# Exercise 5

# Servo Closed Loop Speed Control – Transient Characteristics and Disturbances

**EXERCISE OBJECTIVE**

When you have completed this exercise, you will be familiar with the transient behavior of a servo system in a closed loop speed-control mode. You will understand the effects of controller gain variation on the step response of the closed loop servo system. You will also know the effects of load disturbance on the operation of the closed loop servo system.

**DISCUSSION OUTLINE**

The Discussion of this exercise covers the following points:

- Response to changes in the reference speed
- Effect of disturbances

**DISCUSSION**

## Response to changes in the reference speed

Figure 28 shows a simplified block diagram of the servo closed loop speed-control system with a first-order motor model. The controller is in proportional only mode (constant gain term).

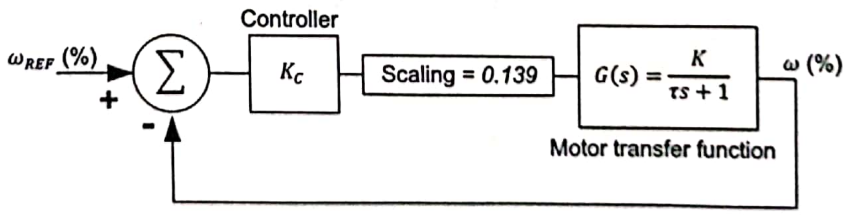


**Figure 28. Block diagram of a servo motor in closed loop speed-control mode.**

For the purposes of step changes to reference speed analysis, the block diagram in Figure 28 can be reduced to Equation (28) (see Appendix B for the complete equation development):

$$\omega_{REF} \rightarrow \frac{\frac{0.139K_CK}{1+0.139K_CK}}{\left(\frac{\tau}{1+0.139K_CK}\right)s+1} \rightarrow \omega \tag{28}$$

where $\omega$ is the motor speed (controlled or process variable)
$\omega_{REF}$ is the desired or reference motor speed (set point)
$K$ is the ratio of motor speed (rad/s) to motor dc supply voltage
$K_C$ is the controller gain (adjustable)
$0.139$ is the scaling factor that accounts for the unit conversions identified in Exercise 4
$\tau$ is the time constant

The corresponding system reduced form is thus:

$$\frac{\omega}{\omega_{REF}} = \frac{K_{SYS}}{\tau_{SYS}s+1} \tag{29}$$

where $K_{SYS}$ is equal to $\frac{0.139K_CK}{1+0.139K_CK}$
$\tau_{SYS}$ is equal to $\frac{\tau}{1+0.139K_CK}$

The Laplace transform transfer function applied to Equation (30) shows that the time constant $\tau$ is reduced to the term $1+0.139K_CK$. This means that increasing the controller gain $K_C$ lowers the time constant $\tau$ and thus shortens the step response.

The step response for the block diagram shown in Figure 28 is equal to (see Appendix B for the complete equation development):

$$\omega = \left(\frac{0.139K_CK}{1+0.139K_CK}\right)\omega_{REF}\left(1-e^{-\frac{1+0.139K_CKt}{\tau}}\right) \tag{30}$$

The first-order time equation is thus:

$$\omega = K_{SYS}\omega_{REF}\left(1-e^{\frac{t}{\tau_{SYS}}}\right) \tag{31}$$

It is very important to note that this analysis is for the simplified first-order model of the system and ignores secondary effects. In practice, these additional effects limit how high the controller gain $K_C$ can be increased before the system begins to show oscillatory behavior of the speed variable and become unstable (continuous **oscillation**).

In addition, as was previously discussed in Exercise 4, the **steady state error** is reduced by increasing $K_C$. This means that when the controller gain $K_C$ increases, the ratio $\left(\frac{0.139K_CK}{1+0.139K_CK}\right)$ $K_{SYS}$ tends towards 1.

## Effect of disturbances

Until now, we have studied the functioning of the Digital Servo in ideal conditions. However, such conditions do not exist in practice. System disturbances of all kinds tend to alter the measured motor speed $\omega$. Following is a short list of possible disturbances:

- Variations of the mechanical load.
- Fluctuations of the supply voltage $E$, which results in armature voltage fluctuations and thus, speed variations.
- Changes in the ambient and motor temperatures, resulting in speed changes.
- Motor power amplifier properties changes caused by different factors (e.g., aging, dust, rust, etc.).

To represent these disturbances, a disturbance component $D$ has been added to the block diagram in Figure 29 and is shown acting on the speed term $\omega$.

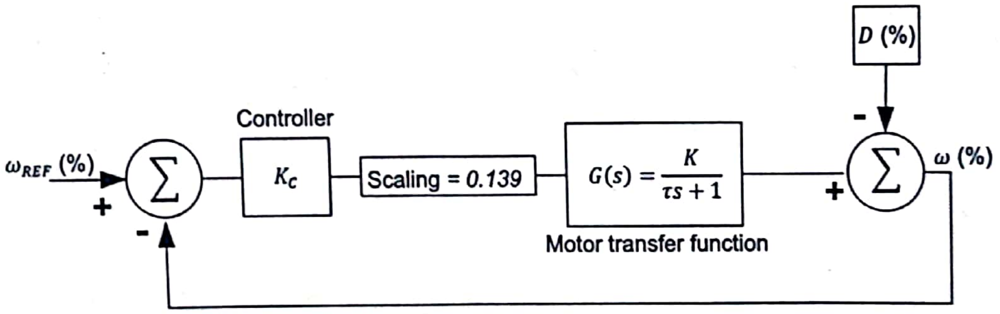


**Figure 29. Block diagram of a servo motor in closed loop speed-control mode with a disturbance component *D*.**

This disturbance component $D$ results in a steady state change in speed $\Delta\omega_{SS}$. The value of $\Delta\omega_{SS}$ can be determined using the following equation:

$$\Delta\omega_{SS} = D\left(\frac{1}{1 + 0.139K_C K}\right) \tag{32}$$

where $\Delta\omega_{SS}$ is the steady state change in speed (%)
$D$ is the disturbance component (%)

Equation (32) shows that as the controller gain $K_C$ increases, the effect of the disturbance $D$ on the speed $\omega$ decreases.

The transient change in speed $\omega$ caused by the disturbance $D$ is equal to:

$$\Delta\omega_{SS} = \frac{\pm D}{1 + 0.139K_C K}\left[1 - 0.139K_C K \cdot e^{\frac{(1+0.139K_C K)t}{\tau}}\right] \tag{33}$$

The first-order equation form is:

$$\omega = \pm D \cdot K_{DSYS}\left(1 - K_C K \cdot e^{\frac{t}{\tau_{SYS}}}\right) \tag{34}$$

where $K_{DSYS}$ is equal to $\frac{1}{1+0.139K_C K}$

The development of these equations is given in Appendix B.

A plot of the response to a 20% step disturbance is given in Figure 30. The plot shows the response for $K_p$ values of 2 and 5. The figure shows that, as the gain $K_p$ increases, the magnitude of the response steady state value and the response time decrease.

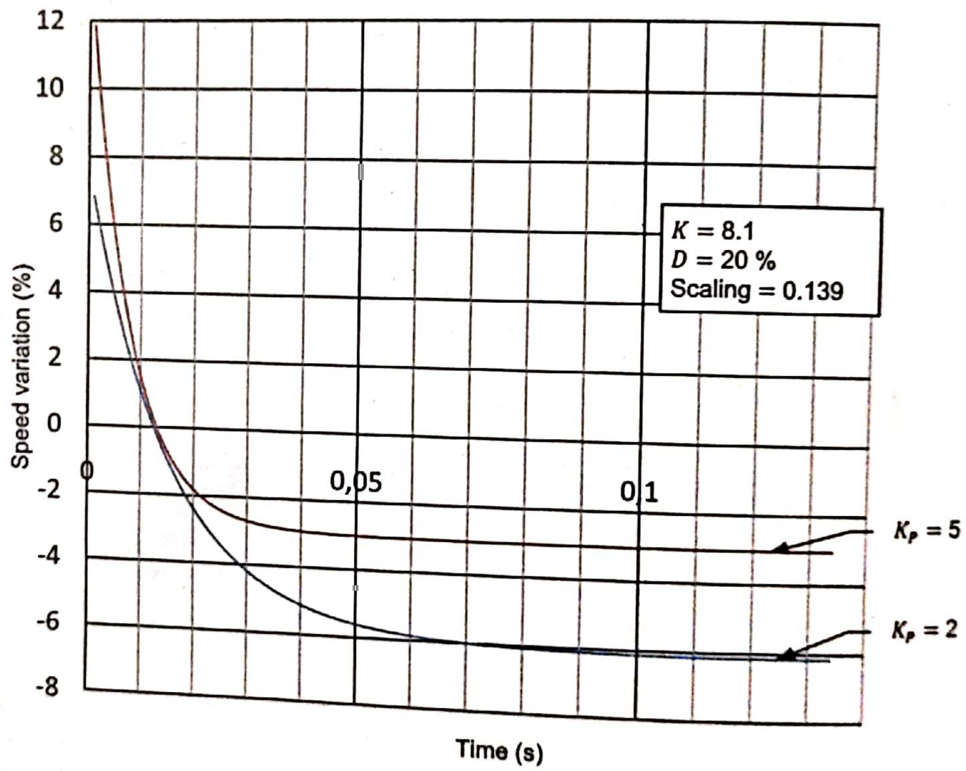


**Figure 30. Disturbance transient response.**

## Procedure Outline

The Procedure is divided into the following sections:

- Setup and connections
- Step response data acquisition
- Time constant approximation
- Observing the effects of load disturbances
- Servo system oscillation

## Procedure

### Setup and connections

In this section, you will setup the Digital Servo for closed-loop speed-control.

1. Make the following settings on the Digital Servo system:

    - Setup the servo system for speed control, i.e., disengage the platform.
    - Set the belt tension to allow the belt to be lifted of the pulley connected to the motor shaft and slipped on the two pins to the rear of the pulley, allowing the shaft to run uncoupled from the belt.
    - Secure the flywheel to the shaft using the appropriate hex key.

2. Run LVServo, and click on the **Device Controlled** button in the **Speed Loop** menu. Make sure the settings are initially as shown in Table 15:

**Table 15. Settings for step response data acquisition.**

| **Function Generator** | | **Trend Recorder** | |
|---|---|---|---|
| Signal Type | *Square* | Reference | *Checked* |
| Frequency | *0.25 Hz* | Speed | *Checked* |
| Amplitude | *10%* | Current | *Unchecked* |
| Offset | *20%* | Voltage | *Unchecked* |
| Power | *Off* | Error | *Unchecked* |
| **PID Controller** | | $K_p$ x Error | *Unchecked* |
| Gain ($K_p$) | *1* | Error Sum / $t_i$ | *Unchecked* |
| Integral Time ($t_i$) | *Inf (Off)* | $t_d$ x Delta Error | *Unchecked* |
| Derivative Time on E ($t_d$ (E)) | *0* | PID Output | *Unchecked* |
| Derivative Time on PV ($t_d$ (PV)) | *0* | Display Type | *Sweep* |
| Timebase | *10 ms* | Show and Record Data | *On* |
| Anti-Reset Windup | *On* | Measured Gain (rpm) | *3000* |
| Upper Limit | *100%* | Measured Gain (A) | *7* |
| Lower Limit | *-100%* | Measured Gain (V) | *48* |
| Open or Closed Loop | *Closed* | | |
| **PV Speed Scaling** | | | |
| 100% Value | *3000 rpm* | | |

3. Set the function generator **Power** switch to ON.

### Step response data acquisition

In this section, you will plot the step response for a square wave speed reference (set point) whose maximum and minimum values are 30% and 10%, respectively. You will then plot the step response for a gain $K_p$ value of 2.

4. Capture a complete positive half cycle and export it to a spread sheet.

5. Set the function generator **Power** switch to OFF.

6. Set the gain $K_p$ value to 2 and repeat the two previous steps to provide step response data for the servo system operating in proportional only mode with a gain of 2.

7. Plot the two step responses in the same graph.

## Time constant approximation

In this section, you will approximate the time constant $\tau$ for the acquired step responses using the time constant approximation method. Since the time constant $\tau$ corresponds to the time when the speed $\omega$ reaches 63.2% of its steady state value, determining the 63.2%-speed time provides a good approximation of the time constant. The time constant approximation method assumes that the step responses are approximately first-order equations.

### *Time constant approximation method*

A time constant $\tau$ can be approximated from captured data using the following method:

- Determine the maximum steady state speed value $\omega_{MAX}$ of the step response in percentage.
- Determine the minimum (initial) speed value $\omega_{MIN}$ of the step response in percentage.
- Subtract the minimal speed value from the maximum speed value ($\omega_{MAX} - \omega_{MIN}$) and multiply the result by 0.632 ($\approx 1 - e^{-1}$).
- Add $\omega_{MIN}$ to the final result. The calculated value is the 63.2%-speed point of the step change ($\omega_{63.2\%}$). The complete operation is summarized in Equation (35):

$$\omega_{63.2\%} = 0.632(\omega_{MAX} - \omega_{MIN}) + \omega_{MIN} \tag{35}$$

where $\omega_{63.2\%}$ is the 63.2%-speed point of the step change

### *Time constant approximation example*

In this section, you will see an example showing how to use the time constant approximation method to find a time constant $\tau$. The data used for this example is given in Table 16. The second time stamp column was added to adjust the starting time to 0 s. Time starts when the reference changes from 10% to 30%.

8. Using your results, like in Table 16, and the time constant approximation method, calculate the motor 63%-speed value $\omega_{63.2\%}$, and then find its corresponding time in Table 16. You may have to interpolate between samples to get a more accurate time.

**Table 16. Time constant approximation captured data.**

$\omega_{MIN}$

$\omega_{63.2\%} = 6.81\%$

$\omega_{MAX}$

| Timestamp | Timestamp' | Reference | Speed | Current | Voltage | Error |
|---|---|---|---|---|---|---|
| 284.01 | 0.00 | 30.001 | 0.250 | 2.847 | 26.607 | 29.769 |
| 284.02 | 0.01 | 30.001 | 1.750 | 32.797 | 25.265 | 28.270 |
| 284.03 | 0.02 | 30.001 | 4.500 | 26.936 | 22.782 | 25.502 |
| 284.04 | 0.03 | 30.001 | 5.625 | 19.920 | 21.788 | 24.404 |
| 284.05 | 0.04 | 30.001 | 7.250 | 14.420 | 20.349 | 22.768 |
| 284.06 | 0.05 | 30.001 | 8.125 | 11.167 | 19.555 | 21.901 |
| 284.07 | 0.06 | 30.001 | 8.500 | 8.995 | 19.208 | 21.501 |
| 284.08 | 0.07 | 30.001 | 9.375 | 7.404 | 18.414 | 20.635 |
| 284.09 | 0.08 | 30.001 | 9.500 | 6.103 | 18.315 | 20.504 |
| 284.10 | 0.09 | 30.001 | 9.625 | 5.258 | 18.216 | 20.403 |
| 284.11 | 0.10 | 30.001 | 10.125 | 4.775 | 17.769 | 19.902 |
| 284.12 | 0.11 | 30.001 | 9.875 | 4.291 | 17.968 | 20.134 |
| 284.13 | 0.12 | 30.001 | 10.125 | 4.220 | 17.770 | 19.902 |
| 284.14 | 0.13 | 30.001 | 10.250 | 3.906 | 17.670 | 19.768 |
| 284.15 | 0.14 | 30.001 | 10.125 | 3.715 | 17.770 | 19.902 |
| 284.16 | 0.15 | 30.001 | 10.500 | 3.906 | 17.422 | 19.503 |
| 284.17 | 0.16 | 30.001 | 10.500 | 3.256 | 17.422 | 19.503 |
| 284.18 | 0.17 | 30.001 | 10.375 | 3.160 | 17.521 | 19.637 |
| 284.19 | 0.18 | 30.001 | 10.625 | 3.304 | 17.323 | 19.402 |
| 284.20 | 0.19 | 30.001 | 10.625 | 3.184 | 17.323 | 19.402 |
| 284.21 | 0.20 | 30.001 | 10.625 | 3.184 | 17.323 | 19.402 |

$\omega_{63.2\%} =$ ______ %

$\tau =$ between ______ and ______ ms

9. It is possible to interpolate the time constant $\tau$ value using the following formula:

$$\tau = 10\frac{(\omega_{63.2\%} - \omega_{LOW})}{(\omega_{UP} - \omega_{LOW})} + t_{LOW} \tag{36}$$

where $\tau$ is the time constant (ms)

$t_{LOW}$ is the time associated with $\omega_{LOW}$ (ms)

$\omega_{LOW}$ is the speed at time interval below $\omega_{63.2\%}$

$\omega_{UP}$ is the speed at time interval above $\omega_{63.2\%}$

To calculate the time constant $\tau$, you will thus have to first determine $\omega_{LOW}$, $\omega_{UP}$, and $t_{LOW}$:

$\omega_{LOW}$ = ______ rad/s

$\omega_{UP}$ = ______ rad/s

$t_{LOW}$ = ______ ms

You can now interpolate the time constant $\tau$ from these values and using Equation (26). Record the result below.

$\tau$ = ______ ms

10. Using the time constant approximation method and the corresponding example, complete Table 17 by calculating the time constants $\tau$ and steady state speeds $\omega_{SS}$ for the step responses acquired in Steps 4) to 7).

**Table 17. Calculated time constants and steady state speeds.**

| Reference Speed $\omega_{REF}$ (%) | Gain $K_p$ | Time Constant $\tau$ (ms) | Steady State Speed $\omega_{SS}$ (%) |
|---|---|---|---|
| 30 | 1 | | |
| 30 | 2 | | |

11. Describe the effects of increasing the gain $K_p$ on the time constant $\tau$ and the steady state error.

______________________________________________

______________________________________________

## Observing the effects of load disturbances

In this section, you will measure the effects of a load disturbance $D$ on the motor speed $\omega$ by comparing the motor speed with a load to the motor speed without load. You will measure the effects of load disturbances on the speed error. You will observe the effects of gain $K_p$ variations on the servo system operating with and without disturbances.

**12.** Setup the servo system as shown in Table 18.

**Table 18. Settings for observing the effects of disturbances.**

| **Function Generator** | | **Trend Recorder** | |
|---|---|---|---|
| Signal Type | *Constant* | Reference | *Checked* |
| Frequency | *1 Hz* | Speed | *Checked* |
| Amplitude | *0%* | Current | *Unchecked* |
| Offset | *50%* | Voltage | *Unchecked* |
| Power | *Off* | Error | *Unchecked* |
| **PID Controller** | | $K_p$ x Error | *Unchecked* |
| Gain ($K_p$) | *1* | Error Sum / $t_i$ | *Unchecked* |
| Integral Time ($t_i$) | *Inf (Off)* | $t_d$ x Delta Error | *Unchecked* |
| Derivative Time on E ($t_d$ (E)) | *0* | PID Output | *Unchecked* |
| Derivative Time on PV ($t_d$ (PV)) | *0* | Display Type | *Sweep* |
| Timebase | *10 ms* | Show and Record Data | *On* |
| Anti-Reset Windup | *On* | Measured Gain (rpm) | *3000* |
| Upper Limit | *100%* | Measured Gain (A) | *7* |
| Lower Limit | *-100%* | Measured Gain (V) | *48* |
| Open or Closed Loop | *Closed* | | |
| **PV Speed Scaling** | | | |
| 100% Value | *3000 rpm* | | |

**13.** Set the function generator **Power** switch to ON.

**14.** Set the gain $K_p$ value to 1 and record the unloaded motor speed $\omega$. Enter this value in Table 19 below.

**15.** Set the gain $K_p$ value to 2 and record the unloaded motor speed $\omega$. Enter this value in Table 19 below.

16. Set the gain $K_p$ value back to 1 and adjust the braking set screw (see Figure 31) until the motor is engaged in full breaking and its speed is approximately 300 rpm ± 50. Enter the loaded motor speed $\omega_{LOAD}$ in Table 19 below.

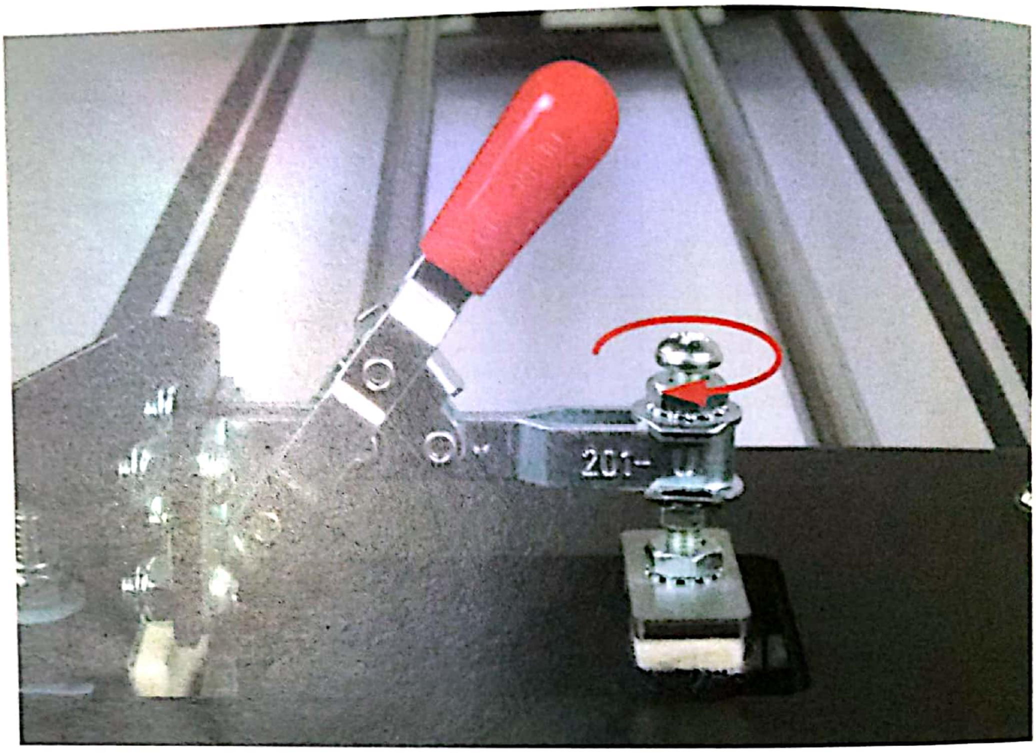


**Figure 31. Breaking set screw adjustment for loaded motor measurements.**

17. Set the gain $K_p$ value to 2 and leave the full breaking engaged. Enter the loaded motor speed $\omega_{LOAD}$ in Table 19 below.

18. Calculate the error value with and without load and enter both values in Table 19, knowing that the reference speed $\omega_{REF}$ value is 1500 rpm. Note that all values in Table 19 are expressed in rpm.

**Table 19. Calculated speed and error with and without load.**

| | Speed $\omega_{LOAD}$ | Error $\omega_{REF} - \omega_{LOAD}$ | Speed $\omega$ | Error $\omega_{REF} - \omega$ |
|---|---|---|---|---|
| | With load | | Without load | |
| Gain $K_p = 1$ | | | | |
| Gain $K_p = 2$ | | | | |

19. What is the effect of applying a load disturbance $D$ on the error value?

**20.** What happens to the error value under load as the gain $K_p$ increases from 1 to 2?

---

**21.** Disengage the break completely and then apply the break again, observing the motor speed on the strip chart recorder. Describe the motor transient response.

---

**22.** Disengage the break completely.

## Servo system oscillation

In this section, you will determine at which moment oscillation begins on the servo system.

**23.** Make sure the settings are initially as shown in Table 20 below:

**Table 20. Settings for measuring the beginning of oscillation.**

| **Function Generator** | | **Trend Recorder** | |
|---|---|---|---|
| Signal Type | *Square* | Reference | *Checked* |
| Frequency | *0.25 Hz* | Speed | *Checked* |
| Amplitude | *10%* | Current | *Unchecked* |
| Offset | *20%* | Voltage | *Unchecked* |
| Power | *Off* | Error | *Unchecked* |
| **PID Controller** | | $K_p$ x Error | *Unchecked* |
| Gain ($K_p$) | *1* | Error Sum / $t_i$ | *Unchecked* |
| Integral Time ($t_i$) | *Inf (Off)* | $t_d$ x Delta Error | *Unchecked* |
| Derivative Time on E ($t_d$ (E)) | *0* | PID Output | *Unchecked* |
| Derivative Time on PV ($t_d$ (PV)) | *0* | Display Type | *Sweep* |
| Timebase | *10 ms* | Show and Record Data | *On* |
| Anti-Reset Windup | *On* | Measured Gain (rpm) | *3000* |
| Upper Limit | *100%* | Measured Gain (A) | *7* |
| Lower Limit | *-100%* | Measured Gain (V) | *48* |
| Open or Closed Loop | *Closed* | | |
| **PV Speed Scaling** | | | |
| 100% Value | *3000 rpm* | | |

**24.** Make sure the function generator **Power** switch to ON.

**25.** Slowly increase the gain $K_p$ in increments of 1 until the system begins to oscillate. Enter the gain value at which oscillation begins.

$K_p =$ ______

## CONCLUSION

In this exercise, you familiarized yourself with the transient behavior of a servo system in a closed loop speed control. You learned the effects of controller gain variations on the effective time constant of the servo system speed as well as on the steady state error. You observed the effects of disturbances on the operation of a closed loop servo system.

## REVIEW QUESTIONS

1. Consider a dc servo motor that has a time constant $\tau$ of 25 ms, a speed constant $K$ value of 5 (rad/s)/V and a scaling factor of 0.139. Calculate the motor closed loop servo system effective time constant $\tau_{SYS}$ for a step change in reference speed when the gain $K_p$ is set to 2. Refer to Equation (28) and Equation (29).

______________________________________________

______________________________________________

2. Using Equation (28), plot the step response of a closed-loop servo speed-control system. The gain $K_p$ value is 2, (no integral action is used), the motor $K$ value [(rad/s)/V] is 5, the scaling factor is 0.139, the motor time constant is 50 ms, and the step change is 0 to 1500 rpm.