Symmetrical Components

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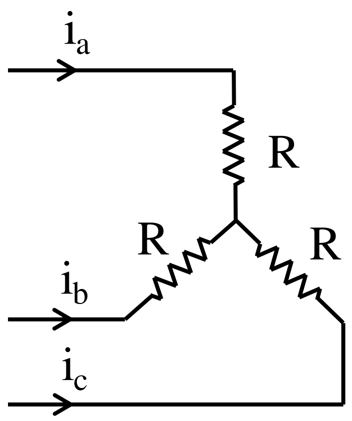
Analysis of Balanced and Unbalanced System

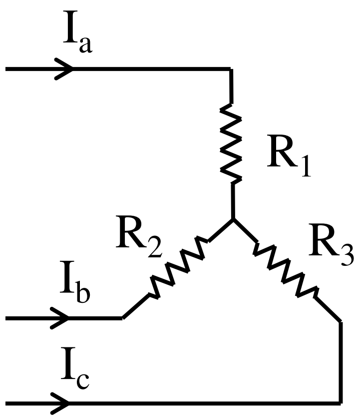
Balanced load



$$|i_a| = |i_b| = |i_c|$$
$$\bar{i}_a + \bar{i}_b + \bar{i}_c = 0$$

For balanced system, voltages and currents are balanced and per phase analysis is sufficient.

Unbalanced load



$$|I_a| \neq |I_b| \neq |I_c|$$

For unbalanced system, voltages and currents are not balanced and per phase analysis is not sufficient. 3-phase analysis has to be done.

Faults in Power System

Type of Fault	Probability
3-φ Fault - Balanced	5%
LLG - Unbalanced	10%
LL - Unbalanced	15%
LG - Unbalanced	70%

Symmetrical Components

- For balanced faults $|I_a| = |I_b| = |I_c|$. So, analysis can be made for one phase only and total real and reactive powers are simply 3 times the corresponding phase values.

- During unbalanced loading or unbalanced faults, analysis has to be carried out on 3-phase basis.

- Alternately a more convenient method of analyzing unbalanced operation is through symmetrical components where three voltages (and currents) are transformed into three sets of balanced voltages (and currents) called symmetrical components

The method of symmetrical components, first developed by **C.L. Fortescue** in 1918.

Analysis by symmetrical components is a powerful tool which makes the calculation of unsymmetrical faults almost as easy as the calculation of symmetrical three-phase faults.

Symmetrical Components…Fortescue's theorem

Fortescue's theorem

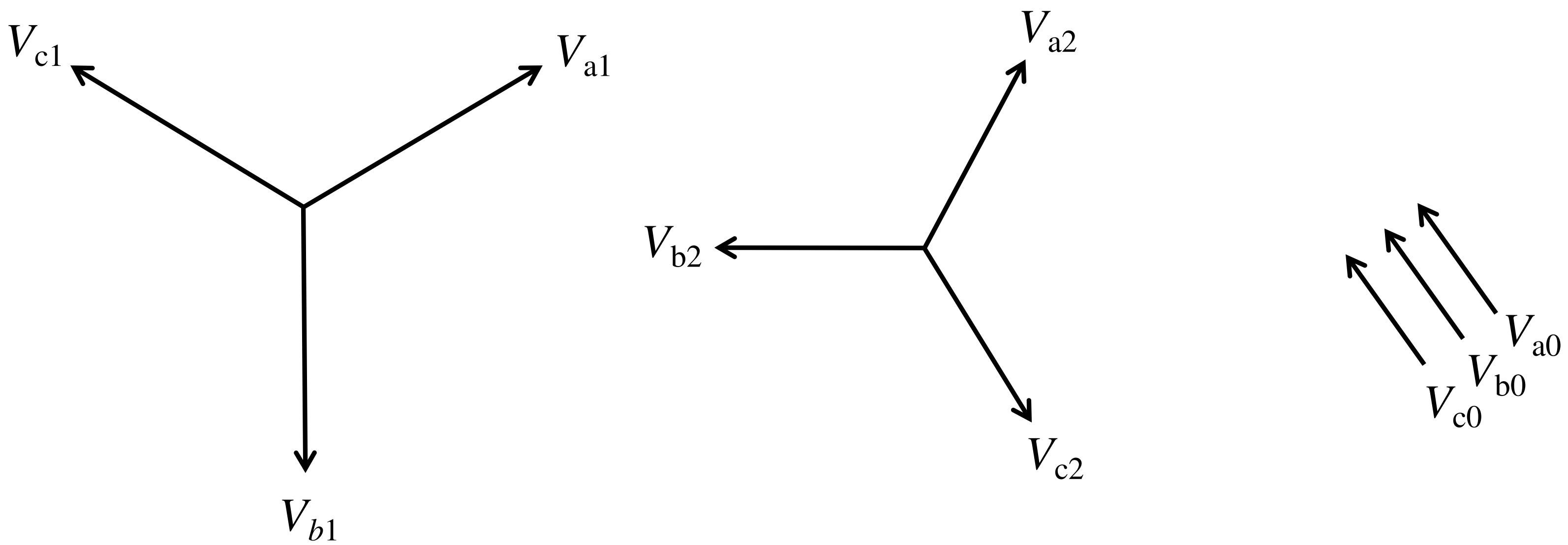
Three unbalanced phasors of a three-phase system can be resolved into three balanced systems of phasors . The balanced sets of components are:

1. ***Positive-sequence components*** consisting of three phasors equal in magnitude, displaced from each other by 120° in phase, and having the same phase sequence as the original phasors,

2. ***Negative-sequence components*** consisting of three phasors equal in magnitude, displaced from each other by 120° in phase, and having the phase sequence opposite to that of the original phasors, and

3. ***Zero-sequence components*** consisting of three phasors equal in magnitude and with zero phase displacement from each other.

Symmetrical Components…Fortescue's theorem



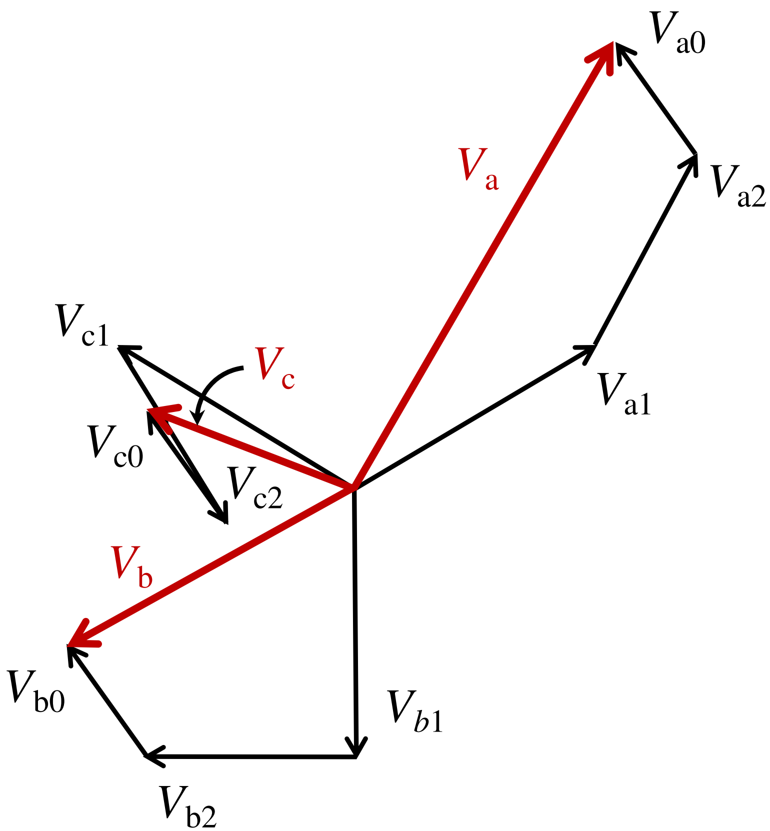
Positive-sequence components

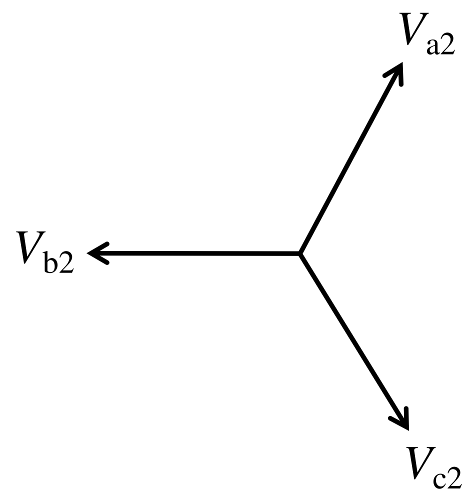
Negative-sequence components

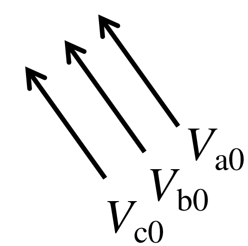
Zero-sequence components

Fig. 3-1 Three sets of balanced phasors which are the symmetrical components of three unbalanced phasors

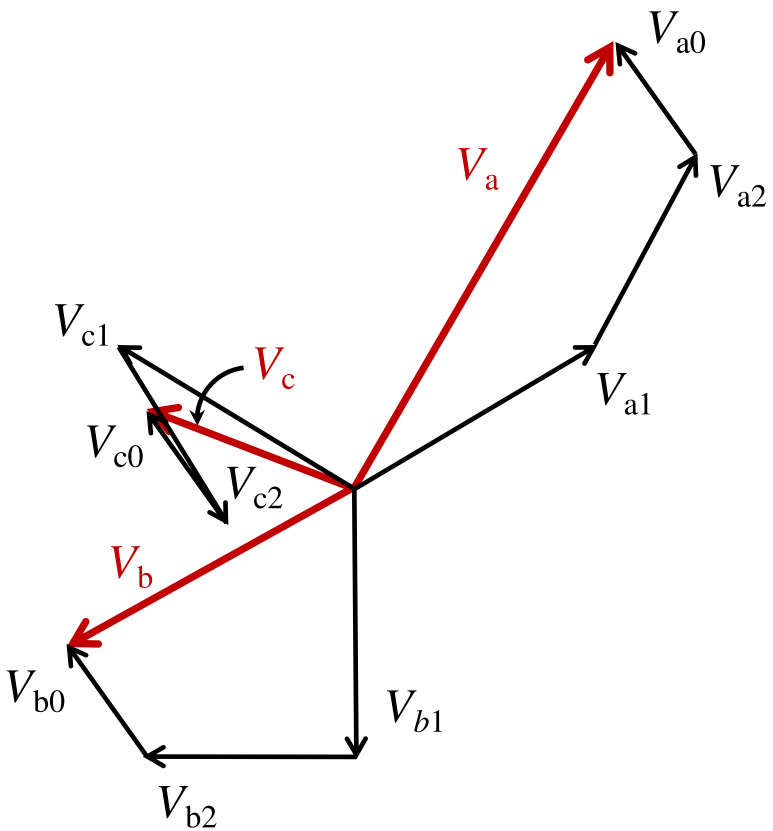
Symmetrical Components…Fortescue's theorem







Symmetrical Components…Fortescue's theorem



Symmetrical Components…Fortescue's theorem

The original phasors expressed in terms of their components are:

$$V_a = V_{a0} + V_{a1} + V_{a2} \text{ ....(3.1)}$$

$$V_b = V_{b0} + V_{b1} + V_{b2} \text{ ....(3.2)}$$

$$V_c = V_{c0} + V_{c1} + V_{c2} \text{ ....(3.3)}$$

Operators

Shorthand method of indicating the rotation of a phasor through 120°.

The letter a is commonly used to designate the operator that causes a rotation of 120° in the counterclockwise direction.

$$a = 1\angle 120° = 1\varepsilon^{j2\pi/3} = -0.5 + j0.866$$

$$a^2 = 1\angle 240° = -0.5 - j0.866$$

$$a^3 = 1\angle 360° = 1\angle 0° = 1$$

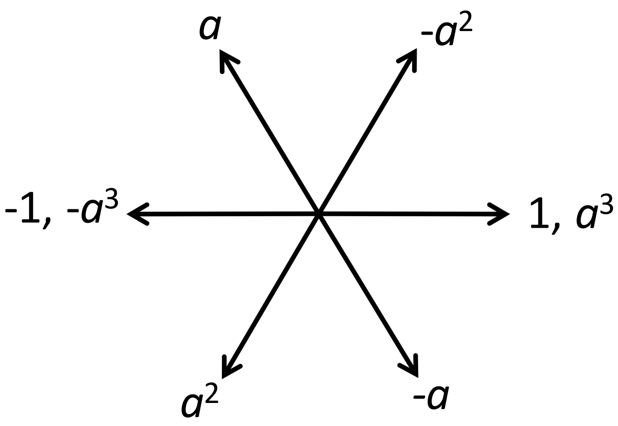
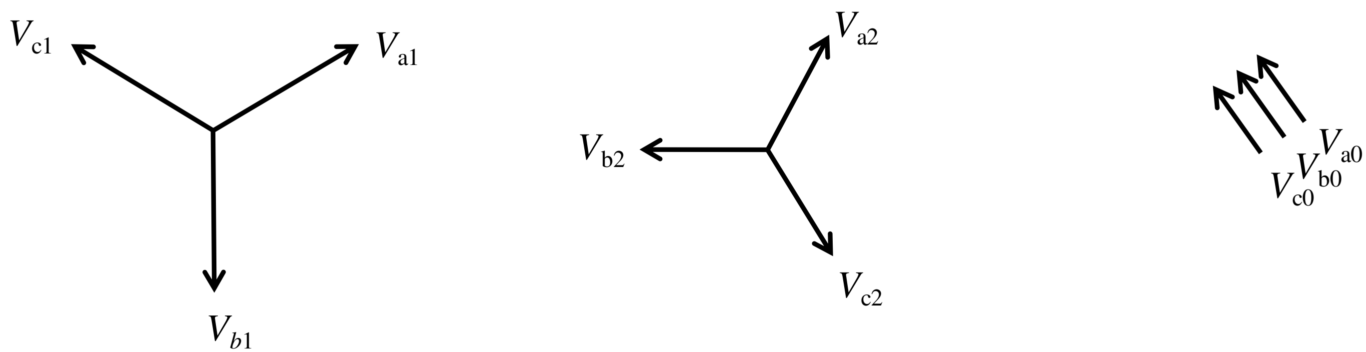


Fig. 3-12 Phasor diagram of the various powers of the operator a

The Symmetrical Components of Unsymmetrical Phasors

Resolve three unsymmetrical phasors into their symmetrical components



$$V_{b1} = a^2 V_{a1} \qquad V_{c1} = aV_{a1}$$

$$V_{b2} = aV_{a2} \qquad V_{c2} = a^2 V_{a2} \quad \text{.....................}(3.4)$$

$$V_{b0} = V_{a0} \qquad V_{c0} = V_{a0}$$

The Symmetrical Components of Unsymmetrical Phasors

From equations (3.1) – (3.4)

$$V_a = V_{a0} + V_{a1} + V_{a2} \qquad \text{..................}(3.5)$$

$$V_b = V_{a0} + a^2 V_{a1} + aV_{a2} \text{ ....}(3.6)$$

$$V_c = V_{a0} + aV_{a1} + a^2 V_{a2} \text{ ....}(3.7)$$

In matrix form

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \text{...............}(3.8)$$

The Symmetrical Components of Unsymmetrical Phasors

Let

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \text{..............................}(3.9)$$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \text{..............................}(3.10)$$

Premultiplying both sides of eq. (3.8) by A^{-1}

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \text{...............}(3.11)$$

The Symmetrical Components of Unsymmetrical Phasors

From eq. (3.11)

$$V_{a0} = \frac{1}{3}\left(V_a + V_b + V_c\right) \text{.......................} (3.12)$$

$$V_{a1} = \frac{1}{3}\left(V_a + aV_b + a^2V_c\right) \text{.................} (3.13)$$

$$V_{a2} = \frac{1}{3}\left(V_a + a^2V_b + aV_c\right) \text{.................} (3.14)$$

If required, the components V_{b0}, V_{b1}, V_{b2}, V_{c0}, V_{c1} and V_{c2} can also be found.

The Symmetrical Components of Unsymmetrical Phasors

$$V_{a0} = \frac{1}{3}\left(V_a + V_b + V_c\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.12)$$

Eq. (3.12) shows that no zero sequence components exist if the sum of the unbalanced phasors is zero. Since the sum of the line-to-line voltage phasors in a three-phase system is always zero, **zero-sequence components are never present in the line voltages**, regardless of the amount of unbalance.

The sum of the three line-to-neutral voltage phasors is not necessarily zero, and **voltages to neutral may contain zero-sequence components**.

The Symmetrical Components of Unsymmetrical Phasors

The equations for currents can be written as

$$I_a = I_{a0} + I_{a1} + I_{a2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.15)$$

$$I_b = I_{a0} + a^2 I_{a1} + a I_{a2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.16)$$

$$I_c = I_{a0} + a I_{a1} + a^2 I_{a2} \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.17)$$

$$I_{a0} = \frac{1}{3}\left(I_a + I_b + I_c\right) \ldots\ldots\ldots\ldots\ldots\ldots(3.18)$$

$$I_{a1} = \frac{1}{3}\left(I_a + a I_b + a^2 I_c\right) \ldots\ldots\ldots\ldots\ldots\ldots(3.19)$$

$$I_{a2} = \frac{1}{3}\left(I_a + a^2 I_b + a I_c\right) \ldots\ldots\ldots\ldots\ldots\ldots(3.20)$$

The Symmetrical Components of Unsymmetrical Phasors

In a three-phase Y-connected system, the neutral current I_n is the sum of the line currents:

$$I_a + I_b + I_c = I_n \text{ ....}(3.21)$$

Comparing eqs. (3.18) and (3.21) gives

$$I_{a0} = \frac{1}{3}\left(I_a + I_b + I_c\right) \text{.............}(3.18)$$

$$I_n = 3I_{a0} \text{ ....}(3.22)$$

- In the absence of a path through the neutral of a three-phase system, I_n is zero, and the line currents contain no zero-sequence components.
- A Δ-connected load provides no path to neutral, and line currents flowing to a Δ-connected load contain no zero-sequence components

The Symmetrical Components of Unsymmetrical Phasors

Example:

The line to ground voltages on the high voltage side of a step up transformer are 100 kV, 35 kV & 40 kV on phases *a*, *b* and *c* respectively. The voltage of phase *a* leads *b* 90° and lags that of phase *c* by 180°. Determine the symmetrical components of voltage.

Solution:

$V_a = 100\angle 0°$ $\qquad$ $V_b = 35\angle\text{-}90°$ $\qquad$ $V_c = 40\angle 180°$

$a = 1\angle 120°$ $\qquad$ $a^2 = 1\angle -120° = 1\angle 240°$

$$V_{a1} = \frac{1}{3}\left(V_a + aV_b + a^2V_c\right)$$

$$= \frac{1}{3}\left(100\angle 0° + 35\angle -90°\angle 120° + 40\angle 180°\angle -120°\right)$$

$$= \frac{1}{3}\left(100\angle 0° + 35\angle 30° + 40\angle 60°\right)$$

$$= \frac{1}{3}\left(100 + j0 + 30.31 + j17.5 + 20 + j14.641\right)$$

$$= 50.1 + j17.38$$

The Symmetrical Components of Unsymmetrical Phasors

Example:

The line to ground voltages on the high voltage side of a step up transformer are 100 kV, 35 kV & 40 kV on phases *a*, *b* and *c* respectively. The voltage of phase *a* leads *b* 90° and lags that of phase *c* by 180°. Determine the symmetrical components of voltage.

Solution:

$$a = 1\angle 120^\circ \qquad a^2 = 1\angle -120^\circ = 1\angle 240^\circ$$

$$V_{a2} = \frac{1}{3}\left(V_a + a^2 V_b + aV_c\right)$$

$$= \frac{1}{3}\left(100\angle 0^\circ + 35\angle -90^\circ \angle 240^\circ + 40\angle 180^\circ \angle 120^\circ\right)$$

$$= \frac{1}{3}\left(100\angle 0^\circ + 35\angle 150^\circ + 40\angle 300^\circ\right)$$

$$= \frac{1}{3}\left(100 + j0 + (-30.31) + j17.5 + 20 - j34.64\right)$$

$$= 29.9 - j5.71$$

The Symmetrical Components of Unsymmetrical Phasors

Example:

The line to ground voltages on the high voltage side of a step up transformer are 100 kV, 35 kV & 40 kV on phases *a*, *b* and *c* respectively. The voltage of phase *a* leads *b* 90° and lags that of phase *c* by 180°. Determine the symmetrical components of voltage.

Solution:

$$V_{a0} = \frac{1}{3}\left(V_a + V_b + V_c\right)$$

$$= \frac{1}{3}\left(100 + j0 + 0 - j15 - 40 + j0\right)$$

$$= 20 - j11.67$$

The Symmetrical Components of Unsymmetrical Phasors

EXAMPLE 8.1 Sequence components: balanced line-to-neutral voltages

Calculate the sequence components of the following balanced line-to-neutral voltages with *abc* sequence:

$$V_p = \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} 277\underline{/0^\circ} \\ 277\underline{/-120^\circ} \\ 277\underline{/+120^\circ} \end{bmatrix} \quad \text{volts}$$

EXAMPLE 8.2 Sequence components: balanced *acb* currents

A Y-connected load has balanced currents with *acb* sequence given by

$$I_p = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 10\underline{/0^\circ} \\ 10\underline{/+120^\circ} \\ 10\underline{/-120^\circ} \end{bmatrix} \quad \text{A}$$

Calculate the sequence currents.

The Symmetrical Components of Unsymmetrical Phasors

EXAMPLE 8.3 Sequence components: unbalanced currents

A three-phase line feeding a balanced-Y load has one of its phases (phase *b*) open. The load neutral is grounded, and the unbalanced line currents are

$$I_p = \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 10\angle 0^\circ \\ 0 \\ 10\angle 120^\circ \end{bmatrix} \quad \text{A}$$

Calculate the sequence currents and the neutral current.

Book: Power System Analysis and Design
by: Glover and Sarma

3-Phase Power In Terms of Symmetrical Components

The total complex power flowing into a three-phase circuit through three lines a, b and c is

$$S = P + jQ = V_a I_a^* + V_b I_b^* + V_c I_c^*$$

$$= \begin{bmatrix} V_a & V_b & V_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$

$$= V_p^T I_p^*$$

Now, $\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}$ and $\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$

or, $V_p = AV_s$ and $I_p = AI_s$

3-Phase Power In Terms of Symmetrical Components

$$S = V_p^T I_p^* = (AV_s)^T (AI_s)^*$$
$$= V_s^T A^T A^* I_s^*$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$S = 3V_s^T U I_s^* = 3V_s^T I_s^*$$
$$= 3\begin{bmatrix} V_{a0} & V_{a1} & V_{a2} \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$
$$= 3V_{a0} I_{a0}^* + 3V_{a1} I_{a1}^* + 3V_{a2} I_{a2}^*$$
$$= 3V_0 I_0^* + 3V_1 I_1^* + 3V_2 I_2^*$$

$$A^T A^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$
$$= \begin{bmatrix} 1+1+1 & 1+a+a^2 & 1+a^2+a \\ 1+a^2+a & 1+a^3+a^3 & 1+a^4+a^2 \\ 1+a+a^2 & 1+a^2+a^4 & 1+a^3+a^3 \end{bmatrix}$$
$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 3U$$

Thank You