Open Loop Servo Motor Transient Characteristics

EXERCISE OBJECTIVE

When you have completed this exercise, you will be familiar with the transient behavior of a dc servo motor in open loop mode. You will know how to calculate and measure the motor time constant. You will be able to distinguish a first-order from a second-order system and to approximate a second-order system using its first-order model.

DISCUSSION OUTLINE

The Discussion of this exercise covers the following points:

- Motor steady state and transient response
- Servo motor steady state and dynamic characteristics
- Servo motor transient response
- Significance of the transient response equation

DISCUSSION

Motor steady state and transient response

When an electric supply voltage E is initially connected to a dc servo motor, the counter-EMF voltage V is at its minimal value of 0 V. The armature current I_A on the other hand, is at a maximum and is only limited by the resistance R. Since the electrical **time constant** L/R is very small relative to the motor mechanical time constant, we can ignore the current lag due to the inductance L. This results in a high armature current I_A that produces a proportionally high torque T, causing the motor to rotate.

As the motor speed ω increases, the counter-EMF voltage V increases in proportion. This, in turn, causes both the armature current I_A and the resultant torque T to decrease. Eventually, the motor reaches a constant speed ω at the moment when the counter-EMF voltage is less than the supply voltage E by a value that results in an armature current I_A producing a motor torque T just high enough to overcome the motor losses (e.g., magnetic hysteresis, eddy currents, windage, bearing, and brush friction). This moment when the motor reaches an equilibrium is defined as the motor steady state. In contrast, the phase occurring before the motor steady state moment is reached and after the motor is applied a supply voltage is called the motor **transient response**. In other words, a motor transient response is the motor state that begins just after a change in its equilibrium occurs (when the motor is powered on for example) and that ends when the motor reaches a new equilibrium, i.e., its steady state. The entire process, that is, the motor's transient response and its subsequent steady state is referred to as a motor **step response**.

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Servo motor steady state and dynamic characteristics

The steady state and **dynamic characteristics** of a dc servo motor can be defined using the following basic equations:

$$E = IR + V + L\frac{di}{dt} \tag{11}$$

$$V = K_E\omega \tag{12}$$

$$T = K_T I \tag{13}$$

$$T = J_M\frac{d\omega}{dt} + B\omega \tag{14}$$

where J_M is the motor inertia (kg·m²)

Servo motor transient response

Using Equation (11), Equation (12), Equation (13), Equation (14), and Laplace transforms, the **transfer function** relating motor speed ω to the supply voltage E, expressed in the frequency domain, is given as:

$$\frac{\omega(s)}{E(s)} = \frac{\frac{K_T}{RB + K_E K_T}}{\frac{LJ_M}{RB + K_E K_T}s^2 + \frac{RJ_M + BL}{RB + K_E K_T}s + 1} \tag{15}$$

This **second-order** system has two time constants, found by evaluating the roots of the denominator. The reciprocal of each root represents the time constant in seconds. They are expressed below in Equation (16).

$$\tau_1, \tau_2 = \frac{2LJ_M}{RJ_M + BL \pm \sqrt{(RJ_M + BL)^2 - 4LJ_M(RB + K_E K_T)}} \tag{16}$$

where τ_1 is the first time constant (s)
τ_2 is the second time constant (s)

Using inverse Laplace transforms, the motor transient speed response for a step change in voltage from 0 to E volts is:

$$\omega = EK_S\left[1 - \left(\frac{\tau_1}{\tau_1 - \tau_2}\right)e^{-\frac{t}{\tau_1}} + \left(\frac{\tau_2}{\tau_1 - \tau_2}\right)e^{-\frac{t}{\tau_2}}\right] \tag{17}$$

In Equation (17), the steady state speed constant K_S is equal to:

$$K_S = \frac{K_T}{RB + K_E K_t} \tag{18}$$

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The development of these equations is given in Appendix B.

Significance of the transient response equation

Equation (17) represents the transient behavior of the servo motor as it increases its speed from 0 rad/s to its final steady state speed after a step change in dc voltage E is applied to the motor.

By substituting t=0 into Equation (17), it can be shown that motor speed at time t=0 is:

$$\omega_{(t=0)} = 0 \tag{19}$$

where $\omega_{(t=0)}$ is the motor speed when t=0 (rad/s)

Using Equation (17), it can also be shown that :

$$\omega_{(t\rightarrow\infty)} = EK_S \tag{20}$$

where $\omega_{(t\rightarrow\infty)}$ is the motor speed in rad/s when $t\rightarrow\infty$ (rad/s)

This means that the steady state speed ω_{SS} of a dc servo motor is equal to the product of K_S and the supply voltage E. As has been seen in Exercise 2, this relationship holds for an ideal dc motor with no static friction.

Example

Given the following dc servo motor characteristics for a brush-type permanent-magnet dc motor:

Table 7. Servo motor characteristics for a brush-type permanent-magnet dc motor.

Parameter	Value
K_T (N·m/A)	0.105
K_E [V/(rad/s)]	0.105
R (Ω)	2.03
L (H)	0.0052
J_M (kg·m^2)	0.0000438
J_{LOAD} (kg·m^2)	0.0001897
B [N·m/(rad/s)]	0.0000708
K_S [(rad/s)/V]	9.4

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The two time constants τ_1 and τ_2 can be calculated using Equation (16). In these equations, the moment of inertia is equal to the sum of the motor J_M and J_{LOAD} moments of inertia. Thus, using the values in Table 7, we obtain:

$\tau_1 = 2.7$ ms

$\tau_2 = 40.0$ ms

Substituting values of time from 0 to 200 ms into Equation (17) and substituting the values of τ_1, and τ_2 calculated above, the motor speed response can be plotted for a 48 V step change to the motor. Note that Equation (17) yields results of speed in rad/s. These must be converted to rpm by multiplying the result in rad/s by 30/π. Figure 20 shows the plot resulting from these values:

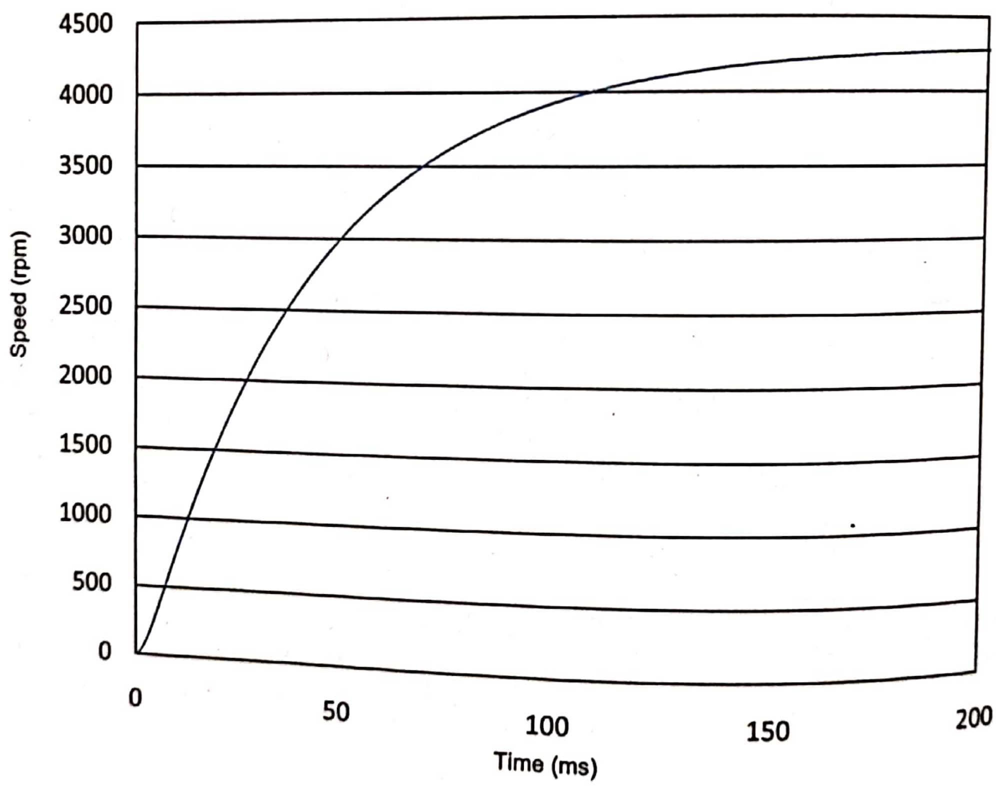

Figure 20. Second-order step response.

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Some typical values selected from a table of values used for plotting the above curve are shown below:

Table 8. Speed ω in rad/s and speed ω_{RPM} in rpm of a dc motor in relation to time.

Time (s)	Speed ω (rad/s)	Speed ω_{RPM} (rpm)
0.000	0.0	0
0.001	1.8	17
0.005	29.1	278
0.010	74.9	715
0.015	118.5	1132
0.020	157.5	1504
0.025	192.0	1833
0.030	222.5	2124
0.035	249.3	2381
0.040	273.1	2608
0.050	312.5	2984
0.060	343.2	3277
0.070	367.1	3505
0.080	385.7	3683
0.090	400.2	3822
0.100	411.5	3930

Note that because the second time constant τ_2 has a much higher value (40 ms) than the first time constant τ_1 (2.7 ms), the plot appears to closely resemble a **first-order** step response. This explains why brush-type permanent-magnet motors with a very low inductance often are modeled as first-order systems where $L \approx 0$.

The simplified first-order equations are:

$$\frac{\omega(s)}{E(s)} = \frac{K_S}{\tau s + 1} \tag{21}$$

$$K_S = \frac{K_T}{RB + K_E K_T} \tag{22}$$

$$\tau = \frac{RJ_M}{RB + K_E K_T} \tag{23}$$

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Using Equation (21), Equation (22), Equation (23), and inverse Laplace transforms, the motor transient speed response for a step change in voltage from 0 to E volts is equal to:

$$\omega = EK_S(1 - e^{-\frac{t}{\tau}}) \tag{24}$$

A comparison of the actual second-order step response and the first-order approximation is shown in Figure 21.

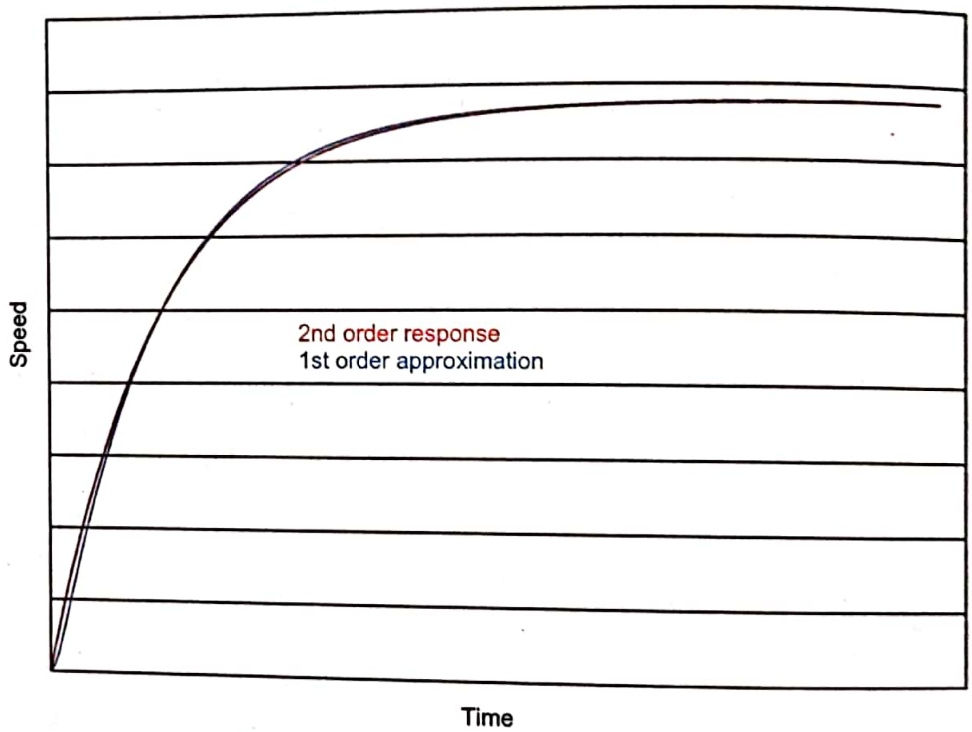

Figure 21. First and second-order step response comparison.

As the plot shows, the first-order approximation is a very good approximation of the actual second-order response. This correlation is due to the fact that the motor in this example has a very low inductance.

PROCEDURE OUTLINE

The Procedure is divided into the following sections:

- Setup and connections
- Calculating the time constant
- Measuring the time constant

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PROCEDURE

Setup and connections

In this experiment, you will determine the first-order approximation of the motor time constant τ. The motor data is shown in Table 9. These parameters are from the motor manufacturer's data sheet. You will have to use Equation (23) to calculate the theoretical value of τ. The value of the viscous friction coefficient B and the steady state speed constant K_S are taken from Exercise 2. Again, note that there is a large tolerance in calculating these parameters as they can vary by as much as ± 15%.

Table 9. Motor parameters from the manufacturer's data sheet.

Parameter	Value
ω_{RPM} (rpm)	3400
L (mH)	6.4
R (Ω)	2.23
K_T (N·m/A)	0.121
K_E [V/(rad/s)]	0.121
J_M (kg·m^2)	0.00006286

1. Make the following settings on the Digital Servo system:

   - Setup the servo system for speed control, i.e., disengage the platform.
   - Set the belt tension to allow the belt to be lifted of the pulley connected to the motor shaft and slipped on the two pins to the rear of the pulley, allowing the shaft to run uncoupled from the belt.
   - Secure the flywheel to the shaft using the appropriate hex key.

Calculating the time constant

In this section, you will calculate the time constant τ of the motor and plot the first-order step response of the motor speed versus time.

2. Calculate the time constant τ using mathematical tools such as a spread sheet and Equation (23). Use the viscous friction coefficient B that you calculated in Exercise 2. The total inertia J_T for calculating the time constant τ is the sum of the individual inertias of the system (i.e., the shaft, motor to shaft coupling, two lock nuts, brake disc, pulley, flywheel, motor, and encoder) and is equal to 0.000333 kg·m^2.

   $\tau =$ ______ ms

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3. Use mathematical tools such as a spread sheet to plot the first-order step response of the motor speed versus time for a supply voltage to the motor of 48 V dc. You will need to use Equation (24) and your calculated time constant τ value, as well as the steady state speed constant K_S you calculated in Exercise 2.

Your plot should look like the one shown in Figure 22:

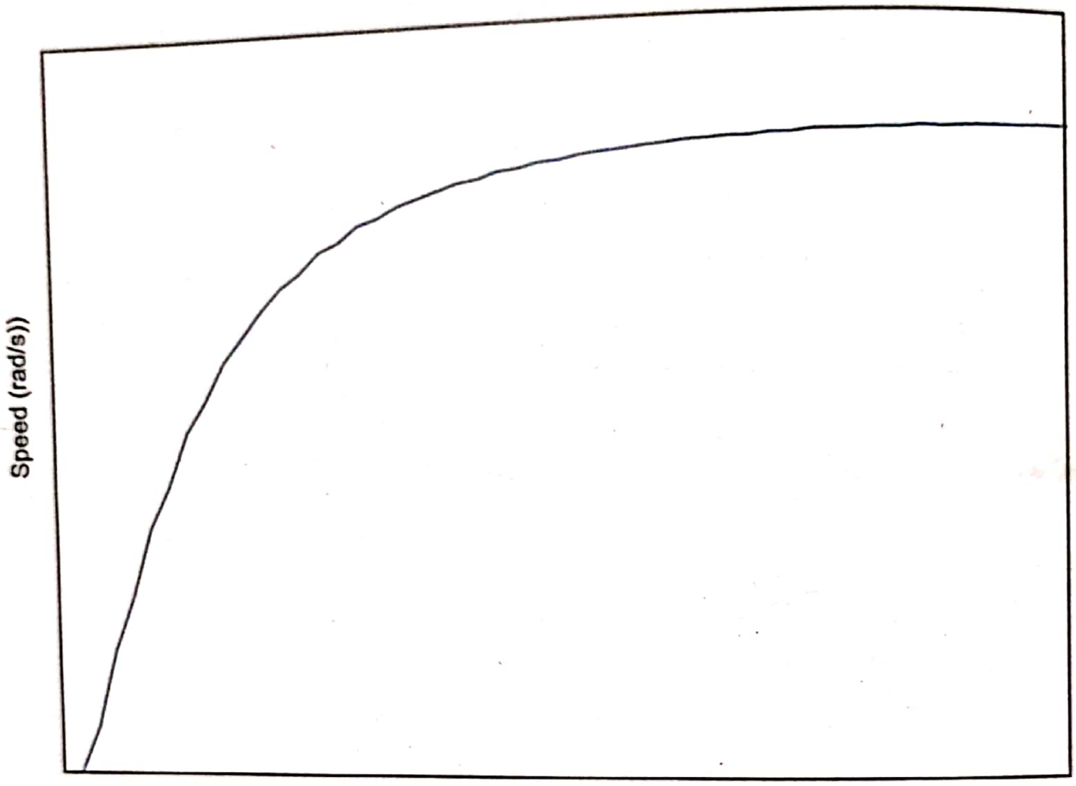

Figure 22. First-order step response characteristic curve.

Measuring the time constant

In this section, you will determine the time constant τ by plotting a motor speed versus time step response curve from data obtained by running the servo motor. You will use the steady state speed constant K_S value that you calculated in Exercise 2.

4. Run LVServo, and click on the **Device Controlled** button in the **Speed Loop** menu. Make sure the settings are initially as shown in Table 10.

The speed provided in the exported data is in percentage of 3000 rpm. To convert to rad/s multiply speed in rpm (%) by π (see Exercise 2). Note also that the voltage in the exported data is a percentage of 48 V.

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Table 10. Settings for measuring the time constant τ.

Function Generator		**Trend Recorder**	
Signal Type	*Square*	Reference	*Checked*
Frequency	*0.25 Hz*	Speed	*Checked*
Amplitude	*40%*	Current	*Unchecked*
Offset	*40%*	Voltage	*Unchecked*
Power	*Off*	Error	*Unchecked*
PID Controller		K_p x Error	*Unchecked*
Gain (K_p)	*1*	Error Sum / t_i	*Unchecked*
Integral Time (t_i)	*0.05*	t_d x Delta Error	*Unchecked*
Derivative Time on E (t_d (E))	*0*	PID Output	*Unchecked*
Derivative Time on PV (t_d (PV))	*0*	Display Type	*Sweep*
Timebase	*10 ms*	Show and Record Data	*On*
Anti-Reset Windup	*On*	Measured Gain (rpm)	*3000*
Upper Limit	*100%*	Measured Gain (A)	*7*
Lower Limit	*-100%*	Measured Gain (V)	*48*
Open or Closed Loop	*Open*		
PV Speed Scaling			
100% Value	*3000 rpm*		

5. Capture a complete positive half cycle and export it to a spread sheet.

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6. Using the spread sheet, plot the motor speed ω in rad/s versus time.Your plot should look similar to the following:

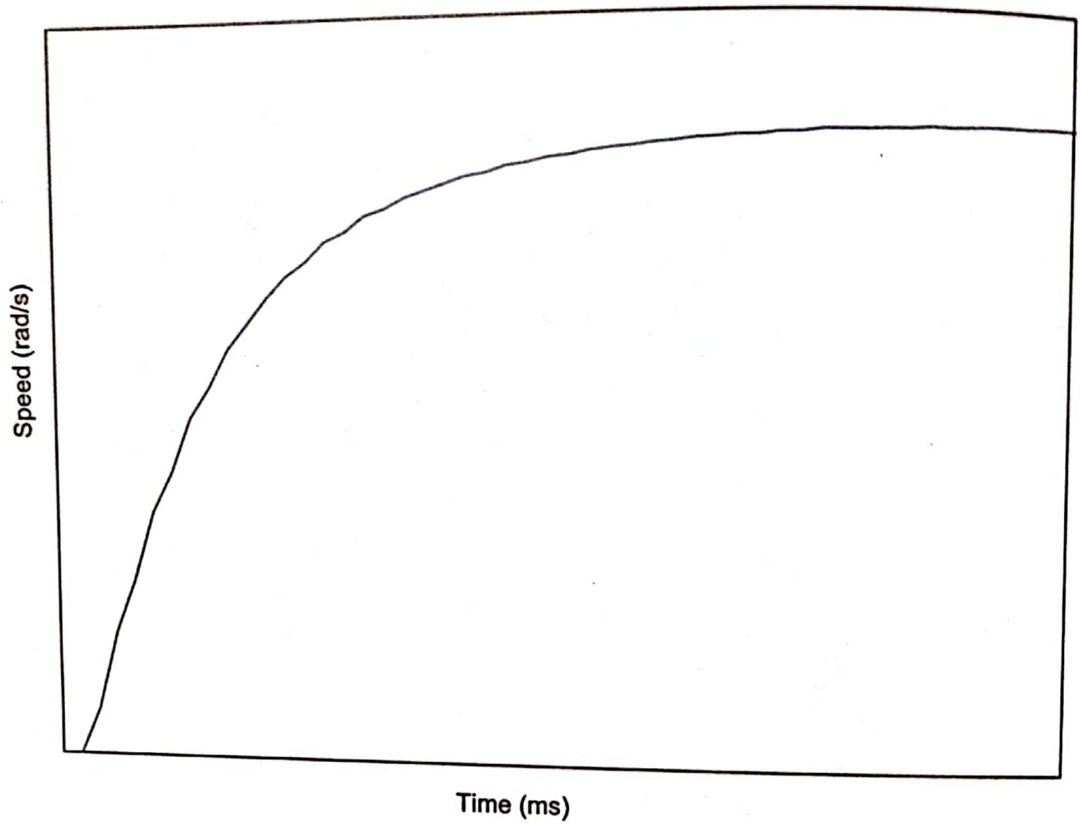

Figure 23. Measured step response characteristic curve.

7. Determine graphically the motor time constant τ as follows: from Equation (24), when $t = \tau$, the speed should be at 63% of the steady state value, that is:

$$\omega = EK_S(1 - e^{-1}) = 0.63EK_S \tag{25}$$

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8. Determine at which point on the curve the steady state speed value is equal to 63%. The time constant value is equal to the time between the beginning of acceleration and the 63%-speed point. Figure 24 shows how to approximate the 63%-speed point.

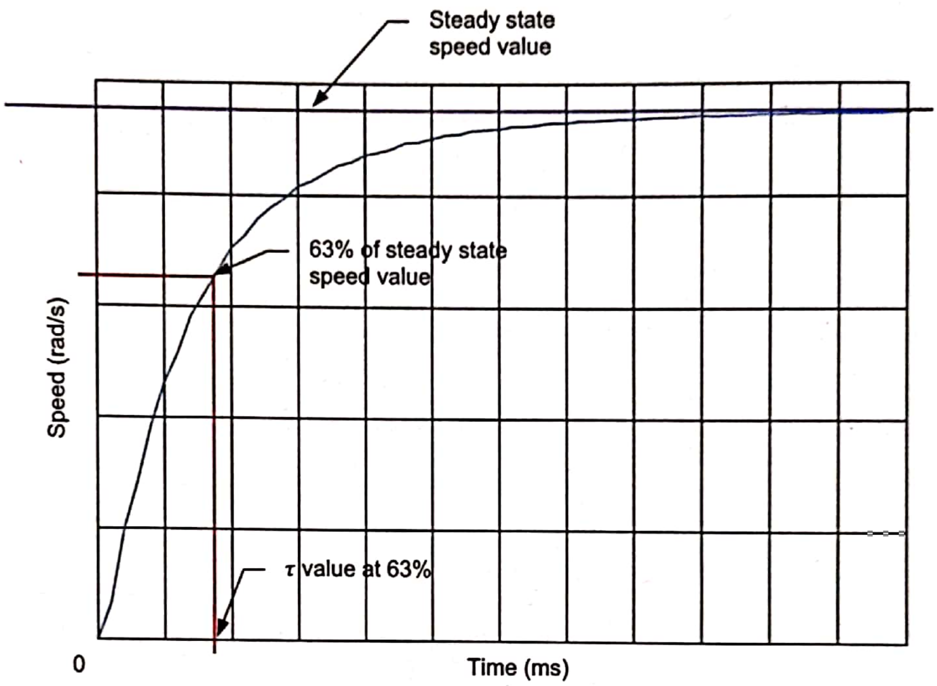

Figure 24. First-order time constant approximation.

9. Complete Table 11 using your calculated and measured steady state speed constant K_S and time constant τ.

Table 11. Calculated and measured K_S and τ of a dc motor.

Parameter	Calculated value	Measured value
K_S [(rad/s)/V]		
τ (ms)		

CONCLUSION

In this exercise, you analyzed the transient characteristics of a servo motor system operating in open loop control. Using experimental measurements, you were able to determine the time constant of the servo motor and compare it with the time constant you calculated from the manufacturer's data. You also learned the distinction between a first-order and a second-order system step response.

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REVIEW QUESTIONS

1. Using the motor data obtained in this exercise and Equation (16), find the time constants τ_1 and τ_2. In addition, plot the servo motor transient response for a step change of $E = 48$ V. Use the first-order model value of K_S measured in Exercise 2. Plot the response times from 0 to 250 ms using a spread sheet or any other convenient mathematical tool.

2. Describe the step response curve you obtained in question 1 and compare it to the one you plotted in step 6 of the procedure section.

3. What can you conclude about the largest time constant calculated in question 1 and the time constant you calculated for the simplified model of the servo motor?

4. On the curve you plotted in question 1, determine at which time the slope is at a maximum?

5. Does the maximum slope time of the response curve plotted in question 1 correspond to the same maximum slope time of the curve plotted in step 6?

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6. What distinguishes a second-order step response from a first-order step response?

7. What are the effects on the speed-vs.-time step response curve if the total moment of inertia J_T increases?

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