Optimal Frequency Regulation of a Two-area Power System

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***Abstract*—Load frequency controllers are required to operate effectively under increasingly restrictive conditions. These include highly varying random fluctuations in power demand which changes the operating point of the power system, in turn resulting in parametric changes and uncertainties in the power system. The frequency and tie-line power deviations need to be regulated in minimum time within narrow limits taking into account the associated constraints on the actuators. In this paper, we consider the application of an Linear Quadratic Gaussian (LQG) controller to a two-area system which suffers from big changes in the parameters of the plant. Using appropriate model reduction techniques, the performance of a reduced-order controller is evaluated under sudden load changes represented as constant disturbances and under parametric changes. We show that the LQG controller performs significantly better and provides greater margin of stability than a pair of optimized PID controller, designed for each area separately.**

***Index Terms*—Optimal control, Power system, Integral action, Frequency regulation**

I. Introduction

Electric utilities are constantly faced with increasing exigencies on the quality and reliability of electrical power that they supply to consumers. Varying load demands are required to be matched continuously, while maintaining frequency and voltage within narrow bounds. Voltage regulation is normally achieved by automatic voltage regulators at generator level, or controlling the reactive power at a given bus through shunt capacitors, static VAr compensators and synchronous condensers. The bus voltages are affected to a lower extent by changes in real load power. On the other hand, the system frequency needs to be controlled within tighter tolerances, since it is a measure of the mismatch between real generated power and real power demand plus losses within the network. Moreover, the speed of line-connected ac motors is directly affected by the supply frequency.

Dynamic changes in real power demand are reflected on the load torque at the shaft of individual synchronous generators, hence affecting their operating speed and resulting frequency output. Under normal conditions, the load power continuously varies, and unpredictable events like sudden loss of generation or heavy load switching may cause drastic frequency deviations from the reference value. In systems where the prime-movers consist of steam or hydro turbines, speed governing schemes ensure that frequency deviations are minimized in the steady state. In interconnected power systems, the Area Control Error (ACE) is applied to an Automatic Generation Control (AGC) scheme to regulate both the frequency and tie-line power deviations for each prescribed area. Such Load Frequency Control (LFC) systems are also required to restore the frequency towards the nominal value in minimum time, while keeping the instantaneous frequency changes within acceptable margins.

It is critical that the LFC system takes into account parameter uncertainties that characterize the power system model, as well as measurement noises that can significantly affect the quality of signals in the feedback loops. While frequency regulation has been achieved conventionally by the use of Proportional-Integral-Derivative (PID) controllers [1], parameter variations or uncertainties may degrade the controller performance under changing operating conditions. In particular, since the PID scheme is based on a linearized model of the power system, deviations from this linearized model by an unavoidable change in operating conditions can severely affect performance and also make the closed-loop system unstable. Recently, several schemes have been proposed to improve the performance of the LFC system. In [2], an adaptive controller based on temporal difference learning neural networks is presented to address parameter uncertainties and changes in the operating point of the power system. The proposed controller requires as measurement the tie-line frequency and power deviations, as well as an estimate of the load perturbation. The neural network proposed also requires a complex training procedure. A Linear Quadratic Regulator (LQR) with reduced order observer for a deregulated power system is presented in [3], where the sensitivity to plant parameter variations is reduced. A discrete-time sliding mode controller for load frequency control is proposed in [4] which in particular addresses time delays in control signal transmission over long distances. The controller requires the development of state estimator and predictor stages for computing the control signals. The ability of the resulting system to cope with parameter uncertainties is however not demonstrated. The use of Superconducting Magnetic Energy Storage (SMES) units with fuzzy gain scheduling are proposed

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in [5], as a supplement for improving the performance of LFC systems. SMES and their associated power conversion system need to be installed in each area of the power system, hence considerably increasing cost and feasibility constraints. Finally, in [6], a multi-objective PID controller based on adaptive weighted particle swarm algorithm is used to tune the controller gains against parameter variations. The performance of the proposed scheme in an interconnected power system is demonstrated for small disturbances.

The Linear Quadratic Gaussian (LQG) control technique is well suited for systems which are subjected to random disturbances and where it is assumed that a ball of uncertainty exists around most of the parameters of the system. These features of the approach make it particularly suitable for load frequency control problem in power systems. Indeed, its application to the LFC problem in [7], [8] and [9] have demonstrated the suitability of the LQG design approach for small load power changes. Development of the controller is based on the separation theorem [10], wherein a Kalman filter is used to generate the minimum mean square linear estimates of the states of the system, which are then used by an optimal state feedback control law to regulate the ACE to zero. One drawback of the approach however is that the complexity of the controller designed is of the same order as that of the plant. For power system models, the order of the system increases linearly with the number of area subsystems. In this paper, we propose a reduced order LQG controller which provides optimal frequency regulation for a two-area interconnected system, while successfully correcting for constant disturbances. Moreover, to drive both the frequency error and the area control error to zero, it is necessary to include some form of integral action. By including integral action in our design, an additional variable is included in a modified cost function, which is also minimized. The proposed scheme caters for process noise and parameter uncertainties inherent in the main blocks of the power system model. We compare the performance of the multivariable LQG controller with optimally designed PID controllers for each area. Moreover, we also compare the effect of parametric uncertainties on the performance and stability of the system. The paper is organized as follows: Section II describes the model of the two-area interconnected power system. In Section III, we present the control setup and scheme used for designing a multivariable LQG controller which includes integral action. Simulation results are shown in Section IV and we conclude with a summary of the paper.

II. Power System Model

The model used to describe the dynamics of the power system is derived by linearizing about a steady-state operating point around which perturbations occur in the frequency and power variables. Since the system frequency is primarily affected by disturbances in the real power demand [11], the interaction between the real power/ frequency control loop and reactive power/voltage control loop is neglected. Furthermore, voltage disturbances are corrected much faster than the power/frequency disturbances, which depend on the slow dynamics of the turbine and generator. Hence the model is based on the power and frequency interactions in the power system. Furthermore, the steam turbines driving generators are of the non-reheat type with first-order characteristics.

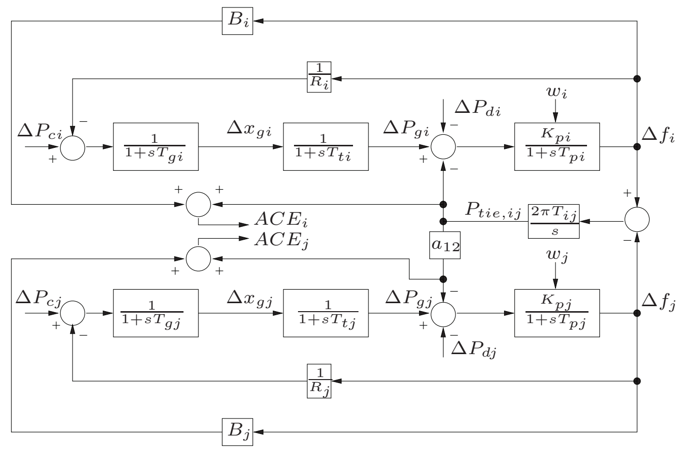

Fig. 1. Two-area system.

The uncontrolled two-area interconnected power system model is shown in shown in Fig. 1, where, for the subsystem area i:

T_{pi}	Power system time constant(s)
K_{pi}	Steady-state gain of power subsystem (Hz/ p.u. MW)
a_{ij}	Tie line coupling coeff. between areas i and j
T_{gi}	Governor time constant(s)
R_i	Frequency droop sensitivity due to governor action (Hz/ p.u. MW)
T_{ti}	Time constant of non-reheat type steam turbine(s)
T_{ij}	Tie line synchronizing coefficient(s)
Δf_i	Frequency deviation(Hz)
Δx_{gi}	Change in governor valve position (p.u.)
ΔP_{gi}	Change in real power generated (p.u. MW)
$\Delta P_{tie,ij}$	Change in tie line real power between areas i and j (p.u. MW)
ΔP_{ci}	Change in real power command at speed governor (p.u. MW)
ΔP_{di}	Change in real power demand (p.u. MW)
B_i	Frequency bias factor(p.u. MW/ Hz)

Similar notation applies for other areas. The Area Control Error (ACE) of area i can be expressed as

$$ACE_i = \Delta P_{tie,ij} + B_i \Delta f_i. \tag{1}$$

By setting the appropriate power commands at the governors, it is required to regulate the frequency and tie-line power deviations so that each ACE is driven to zero. The uncontrolled two-area system can then be described in state-space form by a 7th order linear model as:

$$\dot{x} = Ax + B_1 u + B_2 w; \tag{2}$$

$$y = [ACE_i \quad ACE_j]^T. \tag{3}$$

where

$$\begin{aligned} x &= [\Delta f_i \;\; \Delta f_j \;\; \Delta x_{gi} \;\; \Delta x_{gj} \;\; \Delta P_{gi} \;\; \Delta P_{gj} \;\; \Delta P_{tie,ij}]^T ; \\ u &= [\Delta P_{ci} \;\; \Delta P_{cj}]^T ; \\ w &= [w_i \;\; w_j]^T . \end{aligned}$$

Here, w represents the noise sources which feed into the system in the form of parameter uncertainties in the generating units. Also, P_{di} represents changes in the loading conditions in each control area and can be modeled as low frequency or fixed D.C. disturbances. Traditionally, each individual ACE is applied to an AGC to compute the real power commands, i.e., ΔP_{ci} and ΔP_{cj}, so as to restore the balance between load and generated powers. Thus, each AGC acts independently of other measurements made on the power system and thus cannot fully take advantage of potential sharing of loads that may occur with changing power demands. In this case, we propose a multivariable controller which takes measurements from both control areas and generate optimal control signals to be applied to each area.

III. Linear Quadratic Gaussian Control

In the papers [12], [13], the linear filtering problem was solved and the resulting optimal estimator is known as the Kalman-Bucy filter. This result allows us to design an estimator for a linear system, which given the probabilistic distribution (statistics) of noise(s) entering the system, provides the best possible estimate of the states of the system. By "best possible" here, we roughly mean an estimator whose output is closest to what it should be, despite the noise(s). An important assumption in the realisation of this optimal estimator is the nature of the noise(s) present. It is assumed that the noise(s) are white, Gaussian and have zero mean. White noise implies that the noise is uncorrelated from one instant of time to another while the second property implies that the covariance of the noise, $\mathbf{E}[v(t)\; v'(\tau)]$, where $v(t)$ is one source of noise feeding into the system, provides all the necessary probabilistic information about the noise. This mathematical assumption turns out to be quite convenient as most naturally occurring physical systems are indeed afflicted by Gaussian processes with zero mean. For the interested reader, a more detailed description of filtering theory is provided in [14].

The state feedback quadratic optimal control problem on the other hand was solved in terms of Riccati equations in [15]. Combining the optimal state-feedback law and the state-estimation method leads to the solution of the LQG optimal control problem. This result is also commonly referred to in the control literature as the "separation theorem" as the two separate problems are tackled independently before being combined; see also [16].

The LQG problem is then one where a controller is designed to minimize a cost-function of the form:

$$V = \mathbf{E}\left[\int_0^T \left[x^T Q x + u^T R u\right] dt\right], \tag{4}$$

where $Q \geq 0$ and $R > 0$.

An LQG performance criterion as described in (4) will result in a closed-loop system where the state x and the control signal u will be stationary random processes. The nature of the problem at hand is such that it is subjected to disturbances with high-frequency as well as very-low frequency components. In most cases, the low frequency disturbances can be regarded as being D.C. components as they represent constant changes in load which are maintained for long periods of time. This naturally implies a need for a constant non-zero control signal u which will drive the cost function as defined by (4) to a very large value in a short time. Minimizing the cost function to obtain an appropriate controller under such circumstances will not be feasible. We know however that to provide a constant control signal to deal with constant fixed D.C. disturbances would require integral action in the controller. We deal with this problem by introducing integral action in the standard LQG controller design process. Integral action is included by introducing an additional term in the cost function which involves the integral of the output. Furthermore, we also generate an additional fictitious output of the system by integrating the output y. This new output $\int y\, dt$ is also fed to the Kalman filter, which when combined with an optimal state feedback control law leads to an LQG integral optimal controller. This controller will then meet the desired performance requirements as described above as well as reject low frequency disturbances. Fig. 2 shows the multivariable integral LQG controller design setup.

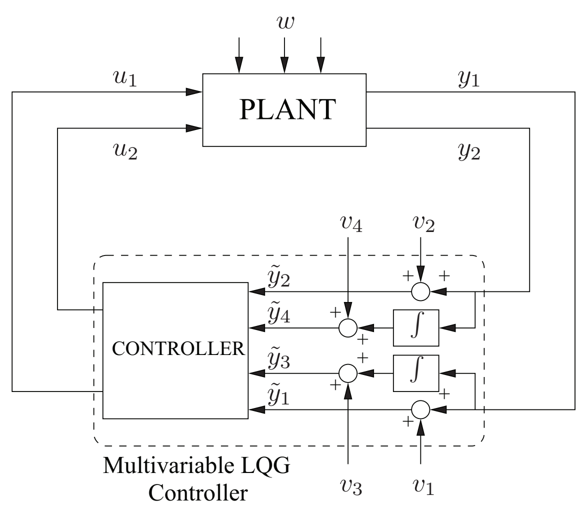

Fig. 2. Multivariable integral LQG controller design configuration.

In this problem, there are two sources of low frequency noises ΔP_{di} and ΔP_{dj}, as described in Sec. II. The standard LQG cost functional is thus modified to include an additional term which involves the integral of the outputs. Let us introduce the integral operator L where,

$$L(y) = \begin{bmatrix} L(y_1) \\ L(y_2) \end{bmatrix} = \begin{bmatrix} \int_0^T y_1(\tau)\, d\tau \\ \int_0^T y_2(\tau)\, d\tau \end{bmatrix}. \tag{5}$$

We can now formulate the integral LQG performance criterion as the minimisation of the following cost function:

$$\mathcal{J} = \lim_{T\to\infty} \mathbf{E}\left[\frac{1}{T}\int_0^T x^T Q x + L(y)^T \bar{Q} L(y) + u^T R u \, dt\right]. \tag{6}$$

We choose the matrices Q, $\bar{Q}$, and R such that:

$$x^T Q x = x^T C^T \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} C x = q_1|z_1|^2 + q_2|z_2|^2;$$

$$L(y)^T \bar{Q} L(y) = L(y)^T \begin{bmatrix} q_3 & 0 \\ 0 & q_4 \end{bmatrix} L(y)$$
$$= q_3|L(y_1)|^2 + q_4|L(y_2)|^2;$$
$$u^T R u = u^T \begin{bmatrix} r_{w1} & 0 \\ 0 & r_{w2} \end{bmatrix} u = r_{w1}|u_1|^2 + r_{w2}|u_2|^2.$$

Here, $q_1, q_2, q_3, q_4, r_{w1}, r_{w2} > 0$ are treated as design parameters. The expectation in (6) is with respect to the Gaussian quantum and classical noise processes, and the assumed Gaussian initial conditions. In order to apply the standard LQG technique to this system, we include the integrators as part of an augmented system which we define next. If we let

$$\tilde{x} = \begin{bmatrix} x \\ L(y) \end{bmatrix}; \quad \tilde{y} = \begin{bmatrix} \tilde{y}_1 \\ \tilde{y}_2 \\ \tilde{y}_3 \\ \tilde{y}_4 \end{bmatrix}; \text{ and } \tilde{w} = \begin{bmatrix} w \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix};$$

we can rewrite the dynamics of the complete system as

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}_1 u + \tilde{B}_2 \tilde{w}; \tag{7}$$
$$\tilde{y} = \tilde{C}\tilde{x} + \tilde{D}\tilde{w}. \tag{8}$$

Here,

$$\tilde{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}; \quad \tilde{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}; \quad \tilde{B}_2 = \begin{bmatrix} B_2 & 0 \\ D & \end{bmatrix};$$
$$\tilde{C} = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}; \quad \tilde{D} = \begin{bmatrix} D & I_{4\times 4} \\ 0 & \end{bmatrix}.$$

After the controller design, the integrators are pulled back from the augmented system and are included as part of the controller, as shown in Fig. 2. The performance criterion can now be reformulated as:

$$\mathcal{J} = \lim_{T\to\infty} \mathbf{E}\left[\frac{1}{T}\int_0^T \tilde{x}^T \tilde{Q}\tilde{x} + u^T R u \, dt\right], \tag{9}$$

where

$$\tilde{Q} = \begin{bmatrix} Q & 0 \\ 0 & \bar{Q} \end{bmatrix}. \tag{10}$$

The multivariable controller is constructed by estimating the states of the augmented system described by (7) using a Kalman filter and combining it with an optimal state feedback control law as follows:

$$u = -K\hat{\tilde{x}}. \tag{11}$$

The optimal feedback gain matrix is given by

$$K = R^{-1}\tilde{B}_1^T X, \tag{12}$$

where X satisfies the matrix Riccati equation:

$$\bar{Q} - R^{-1} X \tilde{B}_1 \tilde{B}_1^T X + X\tilde{A} + \tilde{A}^T X = 0. \tag{13}$$

The optimal observer dynamics (Kalman filter) are described by

$$d\hat{\tilde{x}} = \tilde{A}\hat{\tilde{x}}\, dt + \tilde{B}_1 u\, dt + L[d\tilde{y} - \tilde{C}\hat{\tilde{x}}\, dt]; \tag{14}$$

and for the case of uncorrelated process and measurement noises, the solution of the optimal observer is obtained by choosing the gain matrix

$$L = P\tilde{C}^T V_2^{-1}, \tag{15}$$

where

$$0 = \tilde{A}P + P\tilde{A}^T + V_1 - P\tilde{C}^T V_2^{-1}\tilde{C}P. \tag{16}$$

Here,

$$V_1 = \tilde{B}_2\, \mathbf{E}[w\, w^T]\, \tilde{B}_2^T = \tilde{B}_2 \begin{bmatrix} \epsilon_1^2 & 0 \\ 0 & \epsilon_2^2 \end{bmatrix} \tilde{B}_2^T; \tag{17}$$

$$\text{and } V_2 = \mathbf{E}[n\, n^T] = \begin{bmatrix} \epsilon_3^2 & 0 & 0 & 0 \\ 0 & \epsilon_4^2 & 0 & 0 \\ 0 & 0 & \epsilon_5^2 & 0 \\ 0 & 0 & 0 & \epsilon_6^2 \end{bmatrix}; \tag{18}$$

define the covariance of the process and measurement noises respectively. The design parameters for the regulator are selected for good performance as $r_{w1} = r_{w2} = 3$, and $q_1 = q_2 = 1$ and $q_3 = q_4 = 100$. Similarly, for the filter, the parameters are chosen as $\epsilon_1 = \epsilon_2 = 0.1$ and $\epsilon_3 = \epsilon_4 = \epsilon_5 = \epsilon_6 = 0.01$. The controller designed is of 11th order in this case. We reduce the order of the controller by first computing the Hankel singular values of the full-order controller. The Hankel singular values provide a relativistic measure of the contribution of each state of a linear time-invariant system to the input/output behaviour of the system. The small Hankel singular values can be discarded to reduce the order of the system. The Hankel singular value plot for the designed controller is shown in Fig. 3. The first two states in red show the two "unstable" poles at the origin with which the original controller was augmented with. Then we selected to use 2 more states to approximate the full-order controller using a balanced reduction approach; see [17]. The stability of the system with the reduced controller was then determined and tested.

IV. Simulation Results

The parameters of the power system are given in Table I.

For comparison purposes with conventional controllers used for this type of problem, we also designed optimized proportional-integral (PI) controllers for each control area separately. The "robust response time" tuning algorithm was used and the controllers were then fine-tuned to obtain satisfactory gain and phase margin. The transfer function of the resulting PI controllers are given as:

$$K_{PI,1}(s) = \frac{0.30398}{s} \quad \text{and} \quad K_{PI,2}(s) = \frac{0.65491}{s}. \tag{19}$$

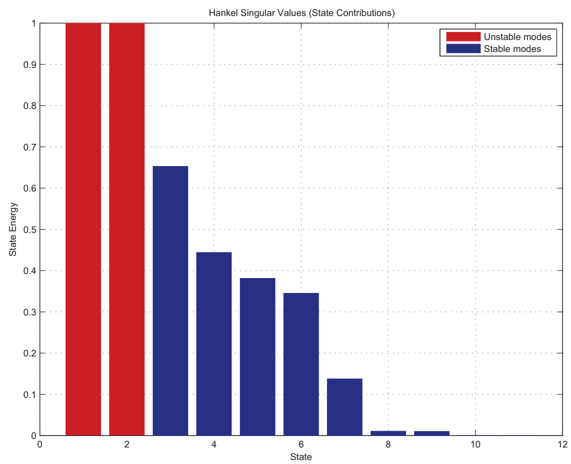

Fig. 3. Hankel singular values of the full controller.

TABLE I
MODEL PARAMETERS

	Values		Values
T_{p1}	19.57 sec	T_{p2}	10.67 sec
K_{p1}	65.22 Hz/p.u.MW	K_{p2}	55.56 Hz/p.u.MW
T_{g1}	0.09 sec	T_{g2}	0.07 sec
R_1	2.0 Hz/p.u.MW	R_2	2.0 Hz/p.u.MW
T_{t1}	0.2 sec	T_{t2}	0.13 sec
B_1	1.0 p.u.MW/Hz	B_2	1.0 p.u.MW/Hz
a_{12}	-1.5	T_{12}	0.094 sec

The power system is then simulated with a change in loading conditions applied in each area. To show the behavior of the closed-loop system, we apply a change of 10% in the load or 0.1 per-unit at times t=3 seconds in area 1 and a change of -10% in area 2 at t=10 seconds. The response of the power system with the LQG controller and with the PI controllers are shown in Fig. 4 and Fig. 5 respectively. The controllers provides good performance allowing a maximum deviation of about ± 0.14 Hz and the frequency error returns to zero in a short time of 4 seconds. It can be noted however that the PI controllers tend to provide a more oscillatory performance, especially when the disturbance is applied in area 2. In both cases, the control energy applied are also comparable although the PI controllers tend to be a bit more aggressive. The controlled system also shows that although a change in control energy is noted in both areas after the application of a disturbance in any given area, the system ultimately stabilizes with most control energy coming from or being taken from the control area in which the disturbance occurred.

Next, we introduce parameter variations in our simulation model and compare the performance of the controllers. In particular, we allow variations in the parameters of the two generating units. We choose the parameters in the generating

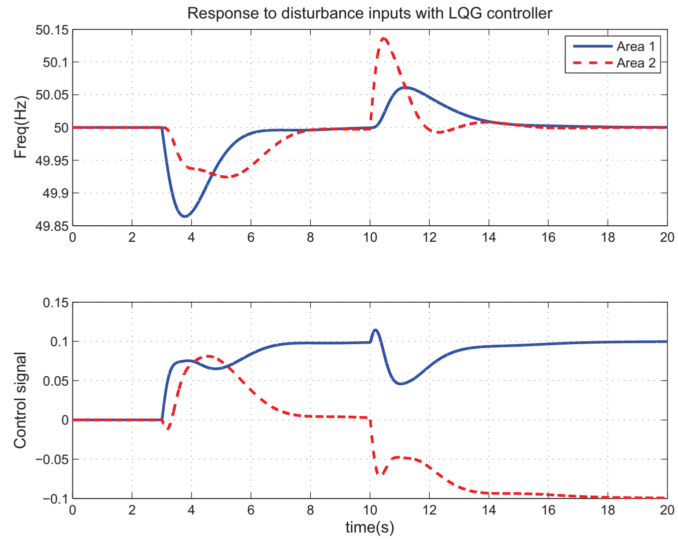

Fig. 4. Time-response of power system with multivariable LQG controller.

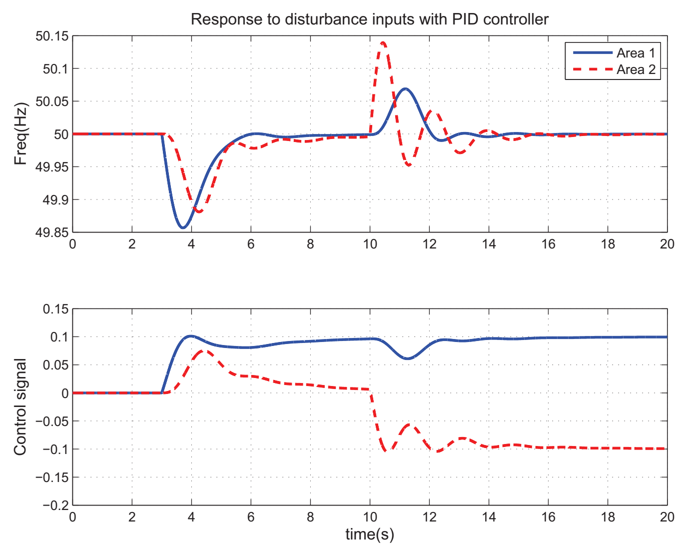

Fig. 5. Time-response of power system with optimized PI controllers.

units as we allowed for process noise only for those parameters during the design stage. The parameters are modified as shown in Table II. The response of the perturbed power system

TABLE II
NEW MODEL PARAMETERS

Parameters	Change	New Value
T_{p1}	-65%	6.85 sec
K_{p1}	$+100\%$	130.44 Hz/p.u.MW
T_{p2}	$+100\%$	21.34 sec
K_{p2}	-65%	55.56 Hz/p.u.MW

with the LQG controller and the PI controllers can be seen in Fig. 6 and Fig. 7 respectively. The superiority of the

LQG controller is clear in this case, restricting the frequency variation within tighter bounds while at the same time limiting the control energy used. Moreover, it can be seen that with the PI controllers in place, the system is also on the verge of instability.

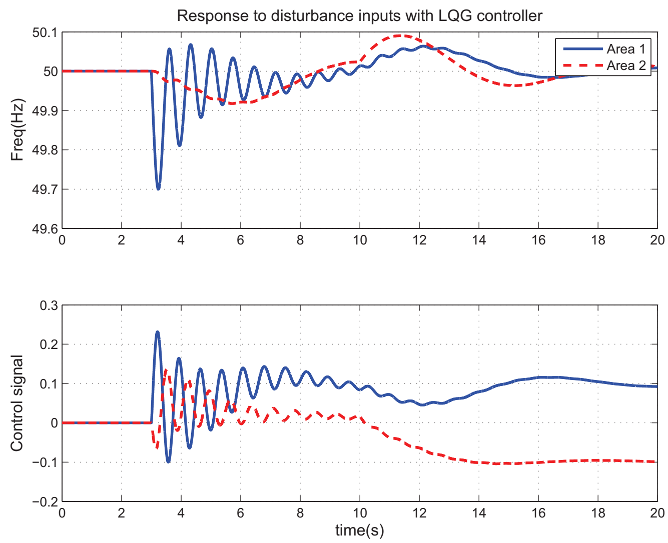

Fig. 6. Time-response of perturbed power system with multivariable LQG controller.

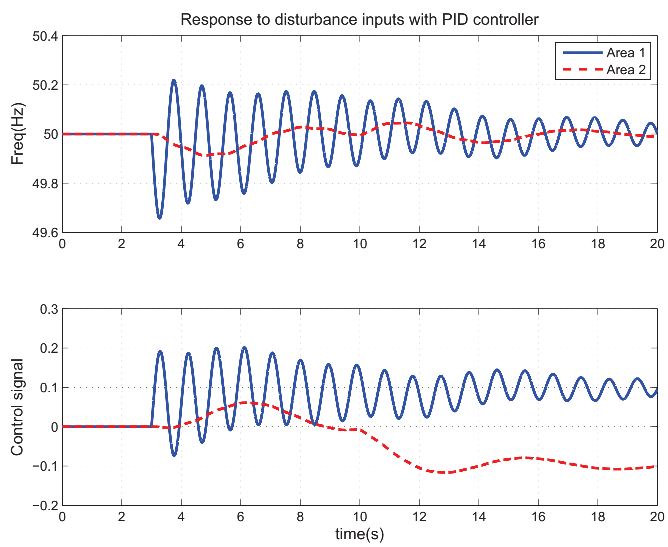

Fig. 7. Time-response of perturbed power system with PI controllers.

V. Conclusion

A two-area power system has been modeled with the aim of minimizing frequency deviation and regulating the frequency of each area of the system. A reduced-order integral LQG controller was designed taking into account uncertainties present in the system. In particular, parametric variation in the generation units are considered as they represent the main source of uncertainty when the operating point of the system changes; see [11]. The closed-loop system is simulated using the nominal plant and a perturbed plant with the application of realistic load disturbances. To provide a measure of comparison, the system is also simulated with a pair of optimized PI controllers. The response of the power system with the LQG controller is shown to be superior to that of the PI controllers both with the nominal and the perturbed plant. Moreover, the PI controllers seem to struggle to maintain stability when the perturbed plant is used for simulation. It is of course possible to extend the idea to multi-area systems where control commands are sent in synchronism to all the generators but as the size of the system increases, both the controller complexity increases as well as the delay in communicating with numerous control areas. We aim to investigate the effect of these in our future work.

References

[1] Y. Moon, H. Ryu, J. Lee, and S. Kim, "Power system load frequency control using noise-tolerable PID feedback," in *Proc. IEEE Int. Symposium on Industrial Electronics*, 2001, pp. 1714–1718.

[2] M. Rashidi, F. Rashidi, A. Saheb, and A. J. Sahragard, "Design of a Robust and Adaptive Load Frequency Controller for multi-area power networks with system parametric uncertainties using TDMLP Neural network," in *Proc. IEEE International conference on Systems, Man and Cybernetics*, 2004, pp. 3698–3703.

[3] E. Rakhshani and J. Sadeh, "Reduced-order estimator with prescribed degree of stability for two-area LFC system in a deregulated environment," in *Proc. IEEE Power Systems Conference and Exposition*, 2009, pp. 1–8.

[4] K. Vrdoljak, I. Petrovic, and N. Peric, "Discrete-time sliding mode control of load frequency in power systems with input delay," in *Proc. 12th Int. Power Electronics and Motion Control Conf.*, 2006, pp. 567–572.

[5] M. R. I. Sheikh, S. M. Muyeen, R. Takahashi, T. Murata, and J. Tamura, "Improvement of load frequency control with fuzzy gain scheduled superconducting magnetic energy storage unit," in *Proc. 18th International Conference on Electrical Machines*, 2008, pp. 1–6.

[6] A. Sharifi, K. Sabahi, M. A. Shoorehdeli, M. A. Nekoui, and M. Teshnehlab, "Load frequency control in interconnected power system with multi-objective PID controller," in *Proc. IEEE Conf. on Soft Computing in Industrial Applications*, 2008, pp. 217–221.

[7] M. Ghazal and J. Poshtan, "Application of LQG/LTR method in load frequency control," in *Proc. 25th Int. Power Systems Conference*, 2010, pp. 1–11.

[8] Y. Oguz, H. Erdal, and S. Taskin, "Design and simulation of a LQG robust controller for an electrical power system," *Journal of Electrical and Electronics Engineering*, vol. 8, no. 2, pp. 693–698, 2008.

[9] X. Yang and O. Marjanovic, "LQG control with extended Kalman filter for Power Systems with unknown time-delays," in *Proc. 18th IFAC World Congress*, 2011, pp. 3708–3713.

[10] P. J. Antsaklis and A. N. Michel, *A Linear Systems Primer*. Birkhaüser, 2007.

[11] O. I. Elgerd, "Control of electric power systems," *IEEE Control Syst. Mag.*, vol. 1, no. 2, pp. 4–16, 1981.

[12] R. E. Kalman, "Contributions to the Theory of Optimal Control," *Boletin de la Sociedad Matematica Mexicana*, vol. 5, pp. 102–119, 1960.

[13] ——, "A New Approach to Linear Filtering and Prediction Problems," *Transactions of the ASME–Journal of Basic Engineering*, vol. 82, no. Series D, pp. 35–45, 1960.

[14] B. D. O. Anderson and J. B. Moore, *Optimal Filtering*. Englewood Cliffs, New Jersey: Prentice-Hall, 1979.

[15] R. E. Kalman and R. S. Bucy, "New Results in Linear Filtering and Prediction Theory," *ASME Journal of Basic Engineering*, vol. 83, pp. 95–108, 1961.

[16] B. D. O. Anderson and J. B. Moore, *Optimal Control: Linear Quadratic Methods*. Prentice-Hall, Englewood Cliffs, NJ, 1990.

[17] S. Gugercin and A. C. Antoulas, "A survey of model reduction by balanced truncation and some new results," *Int. J. Control*, vol. 77, no. 8, pp. 748–766, May 2004.