Modeling, Control and Simulation of a Wave Energy Converter

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Abstract—In this paper, a two-point absorber ocean wave energy converter is modelled and analysed with a view to design an optimal controller to extract maximum mechanical wave power and convert it to electrical power. The control strategy employed is the Linear Quadratic Gaussian (LQG) control which will be designed in two stages. Firstly, the Kalman Filter is designed to estimate the non-measurable states and these are then fed to a linear quadratic regulator (LQR). The controller is then simulated with the wave energy converter (WEC) and the performance of controlled system is compared with the uncontrolled one.

I. Introduction

Ocean waves are formed due to wind energy and wind arise due to the uneven heating of the earth surface. Thus, wave energy is considered as a condensed form of solar energy. As the sun heats some areas of the earth surface more than others, it causes the heated air to expand and rise which in turn is swapped by cold air to fill up the space hence causing a circulation of air which we term as wind. As wind blows over the ocean's surface, it causes the creation of ocean waves which once transformed can travel very long distances with very little energy loss. [1]

Wave energy devices are perceived to be more environmentally friendly compared to other sources of power generation as it does not emit any gaseous, liquid or solid particles while in use, which makes it a practically non-polluting energy source [2]. The input to a wave energy conversion system is obviously the ocean waves and the output is the electrical energy. Hence, keeping these two parameters in mind, a wave energy converter needs to meet some requirements namely, (1) to maximize the amount of energy absorbed from ocean waves; (2) have a good energy conversion efficiency and (3) to generate electrical power that can be brought to the grid without issues. The third requirement is met by the use of appropriate power electronic converters [3].

Wave energy converters can be classified according to:

- Location- The WEC can be placed either off-shore or near-shore which can be either floating, submerged or bottom-standing.
- The working principle of the primary capture system.
- The type of wave energy conversion technique- A WEC can have a mechanical system, hydraulic system, a pneumatic system (gas or pressurized air operating system) or an electric power conversion system.
- Type of energy for end use, that is, the product of the WEC can be used for electricity, water heating, water pumping, desalination of sea water, propulsion or even refrigeration.

They can also be classified according to their horizontal extension and orientation. A wave energy device is known as a "point absorber" if its extension is smaller than a typical wavelength of an ocean waves and on the other hand, a WEC is known as a "line absorber" if its extension is much larger compared to a typical wavelength of an ocean wave. [2]

II. Modelling of the System

The model that has been focused on for this study is known as the L-10 wave energy conversion system which is a heave-oscillating two-body point absorber type WEC. It was developed by the Oregon State University (OSU) in collaboration with Columbia Power Technologies (CPT). The L-10 is a dual point absorber, 10 kW prototype which is designed and built using a permanent magnet synchronous linear generator (LPMG) for power take off. The L-10 consists of two main parts:

- Taurus shaped floating buoy
- Damping body.

As can be seen in Fig. 1, the damping body is a combination of a spar and a ballast tank which is denoted simply as the spar. The buoy and spar are connected together using a power take-off system which helps in control and generation of electrical energy. As ocean waves contact the buoy, the wave forces are transferred to the buoy which are then transferred to the spar, using a contact-less force transmission system. The spar encompasses the linear generator. Henceforth, the force that is imposed on the spar together with the relative velocity of the two floating body is converted into electrical energy by the use of a linear permanent magnet generator. Moreover, the spar is moored to the sea bed to ensure stationary-keeping of the system. Considering the practicality in the construction and hydrodynamic stability of the system, the buoy is made in a wide saucer shape that ensure buoyancy. However, the system is designed in such a way that it does not resonate with the ocean waves but rather just 'follows' the water surface in a vertical position [5], [6]. Fig 1 and 2 show the L-10 wave energy converter and its schematic diagram respectively with its dimensions shown in Table 1.

TABLE I. Dimensions of L-10 WEC

	Diameter	Height
Buoy	3.5 m	0.76 m
Spar	1.1 m	7.03 m

The modelling of the two-point absorber system is based on the work of Eidsmoem [7] and Ruhel [8]. Some assumptions are made to be able to model the system as a linear system in time-domain.

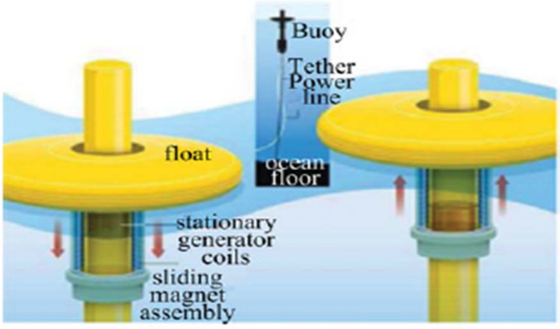

Fig. 1. L-10 Wave Energy Converter [4]

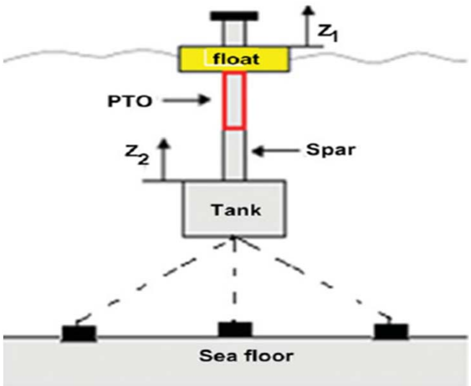

Fig. 2. Schematic diagram of L-10 Wave Energy Converter [4]

In Table II, the mass and damping of the wave energy converter under consideration is documented:

TABLE II. SYSTEM PARAMETERS FOR L-10 WAVE ENERGY CONVERTER. [4]

Variable	Value
m_1	2625.3 Kg
m_2	2650.4 Kg
A_1	8866.7 Kg
A_{12}	361.99 Kg
A_2	361.99 Kg
A_{21}	361.99 Kg
b_1	5000 N/(m/s)
b_2	50000 N/(m/s)
K_1	96743 N/m

Where:

m_1 is the mass of buoy,

m_2 is the mass of spar,

A_1 is the buoy's added mass at infinite frequency,

A_2 is the spar's added mass at infinite frequency,

b_1 is the buoy's radiation damping,

b_2 is the spar's radiation damping,

K_1 is a constant which equals to $g\rho\pi {r_{buoy}}^2$

As can be seen in Fig. 2, z_1 and z_2 corresponds to the position of the buoy and spar respectively. Using Newton's second law, the dynamic equations for the buoy and the spar can be written as follows:

$$F_{PTO} + F_{e1} - F_{r1} - F_{h1} - F_{r12} = m_1 \ddot{z}_1 \qquad (1)$$

$$-F_{PTO} + F_{e2} - F_{r2} - F_{21} + F_m = m_2 \ddot{z}_2 \qquad (2)$$

Where $\ddot{z}_1$ and $\ddot{z}_2$ are the buoy and spar acceleration respectively, m_1 and m_2 are the mass of the buoy and spar respectively. F_{e1} and F_{e2} represent the non-controllable system disturbances, that are the excitation forces induced by the incoming waves whereas F_{PTO} is a controllable input force that is used to optimize the motion of the device. Furthermore, the radiation force can be calculated as shown in [9] using:

$$F_{r1} = A_1 \ddot{z}_1 + b_1 \dot{z}_1 \qquad (3)$$

where A_1 is the buoy's added mass at infinite frequency and b_1 is its radiation damping. The hydrostatic force also considered as the restoring force of the water can be written as:

$$F_{h1} = g\rho\pi {r_{buoy}}^2 \ z_1 = K_1 z_1 \qquad (4)$$

where g, ρ and r_{buoy} are the acceleration due to gravity, density of seawater and radius of buoy respectively. Hence, K_1 is a constant whose linearity depends on the diameter of the buoy and the magnitude of the motions of the buoy. Furthermore, after neglecting the cross-coupling terms, the coupling radiation force, F_{r12} acting from the spar on the buoy can be written as:

$$F_{r12} = A_{12} \ddot{z}_2 \qquad (5)$$

Likewise, applying the same principle to the spar of the system we obtain the following equations:

$$F_{r2} = A_{22} \ddot{z}_2 + b_2 \dot{z}_2 \qquad (6)$$

$$F_{r21} = A_{21} \ddot{z}_1 \qquad (7)$$

Referring to Fig. 2, the system is moored to the sea bed using 3 cables configured like a tripod with an angle α with respect to the sea floor. According to Hooke's law the total mooring force F_C can be written as follows:

$$F_C = -3K(l' - l) = -3K\left(\sqrt{l^2 + {z_2}^2 - 2lz_2 \cos(90+\alpha)} - l\right) \qquad (8)$$

Therefore, the non-linear vertical mooring force F_m for all 3 cables in total is:

$$F_{m(non-linear)} = -3F_C \sin\alpha \qquad (9)$$

which can be rewritten as:

$$F_{m(nonlinear)} = -3K \sin\alpha \left(\sqrt{l^2 + {z_2}^2 - 2lz_2 \cos(90+\alpha)} - l\right) \qquad (10)$$

The system nevertheless needs to be linearized for the application of an optimal feedback controller. This can be done by linearizing equation (10). Since $z_2 \ll 1$, we can choose our linearization point at $z_2=0$ and still obtain good results. Also, since the displacements in the cables compared to the length of the cables are relatively small, these changes are neglected. Hence assuming that the angle α is constant, the new linearized equation is as follows [4]:

$$F_{m(linear)} = (F_m + F'_m \Delta z_2)|_{z_2=0}$$

$$= -3K\sin(\alpha)\frac{z_2 - l\cos(90^0+\alpha'}{\sqrt{l^2+z_2^2-2lz_2\cos(90^0+\alpha)}}\Delta z_2\Bigg|_{(z_2=0)}$$

$$= -3K(\sin\alpha)^2 z_2 = -K_m z_2 \qquad (11)$$

The above equation holds true even if $z_2 > 0$, even for large displacements such as 0.5m. The difference between the non-linear and linear mooring force is only 0.04% for a cable length of 170m. Therefore, the linear case approximation can be considered to be very good.

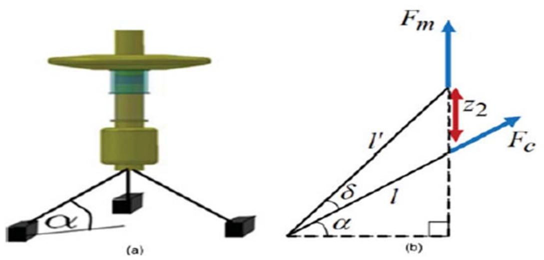

Fig.3. Mooring system for WEC (left) and schematic diagram of forces present in the mooring system(right). [4]

In order to control the system, its state-space representation is derived. For simpler notation, we here combine and rename some constant terms as follows:

$$m_{e1} = \left(m_1 + A_1 - \frac{A_{12}A_{21}}{m_2+A_2}\right) \tag{12}$$

$$m_{e2} = \left(m_2 + A_2 - \frac{A_{21}A_{12}}{m_1+A_1}\right) \tag{13}$$

Hence, we can represent the system in such a form:

$$\dot{x} = Ax + BF_{PTO} + E_1F_{e_1} + E_2F_{e_2} \tag{14}$$

where

x_1 is the buoy's displacement,

x_2 is the spar's displacement,

x_3 is the buoy's velocity,

x_4 is the spar's velocity,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K_1}{m_{e1}} & -\frac{b_1}{m_{e1}} & \frac{A_{12}K_m}{m_{e1}\left(m_2 + A_2\right)} & \frac{A_{12}b_2}{m_{e1}\left(m_2 + A_2\right)} \\ 0 & 0 & 0 & 1 \\ \frac{A_{21}K_1}{m_{e2}\left(m_1 + A_1\right)} & \frac{A_{21}b_1}{m_{e2}\left(m_1 + A_1\right)} & -\frac{K_m}{m_{e2}} & -\frac{b_2}{m_{e2}} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ \frac{1}{m_{e1}} + \frac{A_{12}}{m_{e1}(m_2 + A_2)} \\ 0 \\ -\frac{1}{m_{e2}} - \frac{A_{21}}{m_{e2}(m_1 + A_1)} \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 0 \\ \frac{1}{m_{e1}} \\ 0 \\ -\frac{A_{21}}{m_{e2}(m_1 + A_1)} \end{bmatrix}$$

$$E_2 = \begin{bmatrix} 0 \\ -\frac{A_{12}}{m_{e1}(m_2 + A_2)} \\ 0 \\ \frac{1}{m_{e2}} \end{bmatrix}$$

III. Optimal Control Strategy

As mentioned by K. Budal and J. Falnes in [10], "for an oscillating system to be a good absorber it should be a good wave generator too", that is the device must generate a wave which interferes destructively with sea waves. Hence, the phase of the wave generated by the WEC, relative to the incident sea waves, is of high importance. Optimum destructive interference will result in maximum power absorption which is obtained through optimum phase control of the absorber's oscillation [2]. Therefore, the energy absorbed by the WEC's body is directly determined by the wave parameters and the motion of the WEC's body. Thus, it is important to control the motion of the body so that maximum power absorbed is converted to useful power. However, conversion of power is highly dependent on the type of machinery used, that is on its efficiency. Hence, to enhance the amount of useful power, we need to optimise the amount of power absorbed which can be done through the control of the WEC's body. This study is focused on an all-electric PTO WEC system. PTO control directly affects the performance of the electrical components and the primary capture system of the WEC. PTO control strategies can be applied to different parts of a WEC. According to [8], PTO control is divided into two categories, (1) resistive control and (2) reactive control.

Resistive control, also known as passive control, deals with only the PTO damping force of the WEC. Control strategies such as latching control [11] and clutching control [12] are examples of passive control strategies that can be employed for WECs. On the other hand, reactive control requires the device to use some of its generated power to keep the velocity and the excitation force of the device in phase hereafter controlling its motion. To understand the principle of resistive and reactive control, let us take a look at the mathematical model described for the heave force for a WEC PTO [3]:

$$F_{PTO} = K_{PTO}X(t) + C_{PTO}\dot{X}(t) \tag{15}$$

where X is displacement of the point absorber structure.

F_{PTO} is the PTO force.

K_{PTO} is the system's spring coefficient.

C_{PTO} is the system's damping coefficient..

From the above equation, it can be seen that by controlling C_{PTO} and K_{PTO}, we can change the PTO force which results in an amplitude and phase change of the WEC's motion. This implies that power extraction from waves can be increased if we have control over the system's damping and spring coefficients. Controlling the damping and spring coefficient of the system is referred to as reactive control which keeps the phase and amplitude of the point absorber's motion in check. On the other hand, resistive control refers to only controlling the PTO damping coefficients. [3]

Most generators convert linear movement into a rotating movement to be able to extract energy and convert it into electricity. However, robustness is quite difficult to achieve as well as it is difficult to build a maintenance free mechanism for such a system. That is why linear generators are preferred where we can convert linear motion directly into electrical energy. By the use of a converter, the linear generator can be easily connected to the grid.

IV. Linear Quadratic Gaussian Control

Linear quadratic Gaussian control originates from the optimal stochastic control theory and has a variety of applications in modern technology such as navigation control systems, medical processes controllers and in nuclear power plants. Also known as an optimal feedback controller, the latter is mainly used to minimize tracking errors in the outputs of a system [15]. The LQG controller consists of two main parts:

- A linear quadratic estimator (LQE).
- A linear quadratic regulator (LQR).

The LQG control technique has the advantage that it enables direct trade-off between regulation performance and control efforts while also considering system and measurement noises that follow a Gaussian distribution. This means that the controller can account for unpredicted effects, process and sensor noises as well as other effects on the system outputs that are not caused by the inputs, hence making the LQG controller reliable and useful in different surroundings [15].

A. Kalman Filter

Considered as one of the greater discoveries in the history of statistical estimation and designed by R. E. Kalman, the Kalman filter is an estimation system used to solve the problem of estimating instantaneous states of a linear dynamic system which is affected by white Gaussian noise. The result is an estimator which is optimum for any quadratic function of estimation error. Since all of a system's true state can be difficult to measure due to limitations because of cost and availability of sensors and materials, the Kalman filter performs a state estimation which is then used to generate control inputs for the system. This estimation is refined as the filter learns the actual states by comparing it to the predicted output using the estimated states and the measured output.

As both estimation and generation of system inputs from the estimator happen at the same time, its accuracy increases over time hence stabilizing the system outputs at the desired values quicker [15]. For the application of Kalman filtering over a system, the latter need to satisfy the observability criterion. A linear dynamic system is considered to be observable if and only if the system's states can be solely determined from the system's defined inputs and from its outputs.

Consider a system defined by:

$$\dot{x} = Ax + Bu + Gw \tag{16}$$

$$y = Cx + v \tag{17}$$

Where u is known inputs, w is the white gaussian process noise and v is the white gaussian measurement noise. To determine whether a system is observable, we need to determine the observability matrix which is given by:

$$O_m = \begin{bmatrix} C \\ CA^1 \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \tag{18}$$

Next, consider our kalman filter system to be defined as:

$$\dot{\hat{x}} = A\hat{x} + Bu \tag{19}$$

$$\hat{y} = C\hat{x} \tag{20}$$

where $\hat{x}$ represents the estimated states.

To obtain the dynamics of the difference between our actual system and that of the Kalman filter, we subtract equations (19) and (20) from equations (16) and (17) respectively to obtain:

$$\dot{x} - \dot{\hat{x}} = A(x - \hat{x}) + Gw \tag{21}$$

$$y - \hat{y} = C(x - \hat{x}) + v \tag{22}$$

However, if we decide to ignore the process and measurement noises affecting our system or consider them to be zero, we obtain:

$$\dot{x} - \dot{\hat{x}} = A(x - \hat{x}) \tag{23}$$

$$y - \hat{y} = C(x - \hat{x}) \tag{24}$$

As can be seen from equations (23) and (24), there are no external inputs affecting the dynamics of difference between the actual system and the estimated system except for the Gaussian noises. This implies that the speed of convergence for both systems remains identical since both systems have the same characteristic equation which is defined by $\det(\lambda I - A)$.

In order to increase the speed of convergence, feedback is used to design a transient response which is introduced into the estimator which drives the error between the actual states and the estimated states to zero [16]. If we let $e = (x - \hat{x})$, we obtain the following set of equations:

$$\dot{e} = (A - LC)e + Lv + Gw \tag{29}$$

$$(y - \hat{y}) = Ce + v \tag{30}$$

The estimated state vector e is still unforced and will decay to zero if all its eigenvalues are negative. Therefore, to design an observer system, we need to design for the values of the Kalman gain L that will result into our desired characteristic equation which can be found by $\det(\lambda I - (A - LC))$ [16]. However, we can also notice that our estimator is affected by process and measurement noises. To account for these noises, the Kalman filter requires two additional parameters as described below:

- $Q_n = E\{ww'\}$
- $R_n = E\{vv'\}$

Q_n and R_n are the covariances for the process noise, w, and measurement noise, v, respectively and w' and v' is the transpose of the process noise and measurement noise respectively. Both Q_n and R_n can be positive definite or semi-Hermitian or real symmetric matrices. Once the Kalman filter has been designed, it is combined with the optimal feedback gain to obtain the LQG controller.

B. Linear Quadratic Regulator

The linear quadratic regulator is simply a state feedback gain that is used to optimize a plant's outputs. The gain is connected as a negative feedback to the plant and acts as an input to the plant. The values of the feedback gain compensate for the states that are estimated and hence drive the system optimally as long as full state estimation is possible and provided that the system is fully controllable. Let us consider a linear dynamic system which is defined over a finite time interval of $t_0 \leq t \leq t_f$.

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{31}$$

The system above is considered to be controllable if at time $t = t_0$, for any desired final state $x_f(t)$, there exists an input function that drives the state $x_f(t)$. Therefore, if every initial state of the system is controllable over a finite duration, then the system is said to be fully controllable [17]. A controllability matrix can be determined by using the A and B matrix of the system as shown below:

$$C_m = [B \; AB \; A^2B \ldots A^{n-1}B] \tag{32}$$

where if the rank of the controllability matrix is the same as the rank of A, then the system is considered to be fully controllable. The LQR deals with the optimization of a cost function. The latter can be minimized by using the inputs generated from the estimator. The cost function used is the sum of two costs: (1) penalty for not achieving the desired output and (2) penalty on the control input. Both of these costs have weights which can be specified by the designer. The equation below represents the cost function [4]:

$$H = \int_0^{\infty} (x^T Q x + u^T R u) dt \tag{33}$$

$$u = -Kx \tag{34}$$

where K is the optimal feedback gain, Q is a diagonal matrix with positive values that incorporate the relative importance of achieving different output targets and R is also a diagonal matrix that determine the relative preference for the control inputs [15]. Once the LQR gain has been designed, we can then combine it with our observer to form our LQG controller. Forming the LQG is as simple as feeding the estimated states from the Kalman filter through the optimal feedback gain to our plant.

V. LQG Design and Simulation Results

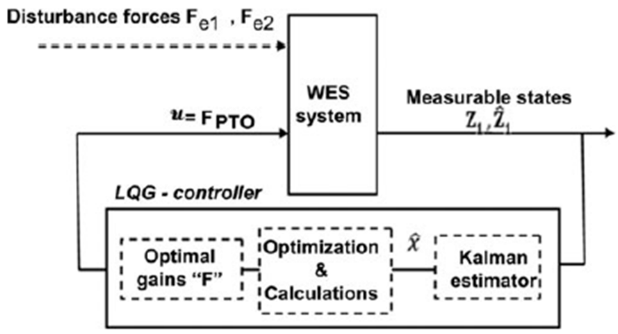

Fig.4. Wave Energy system combined with LQG controller. [4]

The WEC virtual model is simulated using MATLAB/SIMULINK software package. Beginning with the design of the estimator, the function *'lqe'* from MATLAB function list is used to obtain the kalman filter gain L, which requires the process noise covariance matrix Q_n, and the measurement noise covariance matrix, R_n. They were set as follows:

$$Q_n = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad R_n = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Having estimated the full states of the WEC, the optimal feedback gain is designed by using the '*lqr*' function that provides the control input for our system which in turn helps in maximum power extraction from the waves. Using this function, we can directly compute our optimal state feedback gain given our state matrix A and input matrix B by optimizing the cost function (33), for the system together with the Q and R matrices. Since they directly affect the optimal feedback gain, the matrices Q and R need to be set appropriately. Matrix Q and R needs to be a positive-definite or semi-definite Hermittian or real symmetric matrices and are chosen as follows:

$$Q = \begin{bmatrix} 20 & 0 & 0 & 0 \\ 0 & 10 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 10000 \end{bmatrix}; R = 10$$

The full state feedback gain is determined as:

$$K = [-1.8826 \quad 0.6898 \quad -19.8948 \quad -30.9666]$$

For simulation purposes, we first observe the WEC without any control when an irregular waveform is applied to it as shown in Fig. 3. We can observe a maximum elevation of 2.5 m and a peak to peak wave height of 4.78 m. The initial disturbance to the WEC causes the buoy to move significantly to a height of 4 m but after stabilizing itself, the buoy moves between the limits of ± 2.6 m. The buoy velocity also rises significantly in the beginning but then stabilizes itself to be within the limits of ±2.8 m/s. We can also notice that the buoy lags the wave by a very small margin and the buoy displacement is larger than that of the wave elevation itself. This is because the buoy just 'follows' the wave elevation and is not designed to resonate with the waves.

Since the buoy is the main driver of the generator for our system, the effect of having the controller is investigated and how the controller affects the movement of the buoy. From Fig. 4, it can be seen that with the control input, we have a greater heave motion in buoy which in turns allows for an increased motion inside the generator's coils producing a greater power production. Fig. 5 shows the effect that the control input action has on the dynamics of the Wave Energy System. As the ocean wave makes contact with the WEC, a controlled input force is initiated to drive both the buoy and spar of the L-10 WEC. The buoy and spar move in opposite direction to maximize the motion of the iron core inside the Linear Permanent Magnet Generator which increases the efficiency of mechanical power transfer from the ocean wave to electrical power generation.

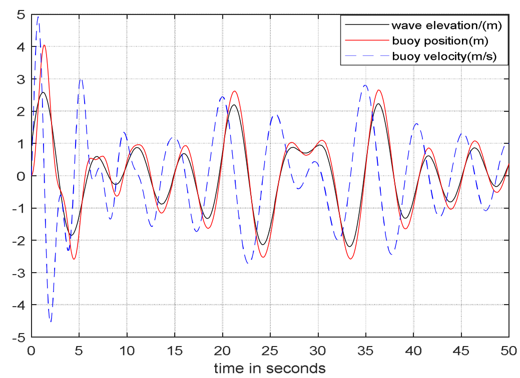

Fig. 3. Outputs of open loop plant with no control action and compared to wave elevation.

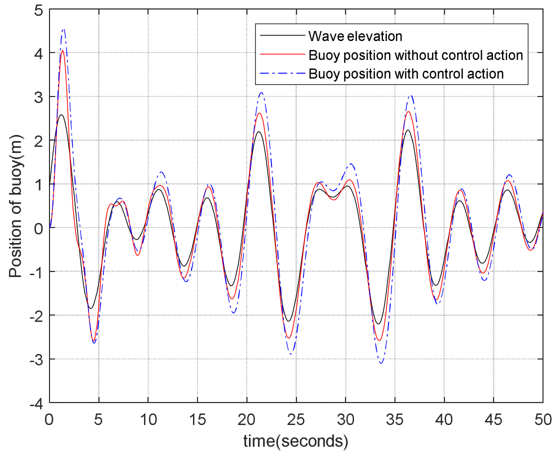

Fig. 4. Comparison of buoy motion with regards to the controller's input action.

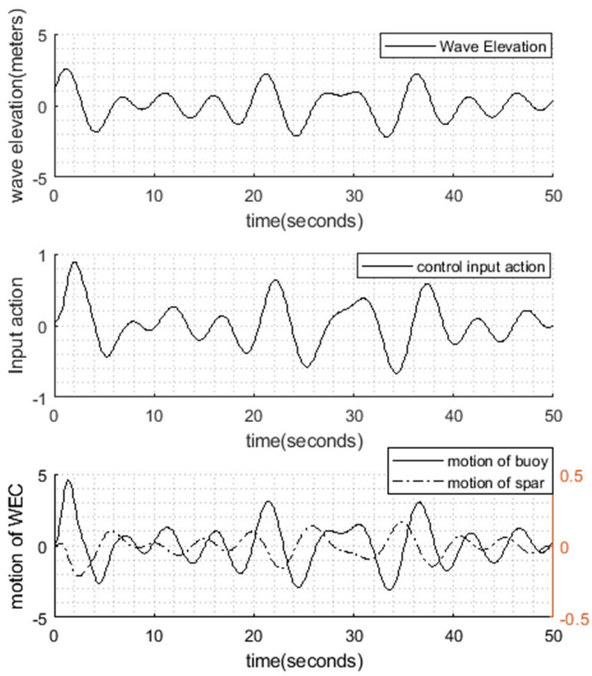

Fig. 5. Action of controlled input force.

VI. Conclusion

From the results obtained, it can be claimed that the LQG controller employed for the L-10 WEC yielded very good results. The Kalman filter is shown to be robust where despite having very noisy inputs the system is still able to function properly and provide the regulator system with accurate estimation for the actual states. On the other hand, the regulator optimizes each of the estimated states by applying appropriate weights to them. The research carried out in this paper showed that a system can be controlled even if not all of its states are available for measurement, simply just by estimating the missing states and applying the appropriate feedback to it given that the system is both controllable and observable.

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