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LQG control of an optical squeezer

S. Z. Sayed Hassen, I. R. Petersen, E. H. Huntington, M. Heurs and M. R. James

Abstract— **In this paper, we consider the application of linear quadratic Gaussian (LQG) control to the problem of optimizing the level of squeezing in one of the quadratures of an optical field. Squeezed states of light can be generated when two optical fields (at fundamental and second-harmonic frequencies) interact in an optical cavity in the presence of a second-order nonlinear crystal. Our system is an optical squeezer which is modelled as a nonlinear quantum system. Suitable models for the quantum and classical noises present in the system are used and laser phase noise which arises due to mechanical vibration of the mirror(s) in the beam path is modeled as (approximately) integrated white noise. An LQG controller is synthesized and the closed loop system is simulated to validate our design.**

I. Introduction

Experimental quantum optics has enabled many fundamental theories of quantum mechanics to be tested at unprecedented level. It is well known in the physics literature that the quantum noise limit (QNL) puts a bound on the signal to noise ratio that can be achieved in the absence of non-classical states. This bound ultimately restricts the usefulness of applications involving quantum technology. It is fortunately possible however to partly circumvent this problem by concentrating the quantum noise in a specific quadrature of light [1]. In other words, we can rearrange the noise distribution between the quadratures, without violating the Heisenberg uncertainty principle. Light which has this property of asymmetric quadrature noise distribution is known as "squeezed" light and one of the most widely used devices in quantum optics by which coherent light (laser light) is converted to squeezed light is known as the "Optical Parametric Oscillator (OPO)". At the current time of writing, the OPO is known to be the best method for the generation of squeezed vacuum states, achieving the largest suppression level of approximately 10 dB [2].

In this paper, we address the problem of optimally squeezing a specific quadrature of light using an OPO. We consider the application of systematic LQG optimal control to an OPO driven by two optical fields; see Fig 1. One of the optical input fields is controlled by a mirror connected to a piezoelectric actuator. The actuator adjusts the phase quadrature of the incident optical field, thus minimizing the effect of phase noise and other sources of noise, including quantum noises. In turn, this regulates the phase angles of the fundamental and second-harmonic intra-cavity fields, allowing for optimal quadrature squeezing of light. The available measurement is the phase quadrature of the output harmonic field which is measured using the homodyne detection method [1]. A novel feature of our approach is that we propose a control law that minimizes a quadrature of the fundamental output field using both measurement and actuation on the second harmonic field.

We begin by calculating the steady-state parametric amplification/phase sensitive gain (see [3]) required for specific quadratures of the fundamental output field to be squeezed. Corresponding region(s) of operation are analytically determined for optimal squeezing and the optical system is linearized about a given point inside this region. The complete model we use for the controller design and subsequent simulations is derived from the linearized quantum dynamics of the OPO (optical subsystem) and from a piezoelectric actuator (mechanical subsystem) approximated with linear second-order dynamics. We use a standard LQG approach in which a Kalman filter produces an optimal estimate of all the relevant variables of the system from the available measurement. The estimate of the variables is combined with an optimal state feedback control law, generating an LQG controller for the system. The requirement to minimize the variance of the quadrature of the fundamental output field is reflected in an LQG cost functional. Typical parameter values of an experimental squeezer are used for the controller design and we validate our design through simulation results.

II. The Optical Parametric Amplification Problem

The optical system under consideration consists of a second-order nonlinear optical medium enclosed within an optical resonator; see, e.g. [1], [4]. Materials showing second-order nonlinearities $\chi^{(2)}$ have the ability to couple a fundamental field (f) to a second harmonic field ($2f$). This coupling forms the basis of operation of the OPO which works by converting an input laser wave into two output waves of lower frequency through nonlinear optical interaction; see, e.g. [3]. When the crystal is contained in a resonator cavity, feedback results in the build-up of the waves in a process similar to that seen in a laser cavity and the output beams become quantum correlated producing squeezed light.

The quantum mechanical Hamiltonian describing such $\chi^{(2)}$ interactions is given by

$$H = i\chi^{(2)}(\hat{b}^{\dagger}\hat{a}^2 - \hat{a}^{\dagger 2}\hat{b}), \tag{1}$$

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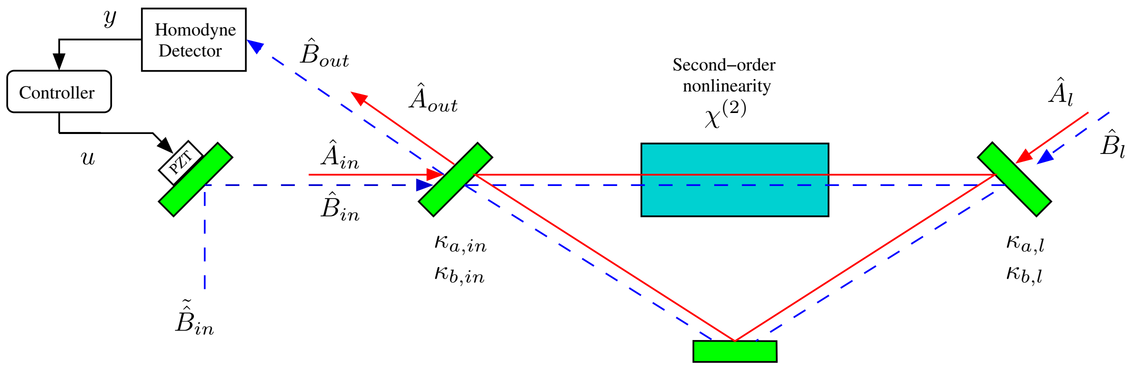

Fig. 1. Schematic for control of an optical parametric oscillator

where $a(a^\dagger)$ and $b(b^\dagger)$ are the annihilation (creation) operators for the fundamental and second harmonic fields respectively. Fig. 1 shows the setup used for the OPO control problem. The aim is to maximize the amplitude quadrature squeezing observed at a given operating point by optimally suppressing the different sources of noise feeding into the system. This aim is translated into a specific quadratic cost functional to define an LQG optimal control problem; the underlying assumption being that the conditional state associated with a quantum system can be equivalently modelled using a classical subsystem; see [5], [6]. The measurement signal used is the phase quadrature of the second harmonic output field $\hat{B}_{out}$ which is measured using homodyne detection. The measured electrical signal is fed to a controller which determines the position of a mirror attached to a piezoelectric actuator, thus manipulating the phase quadrature of the input field $\hat{B}_{in}$ as desired.

III. Modelling

A. Optical Resonator Cavity

The dynamics of the OPO system can be described as [1]:

$$\dot{\hat{a}} = -\kappa_a \hat{a} + \chi^{(2)}\hat{a}^\dagger \hat{b} + \sqrt{2\kappa_{a,in}}\hat{A}_{in} + \sqrt{2\kappa_{a,l}}\delta\hat{A}_l; \qquad (2)$$

$$\dot{\hat{b}} = -\kappa_b \hat{b} - \frac{1}{2}\chi^{(2)}\hat{a}^2 + \sqrt{2\kappa_{b,in}}\hat{B}_{in} + \sqrt{2\kappa_{b,l}}\delta\hat{B}_l; \qquad (3)$$

where $\kappa_{a,in}$ and $\kappa_{b,in}$ are the loss rates of the input/output mirrors for the $\hat{a}$ and $\hat{b}$ fields respectively. The parameters $\kappa_{a,l}$ and $\kappa_{b,l}$ are the internal loss rates for the corresponding two fields. Also, $\kappa_a = \kappa_{a,in} + \kappa_{a,l}$ and $\kappa_b = \kappa_{b,in} + \kappa_{b,l}$ are the associated total resonator decay rates. The quantities $\hat{A}_{in}$ and $\hat{B}_{in}$ are the input fields and $\delta\hat{A}_l$ and $\delta\hat{B}_l$ are the vacuum fields due to the internal losses. The output fields are given by:

$$\begin{aligned}\hat{A}_{out} &= \sqrt{2\kappa_{a,in}}\hat{a} - \hat{A}_{in};\\ \hat{B}_{out} &= \sqrt{2\kappa_{b,in}}\hat{b} - \hat{B}_{in}.\end{aligned}$$

B. Quadrature Operator

Quadrature operators provide a practical description of the properties of light and we shall use two special operators called the amplitude quadrature and the phase quadrature defined as follows for the fields $\hat{a}$ and $\hat{b}$:

$$X_a^+ = \hat{a} + \hat{a}^\dagger; \qquad X_a^- = i(\hat{a}^\dagger - \hat{a}); \qquad (4)$$

$$X_b^+ = \hat{b} + \hat{b}^\dagger; \qquad X_b^- = i(\hat{b}^\dagger - \hat{b}). \qquad (5)$$

The quadratures of the input and noise fields can similarly be defined as $X^\pm_{Ain}, X^\pm_{Bin}, X^\pm_{\delta A,l}$ and $X^\pm_{\delta B,l}$. Then, the dynamics of the optical subsystem can be rewritten in terms of the quadratures as:

$$\begin{aligned}\dot{X}_a^+ &= -\kappa_a X_a^+ + \frac{1}{2}\chi^{(2)}(X_a^+X_b^+ + X_a^-X_b^-)\\ &\quad + \sqrt{2\kappa_{a,in}}X^+_{Ain} + \sqrt{2\kappa_{a,l}}X^+_{\delta A,l}; \qquad (6)\\ \dot{X}_a^- &= -\kappa_a X_a^- + \frac{1}{2}\chi^{(2)}(X_a^+X_b^- - X_a^-X_b^+)\\ &\quad + \sqrt{2\kappa_{a,in}}X^-_{Ain} + \sqrt{2\kappa_{a,l}}X^-_{\delta A,l}; \qquad (7)\\ \dot{X}_b^+ &= -\kappa_b X_b^+ - \frac{1}{4}\chi^{(2)}({X_a^+}^2 - {X_a^-}^2)\\ &\quad + \sqrt{2\kappa_{b,in}}X^+_{Bin} + \sqrt{2\kappa_{b,l}}X^+_{\delta B,l}; \qquad (8)\\ \dot{X}_b^- &= -\kappa_b X_b^- - \frac{1}{2}\chi^{(2)}X_a^+X_a^-\\ &\quad + \sqrt{2\kappa_{b,in}}X^-_{Bin} + \sqrt{2\kappa_{b,l}}X^-_{\delta B,l}. \qquad (9)\end{aligned}$$

The output fields are similarly expressed again as

$$\begin{aligned}X^+_{Aout} &= \sqrt{2\kappa_{a,in}}X_a^+ - X^+_{Ain}; \qquad (10)\\ X^-_{Aout} &= \sqrt{2\kappa_{a,in}}X_a^- - X^-_{Ain}; \qquad (11)\\ X^+_{Bout} &= \sqrt{2\kappa_{b,in}}X_b^+ - X^+_{Bin}; \qquad (12)\\ X^-_{Bout} &= \sqrt{2\kappa_{b,in}}X_b^- - X^-_{Bin}. \qquad (13)\end{aligned}$$

C. Linearization

The dynamics of the optical subsystem (6)–(9) contain nonlinear terms and we use a standard linearization method to obtain linear dynamics to which we can apply LQG control design methods. We expand the field quadratures in terms of their coherent amplitude and quantum noise operator, so that a quadrature $X_i^\pm = \bar{X}_i^\pm + \delta X_i^\pm, i = a, b$ where $\bar{X}_i^\pm$ and $\delta X_i^\pm$ represent the expectation value (steady-state value) and small variations of the field quadratures respectively. The steady-state solution of the fields $\hat{a}$ and $\hat{b}$ are expressed in polar coordinates as $\bar{a} = |\bar{a}|e^{i\bar{\theta}_a}$ and $\bar{b} = |\bar{b}|$.

Also, we write the input fields as $\bar{A}_{in} = |\bar{A}_{in}|e^{i\bar{\theta}_{a,in}}$ and $\bar{B}_{in} = |\bar{B}_{in}|e^{i\bar{\theta}_{b,in}}$.

The linearized dynamics of the optical subsystem is given by

$$\begin{aligned}\delta\dot{X}_a^+ &= -\kappa_a\delta X_a^+ + \frac{1}{2}\chi^{(2)}(\bar{X}_b^+\delta X_a^+ + \bar{X}_b^-\delta X_a^- \\ &\quad + \bar{X}_a^+\delta X_b^+ + \bar{X}_a^-\delta X_b^-) \\ &\quad + \sqrt{2\kappa_{a,in}}\delta X_{Ain}^+ + \sqrt{2\kappa_{a,l}}X_{\delta A,l}^+; \quad (14)\\ \delta\dot{X}_a^- &= -\kappa_a\delta X_a^- + \frac{1}{2}\chi^{(2)}(\bar{X}_b^-\delta X_a^+ - \bar{X}_b^+\delta X_a^- \\ &\quad - \bar{X}_a^-\delta X_b^+ + \bar{X}_a^+\delta X_b^-) \\ &\quad + \sqrt{2\kappa_{a,in}}\delta X_{Ain}^- + \sqrt{2\kappa_{a,l}}X_{\delta A,l}^-; \quad (15)\\ \delta\dot{X}_b^+ &= -\kappa_b\delta X_b^+ - \frac{1}{2}\chi^{(2)}(\bar{X}_a^+\delta X_a^+ - \bar{X}_a^-\delta X_a^-) \\ &\quad + \sqrt{2\kappa_{b,in}}\delta X_{Bin}^+ + \sqrt{2\kappa_{b,l}}X_{\delta B,l}^+; \quad (16)\\ \delta\dot{X}_b^- &= -\kappa_b\delta X_b^- - \frac{1}{2}\chi^{(2)}(\bar{X}_a^-\delta X_a^+ + \bar{X}_a^+\delta X_a^-) \\ &\quad + \sqrt{2\kappa_{b,in}}\delta X_{Bin}^- + \sqrt{2\kappa_{b,l}}X_{\delta B,l}^-. \quad (17)\end{aligned}$$

The corresponding linearized output fields are expressed as

$$\begin{aligned}\delta X_{Aout}^+ &= \sqrt{2\kappa_{a,in}}\delta X_a^+ - \delta X_{Ain}^+; \quad (18)\\ \delta X_{Aout}^- &= \sqrt{2\kappa_{a,in}}\delta X_a^- - \delta X_{Ain}^-; \quad (19)\\ \delta X_{Bout}^+ &= \sqrt{2\kappa_{b,in}}\delta X_b^+ - \delta X_{Bin}^+; \quad (20)\\ \delta X_{Bout}^- &= \sqrt{2\kappa_{b,in}}\delta X_b^- - \delta X_{Bin}^-. \quad (21)\end{aligned}$$

D. Analysis of Optical gain of fundamental field $\hat{A}$

In this section, we investigate the variation of the steady-state gain of the OPO under some specific conditions which are satisfied in our setup shown in Fig. 1. If $\kappa_a << \kappa_b$, and the harmonic field $\hat{B}$ is much more intense than the fundamental field $\hat{A}$, then energy will be predominantly transferred from field $\hat{B}$ to field $\hat{A}$. We thus expect the steady-state optical gain with respect to the corresponding field $\hat{A}$, defined as $|\hat{A}_{out}|^2/|\hat{A}_{in}|^2$, to be of much greater magnitude than the corresponding gain with respect to the field $\hat{B}$. We determine the variation of the gain $|\hat{A}_{out}|^2/|\hat{A}_{in}|^2$ with respect to the quantities $\bar{\theta}_{a,in}$ and $\bar{\theta}_{b,in}$ as shown in Fig. 2. The parameter values we use for our model are chosen to reflect those of a resonator cavity currently in operation in our optics laboratory; see Table I.

Model parameter	Value	Units
κ_a	1×10^5	rad/s
κ_b	1×10^9	rad/s
$\kappa_{a,l}$	5×10^3	rad/s
$\kappa_{b,l}$	5×10^7	rad/s
$\kappa_{a,in}$	9.5×10^4	rad/s
$\kappa_{b,in}$	9.5×10^8	rad/s
$\chi^{(2)}$	3×10^{-2}	–
$\bar{A}_{in}$	2×10^6	$\sqrt{\text{rad/s}}$
$\bar{B}_{in}$	2×10^{10}	$\sqrt{\text{rad/s}}$

TABLE I
MODEL PARAMETER VALUES

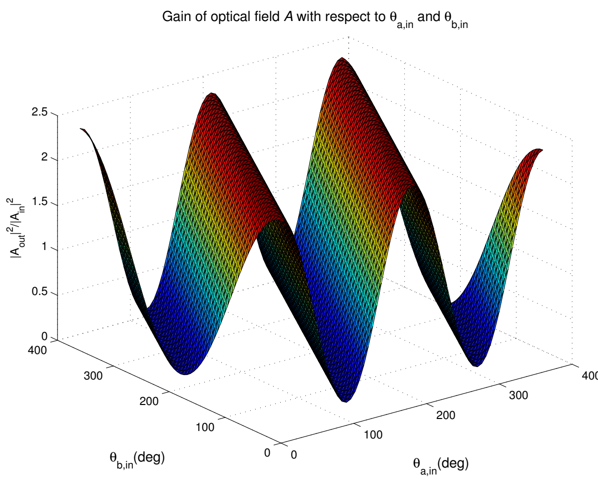

Fig. 2. Variation of the $|\hat{A}_{out}|^2/|\hat{A}_{in}|^2$ with $\bar{\theta}_{a,in}$ and $\bar{\theta}_{b,in}$

The variation of the gain $|\hat{A}_{out}|^2/|\hat{A}_{in}|^2$ with respect to the angles of the input fields, as shown in Fig. 2 is used in the determination of the operating point we would choose to obtain maximum squeezing. Depending on the quadrature we would like to squeeze, we choose appropriate operating point(s) corresponding to a given pair of input field angles $\bar{\theta}_{a,in}$ and $\bar{\theta}_{b,in}$ such that we have the appropriate level of steady-state gain $|\hat{A}_{out}|^2/|\hat{A}_{in}|^2$.

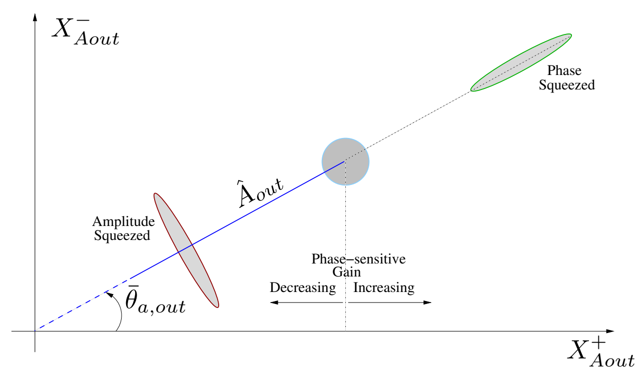

Fig. 3. Representation of phase sensitive squeezing

Fig. 3 illustrates the idea that increasing the steady-state gain results in phase squeezing while decreasing the gain gives rise to amplitude squeezing. In our case, we are interested in maximizing the level of amplitude squeezing and we would therefore be ideally operating in a region where the optical steady-state gain is at its minimum. From Fig. 2, we can also determine the relationship that exists between $\bar{\theta}_{a,in}$ and $\bar{\theta}_{b,in}$ when the optical gain is at its minimum. To achieve optimal amplitude squeezing, we would choose the operating point so that

$$\bar{\theta}_{b,in} - 2\bar{\theta}_{a,in} = n\pi; \quad \text{where } n \in \mathbb{Z}. \quad (22)$$

Furthermore, it can be shown that the phase of the applied input field $\bar{A}_{in}$ corresponds approximately to the phase of the field $\bar{a}$ inside the optical cavity. A similar correspondence exists between the phase of the input field $\bar{B}_{in}$ and the phase of the field $\bar{b}$ inside the optical cavity. Thus, an equivalent condition to (22) for maximum amplitude squeezing of the fundamental field $\hat{A}$ would be

$$\bar{\theta}_b - 2\bar{\theta}_a = n\pi. \tag{23}$$

E. Complete Model

Given the effect of the operating point as explained previously, we linearize the optical system about a set point defined by setting $\bar{A}_{in}$ and $\bar{B}_{in}$ so as to obtain a suitable pair $\bar{\theta}_a$ and $\bar{\theta}_b$ satisfying $\bar{\theta}_b - 2\bar{\theta}_a = \pi$. The results presented in Fig. 2 show that these values will lead to maximum observed amplitude quadrature squeezing of the field $\hat{A}$. One such pair could be $\bar{\theta}_a = \frac{\pi}{3}$ and $\bar{\theta}_b = \frac{5\pi}{3}$. Also, as stated before, the optical subsystem is controlled by regulating the phase quadrature of the second-harmonic input field (δX^-_{Bin}) using the phase quadrature of the second-harmonic output field (δX^-_{Bout}) as measurement.

We next model the piezo-actuator attached to the mirror (the mechanical subsystem) as a second order system:

$$\begin{aligned} \begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ -r_2 & -r_1 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} \\ &\quad + \begin{bmatrix} 0 \\ 1 \end{bmatrix} w_1 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u; \\ \delta X^-_{Bin} &= \begin{bmatrix} c_2 & c_1 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} + p_0. \end{aligned} \tag{24}$$

Here, ξ_1 represents deviation in the position of the mirror which controls the phase quadrature of the input field $\hat{B}_{in}(\delta X^-_{Bin})$. Also, p_0 represents the phase quadrature of the quantum noise present in the input field $\hat{B}_{in}$. Furthermore, w_1 is used to represent laser phase noise which arises due to mechanical fluctuations in the beam path. This noise is modelled as approximately integrated white noise using the state equations

$$\dot{\xi} = -\epsilon\xi + \tilde{w}_1, \qquad w_1 = k_1\xi; \tag{25}$$

where $\tilde{w}_1$ is Gaussian white noise with variance ϵ^2_{ln}, ϵ defines the noise model pole location and k_1 is a gain parameter denoting the size of the mechanical noise signal w_1. In the sequel, the value of ϵ_{ln} will be treated as a design parameter.

The complete linearized model can then be modelled in state-space form as:

$$\begin{aligned} \dot{x} &= Ax + B_1 u + B_2 w; \\ y &= Cx + Dw, \end{aligned}$$

where

$$\begin{aligned} x &= \begin{bmatrix} \delta X^+_a & \delta X^-_a & \delta X^+_b & \delta X^-_b & \xi_1 & \xi_2 & \xi \end{bmatrix}^T; \\ w &= \Big[\tilde{w}_1 \;\; p_0 \;\; \delta X^+_{Ain} \;\; \delta X^-_{Ain} \;\; \delta X^+_{Bin} \\ &\qquad X^+_{\delta A,l} \;\; X^-_{\delta A,l} \;\; X^+_{\delta B,l} \;\; X^-_{\delta B,l} \;\; w_2\Big]^T, \end{aligned}$$

and w_2 is used to represent sensor noise.

The plant to be controlled is represented as shown in Fig. 4.

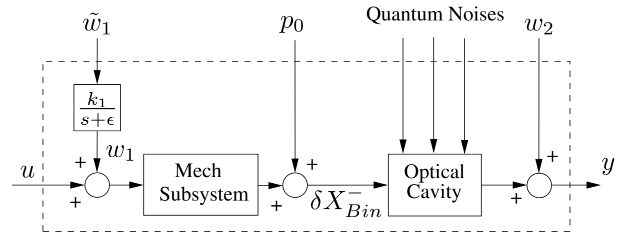

Fig. 4. Block diagram of system interconnections.

IV. LQG Controller Design

A. Performance Criterion and Kalman Filtering

The performance requirement for our control system is to minimize the variance of the amplitude quadrature of the output field $\hat{A}_{out}$, and to achieve squeezing in the presence of both quantum and classical noises. The controlled variable is chosen such that the variations of the quadratures are measured along the direction of the steady-state output field, in particular along the phase angle $\bar{\theta}_{a,out}$. We choose the appropriate quadrature by rotating the reference axis. In general, an arbitrary quadrature, X^θ, can be described as a function of the phase angle θ, according to

$$X^\theta = X^+ \cos\theta + X^- \sin\theta. \tag{26}$$

Thus, to control the amplitude quadrature, we consider the controlled variable

$$z = \delta X^+_{Aout} \cos\bar{\theta}_{a,out} + \delta X^-_{Aout} \sin\bar{\theta}_{a,out}. \tag{27}$$

Moreover, we are also limited by the bandwidth of the measuring device. The measuring device can be modelled as a first order low pass filter

$$\begin{aligned} \tau\dot{x}_f &= -x_f + z; \\ \tilde{z} &= x_f. \end{aligned} \tag{28}$$

Thus, rather than penalizing variations at all frequencies equally, we limit our interest to the lower frequency regions defined by the cut-off frequency of the low pass filter. We shall then consider the filtered controlled variable $\tilde{z}$ in the LQ performance criterion (29). The augmented system we use for the controller design is then completely characterized by

$$\begin{aligned} \dot{\tilde{x}} &= \tilde{A}\tilde{x} + \tilde{B}_1 u + \tilde{B}_2 w; \\ y &= \tilde{C}\tilde{x} + Dw, \end{aligned}$$

where

$$\tilde{x} = \begin{bmatrix} x \\ x_f \end{bmatrix}.$$

The control objective can then be formulated as the minimization of the following cost function:

$$\mathcal{J} = \lim_{T\to\infty} \mathbf{E}\left[\frac{1}{T}\int_0^T \tilde{x}(t)^T Q\tilde{x}(t) + u(t)^T Ru(t)\, dt\right], \tag{29}$$

where Q and R are positive definite matrices. The matrix Q is chosen such that $\tilde{x}^T Q\tilde{x} = |\tilde{z}|^2$ and R is chosen to weight the control energy required. The expectation in (29) is with respect to the Gaussian quantum and classical noise processes, and the assumed Gaussian initial conditions.

The LQG optimal controller is constructed from the solution to a deterministic regulator problem and an optimal observer problem as follows (see, e.g., [7]):

$$u = -R^{-1}\tilde{B}_1^T X\hat{\tilde{x}}, \tag{30}$$

where X satisfies the matrix Riccati equation:

$$Q - X\tilde{B}_1 R^{-1}\tilde{B}_1^T X + X\tilde{A} + \tilde{A}^T X = 0. \tag{31}$$

The optimal observer dynamics (Kalman filter) are described by

$$d\hat{\tilde{x}} = \tilde{A}\hat{\tilde{x}}\, dt + \tilde{B}_1 u\, dt + K[dy - \tilde{C}\hat{\tilde{x}}\, dt]; \tag{32}$$

and the solution of the optimal observer is obtained by choosing the gain matrix

$$K = P_e\tilde{C}^T(DD^T)^{-1}, \tag{33}$$

where P_e is the solution to the matrix Riccati equation

$$\tilde{A}P_e + P_e\tilde{A}^T - P_e\tilde{C}^T(DD^T)^{-1}\tilde{C}P_e + \tilde{B}_2\tilde{B}_2^T = 0. \tag{34}$$

B. Model and Design Parameters

The parameter values of the piezoelectric actuator, the low pass filter and the steady-state phase angles of the input fields are shown in Table II.

Model parameter	Value
r_1	1×10^5
r_2	1×10^9
c_1	-5×10^3
c_2	1.5×10^{10}
τ	1×10^{-5}
$\bar{\theta}_{a,in}$	$\pi/3$
$\bar{\theta}_{b,in}$	$5\pi/3$

TABLE II
MODEL PARAMETERS

In designing the LQG controller, the design parameters ϵ_{ln}^2 (laser noise variance), ϵ_1^2 (quantum noise variance), ϵ_2^2 (sensor noise variance), ϵ (phase noise model pole location), k_1 (phase noise gain parameter), and R (the control energy weighting in the cost functional) were adjusted until good performance was obtained in the controller simulations. The values used were $\epsilon = 3\times10^{-5}$, $\epsilon_1 = 1$, $\epsilon_2 = 0.1$, $\epsilon_{ln} = \sqrt{10}$, $k_1 = 1\times10^6$ and $R = 1\times10^{-10}$. These parameters led to an LQG controller with Bode plots as shown in Fig. 5. The corresponding loop gain Bode plot is also shown in Fig. 6.

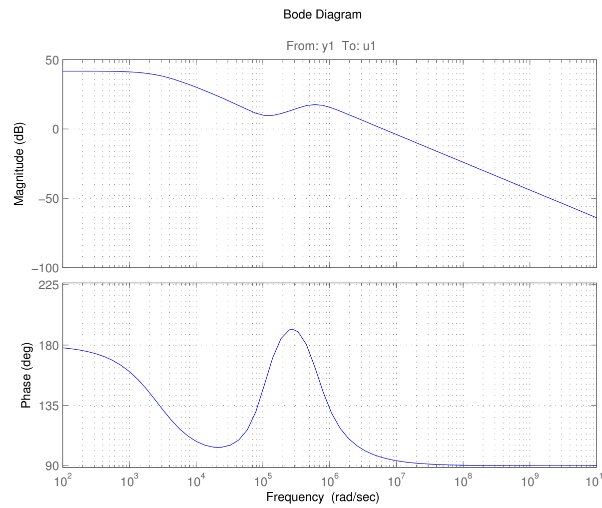

Fig. 5. Bode plots of LQG controller

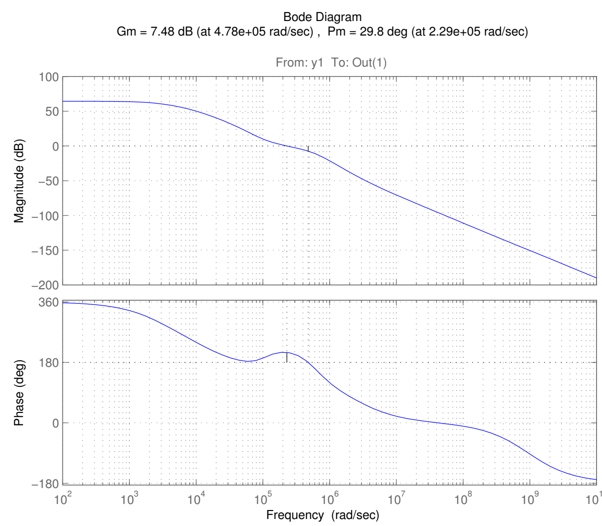

Fig. 6. Bode plot of loop gain transfer function

V. SIMULATION RESULTS

Using the model parameters specified in Table I and II, we simulate the system for different scenarios using the Simulink block diagram shown in Fig. 7.

First, we show the effect of the nonlinearity $\chi^{(2)}$ on the variance of the controlled output $\tilde{z}$. We simulate the open-loop system with only quantum noises exciting the system and with $\chi^{(2)} = 0$. This corresponds to the case where there is no interaction between the fundamental and second harmonic fields inside the optical cavity. The controlled variable $\tilde{z}$ is then representative of the standard quantum noise limit. We repeat the process with $\chi^{(2)} = 0.03$ and the respective plots are shown in Fig. 8. As expected, the interaction between the optical fields results in a reduction of the variance of the controlled variable $\tilde{z}$. Physically, a non-zero value of $\chi^{(2)}$ causes squeezing of the amplitude quadrature, which in turn reduces the degree of uncertainty

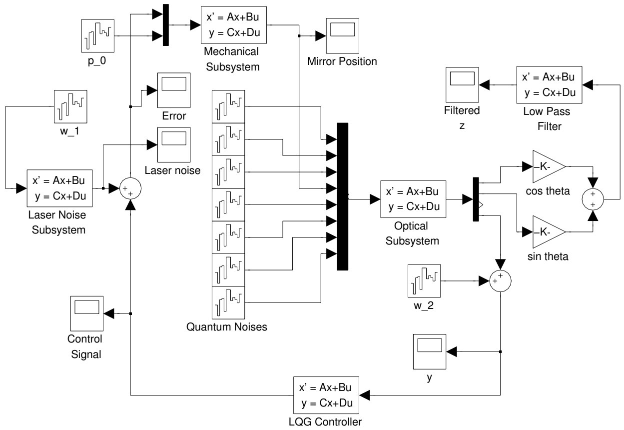

Fig. 7. Block diagram used for simulation

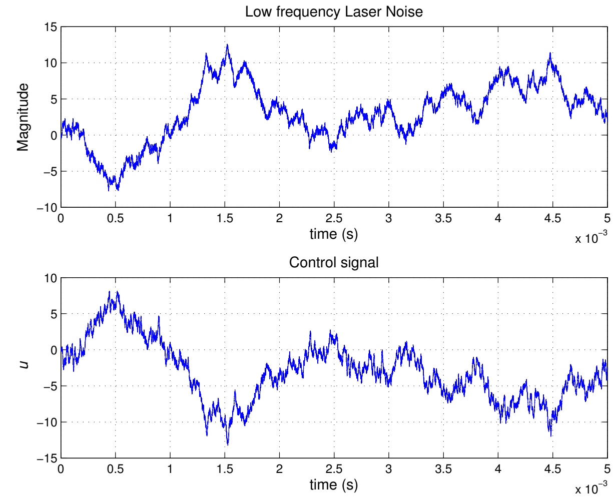

Fig. 9. Laser phase noise w_1 and control signal u

along the chosen axis of measurement as defined by (27).

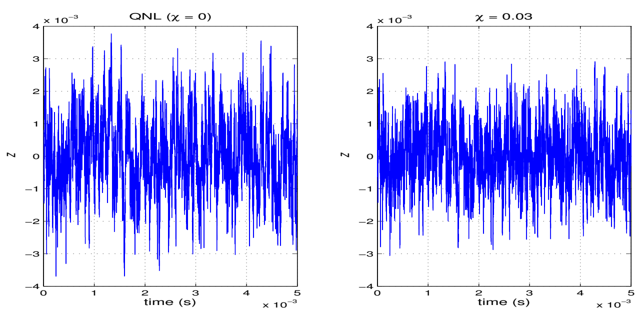

Fig. 8. Quantum noise limit ($\chi^{(2)} = 0$) and squeezed quantum noise limit ($\chi^{(2)} = 0.03$).

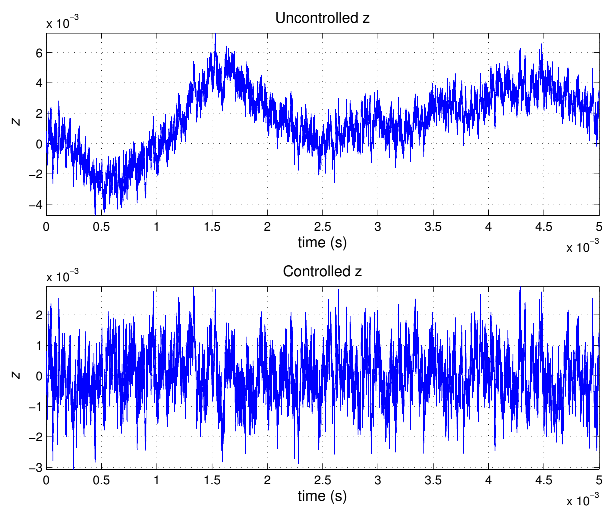

Fig. 10. The uncontrolled and controlled variable $\tilde{z}$

We next simulate the closed-loop system to determine the effectiveness of the designed controller. We consider the case where the system is excited with both classical and quantum noises for $\chi^{(2)} = 0.03$. Fig. 9 shows the laser phase noise w_1 and the corresponding control signal generated by the controller to counteract the effect of this noise. Fig. 10 compares the time history of the filtered controlled variable $\tilde{z}$ for the open-loop and the closed-loop case. It can be seen that the controller successfully minimizes the variance of the controlled variable to a level below the quantum noise limit and close to the squeezed quantum noise limit; cf. Fig. 8.

VI. Conclusion and Future Work

In this paper, we have applied LQG control to the problem of optimal quadrature squeezing of light in an optical cavity. We have modelled the optical system, which is inherently nonlinear and determined the steady-state conditions required to achieve maximum amplitude quadrature squeezing of the fundamental field. At a suitable operating point, the system is linearized to fit into the framework of the LQG control design technique and an optimal state feedback control law is determined from a given cost functional. The cost functional is chosen to reflect our goal of minimizing the variance of the amplitude quadrature of the fundamental output field. Simulation results obtained show the effectiveness of the controller in cancelling out the laser phase noise and the control system performance is limited only by the squeezed quantum noises. Future work would involve applying the proposed control approach to an experimental test bed.

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