

## Servo Closed Loop Speed Control – Steady State Characteristics

**EXERCISE OBJECTIVE** When you have completed this exercise, you will be familiar with servo operation in closed loop speed control. You will know how to calculate and measure the steady state speed of the Digital Servo in closed loop speed control for various controller gains both theoretically and experimentally and be able to compare the two.

**DISCUSSION OUTLINE** The Discussion of this exercise covers the following points:

- Components of the Digital Servo operating under closed loop speed control
- Sensor and power amplifier gain

### DISCUSSION

#### Components of the Digital Servo operating under closed loop speed control

The Digital Servo closed loop speed-control system consists of the following.

- The dc brush-type servo motor.
- A speed sensor, i.e., an incremental encoder directly coupled to the motor shaft.
- The system controller.
- The human machine interface (HMI), which is used for setting the controller parameters, function generator and recorder functions.

From now on, we will refer to the motor steady state speed constant  $K_s$  as the general motor speed constant  $K$  for simplification.

Figure 25 shows a simplified block diagram of the servo closed loop speed-control system with a first-order motor model (developed in Exercise 2). The controller is proportional only, which means that it can only have a constant gain term (proportional action is discussed in more detail in Exercise 9).

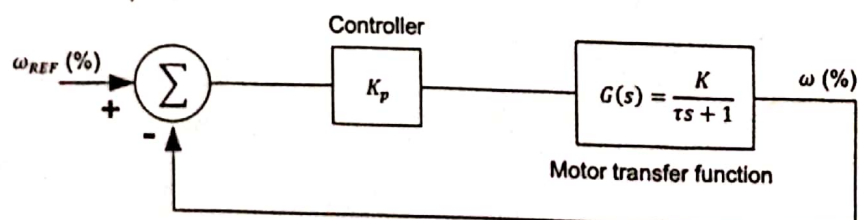


Figure 25. Block diagram of a servo motor in closed loop speed-control mode.

The controller gain  $K_p$  is the result of the PID controller three different gains: the proportional gain  $K_p$ , the integral gain  $K_i$  and the derivative gain  $K_d$ . In most exercises in this manual, the controller gain is equivalent to the proportional gain  $K_p$ , as only proportional action is present in the system.

Analysis of the block diagram components in Figure 25 (see Appendix B for the detailed analysis), shows that the steady state speed  $\omega_{ss}$  of the closed loop system can be given as follows:

$$\omega_{ss} = \left( \frac{K_c K}{1 + K_c K} \right) \omega_{REF} \quad (26)$$

where  $\omega_{ss}$  is the motor steady state speed (controlled or process variable)  
 $\omega_{REF}$  is the desired or reference motor speed (set point)  
 $K$  is the motor speed constant [(rad/s)/V]  
 $K_c$  is the controller gain (adjustable)

Equation (26) shows that as the controller gain (proportional only)  $K_c$  increases, the  $(K_c K)/(1 + K_c K)$  ratio approaches 1. That is, with higher gain, the motor steady state speed  $\omega_{ss}$  approaches the desired or reference speed  $\omega_{REF}$ . The difference between the reference speed and the actual speed ( $\omega_{REF} - \omega$ ) is referred to as the error (or offset). Thus, increasing the proportional gain decreases the error. This means that, theoretically, the proportional gain could be set to a very large value in order to minimize the error. In practice, however, increasing the proportional gain renders the servo system unstable and produces speed changes and oscillations. This is discussed in more details in Exercise 5.

### Sensor and power amplifier gain

In real servo system implementation, the analysis must consider the gains of both the servo system power amplifier and the speed sensor. The reference speed  $\omega_{REF}$  for the servo system as well as the controlled variable speed  $\omega_{ss}$  is often expressed in percentage, as is the case for the Digital Servo controller. The conversion between percentage and speed must be taken into account and can be seen as another gain term. All gain terms can then be grouped as one single term by multiplying them together.

The following gain terms are determined for the Digital Servo:

- A 100% output from the controller output is equivalent to 48 V. The gain for converting percentage output is thus 0.48 V/%.
- The power amplifier gain is of approximately 0.91. A 100% output from the controller thus results in only 0.91 x 48 V being applied to the dc motor. The power amplifier gain is due to the motor output electronic design. This means that it cannot output more than 91% of its entry value of 48 V dc.
- The conversion of rad/s to rpm can be represented as a gain term of  $\frac{30}{\pi}$  rpm/(rad/s).

- A 100% speed is equal for the Digital Servo to 3000 rpm. The gain term for converting rpm to percentage is thus (1/30)%/rpm.

The block diagram in Figure 26 shows all of these gain terms:

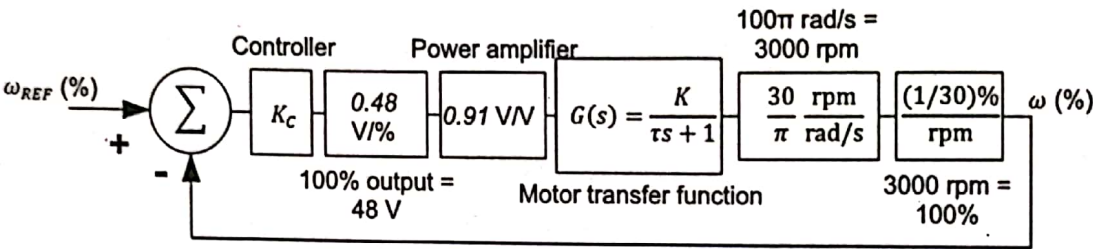


Figure 26. Block diagram of a servo motor in closed loop speed-control mode showing all gain terms.

All the gain terms in Figure 26 can be grouped together as a total product of all terms. In this case, the product is  $0.139 \text{ V/(\text{rad/s})}$  ( $0.48 \times 0.91 \times 30/\pi \times 1/30$ ). This gain will be referred to as a **scaling factor**. A block diagram that shows the grouping of these gains is given in Figure 27.

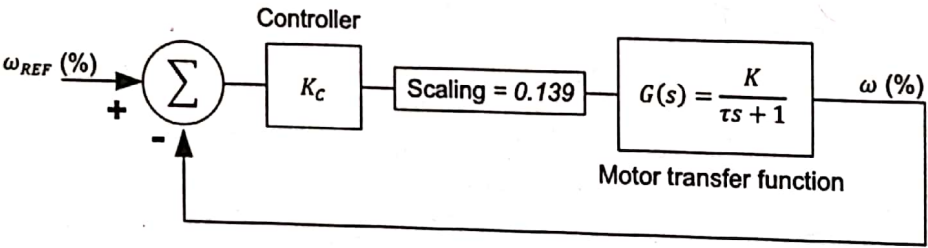


Figure 27. Block diagram of a servo motor in closed loop speed-control mode showing the simplified gain term.

From the above, it can be seen that for this particular system, Equation (26) has to be modified to the following:

$$\omega_{SS} = \left( \frac{0.139 K_C K}{1 + 0.139 K_C K} \right) \omega_{REF} \tag{27}$$



## PROCEDURE OUTLINE

The Procedure is divided into the following sections:

- Setup and connections
- Closed loop speed-control measurements

## PROCEDURE

### Setup and connections

In this section, you will setup the Digital Servo for closed-loop speed-control measurements.

1. Make the following settings on the Digital Servo system:

- Setup the servo system for speed control, i.e., disengage the platform.
- Set the belt tension to allow the belt to be lifted of the pulley connected to the motor shaft and slipped on the two pins to the rear of the pulley, allowing the shaft to run uncoupled from the belt.
- Secure the flywheel to the shaft using the appropriate hex key.

2. Run LVServo, and click on the **Device Controlled** button in the **Speed Loop** menu. Make sure the settings are initially as shown in Table 12:

Table 12. Settings for closed loop speed-control measurements.

Function Generator		Trend Recorder	
Signal Type	Constant	Reference	Checked
Frequency	1 Hz	Speed	Checked
Amplitude	0%	Current	Unchecked
Offset	0%	Voltage	Checked
Power	Off	Error	Unchecked
<b>PID Controller</b>		$K_p \times \text{Error}$	Unchecked
Gain ( $K_p$ )	1	Error Sum / $t_i$	Unchecked
Integral Time ( $t_i$ )	Inf (Off)	$t_d \times \text{Delta Error}$	Unchecked
Derivative Time on E ( $t_d$ (E))	0	PID Output	Unchecked
Derivative Time on PV ( $t_d$ (PV))	0	Display Type	Sweep
Timebase	10 ms	Show and Record Data	On
Anti-Reset Windup	On	Measured Gain (rpm)	3000
Upper Limit	100%	Measured Gain (A)	7
Lower Limit	-100%	Measured Gain (V)	48
Open or Closed Loop	Closed		
<b>PV Speed Scaling</b>			
100% Value	3000 rpm		

3. Set the function generator **Power** switch to ON.

### Closed loop speed-control measurements

In this section, you will calculate the steady state speed  $\omega_{SS}$  and the speed constant  $K$  value of the motor operating under closed loop speed control for various gains using Equation (27). You will then measure experimentally the motor  $\omega_{SS}$  and  $K$  values for various gains and compare the theoretical and experimental results. You will eliminate the calculated error value by means of integral action. Only proportional action will be used. The controller gain value is thus equal to the proportional gain value and will be referred to as  $K_p$ .

4. Slowly increase the offset value until the motor voltage reading reaches 40%, which corresponds to a voltage of 19.2 V. Record the actual speed  $\omega$  (rpm) and the reference speed  $\omega_{REF}$  (%) in Table 13:

Table 13. Speed readings during closed loop speed-control measurements.

Gain	Voltage		Reference Speed		Actual Speed		Error	Speed/Voltage Ratio		Steady State Speed Ratio	Steady State Speed
$K_C$	$E$	$E$	$\omega_{REF}$	$\omega_{REF}$	$\omega$	$\omega$	$\omega$	$\Delta\omega = \omega_{REF} - \omega$	$K = \omega/E$	$\omega_{SS}/\omega_{REF}$	$\omega_{SS}$
	%	V	%	rad/s	rpm	rad/s	%	%	(rad/s)/V		%
1	40	19.2									
2	40	19.2									
3	40	19.2									
4	40	19.2									
5	40	19.2									

5. Decrease the offset to 0%, increase the proportional gain  $K_p$  to 2 and repeat the previous operation. Do the same thing for  $K_p$  values of 3, 4, and 5.
6. Fill out the rest of Table 13 using Table 14 as a quick reference for speed unit conversion. Keep in mind the following while completing Table 13:
- $K$  is the ratio of speed  $\omega$  to supply voltage  $E$  and is calculated by dividing the speed value in rad/s by the supply voltage (V).
  - The error value is calculated by subtracting the speed  $\omega$  value to the reference speed  $\omega_{REF}$ .
  - The steady state speed ratio  $\omega_{SS}/\omega_{REF}$  and speed value  $\omega_{SS}$  (%) are calculated using Equation (27).



Table 14. Speed unit conversion quick reference.

Speed unit type	Multiply by
rpm $\rightarrow$ rad/s	$\frac{\pi \text{ rad/s}}{30 \text{ rpm}}$
rad/s $\rightarrow$ rpm	$\frac{30 \text{ rpm}}{\pi \text{ rad/s}}$
% $\rightarrow$ rad/s	$\pi \text{ rad/s}$
rad/s $\rightarrow$ %	$\frac{1}{\pi \text{ rad/s}}$

7. Compare the calculated steady state speed with the measured steady state speed.  
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8. Describe what happens to the error as the proportional gain  $K_p$  value increases.  
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9. Set the proportional gain  $K_p$  back to 1 and enter 0.1 s into the integral time  $t_i$ . Describe what happens to the error when integral action is introduced into the controller.  
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## CONCLUSION

In this exercise, you familiarized yourself with servo system operation in closed loop speed control. You learned how to calculate and measure the steady state speed of the Digital Servo in closed loop speed control. You also learned to calculate the error value between the reference speed and the actual speed and how to minimize it by increasing the controller gain.

## REVIEW QUESTIONS

1. Consider a dc motor system having a supply voltage  $E$  of 35 V. The dc motor system proportional gain  $K_p$  is set to 3 and its steady state motor speed  $\omega_{ss}$  is 1960 rpm. Find the motor system speed constant  $K$  (steady state speed to voltage ratio):  
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2. Given the same motor parameters as in question 1, find the motor reference speed  $\omega_{REF}$  in rad/s, rpm, and percentage, as well as the steady state closed loop system value ( $\omega_{SS}/\omega_{REF}$ ).

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3. Given the same motor parameters as in question 1, find the system error value in percentage.

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4. Given the same motor parameters as in question 1, calculate what happens to the reference speed  $\omega_{REF}$  (rad/s, rpm, and percentage) when the proportional gain  $K_p$  value is set to 4, as well as the resulting steady state closed loop system value ( $\omega_{SS}/\omega_{REF}$ ).

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5. Given the same motor parameters as in question 4, find the system error value in percentage.

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6. Compare both calculated error values from question 3 and 5. Which one is lower and why?

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