\text{ego}}) \land \text{keeps\_type\_speed\_limit}(x_{\text{ego}}) \land \text{keeps\_braking\_speed\_limit}(x_{\text{ego}})\big)$
R_G4	§ 1(2), § 3(2) StVO; [32] StVO § 3 Rn. 48	$\mathrm{G}\big(\neg\, \text{slow\_leading\_vehicle}(x_{\text{ego}}, \mathcal{X}_{\neg\text{ego}}) \implies \text{preserves\_flow}(x_{\text{ego}})\big)$
R_I1	§ 12(1), § 18(8) StVO; [32] StVO § 18 Rn. 22	$\mathrm{G}\Big(\neg\big(\text{in\_congestion}(x_{\text{ego}}, \mathcal{X}_{\neg\text{ego}}) \lor \text{exist\_standing\_leading\_vehicle}(x_{\text{ego}}, \mathcal{X}_{\neg\text{ego}})\big) \implies \neg\, \text{in\_standstill}(x_{\text{ego}})\Big)$
R_I2	§ 7(2), § 7(2a), § 7a StVO; [32] StVO § 5 Rn. 58, [32] StVO § 18 Rn. 10-11	$\mathrm{G}\Big(\text{left\_of}(x_{\text{o}}, x_{\text{ego}}) \land \text{drives\_faster}(x_{\text{ego}}, x_{\text{o}}) \implies \Big(\big(\text{in\_vehicle\_queue}(x_{\text{o}}, \mathcal{X}_{\neg\text{o}}) \lor \text{in\_slow\_moving\_traffic}(x_{\text{o}}, \mathcal{X}_{\neg\text{o}}) \lor \text{in\_congestion}(x_{\text{o}}, \mathcal{X}_{\neg\text{o}})\big) \land \text{slightly\_higher\_speed}(x_{\text{ego}}, x_{\text{o}})\Big) \lor \Big(\text{right\_of\_broad\_marking}(x_{\text{ego}}) \land \text{left\_of\_broad\_marking}(x_{\text{o}})\Big) \lor \Big(\text{on\_access\_ramp}(x_{\text{ego}}) \land \text{on\_main\_carriageway}(x_{\text{o}}) \land \neg\big(\text{in\_congestion}(x_{\text{o}}, \mathcal{X}_{\neg\text{o}}) \lor \text{in\_slow\_moving\_traffic}(x_{\text{o}}, \mathcal{X}_{\neg\text{o}}) \lor \text{in\_vehicle\_queue}(x_{\text{o}}, \mathcal{X}_{\neg\text{o}})\big)\Big)\Big)$
R_I3	§ 18(7) StVO	$\mathrm{G}\big(\neg\, \text{makes\_u\_turn}(x_{\text{ego}}) \land \neg\, \text{reverses}(x_{\text{ego}})\big)$
R_I4	§ 11(2) StVO; [32] StVO § 11 Rn. 3	$\mathrm{G}\Big(\text{in\_congestion}(x_{\text{ego}}, \mathcal{X}_{\neg\text{ego}}) \lor \text{in\_slow\_moving\_traffic}(x_{\text{ego}}, \mathcal{X}_{\neg\text{ego}}) \implies \Big(\text{interstate\_broad\_enough}(x_{\text{ego}}) \implies \neg\, \text{on\_shoulder}(x_{\text{ego}}) \land \big(\text{leftmost\_lane}(x_{\text{ego}}) \implies \text{drives\_leftmost}(x_{\text{ego}}, \mathcal{X}_{\neg\text{ego}}) \land \text{single\_lane}(x_{\text{ego}})\big) \land \big(\neg\, \text{leftmost\_lane}(x_{\text{ego}}) \implies \text{drives\_rightmost}(x_{\text{ego}}, \mathcal{X}_{\neg\text{ego}})\big)\Big) \land \Big(\neg\, \text{interstate\_broad\_enough}(x_{\text{ego}}) \implies \text{drives\_rightmost}(x_{\text{ego}}, \mathcal{X}_{\neg\text{ego}}) \land \text{rightmost\_lane}(x_{\text{ego}}) \land \text{single\_lane}(x_{\text{ego}})\Big)\Big)$
R_I5	§ 1(2) StVO; [32] StVO § 18 Rn. 11	$\mathrm{G}\Big(\text{on\_main\_carriageway}(x_{\text{ego}}) \land \text{in\_front\_of}(x_{\text{ego}}, x_{\text{o}}) \land \text{on\_access\_ramp}(x_{\text{o}}) \land \mathrm{F}\big(\text{on\_main\_carriageway}(x_{\text{o}})\big) \implies \neg\Big(\neg\, \text{main\_carriageway\_right\_lane}(x_{\text{ego}}) \land \mathrm{F}\big(\text{main\_carriageway\_right\_lane}(x_{\text{ego}})\big)\Big)\Big)$

their textual concretization as well as additional comments. The temporal logic formula and the related laws and traffic signs for each rule are listed in Tab. I. The predicates and functions required for the temporal logic formulas are introduced in Sec. IV. The predicates are evaluated before they are provided to the MTL monitor. We present general rules first and interstate-specific rules afterward. The rules are written from the viewpoint of the autonomous ego vehicle. We select the following rules, since they must be frequently considered on interstates and combine different legal sources. We provide a complete list of our formalized traffic rules for interstates at gitlab.lrz.de/tum-cps/traffic-rules.

A. General Traffic Rules

1) R_G1 - *Safe distance to preceding vehicle:* The ego vehicle following vehicles within the same lane must maintain a safe distance to ensure collision freedom for all of them, even if one or several vehicles suddenly stop. If another vehicle causes a safe distance violation of the ego vehicle due to a cut-in maneuver, the ego vehicle must recover the safe distance within a predefined time t_{c} after the start of the cut-in. This rule must be evaluated with respect to every other vehicle within sensor range. Outside vehicles are taken care of by limiting the allowed velocity in rule R_G3.

2) R_G2 - *Unnecessary braking:* The ego vehicle is not allowed to brake abruptly without reason. The ego vehicle brakes not abruptly if: a) there exist no leading vehicle and the ego vehicle's acceleration does not violate an acceleration threshold; b) there is an obstacle in front of the ego vehicle and the acceleration difference does not violate a predefined threshold; c) the safe distance to a leading vehicle within the same lane is violated. All possible braking situations that are not covered by the three cases are unnecessary.

3) R_G3 - *Maximum speed limit:* The ego vehicle is not allowed to exceed a) the speed limit of the lanes it drives on, b) the speed limit necessary to ensure collision freedom with respect to vehicles outside the field of view, c) the maximum velocity allowed for its vehicle type, or d) the speed necessary to comfortably react to traffic regulations. Weather restrictions are considered by case b) and by rule R_G1.

4) R_G4 - *Traffic flow:* The ego vehicle is not allowed to travel so slowly that it impedes the traffic flow. If no leading vehicle exists, we assume that the ego vehicle should drive with its maximum allowed velocity or with the speed suggested for this type of road.

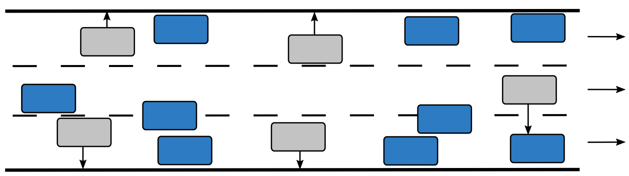

Fig. 3: Congestion in which blue vehicles obey the emergency lane rule (R_I4) and gray vehicles violate it. The vertical arrows indicate the direction in which the gray vehicles must drive to obey the rule.

B. Interstate-specific Traffic Rules

1) R_I1 - *Stopping:* If the ego vehicle is not part of congestion or if the directly leading vehicle is not standing, then stopping on the main carriage way, shoulder lane, or ramps is forbidden.

2) R_I2 - *Driving faster than left traffic:* The ego vehicle is not allowed to drive faster than any vehicle in the lanes to the left of it. Exceptions are: a) the vehicle in the left lane is part of a queue of vehicles, slow-moving traffic, or congestion and the ego vehicle drives with only slightly higher speed; b) the vehicle in the left lane and the ego vehicle are separated by a broad lane marking; c) the vehicle in the left lane is on the main carriageway, but is not part of a queue of vehicles, slow-moving traffic, or congestion and the ego vehicle is on an access ramp. This rule must be evaluated with respect to every other vehicle within the sensor range.

3) R_I3 - *No reversing and turning:* Making U-turns and reversing is prohibited.

4) R_I4 - *Emergency lane:* The ego vehicle forming part of congestion or slow-moving traffic must drive on the leftmost if it is in the leftmost lane and must drive rightmost otherwise. If the interstate is not broad enough to create an emergency lane, all vehicles should drive in the rightmost lane. Using the shoulder lane is allowed if the interstate is not broad enough. Figure 3 shows a traffic situation in which some vehicles obey the rule and some violate it.

5) R_I5 - *Consider entering vehicles:* The ego vehicle is not allowed to move to the rightmost main carriageway lane if another vehicle in front is about to enter the main carriageway from the access ramp. This rule must be evaluated with respect to every other vehicle within sensor range.

IV. Predicates and Functions

The predicates and functions necessary to describe the presented traffic rules are categorized in terms of position, velocity, braking, and general elements. We use higher-order logic to define the predicates and functions.

A. Position Elements

The occupied lanelets of a state x_k are defined by the intersection of its occupancy with the corresponding lanelets:

$$\mathrm{lanelets}(x_\mathrm{k}) = \left\{ l \in \mathcal{L} \,\middle|\, \mathrm{occ}(l) \cap \mathcal{O}(x_\mathrm{k}) \neq \emptyset \right\}.$$

The lanes occupied by a vehicle are defined as:

$$\mathrm{lanes}(x_\mathrm{k}) = \{\mathrm{lane}(l) \,|\, l \in \mathrm{lanelets}(x_\mathrm{k})\}.$$

This information is used to determine whether the k^th vehicle shares a lane with the p^th vehicle:

$$\mathrm{in\_same\_lane}(x_\mathrm{k}, x_\mathrm{p}) \iff \mathrm{lanes}(x_\mathrm{k}) \cap \mathrm{lanes}(x_\mathrm{p}) \neq \emptyset.$$

The k^th vehicle is in front of the p^th vehicle if its rear position is larger than the front position of the p^th vehicle:

$$\mathrm{in\_front\_of}(x_\mathrm{p}, x_\mathrm{k}) \iff \mathrm{front}(x_\mathrm{p}) < \mathrm{rear}(x_\mathrm{k}).$$

Information on whether a vehicle is on the main carriageway is given by

$$\begin{aligned}&\mathrm{on\_main\_carriageway}(x_\mathrm{k}) \iff \\ &\quad \mathit{main\_carriageway} \in \{\mathrm{type}(l) \,|\, l \in \mathrm{lanelets}(x_\mathrm{k})\}.\end{aligned}$$

Other lanelet types can be evaluated similarly. A vehicle is in the rightmost lane of the main carriageway if any occupied lanelet is of type $\mathit{main\_carriageway}$ and this lanelet's right adjacent lanelet is not of the type $\mathit{main\_carriageway}$:

$$\begin{aligned}&\mathrm{main\_carriageway\_right\_lane}(x_\mathrm{k}) \iff \\ &\quad \exists l \in \mathrm{lanelets}(x_\mathrm{k}) : \mathrm{type}(l) = \mathit{main\_carriageway} \\ &\quad\quad \wedge \mathrm{type}(\mathrm{adj_r}(l)) \neq \mathit{main\_carriageway}.\end{aligned}$$

The k^th vehicle is to the left of the p^th vehicle if

$$\begin{aligned}&\mathrm{left\_of}(x_\mathrm{k}, x_\mathrm{p}) \iff \mathrm{left}(x_\mathrm{p}) < \mathrm{right}(x_\mathrm{k}) \\ &\quad \wedge \big(\mathrm{rear}(x_\mathrm{p}) \leq \mathrm{rear}(x_\mathrm{k}) \leq \mathrm{front}(x_\mathrm{p}) \\ &\quad\quad \vee \mathrm{front}(x_\mathrm{p}) < \mathrm{front}(x_\mathrm{k}) \wedge \mathrm{rear}(x_\mathrm{k}) < \mathrm{rear}(x_\mathrm{p}) \\ &\quad\quad \vee \mathrm{rear}(x_\mathrm{p}) \leq \mathrm{front}(x_\mathrm{k}) \leq \mathrm{front}(x_\mathrm{p})\big),\end{aligned}$$

evaluates as true. To determine whether a vehicle is to the right of a broad line marking, the following predicate is introduced:

$$\begin{aligned}&\mathrm{right\_of\_broad\_marking}(x_\mathrm{k}) \iff \\ &\quad \big\{l \in \mathrm{lanelets}(x_\mathrm{k}) \,\big|\, \mathrm{mark_r}(l) \in \{\mathit{broad\_solid}, \\ &\quad\quad \mathit{broad\_dashed}\}\big\} = \emptyset \,\wedge \\ &\quad \big\{l \in \mathrm{lanelets\_left\_of\_vehicle}(x_\mathrm{k}) \,\big| \\ &\quad\quad \mathrm{mark_r}(l) \in \{\mathit{broad\_solid}, \mathit{broad\_dashed}\}\big\} \neq \emptyset,\end{aligned}$$

where

$$\begin{aligned}&\mathrm{lanelets\_left\_of\_vehicle}(x_\mathrm{k}) = \\ &\quad \bigcup \{\mathrm{lanelets\_left\_of\_lanelet}(l) \,|\, l \in \mathrm{lanelets}(x_\mathrm{k})\},\end{aligned}$$

returns the set containing all lanelets to the left of the k^th vehicle and the recursively defined function

$$\begin{aligned}&\mathrm{lanelets\_left\_of\_lanelet}(l) = \\ &\quad \big(\{\mathrm{adj_l}(l)\} \cup \mathrm{lanelets\_left\_of\_lanelet}(\mathrm{adj_l}(l))\big) \setminus \{\perp\},\end{aligned}$$

returns all lanelets to the left of a given lanelet, where $\mathrm{lanelets\_left\_of\_lanelet}(\perp) = \emptyset$. Similarly, the predicates left_of_broad_marking, lanelets_right_of_vehicle, and lanelets_right_of_lanelet can be defined. The width of a road given a lanelet and a longitudinal position s_k is defined as:

$$\mathrm{road\_width}(l, s_\mathrm{k}) = \sum_{l' \in \mathrm{adj\_lanelets}(l)} \mathrm{width_l}(l', s_\mathrm{k}),$$

where $\mathrm{road\_width}(\perp, s_\mathrm{k}) = 0$ and

$$\mathrm{adj\_lanelets}(l) = \big(\{l\} \cup \mathrm{lanelets\_right\_of\_lanelet}(l) \cup \mathrm{lanelets\_left\_of\_lanelet}(l)\big) \setminus \{\perp\},$$

with $\mathrm{adj\_lanelets}(\perp) = \emptyset$. If the width of a road is at least $w^\mathrm{min}_\mathrm{road}$, the road is declared to be broad enough:

$$\mathrm{interstate\_broad\_enough}(x_\mathrm{k}) \iff \forall l \in \mathrm{lanelets}(x_\mathrm{k}) : \mathrm{road\_width}(l, \mathrm{proj}_\mathrm{s}(x_\mathrm{k})) \geq w^\mathrm{min}_\mathrm{road}.$$

A vehicle is in the leftmost lane if any of its occupied lanelets has no left adjacent lanelet:

$$\mathrm{leftmost\_lane}(x_\mathrm{k}) \iff \exists l \in \mathrm{lanelets}(x_\mathrm{k}) : \mathrm{adj}_\mathrm{l}(l) = \perp.$$

The predicate $\mathrm{rightmost\_lane}$ is defined analogously. A vehicle drives as far left as possible if the distance to the left lanelet boundary is at most d_bound, or if there exists a vehicle on the left side to which the distance is at most d_veh:

$$\begin{aligned}\mathrm{drives\_leftmost}(x_\mathrm{k}, \mathcal{X}_{\neg\mathrm{k}}) \iff &\big(\forall l \in \mathrm{lanelets}(x_\mathrm{k}) : \\ &0 < \frac{1}{2}\mathrm{width}_\mathrm{l}(l, \mathrm{proj}_\mathrm{s}(x_\mathrm{k})) - \mathrm{left}(x_\mathrm{k}) \leq d_\mathrm{bound}\big) \\ &\vee \big(\exists x_\mathrm{p} \in \mathcal{X}_{\neg\mathrm{k}} : \mathrm{left\_of}(x_\mathrm{p}, x_\mathrm{k}) \\ &\wedge \mathrm{right}(x_\mathrm{p}) - \mathrm{left}(x_\mathrm{k}) \leq d_\mathrm{veh}\big),\end{aligned}$$

with $\mathrm{drives\_rightmost}$ being defined analogously. A vehicle occupies a single lane if no adjacent lanelet falls within the set of occupied lanelets:

$$\mathrm{single\_lane}(x_\mathrm{k}) \iff \forall l \in \mathrm{lanelets}(x_\mathrm{k}) : \{\mathrm{adj}_\mathrm{l}(l), \mathrm{adj}_\mathrm{r}(l)\} \cap \mathrm{lanelets}(x_\mathrm{k}) = \emptyset.$$

B. Velocity Elements

The speed limit regulated by traffic signs is the minimum speed limit of all lanelets that a vehicle occupies:

$$v^\mathrm{max}_\mathrm{sl}(x_\mathrm{k}) = \min(\mathrm{speed\_limit}(\mathrm{lanelets}(x_\mathrm{k}))).$$

A vehicle must be able to stop within the field of view s_fov. Hence, we search for the maximum value of v_fov that fulfills the condition:

$$\mathrm{front}(\xi(t_s(u_\mathrm{br}(\cdot)); x^\mathrm{sr}_\mathrm{lon}, u_\mathrm{br}(\cdot))) \leq s_\mathrm{fov},$$

where $x^\mathrm{sr}_\mathrm{lon} = [0, v_\mathrm{fov}, a^\mathrm{max}_\mathrm{k}, j^\mathrm{max}_\mathrm{k}]^T$ is the worst-case initial state. Additionally, a vehicle must reduce its velocity to an upcoming speed limit such that it does not brake abruptly. Therefore, the speed limit v_br is introduced, ensuring that a trajectory for the velocity reduction exists that does not exceed the predefined acceleration threshold $a_\mathrm{abrupt} < 0$ or jerk threshold $j_\mathrm{abrupt} < 0$. We calculate v_br analogously to v_fov, where the following two conditions must be fulfilled:

$$\begin{aligned}&x^\mathrm{f}_\mathrm{lon} = \xi(t^*; x^0_\mathrm{lon}, u(\cdot)), \\ &\forall t \in [t_0, t^*] : a_\mathrm{k}(t) \geq a_\mathrm{abrupt} \wedge j_\mathrm{k}(t) \geq j_\mathrm{abrupt},\end{aligned}$$

where $x^0_\mathrm{lon} = [0, v_\mathrm{br}, a^\mathrm{max}_\mathrm{k}, j^\mathrm{max}_\mathrm{k}]^T$ and $x^\mathrm{f}_\mathrm{lon} = [s_\mathrm{fov}, v^*_\mathrm{sl}, 0, 0]^T$ are the worst-case initial and final states at which the upcoming speed limit v^*_sl is reached. A vehicle maintains the speed limit given by traffic signs if its velocity is at most $v^\mathrm{max}_\mathrm{sl}$:

$$\mathrm{keeps\_lane\_speed\_limit}(x_\mathrm{k}) \iff \mathrm{proj}_\mathrm{v}(x_\mathrm{k}) \leq v^\mathrm{max}_\mathrm{sl}(x_\mathrm{k}).$$

The other speed limit predicates are defined analogously. To ensure high traffic throughput on interstates, vehicles must not impede traffic flow:

$$\mathrm{preserves\_flow}(x_\mathrm{k}) \iff \mathrm{v\_max}_1(x_\mathrm{k}) - \mathrm{proj}_\mathrm{v}(x_\mathrm{k}) < \Delta v_\mathrm{fl},$$

where $\mathrm{v\_max}_1(x_\mathrm{k}) = \min(v_\mathrm{br}, v_\mathrm{fov}, v^*_\mathrm{sl}(x_\mathrm{k}), v_\mathrm{type}(x_\mathrm{k}))$,

$$v^*_\mathrm{sl}(x_\mathrm{k}) = \begin{cases} v^\mathrm{max}_\mathrm{sl}(x_\mathrm{k}) & \text{if } v^\mathrm{max}_\mathrm{sl}(x_\mathrm{k}) \neq \mathrm{inf} \\ v_\mathrm{su} & \text{otherwise.}\end{cases}$$

Here, v_su is the suggested velocity to be assumed if no speed limit is present, $\Delta v_\mathrm{fl} \geq 0$ is a threshold indicating whether a vehicle drives slowly, and inf indicates that no speed limit exists. A slow leading vehicle exists for the k^th vehicle if the following predicate evaluates to true:

$$\begin{aligned}\mathrm{slow\_leading\_vehicle}(x_\mathrm{k}, \mathcal{X}_{\neg\mathrm{k}}) \iff &\exists x_\mathrm{p} \in \mathcal{X}_{\neg\mathrm{k}} : \mathrm{v\_max}_2(x_\mathrm{p}) - \mathrm{proj}_\mathrm{v}(x_\mathrm{p}) \geq \Delta v_\mathrm{fl} \\ &\wedge \mathrm{in\_same\_lane}(x_\mathrm{k}, x_\mathrm{p}) \wedge \mathrm{in\_front\_of}(x_\mathrm{k}, x_\mathrm{p}),\end{aligned}$$

where $\mathrm{v\_max}_2(x_\mathrm{k}) = \min(v^*_\mathrm{sl}(x_\mathrm{k}), v_\mathrm{type}(x_\mathrm{k}))$. A vehicle is at a standstill if its velocity is close to zero:

$$\mathrm{in\_standstill}(x_\mathrm{k}) \iff -v_\mathrm{err} \leq \mathrm{proj}_\mathrm{v}(x_\mathrm{k}) \leq v_\mathrm{err},$$

where $v_\mathrm{err} \geq 0$ captures measurement uncertainties and is close to zero. This predicate can be used to evaluate whether leading vehicles exist in a standstill:

$$\begin{aligned}\mathrm{exist\_standing\_leading\_vehicle}(x_\mathrm{k}, \mathcal{X}_{\neg\mathrm{k}}) \iff &\exists x_\mathrm{p} \in \mathcal{X}_{\neg\mathrm{k}} : \mathrm{in\_same\_lane}(x_\mathrm{k}, x_\mathrm{p}) \\ &\wedge \mathrm{in\_front\_of}(x_\mathrm{k}, x_\mathrm{p}) \wedge \mathrm{in\_standstill}(x_\mathrm{p}).\end{aligned}$$

The k^th vehicle drives with only slightly higher speed than the p^th vehicle if the following predicate is true:

$$\mathrm{slightly\_higher\_speed}(x_\mathrm{k}, x_\mathrm{p}) \iff 0 < \mathrm{proj}_\mathrm{v}(x_\mathrm{k}) - \mathrm{proj}_\mathrm{v}(x_\mathrm{p}) < v_\mathrm{so},$$

where v_so indicates a slightly higher speed. A backward-driving vehicle has a negative velocity considering uncertainties:

$$\mathrm{reverses}(x_\mathrm{k}) \iff \mathrm{proj}_\mathrm{v}(x_\mathrm{k}) < -v_\mathrm{err}.$$

The following predicate indicates that the k^th vehicle drives faster than the p^th vehicle:

$$\mathrm{drives\_faster}(x_\mathrm{k}, x_\mathrm{p}) \iff \mathrm{proj}_\mathrm{v}(x_\mathrm{k}) > \mathrm{proj}_\mathrm{v}(x_\mathrm{p}).$$

C. Braking Elements

The safe distance that must be kept by the k^th vehicle to the p^th vehicle is based on the definitions in [7] and [8]:

$$d_\mathrm{safe}(v_\mathrm{k}, v_\mathrm{p}) = \frac{v_\mathrm{p}^2}{-2|a^\mathrm{min}_\mathrm{p}|} - \frac{v_\mathrm{k}^2}{-2|a^\mathrm{min}_\mathrm{k}|} + v_\mathrm{k} t_\mathrm{d},$$

where t_d denotes the reaction time of the k^th vehicle. We assume that preceding vehicles can brake more strongly than succeeding vehicles: $a_{\text{p}}^{\text{min}} < a_{\text{k}}^{\text{min}} < 0$. The safe distance is maintained if the relative distance between vehicles is larger than the safe distance:

$$\begin{aligned}&\text{keeps\_safe\_distance\_prec}(x_{\text{k}}, x_{\text{p}}) \iff \\ &\quad d_{\text{safe}}(\text{proj}_{\text{v}}(x_{\text{k}}), \text{proj}_{\text{v}}(x_{\text{p}})) < (\text{rear}(x_{\text{p}}) - \text{front}(x_{\text{k}})).\end{aligned}$$

Unnecessary braking is evaluated based on the vehicle's acceleration:

$$\begin{aligned}&\text{unnecessary\_braking}(x_{\text{k}}, \mathcal{X}_{\neg\text{k}}) \iff \\ &\quad \text{proj}_{\text{a}}(x_{\text{k}}) < 0 \implies \big(\nexists x_{\text{p}} \in \mathcal{X}_{\neg\text{k}} : \text{in\_front\_of}(x_{\text{k}}, x_{\text{p}}) \\ &\qquad \wedge \text{in\_same\_lane}(x_{\text{k}}, x_{\text{p}}) \wedge \text{proj}_{\text{a}}(x_{\text{k}}) < a_{\text{abrupt}}\big) \\ &\quad \vee \big(\exists x_{\text{p}} \in \mathcal{X}_{\neg\text{k}} : \text{keeps\_safe\_distance\_prec}(x_{\text{k}}, x_{\text{p}}) \\ &\qquad \wedge \text{in\_front\_of}(x_{\text{k}}, x_{\text{p}}) \wedge \text{in\_same\_lane}(x_{\text{k}}, x_{\text{p}}) \\ &\qquad \wedge (\text{proj}_{\text{a}}(x_{\text{k}}) - \text{proj}_{\text{a}}(x_{\text{p}})) < a_{\text{abrupt}}\big).\end{aligned}$$

D. General Elements

The k[th] vehicle is part of a congestion if there is a predefined number of slow-driving vehicles in front of it:

$$\begin{aligned}&\text{in\_congestion}(x_{\text{k}}, \mathcal{X}_{\neg\text{k}}) \iff \\ &\quad \big|\big\{x_{\text{p}} \in \mathcal{X}_{\neg\text{k}} \,\big|\, \text{in\_same\_lane}(x_{\text{k}}, x_{\text{p}}) \\ &\qquad \wedge \text{in\_front\_of}(x_{\text{k}}, x_{\text{p}}) \wedge \text{proj}_{\text{v}}(x_{\text{p}}) \le v_{\text{con}}\big\}\big| \ge n_{\text{con}},\end{aligned}$$

where v_{con} indicates the maximum velocity for vehicles in a congestion and n_{con} is the minimum required number of vehicles that are part of a congestion within one lane. The predicates in_slow_moving_traffic (with v_{smt} and n_{smt}) and in_vehicle_queue (with v_{qv} and n_{qv}) are analogously defined. A turning vehicle violates the difference between the vehicle's and the reference path orientation by $\Delta\theta_{\text{uturn}}$:

$$\begin{aligned}&\text{makes\_u\_turn}(x_{\text{k}}) \iff \exists l \in \text{lanelets}(x_{\text{k}}) : \\ &\quad \Delta\theta_{\text{uturn}} < |\text{proj}_{\theta}(x_{\text{k}}) - \text{ori}_{\text{l}}(l, \text{proj}_{\text{s}}(x_{\text{k}}))|.\end{aligned}$$

A cut-in by the k[th] vehicle into the p[th] vehicle's lane occurs if the k[th] vehicle occupies more than one lane, if it intersects the p[th] vehicle's lane, and if its lateral position and orientation are towards the p[th] vehicle's lane:

$$\begin{aligned}&\text{cut\_in}(x_{\text{k}}, x_{\text{p}}) \iff \neg\,\text{single\_lane}(x_{\text{k}}) \\ &\quad \wedge \Big(\big(\text{proj}_{\text{d}}(x_{\text{k}}) < \text{proj}_{\text{d}}(x_{\text{p}}) \wedge \text{proj}_{\theta}(x_{\text{k}}) > 0\big) \\ &\qquad \vee \big(\text{proj}_{\text{d}}(x_{\text{k}}) > \text{proj}_{\text{d}}(x_{\text{p}}) \wedge \text{proj}_{\theta}(x_{\text{k}}) < 0\big)\Big) \\ &\quad \wedge \text{in\_same\_lane}(x_{\text{k}}, x_{\text{p}}).\end{aligned}$$

V. Numerical Experiments

We evaluate formalized traffic rules for vehicles from a CommonRoad scenario[2] [33] and from the highD dataset [34]. The parameters used for the evaluation are listed in Tab. II. We have evaluated over 2,500 trajectories, each several seconds long. Compliance with all rules is monitored individually and together (rule R_G0) for the highD dataset, except for rule R_I5, which was neglected because the absence of access ramps in the scenarios make it irrelevant. The results of the evaluation are shown in Fig. 4. No vehicle stops (R_I1), turns or reverses (R_I3), or violates the creation of an emergency lane (R_I4). Since these situations do not occur in the tested scenarios, it is apparent that the rules do not create false-negative results. Almost no vehicles brake unnecessarily (R_G2), impede traffic flow (R_G4), or overtake other vehicles illegally on the right-hand side (R_I2). Less than 65% of vehicles maintained the safe distance to one of their leading vehicles (R_G1), and less than 78% considered the speed limit (R_G3). Approximately 37% of vehicles obeyed all evaluated rules (R_G0). Figure 5 shows a snippet of a CommonRoad scenario for which rule R_I5 has been evaluated. In this scenario, one vehicle violates the rule and two vehicles consider it.

[2]https://commonroad.in.tum.de/scenarios/

TABLE II: User-defined parameters for the traffic rule evaluation.

Parameter	Value	Parameter	Value
v_{br}	50.0 m/s	v_{fov}	50.0 m/s
v_{err}	0.01 m/s	v_{su}	36.66 m/s
v_{so}	5.55 m/s	v_{type} (truck)	22.22 m/s
v_{con}	2.78 m/s	v_{smt}	8.33 m/s
v_{qv}	16.67 m/s	Δv_{fl}	15.0 m/s
$a_{\text{ego}}^{\text{min}}$	-10.0 m/s²	$a_{\text{o}}^{\text{min}}$	-10.5 m/s²
$a_{\text{ego}}^{\text{max}}$	5.0 m/s²	a_{abrupt}	-2.0 m/s²
$j_{\text{ego}}^{\text{max}}$	10.0 m/s³	j_{abrupt}	-2.0 m/s³
s_{fov}	200.0 m	$n_{\text{con}}/n_{\text{smt}}/n_{\text{qv}}$	3
t_{c}	3.0 s	t_{d}	0.3 s
d_{veh}	0.75 m	d_{bound}	0.1 m
$w_{\text{road}}^{\text{min}}$	7.0 m	$\Delta\theta_{\text{uturn}}$	1.57 rad

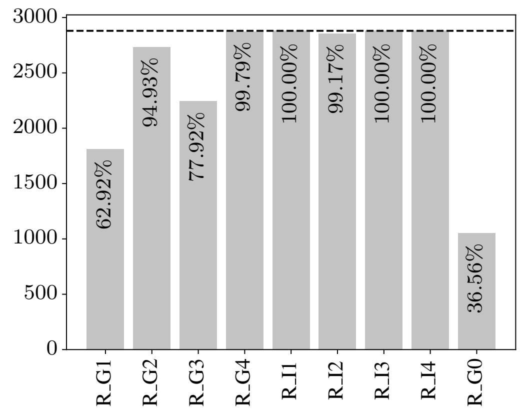

Fig. 4: Rule compliance by the evaluated vehicles. The horizontal dashed line indicates the number of evaluated vehicles.

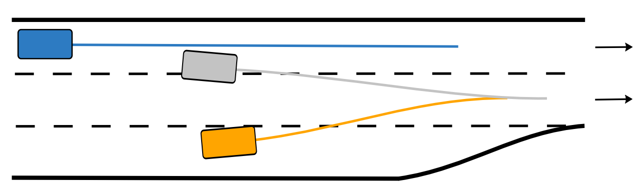

Fig. 5: The orange vehicle is about to enter the main carriageway from the access ramp. The grey vehicle does not consider the orange vehicle and therefore violates rule R_I5, whereas the blue vehicle considers the gray vehicle and obeys rule R_I5, which also evaluates to true for the orange vehicle itself (CommonRoad ID: S=ZAM_Ramp-1_1_T-1:2020a).

VI. Conclusions

This paper presented a set of formalized traffic rules for autonomous vehicles driving on German interstates. Each rule has a concretization in natural language for lawyers. The formalization in MTL allows one to monitor traffic rules for trajectories generated by a motion planner. Future work includes extension to urban roads and online evaluation of vehicle trajectories. We encourage developers and lawyers all over the world to send their formalizations of rules for autonomous vehicles to *commonroad-i06@in.tum.de*, so that we can integrate them into our set of rules to establish a better worldwide understanding of required traffic rules for autonomous vehicles. Our formalized rule set will be continuously updated and made available online at gitlab.lrz.de/tum-cps/traffic-rules.

Acknowledgments

The authors gratefully acknowledge partial financial support by the BMW Group within the CAR@TUM project and the German Research Foundation (DFG) grant AL 1185/7-1.

References

[1] United Nations Economic Commission for Europe, "Convention on road traffic," United Nations Conference on Road Traffic, 1968, (consolidated version of 2006). [Online]. Available: https://www.unece.org/fileadmin/DAM/trans/conventn/Conv_road_traffic_EN.pdf

[2] A. Rizaldi and M. Althoff, "Formalising traffic rules for accountability of autonomous vehicles," in *Proc. of the IEEE Int. Conf. on Intelligent Transportation Systems*, 2015, pp. 1658–1665.

[3] M. Althoff and R. Lösch, "Can automated road vehicles harmonize with traffic flow while guaranteeing a safe distance?" in *Proc. of the IEEE Int. Conf. on Intelligent Transportation Systems*, 2016, pp. 485–491.

[4] M. J. Sergot, F. Sadri, R. A. Kowalski, F. Kriwaczek, P. Hammond, and H. T. Cory, "The British Nationality Act as a logic program," *Communications of the ACM*, vol. 29, no. 5, pp. 370–386, 1986.

[5] M. Koschi, C. Pek, M. Beikirch, and M. Althoff, "Set-based prediction of pedestrians in urban environments considering formalized traffic rules," in *Proc. of the IEEE Int. Conf. on Intelligent Transportation Systems*, 2018, pp. 2704–2711.

[6] K. Esterle, V. Aravantinos, and A. Knoll, "From specifications to behavior: Maneuver verification in a semantic state space," in *Proc. of the IEEE Intelligent Vehicles Symposium*, 2019, pp. 2140–2147.

[7] C. Pek, P. Zahn, and M. Althoff, "Verifying the safety of lane change maneuvers of self-driving vehicles based on formalized traffic rules," in *Proc. of the IEEE Intelligent Vehicles Symposium*, 2017, pp. 1477–1483.

[8] A. Rizaldi, F. Immler, and M. Althoff, "A formally verified checker of the safe distance traffic rules for autonomous vehicles," in *Proc. of the NASA Formal Methods Symposium*, 2016, pp. 175–190.

[9] B. Vanholme, D. Gruyer, B. Lusetti, S. Glaser, and S. Mammar, "Highly automated driving on highways based on legal safety," *IEEE Transactions on Intelligent Transportation Systems*, vol. 14, no. 1, pp. 333–347, 2013.

[10] A. Censi, K. Slutsky, T. Wongpiromsarn, D. Yershov, S. Pendleton, J. Fu, and E. Frazzoli, "Liability, ethics, and culture-aware behavior specification using rulebooks," in *Proc. of the IEEE Int. Conf. on Robotics and Automation*, 2019, pp. 8536–8542.

[11] A. Rizaldi, J. Keinholz, M. Huber, J. Feldle, F. Immler, M. Althoff, E. Hilgendorf, and T. Nipkow, "Formalising and monitoring traffic rules for autonomous vehicles in Isabelle/HOL," in *Proc. of the Int. Conf. on Integrated Formal Methods*, 2017, pp. 50–66.

[12] D. M. Costescu, "Keeping the autonomous vehicles accountable: Legal and logic analysis on traffic code," in *Proc. of the Conf. Vision Zero for Sustainable Road Safety in Baltic Sea Region*, 2020, pp. 21–33.

[13] H. Beck, T. Eiter, and T. Krennwallner, "Inconsistency management for traffic regulations: Formalization and complexity results," in *Proc. of Logics in Artificial Intelligence*, 2012, pp. 80–93.

[14] D. Nienhüser, T. Gumpp, and J. M. Zöllner, "Relevance estimation of traffic elements using markov logic networks," in *Proc. of the IEEE Int. Conf. on Intelligent Transportation Systems*, 2011, pp. 1659–1664.

[15] S. Shalev-Shwartz, S. Shammah, and A. Shashua, "On a formal model of safe and scalable self-driving cars," *arXiv preprint arXiv:1708.06374*, pp. 1:1–37, 2017.

[16] M. Hekmatnejad, S. Yaghoubi, A. Dokhanchi, H. B. Amor, A. Shrivastava, L. Karam, and G. Fainekos, "Encoding and monitoring responsibility sensitive safety rules for automated vehicles in signal temporal logic," in *Proc. of the ACM-IEEE Int. Conf. on Formal Methods and Models for System Design*, 2019, pp. 1–11.

[17] N. Arechiga, "Specifying safety of autonomous vehicles in signal temporal logic," in *Proc. of the IEEE Intelligent Vehicles Symposium*, 2019, pp. 58–63.

[18] K. Havelund and G. Roşu, "Synthesizing monitors for safety properties," in *Proc. of Tools and Algorithms for the Construction and Analysis of Systems*, 2002, pp. 342–356.

[19] M. Leucker and C. Schallhart, "A brief account of runtime verification," *The Journal of Logic and Algebraic Programming*, vol. 78, no. 5, pp. 293 – 303, 2009.

[20] L. I. Reyes Castro, P. Chaudhari, J. Tumova, S. Karaman, E. Frazzoli, and D. Rus, "Incremental sampling-based algorithm for minimum-violation motion planning," in *Proc. of the IEEE Conf. on Decision and Control*, 2013, pp. 3217–3224.

[21] J. Tumova, G. C. Hall, S. Karaman, E. Frazzoli, and D. Rus, "Least-violating control strategy synthesis with safety rules," in *Proc. of the Int. Conf. on Hybrid Systems: Computation and Control*, 2012, pp. 1–10.

[22] L. Zhao, R. Ichise, Y. Sasaki, Z. Liu, and T. Yoshikawa, "Fast decision making using ontology-based knowledge base," in *Proc. of the IEEE Intelligent Vehicles Symposium*, 2016, pp. 173–178.

[23] M. Buechel, G. Hinz, F. Ruehl, H. Schroth, C. Gyoeri, and A. Knoll, "Ontology-based traffic scene modeling, traffic regulations dependent situational awareness and decision-making for automated vehicles," in *Proc. of the IEEE Intelligent Vehicles Symposium*, 2017, pp. 1471–1476.

[24] L. Wellhausen and M. G. Jacob, "Map-optimized probabilistic traffic rule evaluation," in *Proc. of the IEEE Int. Conf. on Intelligent Robots and Systems*, 2016, pp. 3012–3017.

[25] S. M. Thornton, S. Pan, S. M. Erlien, and J. C. Gerdes, "Incorporating ethical considerations into automated vehicle control," *IEEE Transactions on Intelligent Transportation Systems*, vol. 18, no. 6, pp. 1429–1439, 2017.

[26] A. Best, S. Narang, D. Barber, and D. Manocha, "Autonovi: Autonomous vehicle planning with dynamic maneuvers and traffic constraints," in *Proc. of the IEEE Int. Conf. on Intelligent Robots and Systems*, 2017, pp. 2629–2636.

[27] R. Kohlhaas, T. Bittner, T. Schamm, and J. M. Zollner, "Semantic state space for high-level maneuver planning in structured traffic scenes," in *Proc. of the IEEE Int. Conf. on Intelligent Transportation Systems*, 2014, pp. 1060–1065.

[28] R. Alur and T. Henzinger, "Real-time logics: Complexity and expressiveness," *Information and Computation*, vol. 104, no. 1, pp. 35 – 77, 1993.

[29] P. Thati and G. Rou, "Monitoring algorithms for metric temporal logic specifications," *Electronic Notes in Theoretical Computer Science*, vol. 113, pp. 145 – 162, 2005.

[30] P. Bender, J. Ziegler, and C. Stiller, "Lanelets: Efficient map representation for autonomous driving," in *Proc. of the IEEE Intelligent Vehicles Symposium*, 2014, pp. 420–425.

[31] E. Héry, S. Masi, P. Xu, and P. Bonnifait, "Map-based curvilinear coordinates for autonomous vehicles," in *Proc. of the IEEE Int. Conf. on Intelligent Transportation Systems*, 2017, pp. 1–7.

[32] M. Burmann, R. Heß, K. Hühnermann, and J. Jahnke, "Straßenverkehrsrecht: Kommentar," 2018.

[33] M. Althoff, M. Koschi, and S. Manzinger, "CommonRoad: Composable benchmarks for motion planning on roads," in *Proc. of the IEEE Intelligent Vehicles Symposium*, 2017, pp. 719–726.

[34] R. Krajewski, J. Bock, L. Kloeker, and L. Eckstein, "The highD dataset: A drone dataset of naturalistic vehicle trajectories on German highways for validation of highly automated driving systems," in *Proc. of the IEEE Int. Conf. on Intelligent Transportation Systems*, 2018, pp. 2118–2125.