2018 学年概率论期中考试答案

一、选择题

1-5 CBDDB　6-10 DACCD

二、填空题

1、2.4

2、0.25

3、0.140625

4、$F(x)=\begin{cases} 0 & x<0 \\ x & 0\le x<1 \\ 1 & x\ge 1 \end{cases}$

5、N(24,16)

三、计算题

1. A表示观察时间属于高峰时间　$P(A)=\frac{3}{24}$　$A\sim \pi(20)$

B表示观察时间属于平时时间　$P(B)=\frac{10}{24}$　$B\sim \pi(15)$

C表示观察时间属于其余时间　$P(C)=\frac{11}{24}$　$C\sim \pi(5)$

D表示一段时间内有10辆车进出

则 $P(D|A)=\frac{20^{10}e^{-20}}{10!}$　$P(D|B)=\frac{15^{10}\cdot e^{-15}}{10!}$　$P(D|C)=\frac{5^{10}e^{-5}}{10!}$

由贝叶斯公式 $P(A|D)=\frac{P(D|A)\cdot P(A)}{P(D|A)\cdot P(A)+P(D|B)\cdot P(B)+P(D|C)\cdot P(C)}$

$$=\frac{\frac{20^{10}\cdot e^{-20}}{10!}\cdot\frac{3}{24}}{\frac{20^{10}\cdot e^{-20}}{10!}\cdot\frac{3}{24}+\frac{15^{10}\cdot e^{-15}}{10!}\cdot\frac{10}{24}+\frac{5^{10}\cdot e^{-5}}{10!}\cdot\frac{11}{24}}\approx 0.025$$

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2. (1). $E(X)=\int_{-\infty}^{+\infty} x\frac{1}{2}e^{-|x|}\,dx$

$=\frac{1}{2}\int_{-\infty}^{+\infty} xe^{-|x|}dx$

$=\frac{1}{2}\int_{-\infty}^{0} xe^{x}dx+\frac{1}{2}\int_{0}^{+\infty} xe^{-x}dx.$

$=\frac{1}{2}\int_{-\infty}^{0} x\,d(e^{x})-\frac{1}{2}\int_{0}^{+\infty} x\,d(e^{-x}).$

$=\frac{1}{2}\left(\cancel{xe^{x}\Big|_{-\infty}^{0}}-\int_{-\infty}^{0} e^{x}dx\right)-\frac{1}{2}\left(\cancel{xe^{-x}\Big|_{0}^{+\infty}}-\int_{0}^{+\infty} e^{-x}dx\right)$

$=-\frac{1}{2}\int_{-\infty}^{0} e^{x}dx+\frac{1}{2}\int_{0}^{+\infty} e^{-x}dx$

$=-\frac{1}{2}e^{x}\Big|_{-\infty}^{0}-\frac{1}{2}e^{-x}\Big|_{0}^{+\infty}$

$=-\frac{1}{2}+\frac{1}{2}=0$

(2). $E(X^2)=\int_{-\infty}^{+\infty} x^2\frac{1}{2}e^{-|x|}\,dx$

$=\frac{1}{2}\int_{-\infty}^{+\infty} x^2e^{-|x|}dx$

$=\frac{1}{2}\int_{-\infty}^{0} x^2e^{x}dx+\frac{1}{2}\int_{0}^{+\infty} x^2e^{-x}dx.$

$=\frac{1}{2}\int_{-\infty}^{0} x^2d(e^{x})-\frac{1}{2}\int_{0}^{+\infty} x^2d(e^{-x}).$

$=\frac{1}{2}\left(\cancel{x^2e^{x}\Big|_{-\infty}^{0}}-\int_{-\infty}^{0} e^{x}\cdot 2x\,dx\right)-\frac{1}{2}\left(\cancel{x^2e^{-x}\Big|_{0}^{+\infty}}-\int_{0}^{+\infty} e^{-x}\,2x\,dx\right).$

$=-\int_{-\infty}^{0} e^{x}x\,dx+\int_{0}^{+\infty} e^{-x}x\,dx=2.$

$\therefore D(X)=E(X^2)-E^2(X)=2.$

(3). $Y=|X|$ $\quad f_Y(y)=\begin{cases} e^{-y} & y>0 \\ 0. & 其它 \end{cases}$ $\quad f_X(x)=\frac{1}{2}e^{-|x|}\quad x\in(-\infty,+\infty)$

$E(Y)=\int_{0}^{+\infty} ye^{-y}dy=-\int_{0}^{+\infty} y\,d(e^{-y})=\cancel{-ye^{-y}\Big|_{0}^{+\infty}}+\int_{0}^{+\infty} e^{-y}dy=1$

$\because E(XY)=E(X|X|)=\int_{-\infty}^{+\infty} x|x|\cdot\frac{1}{2}e^{-|x|}\,dx$

$=\frac{1}{2}\int_{-\infty}^{0} -x^2e^{x}dx+\frac{1}{2}\int_{0}^{+\infty} x^2\cdot e^{-x}dx$

$=-\frac{1}{2}\int_{-\infty}^{0} x^2e^{x}dx+\frac{1}{2}\int_{0}^{+\infty} x^2e^{-x}dx=0.$

$\therefore \mathrm{cov}.(X,|X|)=E(X|X|)-E(X)\cdot E(|X|)$

$=E(XY)-E(X)E(Y)=0-0=0.$

(4). $E(\sqrt{|X|})=\int_{-\infty}^{+\infty}\sqrt{|x|}\cdot\frac{1}{2}e^{-|x|}\,dx=\int_{0}^{+\infty}\sqrt{x}\cdot e^{-x}dx=\int_{0}^{+\infty} x^{\frac{1}{2}}e^{-x}dx$

$=\Gamma(\frac{3}{2})=\frac{\sqrt{\pi}}{2}$

$\int_{0}^{+\infty} x^{\frac{1}{2}}e^{-x}dx \quad x=\frac{t^2}{2}$

$=\int_{0}^{+\infty}\frac{t}{\sqrt{2}}e^{-\frac{t^2}{2}}\,t\,dt$

$=\frac{\sqrt{\pi}}{2}\int_{-\infty}^{+\infty} t^2\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt$

$=\frac{\sqrt{\pi}}{2}E(X^2)=\frac{\sqrt{\pi}}{2}$

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$*\ \Gamma(r)=\int_{0}^{+\infty} x^{r-1}e^{-x}dx$

3. (1).

X	0	1
p_k	0.3	0.7

$X-1$	-1	0
p_k	0.3	0.7

$E(X-1) = -1\cdot 0.3 + 0\cdot 0.7 = -0.3$

(2).

XY	0	1
p_k	0.6	0.4

$E(XY) = 0\cdot 0.6 + 1\cdot 0.4 = 0.4$

(3).

Y	0	1
p_k	0.4	0.6

$E(Y) = 0\cdot 0.4 + 1\cdot 0.6 = 0.6$. $E(Y^2) = 0\cdot 0.4 + 1\cdot 0.6 = 0.6$

$E(X) = 0\cdot 0.3 + 1\cdot 0.7 = 0.7$ $\quad E(X^2) = 0\cdot 0.3 + 1\cdot 0.7 = 0.7$

$D(Y) = E(Y^2) - E(Y)^2 = 0.6 - 0.6^2 = 0.24$

$D(X) = E(X^2) - E^2(X) = 0.7 - 0.7^2 = 0.21$

$\therefore \rho_{XY} = \dfrac{cov(X,Y)}{\sqrt{D(X)}\cdot\sqrt{D(Y)}} = \dfrac{E(XY) - E(X)E(Y)}{\sqrt{D(X)}\cdot\sqrt{D(Y)}} = -\dfrac{\sqrt{14}}{42} \approx -0.09$

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4. a) $f(x,y) = \begin{cases} 1 & 0 \le y \le 1-|x| \\ 0. & 其他 \end{cases}$

$f_X(x) = \int_0^{1-|x|} 1\, dy = 1-|x| \quad (-1 \le x \le 1)$

$f_Y(y) = \int_{y-1}^{1-y} 1\, dx = 2(1-y) \quad (0 \le y \le 1)$

$\therefore f_X(x) = \begin{cases} 1-|x| & -1 \le x \le 1 \\ 0. & 其它 \end{cases}$

$f_Y(y) = \begin{cases} 2(1-y) & 0 \le y \le 1 \\ 0 & 其它 \end{cases}$

b) $f_{Y|X}(y|x) = \dfrac{f(x,y)}{f_X(x)} = \dfrac{1}{1-|x|} \quad 0 \le x < 1 \quad 0 \le y \le 1-|x|$ $\therefore f_{Y|X}(y|x) = \begin{cases} \dfrac{1}{1-|x|} & 0 \le y \le 1-|x| \\ 0. & 其他 \end{cases}$

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5. (1). $f(x,y) = f_X(x)\cdot f_Y(y) = \begin{cases} 2x\cdot e^{-y} & 0<x<1,\ y>0 \\ 0. & 其它 \end{cases}$

(2). $f_Z(z) = \int_{-\infty}^{+\infty} f(x, z-x)\, dx = \int_{-\infty}^{+\infty} 2xe^{x-z}\, dx$

$0<x<1 \quad z-x>0 \quad \therefore 0<x<\min(z,1)$

$z>1$: $\quad f_Z(z) = \int_0^1 2xe^{x-z}dx = \int_0^1 2x\, d(e^{x-z}) = 2xe^{x-z}\Big|_0^1 - \int_0^1 2\cdot e^{x-z}dx$

$= 2e^{1-z} - 2\int_0^1 e^{x-z}dx = 2\cdot e^{1-z} - 2e^{x-z}\Big|_0^1 = 2\cdot e^{-z}$

$0<z\le 1$ $\quad f_Z(z) = \int_0^z 2xe^{x-z}dx = \int_0^z 2x\, d(e^{x-z}) = 2xe^{x-z}\Big|_0^z - \int_0^z 2\cdot e^{x-z}dx$

$= 2z - 2\int_0^z e^{x-z}dx = 2z - 2\cdot e^{x-z}\Big|_0^z = 2z - 2 + 2e^{-z}$.

$\therefore f_Z(z) = \begin{cases} 0. & z \le 0 \\ 2z-2+2e^{-z} & 0<z\le 1 \\ 2\cdot e^{-z}. & z>1 \end{cases}$

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6. 设P表示售出一盒的收入

则

P	4	5	6
p_k	0.3	0.5	0.2

则 $E(P) = 4\cdot 0.3 + 5\cdot 0.5 + 6\cdot 0.2 = 4.9$

$E(P^2) = 16\cdot 0.3 + 25\cdot 0.5 + 36\cdot 0.2 = 24.5$

$\therefore D(P) = E(P^2) - E^2(P) = 0.49$

$\therefore P\{\sum P_i > 1925\} = P\left\{\frac{\sum P_i - n\cdot E(P)}{\sqrt{n\cdot D(P)}} > \frac{1925 - n\cdot E(P)}{\sqrt{n\cdot D(P)}}\right\}$

$= P\left\{\frac{\sum P_i - nE(P)}{\sqrt{n\cdot D(P)}} > \frac{1925 - 4.9\cdot 400}{\sqrt{400\cdot 0.49}}\right\}$

$= P\left\{\frac{\sum P_i - nE(P)}{\sqrt{n D(P)}} > -\frac{5}{2}\right\}$

$= 1 - \Phi(-\frac{5}{2}) = 1 - (1 - \Phi(\frac{5}{2})) = \Phi(\frac{5}{2}) = 0.994$

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