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Energy optimization is a major objective in the real-time control of step width in human walking



Sabrina J. Abram^a, Jessica C. Selinger^b, J. Maxwell Donelan^{c,*}

- ^a School of Engineering Science, Simon Fraser University, Burnaby, British Columbia, Canada
- ^b School of Kinesiology and Health Studies, Queen's University, Kingston, Ontario, Canada
- ^c Department of Biomedical Physiology and Kinesiology, Simon Fraser University, Burnaby, British Columbia, Canada

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ABSTRACT

People prefer to move in energetically optimal ways during walking. We recently found that this preference arises not just through evolution and development, but that the nervous system will continuously optimize step frequency in response to new energetic cost landscapes. Here we tested whether energy optimization is also a major objective in the nervous system's real-time control of step width using a device that can reshape the relationship between step width and energetic cost, shifting people's energy optimal step width. We accomplished this by changing the walking incline to apply an energetic penalty as a function of step width. We found that people didn't spontaneously initiate energy optimization, but instead required experience with a lower energetic cost step width. After initiating optimization, people adapted, on average, 3.5 standard deviations of their natural step width variability towards the new energy optimal width. Within hundreds of steps, they updated this as their new preferred width and rapidly returned to it when perturbed away. This new preferred width reduced energetic cost by roughly 14%, however, it was slightly narrower than the energetically optimal width, possibly due to non-energy objectives that may contribute to the nervous system's control of step width. Collectively, these findings suggest that the nervous systems of able-bodied people can continuously optimize energetic cost to determine preferred step width.

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1. Introduction

When we walk, we tend to prefer a particular step width and execute this preference with remarkably small variability. For young healthy walkers, preferred step width is approximately 12 cm and varies between steps by less than two centimeters (Donelan et al., 2001, 2004). However, this preference can be influenced by walking context. For example, increases in speed result in decreases in step width and increases in its variability (Stimpson et al., 2018). Another context is stability—preferred width and its variability decrease with external lateral stabilization (Dean et al., 2007; Donelan et al., 2004) and increase with visual field perturbations (Bauby and Kuo, 2000; McAndrew et al., 2010; O'Connor and Kuo, 2009). Step width has also been found to increase with both age and obesity (Browning and Kram, 2007; Owings and Grabiner, 2004a; 2004b). One way to understand how the nervous system controls step width is to identify the relative importance of

different walking objectives in response to changes in walking contexts.

One well established objective of the nervous system is to optimize metabolic energetic cost. Decades of research show that people naturally prefer to move in energetically optimal ways (Alexander, 1996; Ralston, 1958; Zarrugh et al., 1974). In the case of step width, people prefer to walk near the energetically optimal width, avoiding high step-to-step transition costs at wide widths, as well as high lateral limb swing and active lateral stabilization costs at narrow widths (Donelan et al., 2001, 2002, 2004; Shipman et al., 2002; Shorter et al., 2017). When steps become wider, step-to-step transition costs increase because of increases in the mechanical work required to redirect the center of mass velocity from one step to the next (Donelan et al., 2002). Whereas when steps become narrower, active lateral stabilization costs increase because of increases in the control required to remain stable (Donelan et al., 2004). And when steps become narrower than the width of the foot, lateral limb swing costs increase because of the mechanical work required to laterally move the swing leg to avoid the stance leg (Shorter et al., 2017). These individual costs, and as a consequence, the energetically optimal step

^{*} Corresponding author at: Department of Biomedical Physiology and Kinesiology, Simon Fraser University, 8888 University Drive, Burnaby, BC V5A 1S6, Canada. E-mail address: mdonelan@sfu.ca (J.M. Donelan).

width, depend upon many factors including some that change with everyday circumstances. For example, the energetically optimal step width may differ with each shoe change as a result of the shoe compliance reshaping the transition cost, and the shoe width and mass reshaping the limb swing cost.

Although the preferred width is near the energy optimal width in familiar walking conditions, this does not necessarily mean that energy optimality drives the nervous system's real-time control of width. First, this preference may arise from long time-scale processes such as evolution and development (Alexander, 2001; 1996; Rodman and McHenry, 1980; Sockol et al., 2007; Thompson et al., 2018). For example, recent evidence suggests that narrow step widths are more feasible in humans relative to our closest living relatives, chimpanzees, as a result of adaptations to our hips and knees (Thompson et al., 2018). Second, the nervous system likely considers objectives other than energy in determining preferred step width. As one example of a step width objective function, the nervous system may seek to simultaneously optimize energy, stability, and maneuverability (Huang and Ahmed, 2011). These objectives may be independently weighted, and their weightings may depend upon walking contexts. For example, the priority for stability may be increased when the consequences of falling are more severe (Rogers et al., 2001; Rogers and Mille, 2003). Some objectives may also be treated as constraints, such as walking on stepping stones where only some widths result in stepping on the stones. We can gain insight into if and how the nervous system represents the real-time control of step width as a multi-term optimization by determining how changes in individual objectives, relative to all other objectives, influence the control of width. If the nervous system weights other objectives higher than energy in its control of step width, then we expect people to persist at or near their original preferred width when the energetic context changes.

The purpose of this study was to test if energy optimization is a major objective in the real-time control of step width. Our lab previously found that people adapt their step frequency to converge on new energy optima, demonstrating that energy is indeed a major objective in the real-time control of some aspects of walking (Selinger et al., 2015). It is not clear, however, that this finding will apply to step width as the nervous system's control of these two gait parameters is typically thought of differently (Bauby and Kuo, 2000; Shipman et al., 2002). And while there is evidence that both the preferred frequency and width are energetically optimal (Donelan et al., 2001; Umberger and Martin, 2007), a common alternative view is that step width is primarily determined by stability (Townsend, 1985). Here we hypothesize that step frequency and width are similarly controlled to optimize energy. To test this hypothesis, we shifted the energy optimal width and observed the nervous system's response. To accomplish this, we built a custom device that applies energetic penalties, in real-time, as a function of measured step width. With this study, we aim to test the generality of energy optimization by using a second method of applying energetic penalties and by studying a different gait parameter.

2. Methods

2.1. Experimental design

After Simha et al., we built a simple mechatronic system to shift people's energy optimal step width (2018). In this system, subjects walked on an instrumented split-belt treadmill at 1.25 m/s (FIT, Bertec Corporation, Columbus, OH, USA). To shift people's energy optimal width, we commanded the treadmill incline based on the desired energetic penalty for the step width measured from the previous step (Fig. 1a). We implemented this closed loop con-

trol of incline based on measured step width using Simulink Real-Time Workshop running at 200 Hz (Simulink Real-Time Workshop, MathWorks Inc., Natick, MA, USA). We first filtered measured forces and moments using a second-order, low-pass, one-way, digital Butterworth filter (15 Hz cut-off), and then calculated the left and right lateral centers of pressure by dividing the left and right lateral moments by their vertical forces (Verkerke et al., 2005). We identified foot contact events from the measured ground reaction forces and moments at the beginning of double support, and estimated step width by taking the difference between the lateral centers of pressure for consecutive steps. The incline changes were limited to 0.5 degrees per second, equating to a shift in vertical position under the subject of approximately 8 mm per second. This change was subtle, and no subjects reported feeling destabilized.

To eliminate the concern that subjects may be adapting their width to achieve level walking rather than to optimize energy, we first modified our design to have subjects always walking at an incline, and then decided to compensate for this additional energetic penalty above level walking with a forward horizontal force that acted as an energetic reward. The combination of controllable walking incline with a near constant forward horizontal force added a degree of novelty to the experimental design, perhaps making it more likely that the nervous system would search for a new optimal width rather than rely on a prediction of the preferred width. To apply the forward horizontal force, we connected a tensioned cable in series with long rubber tubing to a hip belt worn by the user-the long and compliant rubber tubing allowed for small changes in the walking position on the treadmill without large changes in the applied horizontal force. We adjusted the tension to apply a 10% body weight force, on average, and monitored the applied horizontal force with a load cell mounted on a hip-belt (LCM201, Omega Engineering, Norwalk, CT, USA). This forward horizontal force did not depend upon step width or incline and remained nearly perpendicular to the ground.

We used this system to reshape the relationship between step width and energetic cost by providing energetic penalties as a function of step width. We define the relationship between gait and total energetic cost as 'cost landscape' and the relationship between gait and energetic penalties as 'control function'. We simulated our new cost landscape (Fig. 1c, red) by adding literature values for energetic costs associated with walking at different step widths (Fig. 1c, grey) with our control function (Fig. 1d). Similar to Simha et al. (2018), we designed our control function (Fig. 1d) by solving for the walking incline (Fig. 1e) (Margaria, 1968), combined with a 10% body weight forward horizontal force (Fig. 1f) (Gottschall and Kram, 2005) to achieve an energetic penalty of 1 W/kg at a step width of 3 standard deviations (SD) narrower than initial preferred, decreasing with a constant slope to 0 W/kg at the new energy optimal width of 3 SD wider than initial preferred:

$$\label{eq:Incline} \textit{Incline} = \left\{ \begin{array}{ll} -0.49*sw + 5.16 & sw \leq 3 \\ -0.49*3 + 5.16 & sw > 3 \end{array} \right.$$

Here, step width (*sw*) is in units of standard deviations from preferred. We normalized step width by the variability in width to allow us to distinguish between a shift in step width occurring as a result of energy optimization, and a shift occurring by random chance. We chose to shift the energy optimal width wider than that initially preferred in order to be in the opposite direction of people's tendency to narrow their step width as they become more comfortable walking on a treadmill (Zeni and Higginson, 2010).

2.2. Experimental protocol

Eight subjects (female: n = 5; male: n = 3; body mass: 62.6 ± 9.0 kg; height: 165.3 ± 9.5 cm; mean \pm SD) participated in

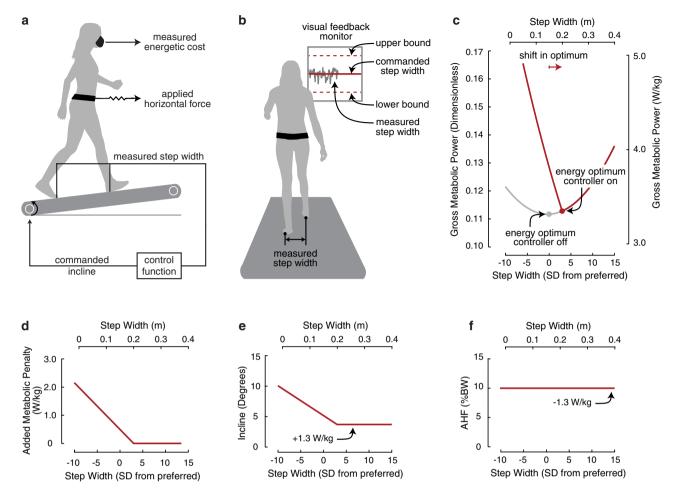


Fig. 1. Experimental setup and design. (a) Our real-time controller uses forces and moments measured by the treadmill to identify foot contact events, calculate step width, and then command the appropriate treadmill incline based on the desired energetic penalty for the measured step width. (b) Our real-time visual feedback system allows us to enforce specific widths at times during the protocol by instructing subjects to keep their measured width signal within small bounds about the commanded width. (c) We simulated our new cost landscape (red) by adding literature values for energetic costs of walking at different step widths (grey) with (d) our control function that applies energetic penalties as a function of step width. To achieve this control function, we applied (e) an energetic penalty of walking incline as a function of step width and combined this with (f) a near constant energetic reward of a forward horizontal force applied to the user. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

the study. All subjects were healthy and exhibited no clinical gait abnormalities. The Simon Fraser University Research Ethics board approved the protocol and participants gave their written, informed consent before participating in the experiment.

First, subjects completed a baseline trial to measure their initial preferred width and width variability (Fig. 2a). They did this while walking on the level, without treadmill incline control or forward pulling force. We calculated initial preferred width as the average width during the final 3 min of this 12-minute baseline trial. We calculated each subject's width variability as the standard deviation during the same averaging window. For each subject, we used their initial preferred width and width variability to design the previously described control function.

Next, we turned the incline controller on, applied the forward horizontal force, and measured whether subjects adapted their step width towards the new energy optimal width. First, we measured whether subjects would adapt their width spontaneously, calculated as the average width during the final 3 min of the first 6-minute after the controller was turned on (Fig. 2b, pre). We estimated that 6 min would be sufficient to test for spontaneous energy optimization of step width as we have previously observed a time constant of roughly 66 s in step frequency optimization (Selinger et al., 2018). Second, we measured whether subjects

would adapt their width when given enforced experience with the new cost landscape. This enforced experience consisted of eight perturbations-each perturbation included a 5-minute hold at either higher costs (6 SD narrower than initial preferred) or lower costs (6 SD wider than initial preferred), followed by a 5minute release, during which subjects self-selected their widths (Fig. 2b). During each hold, we instructed subjects to keep their real-time calculated width, normalized to the commanded width, within small bounds about 1 presented visually on a computer monitor (Fig. 1b). When the visual feedback targets disappeared, we instructed them to walk however they liked, with any step width. We did not provide subjects with any information about how the controller worked. We calculated the change in preferred width at each perturbation as the average width during the final three minutes of the release, and final preferred width as the average width during the last period of enforced experience (Fig. 2b, post).

Lastly, subjects completed a cost mapping trial with the incline controller on and forward horizontal force applied to the participant. We used visual feedback to enforce steady-state walking at different widths (in random order) for five minutes each (Fig. 2c). We used respiratory gas analysis (Vmax Encore Metabolic Cart, ViaSys, Conshohocken, PA, USA) to measure rates of oxygen

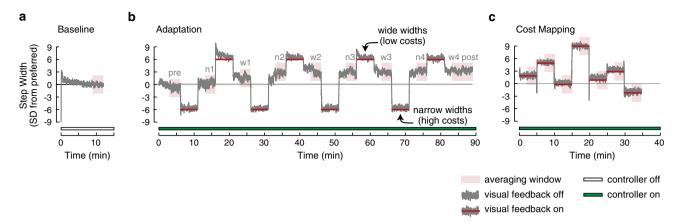


Fig. 2. Experimental protocol. Each subject completed all three trials. We averaged measured step width across subjects. (a) First, before we engaged the controller, each subject completed a baseline trial where we determined their preferred width and width variability. (b) Next, each subject completed an experience period where we measured whether they would adapt their preferred width towards the new energy optimal width spontaneously, and when given enforced experience with the new cost landscape through perturbations to different widths. During the periods denoted by the red horizontal lines, we used visual feedback to command either narrow widths (higher costs) or wider widths (lower costs). During the periods without the red horizontal lines, subjects walked at self-selected widths. (c) Lastly, each subject completed a cost mapping trial where we measured their energetic cost at different widths in the new cost landscape. The regions of red shading illustrate the averaging window for steady-state step widths. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

consumption and carbon dioxide production. Subjects walked at widths both about the initial preferred width (-2, 0, +2 SD from initial preferred) and about the final preferred width (-2, 0, +2, +6 SD from final preferred). We measured energetic cost at these particular step widths to give an expansive estimate of the new cost landscape, while measuring the energetic cost about both initial and final preferred widths. We calculated metabolic power using the Brockway equation (Brockway, 1987) and determined metabolic power at specific step widths by averaging over the final three minutes of each 5-minute walking period (Selinger and Donelan, 2014).

2.3. Analysis

We compared the new cost landscape to an original cost landscape to verify that we had shifted the energy optimal width wider than that initially preferred. For the original cost landscape, we used existing data collected in different subjects (n = 10)(Donelan et al., 2001). We performed all energetic cost analyses in dimensionless units. We made energetic cost, measured in W/kg, dimensionless by multiplying by a normalization factor of $g^{3/2}L^{1/2}$ (g = 9.81 m/s²; L = 0.53 × H, where H = height) (Drillis et al., 1969). We calculated each subject's energy optimum by averaging step width and energetic cost over each steady-state walking period during the cost mapping trial, fitting a secondorder polynomial to these points, and then calculating the minimum of this polynomial. We calculated the original and new energy optimal widths as the average fitted minima across subjects in the original and new cost landscapes, respectively. We calculated the variability in the energy optimal width as the standard deviation of the energy optimal width determined by the fitted minimum. We used a one-tailed paired Student's t-test to test if the new energy optimal widths were wider than initial preferred widths in each cost landscape.

We also tested for adaptation towards the energy optimal width throughout the protocol. We used one-tailed paired Student's ttests to determine if each subject's self-selected width during each release was wider than their initial preferred width. To determine the rate of adaptation, we averaged self-selected widths across subjects during the release periods after optimization was initiated and then used a first-order exponential to determine the time constant. We used two-tailed paired Student's t-tests to test for differ-

ences in the final preferred widths compared to the new energy optimal widths, as well as for differences in the costs of these widths. We calculated the cost of each subject's final preferred width by commanding this width during the cost mapping trial, and the cost of each subject's energy optimal width as the minimum metabolic power of the fitted new cost landscape.

3. Results

Our control system successfully shifted the energy optimal step width wider than that initially preferred (p = 1.3×10^{-5} ; Fig. 3). In the previously collected original cost landscape, the average energy optimal width (0.10 ± 0.05 m; mean \pm SD) was 1.1 ± 3.5 SD slightly narrower than the original preferred width (0.12 ± 0.01 m) (Donelan et al., 2001). In our new cost landscape, the average energy optimal width (0.24 ± 0.03 m) was 4.9 ± 1.1 SD wider than the initial preferred width (0.15 ± 0.04 m).

We found that subjects needed experience with a lower cost width in the new cost landscape to initiate optimization and adapt towards the new energy optimal width. They did not spontaneously adapt towards the new energy optimal width (p = 0.9; Fig. 4b pre) and returned back to their initial preferred width in self-selected steps after being held at a higher cost width (narrower width) (p = 0.2; Fig. 4b n1). Subjects only initiated optimization in self-selected steps after being held at a lower cost width (wider width), where they walked at an average width of 1.6 ± 0.6 SD from initial preferred (p = 9.2×10^{-5} ; Fig. 4b w1). After optimization was initiated, subjects gradually converged on their final preferred width (Fig. 4b, w1-post) with an average time constant of 248 s (95% CI [247 249]), or about 468 steps (Fig. 4a). And after converging on the final preferred width that was towards the energy optimal width, subjects quickly returned to it when perturbed away (Fig. 4b, w1-post).

Subjects adapted their step width to a width that reduced energetic cost. On average, subjects adapted their width 3.5 ± 0.8 SD wider than that initially preferred (p = 3.1×10^{-6} ; Fig. 4b post) and reduced energetic cost by $14.4\% \pm 6.1\%$ relative to the cost of the initial preferred width in the new cost landscape (p = 2.4×10^{-4} ; Fig. 3). This adaptation is not by random chance—a step width 3.3 standard deviations wider than preferred is likely to happen only once in every 1000 steps during normal walking. And although this final preferred width is significantly

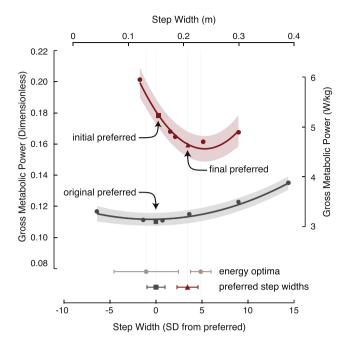


Fig. 3. Original cost landscape (grey) from existing data in different subjects (Donelan et al., 2001) and measured new cost landscape (red). In each cost landscape, we averaged measured step width and energetic cost across subjects. We fit second-order polynomial curves to both the original and new cost landscapes. The shading shows their 95% confidence intervals. We calculated the original preferred width (grey square) as the average width across subjects in familiar walking conditions (Donelan et al., 2001), the initial preferred width (red square) as the average width across subjects when the controller is off, and the final preferred width (red triangle) as the average width across subjects when the controller is on and following enforced experience. We calculated the variability in these preferred widths as the standard deviation of the original and final preferred widths across subjects. We calculated the original (grey circle) and new (red circle) energy optimal widths as the average fitted minima across subjects in the original and new cost landscapes. We calculated the variability in these energy optimal widths as the standard deviation of the fitted minima across subjects in the original and new cost landscapes. Error bars represent 1 standard deviation. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

narrower than the new energy optimal width (p = 2.2×10^{-4}), the costs of these widths are not significantly different (p = 0.6).

4. Discussion

We tested the hypothesis that energy optimization is a major objective in the real-time control of step width by creating a new cost landscape and then measuring whether people adapt towards the new energy optimal width. Our previous understanding of preferred width was that it is energetically optimal in familiar walking conditions, yet we did not know the timescale by which this preference is established (Donelan et al., 2001). Here we find that the nervous system establishes this preferred width by optimizing energetic cost in real-time.

Our experiment has several limitations. One concern is that, while walking at an incline or with a forward horizontal force, people may naturally prefer to walk with a wider width and thus may be adapting in response to our setup rather than to minimize energy. However, others have found that step width changes minimally within the range of inclines that subjects experience in our study (Kawamura et al., 1991). And in pilot experiments, we found no clear adjustments to step width in response to either constant incline or constant forward horizontal force. Another concern may be that the incline controller perturbs walking subjects and that this causes subjects to increase width. Indeed, some studies that perturb level walking with changes in terrain do find that people spontaneously widen their step widths (Gottschall et al., 2011). However, we suspect that this does not explain our results for several reasons. First, this is not a universal finding as others find no systematic effect of changing terrain on step width (Gates et al., 2012; Voloshina et al., 2013). Second, as we describe in the methods section, our incline control is not a large perturbation-the incline changed slowly, and no subjects reported feeling destabilized. Finally, our subjects did not spontaneously widen their step width in response to the incline control—they first required experience with lower cost step widths. Another concern is that subjects may also be adapting their width to achieve level walking rather than to optimize energy. We partially addressed this by

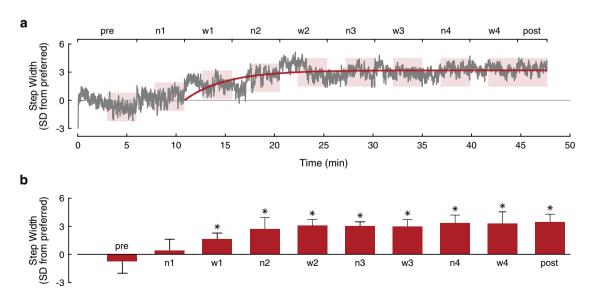


Fig. 4. Rate of adaptation. (a) Step width, averaged across subjects, when allowed to self-select widths prior to enforced experience (pre), during narrow (n1-n4) and wide releases (w1-w4), and following enforced experience (post). We determined the rate of adaptation after optimization was initiated (w1-post) by first removing steps during holds and only considering self-selected steps during releases, fitting an exponential model to these steps, and then calculating the time constant of this fitted exponential. The regions of red shading illustrate the averaging windows used to calculate step width during specific periods throughout the protocol. (b) Average widths during period prior to enforced experience, narrow releases, wide releases, and period following enforced experience. Error bars represent 1 standard deviation. Asterisks indicate statistically significant differences in width when compared to initial preferred width. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

making the minimum incline occur at a non-zero treadmill slope $(\sim 4^{\circ})$, however, the minimum incline is still the energy optimal incline. Thus, we cannot distinguish between a nervous system objective of walking at the shallowest incline from an objective of minimizing energy. Experiments like ours can never entirely rule out other possible explanations—it is always possible that the nervous system has some other objective that happens to produce a gait that is the same as the new energy optimal gait.

The nervous system's criteria for initiating step width optimization in this experiment, and the process used to converge towards energy optimal widths, is consistent with what we have previously observed for step frequency optimization using knee exoskeletons. The first similarity between studies is that perturbations towards lower cost gaits initiated the optimization process (Selinger et al., 2015, 2018). Experience with this lower cost gait may cue the nervous system to explore the new cost landscape by indicating that its preferred gait is now energetically suboptimal. The second similarity between studies is that the nervous system learned to predict the energetically optimal width and rapidly returned to it when perturbed away (Selinger et al., 2015). These similarities suggest that continuous energy optimization is a dominant and general objective of nervous systems in young healthy walking, and that our observed behaviors are common characteristics of the nervous system's energy optimization algorithms.

There are, however, some differences between the two experimental findings. One difference is that no subjects in our current experiment spontaneously optimized their step width, whereas a small subset of people spontaneously optimize step frequency (Selinger et al., 2015, 2018). One possible explanation for this difference is that the general population does have spontaneous step width optimizers, but by chance, we did not have any in our random sample. A more mechanistic candidate explanation is that the nervous system is not primed to identify differences in energetic cost with step width because they are normally relatively small due to the shallowness of the original cost landscape around the preferred width. The second difference is that subjects converged on the energy optimum slower in our step width study compared to our step frequency study. In our step width study, people gradually converged on the energy optimal width with a time constant of 248 s (Fig. 4a). Others have found similar rates of adaptation (i.e. hundreds of seconds) in converging on energy optimal movements in both split-belt walking and reaching paradigms (Finley et al., 2013; Huang et al., 2012). However, in our step frequency study, people converged on the new energy optimal frequency with a time constant of 11 s once optimization was initiated (Selinger et al., 2015). One explanation may be that the nervous system takes longer to learn the new step width cost landscape because of the slow rate of change of our incline controller, resulting in an error between what we design the new cost landscape to be and what the nervous system senses. Simple reinforcement learning models also predict these different rates of adaptation, ranging from tens to hundreds of seconds, given a change to the learning rate (Selinger et al., 2018). Given this flexibility, we don't consider the differences in convergence rates to be evidence against a shared adaptation process.

The nervous system likely determines step width by optimizing energy simultaneously with other objectives such as stability and maneuverability (Acasio et al., 2017; Bauby and Kuo, 2000; Dean et al., 2007; Jindrich and Qiao, 2009; Patla et al., 1991; Townsend, 1985; Wu et al., 2015). Our study suggests that, in these walking conditions, the relative contributions of other objectives are small as people converged on a final preferred width that was near the new energy optimal width. However, it is possible that differences in the adaptation rates we observed between our step frequency and step width experiments reflects differences in the complexity of the optimization problem—the addition of other objectives in determining step width may slow the nervous system's refinement of energy. And although the preferred widths are near their energy optimal widths, they do not perfectly coincide, and the influence of other objectives may explain these small differences. In everyday walking, the contribution from energy and non-energy objectives to the real-time control of gait will depend not only on biomechanics, but also on how heavily the nervous system weights their individual importance.

Declaration of Competing Interest

The authors have no conflicts of interest to declare.

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