

## Motor Shaft Angular Position Control

### EXERCISE OBJECTIVE

When you have completed this exercise, you will be able to associate the pulses generated by a position sensing incremental encoder with the angular position of a motor shaft positioning system. You will know how to analyze and model a servo position control system. You will be familiar with the effects of changes in the damping ratio on the transient operation of the servo positioning system used to control the flywheel angular position. You will also know how to do a basic tuning.

### DISCUSSION OUTLINE

The Discussion of this exercise covers the following points:

- Angular position control block diagram and fundamentals
- Angular position control system equations
- Damping fundamentals
- Damping ratio cases analysis
- Digital Servo damping ratio and damped frequency
- The PID controller
- Servo-system manual tuning

### DISCUSSION

#### Angular position control block diagram and fundamentals

The motor shaft incremental encoder generates 4000 counts per revolution. The Digital Servo system default range for position sensing measurement is  $\pm 5000$  counts, which is the  $\pm 100\%$  position. A 100% position travel is thus equivalent to 1.25 motor shaft revolutions ( $5000/4000$ ). An angular position of 90 degrees, for example, is equivalent to 1000 counts ( $(90/360) \times 4000$ ). The resulting position in percentage is 20% ( $(1000/5000) \times 100$ ).

Figure 32 shows the Digital Servo positioning system first order block diagram. The controller is set to use proportional action only, which means that the controller gain  $K_C$  is equivalent to the proportional gain  $K_p$ .

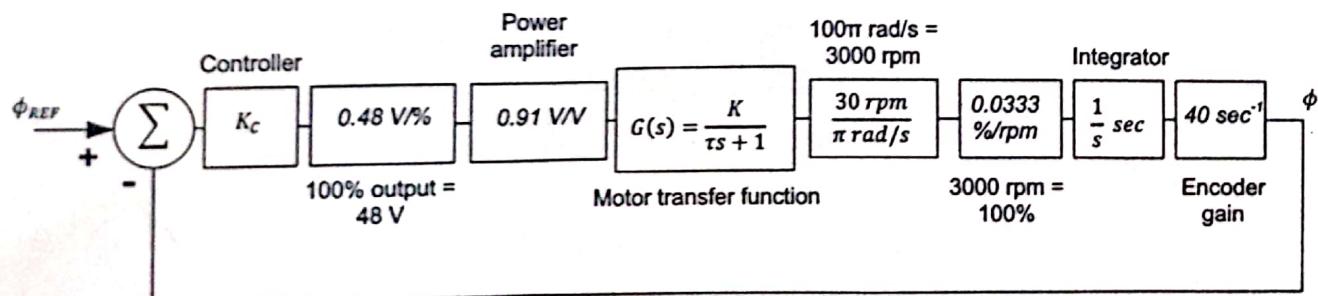


Figure 32. Block diagram of a position-control servo system with motor shaft incremental encoder.

Notice the many similarities with the first order model for the closed loop speed control system. The motor transfer function and the scaling blocks are the same. The difference is that an integrator term  $1/s$  and an encoder block have been added. The integrator term essentially models the conversion from velocity to position. The encoder block provides another scaling factor. For the motor shaft positioning system the encoder gain is 40. This gain is calculated as follows:

At 3000 rpm ( $100\pi$  rad/s or 100% motor speed), the encoder generates 5000 counts (1.25 revolutions) in a time of:

$$t = \frac{5000 \text{ counts in rad}}{\text{speed at } 100\%} = \frac{1.25 \times 2\pi}{100\pi} = 0.025\text{s} \quad (37)$$

0.025 s thus represents the time needed to integrate, i.e., to accumulate, 5000 counts at 100% motor speed. This time value is the reciprocal of the encoder gain. This results in an encoder gain of  $1/0.025$  s or  $40 \text{ s}^{-1}$ .

We have calculated in previous exercises that the scaling factor is equal to 0.139. The addition of the incremental encoder gain of 40 in the block diagram results in an overall scaling factor of 5.56 ( $0.139 \times 40$ ).

A simplified block diagram for the proportional only position control servo system with the motor shaft incremental encoder is shown below:

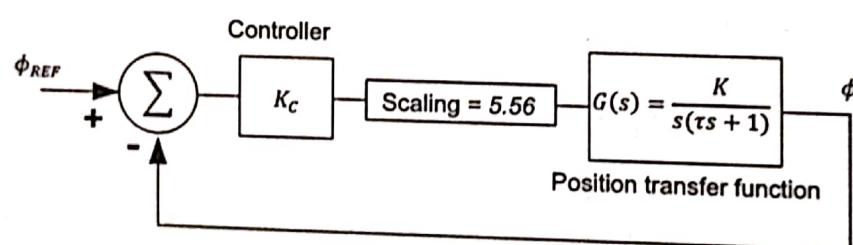


Figure 33. Simplified position control servo system block diagram with overall scaling factor.

For analysis purposes, a further simplification can be made by combining the scaling factor 5.56, controller gain  $K_c$ , and the speed constant  $K$  into the term  $K'$ . Figure 34 shows the resulting diagram:

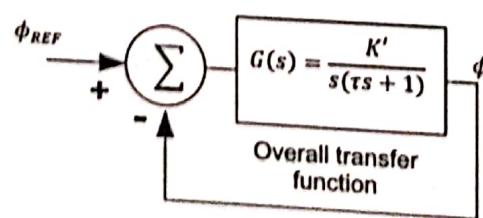


Figure 34. Simplified position-control servo system block diagram.

### Angular position control system equations

The transfer function that relates position to reference position for the block diagram in Figure 34 is given below. The development of the transfer function is given in Appendix B.

$$\phi = \phi_{REF} \left( \frac{1}{\frac{\tau}{K'} s^2 + \frac{1}{K'} s + 1} \right) \quad (38)$$

If we include the step input in the equation, we obtain:

$$\phi = \phi_{REF} \frac{1}{s} \left( \frac{1}{\frac{\tau}{K'} s^2 + \frac{1}{K'} s + 1} \right) \quad (39)$$

In Equation (38) and Equation (39), the ratio  $K_{SYS}$  is equal to one. The step input is  $E(s) = 1/s$  and the equation form is:

$$H(s) = \frac{K_{SYS}}{\frac{s^2}{\omega_n^2} + \frac{2\rho}{\omega_n} s + 1} \quad (40)$$

where  $\omega_n$  is the system natural frequency  
 $\rho$  is the damping ratio

The transfer function for the system can be expressed in standard format as shown below:

$$\phi = \phi_{REF} \left( \frac{\frac{K'}{\tau}}{s^2 + \frac{1}{\tau} s + \frac{K'}{\tau}} \right) \quad (41)$$

With the step input included, we obtain:

$$\phi = \phi_{REF} \frac{1}{s} \left( \frac{1}{s^2 + 2\rho\omega_n s + \omega_n^2} \right) \quad (42)$$

In Equation (41) and Equation (42), the ratio  $K_{SYS}$  is equal to one and the equation form is:

$$H(s) = \frac{K_{SYS}\omega_n^2}{s^2 + 2\rho\omega_n s + \omega_n^2} \quad (43)$$

For the Digital Servo system, the equation when  $K_{SYS} = 1$  results in:

$$K_{SYS}\omega_n^2 = 1\omega_n^2 = \frac{K'}{\tau} \quad (44)$$

We thus obtain:

$$\omega_n = \sqrt{\frac{K'}{\tau}} \quad (45)$$

And:

$$2\rho\omega_n = \frac{1}{\tau} \quad (46)$$

$$\rho = \frac{1}{2\omega_n\tau} = \frac{1}{2\sqrt{\frac{K'}{\tau}} \cdot \tau} = \frac{1}{2\sqrt{K'\tau}} \quad (47)$$

### Damping fundamentals

The damping ratio  $\rho$  is very important to the analysis of angular position control step response. The higher the damping ratio, the more damped a system is. Increasing the damping ratio value tends to increase the step response rise time and reduce the overshoot, but can also affect the settling time and the steady state error value.

A step response overshoot corresponds to the highest point at which the response signal exceeds the reference point. The less a system is damped, the higher this value and the greater the overshoot amplitude. In servo system control, reducing the overshoot is desirable and in some cases, crucial, to prevent more delicate systems from being damaged.

A step response rise time is the time needed for a step response to rise from a specified low value (usually 10% of the reference value) to a specified high value (usually 90% of the reference value). A high rise time is important in servo system control to ensure that the process responds quickly, but can also be problematic as it is often associated with overshoot.

A step response settling time corresponds to the time required for the signal before it "settles" within a given range (10% for example) of the reference point. In servo system control, obtaining a low settling time value is usually desired as the system thus takes less time before reaching its steady state.

A step response steady state error value (also called ringing) corresponds to the amplitude of the normal oscillation that occurs when the steady state is reached. In servo system control, a low steady state error value is desirable to ensure that the actual measured value is close at all times to the reference value.

### Damping ratio cases analysis

We will now examine three cases, each using a different damping ratio to determine the complex state of the variable  $s$ . Case 1 will deal with a damping ratio  $\rho > 1$ , case 2 will deal with a damping ratio  $\rho = 1$ , and case 3 will deal with a damping ratio  $\rho < 1$ . In all three cases,  $K_{sys}$  is equal to one, the input step

is  $e(t) = A \cdot u(t)$ , and the Laplace transform is  $E(s) = A/s$  where  $A = 1$ . The equation form of the three cases is:

The mathematical equations in this section demonstrate the effects of the damping ratio on the time constants. They are proposed only as additional information to the reader and are not essential to the comprehension of this exercise.

$$E(s)H(s) = \frac{A}{s} \left( \frac{K_{SYS}}{\frac{s^2}{\omega_n^2} + \frac{2\rho}{\omega_n} s + 1} \right) \quad (48)$$

This becomes:

$$E(s)H(s) = \frac{A}{s} \left[ \frac{K_{SYS} s_1 s_2}{(s - s_1)(s - s_2)} \right] = \frac{A}{s} \left[ \frac{K_{SYS}}{(1 + T_1 s)(1 + T_2 s)} \right] \quad (49)$$

### Case 1

In this case,  $\rho > 1$ , which means that the system is overdamped and has real separate poles ( $s_1 = -\frac{1}{T_1}, s_2 = -\frac{1}{T_2}$ ) whose real form is:

$$s = -\rho \omega_n \pm \omega_n \sqrt{\rho^2 - 1} \quad (50)$$

The equation for case 1 is thus:

$$\frac{\phi(t)}{\phi_{REF}(t)} = K_{SYS} A + \frac{K_{SYS} A}{T_2 - T_1} \left( T_1 e^{-\frac{t}{T_1}} - T_2 e^{-\frac{t}{T_2}} \right) \quad (51)$$

### Case 2

In this case,  $\rho = 1$ , which means that the system is critically damped and has superposed poles ( $s_1 = s_2 = -\frac{1}{T}$ ) whose real form is:

$$s_1 = s_2 = -\omega_n \quad (52)$$

The equation for case 2 is thus:

$$\frac{\phi(t)}{\phi_{REF}(t)} = K_{SYS} - K_{SYS} \left( 1 + \frac{1}{T} \right) e^{-\frac{t}{T}} \quad (53)$$

### Case 3

In this case,  $\rho < 1$ , which means that the system is underdamped and has conjugated complex system poles whose complex form is:

$$s = -\rho \omega_n \pm j \omega_n \sqrt{1 - \rho^2} \quad (54)$$

The equation for case 3 is thus:

$$\frac{\phi(t)}{\phi_{REF}(t)} = K_{SYS}A - \frac{K_{SYS}A}{\sqrt{1-\rho^2}} e^{-\rho\omega_n t} \sin \left[ \omega_n \sqrt{1-\rho^2} \cdot t - \tan^{-1} \left( \frac{\sqrt{1-\rho^2}}{-\rho} \right) \right] \quad (55)$$

$$\frac{\phi(t)}{\phi_{REF}(t)} = K_{SYS}A - \frac{K_{SYS}A}{\sqrt{1-\rho^2}} e^{-\rho\omega_n t} \sin \left[ \rho\omega_n \sqrt{\left(\frac{1}{\rho}\right)^2 - 1} \cdot t + \cos^{-1} \rho \right] \quad (56)$$

Note that the following term must be evaluated in the second quadrant (positive sin, negative cos):

$$Angle = \tan^{-1} \left( \frac{\sqrt{1-\rho^2}}{-\rho} \right) \quad (57)$$

Using reverse Laplace transforms for a step response of  $s(t) = a \cdot u(t)$ , where  $A = 1$  and  $K_{SYS} = 1$ , we find that the above equation is equivalent to:

$$\phi = \phi_{REF} \left\{ 1 - \frac{1}{\sqrt{1-\frac{1}{4K'\tau}}} e^{\frac{-t}{2\tau}} \sin \left[ \frac{1}{2\tau} \sqrt{4K'\tau - 1} \cdot t + \cos^{-1} \left( \frac{1}{2\sqrt{K'\tau}} \right) \right] \right\} \quad (58)$$

Equation (58) can be written using the parameters  $\omega_n$  and  $\rho$  as shown below:

$$\phi = \phi_{REF} \left[ 1 - \frac{1}{\sqrt{1-\rho^2}} e^{-\rho\omega_n t} \sin \left( \omega_n \sqrt{1-\rho^2} \cdot t + \cos^{-1} \rho \right) \right] \quad (59)$$

Equation (59) is valid for  $0 < \rho < 1$ . See Appendix B for the equations development.

Figure 35 illustrates the previous calculations with a 40% reference position step change for the motor of the Digital Servo system. The plot was obtained with a gain  $K_p$  of 2 and a speed constant  $K$  of 7.6.

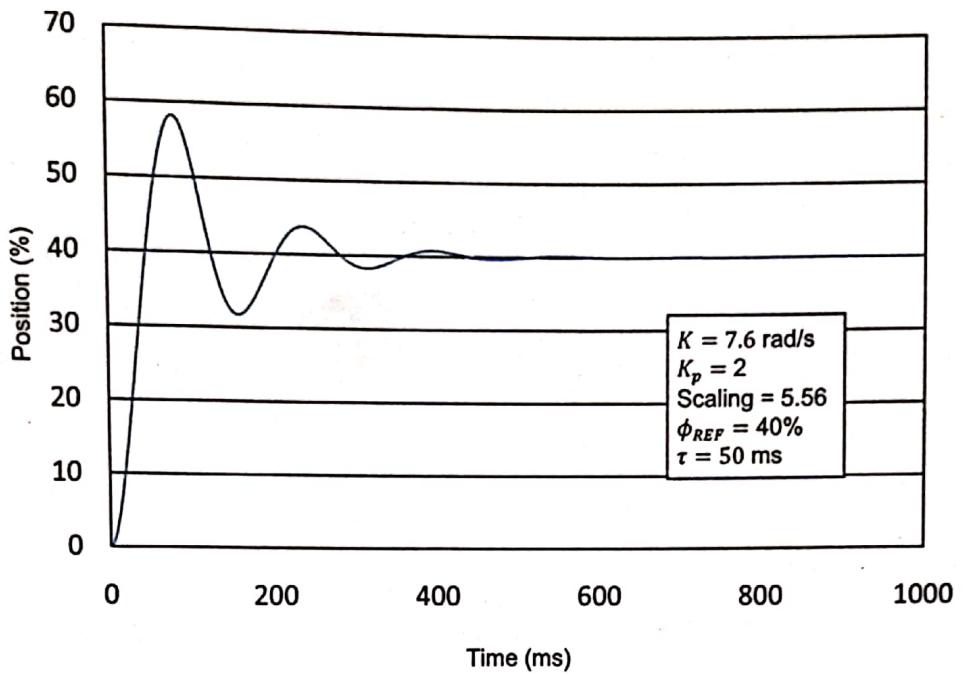


Figure 35. Step response to a 40% step change in reference position  $\phi_{REF}$ .

Figure 35 is an approximation of the true response as the calculation is a simplified first-order model. This type of response is categorized as underdamped due to the response rising above and falling below the steady state value for a number oscillatory cycles of decreasing amplitude. The step response shows such overshoot and undershoot cycles only when the damping ratio  $\rho$  is less than 1.

### Digital Servo damping ratio and damped frequency

For the Digital Servo system, the damping ratio is equal to:

$$\rho = \frac{1}{2\sqrt{K'\tau}} \quad (60)$$

Remember that  $K'$  is the product of the gain  $K_p$  by the scaling factor 5.56, and the motor speed constant  $K$ . Thus, as the gain increases,  $K'$  increases too and consequently, the damping ratio  $\rho$  decreases.

The equation below shows that the damped frequency  $\omega_d$  is equal to:

$$\omega_d = \frac{1}{2\tau} \sqrt{4K'\tau - 1} \quad (61)$$

This means that, as the gain  $K_p$  increases, the damped frequency  $\omega_d$  increases too. This equation can be expressed using  $\omega_n$  and  $\rho$ , as shown below:

$$\omega_d = \omega_n \sqrt{1 - \rho^2} \quad (62)$$

As the gain  $K_p$  decreases, the damping ratio  $\rho$  increases. When the damping ratio reaches a value of 1, the step response no longer exhibits overshoot. This damping ratio value results in a critically damped step response (case 2 above). A critically damped response thus has the fastest rise to the steady state and produces no overshoot. Further decreasing the gain  $K_p$  results in an overdamped system. Figure 36 illustrates the response of the servo position system for gains  $K_p$  of 0.1185, 2, and 10. Notice that the overshoot and damped frequency increases with the gain. The results displayed in Figure 36 are calculated using Equation (58). These results approximate the actual servo positioning system.

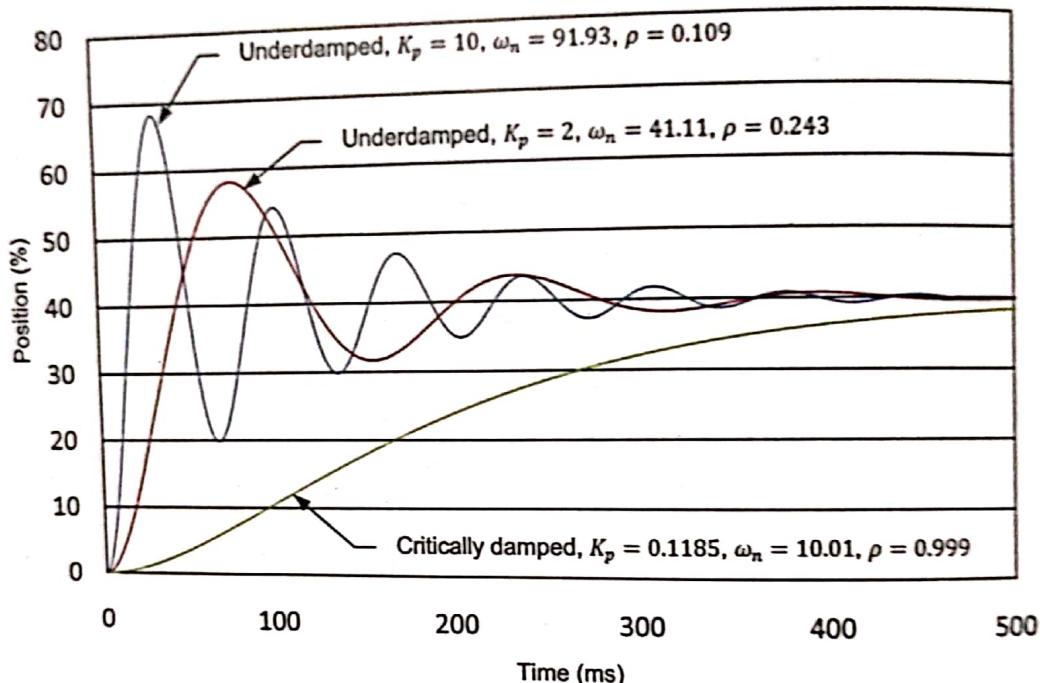


Figure 36. Underdamped and critically damped step responses.

Note that this exercise's discussion is true for an angular position control system in proportional only mode. The addition of integral or derivative terms results in a much more complex time domain analysis. We will now see how these terms are calculated in the Digital Servo system.

### The PID controller

In order to manage the proportional, integral and derivative terms, the Digital Servo system uses a PID controller. A PID controller (Proportional, Integral, Derivative) is a control loop feedback controller that attempts to correct the error between a measured process variable and a desired set point by calculating and then instigating a corrective action that can adjust the process to keep the error minimal. In the case of the Digital Servo system, the measured variable is either the motor speed (speed loop) or the platform position (position loop). The Digital Servo controller thus measures the error between the reference speed or position reference and the actual speed or platform position and attempts to correct it.

The PID controller calculates three separate parameters in order to correct the error: proportional, integral, and derivative action. Proportional action determines the controller reaction to the current error. Integral action determines the controller reaction based on the sum of the last measured error values.

Derivative action determines the controller reaction based on the rate at which the error changes. These actions are then added together to correct the system output (motor speed or platform position) in order to reduce the difference between the reference value and the actual value, i.e., the system error.

In previous exercises, you set the Digital Servo PID controller so that only proportional action was used when acquiring data. In the next exercises, you will also use the PID controller integral and derivative actions. These can be adjusted by modifying the integration time  $t_i$  value in the case of the integral action, and the derivative time  $t_d$  value in the case of derivative action.

### Servo-system manual tuning

When using a PID controller, it is necessary for the proportional, integral and derivative terms to be set properly, otherwise, the controlled process can become unstable. There are many different tuning methods to find the parameters giving an optimal response. The optimal response itself varies depending on the type of application. There are usually four main points to an optimal response:

- A minimal overshoot or no overshoot at all.
- A quick rise time.
- A quick settling time.
- A low steady state error.

The most basic tuning method is the manual tuning. This type of tuning can only be performed by people who are experienced with the process type of the application. There are three main tuning parameters: the proportional gain  $K_p$ , the integral time  $t_i$ , and the derivative time  $t_d$ . Each tuning parameters has different effects on the response characteristics and so it is important to know when to use one instead of the other. Table 21 summarizes the effects of  $K_p$ ,  $t_i$  or  $t_d$  variations on rise time, overshoot, settling time and steady-state error.

Table 21. Effects of PID parameters variations on a step response.

PID parameters	Rise time	Overshoot	Settling time	Steady state error
Increasing $K_p$	Decrease	Increase	Negligible change	Decrease
Decreasing $t_i$	Decrease	Increase	Increase	Eliminated
Increasing $t_d$	Minor decrease	Minor decrease	Minor decrease	No change

A quick and easy alternative to the manual tuning method is the Ziegler-Nichols method. It requires first to deactivate the integral and derivative terms. The proportional gain  $K_p$  is then increased (starting from 0) until the system reaches a constant oscillation with a constant amplitude and period. This gain value is

referenced as the ultimate gain  $K_u$ . The oscillation period is also measured and is called the oscillation period  $t_u$ . Using these two measured values, the three tuning parameters are then calculated using the equations, depending on the controller type: (P, PI, or PID).

This type of tuning produces a quarter amplitude decay response which is acceptable, but not optimal. This method cannot be used in applications where oscillation could cause the system to go out of control.

Proper control system tuning is to a great extent application dependant. What works for one system can be totally irrelevant for another. For example, a correctly tuned temperature control system that produces a quarter amplitude response may be entirely incorrect for a robot arm system where a much more damped response is required and no overshoot is tolerated.

To summarize the different effects of proportional, integral and derivative action on a servo system in angular position control, we can say that:

- Increasing the proportional gain  $K_p$ , causes the positional step response of an overdamped system to become critically damped. If the proportional gain is increased further, the critically damped system will become underdamped. As the gain increases, an oscillatory component that increases in frequency and amplitude will appear and eventually cause the system to become unstable.
- Adding integral action eliminates static friction error, but also increases the tendency of the step response to oscillate and can cause the system to become unstable.
- Adding derivative action can dampen or suppress an oscillating step response. Derivative action, however, is very sensitive to noise and can cause erratic behaviors when set at high values.

#### PROCEDURE OUTLINE

The Procedure is divided into the following sections:

- Setup and connections
- Effect of the proportional gain on the step response
- Tuning the controller with the Ziegler-Nichols method
- Quarter amplitude decay step response
- Significantly damped step response

#### PROCEDURE

##### Setup and connections

In this section, you will setup the Digital Servo for studying angular position control.

1. Make the following settings on the Digital Servo system:

- Setup the servo system for position control and leave the platform disengaged.

- Set the belt tension to allow the belt to be lifted off the pulley connected to the motor shaft and slipped on the two pins to the rear of the pulley, allowing the shaft to run uncoupled from the belt.
- Secure the flywheel to the shaft using the appropriate hex key.

### Effect of the proportional gain on the step response

In this section, you will acquire three plots having different proportional gains  $K_p$ , and compare the measured step responses.

2. Run LVServo, and click on the **Device Controlled** button in the **Position Loop** menu. Make sure the settings are initially as shown in Table 22:

Table 22. Settings for step response data acquisition.

Function Generator	Trend Recorder
Signal Type	Square
Frequency	0.20 Hz
Amplitude	80%
Offset	0%
Power	Off
<b>PID Controller</b>	Error
Gain ( $K_p$ )	0.14
Integral Time ( $t_i$ )	Inf (Off)
Derivative Time on E ( $t_d$ (E))	0
Derivative Time on PV ( $t_d$ (PV))	0
Timebase	10 ms
Anti-Reset Windup	On
Upper Limit	100%
Lower Limit	-100%
Open or Closed Loop	Closed
<b>PV Speed Scaling</b>	Encoders
100% Value	5000 cnt
-100% Value	-5000 cnt

3. Reset the position count, and set the function generator **Power** switch to **ON**.
4. Acquire a complete positive half cycle using the trend recorder and export the data to a spread sheet.
5. Set the function generator **Power** switch to off.

6. Set the proportional gain  $K_p$  to 2, and the square wave amplitude to 15% and repeat steps 2 to 5.
7. Set the proportional gain  $K_p$  to 3, maintain the square wave amplitude at 15% and repeat steps 2 to 5.
8. Plot the three resulting step responses.
9. From the obtained plots, describe the effects of increasing the proportional gain  $K_p$  on oscillatory frequency and overshoot.

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10. How would you describe the first plot ( $K_p = 0.14$ )? Why is there such a large error in the steady state position?

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#### Tuning the controller with the Ziegler-Nichols method

In this section, you will cause the servo system to almost continuously oscillate. You will achieve this by slowly increasing the proportional gain  $K_p$ . You will then tune the controller using the period of oscillation  $t_u$  and the measured ultimate gain  $K_u$  value at which continuous oscillation occurs.

11. Run LVServo, and click on the **Device Controlled** button in the **Position Loop** menu. Make sure the settings are initially as shown in Table 23:

Table 23. Settings for the Ziegler-Nichols tuning method.

Function Generator	Trend Recorder
Signal Type	Square
Frequency	0.10 Hz
Amplitude	10%
Offset	0%
Power	Off
PID Controller	Error
Gain ( $K_p$ )	1
Integral Time ( $t_i$ )	Inf (Off)
Derivative Time on E ( $t_d$ (E))	0
Derivative Time on PV ( $t_d$ (PV))	0
Timebase	10 ms
Anti-Reset Windup	On
Upper Limit	100%
Lower Limit	-100%
Open or Closed Loop	Closed
PV Speed Scaling	Encoders
100% Value	5000 cnt
-100% Value	-5000 cnt

12. Reset the position count, and set the function generator **Power** switch to **ON**.

13. Starting with a value of 1, slowly increase the proportional gain  $K_p$  by 0.25 steps. If an error occurs because of a current overshoot, reduce the gain back to the previous value, reposition the wheel to position 0 and **RESET** the error. Try to refine your search by steps of 0.05 instead. When the system begins to oscillate continuously, stop the process and note the corresponding ultimate gain  $K_u$  value.

$$K_u = \underline{\hspace{2cm}}$$

14. Acquire a complete positive half cycle using the trend recorder and export the data to a spread sheet.

15. Set the function generator **Power** switch to off.

16. Plot the continuously oscillating response (2 or 3 full circles) and determine the period of oscillation  $t_U$  in seconds.

$$t_U = \underline{\hspace{2cm}} \text{ s}$$

17. Using the ultimate gain  $K_U$  and the oscillation period  $t_U$ , calculate the tuning constants  $K_p$ ,  $t_l$ , and  $t_d$  using the Ziegler-Nichols formulae shown below:

$$K_p = 0.6K_U = \underline{\hspace{2cm}}$$

$$t_l = \frac{t_U}{2K_p} = \underline{\hspace{2cm}} \text{ s}$$

$$t_d = \frac{t_U K_p}{8} = \underline{\hspace{2cm}} \text{ s}$$

### Quarter amplitude decay step response

In this section, you will acquire a quarter amplitude decay step response using the parameters calculated in the previous steps.

18. Run LVServo, and click on the **Device Controlled** button in the **Position Loop** menu. Make sure the settings are initially as shown in Table 24:

Table 24. Settings for the quarter amplitude decay response.

Function Generator	Trend Recorder
Signal Type	Square
Frequency	0.20 Hz
Amplitude	15%
Offset	0%
Power	Off
<b>PID Controller</b>	Error
Gain ( $K_p$ )	Calculated $K_p$
Integral Time ( $t_l$ )	Calculated $t_l$
Derivative Time on E ( $t_d$ (E))	0
Derivative Time on PV ( $t_d$ (PV))	Calculated $t_d$
Timebase	10 ms
Anti-Reset Windup	On
Upper Limit	100%
Lower Limit	-100%
Open or Closed Loop	Closed
<b>PV Speed Scaling</b>	Encoders
100% Value	5000 cnt
-100% Value	-5000 cnt

19. Reset the position count, and set the function generator Power switch to ON.
20. Adjust the derivative term if necessary to obtain a quarter amplitude decay response. The ratio of the peak overshoot ( $\phi_{PEAK} - \phi_{REF}$ ) to the peak overshoot of the second successive cycle must thus be close to 4 to 1. An approximate quarter decay response is shown in Figure 37:

Note down the modified tuning constants:

$$K_p = \underline{\hspace{2cm}}$$

$$t_i = \underline{\hspace{2cm}} \text{ s}$$

$$t_d = \underline{\hspace{2cm}} \text{ s}$$

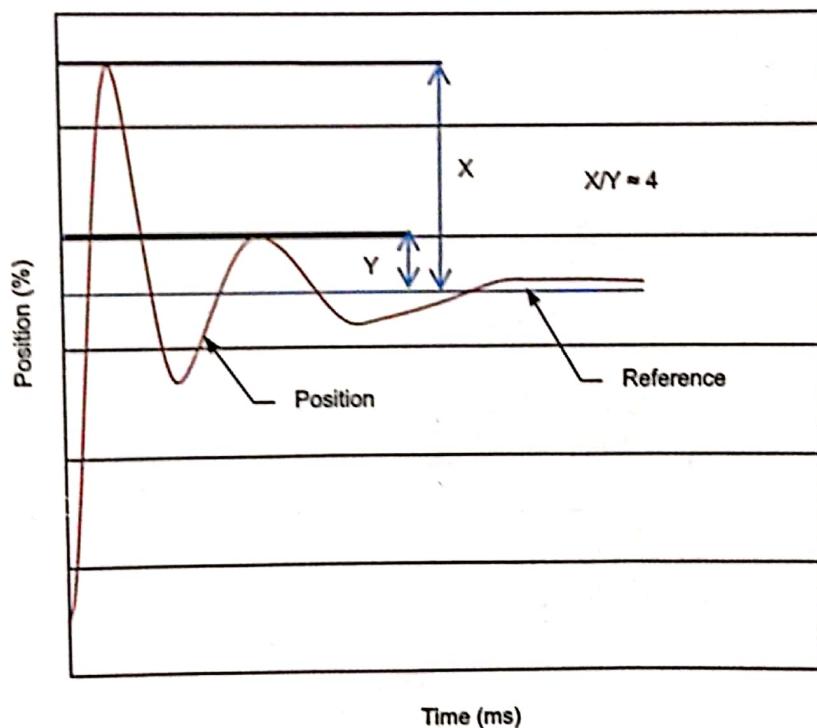


Figure 37. Quarter amplitude decay response.

21. Acquire a complete positive half cycle using the trend recorder and export the data to a spread sheet.
22. Set the function generator Power switch to off.
23. Plot the Quarter Amplitude Decay step response.

24. At approximately what time after the position reference changes from -15% to +15%, does the position first reach 15%?

Time = \_\_\_\_\_ s

### Significantly damped step response

In this section, you will acquire a significantly damped step response by modifying the parameters you used to obtain the quarter amplitude decay response. You will then compare the resulting plot with the plot obtained for the quarter amplitude decay response.

25. Adjust the parameters as follows to produce a damping effect on the system:

$$K_p \times 4$$

$$t_l \times 16$$

$$t_d \times 4$$

Note down the new values:

$$K_p = _____$$

$$t_l = _____ \text{ s}$$

$$t_d = _____ \text{ s}$$

26. Reset the position count, and set the function generator **Power** switch to **ON**.

27. Run LVServo, and click on the **Device Controlled** button in the **Position Loop** menu. Make sure the settings are initially as shown in Table 25:

**Table 25. Settings for capturing a significantly damped step response.**

Function Generator		Trend Recorder	
Signal Type	Square	Reference	Checked
Frequency	0.20 Hz	Position	Checked
Amplitude	20%	Speed	Unchecked
Offset	0%	Current	Unchecked
Power	Off	Voltage	Unchecked
PID Controller		Error	Unchecked
Gain ( $K_p$ )	Calculated $K_p$	$K_p \times$ Error	Unchecked
Integral Time ( $t_i$ )	Calculated $T_i$	Error Sum / $t_i$	Unchecked
Derivative Time on E ( $t_d$ (E))	0	$t_d \times$ Delta Error	Unchecked
Derivative Time on PV ( $t_d$ (PV))	Calculated $T_d$	PID Output	Unchecked
Timebase	10 ms	Display Type	Sweep
Anti-Reset Windup	On	Show and Record Data	On
Upper Limit	100%	Measured Gain (rpm)	3000
Lower Limit	-100%	Measured Gain (A)	7
Open or Closed Loop	Closed	Measured Gain (V)	48
PV Speed Scaling		Encoders	
100% Value	5000 cnt	Motor or Rail	Motor
-100% Value	-5000 cnt		

28. With the trend recorder, acquire a full positive half cycle and export it to a spread sheet.

- 29 Set the function generator Power switch to off.

- 30 Plot the obtained step response.

31. When does the position first reaches the 15% point after the position reference changes from -15% to +15%?

Time = \_\_\_\_\_ s

32. Compare the quarter amplitude decay responses generated using the Ziegler-Nichols tuning method with the significantly damped step response generated using the higher tuning constants. Which response reaches first the 15% position reference?

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33. Which response of the quarter amplitude decay response and the significantly damped response is more appropriate for delicate applications (moving a robot's arm, for example)?

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## CONCLUSION

In this exercise, you learned the effects of the proportional, integral and derivative terms on the transient operation of a servo system used for angular position control. You measured the effects of varying the proportional gain and the damping ratio on a servo system step response in angular position control. You familiarized yourself with different servo-system tuning methods.

## REVIEW QUESTIONS

1. The flywheel coupled to the motor shaft of the Digital Servo has a reference mark at the perimeter. Suppose that the shaft is currently positioned so that the marker is at the 12:00 position ( $0^\circ$  and 0% position reference). In order to reposition the flywheel to  $270^\circ$ , which reference position in counts should be set? What position in percentage does this corresponds to?

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2. The Digital Servo flywheel's steady state position indicates an error exists between the reference position and the actual position. Knowing that only proportional action is used, select the best option from below to eliminate the error:

- a) Add derivative action
- b) Add integral action
- c) Increase the proportional gain
- d) Decrease the proportional gain

3. Suppose a servo system has the following characteristics:

Table 26. Servo system characteristics.

Parameter	Value
$K$ [(rad/s)/V]	7.6
$\tau$ (s)	0.05
Scaling factor	5.56
$K_p$	2.5
$\phi_{REF}$ (%)	40

Use the system data and Equation (58) to plot to a spread sheet this system step response.

4. How would the step response obtained in question 3 react to a proportional gain  $K_p$  increase?

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5. In a position-control system, the encoder acts as an integrator by totalling the pulses generated as the motor shaft rotates. Why is integral action from the controller still necessary to the position control system?

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