



## Original Article

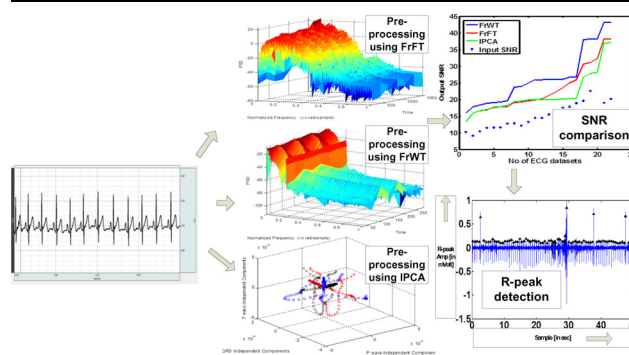
## A Comparison of ECG Signal Pre-processing Using FrFT, FrWT and IPCA for Improved Analysis

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## HIGHLIGHTS

- Fractional wavelet transform (FrWT) as a pre-processing technique.
- Fractional Fourier transform (FrFT) as a pre-processing technique.
- Independent Principal Component Analysis (IPCA) as a pre-processing technique.
- Signal portrayals in the time-fractional-frequency plane using FrFT and FrWT.
- R-peaks detection of the ECG signal acquired using the low cost electrocardiogram device.

## GRAPHICAL ABSTRACT



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## ABSTRACT

**Objective:** Electrocardiogram (ECG) is a diagnostic tool for recording electrical activities of the human heart non-invasively. It is detected by electrodes placed on the surface of the skin in a conductive medium. In medical applications, ECG is used by cardiologists to observe heart anomalies (cardiovascular diseases) such as abnormal heart rhythms, heart attacks, effects of drug dosage on subject's heart and knowledge of previous heart attacks. Recorded ECG signal is generally corrupted by various types of noise/distortion such as cardiac (isoelectric interval, prolonged depolarization and atrial flutter) or extra cardiac (respiration, changes in electrode position, muscle contraction and power line noise). These factors hide the useful information and alter the signal characteristic due to low Signal-to-Noise Ratio (SNR). In such situations, any failure to judge the ECG signal correctly may result in a delay in the treatment and harm a subject (patient) health. Therefore, appropriate pre-processing technique is necessary to **improve SNR** to facilitate better treatment to the subject. Effects of different pre-processing techniques on ECG signal analysis (based on **R-peaks detection**) are compared using various **Figures of Merit (FoM)** such as sensitivity (Se), accuracy (Acc) and detection error rate (DER) along with SNR.

**Methods:** In this research article, a new **fractional wavelet transform (FrWT)** has been proposed as a pre-processing technique in order to overcome the **disadvantages of other existing commonly used techniques viz. wavelet transform (WT) and the fractional Fourier transform (FrFT)**. The proposed FrWT technique possesses the properties of multiresolution analysis and represents signal in the fractional domain which consists of representation in terms of rotation of signals in the time–frequency plane. In the literature, ECG signal analysis has been improvised using statistical pre-processing techniques such as **principal component analysis (PCA)**, and **independent component analysis (ICA)**. However, both PCA and ICA are prone to suffer from slight alterations in either signal or noise, unless the basis functions are prepared with a **worldwide set of ECG**. Independent Principal Component Analysis (IPCA) has been used to overcome this shortcoming of PCA and ICA. Therefore, in this paper three techniques viz. FrFT, FrWT and IPCA are selected for comparison in pre-processing of ECG signals.

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**Results:** The selected methods have been evaluated on the basis of SNR, Se, Acc and DER of the detected ECG beats. FrWT yields the best results among all the methods considered in this paper; 34.37dB output SNR, 99.98% Se, 99.96% Acc, and 0.036% DER. These results indicate the quality of biology-related information retained from the pre-processed ECG signals for identifying different heart abnormalities.

**Conclusion:** Correct analysis of the acquired ECG signal is the main challenge for cardiologist due to involvement of various types of noises (high and low frequency). Twenty two real time ECG records have been evaluated based on various FoM such as SNR, Se, Acc and DER for the proposed FrWT and existing FrFT and IPCA preprocessing techniques. Acquired real-time ECG database in normal and disease situations is used for the purpose. The values of FoMs indicate high SNR and better detection of R-peaks in a ECG signal which is important for the diagnosis of cardiovascular disease. The proposed FrWT outperforms all other techniques and holds both analytical attributes of the actual ECG signal and alterations in the amplitudes of various ECG waveforms adequately. It also provides signal portrayals in the time-fractional-frequency plane with low computational complexity enabling their use practically for versatile applications.

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## 1. Introduction

Electrocardiogram (ECG) is a non-invasive diagnostic tool that records the electrical activity of the heart over a time period [1]. It consists of P-QRS-T waves [2]. Among these three waves, QRS-complex alarms to cardiac arrhythmias. Its detection is very important at the early stage for saving the life of someone. It is a weak and noisy electrical signal of the heart conduction system which contains different types of distortions such as baseline wander (BLW), muscle noise, electrode contact noise, Power Line Interference (PLI), Electromyogram (EMG) and body movement artifacts [3]. These noises may appear due to electrical instability in instruments and electromagnetic interference. These artifacts are categorized under the category of low and high frequency components in which muscle noise appears with 10% of regular peak-to-peak ECG amplitude and frequency up to 10 KHz [4]. BLW exists between the frequency spectrum of 0.15 Hz to 0.3 Hz, which may persist due to respiration, bad electrode attachment, etc. during acquisition of ECG datasets. PLI may occur due to improper grounding which is of about 50 Hz or 60 Hz frequency. EMG noise exists due to muscular movement at the time of ECG recording which is of about 10 kHz and Electrode contact noise comes due to disturbance in the contact between electrodes and skin [5]. These distortions change the form of the ECG signal and the chances of wrong diagnosis of the ECG signal increases. A short term variation comes because of modifications in the sympathetic and parasympathetic activity of the central nervous system (CNS) while modifications in intrinsic circadian controls develop long term variations. The interpretation of the modifications in the heart rate gives diagnostic information [6]. Heart specialists view amplitude, frequency and polarity for detecting the underlying disease [7]. They may further identify the following essential knowledge from the ECG signals such as thickening of heart muscle, essential knowledge of the previous heart attack, drug dosage issues, indication of coronary abnormality, etc. [8]. Some people are confused between heart attack and cardiac arrest. In practice, circulation issue leads to heart attack, whereas electrical issue leads to cardiac arrest. In a critical situation, it is very important for automated diagnostic systems to detect ECG signals accurately [9]. The automatic diagnostic system demands reliable, efficient, and stable process [10]. Therefore, ECG signal quality should be enhanced before this automated analysis. The quality of ECG signal is enhanced by performing denoising step that is still an active area in the field of health informatics [11]. Various denoising techniques have been reported in the literature as signal pre-processing techniques to provide the solution. Noise filtering and preserving actual subject (patient) ECG signal information is the principal objective of signal pre-processing [12].

The primary obstacle in the way of automatic ECG signal processing is the undesired frequency components which exist due to

body movement of the subject during recording, PLI appears due to interference of supply frequency with the recorded ECG signal, quantization noise, aliasing and signal processing artifacts such as Gibbs oscillations [13]. The removal of such noises is an essential step of ECG signal processing. It enhances the strength of ECG signal and represents its clear dynamics. Any signal having multiple time-varying frequencies is known as a non-stationary signal. In general, vibrations produced by a jackhammer, telecommunications and biomedical signals come in the class of non-stationary signals. Time-frequency representation (TFR) represents the signal both in time and frequency domains [14]. The signal should be of high SNR for providing accurate results while detecting various cardiac diseases [15]. Detection of QRS complex and R-peak alarms correct heart condition of the subject based on its non-invasive Heart rate variability (HRV) measurement [16]. HRV measurement is a valuable tool in cardiology which refers to the variations in heart rate (RR intervals). The normal changes are noticed due to autonomic neural regulation of the heart and the circulatory system [17]. HRV has good reproducibility and gives diagnostic information on subjects which are under observation for heart disease. HRV gives important knowledge about the working of the nervous control of the heart rate and the heart's ability to respond [18]. The Autonomic nervous system (ANS) has sympathetic and parasympathetic components which modulate the heart rate in ECG at different frequencies. Sympathetic Activity (SA) comes under the category of low frequency range (0.04 Hz–0.15 Hz) while the Parasympathetic Activity (PSA) comes under the category of high frequency range (0.15–0.4 Hz) of the heart rate [19]. Therefore, at this stage reliable pre-processing technique is needed. The primary challenges which were faced in ECG signal pre-processing are spectral overlap between the ECG and muscle noise. The filtering methods were incorporated for removing muscle noise, but they failed due to time varying characteristic of the ECG signal. In some cases, filtering methods are successful when desired information remains undistorted but failed to analyze micro potentials because they overlap with the PLI. The major problem with analog filtering is that ECG frequency spectra overlaps with that of the noise spectra, mainly due to the presence of QRS complex, which is a high frequency component in the ECG. Either digital or analog filters, it has major problems of correct selection of the signal pass band that provides correct pre-processing results. A simple QRS detection algorithm proposed by Pan and Tompkins is mostly employed in recognition of various cardiac abnormalities [20]. Various methods have been considered for pre-processing of ECG signals such as fixed notch filter [21], median filter [22], adaptive Fourier decomposition (AFD) [23], Alexander fractional differential window filter (AFDW) [1], Stockwell transform (S-transform) [14], intrinsic mode functions (IMF) [24], adaptive signal processing (ASP) [25], sinusoidal modeling (SM) [26], Kalman filtering [27],

Hilbert-Huang transform (HHT) [28], discrete wavelet transform (DWT) [29], Hilbert transform [30], hidden Markov model [31], deep recurrent neural networks [4], and adaptive filtering [32]. The Fourier transform (FT) is most often used tools in signal processing, but unable to analyze non-stationary signals effectively with necessarily alteration of the time and frequency domain resolution on the basis of signal attributes [33]. Fast Fourier transform (FFT) is computationally efficient and secures an excellent frequency resolution [34], but FFT does not provide time localization of common frequency components of two signals. FFT is inappropriate technique due to non-stationary nature of ECG signal. Next, short-time Fourier transform (STFT) was utilized for analyzing and processing such non-stationary signals. STFT was applied using different sliding window functions to find spectrogram. Due to fixed duration of the sliding window, the time-frequency resolution is restricted. Therefore, resolution was limited both in time domain and in frequency domain [35]. Wavelet transform (WT) has given the solution to such problems by representing them with small waves in restricted time slot. WT represents the time and frequency resolutions, which vary in time-frequency plane, for obtaining multiresolution analysis [36]. But, WT outcomes suffer due to its shift variant property and oscillatory wavelet coefficient [37]. It also considers a fixed decomposition scale for signal analysis due to its leakage characteristic that exist because of bounded length of wavelet functions [38]. Also, WT is incapable of figuring out signals whose energy is not strongly condensed in the frequency domain [39]. Some authors have used conventional notch filter for ECG signal pre-processing but it filters the ECG signal components present at line frequency i.e. 50 Hz [36]. In the literature, authors have also worked out on digital filters such as infinite impulse response (IIR) filters and finite impulse response (FIR) filters. Digital filters fail due to its incapability to maintain essential characteristics, which exists in the ECG signal [40]. Moreover, FIR filters need massive filter coefficients for the filtering operation. Sometimes, it also removes the essential frequency components of ECG signal. A crucial technique for noise removal inspects the signal spectra and overcomes unwanted frequency components. Noise may exist over the whole frequency range of the signal and previous techniques, those have been used for signal pre-processing are not sufficient. To overcome this problem, ASP technique has been given in [25] that change the filtering parameters adaptively in such way that will enhance their operation. Filtration of PLI using ASP demands reference signal, which contributes to the complication in the system hardware and software. In [41], Ensemble Averaging (EA) has been considered to get the extraction of small cardiac components of the noise corrupted ECG signal. But this technique calls for the averaging of many beats where cardiac cycles are vanished in the averaging process. To overcome the Fourier transform limitation [42] and its capability to remove Gaussian noise, the WT based technique was proposed in [43]. Mostly, accuracy of WT-based pre-processing relies on their correct choice of mother wavelet that provides inherent stable pre-processing results. In a real scenario, it is very cumbersome [44]. In [45], S-transform was proposed for pre-processing of the ECG signal which presented better results than WT. However, it has a low energy convergence rate and trade-off between the time resolution and the frequency resolution. Most of the researchers claim that the linear technique presents a good conduct. These researchers observed less noise and disturbances on its recordings. Also, some researches proved nonlinear methods to be robust in the presence of noise. So, there is a scope to explore further linear and nonlinear techniques. Thus, linear and nonlinear techniques should be examined in high noise and disturbance environment.

Pre-processing is also very essential in the Computer aided diagnosis (CAD) system for doing accurate treatment. The conduct of the CAD system is enhanced using linear and nonlinear techniques

along with potent classifiers. In [24], the IMF was adopted, but the analytic phase functions of IMFs are not always monotonous and due to this, its meaningful analytic instantaneous frequency (IF) cannot be calculated that affects the time-frequency analysis of the IMFs [46]. The important concept of empirical mode decomposition (EMD) in filtering is to decompose the noisy signal into the IMFs. Due to the assumption of noise free primary IMFs, it provides the reconstructed ECG signal with misinterpretation [29]. In [47], the enhanced EMD method has been used for ECG signal pre-processing. In this method, ECG signal was decomposed into IMFs using the windowing technique to retain the QRS complex. The main disadvantage that was noticed during the processing of ECG signal is that it considers fixed window length of 200 ms, which may not be appropriate for all real time applications in which there is variation in width of the QRS complex. It results in the loss of important clinical information. In [1], Alexander fractional differential window (AFDW) filter was proposed. It worked on the basis of averaging the forward and backward filter coefficients. But for getting better outcomes, it should tune its parameters (fractional order  $\beta$  and variable  $t$ ) empirically as well as optimally. Some authors have used principal component analysis (PCA) and independent component analysis (ICA) [48,49]. The main problem faced with PCA and ICA is that they are utterly prone to slight alterations in either the signals or the noises, unless the basis functions are prepared with a worldwide set of ECG. The main problem to adopt ICA in the analysis is the examination of the order of the independent components (ICs). So interference of the human naked eye becomes essential, which is not preferred in CAD for analyzing varieties of heart signals. PCA showed its effectiveness when it is related to the highest variance. Neither PCA nor ICA is sufficient for the high dimensional and noisy environments of ECG recordings. Independent Principal Component Analysis (IPCA) has overcome the existing problems of PCA and ICA. Some authors worked on wearable ECG signal acquiring instruments for ensuring essential health based problems. This system is not efficient for removing motion artifacts. In [27], Kalman filtering was applied to assess heart rate, which can forecast and clarify it continuously. Due to deep body actions, this approach limits its measurement to model associated noise [50]. In [13], nonlocal means (NLM) were used for ECG signal denoising, but this method cannot work effectively for eminently debased ECG signals and the motion distortions still occur [50]. Therefore, techniques proposed in past literature still have imperfections in their outcomes, as they do not provide efficient noise removal and denoising resulting in a deviation from the actual ECG signal. WT in ECG signal pre-processing is not a new technique, various researchers have worked on it. Its operation relies on the choice of the mother wavelet and values of the scale parameter in order to improve the clinical usefulness. In this paper, fractional wavelet transform (FrWT) has been proposed which has good mathematical attributes of WT and fractional Fourier transform (FrFT), along with some gripping attributes of its own. FrWT also gives a prosperous representation of details which alter with the various numbers of transforming orders [51]. Also, FrWT is a transformation to represent multiresolution analysis of signals in the fractional domain.

FrFT is the generalization of the traditional Fourier transform (TFT) which has one additional attribute, i.e. rotation of the plane by an angle  $\alpha = \frac{p\pi}{2}$ , where  $p$  is real parameter ( $R^\alpha$ ). This new rotated axis preserves both time and frequency fact. If  $p$  has unity value, then FrFT behaves like a TFT. If  $p$  is equal to 0 value, then the procedure will be indistinguishable. FrFT uses chirp signal, since basis and computational complexity is similar to TFT. Due to this reason, it can be used in all TFT applications [32]. In this paper, pre-processing of ECG signal using FrFT, FrWT and IPCA has been compared.

### Abbreviation

FrFT: Fractional Fourier transform  
 FrWT: Fractional wavelet transform  
 IPCA: Independent Principal Component Analysis  
 PCA: Principal Component Analysis  
 ICA: Independent Component Analysis  
 SNR: Signal to Noise Ratio  
 BLW: Base Line Wander  
 ECG: Electrocardiography/Electrocardiogram  
 PLI: Power Line Interference  
 EMG: Electromyography  
 MATLAB: Matrix Laboratory  
 FoM: Figures of Merit

TFR: Time–Frequency Representation  
 WT: Wavelet Transform  
 DWT: Discrete Wavelet Transform  
 Se: Sensitivity  
 Acc: Accuracy  
 DER: Detection Error Rate  
 PSD: Power Spectral Density  
 FT: Fourier Transform  
 FrFD: Fractional Fourier domain  
 TP: True Positives  
 FP: False Positives  
 FN: False Negatives

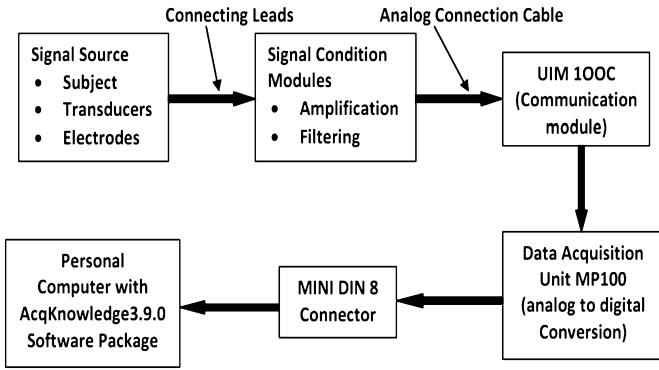


Fig. 1. BIOPAC system (MP100) block diagram [52].



Fig. 2. The Experimental set-up of ECG signal acquisition in laboratory in sitting position using MP100 BIOPAC system.

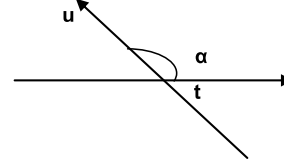


Fig. 3. The angle of rotation  $\alpha$ .

This paper is organized as follows: Section 2 presents the materials and methods proposed in the paper, Section 3 illustrates simulation results and discussion on it, Section 4 concludes the paper by giving the main findings of the problems addressed and Section 5 has given the future trends related to this research work.

## 2. Materials and methods

In this section information about the ECG signal acquisition and ECG signal pre-processing methods will be discussed.

### 2.1. ECG signal recording

ECG data is taken at biomedical lab using BIOPAC machinery MP35/MP100 [52] as shown in Fig. 1 under proposal of one week free ECG signal (data) acquisition. During acquisition of ECG signal, data was selected from the subjects consisting of college guards, outsiders, residents and students. Twenty Two recordings were collected at 360 Hz sampling rate. The experimental set-up of ECG signal acquisition in laboratory in sitting position using MP100 BIOPAC system is shown in Fig. 2.

### 2.2. Pre-processing methods

In this section, different pre-processing methods considered in this paper have been discussed in detail.

#### 2.2.1. Fractional Fourier transform (FrFT)

The fractional Fourier transform (FrFT) is an enhancement of the classical Fourier transform for providing distribution of time–frequency plane and it executes the rotation of signals. Due to its clear and fascinating attributes in Time–Frequency plane, it is deeply used in signal processing applications [53] such as signal restoration, noise removal in fractional domain, optics analysis

[54,55]. It is considered as rotation of the signal in the time–frequency plane by an arbitrary angle  $\alpha = p\pi/2$ , where  $p$  is a real number. It gives both time and frequency information along the intermediate axis, making a certain angle with a time axis. As the angle of rotation is a non-integer multiple of  $\pi/2$ , the transform is called FrFT. Any value of  $\alpha$  lies between  $0 < \alpha < \frac{\pi}{2}$  gives a rotated time–frequency description of the signal [56] and shown in Fig. 3.

The  $p$ th ( $p = 2\alpha/\pi$ ) order FrFT of a signal  $x(t) \in L^2(R)$  is defined as

$$X_p(u) = F^p\{x(t)\}(u) = \int_{-\infty}^{\infty} x(t) K_p(t, u) dt \quad (1)$$

where

$$K_p(t, u) = \sqrt{\frac{1 - j \cot \alpha}{2\pi}} \exp\left(j \frac{t^2 + u^2}{2} \cot \alpha - jut \csc \alpha\right), \quad \text{when } \alpha \neq k\pi \quad (2)$$

$$K_p(t, u) = \delta(u - t), \quad \text{when } \alpha = 2k\pi \quad (3)$$

$$K_p(t, u) = \delta(u + t), \quad \text{when } \alpha = (2k + 1)\pi \quad (4)$$

The kernel has the following properties which will be useful in this paper



$$K_p^*(t, u) = K_{-p}(t, u) \quad (5)$$

$$\int_{-\infty}^{\infty} K_{p1}(t, u) K_{p2}(t, z) du = K_{p1+p2}(t, z) \quad (6)$$

$$\int_{-\infty}^{\infty} K_p(u, t) \cdot K_p^*(t, u') dt = \delta(u - u') \quad (7)$$

$$x(t) = \int_{-\infty}^{\infty} K_p(u) \cdot K_p^*(t, u) du \quad (8)$$

The definition implies that the FrFT is the decomposition into the chirp bases  $K_p^*(t, u)$ . So a proper order FrFT of the chirp signal is an impulse.

### 2.2.2. Fractional wavelet transform (FrWT)

Recently, fractional wavelet transform (FrWT) has been used in various fields such as image compression, signal processing, optics etc. [57]. In R-peak (QRS-complex) detection, problem arises due to morphological variations of P-QRS-T waveforms, position of waveforms and the change in cyclic intervals of the ECG waveforms of different subjects (patients) and noises occurrence at acquiring data. The essential step to provide cardiac physiology is performing spectral analysis. WT describes the act of ECG signal based on smaller parameter numbers in diagnostic analysis. It overcomes the limitations of single scale (time or frequency) by incorporating multi-scale basis. DWT provides the essential features which are not possible with previously discussed methods. Unlike Fourier transform, this step makes DWT as an important tool for non-stationary signals. Two necessary steps are performed in DWT of the ECG signal which are decomposition and reconstruction of the ECG signal. During decomposition, the signal is transformed into filter bank tree. This step is done on the basis of multi-level decomposition, including a low pass filter  $u[k]$  and a high pass filter  $v[k]$ . Therefore, it permits the decomposition of a noisy ECG signal into number of scales using two digital filters and two down samplers by 2 (one down sampler by 2 in every stage). Proper chosen of wavelets and its decomposition level is very important for signal denoising using the WT [58]. In this research, Daubechies wavelet has been chosen because it is capable to extract minuscule information which is not possible with other techniques [59]. Next, the wavelet coefficients have been estimated in order 2 using the MATLAB@2012 software package. Every stage is the combination of two parts; approximation part  $pA_k$  and detailed part  $pD_k$ . These parts represent low frequency and high frequency parts respectively. In general, the  $A_k$  and  $D_k$  are defined as [42,51]

$$pA_{k+1}(m) = \sum_{n=-\infty}^{\infty} u[n-2m] pA_k(m) \quad (9)$$

$$pD_{k+1}(m) = \sum_{n=-\infty}^{\infty} v[n-2m] pD_k(m) \quad (10)$$

where  $k$  reveals the decomposition level.

FrWT performs the reconstruction step using two digital filters and two up samplers by 2 (one up sampler by 2 in each stage). The reconstruction of the ECG signal is obtained by considering a low pass filter  $\bar{u}[k]$  and a high pass filter  $\bar{v}[k]$ .

$$z_{ECG}(t) = pA_k(m) \quad (11)$$

$$z_{ECG}(t) = \sum_{n=-\infty}^{\infty} pA_{k+1}(n) \bar{u}[m-2n] + \sum_{n=-\infty}^{\infty} pD_{k+1}(n) \bar{v}[m-2n] \quad (12)$$

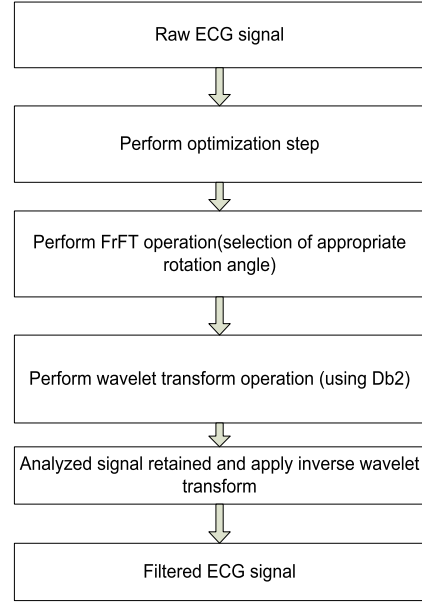


Fig. 4. General steps required to apply FrWT [57].

Wavelet thresholding of acquired ECG signal has been done using soft thresholding function, which is much more stable than hard thresholding [60].

$$f_{soft} = \begin{cases} \text{sign}(f)(|f| - n) & |f| \geq n \\ 0 & |f| < n \end{cases} \quad (13)$$

where  $f$  denotes the wavelet coefficient, and  $n$  represents the threshold.

$$n = \frac{\text{median}}{0.6745} \sqrt{2 \log(w)/w} \quad (14)$$

where  $w$  represents the wavelet coefficients.

FrWT is the combinations of WT and FrFT. It has the advantage of both transforms and avoids the limitations of the WT and the FrFT. It secures the multiresolution analysis capability and signal representations in the fractional domain due to the WT and FrFT [61]. All required steps to apply FrWT have been shown in Fig. 4. The  $\alpha$ -order FrWT of a signal  $x(t)$  is expressed as [61]

$$W_x^\alpha(p, q) = \int_{-\infty}^{\infty} x(t) \varphi_{\alpha, p, q}^*(t) dt \quad (15)$$

where the  $\alpha$ -order Fractional wavelet

$$\varphi_{\alpha, p, q}(t) = e^{-\left(\frac{i}{2}\right)(t^2 - q^2 - \left(\frac{t-q}{p}\right)^2) \cot \alpha} \varphi_{p, q}(t) \quad (16)$$

The rotation matrix is represented as  $[T]_{2 \times 2}$

$$[T]_{2 \times 2} = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix} \quad (17)$$

### 2.2.3. Independent Principal Component Analysis (IPCA)

Independent Principal Component Analysis (IPCA) includes the good characteristic of both PCA and ICA [62] and provides perceptive patterns in the data. It depends on the data being analyzed [63]. IPCA worked on PCA with ICA effectively and it has the capability to provide a good collection of ECG samples on graphical production. Next, kurtosis and percentage variance value have been considered to select the number of independent principal components (IPCs). In IPCA, PCA works as a filtering step to trim the ECG data and create loading vectors. Thus, this step provides filtered

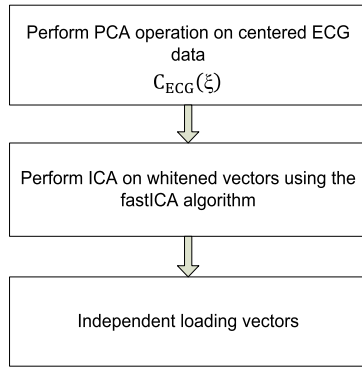


Fig. 5. General steps to find out IPCA.

principal components (PCs) based on ICA as a filtering step of the related loading vectors.

In case of real recording ECGs of high dimensionality, as compared to PCA and ICA, IPCA is more effective to sum up the whole ECG recording with a smaller number of components [64]. After PCA operation on centered ECG data, ECG signal is represented as  $C_{ECG}(\xi) = O(\xi)DR^T$ . Where  $\xi$  represents a single independent variable (e.g. time, location, etc.),  $O$  turns out to be uncorrelated columns matrix,  $D$  represents diagonal matrix and  $R^T$  represents an orthogonal matrix. Fig. 5 shows the general steps to find out IPCA components.

#### 2.2.4. Signal to noise ratio (input and output)

The input SNR and output SNR of the ECG signal are defined as [40,65,66]

SNR (dB) at the input,

$$SNR_{dB} = 10 \log_{10} \frac{[x(t)_{neat}]^2}{(x(t)_{noisy} - x(t)_{neat})^2} \quad (18)$$

SNR (dB) at the output,

$$SNR_{dB} = 10 \log_{10} \frac{[x(t)_{neat}]^2}{(x(t)_{filtered} - x(t)_{neat})^2} \quad (19)$$

where  $x(t)$  denotes the ECG signal.

% disturbance present in filtered ECG signal

$$= \frac{P_{ECG,of} - P_{ECG,in}}{P_{ECG,in}} \quad (20)$$

where  $P_{ECG,of}$  is the filtered output ECG signal power and  $P_{ECG,in}$  is the net input ECG signal power.

#### 2.2.5. Figures of merit (FoM)

Mathematically the considered FoM are defined as [67,68]

$$Se = \frac{TP}{TP + FN} \times 100\% \quad (21)$$

$$Acc = \frac{TP}{TP + FN + FP} \times 100\% \quad (22)$$

$$DER = \frac{FP + FN}{Actual\ beats} \times 100\% \quad (23)$$

where TP is the number of true positives, FN is the number of false negatives, and FP is the number of false positives.

### 3. Results and discussion

The main challenge in ECG signal pre-processing is spectral overlap between the ECG and muscle noise. Various filtering methods were used by different researchers, but these methods failed

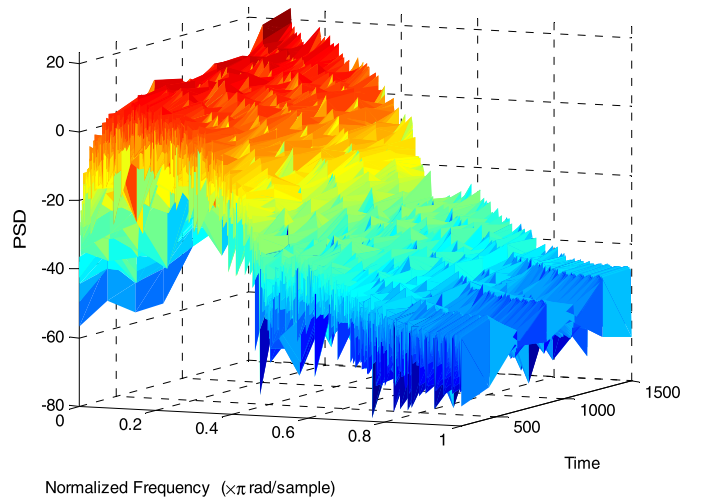


Fig. 6. FrFT of ECG signal at  $\alpha = 0.3\pi$ .

due to time varying characteristic of the ECG signal. In certain cases, filtering methods were successful when desired information remains undistorted. But they failed to analyze micro potentials which were overlapping with the PLI. Therefore, ECG signal needs appropriate pre-processing technique to enhance the quality of ECG signal for correct prediction of the heart condition. Various transformations have been presented in the literature, because a single transformation method is not suitable for all applications; rather each one has its own area of application. WT has succeeded in the biomedical digital signal processing, but its analysis capability is restricted in the time-frequency plane. FrFT has given the solution of above problem and can give signal descriptions in the fractional domain. Using energy preservation property, the signal has been decomposed on the orthonormal basis set of the chirp functions. It is illustrated as the distribution of the signal energy between distinct frequencies. It makes squared magnitude of the FrFT of a signal as the fractional energy spectrum to distribute signal energy in various chirp basis functions [60]. FrFT, FrWT and IPCA have been performed of normal ECGs. In two data sets, high-frequency components were noticed. Detection of P and T waves relies on low frequency response than R-peak detection. The simulation results of FrFT were obtained using MATLAB code available in [69]. The PSD function has been used to represent the important observations of the variations (energy) as a function of frequency. It represents the frequency variations according to their strong or weak signal components. Filtering with FrFT presented attributes of all ECG components, but at rotation angle  $0.5\pi$  most of the information was missing (due to spectral overlapping). At rotation angle  $0.1\pi$ , the attributes of all ECG components were; P-wave: 0.301 mV to 0.344 mV (amplitude), 12.277 Hz to 13.269 Hz (frequency), R-wave (QRS-complex): 0.85 mV to 1.21 mV (amplitude) and frequency ranges between 13.791 Hz to 14.761 Hz, T-wave: 0.401 mV to 0.427 mV (amplitude) and frequency ranges between 6.17 Hz to 7.69 Hz. In Fig. 6, at rotation angle  $0.3\pi$ , the attributes of all ECG components were; P-wave: 0.221 mV to 0.361 mV (amplitude), 11.346 Hz to 12.231 Hz (frequency), R-wave (QRS-complex): 0.92 mV to 1.41 mV (amplitude) and frequency ranges between 13.271 Hz to 14.122 Hz, T-wave: 0.311 mV to 0.431 mV (amplitude) and frequency ranges between 5.67 Hz to 7.11 Hz. These results, which were obtained using FrFT are very important but will not work effectively when motion artifacts are considered and will result into overlapping of QRS complexes and motion artifact frequency components. The main dispute of noise in ECG signal pre-processing exists due to the existence of the large DC offset and different distortion signals. It may increase the R-peak amplitude up to 300 mV for a usual electrode. BLW and PLI make

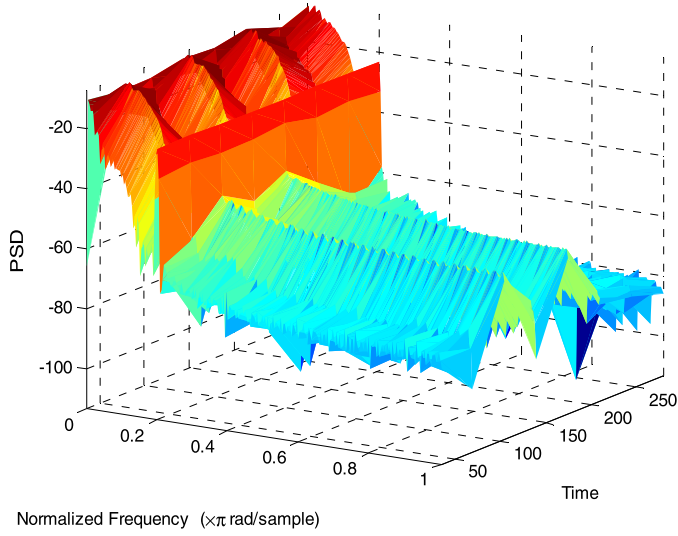


Fig. 7. FrWT of ECG signal at  $\alpha = 0.3\pi$ .

**Table 1**  
Details, cross correlation, and selection of artifact/noise (motion artifact).

Details	r (cross correlation)	Selection
d <sub>1</sub>	3.77	–
d <sub>2</sub>	5.21	–
d <sub>3</sub>	19.78	–
d <sub>4</sub>	38.35	–
d <sub>5</sub>	41.77	In case of motion artifact
d <sub>6</sub>	40.18	–
d <sub>7</sub>	30.71	–
d <sub>8</sub>	23.76	–

the detection of R-peaks more complex, which exists due to respiration or patient movement. R-peaks detection based on FrFT, FrWT and IPCA has been compared in this paper. In case of FrWT, first step is to select rotation angle so that the next step (decomposition) may be effective at low computational complexity.

Filtering using FrWT gave attributes of all ECG-components, but at rotation angle  $0.5\pi$ , some of the information was missing (due to spectral overlapping). At rotation angle  $0.1\pi$ , the attributes of all ECG components were; P-wave: 0.322 mV to 0.377 mV (amplitude), 12.771 Hz to 13.789 Hz (frequency), R-wave (QRS-complex): 0.97 mV to 1.31 mV (amplitude) and frequency ranges between 13.411 Hz to 14.331 Hz, T-wave: 0.381 mV to 0.451 mV (amplitude) and frequency ranges between 6.61 Hz to 7.31 Hz. In Fig. 7, at rotation angle  $0.3\pi$ , the attributes of all ECG components were; P-wave: 0.175 mV to 0.211 mV (amplitude), 11.171 Hz to 12.777 Hz (frequency), R-wave (QRS-complex): 0.73 mV to 1.37 mV (amplitude) and frequency ranges between 13.771 Hz to 15.723 Hz, T-wave: 0.287 mV to 0.437 mV (amplitude) and frequency ranges between 5.22 Hz to 8.77 Hz. These results obtained using FrWT is much important and it is similar to normal attributes of clinical ECG signal. FrWT provided the segregation of QRS complexes and motion artifact frequency component clearly. Also, its results are similar to normal clinical ECG signal. To get the optimal and efficient coefficient, comparison is done between specific decomposed ECG signal and actual ECG signal as shown in Table 1 and Table 2.

$$r = \frac{\sum_{j=1}^M p(j) q(j)}{\sqrt{\sum_{j=1}^M p^2(j) \sum_{j=1}^M q^2(j)}} \quad (24)$$

**Table 2**  
Details, cross correlation, and selection of artifact/noise (PLI, BLW).

Details	r (cross correlation)	Selection
d <sub>1</sub>	2.81	–
d <sub>2</sub>	5.21	In case of PLI, BLW
d <sub>3</sub>	5.02	–
d <sub>4</sub>	4.71	–
d <sub>5</sub>	4.27	–
d <sub>6</sub>	3.78	–
d <sub>7</sub>	3.21	–
d <sub>8</sub>	2.42	–

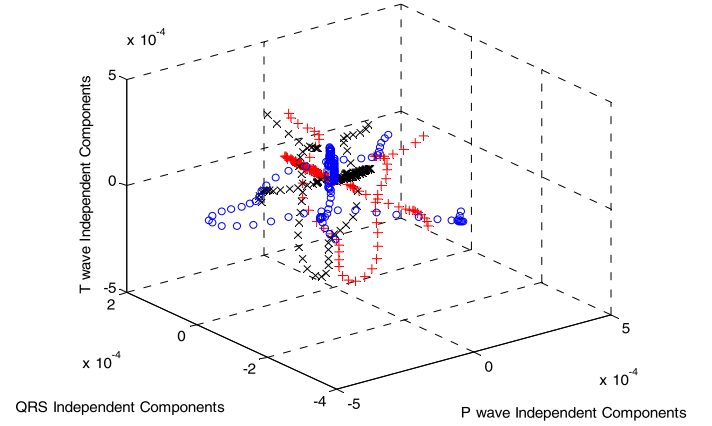


Fig. 8. IPCA of ECG signal where red color indicates P-wave IPCA components, black color indicates QRS-complex IPCA components, and blue color indicates T-wave IPCA components. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

where  $p$  denotes the actual ECG signal and  $q$  denotes the specific decomposed ECG signal. In this research Daubechies wavelet has been chosen because it has the ability to extract minuscule information which is not possible with other techniques. It has given the QRS complexes in the range of 2.5–4.7 Hz and motion artifacts in the range of 2.5–22.5 Hz, respectively using decomposition level 2 and 5. It provides better detection of R-peaks. The detection steps of R-peaks have been completed based on maximum/minimum value and the starting/terminating of each cardiac cycle.

Due to involvement of distortion/noise, the distribution of loading vectors tends to Gaussian distribution (central limit theorem). In this problem, non-gaussianity of loading vectors is increased so that noiseless loading vectors can be produced. This step provides the effective independent principal components (IPCs). IPCA result has been shown in Fig. 8, which is obtained using MATLAB code available in [70]. The eigenvalue matrix, eigenvector matrix and variance were computed for the proposed recordings. In this study, the observed first principal component (PC) eigenvalue variances were 99.13%, 99.69%, 99.65%, 99.74%, 98.37%, 96.81%, 99.62%, 94.67%, 97.87%, 97.81%, 98.79%, 99.32%, 98.67%, 99.32%, 99.54%, 99.23%, 99.23%, 98.87%, 97.76%, 98.56%, 99.77% and 98.77% of the twenty two real-time recordings. The selected twenty two real-time ECG recordings had a kurtosis value of 15.34, 13.23, 2.55, 6.34, 7.34, 12.56, 3.66, 7.11, 3.44, 6.11, 8.44, 9.33, 6.67, 2.33, 3.55, 5.88, 6.99, 6.44, 3.45, 4.12, 5.55 and 6.17 for first independent component (IC) value. Another three ICs have also been calculated and found super-Gaussian signals (in some cases sub-Gaussian signal).

ECG signals, which consist of many data points, can be compressed into a few features by performing spectral analysis of the signals with the WT. These features characterize the behavior of the ECG signals. It has been revealed that WT with FrFT can remove disturbances more efficiently as compared to other techniques. SNR values have been compared and shown in Fig. 9(a)–(d).

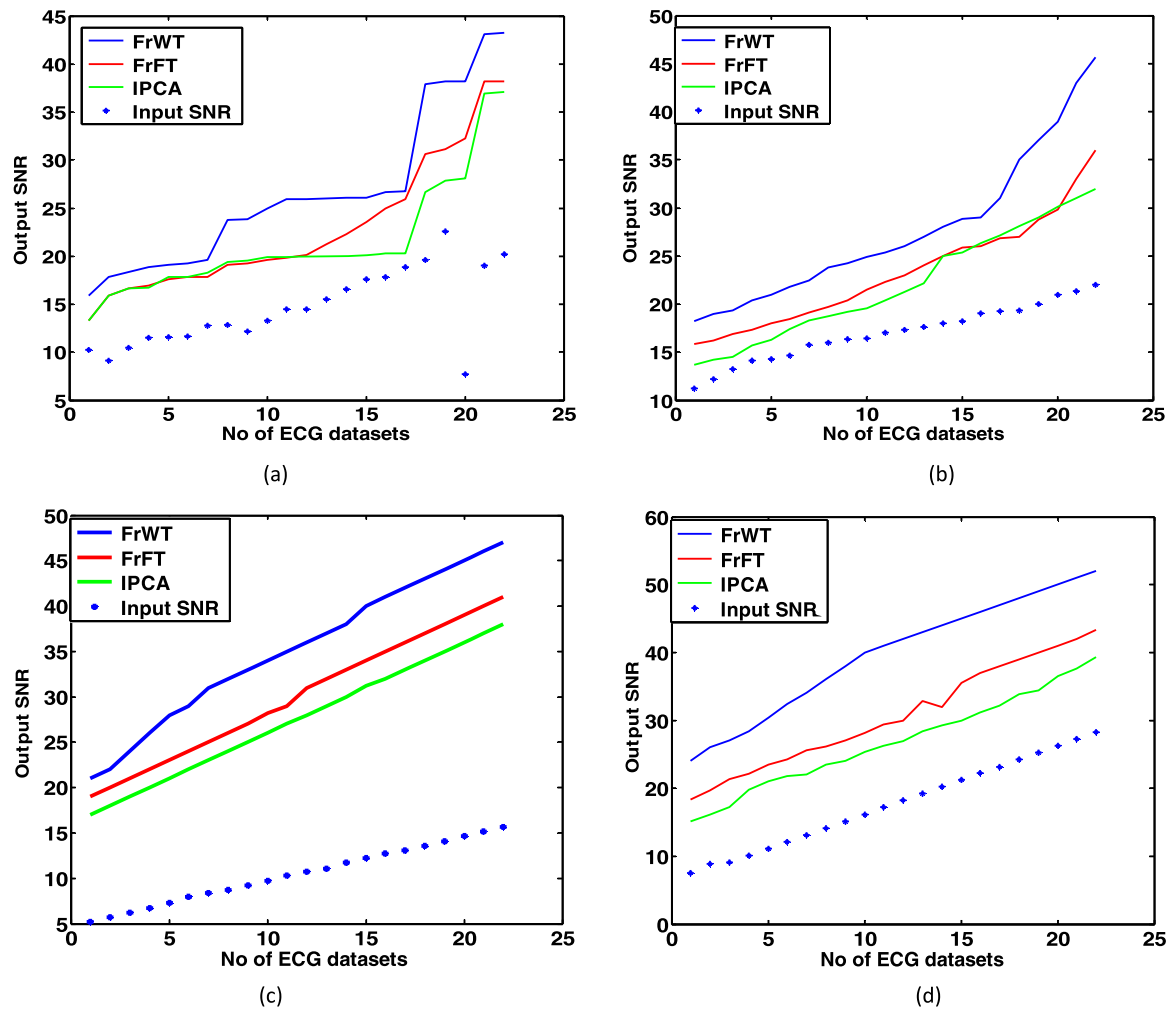


Fig. 9. SNR comparison between Output and Input using FrWT, FrFT and IPCA.

Table 3

Performance evaluation of IPCA denoising based on Se, Acc, DER and output SNR.

Real time database	SNR (Input in dB)	Actual beats	TP	FP	FN	Se (%)	Acc (%)	DER (%)	SNR (Output in dB)
01	14.11	1823	1820	1	2	99.89	99.83	0.164	23.03
02	16.23	1411	1408	1	2	99.85	99.78	0.212	25.11
03	17.11	2161	2158	1	2	99.90	99.86	0.138	26.23
04	19.23	2201	2200	0	1	99.95	99.95	0.045	28.11
05	17.24	1874	1871	1	2	99.89	99.83	0.160	26.78
06	16.87	1278	1275	1	2	99.84	99.76	0.234	25.11
07	15.12	1787	1785	1	1	99.94	99.88	0.111	24.11
08	18.13	1701	1697	2	3	99.82	99.70	0.293	26.45
09	17.34	1789	1788	0	1	99.94	99.94	0.055	27.11
10	17.21	1389	1387	0	0	100	100	0.000	26.76
11	18.11	1911	1909	0	0	100	100	0.000	27.21
12	14.87	1788	1785	1	1	99.94	99.88	0.111	22.22
13	13.34	1289	1287	0	1	99.92	99.92	0.077	21.34
14	12.23	1891	1890	0	0	100	100	0.000	21.11
15	13.24	1289	1286	1	2	99.84	99.76	0.232	21.22
16	14.67	2199	2199	0	0	100	100	0.000	23.78
17	13.09	2108	2107	0	1	99.95	99.95	0.047	22.78
18	17.12	1788	1785	1	2	99.88	99.83	0.167	26.32
19	14.32	2811	2809	1	2	99.92	99.89	0.106	23.12
20	13.11	1198	1197	0	1	99.91	99.91	0.083	22.51
21	14.11	2489	2487	1	2	99.91	99.87	0.120	22.11
22	17.65	1487	1486	0	1	99.93	99.93	0.067	24.78
<b>Total 22 records</b>	<b>15.656</b>	<b>39,662</b>	<b>39,616</b>	<b>13</b>	<b>29</b>	<b>99.92</b>	<b>99.88</b>	<b>0.110</b>	<b>24.42</b>



**Table 4**

Performance evaluation of FrFT denoising based on Se, Acc, DER and output SNR.

Real time database	SNR (Input in dB)	Actual beats	TP	FP	FN	Se (%)	Acc (%)	DER (%)	SNR (Output in dB)
01	14.11	1823	1821	1	2	99.89	99.83	0.164	22.78
02	16.23	1411	1408	1	1	99.92	99.85	0.141	26.23
03	17.11	2161	2158	1	1	99.95	99.90	0.092	27.88
04	19.23	2201	2200	0	1	99.95	99.95	0.045	31.67
05	17.24	1874	1871	1	2	99.89	99.83	0.160	27.12
06	16.87	1278	1275	1	2	99.84	99.76	0.234	28.27
07	15.12	1787	1785	1	1	99.94	99.88	0.111	27.78
08	18.13	1701	1700	0	0	100	100	0.000	29.78
09	17.34	1789	1788	0	0	100	100	0.000	28.97
10	17.21	1389	1389	0	0	100	100	0.000	27.67
11	18.11	1911	1911	0	0	100	100	0.000	29.78
12	14.87	1788	1788	0	0	100	100	0.000	25.89
13	13.34	1289	1287	0	1	99.92	99.92	0.077	23.87
14	12.23	1891	1891	0	0	100	100	0.000	23.89
15	13.24	1289	1286	1	2	99.84	99.76	0.232	24.23
16	14.67	2199	2199	0	0	100	100	0.000	24.23
17	13.09	2108	2108	0	0	100	100	0.000	23.12
18	17.12	1788	1788	0	0	100	100	0.000	26.12
19	14.32	2811	2809	1	2	99.92	99.89	0.106	23.22
20	13.11	1198	1198	0	0	100	100	0.000	25.68
21	14.11	2489	2488	0	1	99.95	99.95	0.040	22.89
22	17.65	1487	1485	0	1	99.93	99.93	0.067	25.12
<b>Total 22 records</b>	<b>15.656</b>	<b>39,662</b>	<b>39,633</b>	<b>8</b>	<b>17</b>	<b>99.95</b>	<b>99.93</b>	<b>0.066</b>	<b>26.19</b>

**Table 5**

Performance evaluation of FrWT denoising based on Se, Acc, DER and output SNR.

Real time database	SNR (Input in dB)	Actual beats	TP	FP	FN	Se (%)	Acc (%)	DER (%)	SNR (Output in dB)
01	14.11	1823	1823	0	0	100	100	0.000	34.13
02	16.23	1411	1410	1	1	99.92	99.85	0.141	37.34
03	17.11	2161	2161	0	0	100	100	0.000	38.99
04	19.23	2201	2201	0	0	100	100	0.000	41.11
05	17.24	1874	1873	0	1	99.94	99.94	0.053	39.23
06	16.87	1278	1276	1	1	99.92	99.84	0.156	38.23
07	15.12	1787	1787	0	0	100	100	0.000	28.89
08	18.13	1701	1700	1	1	99.94	99.88	0.117	31.11
09	17.34	1789	1788	0	0	100	100	0.000	34.11
10	17.21	1389	1389	0	0	100	100	0.000	33.12
11	18.11	1911	1911	0	0	100	100	0.000	35.19
12	14.87	1788	1788	0	1	99.94	99.94	0.055	32.19
13	13.34	1289	1289	1	1	99.92	99.84	0.155	29.56
14	12.23	1891	1891	0	0	100	100	0.000	31.01
15	13.24	1289	1289	0	0	100	100	0.000	32.78
16	14.67	2199	2199	0	0	100	100	0.000	33.09
17	13.09	2108	2108	0	1	99.95	99.95	0.047	31.78
18	17.12	1788	1788	0	0	100	100	0.000	37.89
19	14.32	2811	2810	1	1	99.96	99.92	0.071	33.67
20	13.11	1198	1198	0	0	100	100	0.000	32.78
21	14.11	2489	2489	0	0	100	100	0.000	33.98
22	17.65	1487	1487	0	0	100	100	0.000	36.11
<b>Total 22 records</b>	<b>15.656</b>	<b>39,662</b>	<b>39,655</b>	<b>5</b>	<b>8</b>	<b>99.98</b>	<b>99.96</b>	<b>0.036</b>	<b>34.37</b>

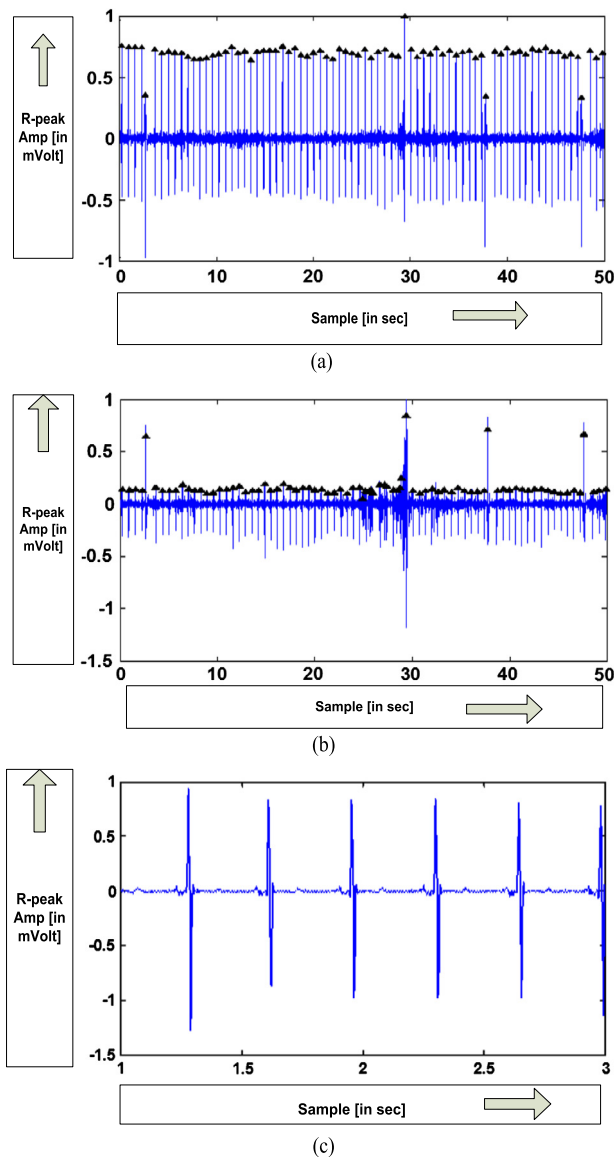
Proposed techniques have also been evaluated on the basis of Se, Acc and DER of the detected ECG beats. FrWT gave fruitful results among all three proposed methods - 34.37dB output SNR, 99.98% Se, 99.96% Acc and 0.036% DER. Table 3–5 has been represented the performance of R-peak detection. Proposed techniques have given percentage disturbance of 2%, 1.7% and 2.3% with FrFT, FrWT and IPCA respectively. Therefore, FrWT has provided better resolution, less percentage distortion, high Se, high Acc and low DER among other proposed techniques. These results indicate the quality of biology-related information retained from the pre-processed ECG signals for identifying different heart abnormalities.

Signals were also acquired using the low cost electrocardiogram device and have been investigated based on R-peaks. This recorded ECG signal is highly corrupted due to the involvement of various

types of noises. Fig. 10 (a)–(b) clearly indicate the detected R peak and Fig. 10 (c) shows a wider view of Fig. 10 (a). The proposed techniques are constructing to contribute superior act and ready to secure the important ECG signal attributes enough exceptional than the reported techniques till date.

#### 4. Conclusion

The FrFT is effective for analyzing the time-varying signals and represents it in time, frequency and intermediate time-frequency descriptions. It is not suitable for detecting FrFD frequency contents, which is an important aspect in pre-processing applications. It cannot provide local structures of the signal. Also, this method has not worked effectively when motion artifacts were noticed in



**Fig. 10.** R-peaks detection in ECG signal acquired using low cost electrocardiogram device.

the signal and resulted into overlapping of QRS complexes and motion artifact frequency components. FrWT exposed the local tendency of the signal and supported harmony with the signal which is under observation. Therefore, it is an effective technique for displaying signals at different resolutions by dilating and compressing its basis functions. The ECG signals which consist of many data points can be compressed into few features point by performing spectral analysis of the signals with the WT. These features characterize the behavior of the ECG signals. It outperforms all other techniques considered in this paper and holds both the analytical attributes of the actual ECG signal and alterations in the amplitudes of various ECG waveforms adequately. It also provides signal portrayals in the time-fractional-frequency plane with low computational complexity enabling their use practically for versatile applications. These techniques have been investigated on the basis of all recorded twenty two real time ECG records in normal and disease situations. The outcome obtained is likely to be efficacious in terms of high SNR and R-peaks of the ECG signals which are essential for the correct diagnosis of cardiovascular disease. It has been revealed that WT with FrFT can remove disturbances more efficiently as compared to other techniques. IPCA conducted PCA

and ICA operation in the super-Gaussian environment effectively. It adds the merits of both PCA and ICA. In high dimensional ECG recordings, IPCA gave better SNR than PCA or ICA. It facilitates the recognition and diagnosis of different heart abnormalities in an ECG signal. In this paper, FrWT has also been used for R-peaks detection of the ECG signal acquired using low cost electrocardiogram device. Therefore, strength of FrWT is also effective to filter highly corrupted ECG signals.

## 5. Future scope

Outcomes of proposed techniques cannot be compared with the existing results of the ECG signal denoising, since in the literature various researchers have worked on the physioNet data that comes in standard category. On the involvement of proposed pre-processing technique, ECG signal has proven its effectiveness more as a representative signal of cardiac physiology, which will be definitely helpful in detecting cardiac arrhythmias. In future, R-peak detection or any classification technique may be used to improve the cardiac arrhythmias detection result, so that effective diagnostic analysis report may be obtained. Additionally, obtaining the diagnostic report timely may help in lowering the casualties happening around the world.

## Human and animal rights

No animal data have been used in this research article.

## Disclosure of interest

No conflict of interest.

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