Case 3. Roots of the Denominator of F(s) Are Complex or Imaginary

$$G(s) = \frac{3}{s(s^2+2s+5)} = \frac{3}{s\{s-(-1+2j)\}\{s-(-1-2j)\}}$$

$$s = \frac{-2 \pm \sqrt{4-20}}{2} = \frac{-2 \pm 4j}{2} = -1 \pm 2j$$

$$G(s) = \frac{3}{s(s+1-2j)(s+1+2j)}$$

$$G(s) = \frac{3}{s(s+1-2j)(s+1+2j)} = \frac{A}{s} + \frac{B}{s+1-2j} + \frac{C}{s+1+2j}$$

$$A = sG(s)\Big|_{s=0} = \frac{3}{s^2+2s+5}\Big|_{s=0} = \frac{3}{5}$$

$$B = (s+1-2j)G(s)\Big|_{s=-1+2j} = \frac{3}{s(s+1+2j)}\Big|_{s=-1+2j}$$

$$= \frac{3}{(-1+2j)\times 4j} = -\frac{3}{10} + \frac{3}{20}j$$

$$B = -\frac{3}{20}(2-j)$$

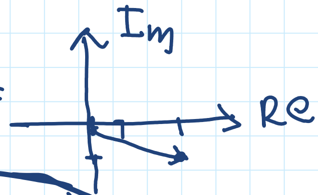
$$C = B^* = \frac{-3}{20}(2+j)$$

$$G(s) = \frac{3}{5}\cdot\frac{1}{s} + \frac{-\frac{3}{20}(2-j)}{s+1-2j} + \frac{-\frac{3}{20}(2+j)}{s+1+2j}$$

$$= \frac{3}{5}\times\frac{1}{s} - \frac{3}{20}\,\frac{2-j}{s+1-2j} - \frac{3}{20}\,\frac{2+j}{s+1+2j}$$

$$= \frac{3}{5}\cdot\frac{1}{s} - \frac{3}{20}\cdot\frac{\sqrt{5}\, e^{j\theta}}{s+(1-2j)} - \frac{3}{20}\cdot\frac{\sqrt{5}\, e^{-j\theta}}{s+(1+2j)}$$

$\sqrt{2^2+1^2} = \sqrt{5}$



$\theta = \tan^{-1}\left(-\frac{1}{2}\right)$

$$= \frac{3}{5}u(t) - \frac{3\sqrt{5}}{20} e^{j\theta}\cdot e^{-(1-2j)t} - \frac{3\sqrt{5}}{20} e^{-j\theta}\, e^{-(1+2j)t}$$

$$= \frac{3}{5}u(t) - \frac{3\sqrt{5}}{20}\left[e^{j\theta}\cdot e^{-t}\cdot e^{2jt} + e^{-j\theta}\cdot e^{-t}\cdot e^{-2jt}\right]$$

$$= \frac{3}{5}u(t) - \frac{3\sqrt{5}}{20} e^{-t}\left[e^{j(2t+\theta)} + e^{-j(2t+\theta)}\right]$$

$$= \frac{3}{5}u(t) - \frac{3\sqrt{5}}{20} e^{-t}\cdot 2\cos(2t+\theta)$$

$$= \frac{3}{5}u(t) - \frac{3\sqrt{5}}{2} e^{-t}\cos(2t+\theta) \quad \text{where } \theta = \tan^{-1}\left(-\frac{1}{2}\right)$$