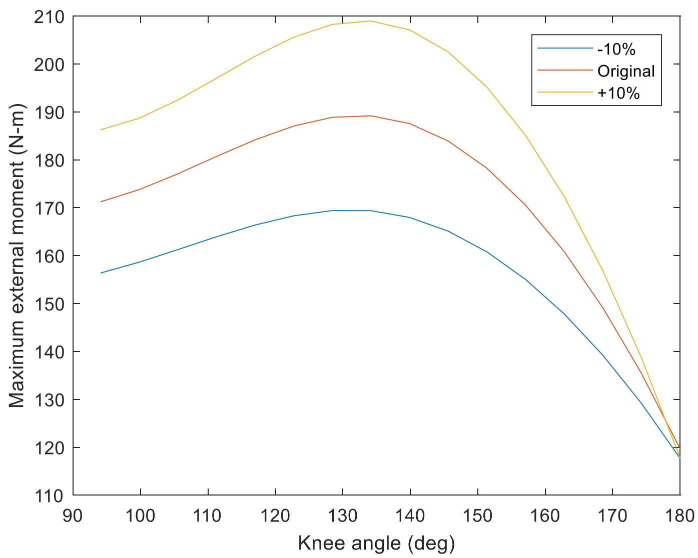
HW03

Sayed Thangal

1A:



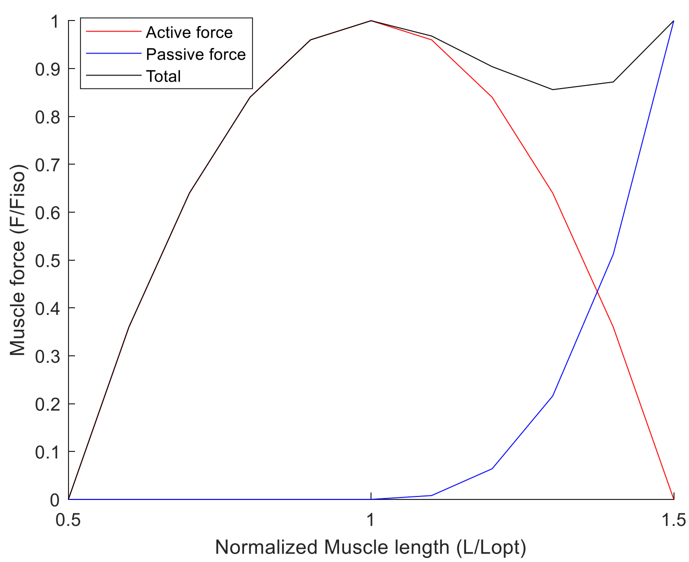
The maximum available moment consists of two terms:

1. The torque generated by the muscle, which is muscle force multiplied by moment arm. $F_{musc} * rf$

2. The torque due to gravity acting on the shank center of mass. $m * g * r_{shankCOM} * sin(\varphi - \frac{\pi}{2})$

Adding them together gives you the free moment available at the knee.

1B.



The active force length curve represents the force generated by the sarcomeres with in the muscle. There is maximum overlap between the actin and myosin bands at the optimal muscle length (L_{opt}).

The passive force length curve represents the elastic connective tissues within the muscle. Here we assume that these elastic tissues only start producing force when the muscle is stretched past its optimal length.

1C. If force velocity was introduced into the model, it would reduce the force generated by the muscle as a function of the velocity of contraction.

1D. Presently, there is a one-to-one relationship between muscle length and knee joint angle. If an elastic tendon would be included, it would stretch due to the forces acting about its ends, i.e. the muscle torque and gravitational torque. This would cause the angle at which the maximum external moment occurs to shift closer to 90 degrees from its present angle of 135 degrees. The stretch in the elastic tendon would also vary with knee joint angle, distorting the present curve.

HW03

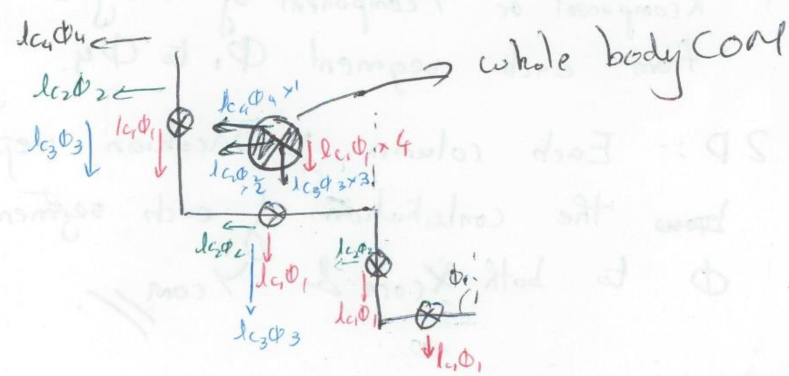
(1)

A2A)
$$X_c = \begin{bmatrix} x_{c1} \\ y_{c1} \\ \phi_{c1} \\ x_{c2} \\ y_{c2} \\ \phi_{c2} \\ \vdots \end{bmatrix} \qquad \Phi = \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \end{bmatrix} \qquad \dot{X}_c = \underbrace{J_{x_c\phi}}_{\text{pose Jacobian}} \dot{\Phi}$$

$$\begin{bmatrix} \dot{X}_{com} \\ \dot{Y}_{com} \end{bmatrix} = \overbrace{\begin{bmatrix} \frac{m_1}{M} & 0 & 0 & \frac{m_2}{M} & 0 & 0 & \frac{m_3}{M} & 0 & 0 & \frac{m_4}{M} & 0 & 0 \\ 0 & \frac{m_1}{M} & 0 & 0 & \frac{m_2}{M} & 0 & 0 & \frac{m_3}{M} & 0 & 0 & \frac{m_4}{M} & 0 \end{bmatrix}}^{M} \cdot \dot{X}_c$$

$$\therefore \begin{bmatrix} \dot{X}_{com} \\ \dot{Y}_{com} \end{bmatrix} = \underbrace{\left[M J_{x_c\phi} \right]}_{\text{Jacobian from whole body COM to segment angles}} \dot{\Phi}$$

2B



2c) constraint $C\dot{X} = 0$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \dot{X}_{com} \\ \dot{Y}_{com} \end{bmatrix} = \begin{bmatrix} 0 \\ \dot{Y}_{com} \end{bmatrix}$$

2D $\dot{X}_{com} = J_{X_{com}\phi}\,\dot{\phi}$

$$T = J^T_{X_{com}\phi} F_{com}$$

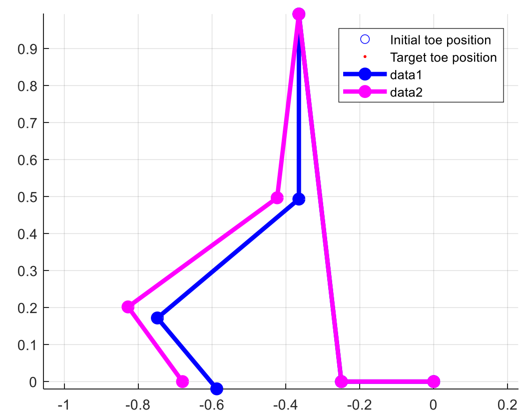
Transformer rule gives forces on COM to joint torques eqn

2E:- $\dot{X}_{com} = \{M \cdot J_{X\phi}\}\,\phi$

Each row represents the contribution to X component or Y component of body COM velocity from each segment Φ_1 to Φ_4.

2D :- Each column of Jacobian represents ~~how~~ the contribution of each segment angle Φ to both $\dot{X}_{com}$ & $\dot{Y}_{com}$ //.

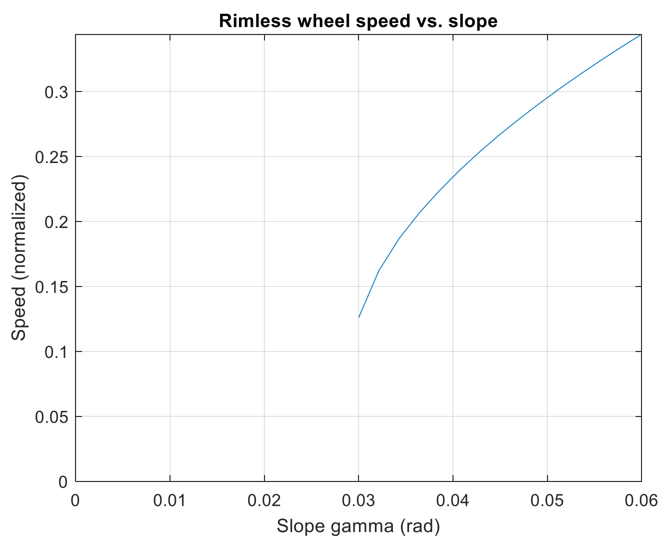
A3A:



Initial guess q = [6.60363 0 -50] degrees
Initial toe error = -0.0926747 0.0195394
Final value   q = [6.60363 -6.74877 -53.8476] degrees
Final toe error = -1.63658e-12 -6.74677e-12

A3B: We can set up an optimization using the single shooting method. The optimization will vary the initial states q1, q2, q3, q1dot, q2dot, q3dot. The cost function would consist of 3 terms:
Cost function = $(SL-SL_{desired})+(q2)+(ST-ST_{desired})$
We can then use the root finder to find the initial states that result in the cost function becoming 0, at which point all the conditions will be met.

A4A:

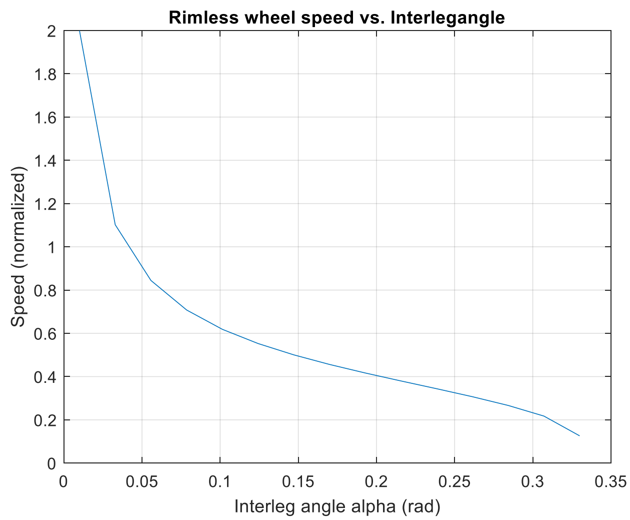


$$\dot{\theta}_*^+ = \sqrt{\frac{4sin\alpha.\gamma}{sin(2\alpha)^2} * cos2\alpha}$$

The steeper the slope, the faster the angular velocity at collision. Assuming that faster angular velocities also mean faster linear velocities, the speed with grow ∝ square root of ground slope.

For a step to take place, the wheel must have enough angular velocity to lift the COM to its peak height before it can fall forward. For ground slope below 0.03 radians and the present initial angular velocities, there is not enough energy to take 1 step.

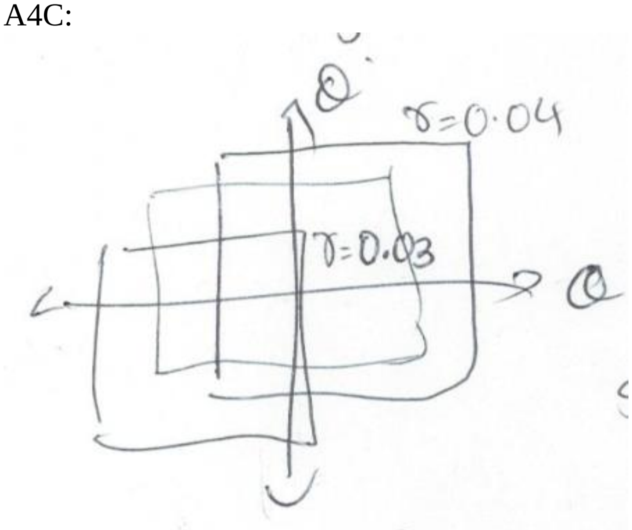
A4B:



$$\dot{\theta}^{+} = cos2\alpha.\dot{\theta}^{-}$$

More interleg angle means less energy transferred at collision. Eventually as α grows and reaches 0.33 radians, the wheel is too slow to take another step.

A4C:



Consider the range of initial conditions that cause the rimless wheel to take a step for a certain value of a parameter, like its basin of attraction. When we vary the parameter slightly, the area of the basin of attraction also shifts slightly. Therefore, if we shift the initial guess for root finding for a parameter P+Δ to the solution of the previous search for P, we have better chances of being within the basin of attraction for the new slightly shifted parameter.