# Following Error in a Linear Position Control System

**EXERCISE OBJECTIVE**

When you have completed this exercise, you will know what a following error is and how to measure it. You will be able to describe the effects of proportional, integral and derivative action on the following error of a servo system. You will know how to tune a PID controller to minimize the following error in linear position control systems.

**DISCUSSION OUTLINE**

The Discussion of this exercise covers the following points:

- PID controller output with triangular ramp error
- Following error
- Tuning the PID controller to minimize the following error

**DISCUSSION**

## PID controller output with triangular ramp error

To provide insight into the operation of proportional, integral and derivative action, it is useful to be able to analyze the controller output for various error signals. In Exercise 8, a square wave error was analyzed. In this exercise, a triangular error is analyzed.

Figure 49 displays the servo system PID controller output. The plots were generated using the **Position Loop (Device Controlled)** mode with the rail incremental encoder and the triangle wave generator. This created a ramping error of ±100%/s in the resulting plotted waveforms. The gain $K_p$ value was of 2, the integral $t_i$ value of 0.1 s, and the derivative $t_d$ value of 0.1 s.

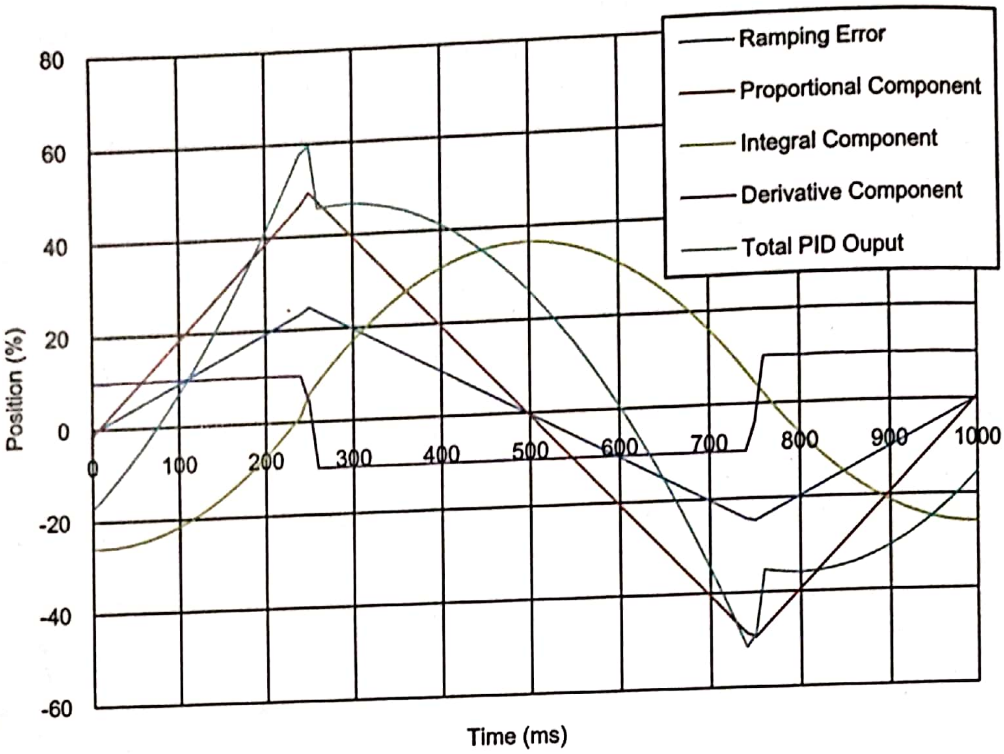


**Figure 49. Servo system PID controller outputs.**

Figure 49 shows that the peak proportional output $P_{OUT}$ is 50% and the peak error is 25%. The peak proportional output can be calculated using Equation (65). The proportional output is thus shown to be equal to 50% (2×25).

The integral output shows a rise from about –26% to 5%. This 31% rise can be calculated using Equation (66). Considering that the positive ramping error has a slope of +100%/s, the calculated integral component in the example is equal to 31.25%.

The derivative output shows a range of about ±10%. This value can be calculated using Equation (68). The derivative output is thus shown to be equal to 10% (0.1×100).

## Following error

Position reference signals are not always stationary. Motion control systems often have to track a changing position reference signal. This signal could change in a manner similar to a triangle or saw tooth wave. A welding machine, for example, might be required to move along a linear track at a constant speed, following extremely closely the required position.

These circumstances require integral action not only to overcome the effects of static friction, but also to eliminate the error that would have existed between the position reference and the actual position if the controller had used proportional only or proportional plus derivative action. The error that occurs when a motion control system has to track a changing position reference signal is called the **following error**.

## Tuning the PID controller to minimize the following error

In order to minimize the following error in a control system, the PID controller must be tuned accordingly. The procedure for tuning the PID controller is given below:

1. Begin by increasing the proportional gain $K_p$ to a value that just starts to cause the position signal to oscillate.

2. Add derivative and integral action to stop the oscillation. Derivative action stabilizes the position signal, but tends to increase the following error, while integral action reduces the time necessary to correct the error but tends to increase instability.

3. Repeat steps 1 and 2 until the error value reaches a minimum, i.e., when it cannot be reduced to a lower value without causing the system to become unstable.

Figure 50 shows a captured response from the Digital Servo system. The proportional gain $K_p$ was set to 3, while the integral was set to infinite, effectively turning it off. The derivative time $t_d$ was changed approximately midway from 0 to 0.5. The position reference was a triangle signal, amplitude 70%, and frequency 0.25 Hz.

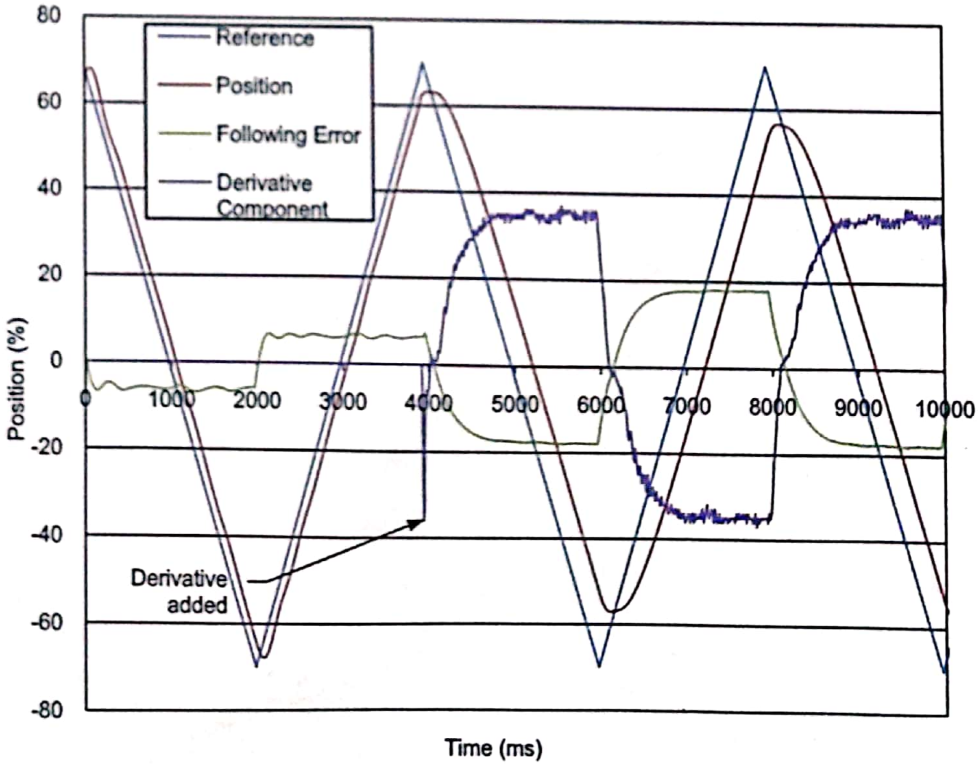


**Figure 50. Response to triangle position reference.**

Notice in Figure 50 how the following error increases when derivative action is added. This can be observed in two ways. Firstly, the amplitude of the following error plot increases. Secondly, the gap between the position signal and the position reference signal also increases. Both of these changes happen as direct consequence of the addition of derivative action.

The explanation to this phenomenon is that, since the position signal is tracking the position reference signal, an error must exist to provide the controller output with a finite value that results in a voltage that in turn drives the motor. If the output was 0, there would be no motion. When derivative is added, it is subtracted from the proportional component resulting from the product of error and controller gain. Therefore, for the motor speed and voltage to be maintained, the following error must increase.

## PROCEDURE OUTLINE

The Procedure is divided into the following sections:

- Setup and connections
- Plotting the position reference and position, the following error, and the speed

## PROCEDURE

### Setup and connections

In this section, you will setup the Digital Servo for studying the following error in a linear position control system and set the platform to mid-position.

**In this experiment, the platform will be engaged and the platform loaded. To prevent equipment damage, it is extremely important to ensure that the position device controller application is used at all times.**

1. Set up the servo system for linear position control of the platform as in Exercise 8.

2. Run LVServo, and click on the **Device Controlled** button in the **Position Loop** menu. Make sure the settings are initially as shown in Table 36:

**Table 36. Settings for data acquisition.**

| | | | |
|---|---|---|---|
| **Function Generator** | | **Trend Recorder** | |
| Signal Type | *Triangle* | Reference | *Checked* |
| Frequency | *0.2 Hz* | Position | *Checked* |
| Amplitude | *0%* | Speed | *Unchecked* |
| Offset | *0%* | Current | *Checked* |
| Power | *Off* | Voltage | *Unchecked* |
| **PID Controller** | | Error | *Unchecked* |
| Gain ($K_p$) | *2* | $K_p$ x Error | *Unchecked* |
| Integral Time ($t_i$) | *Inf (off)* | Error Sum / $t_i$ | *Unchecked* |
| Derivative Time on E ($t_d$ (E)) | *0* | $t_d$ x Delta Error | *Unchecked* |
| Derivative Time on PV ($t_d$ (PV)) | *0* | PID Output | *Unchecked* |
| Timebase | *10 ms* | Display Type | *Sweep* |
| Anti-Reset Windup | *On* | Show and Record Data | *On* |
| Upper Limit | *100%* | Measured Gain (rpm) | *3000* |
| Lower Limit | *-100%* | Measured Gain (A) | *7* |
| Open or Closed Loop | *Closed* | Measured Gain (V) | *48* |
| **PV Speed Scaling** | | **Encoders** | |
| 100% Value | *5000 cnt* | Motor or Rail | *Rail* |
| -100% Value | *-5000 cnt* | | |

3. Set the platform to the mid position (see Exercise 8 for the complete procedure) and reset the position to 0.

4. Set the function generator **Power** switch to ON.

### Plotting the position reference and position, the following error, and the speed

In this section, you will plot the position reference and position, the following error, and the speed of the Digital Servo using data recorded during a step response.

5. Adjust the amplitude of the function generator, the proportional gain $K_p$, the integral time $t_i$, and the derivative time $t_d$ following the step sequence shown in Table 37. After each step in the table, wait several seconds to observe the effect of the change. During the steps, observe the current drawn by the motor and its stability through the steps.

**Table 37. Variations in amplitude, proportional gain $K_p$, integral time $t_i$, and derivative time $t_d$.**

| Step Number | Amplitude (%) | $K_p$ | $t_i$ | $t_d$ (PV) |
|---|---|---|---|---|
| 1 | 0 | 2 | Inf | 0 |
| 2 | 5 | 2 | Inf | 0 |
| 3 | 10 | 2 | Inf | 0 |
| 4 | 15 | 2 | Inf | 0 |
| 5 | 20 | 2 | Inf | 0 |
| 6 | 25 | 2 | Inf | 0 |
| 7 | 30 | 2 | Inf | 0 |
| 8 | 35 | 2 | Inf | 0 |
| 9 | 40 | 2 | Inf | 0 |
| 10 | 45 | 2 | Inf | 0 |
| 11 | 50 | 2 | Inf | 0 |
| 12 | 50 | 4 | Inf | 0 |
| 13 | 50 | 6 | Inf | 0 |
| 14 | 50 | 8 | Inf | 0 |
| 15 | 50 | 8 | Inf | 0.1 |
| 16 | 50 | 10 | Inf | 0.1 |
| 17 | 50 | 12 | Inf | 0.1 |
| 18 | 50 | 14 | Inf | 0.1 |
| 19 | 50 | 16 | Inf | 0.1 |
| 20 | 50 | 16 | 0.1 | 0.1 |
| 21 | 50 | 16 | 0.09 | 0.1 |
| 22 | 50 | 16 | 0.08 | 0.1 |
| 23 | 50 | 16 | 0.07 | 0.1 |
| 24 | 50 | 16 | 0.06 | 0.1 |
| 25 | 50 | 16 | 0.05 | 0.1 |
| 26 | 50 | 16 | 0.04 | 0.1 |
| 27 | 50 | 16 | 0.03 | 0.1 |
| 28 | 50 | 16 | 0.02 | 0.1 |
| 29 | 50 | 16 | 0.02 | 0.2 |
| 30 | 50 | 16 | 0.01 | 0.2 |
| 31 | 50 | 16 | 0.009 | 0.2 |
| 32 | 50 | 16 | 0.009 | 0.25 |
| 33 | 50 | 16 | 0.008 | 0.25 |
| 34 | 50 | 16 | 0.008 | 0.3 |
| 35 | 50 | 16 | 0.007 | 0.3 |
| 36 | 50 | 16 | 0.006 | 0.3 |
| 37 | 50 | 16 | 0.006 | 0.35 |
| 38 | 50 | 16 | 0.005 | 0.35 |
| 39 | 50 | 16 | 0.004 | 0.35 |
| 40 | 50 | 16 | 0.004 | 0.4 |
| 41 | 50 | 16 | 0.004 | 0.5 |
| 42 | 50 | 16 | 0.004 | 0.6 |

6. With the trend recorder, acquire a complete cycle and export it to a spread sheet.

7. Set the function generator **Power** switch to OFF.

8. Plot the following parameters in three different plots:
   - Position reference and position
   - Following error
   - Speed

## CONCLUSION

In this exercise, you learned what a following error is and how to measure it. You learned how to tune the PID controller to minimize the following error. You were able to observe the effects of controller gain, derivative, and integral variations on a servo system following error.

## REVIEW QUESTIONS

1. The following three plots display the position, position reference, and speed signals generated by the Digital Servo system. The three sets of tuning constants are given in the table. By completing the table, match the correct plot with the tuning constants.

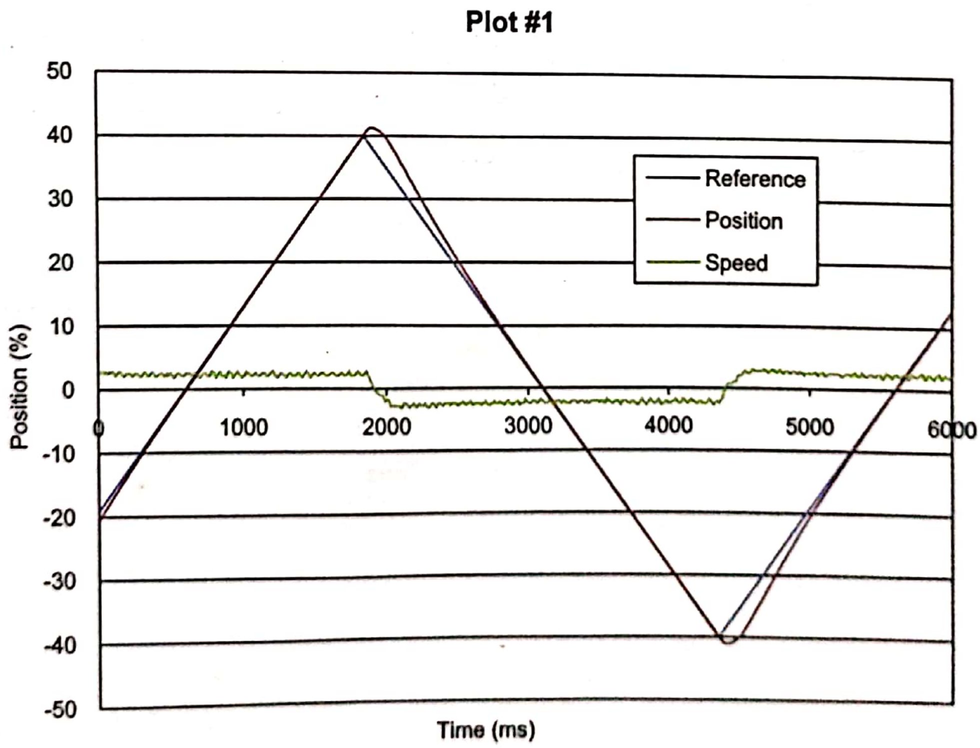


**Figure 51. Digital Servo position reference, position, and speed versus time plot 1.**

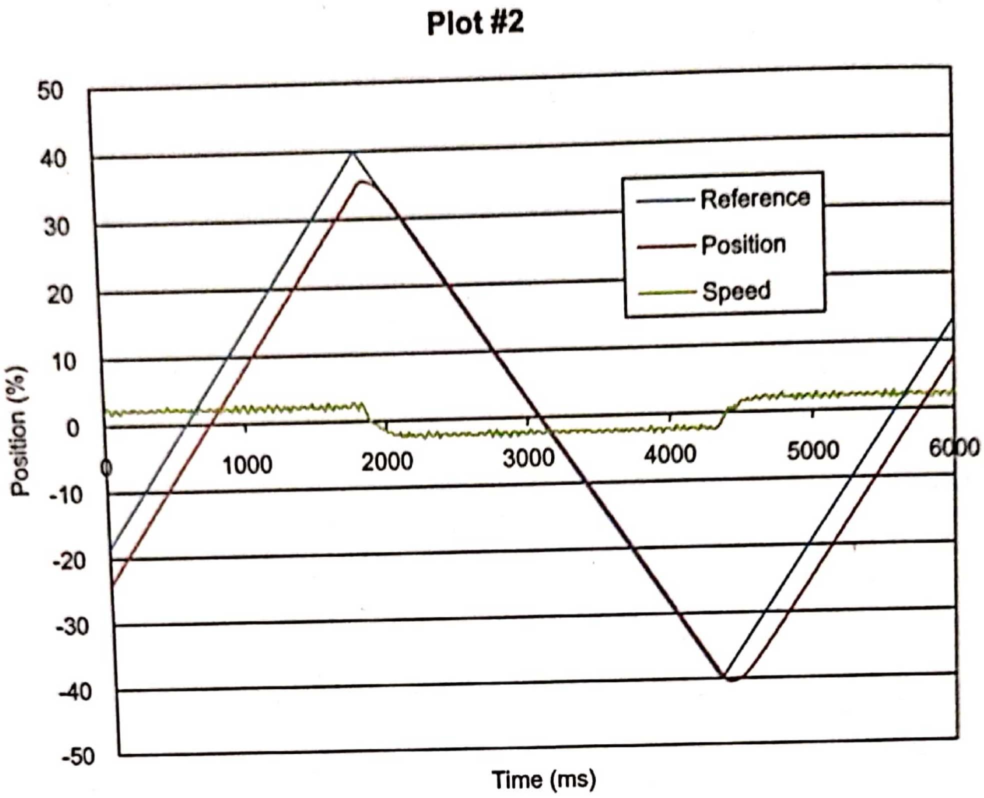


Figure 52. Digital Servo position reference, position, and speed versus time plot 2.

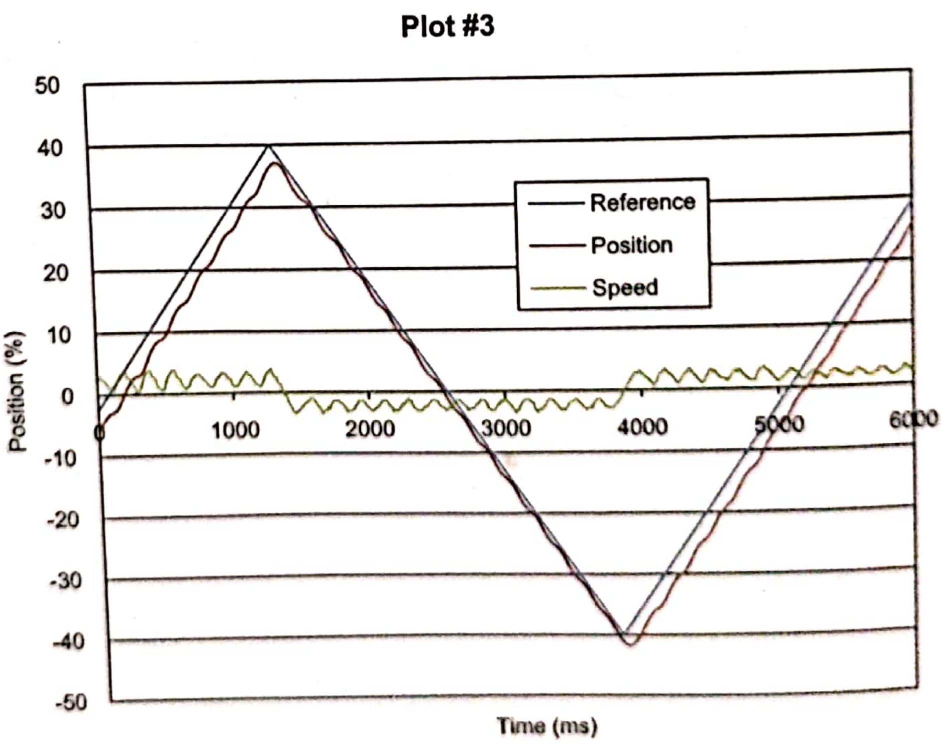


Figure 53. Digital Servo position reference, position, and speed versus time plot 3.

**Table 38.** $K_p$, $t_i$, and $t_d$ **corresponding to the three plots in Figure 51, Figure 52, and Figure 53.**

| Plot number | $K_p$ | $t_i$ | $t_d$ |
|---|---|---|---|
| | 8 | ∞ | 0.5 |
| | 8 | 0.08 | 0.5 |
| | 8 | ∞ | 0 |

2. Figure 54 shows plots that were made by generating a ramping error with a negative slope into the PID controller. Using the plot, determine the proportional gain $K_p$,the integral time $t_i$, and the derivative time $t_d$.

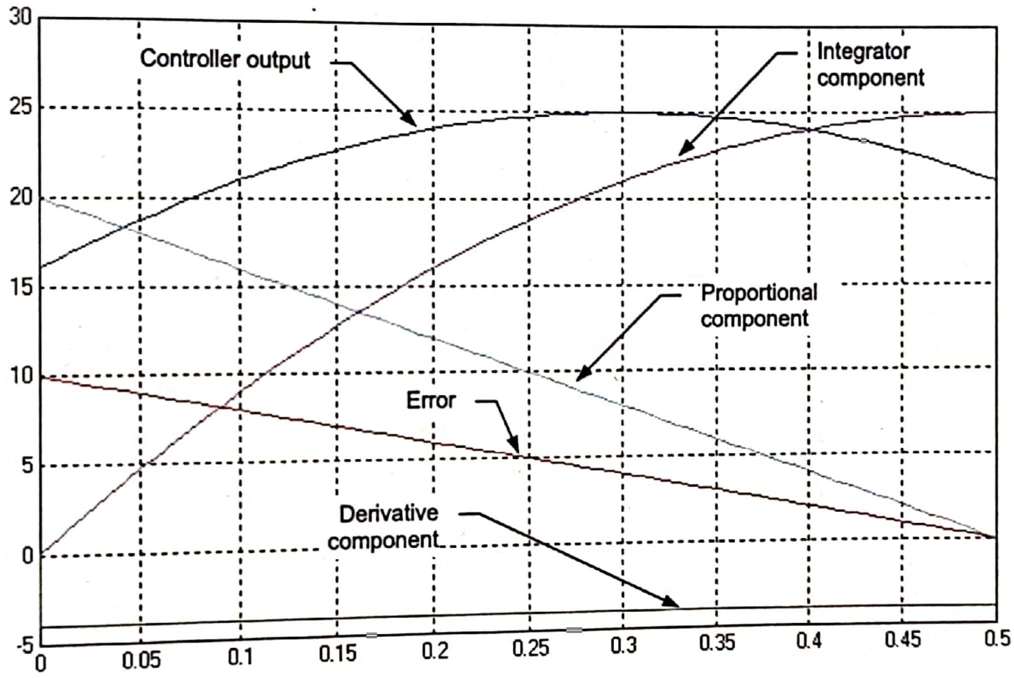


**Figure 54. PID components for a negative slope ramping error.**