L4

Geometry is a branch of mathematics that studies shapes, sizes, and the properties of space. It is concerned with points, lines, angles, surfaces, and solids, and how they interact with each other. Geometry helps us understand the physical space around us and is essential for various fields like architecture, engineering, physics, and even art.

Triangle

A triangle is a three-sided polygon with three vertices (corners) and three edges (sides). It is one of the simplest shapes in geometry. The sum of the interior angles of any triangle is always 180°.

a) Area of a Triangle

The area of a triangle depends on its base and height. The general formula is:

Area=$\frac{1}{2}$×Base×Height

- **Base** is the length of one side of the triangle.
- **Height** is the perpendicular distance from the base to the opposite vertex.

For example, if the base is 6 cm and the height is 4 cm:

Area=$\frac{1}{2}$×6×4=12cm^2

b) Perimeter of a Triangle

The perimeter of a triangle is the sum of the lengths of its sides. For a triangle with side lengths a, b, and c:

Perimeter=a+b+c

For example, if the sides are 3 cm, 4 cm, and 5 cm:

Perimeter=3+4+5=12 cm

c) Special Triangles:

1. **Equilateral Triangle** (all sides are equal):
    - **Area**: Area= $\frac{\sqrt{3}}{4} \times side^2$
    - **Perimeter**: Perimeter=3×Side
2. **Isosceles Triangle** (two sides are equal):
    - Perimeter: 2×Equal Side+Base

Rectangle

A rectangle is a four-sided polygon (quadrilateral) where opposite sides are equal in length, and all angles are 90°. It is a specific type of parallelogram where the angles are right angles.

a) Area of a Rectangle

The area of a rectangle is the product of its length and width:

Area=Length×Width

For example, if the length is 8 cm and the width is 5 cm:

Area=8×5=40 cm^2

b) Perimeter of a Rectangle

The perimeter of a rectangle is the total distance around the shape:

Perimeter=2×(Length+Width)

For example, if the length is 8 cm and the width is 5 cm:

Perimeter=2×(8+5)=2×13=26 cm

Circle

A circle is a round, two-dimensional shape where all points are equidistant from a fixed point known as the center. The distance from the center to any point on the circle is called the radius, and the distance across the circle passing through the center is the diameter (which is twice the radius).

a) Area of a Circle

The area of a circle is determined by its radius r:

Area=π×r^2

Where π≈3.1416.

For example, if the radius is 7 cm:

Area=π×7^2=π×49≈153.94 cm^2

b) Perimeter/Circumference of a Circle

The Perimeter is the distance around the circle:

Perimeter=2×π×r

For a circle with radius 7 cm:

Perimeter=2×π×7≈43.98 cm

Algebraic Expressions and Equations

1. What is an Algebraic Expression?

An algebraic expression is a combination of numbers, variables (letters), and mathematical operations like addition, subtraction, multiplication, or division. Expressions do not have an equal sign (=).

- Variables are symbols (usually letters like x,y,z) that represent unknown values.
- Constants are fixed numbers.

Examples of Algebraic Expressions:

- 3x+5
- 2a−7
- $4y^2$+2y−1

Here's how to understand them:

- In 3x+5, the variable is x, the number next to x (3) is the coefficient, and 5 is a constant.
- In $4y^2$+2y−1, y is the variable, y^2 means y is squared, and each term is either multiplied or added together.

Types of Algebraic Expressions:

1. **Monomial**: An expression with just one term.
    a. Example: 5x, $-3y^2$, or 7
2. **Binomial**: An expression with two terms.
    a. Example: 2x+3, x−5,

3. **Trinomial**: An expression with three terms.
    a. Example: x^2+3x+2, 5a^2−2a+1
4. **Polynomial**: An expression with multiple terms (can be a monomial, binomial, trinomial, etc.).
    a. Example: 3x^3+2y^2−x+5

Simplifying Algebraic Expressions:

You can simplify algebraic expressions by **combining like terms**. Like terms are terms that have the same variable raised to the same power.

- Example: Simplify 3x+2x−5
    - Combine the like terms: (3x+2x)−5=5x−5

2. What is an Algebraic Equation?

An algebraic equation is a statement that two algebraic expressions are equal. It has an equal sign (=), and it shows the relationship between two sides of the equation.

Examples of Algebraic Equations:

- 2x+3=7
- 4a−5=11

In an equation, you can solve for the variable to find its value.

For example, to solve 2x+3=7:

1. Subtract 3 from both sides: 2x=4
2. Divide both sides by 2: x=2

Types of Equations:

1. **Linear Equation**: An equation where the variable has a maximum exponent of 1. The graph of a linear equation is a straight line.
    - Example: 2x+3=7
    - Solving: Subtract 3 from both sides to get 2x=4, then divide both sides by 2: x=2.

2. **Quadratic Equation**: An equation where the variable is squared (exponent of 2). The graph of a quadratic equation is a parabola.
    - Example: $x^2-5x+6=0$
    - You can solve it using factoring, the quadratic formula, or completing the square.
3. **Cubic Equation**: An equation where the variable is raised to the power of 3.
    - Example: $x^3-3x+2=0$

More Complex Equations:

Sometimes, equations have variables on both sides or multiple terms involving variables.

Example: Solve $3x-4=2x+1$

1. Subtract 2x from both sides to get the variables on one side:
   $3x-2x-4=2x-2x+1$, so $x-4=1$
2. Add 4 to both sides:
   $x-4+4=1+4$, so $x=5$

The solution is $x=5$.

Practice Problems:

1. Simplify the expression: $2x+3x-4$
2. Solve the equation: $4y-7=9$
3. Combine like terms: $5a+2a-3a$
4. Solve: $3x+1=10$