Simultaneous positive and negative external mechanical work in human walking

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Abstract

In human walking, the center of mass motion is similar to an inverted pendulum. Viewing double support as a transition from one inverted pendulum to the next, we hypothesized that the leading leg performs negative work to redirect the center of mass velocity, while simultaneously, the trailing leg performs positive work to replace the lost energy. To test this hypothesis, we developed a method to quantify the external mechanical work performed by each limb (*individual limbs method*). Traditional measures of external mechanical work use the sum of the ground reaction forces acting on the limbs (*combined limbs method*) allowing for the mathematical cancellation of simultaneous positive and negative work during multiple support periods. We expected to find that the traditional combined limbs method underestimates external mechanical work by a substantial amount. We used both methods to measure the external mechanical work performed by humans walking over a range of speeds. We found that during double support, the legs perform a substantial amount of positive and negative external work simultaneously. The combined limbs measures of positive and negative external work were approximately 33% less than those calculated using the individual limbs method. At all speeds, the trailing leg performs greater than 97% of the double support positive work while the leading leg performs greater than 94% of the double support negative work. © 2002 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Walking is characterized by center of mass motion similar to that of an inverted pendulum. During the single support phase of human walking, for example, the stance limb behaves much like a rigid strut allowing kinetic energy to be stored as gravitational potential energy and then returned in a nearly conservative manner (Cavagna et al., 1977). For compatibility with this model, the double support phase (Fig. 1a) is perhaps best regarded as a transition from one inverted pendulum to the next. During single support phases, the center of mass moves along an arc dictated by the stance limb and the center of mass velocity is approximately perpendicular to the limb (Fig. 1b). Each transition to a new stance limb requires redirection of

the center of mass velocity from one inverted pendulum arc to the next (Fig. 1c). Since ground reaction forces are directed approximately along each leg, this redirection of the center of mass velocity requires negative work by the leading leg beginning at heel contact.

To maintain a steady walking speed, positive work is needed to replace the energy lost due to this negative work. The positive work can be performed at any time during a step. But, a simple model of bipedal walking predicts that it is most advantageous for the trailing limb to perform the required positive work at the same time that the leading leg performs the necessary negative work (Fig. 1c and d). If the positive work is performed substantially prior to the double support phase, more energy is lost in redirecting the center of mass velocity (Fig. 1d). As a consequence, more positive work is required to maintain the same walking speed.

There are several methods for non-invasively estimating the total mechanical work performed by muscles during locomotion (Winter, 1990). The approach that can be tied more closely to the inverted pendulum model

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(a) Free Body Diagram   **(b) Inverted Pendulum Phase**

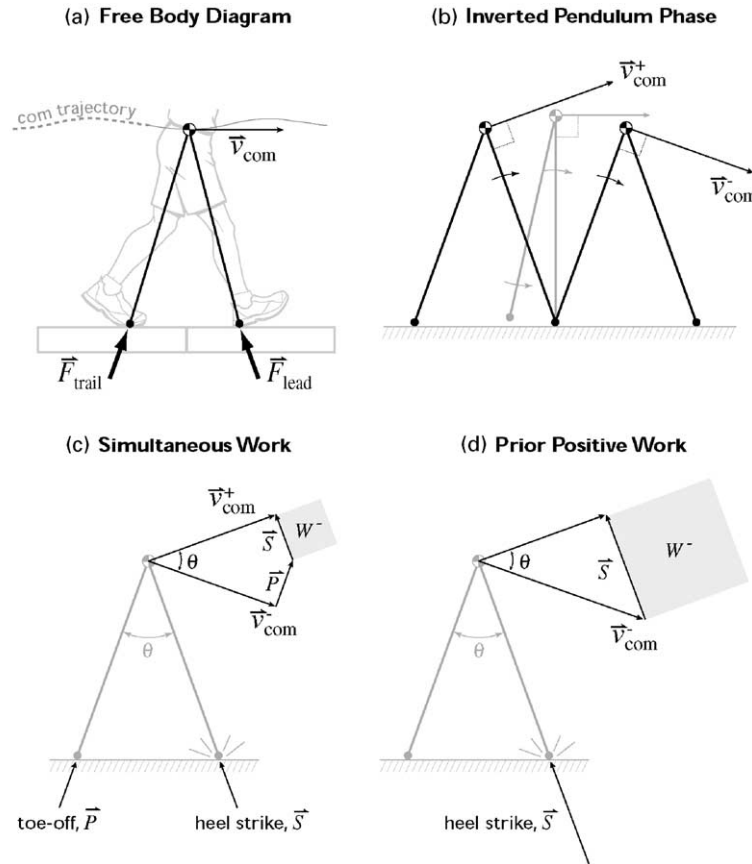**(c) Simultaneous Work**   **(d) Prior Positive Work**

Fig. 1. A simple model of bipedal walking predicts simultaneous positive and negative work by the trailing and leading limbs during double support. (a) Free-body diagram showing the velocity of the center of mass, $\vec{v}_{com}$, and ground reaction forces acting on trailing, F_{trail}, and leading, F_{lead}, legs during double support. (b) The direction of the center of mass velocity is perpendicular to the stance limb during inverted pendulum-like phases. The $+$ and $-$ superscripts denote the center of mass velocity at the beginning and end of the inverted pendulum-like phase, respectively. (c) Each transition to a new stance limb requires redirection of the center of mass velocity from $\vec{v}_{com}^{-}$ to $\vec{v}_{com}^{+}$. This requires negative work by the leading limb, W^{-}. We model the heel strike ground reaction force as an impulse, S, acting along the leading limb. S causes an instantaneous change in the center of mass velocity, with $W^{-} \sim |S|^{2}$. To maintain a steady walking speed, positive and negative work must be equal. The most effective way to power locomotion in this simple model is to apply a toe-off impulse by the trailing limb, P, at the same time or immediately before heel strike. (d) Positive work performed substantially prior to the heel-strike results in greater leading leg negative work, W^{-}, and consequently requires greater positive work. The difference in the area of the shaded squares in (c) and (d) illustrates the difference in the required positive work. A more detailed explanation of this model can be found in Kuo (in press).

is to separate total mechanical work into external and internal work, using Koenig's theorem (Greenwood, 1988; Willems et al., 1995). External work is the work performed by external forces—mainly from the ground—to move the center of mass through a displacement. Internal work is that associated with the motion of the limbs relative to the body center of mass. The muscles are ultimately responsible for the mechanical work performed on the body. Nevertheless, external and internal work are useful abstractions because the stance leg and body center of mass may be viewed as an inverted pendulum, and external work may be viewed as an indication of deviations from the behavior of a perfectly conservative pendulum.

Another advantage of external work is its relative simplicity of measurement. It can be estimated from measured ground reaction forces alone, given knowledge of the mass and speed of the animal (Cavagna,

1975). In contrast, measurements of kinematics and inertial properties of body segments are needed to estimate internal and most other measures of work. As a result, traditional measurements of external work have been applied to humans (Cavagna et al., 1963; Cavagna and Margaria, 1966; Cavagna et al., 1976; Cavagna and Franzetti, 1986; Heglund et al., 1995; Willems et al., 1995; Thys et al., 1996; Lejeune et al., 1998; Griffin et al., 1999) and many other animal species (Cavagna et al., 1977; Heglund et al., 1982; Full, 1989; Full and Tu, 1990; Farley and Ko, 1997). The results imply that animals that differ drastically in shape, size and number of limbs all use the inverted pendulum mechanism to conserve energy during walking (Cavagna et al., 1977; Full, 1989).

Despite these advantages, the traditional method for estimating external mechanical work is flawed when applied to double and multiple support phases. Since it

uses the sum of the ground reaction forces acting on all of an animal's limbs, it allows for the mathematical cancellation of simultaneous positive and negative work performed by the limbs during multiple support periods. This mathematical cancellation is likely not representative of any physical cancellation, which would require a transfer of energy from one limb to the other. This flaw has been recognized for over 20 years (Alexander, 1980; Winter, 1990) but has yet to be quantified.

Viewing double support as a transition from one inverted pendulum to another, we hypothesized that the leading leg performs negative work to redirect the center of mass velocity, while the trailing leg simultaneously performs positive work to replace the lost energy. As a result, we expected to find that the traditional combined limbs method underestimates external mechanical work by a substantial amount. This paper has two parts. First, we outline a method for calculating the mechanical work performed on the center of mass by each leg during bipedal walking. This method uses the individual limb ground reaction forces (*individual limbs method*) to calculate external mechanical work, rather than the traditional method of using the sum of the ground reaction forces acting under the limbs (*combined limbs method*). Second, we report our measurements of the external mechanical work performed by humans walking over a range of speeds calculated with both the individual limbs and combined limbs methods.

2. Methods

The power generated on the body's center of mass (external mechanical power) by a limb is equal to the dot product of two vectors: the external force acting on the limb and the velocity of the center of mass. For a biped

$$P_{\text{trail}} = \vec{F}_{\text{trail}} \cdot \vec{v}_{\text{com}} = F_{z,\text{trail}} v_{z,\text{com}} + F_{y,\text{trail}} v_{y,\text{com}} + F_{x,\text{trail}} v_{x,\text{com}}, \tag{1}$$

$$P_{\text{lead}} = \vec{F}_{\text{lead}} \cdot \vec{v}_{\text{com}} = F_{z,\text{lead}} v_{z,\text{com}} + F_{y,\text{lead}} v_{y,\text{com}} + F_{x,\text{lead}} v_{x,\text{com}}. \tag{2}$$

In these equations, P_{trail} and P_{lead} are the external mechanical powers generated by the trailing and leading limbs, respectively. $\vec{F}_{\text{trail}}$ and $\vec{F}_{\text{lead}}$ are the ground reaction forces acting on the double support trailing and leading limbs, respectively (Fig. 1a). $\vec{v}_{\text{com}}$ is the velocity of the center of mass. The subscripts z, y and x denote the vertical, fore-aft and medio-lateral directions, respectively. The positive vertical direction is up, the positive fore-aft direction is forward and the positive medio-lateral direction is to the left.

The mechanical work performed on the center of mass (external mechanical work) is equal to the cumulative time-integral of the external mechanical power. Positive external mechanical work is calculated by restricting the integral to the intervals over which the integrand is positive (denoted by the domain POS). We found the positive external work performed by the trailing leg (W_{trail}^+) and leading leg (W_{lead}^+) using P_{trail} and P_{lead}, respectively.

$$W_{\text{trail}}^+ = \int_{\text{POS}} P_{\text{trail}} \, dt, \tag{3}$$

$$W_{\text{lead}}^+ = \int_{\text{POS}} P_{\text{lead}} \, dt. \tag{4}$$

Correspondingly, the negative external mechanical work is equal to the cumulative time-integral of external mechanical power, restricting the integral to the intervals over which the integrand is negative (denoted by the domain NEG). We found the negative external work performed by the trailing leg (W_{trail}^-) and leading leg (W_{lead}^-) using P_{trail} and P_{lead}, respectively.

$$W_{\text{trail}}^- = \int_{\text{NEG}} P_{\text{trail}} \, dt, \tag{5}$$

$$W_{\text{lead}}^- = \int_{\text{NEG}} P_{\text{lead}} \, dt. \tag{6}$$

We calculated the positive and negative external mechanical work performed by each limb over particular time intervals (i.e. complete step, double support, single support) by restricting the integration of the positive and negative external mechanical power to these time intervals. The total positive (W_{ILM}^+) or negative (W_{ILM}^-) external mechanical work, calculated using the individuals limbs method, is the sum of the positive or negative external mechanical work performed by each limb.

$$W_{\text{ILM}}^+ = W_{\text{trail}}^+ + W_{\text{lead}}^+, \tag{7}$$

$$W_{\text{ILM}}^- = W_{\text{trail}}^- + W_{\text{lead}}^-. \tag{8}$$

To ease comparison between methods of calculating external mechanical work, we can express the traditional combined limbs method using our individual limbs method notation. External mechanical work has traditionally been calculated using the resultant (vector sum) of the ground reaction forces acting under the limbs ($\vec{F}_{\text{res}}$) to determine the change in mechanical energy of the center of mass (Cavagna, 1975).

$$\vec{F}_{\text{res}} = \begin{bmatrix} F_{z,\text{res}} \\ F_{y,\text{res}} \\ F_{x,\text{res}} \end{bmatrix} = \begin{bmatrix} F_{z,\text{trail}} + F_{z,\text{lead}} \\ F_{y,\text{trail}} + F_{y,\text{lead}} \\ F_{x,\text{trail}} + F_{x,\text{lead}} \end{bmatrix}. \tag{9}$$

The combined limbs method can be expressed using our notation in two different ways. First, it is equivalent to using the sum of the ground reaction forces acting under

the limbs to calculate power

$$P_{\text{CLM}} = \vec{F}_{\text{res}} \cdot \vec{v}_{\text{com}} = F_{z,\text{res}}v_{z,\text{com}} + F_{y,\text{res}}v_{y,\text{com}}$$
$$+ F_{x,\text{res}}v_{x,\text{com}}, \qquad (10)$$

where P_{CLM} is the instantaneous sum of the powers generated by the limbs. The combined limbs method is also equivalent to summing the powers generated by each limb *before* integrating, subject to the positive and negative domains

$$W_{\text{CLM}}^{+} = \int_{\text{POS}} P_{\text{CLM}}\, dt = \int_{\text{POS}} (P_{\text{trail}} + P_{\text{lead}})dt, \qquad (11)$$

$$W_{\text{CLM}}^{-} = \int_{\text{NEG}} P_{\text{CLM}}\, dt = \int_{\text{NEG}} (P_{\text{trail}} + P_{\text{lead}})dt, \qquad (12)$$

where W_{CLM}^{+} and W_{CLM}^{-} are the positive and negative external mechanical work terms calculated in the traditional combined limbs manner. They are equal to the sum of the positive increments and the sum of the negative decrements in the total energy of the center of mass (Cavagna, 1975). For completeness, our calculations include the medio-lateral component (Cavagna, 1975). The magnitude of external mechanical work as calculated using the traditional combined limbs method must be less than or equal to that found using the individual limbs method. This is demonstrated by the following two inequalities:

$$\int_{\text{POS}} P_{\text{trail}}\, dt + \int_{\text{POS}} P_{\text{lead}}\, dt \geqslant \int_{\text{POS}} (P_{\text{trail}} + P_{\text{lead}})dt, \quad (13)$$

$$\int_{\text{NEG}} P_{\text{trail}}\, dt + \int_{\text{NEG}} P_{\text{lead}}\, dt \leqslant \int_{\text{NEG}} (P_{\text{trail}} + P_{\text{lead}})dt, \qquad (14)$$

where W_{ILM} is on the left-hand side and W_{CLM} is on the right. Since the power generated by the trailing and leading limbs are summed before integration in the combined limbs method, simultaneous positive and negative powers mathematically cancel and the magnitude of the calculated work is reduced.

We used the individual limbs and combined limbs methods to calculate the external mechanical work performed by human subjects walking across a range of speeds. The volunteers (5 males, 5 females, body mass 68.9 ± 12.2 kg; leg length 0.93 ± 0.05 m; age 24 ± 5 yr; mean $\pm$ s.d.) read a description of the purpose, risks and basic procedures of the experiment and then gave their informed consent to participate.

Estimating external mechanical work using the individual limbs method requires measuring the individual limb ground reaction forces. We used two separate force platforms for this purpose. The platforms (Model LG6-4-2000, AMTI, Newton, MA) were mounted in series near the midpoint of a 17 m walkway. We collected the three components of the ground reaction force (F_z—vertical, F_y—fore-aft, F_x—medio-lateral)

from both force platforms simultaneously at 1000 Hz per channel. A fourth-order, recursive, zero-phase-shift Butterworth 100 Hz low-pass digital filter conditioned the ground reaction force signals.

We studied six walking speeds (0.75, 1.00, 1.25, 1.50, 1.75 and 2.00 m/s; randomized order). Two infrared photocells, placed on either side of the force platforms (3.0 m apart), measured the average walking speed. We discarded trials if the speed was not within 0.05 m/s of the desired speed or if the individual feet did not fall cleanly on separate force platforms. The average difference between the beginning and ending speed of each step, calculated using the fore-aft impulse, was -0.03 m/s (± 0.06 m/s s.d.). We saved and analyzed data for three acceptable trials from each subject at each speed. We defined a step as beginning with ground contact of one foot and ending with ground contact of the opposite foot. For each trial, we performed all calculations on this defined step.

We determined the center of mass velocities using the vector sum of the ground reaction forces acting under the limbs ($\vec{F}_{\text{res}}$) (Cavagna, 1975). This first required calculating the acceleration of the center of mass from the summed ground reaction forces and then calculating the center of mass velocities using the time-integrals of these center of mass accelerations:

$$v_{z,\text{com}} = \int \frac{F_{z,\text{trail}} + F_{z,\text{lead}} - mg}{m}\, dt, \qquad (15)$$

$$v_{y,\text{com}} = \int \frac{F_{y,\text{trail}} + F_{y,\text{lead}}}{m}\, dt, \qquad (16)$$

$$v_{x,\text{com}} = \int \frac{F_{x,\text{trail}} + F_{x,\text{lead}}}{m}\, dt. \qquad (17)$$

In these equations, m is body mass, and g is gravitational acceleration (9.81 m/s^2). We determined the integration constant for the vertical direction by requiring the average vertical center of mass velocity over a step to be zero. Similarly, we determined the integration constant for the fore-aft direction by requiring the average fore-aft center of mass velocity over a step to be equal to the average walking speed. For the medio-lateral direction, we determined the integration constant by requiring that the medio-lateral velocities at the beginning and end of a step be equal in magnitude but opposite in sign.

For each condition, we calculated the mean values of each variable for three steps per subject. We used repeated-measures analysis of variance (ANOVA, $P < 0.05$) to determine statistical differences.

3. Results

As hypothesized, the individual limbs method revealed that during double support, the trailing leg

(a) Double Support External Work

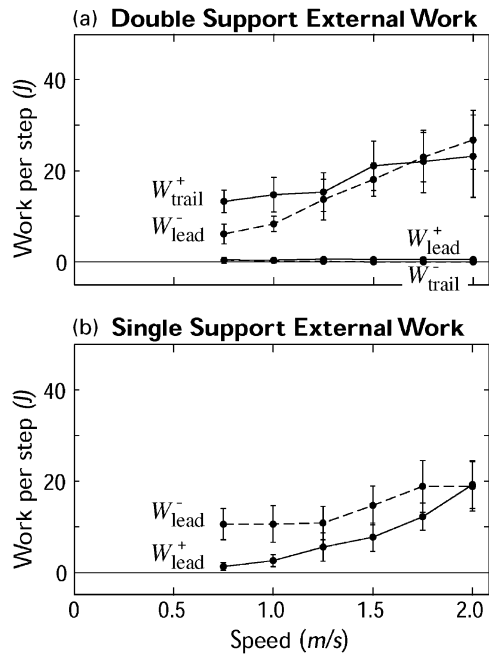## (b) Single Support External Work

Fig. 2. During double support, the trailing leg performed predominantly positive external work while the leading leg performed predominantly negative external work. (a) Positive and negative external work performed during double support by the trailing and leading limbs as a function of walking speed. (b) Positive and negative external work performed during single support as a function of walking speed. Values shown are means $\pm$ standard deviation, $N = 10$. Some error bar ranges are too small to be clearly legible.

performed predominantly positive external work while the leading leg performed predominantly negative external work (Fig. 2a). The positive external work performed during double support increased from 13.7 J at a walking speed of 0.75 m/s to 23.8 J at 2.00 m/s. The trailing leg performed greater than 97% of this double support positive work. The negative external work performed during double support increased from 6.5 J at 0.75 m/s to 26.8 J at 2.00 m/s. The leading leg performed greater than 94% of this double support negative work. From 1.25 to 2.00 m/s, the negative external work performed during double support was similar in magnitude to the positive external work. External work is also performed during single support (Fig. 2b). Single support positive external work increased from 1.3 J at 0.75 m/s to 19.3 J at 2.00 m/s. The corresponding negative work increased from 10.6 to 18.8 J at the same respective speeds.

As hypothesized, the trailing limb performed positive external work at the same time the leading limb performed negative external work. This is illustrated by the power generated during double support (Fig. 3). The legs generated positive and negative power simultaneously because they pushed against each other in both the fore-aft and medio-lateral directions (Fig. 4). While opposing fore-aft forces were relatively small compared to the vertical forces, the fore-aft velocity of the center

of mass was relatively large compared to the vertical velocity. Thus, the contributions of the opposing fore-aft forces to the external mechanical power generated by the legs tended to be large and opposite in sign. The contributions of the opposing medio-lateral forces to the power generated tended to be small because both the medio-lateral ground reaction forces and center of mass velocities were small (Fig. 4).

The individual limbs method revealed a substantial amount of external mechanical work that has been overlooked by the traditional combined limbs method. Over a complete step, the combined limbs measures of positive and negative external work averaged 33% less than that calculated using the individual limbs method ($P < 0.0001$) (Fig. 5). The combined limbs method underestimates external mechanical work because during double support, the simultaneous positive and negative work by the trailing and leading limbs reduces the magnitude of the sum of the powers generated by the limbs (P_{CLM}) (Fig. 3). During double support, the combined limbs measures of positive and negative external work were 47% and 60% less, on average, than the respective individual limbs measures. During single support, the individual limbs method for calculating external mechanical work is identical to the
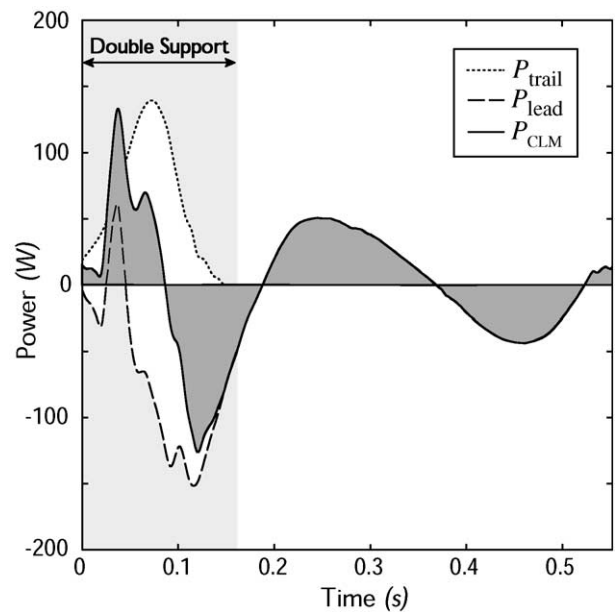

Fig. 3. The trailing limb performed positive external work at the same time that the leading limb performed negative external work. External mechanical power generated by the trailing (P_{trail}) and leading (P_{lead}) legs, and their instantaneous sum (P_{CLM}), for a typical step by one subject walking at 1.25 m/s. Combined limbs measures of positive and negative external mechanical work are represented by the dark gray areas. The white areas under the power curves represent the positive and negative external mechanical work overlooked by the combined limbs method. Thus, individual limbs measures of positive and negative external mechanical work are represented by the sum of the white and dark gray areas.
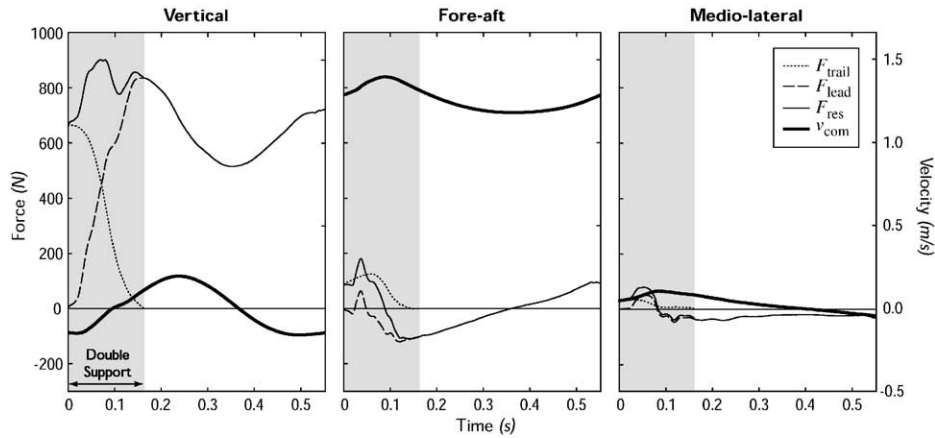
Fig. 4. During the double support phase, the legs generate positive and negative power simultaneously because they push against each other in both the fore-aft and medio-lateral directions. Ground reaction forces and center of mass velocities in the vertical, fore-aft and medio-lateral directions during the same step as shown in Fig. 3. F_{trail} and F_{lead} are the ground reaction forces acting on the trailing and leading legs and F_{res} is their instantaneous sum.

4. Discussion

These results are based on several assumptions that are inherent to the individual limbs method. One assumption is that negative work by one leg cannot be conservatively transferred into positive work at the other leg. There are no muscles that cross from one leg to the other dictating that each leg must perform work separately (Kuo, 2001). Another assumption is that external work satisfactorily estimates the total mechanical work performed on the body. During double support, the angular displacements of the limbs relative to the center of mass are relatively small, indicating that there is little internal work. However, this assumption is less valid during single support, when angular displacements relative to the center of mass are much greater, and external work alone is insufficient to estimate total mechanical work. Interpretation of single support measures of external work should therefore be made with caution. Finally, the relevance of estimates of mechanical work performed on the body to actual work performed by the muscles is based on assumptions that there is little mechanical energy storage and little muscular co-contraction during walking. These latter assumptions are legitimate sources of error, but they also apply to all other non-invasive estimates of mechanical work.

Since the traditional combined limbs method substantially underestimates the external mechanical work performed in walking, its applications should be reevaluated. Combined limbs measures of external work are the basis for measures of "percent recovery". These measures attempt to quantify the degree to which potential and kinetic energy are exchanged in an inverted pendulum-like manner (Cavagna et al., 1976). This application is flawed because combined limbs
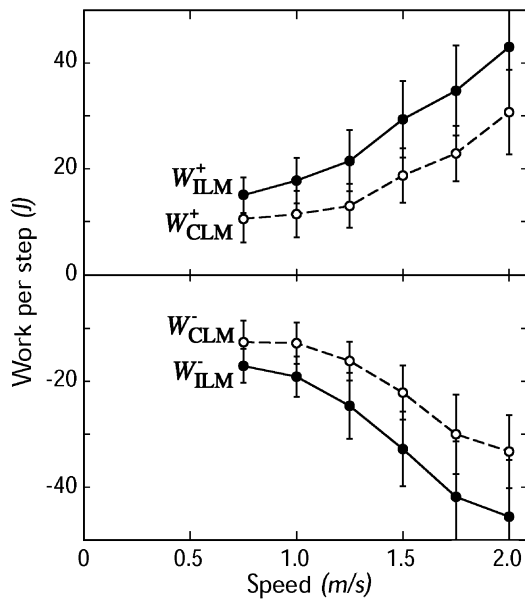

Fig. 5. Combined limbs measures substantially underestimated positive and negative external mechanical work at all speeds. Comparison of the combined limbs method (open circles and dotted lines) and individual limbs method (filled circles and solid lines) for calculating external mechanical work. Values shown are means ± standard deviation, $N = 10$. All changes were statistically significant ($P < 0.0001$).

combined limbs method. Though differing in magnitude, the individual and combined limbs measures exhibited similar increases in the external mechanical work performed per step when subjects walked at faster speeds. From 0.75 to 2.00 m/s, the positive external mechanical work performed during a step, calculated using the individual limbs method, increased by 187% while the combined limbs measures increased by 192% (Fig. 5).

measures improperly quantify the external work done during multi-support phases. Combined limbs measures of external work have also been studied as possible predictors of the metabolic cost of locomotion (Taylor and Heglund, 1982; Full, 1989). If the external mechanical work is an important determinant of the metabolic cost of walking, and the efficiency of muscle mechanical work is fairly constant, it is reasonable to expect measures of external mechanical work to be closely related to metabolic cost. Under these assumptions, however, individual limbs measures should yield more causal predictions of metabolic cost and more accurate estimates of efficiency because they account for the simultaneous performance of positive and negative work during multi-support phases.

The individual limbs method is applicable to all limbed animals. Most walking animals use four or more limbs and generally have no single support phase (Hildebrand, 1985). At some time during each stride, some limbs may generate positive power while other limbs simultaneously generate negative power. The traditional combined limbs method will therefore underestimate the external mechanical work performed (Cavagna et al., 1977; Full, 1989). Application of the individual limbs method to measurement of external mechanical work performed by multi-limbed animals may provide new insight into the general principles underlying walking mechanics (Alexander, 1980).

The mechanical work of walking might be best studied using several methods for estimating mechanical work. Joint power approaches give more direct estimates of total mechanical work, and mechanical power measured from the joints in each limb can be summed and integrated to yield individual limbs measures of mechanical work. But the individual limbs method retains much of the simplicity of the combined limbs method, and is easier to relate to inverted pendulum behavior and motion of the center of mass. This relative simplicity is appealing, especially for measurements of diverse animals for which measurements of kinematics and inertial properties are impractical. The consideration of joint power information, when feasible, makes it possible to quantify internal work and to understand how it is related to the motion of the limbs. By coupling joint power and individual limb approaches it may be possible to exploit the strengths of each method while avoiding some of their intrinsic limitations. Used together, one may study the mechanical work of walking in the context of the overall inverted pendulum behavior.

In summary, the individual limbs method demonstrates that during double support, a substantial amount of positive and negative mechanical work is performed simultaneously by the trailing and leading legs, respectively. This is consistent with our view of double support as a transition from one inverted pendulum to another.

Negative work by the leading leg is used to change the direction of the center of mass velocity between the end of one inverted pendulum phase and the beginning of the next. Positive work is necessary to restore lost energy. While this positive work may be performed at any point in the step, it is most advantageous to restore it during double support. The transition from one stance limb inverted pendulum to the next appears to be a major determinant of the mechanical work of walking.

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