EE 370 L CONTROL SYSTEM LABORATORY

LABORATORY 3: STATE SPACE METHOD

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING
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1. OBJECTIVE

To demonstrate the concept of State Space method of a single-input single-output (SISO) system analysis using MATLAB. Note that, the method can also be used for multi-input multi-output (MIMO) system modeling. In the end, compare the results evaluated through transfer function and LTspice.

2. COMPONENTS & EQUIPMENT

PC with MATLAB and Simulink toolbox installed.

3. BACKGROUND

The state space model represents a physical system as ***n*** first-order coupled differential equations. The general vector-matrix form of the state space model is:

$$\dot{\vec{x}} = \frac{d\vec{x}}{dt} = A\vec{x} + Bu$$
$$y = C\vec{x} + Du \tag{1}$$

where $\vec{\boldsymbol{x}}$ is an n-by-1 vector representing the state (commonly position and/or velocity variable in mechanical systems), ***u*** is a scalar representing the input (commonly a force or torque), and ***y*** is a scalar representing the output. The matrices ***A*** is n-by-n, ***B*** is n-by-1 and ***C*** is 1-by-n, altogether determining the relationships between the state and input and output variable.

Its transfer function is

$$G(s) = Y(s)/U(s) = C(sI - A)^{-1}B + D \tag{2}$$

and the time response is as follows:

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau \tag{3}$$

We use the following RLC circuit as an example to illustrate the state space design method.

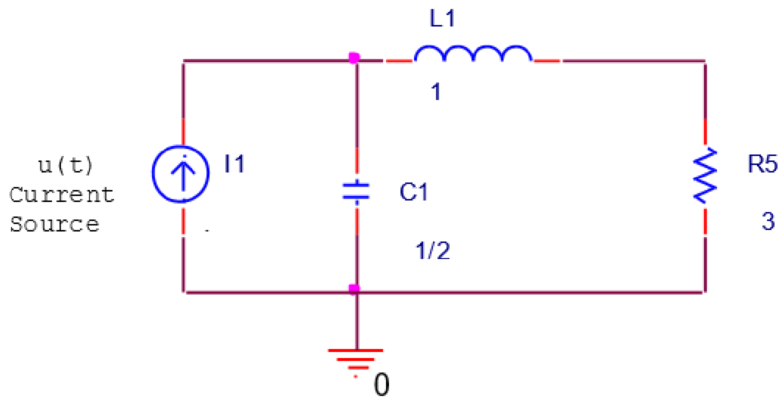


Figure 1. System 1.

From KCL & KVL:

$$\begin{aligned} i_c = C\frac{dV_c}{dt} &= u(t) - i_L \\ L\frac{di_L}{dt} &= -Ri_L + v_c \end{aligned} \tag{4}$$

The output of the system is $v_o = Ri_L$. As $x_1(t) = v_c(t)$ and $x_2(t) = i_L(t)$, we can get:

$$\begin{aligned} \frac{dx_1}{dt} &= \quad -\frac{1}{C}x_2 + \frac{1}{C}u(t) \\ \frac{dx_2}{dt} &= \frac{1}{L}x_1 - \frac{R}{L}x_2 \\ y(t) &= v_o(t) = Rx_2 \end{aligned} \tag{5}$$

Comparing Eq. (1) and Eq. (5), we have

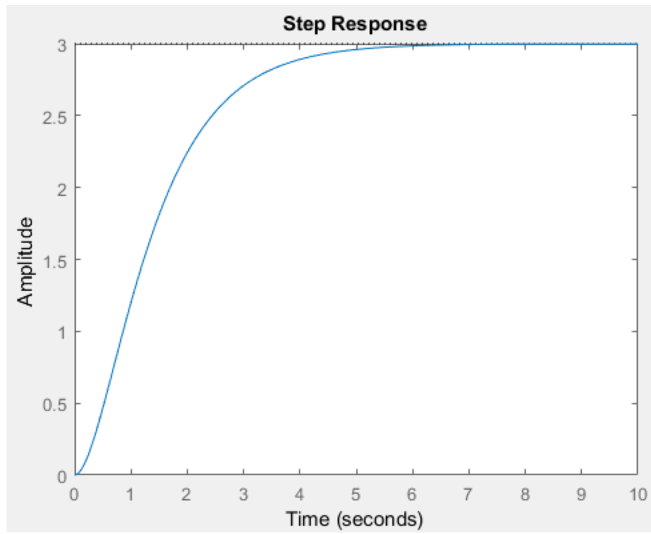
$$A = \begin{bmatrix} 0 & -1/C \\ 1/L & -R/L \end{bmatrix} \qquad B = \begin{bmatrix} 1/C \\ 0 \end{bmatrix} \qquad C = [0 \quad R] \qquad D = [0] \tag{6}$$

MATLAB Example: From the knowledge gained so far we can compute, step response and instability of a system characterized by a state space model. To demonstrate the example, you will need to enter the following in an m-file (left), and MATLAB will output the following plot of the unit step response (right).

MATLAB code example:

```
clear; clc;

A = [0, -2;
      1, -3];

B = [2;
      0];

C = [0, 3];

D = 0;

example_ss = ss(A, B, C, D);

step(example_ss);
axis([0 10 0 3]);
```

Results (Unit Step Response):



To find the stability of the system, we can locate the poles by computing the eigenvalue of A:

```
>> poles = eig(A)

poles =

    -1
    -2
```

4. LAB DELIVERIES

PRELAB:

1. **Review the knowledge of state space, and derive Eq. (2) from Eq. (1).**

2. **Run LTspice simulation on System 1, and compare the results from MATLAB.**

3. **Derive both state space and transfer functions for vout2 and vout3 in System 2, respectively.**

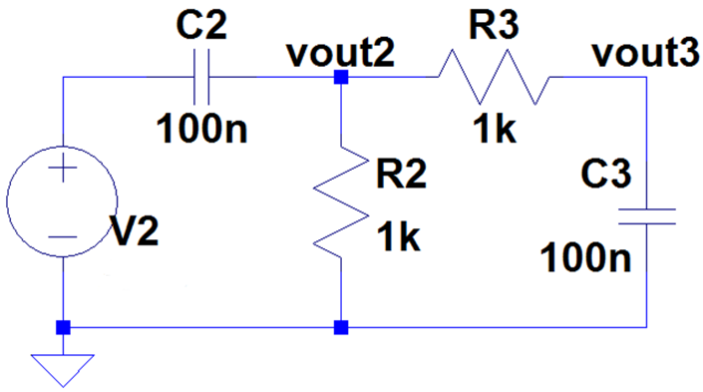


Figure 2. System 2

4. **Derive both state space and transfer functions for vo4 and vo5 in System 3, respectively.**

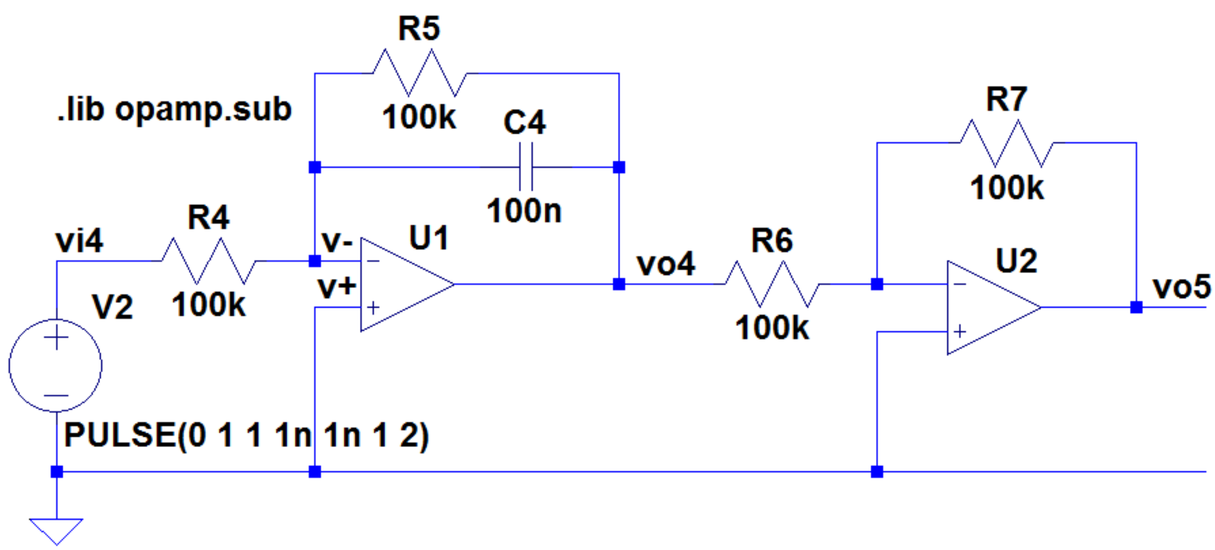


Figure 3. System 3

LAB EXPERIMENTS:

1. **Plot step and impulse response at both vout2 and vout3 of System 2, respectively.**
   - Use transfer function to plot the step and impulse responses for vout2 and vout3, respectively.
   - Use state space method to plot the step and impulse responses for both outputs, respectively.
   - Run the LTspice simulations and compare the results for all simulations.

2. **Plot step and impulse response of both vo4 and vo5 in System 3, respectively.**
    - Use transfer function to plot the step and impulse responses for vo4 and vo5, respectively.
    - Use state space method to plot the step and impulse responses for both outputs, respectively.
    - Run the LTspice simulations and compare the results for all simulations.

POSTLAB REPORT:

Include the following elements in the report document:

<table>
<tr><th>Section</th><th colspan="2">Element</th></tr>
<tr><td>1</td><td colspan="2">Theory of operation
Include a brief description of every element and phenomenon that appear during the experiments.</td></tr>
<tr><td>2</td><td colspan="2">Prelab report
1. Hand calculation results of prelab circuit (a) and (b).
2. LTspice schematics and simulation results of the two circuits, and their Bode plots.</td></tr>
<tr><td rowspan="4">3</td><td colspan="2">Results of the experiments</td></tr>
<tr><td>Experiments</td><td>Experiment Results</td></tr>
<tr><td>1</td><td>MATLAB code and simulation results Experiment 1.</td></tr>
<tr><td>2</td><td>MATLAB code and simulation results Experiment 2.</td></tr>
<tr><td rowspan="3">4</td><td colspan="2">Answer the questions</td></tr>
<tr><td>Questions</td><td>Questions</td></tr>
<tr><td>1</td><td>Question in Experiment 2.</td></tr>
<tr><td>5</td><td colspan="2">Conclusions
Write down your conclusions, things learned, problems encountered during the lab and how they were solved, etc.</td></tr>
<tr><td>6</td><td colspan="2">Images
Paste images (e.g. scratches, drafts, screenshots, photos, etc.) in Postlab report document (only .docx, .doc or .pdf format is accepted). If the sizes of images are too large, convert them to jpg/jpeg format first, and then paste them in the document.

Attachments (If needed)
Zip your projects. Send through WebCampus as attachments, or provide link to the zip file on Google Drive / Dropbox, etc.</td></tr>
</table>

5. REFERENCES & ACKNOWLEDGEMENT

1. Norman S. Nise, “Control Systems Engineering”, 7th Ed.

I appreciate the help from faculty members and TAs during the composing of this instruction manual. I would also thank students who provide valuable feedback so that we can offer better higher education to the students.