Linear Position Control

EXERCISE OBJECTIVE When you have completed this exercise, you will be familiar with the Digital Servo PID controller operation and the effects of the proportional, integral, and derivative terms on the transient operation of a servo positioning system used for linear position control. You will measure the effects of damping on the step response of a servo system in linear position control. You will know the effects of incremental encoder resolution and platform load on the tuning parameters of a servo system and on its transient behavior in linear position control.

DISCUSSION OUTLINE The Discussion of this exercise covers the following points:

- Linear position control block diagram and fundamentals
- Proportional, integral, and derivative action on a linear position control system

DISCUSSION

Linear position control block diagram and fundamentals

Figure 43 shows the block diagram model of the Digital Servo system in linear position control. As already mentioned, the Digital Servo controller is a proportional, integral, derivative controller. Derivative action can be implemented on either the system error value or on the process which, for the Digital Servo, is the platform position. On the diagram, only the error signal is shown as an input to the PID. Notice that the encoder gain of the rail incremental encoder is equal to 14.5 s^{-1}. The difference in inertia J and viscous friction coefficient B of the linear position system compared to the angular position system results in a different first order time constant.

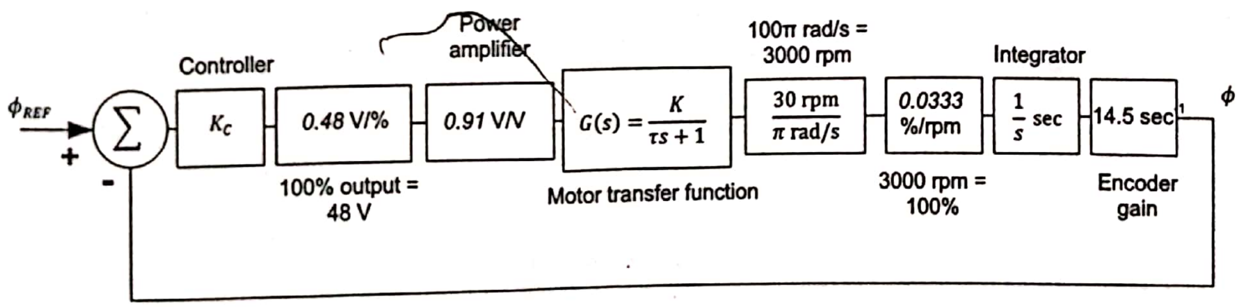

Figure 43. Block diagram of a position-control servo system with rail incremental encoder.

The encoder gain of 14.5 s^{-1} is calculated as follows:

At 3000 rpm (314 rad/s or 100% motor speed) the encoder generates 5000 counts (3.47 revolutions) in a time of:

$$t = \frac{3.47 \times 2\pi \text{ rad}}{100\pi \text{ rad/s}} = 0.069 \text{ s}$$

0.069 s thus represents the time that is needed to integrate, i.e., to accumulate, a count of 5000 at 100% motor speed. This time value is the encoder gain reciprocal. The encoder gain is thus equal to 1/0.069 s or 14.5 s^{-1}.

As was discussed in Exercise 7 using proportional only control, a servo positioning step response can result in a static friction error value. It should be noted that the positioning system (motor plus encoder) has a natural integrating term. Thus, if the system had no static friction, it would yield an error value of 0 (ideal system). In an actual system, however, the steady state error value is significant, but can be reduced by increasing the proportional gain K_p. However, an excessive proportional gain can cause the servo positioning system to become unstable. The addition of integral action can eliminate this error but also tends to cause the step response to become more oscillatory. The addition of derivative action reduces or compensates for this oscillatory tendency. Finding the most balanced settings for all three parameters is referred to as tuning the loop.

Figure 44 displays the Digital Servo system step response to a 20% step using P, PI, and PID control.

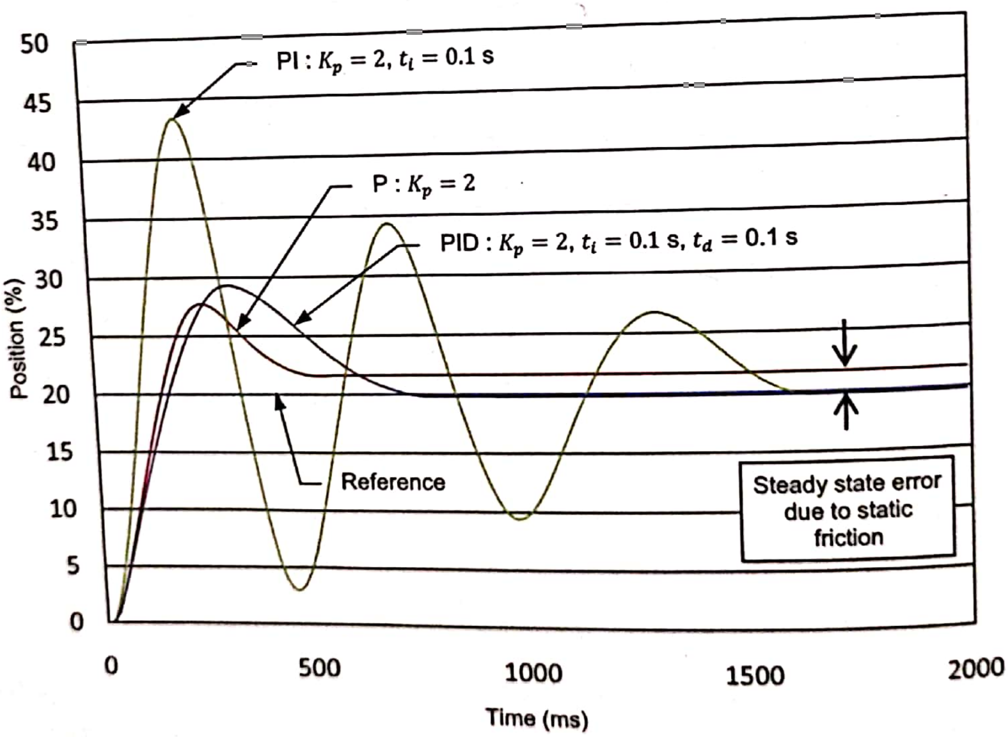

Figure 44. P, PI, and PID response to a 20% step change.

The PID controller used in the Digital Servo system is represented in Figure 45. As illustrated, derivative action can be performed either on the error value or on the process negative. Performing derivative on the process negative eliminates the impulse associated with derivative of error during position reference step transitions.

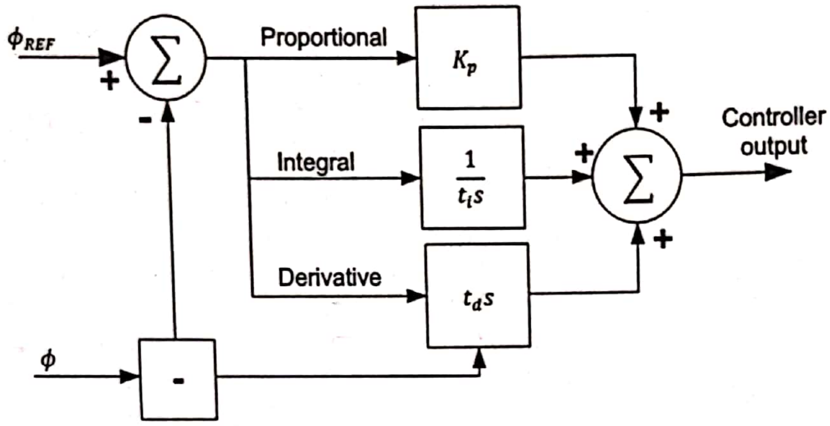

Figure 45. Expanded PID controller block diagram.

Proportional, integral, and derivative action on a linear position control system

The PID equation used in the Digital Servo system can be expressed in the time domain as Equation (63):

$$C_{OUT} = K_p e + \frac{1}{t_i}\int e dt + t_d \frac{de}{dt} \tag{63}$$

where C_{OUT} is the controller output
e is the error value (%)

Equation (63) can also be expressed with the derivative component acting on the process variable PV instead of on the error value, as shown below:

$$C_{OUT} = K_p e + \frac{1}{t_i}\int e dt + t_d \frac{dPV}{dt} \tag{64}$$

where PV is the process variable

Figure 46 displays the Digital Servo PID controller output. The plots were generated by disengaging the platform from the motor using the **Position Loop (Device Controlled)** with the rail incremental encoder and the square wave generator. This created a constant error of ±25% with the resulting plotted waveforms. The following parameters were used: proportional gain $K_p = 2$, integral time $t_i = 0.5$ s, derivative time $t_d = 0.01$ s.

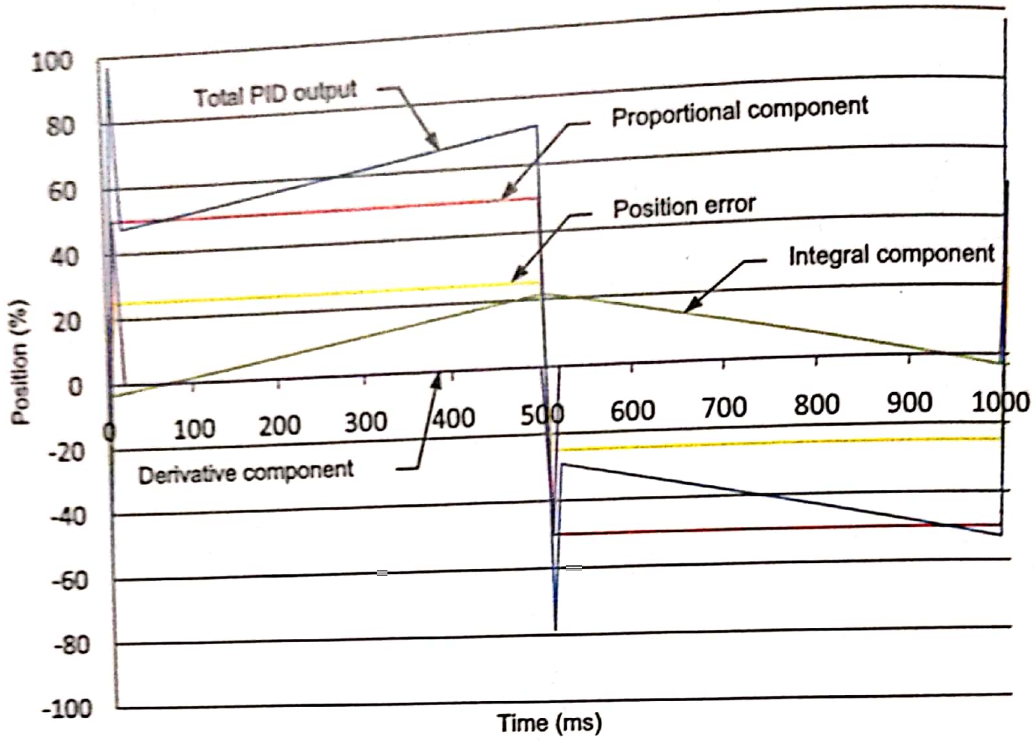

Figure 46. Digital Servo PID controller output with a 25%-constant error step change.

Figure 46 shows that the proportional component varies of ±50% This value can be calculated using the following equation:

$$P_{OUT} = K_p \cdot e_{PEAK} \tag{65}$$

where P_{OUT} is the peak proportional output (%)
e_{PEAK} is the peak error (%)

The Digital Servo proportional output for the plot in Figure 46 is thus equal to:

$$P_{OUT} = 2 \times 25 = 50\%$$

The integral component shows a rise from about –3% to 22%. This total rise of 25% can be calculated using the following equation:

$$I_{OUT} = \frac{1}{t_i} \int_0^{t_0} e dt \tag{66}$$

where I_{OUT} is the integral output (%)
t_i is the integral time (s)
e is the error value (%)

The integral action gain value K_i is equal to $1/t_i$. The resulting equation is shown below. This form of equation is given as reference only, as the Digital Servo uses the integral time t_i instead of the integral gain K_i as the integral tuning parameter.

$$I_{OUT} = K_I \int_0^{t_0} e\,dt \tag{67}$$

The Digital Servo integral output for the plot in Figure 46 is thus equal to:

$$I_{OUT} = \frac{1}{0.5}\int_0^{0.5} 25dt = 25\%$$

The derivative component is represented by a sharp spike occurring at step transitions. The rise shown by the spike is of 50% and it can be calculated using the following equation:

$$D_{OUT} = t_d \frac{de}{dt} \tag{68}$$

where D_{OUT} is the derivative output (%)
t_d is the derivative time (s)
e is the error value (%)

The derivative action gain value K_d is equal to t_d. The resulting equation is shown below. This form of equation is given as reference only, as the Digital Servo uses the derivative time t_d instead of the derivative gain K_d as the derivative tuning parameter.

$$D_{OUT} = K_d \frac{de}{dt} \tag{69}$$

The Digital Servo derivative peak value for the plot in Figure 46 is thus equal to:

$$D_{OUT} = 0.01\ \text{s} \times \frac{50\%}{0.01\ \text{s}} = 50\%$$

When the error is constant, the derivative output is equal to 0. If the derivative component had been set on the process variable PV instead of on the error value, the spikes at step changes would have been eliminated.

The effects of proportional, integral and derivative gain on a servo system in linear position control can be summarized by the following general observations. Notice that they are similar to the observations made for angular position control.

- Increasing the proportional gain K_p causes the positional step response of a significantly damped system to become critically damped. If the proportional gain is increased further, the critically damped system will become underdamped. As the gain increases, an oscillatory component that increases in frequency and amplitude will appear and eventually cause the system to become unstable.

- Adding integral action eliminates static friction error, but also increases the tendency of the step response to oscillate and can cause the system to become unstable.
- Adding derivative action can dampen or suppress an oscillating step response. Derivative action, however, is very sensitive to noise and can cause erratic behaviors when set at high values.
- Implementing derivative action on the process rather than on the error value eliminates the large spike due to sudden errors during position reference step transitions.
- The encoder has a gain that affects the tuning constants. An encoder with a higher gain results in a lower proportional gain, a higher integral output, and a lower derivative output for a given step response.
- Increasing the platform load affects the tuning constants for a given desired response. For quarter amplitude decay, for example, an increase in loading results in a decreased proportional gain, an increased integral time, and a decreased derivative time.

As for angular position control, the correct tuning for a linear position control system is very application dependant. A new system tuning must be performed before each experiment when any of the basic settings (e.g., encoder type, platform load, signal waveform type, etc) has been modified.

Procedure Outline

The Procedure is divided into the following sections:

- Setup and connections
- Tuning the controller with the Ziegler-Nichols method
- Quarter amplitude decay step response
- Significantly damped step response
- Motor shaft incremental encoder step response
- Unloaded platform step response

Procedure

Setup and connections

In this exercise, you will set the Digital Servo for studying linear position control and set the platform to mid position.

In this experiment, the platform will be engaged and the flywheel attached to the platform. To prevent equipment damage, it is extremely important to ensure that the position device controller application is used at all times.

1. Run LVServo, and click on the **Device Controlled** button in the **Position Loop** menu. Make sure the settings are initially as shown in Table 30:

Table 30. Settings for tuning the controller with the Ziegler-Nichols method.

Function Generator		**Trend Recorder**	
Signal Type	*Square*	Reference	*Checked*
Frequency	*0.1 Hz*	Position	*Checked*
Amplitude	*3%*	Speed	*Unchecked*
Offset	*0%*	Current	*Unchecked*
Power	*Off*	Voltage	*Unchecked*
PID Controller		Error	*Unchecked*
Gain (K_p)	*1*	K_p x Error	*Unchecked*
Integral Time (t_i)	*Inf (Off)*	Error Sum / t_i	*Unchecked*
Derivative Time on E (t_d (E))	*0*	t_d x Delta Error	*Unchecked*
Derivative Time on PV (t_d (PV))	*0*	PID Output	*Unchecked*
Timebase	*10 ms*	Display Type	*Sweep*
Anti-Reset Windup	*On*	Show and Record Data	*On*
Upper Limit	*100%*	Measured Gain (rpm)	*3000*
Lower Limit	*-100%*	Measured Gain (A)	*7*
Open or Closed Loop	*Closed*	Measured Gain (V)	*48*
PV Speed Scaling		**Encoders**	
100% Value	*5000 cnt*	Motor or Rail	*Rail*
-100% Value	*-5000 cnt*		

2. Set the platform to mid position as follows:

   Find the center (or 0) reference position of the platform by moving the platform to the right-hand stop position and resetting the position count to 0. Then, move the platform to the left-hand stop position and record the count. This value is the rail length in counts.

   Rail length = _______ counts

3. Reset the count to 0 and manually move the platform until the count is equal to exactly half of the entire rail length. When this point is found, RESET the position count to 0. The platform is now positioned to exactly half way. This also corresponds to the 0% reference position.

Tuning the controller with the Ziegler-Nichols method

In this section, you will tune the controller using the Ziegler-Nichols method, as you have done in Exercise 6.

4. Set the function generator **Power** switch to ON.

5. Starting with a value of 1, slowly increase the proportional gain K_p by 0.25 steps. If an error occurs because of a current overshoot, reduce the gain back to the previous value, reposition the platform to position 0 and RESET the error. Try to refine your search by steps of 0.05 instead. When the system begins to oscillate continuously, stop the process and note the corresponding ultimate gain K_U value.

   $K_U =$ ______

6. Acquire a complete positive half cycle using the trend recorder and export the data to a spread sheet.

7. Set the function generator **Power** switch to OFF.

8. Plot the continuously oscillating response and determine the period of oscillation t_U in seconds:

   $t_U =$ ______ s

9. Using the ultimate gain K_U and the oscillation period t_U, calculate the tuning constants K_p, t_i, and t_d using the Ziegler-Nichols formulae shown below:

   $K_p = 0.6K_U =$ ______

   $t_i = \frac{t_U}{2K_p} =$ ______

   $t_d = \frac{t_U K_p}{8} =$ ______

Quarter amplitude decay step response

In this section, you will acquire a quarter amplitude decay step response for the rail incremental encoder using your calculated parameters.

10. Run LVServo, and click on the **Device Controlled** button in the **Position Loop** menu. Make sure the settings are initially as shown in Table 31:

Table 31. Settings for quarter amplitude decay data acquisition.

Function Generator		Trend Recorder	
Signal Type	*Square*	Reference	*Checked*
Frequency	*0.2 Hz*	Position	*Checked*
Amplitude	*10%*	Speed	*Unchecked*
Offset	*0%*	Current	*Unchecked*
Power	*Off*	Voltage	*Unchecked*
PID Controller		Error	*Unchecked*
Gain (K_p)	*Calculated* K_p	K_p x Error	*Unchecked*
Integral Time (t_i)	*Calculated* t_i	Error Sum / t_i	*Unchecked*
Derivative Time on E (t_d (E))	*0*	t_d x Delta Error	*Unchecked*
Derivative Time on PV (t_d (PV))	*Calculated* t_d	PID Output	*Unchecked*
Timebase	*10 ms*	Display Type	*Sweep*
Anti-Reset Windup	*On*	Show and Record Data	*On*
Upper Limit	*100%*	Measured Gain (rpm)	*3000*
Lower Limit	*-100%*	Measured Gain (A)	*7*
Open or Closed Loop	*Closed*	Measured Gain (V)	*48*
PV Speed Scaling		**Encoders**	
100% Value	*5000 cnt*	Motor or Rail	*Rail*
-100% Value	*-5000 cnt*		

11. Reset the position count, and set the function generator **Power** switch to ON.

12. Adjust the derivative term if necessary to obtain a quarter amplitude decay response. The ratio of the peak overshoot ($\phi_{PEAK} - \phi_{REF}$) to the peak overshoot of the second successive cycle must thus be close to 4 to 1. An approximate quarter decay response is shown in Figure 47.

Note down the modified tuning constants:

K_p = _______

t_i = _______

t_d = _______

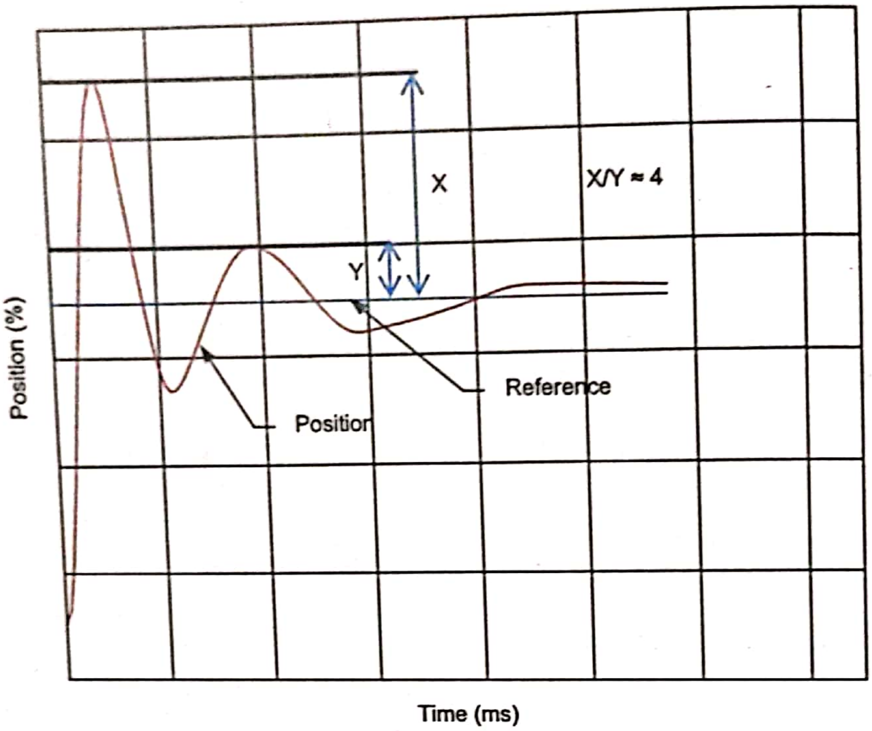

Figure 47.Quarter amplitude decay step response.

13. With the trend recorder, record a complete positive half cycle and export it to a spread sheet.

14. Set the function generator **Power** switch to OFF.

15. Plot the quarter amplitude decay step response.

16. When does the position first reaches the 10% point after the position reference changes from -10% to +10%?

    Time = _______ s

Significantly damped step response

In this section, you will acquire a significantly damped step response for the rail incremental encoder by modifying the parameters you used for the quarter amplitude decay response. You will then compare the resulting plot with the one obtained for the quarter amplitude decay response.

17. Adjust the parameters as follows to produce a damping effect on the system:

$K_p \times 1.5$

$t_i \times 16$

$t_d \times 2$

Note down the new values:

$K_p =$ _______

$t_i =$ _______ s

$t_d =$ _______ s

18. Reset the position count and set the function generator **Power** switch to ON.

19. Run LVServo, and click on the **Device Controlled** button in the **Position Loop** menu. Make sure the settings are initially as shown in Table 32:

Table 32. Setting for a significantly damped step response.

Function Generator		**Trend Recorder**	
Signal Type	*Square*	Reference	*Checked*
Frequency	*0.2 Hz*	Position	*Checked*
Amplitude	*10%*	Speed	*Unchecked*
Offset	*0%*	Current	*Unchecked*
Power	*Off*	Voltage	*Unchecked*
PID Controller		Error	*Unchecked*
Gain (K_p)	*Calculated K_p*	K_p x Error	*Unchecked*
Integral Time (t_i)	*Calculated t_i*	Error Sum / t_i	*Unchecked*
Derivative Time on E (t_d (E))	*0*	t_d x Delta Error	*Unchecked*
Derivative Time on PV (t_d (PV))	*Calculated t_d*	PID Output	*Unchecked*
Timebase	*10 ms*	Display Type	*Sweep*
Anti-Reset Windup	*On*	Show and Record Data	*On*
Upper Limit	*100%*	Measured Gain (rpm)	*3000*
Lower Limit	*-100%*	Measured Gain (A)	*7*
Open or Closed Loop	*Closed*	Measured Gain (V)	*48*
PV Speed Scaling		**Encoders**	
100% Value	*5000 cnt*	Motor or Rail	*Rail*
-100% Value	*-5000 cnt*		

20. Acquire a complete positive half cycle using the trend recorder and export the data to a spread sheet.

21. Set the function generator **Power** switch to OFF.

22. Plot the step response.

23. When does the position first reaches the 10% point after the position reference changes from -10% to +10%?

    Time = ______ s

24. Which response first reaches the position reference of 10% between the quarter amplitude decay response and the significantly damped response?

    ____

    ____

    ____

Motor shaft incremental encoder step response

In this section, you will acquire a quarter amplitude decay step response and a significantly damped step response for the motor shaft encoder using a method similar to the one used for the rail encoder.

25. Switch the incremental encoder from the rail encoder to the motor shaft encoder and repeat steps 1 to 15. Plot the step response and note down the quarter amplitude decay parameters:

    K_U = ______

    t_U = ______ s

    K_p = ______

    t_i = ______ s

    t_d = ______ s

26. Still using the motor shaft incremental encoder, repeat steps 17 to 22,. Plot the significantly damped step response and note the corresponding tuning parameters:

    K_p = ______

    t_i = ______ s

$t_d =$ ______ s

27. Complete the following table with the measured parameters for both the rail encoder and the motor shaft encoder.

Table 33. Tuning parameters for the rail encoder and the motor shaft encoder.

Incremental encoder	Quarter amplitude decay response			Significantly damped response		
	K_p	t_i	t_d	K_p	t_i	t_d
Rail Gain = 14.5						
Motor shaft Gain = 40						

28. Describe the effects on proportional gain K_p, the integral time t_i, and the derivative time t_d when the incremental encoder is switched from the rail incremental encoder to the motor shaft incremental encoder.

____

____

____

Unloaded platform step response

In this section, you will acquire a quarter amplitude decay step response with the platform unloaded. Since the settings have changed, you will have to retune the controller using the Ziegler-Nichols tuning method. You will then compare the results obtained with the loaded platform to the results obtained with the unloaded platform.

29. Remove the flywheel from the platform and make sure that the incremental encoder is set back again to the rail encoder.

30. Acquire another quarter amplitude decay step response using the Ziegler-Nichols tuning method. Plot both the continuous oscillation and the quarter amplitude decay step response. Note down the following modified parameters as well as the time at which the position reaches first the 10% point.

$K_U =$ ______

$t_U =$ ______ s

$K_p =$ ______

$t_i =$ ______ s

$t_d =$ ______ s

Time at 10% point = ______ s

31. Using the measured parameters, complete the following table:

Table 34. Tuning parameters for a loaded platform and an unloaded platform.

Platform state	10%-point time	Quarter amplitude decay parameters		
		K_p	t_i	t_d
Loaded				
Unloaded				

CONCLUSION

In this exercise, you familiarized yourself with the Digital Servo PID controller operation, as well as with the effects of the proportional, integral, and derivative terms on the transient operation of a servo system used for linear position control. You measured the effects of damping on the step response of a servo system in linear position control. You observed the effects of incremental encoder resolution and platform load on the tuning parameters of a servo system, as well as on its transient behavior in linear position control.

REVIEW QUESTIONS

1. Suppose the Digital Servo rail encoder has a resolution of 2880 pulses per revolution. Calculate the resulting encoder gain in s^{-1}. The ±100% position range is of ±4800 counts.

______

______

______

2. Three different incremental encoders have a quarter amplitude decay response that results in the parameters shown in the table below:

Table 35. Three different encoders quarter amplitude decay parameters.

Encoders	Quarter amplitude decay parameters		
	K_p	t_i	t_d
X→	7.1	0.007	0.14
Y→	3.5	0.014	0.05
Z→	5.8	0.012	0.11

Knowing that encoder 1 has a gain of 32, encoder 2 a gain of 42 and encoder 3 a gain of 15, associate encoders X,Y, and Z in the table above to encoders 1,2, and 3 by entering the correct encoder number in the relevant column.

3. Suppose the Digital Servo system shows a significantly underdamped oscillatory response. What changes can be made to the integral and derivative times in order to increase the damping?

4. Suppose a PI controller has a square wave position reference signal of ±10% at a frequency of 1 Hz. The position signal is held at 0% so that an error signal of ±10% is created. The response from the PI controller is shown in Figure 48. Calculate the system proportional gain K_p and the integral time t_i.

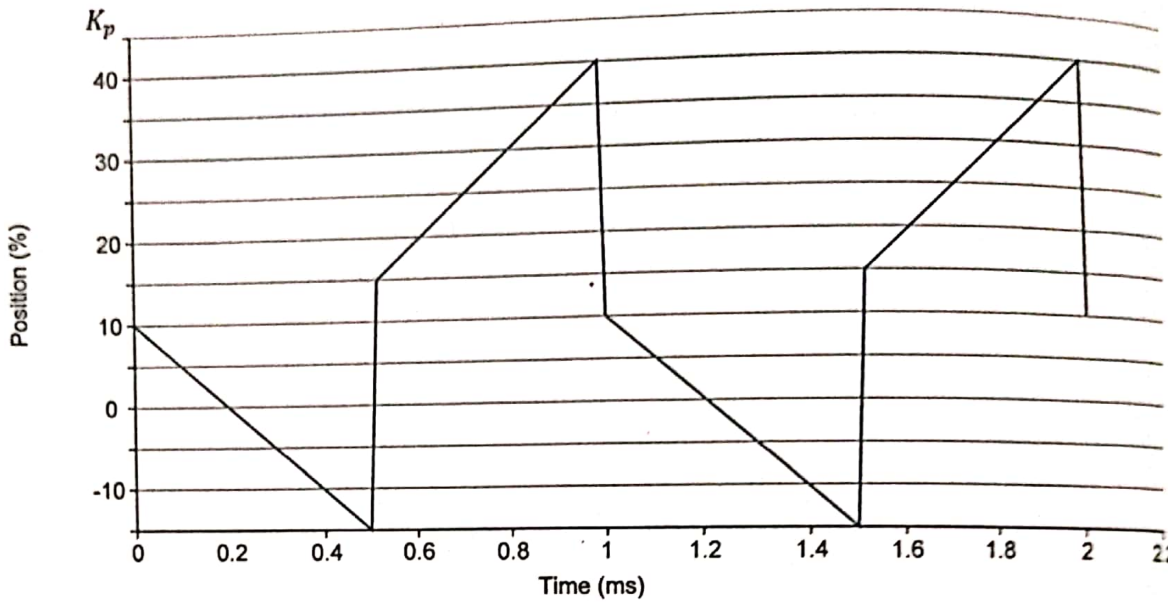

Figure 48. PI controller output having a ±10% error.