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International Islamic University Chittagong

BSE-2301

L-01

Exponents

What is Exponent?

An exponent of a number, represents the number of times the number is multiplied to itself. If 8 is multiplied by itself for n times, then, it is represented as:

$8 \times 8 \times 8 \times 8 \times \ldots\ldots n \text{ times} = 8^n$

The above expression, 8^n, is said as 8 raised to the power n. Therefore, exponents are also called power or sometimes indices.

Examples:

- $2 \times 2 \times 2 \times 2 = 2^4$
- $5 \times 5 \times 5 = 5^3$
- $10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$

General Form of Exponents

The exponent is a simple but powerful tool. It tells us how many times a number should be multiplied by itself to get the desired result. Thus any number ‘a’ raised to power ‘n’ can be expressed as:

$$\boxed{a^n = \underbrace{a \times a \times a \times \ldots\ldots\ldots \times a}_{n-times}}$$

Here *a* is any number and *n* is a natural number.

a^n is also called the *nth* power of *a*.

'a' is the base and 'n' is the exponent or index or power.

'a' is multiplied 'n' times, and thereby exponentiation is the shorthand method of repeated multiplication.

Exponent Laws

Different laws of exponents are described based on the powers they bear.

Multiplication Law: *Bases – multiplying the like ones; add the exponents and keep the base the same.*

When bases are raised with power to another, multiply the exponents and keep the base the same.

Division Law: *Bases – dividing the like ones; subtract the exponent of the denominator from the exponent of the numerator Exponent and keep the base the same.*

Let 'a' be any integer or a decimal number and 'm', 'n' are positive integers, that represent the powers to the bases such that the above laws can be written as:

- $a^m . a^n = a^{m+n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $(a/b)^n = a^n/b^n$
- $a^m/a^n = a^{m-n}$
- $a^m/a^n = 1/a^{n-m}$

These laws referred to the properties of exponents. These are used to simplify complex algebraic expressions and write large numbers in an understandable manner.

Exponent Rules

Exponents have certain rules which we apply in solving many problems in math. Some of the exponent rules are given below.

Zero rule: Any number with an exponent zero is equal to 1.

Example: $8^0 = 1$, $a^0 = 1$

One Rule: Any number or variable that has the exponent of 1 is equal to the number or variable itself.

Example: $a^1 = a$, $7^1 = 1$

Negative Exponent Rule: If the exponent value is a negative integer, then we can write the number as:

$a^{-k} = 1/a^k$

Example: $3^{-2} = 1/3^2 = 1/(3 \times 3) = 1/9$

Exponent Table

The below table shows the values of different expressions in terms of exponents along with their expansions and values. This will help you in understanding the simplification of numbers with exponents in detail.

Type of Exponent	Expression	Expansion	Simplified value
Zero exponent	6^0	1	1

One exponent	4^1	4	4
Exponent and power	2^3	2 × 2 × 2	8
Negative exponent	5^{-3}	$1/5^3 = 1/(5 × 5 × 5)$	1/125
Rational exponent	$9^{1/2}$	√9	3
Multiplication	$3^2 × 3^3$	$3^{(2+3)}$ = 35	273
Quotient	$7^5/ 7^3$	$7^{(5-3)} = 7^2$	49
Power of exponent	$(8^2)^2$	$8^{(2*2)} = 8^4$	4096

Solved Questions

Example 1: Write 7 x 7 x 7 x 7 x 7 x 7 x 7 x 7 in exponent form..

Example 2: **Write below problems like exponents:**

1. 3 x 3 x 3 x 3 x 3 x 3
2. 7 x 7 x 7 x 7 x 7
3. 10 x 10 x 10 x 10 x 10 x 10 x 10

Example 3: Simplify $25^3/5^3$

Example 4: Simplify $(3^2 × 3^{-5})/ 9.^{-2}$

Example 5: Simplify and write the answer in exponential form.

(i) $(2^5 \div 2^8)^5 \times 2^{-5}$

(ii) $(-4)^{-3} \times (5)^{-3} \times (-5)^{-3}$

(iii) $(1/8) \times (3)^{-3}$

Practice Problems

1. Find m such that $(-4)^{m+1} \times (-4)^5 = (-4)^7$.

Exponents and Powers Applications

Scientific notation uses the power of ten expressed as exponents, so we need a little background before we can jump in. In this concept, we round out your knowledge of exponents, which we studied in previous classes.

The distance between the Sun and the Earth is 149,600,000 kilometers. The mass of the Sun is 1,989,000,000,000,000,000,000,000,000,000,000 kilograms. The age of the Earth is 4,550,000,000 years. These numbers are way too large or small to memorize in this way. With the help of exponents and powers, these huge numbers can be reduced to a very compact form and can be easily expressed in powers of 10.

Now, coming back to the examples we mentioned above, we can express the distance between the Sun and the Earth with the help of exponents and powers as following:

Distance between the Sun and the Earth 149,600,000 = $1.496 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1.496 \times 10^8$ kilometers.

Mass of the Sun: 1,989,000,000,000,000,000,000,000,000,000,000 kilograms = 1.989×10^{30} kilograms.

Age of the Earth: 4,550,000,000 years = 4.55×10^9 years

ROOTS

1. What are Roots?

Roots are the opposite of exponents. While exponents tell us how many times to multiply a number by itself, roots tell us which number, when multiplied by itself a certain number of times, gives us the original number. The most common root is the square root, but there are also cube roots, fourth roots, and so on.

2. The Square Root

- Definition: The square root of a number x is a value that, when multiplied by itself, gives x. It is represented as $\sqrt{x}$.
- Example:
  - $\sqrt{9}=3$ because $3\times3=9$
  - $\sqrt{16}=4$ because $4\times4=16$
- Important Points:
  - The square root of 0 is 0.
  - Square roots are usually positive because they refer to the principal (positive) root.
  - For every positive number, there are two square roots: a positive and a negative (e.g., $\sqrt{9}=3$ and -3, since $(-3)\times(-3)=9$).

3. The Cube Root

- Definition: The cube root of a number x is a value that, when used three times in multiplication, gives x. It is represented as $\sqrt[3]{x}$.
- Example:
  - $\sqrt[3]{27}=3$ because $3\times3\times3=27$.
  - $\sqrt[3]{-8}=-2$ because $(-2)\times(-2)\times(-2)=-8$
- Important Points:
  - Cube roots can be positive or negative, depending on the original number.

- Unlike square roots, cube roots of negative numbers are real because a negative times a negative times a negative is negative.

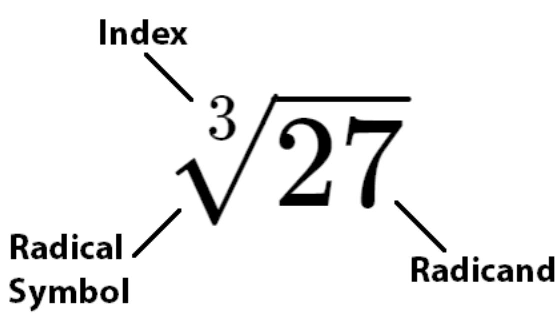
4. Higher Roots

- Definition: Higher roots follow the same principle but involve higher exponents. For instance, the fourth root of a number x (written as $\sqrt[4]{x}$) is a number that, when used four times in multiplication, gives x.
- Example:
  - $\sqrt[4]{16}$=2 because 2×2×2×2=16

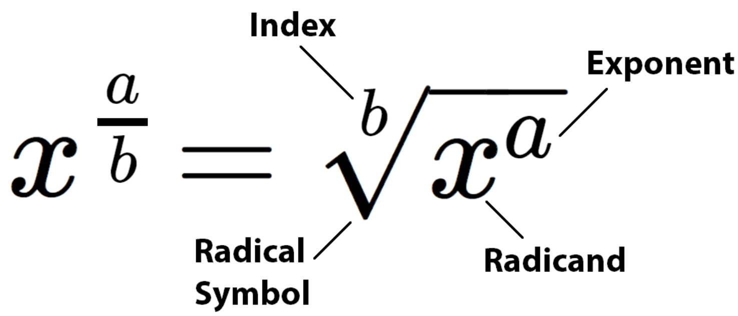
5. How to Simplify Roots

- Perfect Squares: Numbers like 1, 4, 9, 16, 25, etc., are perfect squares because they are squares of whole numbers. Their roots are whole numbers (e.g., $\sqrt{25}$ =5).
- Non-Perfect Squares: For numbers that aren't perfect squares, like 8 or 20, their square roots aren't whole numbers. You can simplify them into a simpler form using factors:
  - For example, $\sqrt{8}$=$\sqrt{4 * 2}$ =$\sqrt{4}$ *$\sqrt{2}$ =2$\sqrt{2}$

To generalize the rule of finding roots, let's introduce the notation of **radicals**. Let's break down this notation into "parts" by looking at the cube root of 27, which looks like this: $\sqrt[3]{27}$

- 

Let's make an important link from radical notation to exponent notation.



A radical is the same as raising a base to a fractional exponent, where the index of the radical becomes the denominator of the fractional exponent and the exponent of the radical is the numerator of the fractional exponent. Here is a generalized example:

Let's say that the radicand of a radical is x^a, and the index of the radical is b. This is equivalent to raising a base,x, to the fractional exponent of a/b.

First, let's practice expressing a radical as a fractional exponent: $\sqrt[3]{x^{2}}$

Remember, the index of the radical becomes the denominator of the fractional exponent, which in this case is 3. The exponent of the radicand becomes the numerator of the fractional exponent, which is 2 in this case. So, our fractional exponent is $x^{2/3}$

Try yourself :1) $\sqrt[4]{3}$ = ?

2) $\sqrt{18}$ = ?

6. Roots and Real-Life Examples

- Area of a Square: If you know the area of a square and want to find the length of one side, you use the square root. For example, if the area is 25 square units, then each side is $\sqrt{25}$=5 units.
- Volume of a Cube: If you know the volume of a cube and want to find the length of one edge, you use the cube root. For example, if the volume is 64 cubic units, then each edge is $\sqrt[3]{64}$=4 units.

Factorization of Whole Numbers

1. What is Factorization?

Factorization is the process of breaking a whole number down into smaller numbers that multiply together to give the original number. These smaller numbers are called factors. For instance, for the number 12, you can write it as:

$12 = 3 \times 4$ or $12 = 2 \times 6$

When you keep breaking these factors down until all are prime numbers, you've completed the prime factorization.

2. Prime Numbers and Composite Numbers

- Prime Numbers: Prime numbers are numbers greater than 1 that have only two factors: 1 and the number itself (e.g., 2, 3, 5, 7, 11, etc.).
- Composite Numbers: Composite numbers are whole numbers greater than 1 that have more than two factors (e.g., 4, 6, 8, 9, 12, etc.). These can be factorized into smaller prime numbers.

3. Steps for Prime Factorization

Prime factorization involves breaking down a composite number into a product of prime numbers. Here's how you can do it:

1. Start with the smallest prime number (2).
2. Check if the number is divisible by that prime. If it is, divide the number by that prime and write down the factor. Repeat until it is no longer divisible by that prime.
3. Move to the next smallest prime number. Continue the process with the next prime (3, then 5, then 7, etc.).
4. Repeat until the result is 1. The prime numbers you used for division are the prime factors.

4. Example of Prime Factorization

Let's take a detailed look at factorizing the number 180.

Step-by-Step Process:

1. Divide by 2 (the smallest prime number):

- $180 \div 2 = 90$
- So, one factor is 2.

2. Divide by 2 again (since 90 is even):
   - $90 \div 2 = 45$
   - Another factor is 2.
3. Move to the next smallest prime, 3 (since 45 is odd):
   - $45 \div 3 = 15$
   - A factor is 3.
4. Divide by 3 again (since 15 is divisible by 3):
   - $15 \div 3 = 5$
   - Another factor is 3.
5. Move to 5, the next prime factor (since the result is 5, which is prime):
   - $5 \div 5 = 1$
   - The last factor is 5.

Thus, the prime factorization of 180 is:

$180 = 2 \times 2 \times 3 \times 3 \times 5$ or $180 = 2^2 \times 3^2 \times 5$

5. Applications and Importance of Factorization

- Simplifying Fractions: By breaking down numbers into prime factors, you can easily simplify fractions by canceling out common factors in the numerator and denominator.
- Finding GCD (Greatest Common Divisor): The GCD of two numbers can be found by taking the lowest powers of all common prime factors.
- Finding LCM (Least Common Multiple): The LCM of two numbers can be found by taking the highest powers of all prime factors involved.
- Solving Equations: In algebra, factorization is used to simplify and solve polynomial equations, especially quadratics. For example, solving x2−5x+6=0x^2 - 5x + 6 = 0x2−5x+6=0 by factorizing to (x−2)(x−3)=0(x - 2)(x - 3) = 0(x−2)(x−3)=0.

Let's find the Greatest Common Factor (GCF) of two numbers using factorization.

Example: Find the GCF of 48 and 180 Using Factorization

Step 1: Factorize Each Number into Prime Factors

Factorize 48:

- Start with the smallest prime, 2: $48 \div 2 = 24$
- Divide 24 by 2: $24 \div 2 = 12$
- Divide 12 by 2: $12 \div 2 = 6$
- Divide 6 by 2: $6 \div 2 = 3$
- Finally, divide 3 by 3: $3 \div 3 = 1$
- So, the prime factorization of 48 is: $48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$

Factorize 180:

- Start with 2: $180 \div 2 = 90$
- Divide 90 by 2: $90 \div 2 = 45$
- 45 is not divisible by 2, so move to 3: $45 \div 3 = 15$
- Divide 15 by 3: $15 \div 3 = 5$
- Finally, divide by 5: $5 \div 5 = 1$
- So, the prime factorization of 180 is: $180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$

Step 2: Identify the Common Prime Factors with the Lowest Powers

Now, list the common prime factors from both numbers with the lowest power for each:

- **Common factor of 2:** 2^2 (the lowest power of 2 between 2^4 from 48 and 2^2 from 180).
- **Common factor of 3:** 3 (the lowest power of 3 between 3 from 48 and 3^2 from 180).

Step 3: Multiply the Common Factors to Find the GCF

$GCF = 2^2 \times 3 = 4 \times 3 = 12$

Conclusion

The GCF of 48 and 180 is **12**.

Explanation

By using prime factorization, we broke down each number into its prime components, compared the factors, and selected the smallest powers of the common primes. This approach ensures that the GCF is the highest number that can divide both original numbers without leaving a remainder.

Let's find the Least Common Multiple (LCM) of two numbers using factorization.

Example: Find the LCM of 48 and 180 Using Factorization

Step 1: Factorize Each Number into Prime Factors

1. **Factorize 48:**
   - The prime factorization of 48 is: $48=2\times2\times2\times2\times3=2^4\times3$
2. **Factorize 180:**
   - the prime factorization of 180 is: $180=2\times2\times3\times3\times5=2^2\times3^2\times5$

Step 2: Identify All Prime Factors Using the Highest Powers

To find the LCM, you take the highest power of each prime number from the factorizations:

- **Prime factor 2:** Highest power is 2^4 (from 48).
- **Prime factor 3:** Highest power is 3^2 (from 180).
- **Prime factor 5:** Highest power is 5 (from 180).

Step 3: Multiply the Highest Powers to Find the LCM

$LCM=2^4\times3^2\times5=16\times9\times5=144\times5=720$

Conclusion

The LCM of 48 and 180 is **720**.

Explanation

To find the LCM using factorization, you:

1. Factorize each number into its prime factors.
2. Identify all the unique prime factors across both numbers.
3. Use the highest power of each prime factor found in either number.
4. Multiply these highest powers together to get the LCM.

This method ensures that the LCM is the smallest number that both original numbers can divide into without leaving a remainder, making it the least common multiple.

Try yourself : Find the GCF and LCM of 48, 180, and 240 Using Factorization