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OPTIMAL FREQUENCY REGULATION OF A SINGLE-AREA POWER SYSTEM

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ABSTRACT
The mismatch between the generating capacity of a power system and the load demand at any given moment in time is reflected in the system's frequency. Large deviations from a chosen nominal value of frequency are undesirable for numerous reasons. Power systems frequency is traditionally regulated using proportional-integral (PI) or other forms of classical controllers. While these controllers are effective, they are far from being optimal. In this paper, we propose a linear quadratic Gaussian (LQG) controller which includes integral action that guarantees optimal performance of a single-area power system. In particular, we show that our proposed controller performs better than an optimized PID controller in the face of constant or low frequency disturbances which accurately model load changes in the power system.

KEY WORDS
Power system, LQG control, Integral action

1 Introduction

Man's demand for and consumption of energy has increased steadily over the years and a major portion of the energy needs of a modern society is supplied in the form of electrical energy. Industrially developed societies need an ever-increasing supply of electrical power, and the demand can double every ten years. As a result, very complex power systems have been built over time and supplying electrical energy presents numerous engineering problems that provide the engineer with a variety of challenges. To predict and regulate the performance of such complex systems, the engineer is forced to seek ever more powerful tools of analysis and synthesis.

Most of the generators and some of the loads are rotating machines and at steady-state they rotate at synchronous speed or 50Hz. The system, however, is never at steady state because the loads are always varying as customers constantly change their electricity consumption. Thus the power system is continuously subjected to random perturbations. If these changes are relatively small compared to the inertia of the total rotational mass of all the machines, the machine rotational speed does not deviate much from synchronous speed, as long as the small imbalances between generation and load are continuously corrected. If the disturbance is large, for e.g., loss of a large generator or load, short-circuit on a high voltage transmission line or the sudden application of a major load such as a steel mill, it is quite possible that some of the machines will deviate significantly from synchronous speed. In some cases, machines deviating too much from synchronous speed may become unstable and lose synchronism. Obviously, this is not desirable but large disturbances do happen and the power system has to be designed and operated to ensure that such credible disturbances do not actually disrupt power supply to customers.

The quality of a power system depends largely on the engineer's ability to provide reliable and uninterrupted service to the loads. This implies much more than merely being available and ideally, the loads must be fed at constant voltage and frequency at all times. In practical terms, this means that both voltage and frequency must be held within close tolerances so that the consumer's equipment may operate satisfactorily. For example, a drop in voltage of 10% or a reduction of the system frequency of only a few hertz may have dramatic effects on connected loads, see [1]. The power system operator must maintain a very high standard of continuous electrical service.

Under normal conditions, the requirements of a power system are to:

- meet the load demands and satisfy the power flow equations, see [2];
- ensure that the operating frequency is constant (50 Hz);
- maintain a bus voltage that is within prescribed limits.

In this paper, we will model the subsystems comprising of a single-area power system, with a view to regulate the frequency of the system in the presence of realistic disturbance models. We model the components of the power system with fixed parameters and an optimal controller is designed for the system. The optimal controller design algorithm takes into account process noises which feed into the system at various points and which can be regarded as uncertainty in the fixed parameters. Also, we include

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sensor noise which arises due to imperfections in the measurement system. These noises are modeled as Gaussian white noises, i.e., serially uncorrelated noises which are normally distributed with zero mean and a given variance. The control problem then becomes one which is based on the "separation theorem"; see [3]. The controller is determined by obtaining *minimum mean square linear* estimates of the states of the system using a Kalman filter which are then combined with an optimal state-feedback control law.

It is well known in the control literature that standard LQG techniques is appropriate for random disturbance and are not able to satisfactorily deal with constant or low frequency disturbances; see, e.g., [4–6]. A novel feature of our approach is that our controller also caters for fixed disturbances. By including integral action in our design and suitably augmenting our system, an additional variable is included in a modified cost function that is then minimized. We shall expand on this feature in the sequel. The stability of the system is investigated and we support our design with suitable simulation. Also, to justify the superior performance of the proposed LQG control scheme, we compare its performance with an optimized PID control scheme designed using Matlab®.

2 Power System Model

Generators supply both *real* (P) and *reactive* (Q) powers. The real and reactive power components can be viewed as separate control inputs acting on the system and the obvious way of keeping a perfect power balance is to continuously keep the generated powers P_G and Q_G in balance with the loads P_L and Q_L. Thus, each generator is equipped with two separate feedback loops to regulate the real and the reactive powers respectively. The voltage in a power system is regulated by controlling the reactive power output and this is achieved by manipulating the field current supplied to the generator. On the other hand, frequency regulation is achieved by controlling the real power output. This is done by adjusting the position of a steam control valve which in turn modifies the turbine torque to match changes in the load torque. Both control loops are designed to operate around an equilibrium point with small excursions tolerated about that point. The system may thus be modelled with linear differential equations and represented using linear transfer functions. We will restrict our attention to the frequency regulation problem from this point onwards.

The frequency regulation loop can be analysed by modelling its building blocks. In general, it comprises of a hydraulic amplifier, a turbine and a generating unit. A reference power setting is fed to the hydraulic amplifier which controls the opening position of a steam valve. The reference power is chosen such that the frequency of generated voltage is 50 Hz. The amplifier can be modelled as a first-order transfer function with a time constant T_H as follows:

$$G_H = \frac{1}{1 + sT_H}.$$

The flow rate of steam regulates the angular speed of rotation of the turbine. The dynamics of turbines vary widely depending on the type used. In this problem, we consider a "Non-Reheat Steam Turbine" where steam enters through a steam-chest, before going through the turbine and back to the condenser. The steam-chest introduces a delay T_T in the system. The turbine can be modelled as:

$$G_T = \frac{1}{1 + sT_T}.$$

The turbine torque together with the loading conditions on the generator then determine the frequency of generated voltage.

Fig. 1 shows the interconnection of the differential blocks that make up a single-area power system. K_p and T_p represents the power system gain and time-constant respectively. Also, P_L represents the local load change or line load change or both.

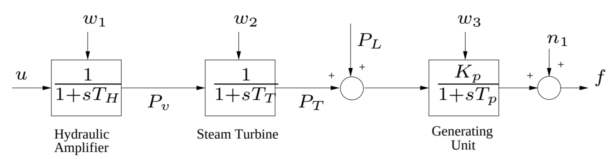

Figure 1. Subsystems representation of a single-area power system.

The system described in Fig. 1 can also be represented in state-space form as follows:

$$\begin{aligned} \dot{x} &= Ax + B_1 u + B_2 w; \\ y &= Cx + n_1. \end{aligned} \tag{1}$$

where the output y represents the power system's frequency f and $w = [w_1\, w_2\, w_3]^T$ represents the three sources of mechanical noises feeding into the system. These can be regarded as uncertainty in the parameters of the individual blocks.

3 Linear Quadratic Gaussian Control

In the papers ([7, 8]), the linear filtering problem was solved and the resulting optimal estimator is known as the Kalman-Bucy filter. This result allows us to design an estimator for a linear system, which given the probabilistic distribution (statistics) of noise(s) entering the system, provides the best possible estimate of the states of the system. By best possible here, we roughly mean an estimator whose output is closest to what it should be, despite the noise(s). An important assumption in the realisation of this optimal estimator is the nature of the noise(s) present. It is assumed that the noise(s) are white, Gaussian and have zero mean. White noise implies that the noise is uncorrelated from one instant of time to another while the second property implies that the covariance of the noise, $\mathbf{E}[v(t)\ v'(\tau)]$, where $v(t)$

is one source of noise feeding into the system, provides all the necessary probabilistic information about the noise. This mathematical assumption turns out to be quite convenient as most naturally occurring physical systems are indeed afflicted by Gaussian processes with zero mean. For the interested reader, a more detailed description of filtering theory is provided in [9].

The state feedback quadratic optimal control problem on the other hand was solved in terms of Riccati equations in [10]. Combining the optimal state-feedback law and the state-estimation method leads to the solution of the LQG optimal control problem. This result is also commonly referred to in the control literature as the "separation theorem" as the two separate problems are tackled independently before being combined, as shown in Fig. 2; see also [5].

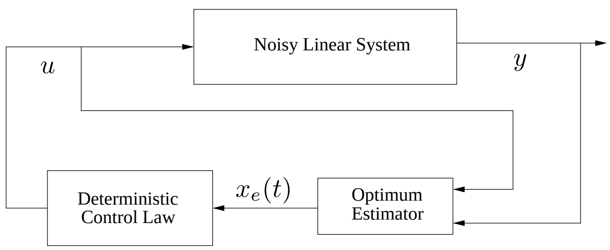

Figure 2. Illustration of Separation Theorem

The LQG problem is then one where a controller is designed to minimize a cost-function of the form:

$$V = \mathbf{E}\left[\int_0^T \left[x^T Q x + u^T R u\right] dt\right], \tag{2}$$

where $Q \geq 0$ and $R > 0$.

An LQG performance criterion as described in (2) will result in a closed-loop system where the state x and the control signal u will be stationary random processes. The nature of our problem here is such that it is subjected to disturbances with high-frequency as well as very-low frequency components. In most cases, the disturbances can be regarded as being d.c. components as they represent constant changes in load which are maintained for long periods of time. This naturally implies a need for a constant non-zero control signal u which will drive the cost function as defined by (2) to a very large value in a short time. We deal with this problem by introducing integral action in our LQG controller design. Integral action is included by adding an additional term in the cost function which involves the integral of the output. Furthermore, we also generate an additional fictitious output of the system by integrating the output y. This new output $\int y\,dt$ is also fed to the Kalman filter, which when combined with an optimal state feedback control law leads to an LQG integral optimal controller. This controller will then meet the desired performance requirements as described above as well as reject low frequency disturbances. Fig. 3 shows the integral LQG controller design setup.

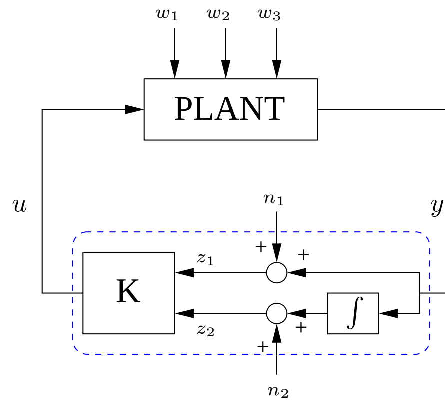

Figure 3. Integral LQG controller design.

The augmented system can be described in state-space form as follows:

$$\begin{aligned} \dot{\tilde{x}} &= \tilde{A}\tilde{x} + \tilde{B}_1 u + \tilde{B}_2 w; \\ \tilde{z} &= \tilde{C}\tilde{x} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}, \end{aligned} \tag{3}$$

where

$$\tilde{x} = \begin{bmatrix} x \\ \int y\,d\tau \end{bmatrix} \text{ and } \tilde{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

and the matrices $\tilde{A}, \tilde{B}, \tilde{C}$ are constructed from the matrices A, B, C in (1) as follows:

$$\tilde{A} = \begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix}, \; \tilde{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \; \tilde{B}_2 = \begin{bmatrix} B_2 \\ 0 \end{bmatrix};$$

$$\text{and } \tilde{C} = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}.$$

The integral LQG performance criterion can be described as:

$$\mathcal{J} = \lim_{T\to\infty} \mathbf{E}\left[\frac{1}{T}\int_0^T [x^T Q x + u^T R u + L(y)^T \bar{Q} L(y)]dt\right];$$

where

$$L(y) = \int_0^t y(\tau)\,d\tau.$$

We choose the matrices Q, R and $\bar{Q}$ such that

$$x^T Q x = |y|^2, \;\; u^T R u = r|u|^2, \text{ and } \; \bar{Q} = \bar{q},$$

where $r > 0$ and $\bar{q} > 0$ are also treated as design parameters.

Given the system as described by (3), the optimal LQG controller is given by (e.g., see [11])

$$u = -r^{-1}\tilde{B}_1^T X \hat{\tilde{x}}, \tag{4}$$

where

$$0 \quad = \quad X\tilde{A} + \tilde{A}^T X + \tilde{Q} - r^{-1} X \tilde{B}_1^T \tilde{B}_1 X, \qquad (5)$$

and

$$\tilde{Q} = \tilde{C}^T \begin{bmatrix} 1 & 0 \\ 0 & \bar{q} \end{bmatrix} \tilde{C}.$$

The observer dynamics are described by

$$d\hat{\tilde{x}} = \tilde{A}\hat{\tilde{x}}\,dt + \tilde{B}_1 u\,dt + K[d\tilde{z} - \tilde{C}\hat{\tilde{x}}\,dt] \qquad (6)$$

and for the case of uncorrelated process and measurement noises, the solution of the optimal observer is obtained by choosing the gain matrix

$$K \quad = \quad P\tilde{C}^T V_2^{-1}, \qquad (7)$$

where

$$0 \quad = \quad \tilde{A}P + P\tilde{A}^T + V_1 - P\tilde{C}^T V_2^{-1}\tilde{C}P. \qquad (8)$$

Here

$$V_1 = \tilde{B}_2\,\mathbf{E}[w\,w^T]\,\tilde{B}_2^T = \tilde{B}_2 \begin{bmatrix} \epsilon_1^2 & 0 & 0 \\ 0 & \epsilon_2^2 & 0 \\ 0 & 0 & \epsilon_3^2 \end{bmatrix} \tilde{B}_2^T;$$

$$\text{and} \quad V_2 = \mathbf{E}[n\,n^T] = \begin{bmatrix} \epsilon_4^2 & 0 \\ 0 & \epsilon_5^2 \end{bmatrix};$$

define the covariance of the process and measurement noises respectively.

4 Simulation Results

The parameters of the plant are given in Table 1.

	Values
Hydraulic Amplifier time-constant, T_H	0.1 sec
Steam Turbine time-constant, T_T	0.3 sec
Power system gain, K_p	75 Hz/p.u.MW
Power system time-constant, T_p	13 sec

Table 1. Model Parameters

The design parameters are chosen as $\epsilon_1 = 0.1$, $\epsilon_2 = 0.1$, $\epsilon_3 = 30$, $\epsilon_4 = 10$, $\epsilon_5 = 1$, $r = 1 \times 10^7$, $\bar{q} = 1 \times 10^{16}$. For comparison purposes with conventional controllers used for this type of problem, we also designed an optimized PID controller using a first-order filter at the output of the derivative term. The filter is chosen to have a frequency that is low enough to restrict high frequency components which may arise due to abrupt changes in the load but high enough to damp the controller output. The robust response time tuning algorithm was used and the controller was then fine-tuned to obtain satisfactory gain and

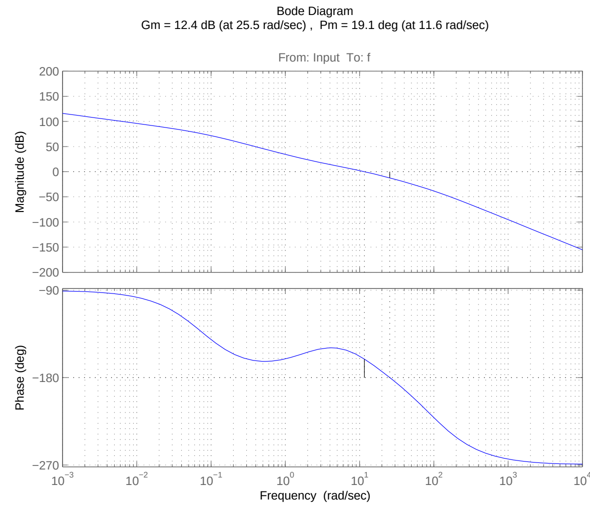

Figure 4. Bode plot of the loop gain transfer function

phase margin. The transfer function of the resulting PID controller is given as:

$$K_{PID}(s) = 8.4617\frac{(0.22s+1)(0.47s+1)}{s(0.01s+1)} \qquad (9)$$

The corresponding loop-gain is shown in Figure 4.

The power system is then simulated with the load being modeled as integrated white noise. To show the behavior of the system due to sudden changes in the load, we model a change of 10% in the load or 0.1 per-unit at times 1s and 5s respectively. The response of the system with the LQG controller and the optimized PID controller is compared in Fig. 5.

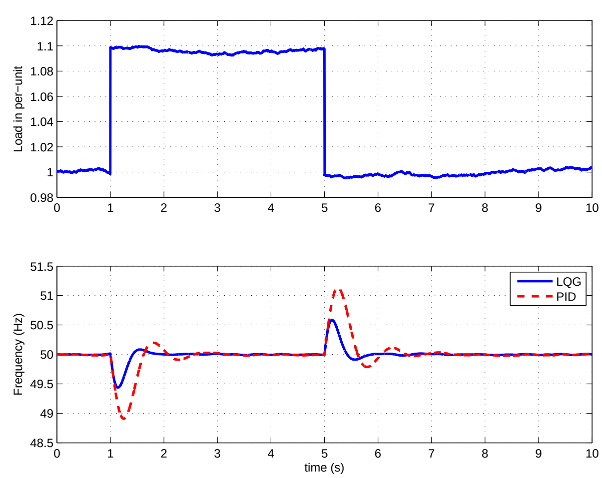

Figure 5. Time-response of power system with LQG and PID controllers

It is clear that the LQG controller shows superior performance to the PID controller. It provides better disturbance rejection with the frequency deviating by only 0.6

Hz, compared with 1.2 Hz for the PID controller. Moreover, it also has a much smaller settling time.

5 Conclusion

We have modelled a single-area power system with the aim of regulating the frequency of the system. An integral LQG controller was designed taking into account uncertainties present in a practical system and the closed-loop system was simulated using realistic disturbance models. The response of the system is shown to be superior to that of an optimized PID controller. While the model developed is valid only for single-area systems, it is possible to extend the idea to multi-area systems where control commands are sent in synchronism to all the generators.

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